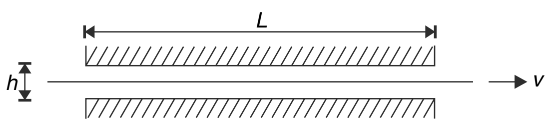
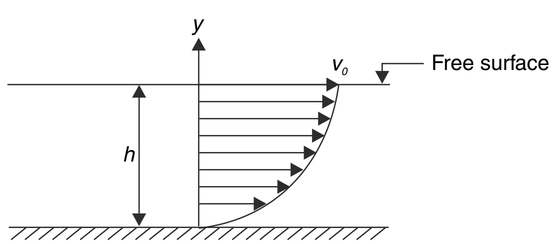
10 VISCOSITY

Level 1

Q.1. During a painting process, a thin, flat tape of width b (dimension perpendicular to the plane of the figure) is pulled through a paint filled channel of length L. The density and viscosity of the paint liquid is ρ and η respectively. The tape is pulled at a constant speed v and width of the channel is h. Find the minimum force needed to pull the tape.



Q.2. A liquid is flowing through a horizontal channel. The speed of flow (v) depends on height (y) from the floor as $v = v_0\left[2\left(\frac{y}{h}\right) - \left(\frac{y}{h}\right)^2\right]$. Where h is the height of liquid in the channel and v_0 is the speed of the top layer. Coefficient of viscosity is η. Calculate the shear stress that the liquid exerts on the floor.



Q.3. A car having cross sectional area of its front equal to A is travelling on a highway at a speed v. The viscous drag force acting on the car is known to be given as $F_v = CA\rho v^2$.Where ρ is density of air and C is a constant which depends on the shape of the car. The petrol used by the car produces E joules of energy per kg of it burnt. Calculate the mileage (in *km/kg*) of the car if the combined efficiency of its engine and transmission is f.

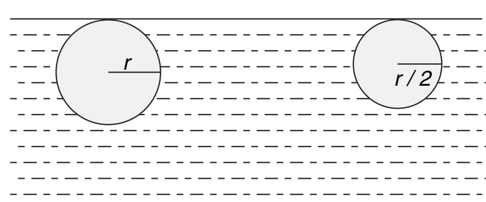
Q.4. An ideal fluid flows through a pipe of circular cross section of radius r at a speed v_0. Now a viscous liquid is made to flow through the pipe at the same volume flow rate (measured in m^3s^{-1}). Find the maximum speed of a fluid particle in the pipe.

Q.5. A near surface earth satellite is in the shape of a sphere of radius r. It encounters cosmic dust in its path. The viscous force experienced by the satellite follows stoke's law. The coefficient of viscosity is η. Mass and radius of the earth are M and R respectively.

(a) Calculate the power of the rocket engine that must be put on to keep the satellite moving as usual.

(b) Calculate the equilibrium temperature of the surface of the satellite assuming that it radiates like a black body and no outer radiation falls on it. Assume that the heat generated due to viscous force is absorbed completely by the satellite body.

Q.6. Two balls of radii r and $\frac{r}{2}$ are released inside a deep water tank. Their initial accelerations are found to be $\frac{g}{2}$ and $\frac{g}{4}$ respectively. Find the velocity of smaller ball relative to the larger ball, a long time after the two balls are released. Coefficient of viscosity is given to be η.



Q.7. The coefficient of viscosity η of a gas depends on mass of the gas molecule, its effective diameter and its average speed. It is known that diameter of helium atom is $2.1 \times 10^{-10}\,m$ and its coefficient of viscosity, η at room temperature is 2.0×10^{-5} $kg\,m^{-1}s^{-1}$. Estimate the effective diameter of CO_2

molecule if it is known that η at room temperature for CO_2 is $1.5 \times 10^{-5}\ kg\ m^{-1}s^{-1}$.

Q.8. When hard brakes are applied (so as to lock the wheels) in a car travelling on a wet road it can "hydro-plane". A film of water is created between the tires and the road and, theoretically, the car can slide a very long distance. [In practice film is destroyed much before such distances can be achieved]. Consider a car of mass M moving on a wet road with speed v_0. Hard brakes are applied. Let the area of film under all four tires be A and thickness of the film be h. Coefficient of viscosity is η.

