CHAPTER SIXTEEN

Cartesian System of Rectangular Coordinates and Straight Lines

RESULTS REGARDING POINTS IN A PLANE

Distance formula

The distance between two points $P(x_1, y_1)$ and $Q(x_2, y_2)$ is given by

$PQ = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$. The distance from the origin O (0, 0) to the point $P(x_1, y_1)$ is $OP = \sqrt{x_1^2 + y_1^2}$.

Illustration 1

Show that the triangle with vertices (3, 0), (–1, –1) and (2, 4) is isosceles and right angled.h

Solution: Let the vertices of the triangle be A (3, 0), B (–1, –1) and C (2, 4) then using distance formula, we have

$AB = \sqrt{(3+1)^2 + (0+1)^2} = \sqrt{17}$

$BC = \sqrt{(-1-2)^2 + (-1-4)^2} = \sqrt{9+25} = \sqrt{34}$

and $CA = \sqrt{(2-3)^2 + (4-0)^2} = \sqrt{17}$.

Since $AB = AC = \sqrt{17}$, the triangle ABC is isosceles. Also $(AB)^2 + (AC)^2 = (BC)^2$ shows the triangle is right angled.

Section formula

If $R(x, y)$ divides the join of $P(x_1, y_1)$ and $Q(x_2, y_2)$ in the ratio $m : n$ ($m, n > 0, m \neq n$) then

$$x = \frac{mx_2 \pm nx_1}{m \pm n} \text{ and } y = \frac{my_2 \pm ny_1}{m \pm n}$$

The positive sign is taken for internal division and the negative sign for external division. The *mid-point* of $P(x_1, y_1)$ and $Q(x_2, y_2)$ is

$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$ which corresponds to internal division when $m = n$. Note that for external division $m \neq n$.

Illustration 2

The line $x - y + k = 0$ passes through the point which divides the segment joining the points (2, 3) and (4, 5) in the ratio 2 : 3. Find the value of k.

Solution: Using the section formula, we get the coordinates of the point of division as $\left(\frac{2 \times 4 + 3 \times 2}{5}, \frac{2 \times 5 + 3 \times 3}{5}\right) = \left(\frac{14}{5}, \frac{19}{5}\right)$

Since it lies on the line $x - y + k = 0$

$$\frac{14}{5} - \frac{19}{5} + k = 0 \Rightarrow k = 1$$

Centroid of a triangle

If $G(x, y)$ is the centroid of the triangle with vertices $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$, then

$$x = \frac{x_1 + x_2 + x_3}{3} \quad \text{and} \quad y = \frac{y_1 + y_2 + y_3}{3}$$

Illustration 3

Show that the centroid of the triangle with vertices $(5 \cos \theta, 4 \sin \theta)$, $(4 \cos \theta, 5 \sin \theta)$ and (0, 0) lies on the circle $x^2 + y^2 = 9$.

Solution: Coordinates of the centroid of the given triangle are,

$$\left(\frac{5\cos\theta + 4\cos\theta + 0}{3}, \frac{4\sin\theta + 5\sin\theta + 0}{3}\right)$$

$= (3\cos \theta, 3\sin \theta)$

which lies on the circle $x^2 + y^2 = 9$

as $(3 \cos \theta)^2 + (3 \sin \theta)^2 = 9$.

Note

- When A, B, C are taken as vertices of a triangle, it is assumed that they are not collinear.

Incentre of a triangle

If $I(x, y)$ is the incentre of the triangle with vertices $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$, then

$$x = \frac{ax_1 + bx_2 + cx_3}{a+b+c} \quad \text{and}$$

$$y = \frac{ay_1 + by_2 + cy_3}{a+b+c}$$

a, b and c being the lengths of the sides BC, CA and AB, respectively of the triangle ABC.

Illustration 4

Find the coordinates of the in center of the triangle with vertices A (–1, 12), B (–1, 0) and C (4, 0).

Solution: We have

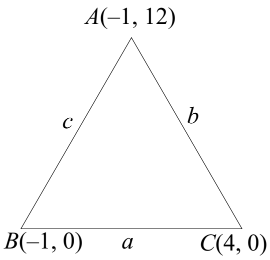


Fig. 16.1

$$a = BC = \sqrt{(-1-4)^2 + 0} = 5$$

$$b = CA = \sqrt{(4+1)^2 + (0-12)^2} = 13$$

$$c = AB = \sqrt{(-1+1)^2 + (12-0)^2} = 12$$

So using the formula for the coordinates of the in-center we get the required coordinates as

$$\left(\frac{5(-1)+13(-1)+12(4)}{5+13+12}, \frac{5(12)+13(0)+12(0)}{5+13+12}\right)$$

= (1, 2)

Area of triangle

ABC with vertices $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ is

$$\frac{1}{2}\left|\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}\right|$$

$$= \frac{1}{2}|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$$

and is generally denoted by Δ. Note that if one of the vertex (x_3, y_3) is at $O(0, 0)$, then $\Delta = \frac{1}{2}|x_1 y_2 - x_2 y_1|$.

Illustration 5

If the area of the triangle with vertices at the points whose coordinates are (2, 5), (0, 3) and (4, k) is 4 units, then find the value of k.

Solution: We have

$$\frac{1}{2}\left|\begin{vmatrix} 2 & 5 & 1 \\ 0 & 3 & 1 \\ 4 & k & 1 \end{vmatrix}\right| = 4$$

$\Rightarrow \quad 2(3 - k) + 4(5 - 3) = \pm 8$

$\Rightarrow 7 - k = \pm 4 \Rightarrow k = 3$ or 11

Condition of collinearity

Three points $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ are collinear if and only if

$$\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$$

Slope of a line

Let $A(x_1, y_1)$ and $B(x_2, y_2)$ $(x_1 \neq x_2)$ be any two points. Then the slope of the line joining A and B is defined as

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \tan\theta$$

where θ is the angle which the line makes with the positive direction of the x-axis, $0° \leq \theta \leq 180°$, except at $\theta = 90°$. Which is possible only if $x_1 = x_2$ and the line is parallel to the y-axis.

Condition for the points $Z_k = x_k + iy_k$ $(k = 1, 2, 3)$ to form an equilateral triangle is

$Z_1^2 + Z_2^2 + Z_3^2 = Z_1Z_2 + Z_2Z_3 + Z_3Z_1$ (See Complex numbers)

STANDARD FORMS OF THE EQUATION OF A LINE

1. An equation of a line parallel to the x-axis is $y = k$ and that of the x-axis itself is $y = 0$.

Illustration 6

Find the equation of the line parallel to x-axis passing through the intersection of the lines $3ax + 2by + 7b = 0$ and $3bx - 2ay - 7a = 0$, where $(a, b) \neq (0, 0)$

Solution: Equation of the line parallel to x-axis is $y = k$. So we need to find the y-coordinates of the point of intersection of the given lines. Multiplying the first equation by b and the second by a and subtracting we get.

$2(b^2 + a^2)y + 7(b^2 + a^2) = 0$

$\Rightarrow y = -\frac{7}{2}$ which is the required equation.

2. An equation of a line parallel to the y-axis is $x = h$ and that of the y-axis itself is $x = 0$.

3. An equation of a line passing through the origin and (a) making an angle θ with the positive direction of the x-axis is $y = x \tan \theta$; (b) having a slope m is $y = mx$; and (c) passing through the point (x_1, y_1) is $x_1 y = y_1 x$.

4. *Slope-Intercept form* An equation of a line with slope m and making an intercept c on the y-axis is $y = mx + c$.

5. *Point-Slope form* An equation of a line with slope m and passing through (x_1, y_1) is $y - y_1 = m(x - x_1)$.

6. *Two-Point form* An equation of a line passing through the points (x_1, y_1) and (x_2, y_2) is

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

7. *Intercept form* An equation of a line making intercepts a and b on the x-axis and y-axis respectively, is

$$\frac{x}{a} + \frac{y}{b} = 1$$

Illustration 7

Find the equation of the straight line passing through the point (10, –7) and making intercepts on the coordinate axes whose sum is 12.

Solution: Let the equation of the line be $\frac{x}{a} + \frac{y}{b} = 1$ where $a + b = 12$.

Since it passes through (10, – 7), $\frac{10}{a} - \frac{7}{b} = 1$

$\Rightarrow 10b - 7a = ab$
$\Rightarrow 10(12 - a) - 7a = a(12 - a)$
$\Rightarrow a^2 - 29a + 120 = 0$
$\Rightarrow a = 5, 24.$

When $a = 5$, $b = 7$; the required equation is $\frac{x}{5} + \frac{y}{7} = 1$

When a = 24, $b = -12$, the required equation is $\frac{x}{24} - \frac{y}{12} = 1$

8. *Parametric form* An equation of a line passing through a fixed point $A(x_1, y_1)$ and making an angle θ, $0 \le \theta \le \pi$, $\theta \ne \pi/2$ with the positive direction of the x-axis is

$$\frac{x - x_1}{\cos\theta} = \frac{y - y_1}{\sin\theta} = r$$

where r is the distance of *any* point $P(x, y)$ on the line from the point $A(x_1, y_1)$. Note that $x = x_1 + r\cos\theta$ and $y = y_1 + r\sin\theta$.

9. *Normal form* An equation of a line such that the length of the perpendicular from the origin on it is p and the angle which this perpendicular makes with the positive direction of the x-axis is α, is $x\cos\alpha + y\sin\alpha = p$.

10. *General form* In general, an equation of a straight line is of the form $ax + by + c = 0$, where a, b and c are real numbers and a and b cannot both be zero simultaneously. From this general form of the equation of the line, we can calculate the following:

(i) The slope is $-a/b$ $(b \ne 0)$.

(ii) The intercept on the x-axis is $-c/a$ $(a \ne 0)$, and the intercept on the y-axis is $-c/b$ $(b \ne 0)$.

(iii) $p = \frac{|c|}{\sqrt{a^2 + b^2}}$, $\cos\alpha = \pm\frac{|a|}{\sqrt{a^2 + b^2}}$ and

$\sin\alpha = \pm\frac{|b|}{\sqrt{a^2 + b^2}}$, the positive sign being taken if c is negative and vice versa.

(iv) If p_1 denotes the length of the perpendicular from (x_1, y_1) on this line, then

$$p_1 = \left|\frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}}\right|$$

(v) The points (x_1, y_1) and (x_2, y_2) lie on the same side of the line if the expressions $ax_1 + by_1 + c$ and $ax_2 + by_2 + c$ have the same sign, and on the opposite side if they have the opposite signs.

SOME RESULTS FOR TWO OR MORE LINES

1. Two lines given by the equations $ax + by + c = 0$ and $a'x + b'y + c' = 0$ are

(i) *parallel* (i.e., the slopes are equal), if $ab' = a'b$

(ii) *perpendicular* (i.e., the product of their slopes is –1), if $aa' + bb' = 0$

(iii) identical if $ab'c' = a'b'c = a'c'b$.

(iv) not parallel, then

(a) angle θ between them at their point of intersection is given by

$$\tan\theta = \pm\frac{m - m'}{1 + mm'} = \pm\frac{a'b - ab'}{aa' + bb'}$$

m, m' being the slopes of the two lines.

(b) the coordinates of their point of intersection are

$$\left(\frac{bc' - b'c}{ab' - a'b}, \frac{ca' - c'a}{ab' - a'b}\right)$$

(c) An equation of any line through their point of intersection is

$$(ax + by + c) + \lambda(a'x + b'y + c') = 0$$

where λ is a real number.

2. An equation of a line parallel to the line $ax + by + c = 0$ is $ax + by + c' = 0$, and the distance between these lines is

$$\frac{|c - c'|}{\sqrt{a^2 + b^2}}$$

Illustration 8

The line L is given by $2x + by = 7$, passes through (8, 3). The line K is parallel to L and has the equation $cx + 2y = c$. Find the distance between L and K.

Solution: Equation of L is $2x - 3y = 7$ as it passes through (8, 3) $\Rightarrow$ slope of L is (2/3).

$\Rightarrow$ slope of $K = -\frac{c}{2} = \frac{2}{3} \Rightarrow c = -\frac{4}{3}$.

and thus the equation of K is $2x - 3y = 2$.