(a) Calculate the distance (x) to which the car will slide before coming to rest.

(b) Calculate the value of x for $M = 10^3\ kg$, $A = 0.2\ m^2$, $h = 0.1\ mm$, $v_0 = 20\ ms^{-1}$, and $\eta = 10^{-3}\ kg\ m^{-1}$

LEVEL 2

Q.9. A spherical ball of radius r and density d is dropped from rest in a viscous fluid having density ρ and coefficient of viscosity η.

(a) Calculate the power (P_1) of gravitational force acting on the ball at a time t after it is dropped.

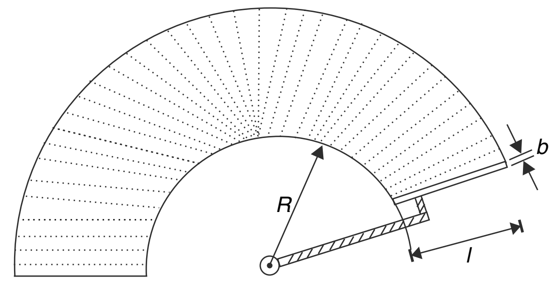
(b) Calculate the rate of heat generation (P_2) due to rubbing of fluid molecules with the ball, at time t after it is dropped.

(c) How do P_1 and P_2 change if the radius of the ball were doubled?

(d) Find P_1 and P_2 when both become equal.

Q.10. Two balls of same material of density ρ but radius r_1 and r_2 are joined by a light inextensible vertical thread and released from a large height in a medium of coefficient of viscosity = η. Find the terminal velocity acquired by the balls. Also find the tension in the string connecting both the balls when both of them are moving with terminal velocity. Neglect buoyancy and change in acceleration due to gravity.

Q.11. A car windshield wiper blade sweeps the wet windshield rotating at a constant angular speed of ω. R is the radius of innermost arc swept by the blade. Length and width of the blade are l and b respectively. Coefficient of viscosity of water is η. Calculate the torque delivered by the motor to rotate the blade assuming that there is a uniform layer of water of thickness t on the glass surface.



LEVEL 3

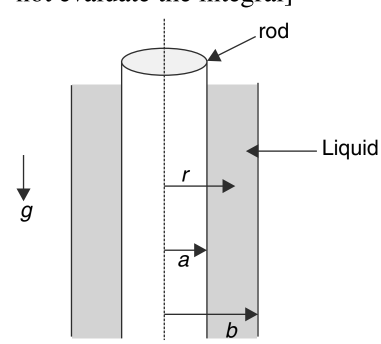
Q.12. A vertical steel rod has radius a. The rod has a coat of a liquid film on it. The liquid slides under gravity. It was found that the speed of liquid layer at radius r is given by

$$v = \frac{\rho g b^2}{2\eta}\ln\left(\frac{r}{a}\right) - \frac{\rho g}{4\eta}\left(r^2 - a^2\right)$$

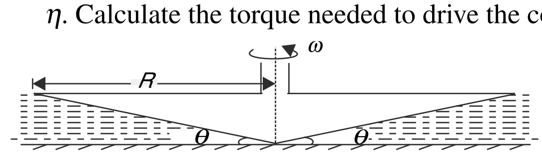
Where b is the outer radius of liquid film, η is coefficient of viscosity and ρ is density of the liquid.

(i) Calculate the force on unit length of the rod due to the viscous liquid?

(ii) Set up the integral to calculate the volume flow rate of the liquid down the rod. [you may not evaluate the integral]



Q.13. A viscometer (an instrument used to study characteristics of a non-ideal fluid) consists of a flat plate and a rotating cone. The cone has a large apex angle and the angle θ shown in figure is very small (typically less than 0.5°). The apex of the cone just touches the plate and a liquid fills the narrow gap between the plate and the cone. The cone has a base radius R and is rotated with constant angular speed ω. Consider the liquid to be ideal and take its coefficient of viscosity to be η. Calculate the torque needed to drive the cone.



ANSWER

1. $\frac{4\eta vLb}{h}$

2. $\frac{2\eta v_0}{h}$

3. $\frac{fE}{CA\rho v^2}$

4. $\frac{3}{2}v_0$

5. (a) $\frac{6\pi GM\eta r}{R}$ (b) $\left[\frac{3\eta GM}{2\sigma rR}\right]^{1/4}$

6. $\frac{11}{54}\frac{r^2\rho g}{\eta}$ upwards

7. $4.4 \times 10^{-10}\, m$

8. (a) $x = \frac{Mhv_0}{\eta A}$ (b) 10 km

9. (a) $P_1 = \frac{8\pi}{27}\frac{d(d-\rho)g^2r^5}{\eta}\left[1-e^{-\frac{9\eta t}{2dr^2}}\right]$

(b) $P_1 = \frac{8\pi}{27}\frac{(d-\rho)^2g^2r^5}{\eta}\left[1-e^{-\frac{9\eta t}{2dr^2}}\right]$

(c) P_1 and P_2 become 32 times.

(d) $P_1 = P_2 = \frac{8\pi}{27}\frac{(d-\rho)^2g^2r^5}{\eta}$

10. $\frac{2}{9}\frac{\rho g}{\eta}\left[r_1^2 - r_1r_2 + r_2^2\right]$; $\frac{4}{3}\pi\rho g\left|r_1^2r_2 - r_2^2r_1\right|$

11. $\frac{\eta b\omega R^3}{3t}\left[\left(1+\frac{L}{R}\right)^3 - 1\right]$

12. (i) $\pi\rho g a^2\left[\left(\frac{b}{a}\right)^2 - 1\right]$ (ii) $Q = \int_a^b v\cdot 2\pi r dr$

13. $\frac{2\pi\eta\omega R^3}{3\sin\theta} \simeq \frac{2\pi\eta\omega R^3}{3\theta}$

SOLUTIONS

1. The paint layer in contact with channel wall is at rest and that in contact with the tape is v. The viscous force acts on two surfaces of the tape. If gap between the tape and upper surface of the channel is x then velocity gradient at the two surfaces of the tape is

$$\left(\frac{dv}{dh}\right)_{upper} = \frac{v}{x} \text{ and } \left(\frac{dv}{dh}\right)_{lower} = \frac{v}{h-x}$$

Total viscous force on the tape is $F_{vis} = F_{upper} + F_{lower}$

$$= \eta(bL)\cdot\frac{v}{x} + \eta(bL)\cdot\frac{v}{h-x} = \eta bLv\left[\frac{h}{x(h-x)}\right]$$

This force is minimum when $x(h-x)$ is maximum, i.e., when $x = \frac{h}{2}$

$$\therefore F_{min} = \frac{4\eta bLv}{h}$$

2. Shear stress is tangential force applied by the liquid on unit area of the floor.

Velocity gradient $= \frac{dv}{dy} = \frac{2v_0}{h} - \frac{2v_0}{h^2}y$

At $y = 0$, $\frac{dv}{dy} = \frac{2v_0}{h}$

$\therefore$ Viscous force per unit area $= \eta \frac{dv}{dy} = \frac{2\eta v_0}{h}$

3. Viscous drag force acting on the car is $F = CA\rho v^2$

$\therefore$ Work required to travel through a distance x is

$W = Fx = CA\rho v^2 \cdot x$

Let mass of petrol burnt = m ; Energy produced = mE

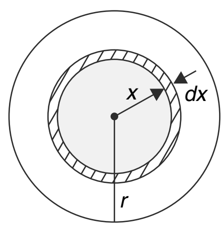
Since efficiency is f $\therefore$ $fmE = W$

$fmE = CA\rho v^2 x$ $\therefore$ $m = \frac{CA\rho v^2 x}{fE}$

$\therefore$ Mileage = Distance travelled per unit mass of fuel burnt $= \frac{x}{m} = \frac{fE}{CA\rho v^2}$

4. Volume flow rate $Q = \pi r^2 . v_0$

When a viscous liquid flows, speed is maximum along the axis of the pipe (let us call this speed as u_0) and decreases to zero at the circumference. The variation in speed will be almost linear if radius of the pipe is small.