So the required distance is $\frac{7-2}{\sqrt{2^2 + (-3)^2}} = \frac{5}{\sqrt{13}}$

3. Three lines $a_1 x + b_1 y + c_1 = 0$, $a_2 x + b_2 y + c_2 = 0$ and $a_3 x + b_3 y + c_3 = 0$ are *concurrent* (intersect at a point) if and only if

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$$

4. Equations of the *bisectors of the angles* between two intersecting lines $ax + by + c = 0$ and $a'x + b'y + c' = 0$ are

$$\frac{ax + by + c}{\sqrt{a^2 + b^2}} = \pm \frac{a'x + b'y + c'}{\sqrt{a'^2 + b'^2}}$$

Any point on the bisectors is equidistant from the given lines. If ϕ is the angle between one of the bisectors and one of the lines $ax + by + c = 0$, such that $|\tan \phi| < 1$, i.e., $-\pi/4 < \phi < \pi/4$, then that bisector bisects the acute angle between the two lines, i.e., it is the *acute angle bisector* of the two lines. The other equation then represents the *obtuse angle bisector* between the two lines.

5. Equations of the lines through (x_1, y_1) and making an angle ϕ with the line $ax + by + c = 0$, $b \neq 0$ are

$$y - y_1 = m_1 (x - x_1) \text{ where } m_1 = \frac{\tan\theta - \tan\phi}{1 + \tan\theta \tan\phi}$$

and $$y - y_1 = m_2 (x - x_1) \text{ where } m_2 = \frac{\tan\theta + \tan\phi}{1 - \tan\theta \tan\phi}$$

where $\tan\theta = -a/b$ is the slope of the given line. Note that $m_1 = \tan(\theta - \phi)$, $m_2 = \tan(\theta + \phi)$ and when $b = 0$, $\theta = \pi/2$.

SOME USEFUL POINTS

To show that A, B, C, D are the vertices of a

Parallelogram

Show that the diagonals AC and BD bisect each other.

Illustration 9

If $P(4, 7)$, $Q(7, 2)$ $R(a, b)$ and $S(3, 8)$ are the vertices of a parallelogram then find the value of a and b.

Solution: Since the diagonals of a parallelogram bisect each other. Mid points of PR and QS are same.

$$\text{so } \left(\frac{4+a}{2}, \frac{7+b}{2}\right) = \left(\frac{7+3}{2}, \frac{8+2}{2}\right)$$

$\Rightarrow a = 6, b = 3$

Rhombus

Show that the diagonals AC and BD bisect each other and a pair of adjacent sides, say, AB and BC are equal.

Square

Show that the diagonals AC and BD are equal and bisect each other, a pair of adjacent sides say AB and BC are equal.

Rectangle

Show that the diagonals AC and BD are equal and bisect each other.

LOCUS OF A POINT

To obtain the equation of a set of points satisfying some given condition(s) called locus, proceed as follows.

1. Let $P(h, k)$ be any point on the locus.
2. Write the given condition involving h and k and simplify. If possible draw a figure.
3. Eliminate the unknowns, if any.
4. Replace h by x and k by y and obtain an equation in terms of (x, y) and the known quantities. This is the required locus.

Illustration 10

Find the locus of the mid point of the portion between the axes of the line $x \cos\alpha + y \sin\alpha = \sin\alpha \tan\alpha$.

Solution: The given line meets x-axis at the point $A(\tan^2\alpha, 0)$ and y-axis at the point $B(0, \tan\alpha)$

Let $P(h, k)$ be the mid point of AB, then

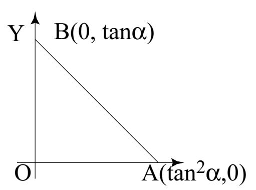


Fig. 16.2

$$h = \frac{\tan^2\alpha}{2}, \ k = \frac{\tan\alpha}{2}.$$

Eliminating α, we get
$2k^2 = h$
So the required locus is $2y^2 - x = 0$.

CHANGE OF AXES

Rotation of Axes

If the axes are rotated through an angle θ in the anticlockwise direction keeping the origin fixed, then the coordinates (X, Y) of a point $P(x, y)$ with respect to the new system of coordinate are given by $X = x\cos\theta + y\sin\theta$ and $Y = y\cos\theta - x\sin\theta$.

Translation of Axes

The shifting of origin of axes without rotation of axes is called *Translation of axes*.

If the origin (0, 0) is shifted to the point (h, k) without rotation of the axes then the coordinates (X, Y) of a point $P(x, y)$ with respect to the new system of coordinates are given by $X = x - h$, $Y = y - k$.

Illustration 11

If origin is shifted to the point (2, 3) and the axes are rotated through an angle $\pi/4$ in the anticlockwise direction, then find the coordinates of the point (7, 11) in the new system of coordinates.

Solution: When origin is shifted to the point (2, 3), the coordinates of $P(7, 11)$ are $(7 - 2, 11 - 3) = (5, 8)$.

Now when the axes are rotated through an angle $\pi/4$ in the anticlockwise direction, the coordinates of P are $(5\cos\pi/4 + 8\sin\pi/4, 8\cos\pi/4 - 5\sin\pi/4.) = \left(\frac{13}{\sqrt{2}}, \frac{3}{\sqrt{2}}\right)$

which are the required coordinates.

EQUATION OF FAMILY OF LINES

A first degree equation $ax + by + c = 0$ represents a straight line involving three constants a, b and c which can be reduced to two by dividing both the sides of the equation by a non zero constant. For instance if $a \neq 0$, we can write the equation as

$$x + \frac{b}{a}y + \frac{c}{a} = 0$$

or $$x + By + C = 0$$

So in order to determine an *equation of a line* we need two conditions on the line to determine these constants. For instance, if we know two points on the line or a point on the line and its slope etc., we know the line. But if we know just one condition, we have infinite number of lines satisfying the given condition. In this case the equation of the line contains an arbitrary constant and for different values of the constant we have different lines satisfying the given condition and the constant is called a parameter.

Some Equations of Family of Lines

1. *Family of lines with given slope* (Family of parallel lines)
   $y = mx + k$, where k is *a* parameter represents a family of lines in which each line has slope m.
2. *Family of lines through a point*
   $y - y_0 = k(x - x_0)$, where k is a parameter represents a family of lines in which each line passes through the point (x_0, y_0).
3. *Family of lines parallel to a given line*
   $ax + by + k = 0$, where k is a parameter represents a family of lines which are parallel to the line $ax + by + c = 0$.
4. *Family of lines through intersection of two given lines*
   $a_1x + b_1y + c_1 + k(a_2x + b_2y + c_2) = 0$; where k is a parameter represents a family of lines, in which each line passes through the point of intersection of two intersecting lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$.

Illustration 12

Two sides of a triangle given by $2x + 3y - 5 = 0$ and $5x - 4y + 10 = 0$ intersect at A. Centroid of the triangle is at the origin. find the equation of the median through A.

Solution: Equation of any line through A is $(2x + 3y - 5) + \lambda(5x - 4y + 10) = 0$. This will represent the median through A if it passes through the centroid (0, 0)

$\Rightarrow -5 + 10\lambda = 0 \Rightarrow \lambda = 1/2$ and the required equation is
$2(2x + 3y - 5) + (5x - 4y + 10) = 0$
$\Rightarrow 9x + 2y = 0$

5. *Family of lines perpendicular to a given line*
   $bx - ay + k = 0$, where k is a parameter represents a family of lines, in which each line is perpendicular to the line $ax + by + c = 0$.
6. *Family of lines making a given intercept on axes*
   (a) $\frac{x}{a} + \frac{y}{k} = 1$, where k is a parameter, represents a family of lines, in which each line makes an intercept a on the axis of x.
   (b) $\frac{x}{k} + \frac{y}{b} = 1$, where k is a parameter, represents a family of lines, in which each line makes an intercept b on the axis of y.
7. *Family of lines at a constant distance from the origin*
   $x\cos\alpha + y\sin\alpha = p$, where α is a parameter, represents a family of lines, in which each line is at a distance p from the origin.

Circumcentre of a triangle

The circumcentre of a triangle is the centre of the circle passing through the vertices of the triangle, so it is equi distant from the vertices of the triangle.

For example–The circumcentre (x, y) of the triangle with vertices (0, 0), (3, 4) and (4, 3) is given by

$$x^2 + y^2 = (x - 3)^2 + (y - 4)^2 = (x - 4)^2 + (y - 3)^2$$

Solving, we get $(x, y) = (25/14, 25/14)$

as the required circumcentre of the triangle.

Orthocentre of a triangle

The orthocentre of a triangle is the point of intersection of any two altitudes of the triangle.

For example–The orthocentre (p, q) of the triangle with vertices (0, 0), (3, 4) and (4, 3) is the point of intersection of the altitudes $y - x = 0$ through (0, 0) and $4x + 3y = 24$ through (3, 4) which is given by $(p, q) = \left(\frac{24}{7}, \frac{24}{7}\right)$

Note

- We know from geometry that circumcentre, centroid and orthocentre of a triangle lie on a line.

We observe that the centroid $\left(\frac{7}{3}, \frac{7}{3}\right)$ circumcentre $\left(\frac{25}{14}, \frac{25}{14}\right)$ and orthocentre $\left(\frac{24}{7}, \frac{24}{7}\right)$ of the triangle in the above example lie on the line $y = x$

SOLVED EXAMPLES

Concept-based

Straight Objective Type Questions

Example 1: The base of an equilateral triangle with side $2a$ lies along the y-axis such that the midpoint of the base is at the origin if the centroid of the triangle lies on the + ve x-axis, coordinates of the vertex are.

(a) $(a, 0)$ (b) $(-a, 0)$

(c) $(a\sqrt{3}, 0)$ (d) $(-a\sqrt{3}, 0)$

Ans. (c)

Solution: Coordinates of the base BC of the triangle are $B\ (0, a)$ and $C\ (0, -a)$ as origin is the mid-point.

Since the triangle is equilateral vertex A lies on the perpendicular bisector of the base which is x-axis. Let the coordinates of A be $(x, 0)$.

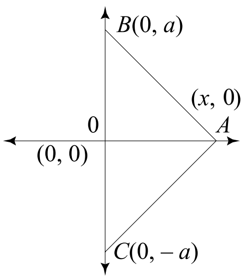


Fig. 16.3

Since the triangle is equilateral $AB = BC$

$\Rightarrow \quad x^2 + a^2 = (2a)^2$

$\Rightarrow \quad x^2 = 3a^2 \Rightarrow x = \pm\sqrt{3}a$

But since the centroid of the triangle lies on positive x-axis, coordinates of the vertex A are $(a\sqrt{3}, 0)$

Example 2: If three points $(h, 0)$, (a, b) and (o, k) lie on a line then

(a) $ah + bk = 1$ (b) $\frac{a}{h} + \frac{b}{k} = 1$

(c) $ak + bh = 1$ (d) $\frac{a}{k} + \frac{b}{h} = 1$

Ans. (b)

Solution: If given points lie on a line, then

$$\begin{vmatrix} h & 0 & 1 \\ a & b & 1 \\ 0 & k & 1 \end{vmatrix} = 0$$

Expanding along first row

$$h(b - k) + 1.\ (ak - b \times 0) = 0$$

$\Rightarrow \quad bh + ak = hk$

$\Rightarrow \quad \frac{a}{h} + \frac{b}{k} = 1$

Alternatively: Equation of the line joining $(h, 0)$ and $(0, k)$ is $\frac{x}{h} + \frac{y}{k} = 1$ as it makes intercepts h on x-axis and k on y-axis

If the point (a, b) also lies on it. then

$\frac{a}{h} + \frac{b}{k} = 1$

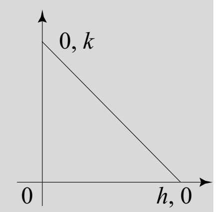


Fig. 16.4

Example 3: The line joining the points (2, x) and (3, 1) is perpendicular to the line joining the points (x, 4) and (7, 5). The value of x is

(a) 2 (b) 3
(c) 4 (d) 7

Ans. (c)

Solution: Slope of the line joining (2, x) and (3, 1) is $\frac{1-x}{3-2}$ and that of the line joining (x, 4) and (7, 5) is $\frac{5-4}{7-x}$.

Since the lines are perpendicular the product of the slopes is – 1

$$\Rightarrow \quad \frac{1-x}{1} \times \frac{1}{7-x} = -1$$

$$\Rightarrow \quad x = 4.$$

Example 4: If the point (a, b) divides a line between the axes in the ratio 2 : 3, the equation of the line is

(a) $ax + by = 5$ (b) $bx + ay = 5$
(c) $\frac{2x}{a} + \frac{3y}{b} = 5$ (d) $2\,ax + 3by = 5$

Ans. (c)

Solution: Let the line meet x-axis at (h, 0) and y-axis at (o, k), then the equation of the line is $\frac{x}{h} + \frac{y}{k} = 1$ (Intercept form)