Speed at a distance x from the axis is $u = \frac{u_0 x}{r}$

$\therefore$ volume flow rate through a region of width dx is $= (2\pi x dx) u = (2\pi x dx)\left(\frac{u_0 x}{r}\right)$

$\therefore$ Total flow rate is $Q = \frac{2\pi u_0}{r} \int_0^r x^2 dx = \frac{2\pi u_0}{3} r^2$

$\therefore$ $\pi r^2 v_0 = \frac{2\pi u_0}{3} r^2$ $\therefore$ $u_0 = \frac{3}{2} v_0$

5. (a) Orbital speed of the satellite $v = \sqrt{\frac{GM}{R}}$

Viscous force $f_v = 6\pi\eta r v$

Power required $P = v . f_v = 6\pi\eta r v^2 = 6\pi\eta r \frac{GM}{R}$

(b) $\sigma A T^4 = 6\pi\eta r \frac{GM}{R}$ $\Rightarrow$ $\sigma 4\pi r^2 T^4 = 6\pi\eta r \frac{GM}{R}$

$\therefore$ $T^4 = \frac{3}{2} \frac{\eta}{\sigma r} \frac{GM}{R} \Rightarrow T = \left[\frac{3}{2} \frac{\eta GM}{\sigma r R}\right]^{1/4}$

6. Initial acceleration for larger ball is given by -

$ma = mg - F_B$

$$V_1 d_1 \frac{g}{2} = V_1 d_1 g - V_1 \rho g$$ [ρ= density of water, d_1= density of ball]

$\Rightarrow d_1 = 2\rho$

Similarly, for second ball $d_2 = \frac{4\rho}{3}$

After long time both of them will acquire terminal speed (u_{01} for larger ball and u_{02} for smaller ball)

$$6\pi\eta r\, u_{01} = mg - F_B \quad \Rightarrow \quad 6\pi\eta r\, u_{01} = \frac{4}{3}\pi r^3 d_1 g - \frac{4}{3}\pi r^3 \rho g$$

$$\Rightarrow u_{01} = \frac{2}{9}\frac{r^2 \rho g}{\eta} \qquad \text{Similarly, } u_{02} = \frac{r^2 \rho g}{54\eta}$$

$\therefore$ Velocity of smaller ball with respect to the larger ball is

$$= \left(\frac{1}{54} - \frac{2}{9}\right) r^2 \frac{\rho g}{\eta} = -\frac{11}{54}\frac{r^2 \rho g}{\eta} = \frac{11}{54}\frac{r^2 \rho g}{\eta}(\uparrow)$$

7. From method of dimensions

$\eta = kmvd^{-2}$ [m = mass, v = average speed, d = diameter of molecule]

But $v \propto \sqrt{\frac{T}{m}}$ $\quad \therefore \eta = k^1 m^{\frac{1}{2}} T^{\frac{1}{2}} d^{-2}$

At a given temperature

$$\left(\frac{d_{CO_2}}{d_{He}}\right)^2 = \left(\frac{m_{CO_2}}{m_{He}}\right)^{1/2} \left(\frac{\eta_{He}}{\eta_{CO_2}}\right)$$

$$d_{CO_2} = d_{He}\left(\frac{m_{CO_2}}{m_{He}}\right)^{1/4} \left(\frac{\eta_{He}}{\eta_{CO_2}}\right)^{1/2}$$

$$= 2.1 \times 10^{-10} \times (11)^{\frac{11}{4}} \times \left(\frac{2}{1.5}\right)^{1/2} = 4.4 \times 10^{-10} m$$

8. (a) Viscous force [v = instantaneous speed of the car]

$$F_v = \eta A \frac{dv}{dh} = \eta A \frac{v}{h}$$

$$\therefore M\frac{dv}{dt} = -\eta A \frac{v}{h} \qquad \text{Or, } Mv\frac{dv}{dx} = -\eta A\frac{v}{h}$$

$$\text{Or, } \int_{v_0}^{0} dv = -\frac{\eta A}{hM}\int_0^x dx \qquad \Rightarrow v_0 = \frac{\eta A}{hM}x$$

$$\Rightarrow x = \frac{hMv_0}{\eta A}$$

(b) $x = \frac{10^4 \times 10^3 \times 20}{10^{-3} \times 0.2} = 10^4 m = 10\ km$ (!)