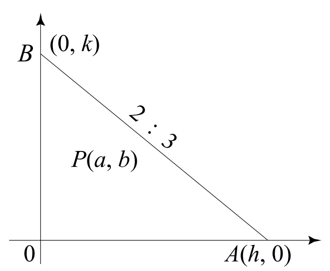


Fig. 16.5

Since $P(a, b)$ divides BA in the ratio 2 : 3 we get

$$\frac{2h + 0 \times 3}{5} = a, \quad \frac{2 \times 0 + 3k}{5} = b$$

$\Rightarrow h = \frac{5a}{2}$, $k = \frac{5b}{3}$ and the required equation is $\frac{2x}{a} + \frac{3y}{b} = 5$

Example 5: P is a point on x-axis, Q is a point on y-axis. Both are equidistant from the points (7, 6) and (3, 4). Distance between P and Q is

(a) $2\sqrt{5}$ (b) $15\sqrt{5}$
(c) $15\sqrt{5}/2$ (d) $5\sqrt{15}/2$

Ans. (c)

Solution: Let the coordinates of P be (x, 0) and of Q be (0, y), then

$(x - 7)^2 + 6^2 = (x - 3)^2 + 4^2 \Rightarrow x = 15/2$

and $7^2 + (y - 6)^2 = 3^2 + (y - 4)^2 \Rightarrow y = 15$

So coordinates of P are (15/2, 0) and of Q are (0, 15)

$$\Rightarrow \quad PQ = \sqrt{\left(\frac{15}{2}\right)^2 + (15)^2} = \frac{15\sqrt{5}}{2}$$

Example 6: The slope of a line is 3 times the slope of the other line and the tangent of the angle between them is 4/13, sum of their slopes is equal to

(a) 2 (b) 4
(c) 6 (d) 8

Ans. (d)

Solution: Let the slopes of the given lines be m and 3 m. if θ is the angle between them then

$$\tan\theta = \frac{|3m - m|}{1 + (3m)(m)} = \frac{4}{13}$$

$$\Rightarrow \quad \pm\, 2m \times 13 = 4\,(1 + 3m^2)$$

$$\Rightarrow \quad 6m^2 \pm 13m + 2 = 0$$

$$\Rightarrow \quad m = \frac{1}{6} \text{ or } 2 \text{ and } m = -\frac{1}{6} \text{ or } -2.$$

For $m = 2$, the sum of the slopes is $4m = 8$

Example 7: A line perpendicular to the line segment joining the points (7, 3) and (3, 7) divides it in the ratio 1 : 3, the equation of the line is

(a) $x + y - 10 = 0$ (b) $x - y + 4 = 0$
(c) $x - y + 2 = 0$ (d) $x - y - 2 = 0$

Ans. (d)

Solution: Slope of the given line is $\frac{7-3}{3-7} = -1$ and the coordinates of the point which divides the given line segment in the ratio 1 : 3

$$\underset{(7,\,3)\qquad\qquad\qquad\qquad(3,\,7)}{\overline{\qquad\qquad 1\ :\ 3 \qquad\qquad}}$$

is $\left(\frac{1 \times 3 + 3 \times 7}{4}, \frac{1 \times 7 + 3 \times 3}{4}\right) = (6, 4)$

So equation of the line passing through the point (6, 4) and perpendicular to the given line is

$$y - 4 = x - 6 \Rightarrow x - y - 2 = 0.$$

Example 8: If the distance between the parallel lines $3x + 4y + 7 = 0$ and $ax + y + b = 0$ is 1, the integral value of b is

(a) 1 (b) 2
(c) 3 (d) 4

Ans. (c)

Solution: Since the given lines are parallel. Slopes of the lines are equal so $a = \frac{3}{4}$ and the two lines are $3x + 4y + 7 = 0$ and $3x + 4y + 4b = 0$.

Distance between them is $\dfrac{|7-4b|}{\sqrt{3^2+4^2}} = 1$

$\Rightarrow 7 - 4b = \pm 5 \Rightarrow b = 1/2$ or 3

Example 9: If P and Q are the lengths of the perpendiculars from the origin to the lines $x \cos\theta + y \sin\theta = k \cos 2\theta$ and $x \sec\theta - y \operatorname{cosec}\theta = k$ respectively then

(a) $p^2 - 4q^2 = k^2$ (b) $p^2 + 4q^2 = k^2$
(c) $4p^2 + q^2 = k^2$ (d) $p^2 + q^2 = k^2$

Ans. (b)

Solution: $p = \dfrac{|k\cos 2\theta|}{\sqrt{\cos^2\theta + \sin^2\theta}} = |k \cos 2\theta|$

$$q = \frac{|k|}{\sqrt{\sec^2\theta + \operatorname{cosec}^2\theta}} = |k \sin\theta \cos\theta|$$

$$= |k\,(1/2) \sin 2\theta|$$

So $\quad p^2 + 4q^2 = k^2 (\cos^2 2\theta + \sin^2 2\theta) = k^2.$

Example: 10: If the lines $3x - y + 1 = 0$ and $x - 2y + 3 = 0$ are equally inclined to the line $y = mx$, then the value of m is given by

(a) $2m^2 - 7m - 7 = 0$ (b) $7m^2 - 7m - 2 = 0$
(c) $7m^2 - 2m - 7 = 0$ (d) $2m^2 - 7m - 2 = 0$

Ans. (c)

Solution: If θ is the angle between the lines then

$\tan\theta = \left|\dfrac{m-3}{1+3m}\right| = \left|\dfrac{m-(1/2)}{1+m(1/2)}\right|$

$\Rightarrow \quad (m-3)(2+m) = \pm(2m-1)(3m+1)$
$\Rightarrow \quad m^2 - m - 6 = \pm(6m^2 - m - 1)$
$\Rightarrow \quad 7m^2 - 2m - 7 = 0$ or $m^2 + 1 = 0$

Example 11: Equation of the line which makes an intercept of length 2 on positive x-axis and an intercept of length 3 on the negative y-axis is

(a) $2x - 3y + 6 = 0$ (b) $3x - 2y - 6 = 0$
(c) $2x - 3y - 6 = 0$ (d) $3x - 2y + 6 = 0$

Ans. (b)

Solution: Equation of the required line is $\dfrac{x}{2} + \dfrac{y}{-3} = 1$

$\Rightarrow \quad 3x - 2y = 6$

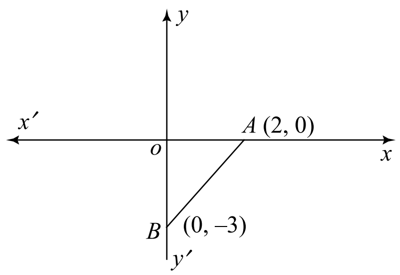


Fig. 16.6

Example 12: The perpendicular from the origin to a line L meets it at the point $(3, -9)$, equation of the line L is

(a) $x - 3y = 30$ (b) $3x - y = 18$
(c) $x + 3y + 24 = 0$ (d) $3x + y = 18$

Ans. (a)

Solution: Slope of the perpendicular line is $\dfrac{-9}{3} = -3$

$\Rightarrow$ slope of L is (1/3) and as it passes through $(3, -9)$

equation of L is

$$y + 9 = (1/3)(x - 3)$$

$\Rightarrow \quad x - 3y = 30$

Example 13: The distance of the point (2, 3) from the line $4x - 3y + 26 = 0$ is same as its distance from the line $3x - 4y + p = 0$. The value of p can be

(a) 5 (b) 25
(c) 31 (d) – 31

Ans. (c)

Solution: We have

$$\left|\frac{4\times 2 - 3\times 3 + 26}{\sqrt{4^2+3^2}}\right| = \left|\frac{3\times 2 - 4\times 3 + p}{\sqrt{4^2+3^2}}\right|$$

$\Rightarrow \quad \pm 25 = 6 - 12 + p$
$\Rightarrow \quad p = \pm 25 + 6$
$\Rightarrow \quad p = 31$ or -19.

Example 14: If the segment of the line between the lines $x - y + 2 = 0$ and $x + y + 4 = 0$ is bisected at the origin, equation of the line is

(a) $y + 3x = 0$ (b) $x + 3y = 0$
(c) $y - 3x = 0$ (d) $x - 3y = 0$

Ans. (c)

Solution: Let the required line meet the given lines at (α_1, β_1) and (α_2, β_2) respectively.
then $\alpha_1 - \beta_1, + 2 = 0$, $\alpha_2 + \beta_2 + 4 = 0$

Also $\dfrac{\alpha_1 + \alpha_2}{2} = 0$, $\dfrac{\beta_1 + \beta_2}{2} = 0$

$\Rightarrow \quad \alpha_2 = -1, \beta_2 = -3$ and $\alpha_1 = 1, \beta_1 = 3$

Equation of the line passing through (1, 3) and (0, 0) is

$$y = 3x \quad \Rightarrow \quad y - 3x = 0$$

Example 15: A ray of light passing through the point (3, 7) reflects on the x-axis at a point A and the reflected ray passes through the point (2, 5), the coordinates of A are

(a) $\left(\dfrac{29}{12}, 0\right)$ (b) $\left(\dfrac{1}{2}, 0\right)$

(c) $\left(-\dfrac{1}{2}, 0\right)$ (d) $\left(-\dfrac{29}{12}, 0\right)$

Ans. (a)

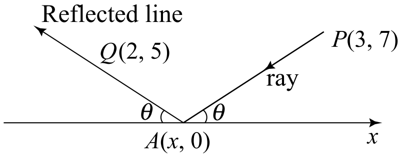


Fig. 16.7

◎ **Solution:** Let P be the given point (3, 7) and the coordinates of A be $(x, 0)$ so, if the ray PA makes an angle θ with the positive direction of x-axis, the reflected ray AQ , where Q is (2, 5), will make the same angle θ with the negative direction of x-axis. So the slope of PA = – (the slope of QA)

$$\Rightarrow \quad \frac{7}{3-x} = -\frac{5}{2-x}$$

$$\Rightarrow \quad 7(2-x) + 5(3-x) = 0$$

$$\Rightarrow \quad x = \frac{29}{12}$$

$\Rightarrow$ coordinates of A are $\left(\frac{29}{12}, 0\right)$.

LEVEL 1

Straight Objective Type Questions

Example 16: If a vertex of a triangle is (1, 1) and the mid points of two sides through this vertex are (– 1, 2) and (3, 2), then centroid of the triangle is

(a) (1, 7/3) (b) (1/3, 7/3)
(c) (– 1, 7/3) (d) (– 1/3, 7/3)

Ans. (a)

◎ **Solution:** Let (x_1, y_1) and (x_2, y_2) be the other two vertices of the triangle, then $\frac{x_1+1}{2} = -1$, $\frac{y_1+1}{2} = 2$

$\Rightarrow (x_1, y_1) = (-3, 3)$; similarly $(x_2, y_2) = (5, 3)$

centroid of the triangle is $\left(\frac{1-3+5}{3}, \frac{1+3+3}{3}\right) = (1, 7/3)$

Example 17: The points $(a, b + c)$, $(b, c + a)$ and $(c, a + b)$ are

(a) vertices of an equilateral triangle
(b) concyclic
(c) vertices of a right angled triangle
(d) none of these

Ans. (d)

◎ **Solution:** As the given points lie on the line $x + y = a + b + c$, they are collinear.

Example 18: If the lines $x + 2ay + a = 0$, $x + 3by + b = 0$ and $x + 4cy + c = 0$ are concurrent, then a, b, c are in

(a) A.P. (b) G.P.
(c) H.P. (d) none of these

Ans. (c)

◎ **Solution:** Since the given lines are concurrent

$$\begin{vmatrix} 1 & 2a & a \\ 1 & 3b & b \\ 1 & 4c & c \end{vmatrix} = 0 \Rightarrow \quad -bc + 2ac - ab = 0$$

$$\Rightarrow \quad b = \frac{2ac}{a+c} \quad \Rightarrow \quad a, b, c \text{ are in H.P.}$$

Example 19: If p is the length of the perpendicular from the origin on the line $\frac{x}{a} + \frac{y}{b} = 1$ and a^2, p^2, b^2 are in A.P., then $a^4 - 2p^2a^2 + 2p^4 =$

(a) – 1 (b) 0
(c) 1 (d) none of these

Ans (b)

◎ **Solution:** We have $p = \frac{|a\,b|}{\sqrt{a^2+b^2}} \quad \Rightarrow \quad p^2 = \frac{a^2b^2}{a^2+b^2}$

Also, since a^2, p^2, b^2 are in A.P.,

$$a^2 + b^2 = 2p^2 \text{ thus } a^2b^2 = p^2(a^2 + b^2) = 2p^4$$

$$\Rightarrow \quad 2p^4 = a^2(2p^2 - a^2) \Rightarrow a^4 - 2p^2a^2 + 2p^4 = 0$$