9. $m\dfrac{dv}{dt} = mg - F_B - F_v$

$$\frac{4}{3}\pi r^3 d\ \frac{dv}{dt} = \frac{4}{3}\pi r^3 d\cdot g - \frac{4}{3}\pi r^3 \rho\cdot g - 6\pi\eta r v$$

$$\frac{dv}{dt} = g - \frac{\rho}{d}g - \frac{9}{2}\frac{\eta v}{dr^2}$$

$$= g\left(1-\frac{\rho}{d}\right) - \frac{9\eta}{2\cdot dr^2}\cdot v \quad = a - bv \qquad \text{(say)}$$

$$\Rightarrow \int_0^v \frac{dv}{a-bv} = \int_0^t dt \qquad \Rightarrow [\ln(a-bv)]_0^v = -bt$$

$$\Rightarrow \ln\left(\frac{a-bv}{a}\right) = -bt \quad \Rightarrow 1-\frac{b}{a}v = e^{-bt}$$

$$\Rightarrow v = \frac{a}{b}\left(1-e^{-bt}\right) \qquad \Rightarrow v = \frac{2g(d-\rho)r^2}{9\eta}\left(1-e^{-\frac{9\eta}{2\cdot d\cdot r^2}t}\right)$$

(a) $P_1 = m\cdot g\cdot v = \dfrac{4}{3}\pi r^3 d\cdot g\cdot\dfrac{2g(d-\rho)r^2}{9\eta}\left[1-e^{-\frac{9\eta}{2dr^2}t}\right]$

$$= \frac{8\pi}{27}\frac{d(d-\rho)g^2\cdot r^5}{\eta}\left[1-e^{-\frac{9\eta t}{2dr^2}}\right]$$

(b) $P_2 = F_v\cdot v = 6\pi\eta r v\cdot v = 6\pi\eta r v^2$

$$= 6\pi\eta r\cdot\frac{4g^2(d-\rho)^2\cdot r^4}{81\eta^2}\left[1-e^{-\frac{9\eta t}{2dr^2}}\right]^2$$

$$= \frac{8\pi}{27}\frac{(d-\rho)^2 g^2\cdot r^5}{\eta}\left[1-e^{-\frac{9\eta t}{2dr^2}}\right]^2$$

(c) $P_1 \propto r^5$ and $P_2 \propto r^5$

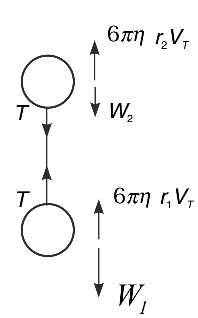
$\therefore$ On doubling the radius, both become 32 times

(d) $P_1 = P_2$, when

$e^{-\frac{9\eta t}{2dr^2}} \to 0$ [i.e., at $t = \infty$]

$$\therefore\ P_1 = P_2 = \frac{8\pi}{27}\ \frac{(d-\rho)^2 g^2 r^5}{\eta}$$

10. Considering both balls together



$$\rho\left(\frac{4}{3}\pi r_1^3\right)g + \rho\left(\frac{4}{3}\pi r_2^3\right) = 6\pi\eta r_1 V_T + 6\pi\eta r_2 V_T$$

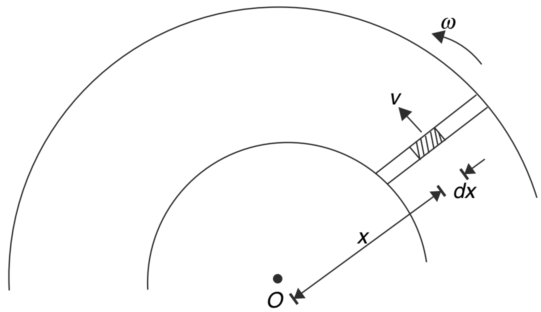
$$\Rightarrow V_T = \frac{2\rho g(r_1^3 + r_2^3)}{9(\eta)(r_1+r_2)} = \frac{2\rho g(r_1^2 - r_1 r_2 + r_2^2)}{9\eta}$$

Consider only ball 1 and write $W_1 = T + 6\pi\eta r_1 V_r$

$$\frac{4}{3}\pi r_1^3 \rho = T + 6\pi\eta r_1 V_T$$

Put the value of V_T and calculate T.

11.