Example 20: If a, x_1, x_2 are in G.P. with common ratio r, and b, y_1, y_2 are in G.P. with common ratio s where $s - r = 2$, then the area of the triangle with vertices (a, b), (x_1, y_1) and (x_2, y_2) is

(a) $|\,ab\,(r^2 - 1)\,|$ (b) $ab\,(r^2 - s^2)$
(c) $ab\,(s^2 - 1)$ (d) $abrs$

Ans. (a)

◎ **Solution:** Area of the triangle

$$= \frac{1}{2}\left|\begin{vmatrix} a & b & 1 \\ ar & bs & 1 \\ ar^2 & bs^2 & 1 \end{vmatrix}\right| = \frac{1}{2}|\,ab\,(r-1)(s-1)(s-r)\,|$$

$$= |ab\,(r-1)\,(r+1)| = |ab\,(r^2-1)|$$

Example 21: The line joining $A\,(b\cos\alpha, b\sin\alpha)$ and $B\,(a\cos\beta, a\sin\beta)$ is produced to the point $M\,(x, y)$ so that $AM : MB = b : a$, then $x\cos\frac{\alpha+\beta}{2} + y\sin\frac{\alpha+\beta}{2} =$

(a) -1 (b) 0
(c) 1 (d) $a^2 + b^2$

Ans. (b)

Solution: As M divides AB externally in the ratio $b : a$

$$x = \frac{b(a\cos\beta) - a(b\cos\alpha)}{b-a} \text{ and}$$

$$y = \frac{b(a\sin\beta) - a(b\sin\alpha)}{b-a}$$

$$\Rightarrow \quad \frac{x}{y} = \frac{\cos\beta - \cos\alpha}{\sin\beta - \sin\alpha} = \frac{2\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}}{2\cos\frac{\alpha+\beta}{2}\sin\frac{\beta-\alpha}{2}}$$

$$\Rightarrow \quad x\cos\frac{\alpha+\beta}{2} + y\sin\frac{\alpha+\beta}{2} = 0.$$

Example 22: If the circumcentre of a triangle lies at the origin and the centroid is the middle point of the line joining the points (a^2+1, a^2+1) and $(2a, -2a)$; then the orthocentre lies on the line

(a) $y = (a^2+1)x$ (b) $y = 2ax$
(c) $x + y = 0$ (d) $(a-1)^2 x - (a+1)^2 y = 0$

Ans. (d)

Solution: We know from geometry that the circumcentre, centroid and orthocentre of a triangle lie on a line. So the orthocentre of the triangle lies on the line joining the circumcentre (0, 0) and the centroid $\left(\frac{(a+1)^2}{2}, \frac{(a-1)^2}{2}\right)$

i.e. $\frac{(a+1)^2}{2} y = \frac{(a-1)^2}{2} x$

or $(a-1)^2 x - (a+1)^2 y = 0$.

Example 23: If p_1, p_2 denote the lengths of the perpendiculars from the origin on the lines $x \sec\alpha + y \operatorname{cosec}\alpha = 2a$ and $x\cos\alpha + y\sin\alpha = a\cos 2\alpha$ respectively, then $\left(\frac{p_1}{p_2} + \frac{p_2}{p_1}\right)^2$ is equal to

(a) $4\sin^2 4\alpha$ (b) $4\cos^2 4\alpha$
(c) $4\operatorname{cosec}^2 4\alpha$ (d) $4\sec^2 4\alpha$

Ans. (c)

Solution: $p_1^2 + p_2^2 = \frac{4a^2}{\sec^2\alpha + \operatorname{cosec}^2\alpha} + \frac{a^2\cos^2 2\alpha}{\cos^2\alpha + \sin^2\alpha}$

$$= a^2(\sin^2 2\alpha + \cos^2 2\alpha) = a^2$$

and $\quad p_1^2 p_2^2 = a^4 \sin^2 2\alpha \cos^2 2\alpha = (1/4)a^4 \sin^2 4\alpha$

$$\therefore \left(\frac{p_1}{p_2} + \frac{p_2}{p_1}\right)^2 = \frac{\left(p_1^2 + p_2^2\right)^2}{p_1^2 p_2^2} = \frac{4}{\sin^2 4\alpha} = 4\operatorname{cosec}^2 4\alpha.$$

Example 24: $OPQR$ is a square and M, N are the middle points of the sides PQ and QR respectively then the ratio of the areas of the square and the triangle OMN is

(a) $4 : 1$ (b) $2 : 1$
(c) $8 : 3$ (d) $4 : 3$

Ans. (c)

Solution: Taking the coordinates of vertices O, P, Q, R as (0, 0), $(a, 0)$, (a, a), $(0, a)$ respectively we get the coordinates of M as $(a, a/2)$ and of N as $(a/2, a)$

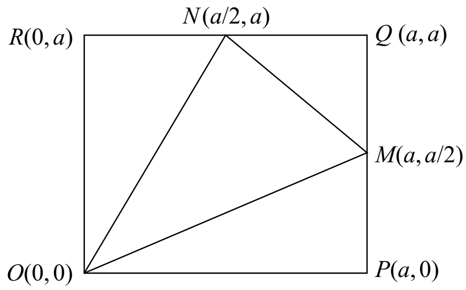


Fig. 16.8

$$\therefore \quad \text{Area of the } \Delta OMN = \frac{1}{2}\left|\begin{vmatrix} 0 & 0 & 1 \\ a & a/2 & 1 \\ a/2 & a & 1 \end{vmatrix}\right| = \frac{3a^2}{8}$$

Area of the square $= a^2$

$\therefore$ the required ratio is $8 : 3$.