Consider an element of length dx on the blade. Speed of this element is $v = \omega x$

Viscous force on this element is

$$dF = \eta A\left(\frac{v}{t}\right) \qquad \left[\frac{v}{t} = \text{velocity gradient}\right]$$

$$= \eta b dx\left(\frac{\omega x}{t}\right) = \eta\frac{b\omega}{t}\cdot x dx$$

Torque due to this force $d\tau = xdF = \dfrac{\eta b\omega}{t}x^2 dx$

$\therefore$ Net viscous torque on the blade is

$$\tau = \frac{\eta b\omega}{t}\int_R^{R+L} x^2 dx \qquad \Rightarrow \quad \tau = \frac{\eta b\omega}{t}\frac{1}{3}\left[x^3\right]_R^{R+L}$$

$$= \frac{\eta b\omega}{3t}\left[(R+L)^3 - R^3\right] \qquad = \frac{\eta b\omega R^3}{3t}\left[\left(1+\frac{L}{R}\right)^3 - 1\right]$$

The same torque must be applied by the motor to keep the blade moving.

12. (a) Velocity of fluid layer at radius r is $v = \dfrac{\rho g b^2}{2\eta}\ln\left(\dfrac{r}{a}\right) - \dfrac{\rho g}{4\eta}\left(r^2 - a^2\right)$

Velocity gradient along radial direction is $\dfrac{dv}{dr} = \dfrac{\rho g b^2}{2\eta}\dfrac{1}{r} - \dfrac{\rho g}{4\eta}2r$

At $r = a$ (i.e., on the surface of the rod)

$$\frac{dv}{dr} = \frac{\rho g b^2}{2\eta a} - \frac{\rho g a}{2\eta} = \frac{\rho g a}{2\eta}\left[\frac{b^2}{a^2} - 1\right]$$

Area of unit length of the rod's surface $A = (2\pi a)(1) = 2\pi a$

$$\therefore\ F_v = \eta A\frac{dv}{dr} = \pi\rho g a^2\left[\frac{b^2}{a^2} - 1\right]$$

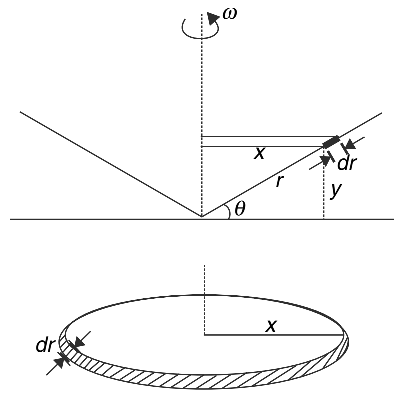
(b) Volume flow rate is

$$Q = \int_{r=a}^{r=b} v \cdot 2\pi r dr = \frac{\pi\rho g b^2}{\eta}\int_a^b r \ln\left(\frac{r}{a}\right) dr - \frac{\pi\rho g}{2\eta}\int_a^b \left(r^2 - a^2\right) r dr$$

You can evaluate the above integral if you want some practice in mathematics. To help you it is being given that

$$\int x \ln x = \frac{x^2}{2}\ln x - \frac{x^2}{4} + c$$

13.



Consider a ring shaped element in the cone as shown in the figure.

$$\frac{x}{r} = \cos\theta \qquad \Rightarrow \; dr = \frac{dx}{\cos\theta}$$

Area of the ring element $dA = 2\pi x \cdot dr = \dfrac{2\pi x \cdot dx}{\cos\theta}$

Velocity gradient at the location of this ring is $= \dfrac{\omega x}{y} = \dfrac{\omega x}{x\tan\theta} = \dfrac{\omega}{\tan\theta}$

∴ Viscous force on ring element

$$dF = \eta \cdot (dA)\frac{\omega}{\tan\theta} = \frac{2\pi\eta\omega}{\sin\theta} x dx$$

Torque on the element $d\tau = x dF = \dfrac{2\pi\eta\omega}{\sin\theta} x^2 dx$

Net torque on the cone

$$\tau = \frac{2\pi\eta\omega}{\sin\theta}\int_0^R x^2 dx = \frac{2\pi\eta\omega}{3\sin\theta} R^3 \qquad \simeq \frac{2\pi\eta\omega R^3}{3\theta} \qquad [\because \theta \text{ is small}]$$