Example 25: The locus of the point of intersection of the lines $x\sin\theta + (1 - \cos\theta)y = a\sin\theta$ and $x\sin\theta - (1 + \cos\theta)y + a\sin\theta = 0$ is

(a) $x^2 - y^2 = a^2$ (b) $x^2 + y^2 = a^2$
(c) $y^2 = ax$ (d) none of these

Ans. (b)

Solution: From the given equations we have

$$\frac{1-\cos\theta}{\sin\theta} = \frac{a-x}{y} \text{ and } \frac{1+\cos\theta}{\sin\theta} = \frac{a+x}{y}$$

Multiplying we get $\frac{1-\cos^2\theta}{\sin^2\theta} = \frac{a^2-x^2}{y^2} \Rightarrow x^2 + y^2 = a^2$

Example 26: Two points $(a, 3)$ and $(5, b)$ are the opposite vertices of a rectangle. If the other two vertices lie on the line $y = 2x + c$ which passes through the point (a, b) then the value of c is

(a) -7 (b) -4
(c) 0 (d) 7

Ans. (a)

Solution: Mid point of the line joining the given points lie on the given line

$$\frac{3+b}{2} = 2\left(\frac{a+5}{2}\right) + c$$

$\Rightarrow \quad 2a + 2c - b + 7 = 0$ (i)

Also since the given line passes through (a, b)

$b = 2a + c$ (ii)

Solving (i) and (ii) we get $c = -7$

Example 27: If every point on the line $(a_1 - a_2)x + (b_1 - b_2)y = c$ is equidistant from the points (a_1, b_1) and (a_2, b_2) then $2c =$

(a) $a_1^2 - b_1^2 + a_2^2 - b_2^2$ (b) $a_1^2 + b_1^2 + a_2^2 + b_2^2$
(c) $a_1^2 + b_1^2 - a_2^2 - b_2^2$ (d) none of these

Ans. (c)

Solution: Let (h, k) be any point on the given line then

$$(h - a_1)^2 + (k - b_1)^2 = (h - a_2)^2 + (k - b_2)^2$$

$\Rightarrow$ $2(a_1 - a_2)h + 2(b_1 - b_2)k = a_1^2 + b_1^2 - a_2^2 - b_2^2$

$\Rightarrow$ $(a_1 - a_2)h + (b_1 - b_2)k$
$= (1/2)(a_1^2 + b_1^2 - a_2^2 - b_2^2)$ (i)

Since (h, k) lies on the given line

$$(a_1 - a_2)h + (b_1 - b_2)k = c \quad \text{(ii)}$$

Comparing (i) and (ii) we get $c = (1/2)(a_1^2 + b_1^2 - a_2^2 - b_2^2)$.

Example 28: Equations of the straight lines passing through the point (4, 3) and making intercepts on the coordinate axes whose sum is – 1 are

(a) $x/2 + y/3 = 1$ and $x/2 + y/1 = 1$
(b) $x/2 - y/3 = -1$ and $x/(-2) + y/1 = -1$
(c) $x/2 + y/3 = -1$ and $x/(-2) + y/1 = -1$
(d) $x/2 - y/3 = 1$ and $x/(-2) + y/1 = 1$

Ans. (d)

Solution: Let the equation of the line be $\dfrac{x}{a} + \dfrac{y}{-1-a} = 1$.

Since it passes through (4, 3), $\dfrac{4}{a} + \dfrac{3}{-1-a} = 1$

$\Rightarrow$ $a^2 = 4$ $\Rightarrow$ $a = \pm 2$

and the required equations are

$$\frac{x}{2} + \frac{y}{-3} = 1 \quad \text{and} \quad \frac{x}{-2} + \frac{y}{1} = 1.$$

Example 29: If non zero numbers a, b, c are in H.P, then the straightline $\dfrac{x}{a} + \dfrac{y}{b} + \dfrac{1}{c} = 0$ always passes through a fixed point. That point is

(a) (1, – 2) (b) (1, – 1/2)
(c) (– 1, 2) (d) (– 1, – 2)

Ans. (a)

Solution: a, b, c are in H.P.

$\Rightarrow$ $\dfrac{1}{b} - \dfrac{1}{a} = \dfrac{1}{c} - \dfrac{1}{b}$

$\Rightarrow$ $\dfrac{1}{a} - \dfrac{2}{b} + \dfrac{1}{c} = 0$

which shows that the given line passes through the point (1, – 2).

Example 30: The line parallel to x-axis passing through the intersection of the lines $ax + 2by + 3b = 0$ and $bx - 2ay - 3a = 0$ where $(a, b) \neq (0, 0)$ is

(a) above x-axis at a distance 3/2 from it.
(b) above x-axis at a distance 2/3 from it.
(c) below x-axis at a distance 3/2 from it.
(d) below x-axis at a distance 2/3 from it.

Ans. (c)

Solution: Eliminating x, we get
$(2b^2 + 2a^2)y + 3b^2 + 3a^2 = 0$

$\Rightarrow$ $y = -3/2$ which is the required line and hence below x-axis at a distance 3/2 from it.

Example 31: A straight line through the point $A(3, 4)$ is such that its intercept between the axes is bisected at A. Its equation is

(a) $3x + 4y = 25$ (b) $x + y = 7$
(c) $3x - 4y + 7 = 0$ (d) $4x + 3y = 24$

Ans. (d)

Solution: Let the equation be $\dfrac{x}{a} + \dfrac{y}{b} = 1$ where $\dfrac{a}{2} = 3$, $\dfrac{b}{2} = 4$

$\Rightarrow$ $a = 6$, $b = 8$ and the required equation is $8x + 6y = 48$ or $4x + 3y = 24$

Example 32: Let $A(h, k)$, $B(1, 1)$ and $C(2, 1)$ be the vertices of a right angled triangle with AC as its hypotenuse. If the area of the triangle is 1, then the set of values which k can take is given by

(a) {1, 3} (b) {0, 2}
(c) {– 1, 3} (d) {– 3, – 2}

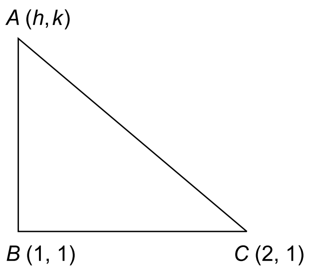


Fig 15.9

Ans. (c)

Solution: Equation of BC is $y = 1$. AB is perpendicular to BC through $B(1, 1)$ so in equation is $x = 1 \Rightarrow h = 1$.

Area of the $\Delta ABC = \dfrac{1}{2} AB \times BC = 1$

$\Rightarrow$ $AB = 2 \Rightarrow |k - 1| = 2$

$\Rightarrow$ $k = -1$ or 3

Example 33: Let $A(2, -3)$ and $B(-2, 1)$ be vertices of a triangle ABC. If the centroid of this triangle moves on the line $2x + 3y = 1$, then the locus of the vertex C is the line

(a) $3x + 2y = 5$ (b) $2x - 3y = 7$
(c) $2x + 3y = 9$ (d) $3x - 2y = 3$

Ans. (c)

Solution: Let $C(h, k)$ be the vertex, then the centroid is $\left(\frac{h+2-2}{3}, \frac{k-3+1}{3}\right)$ i.e. $(h/3, (k-2)/3)$ which lies on $2x + 3y = 1$

$\Rightarrow \quad 2\,\frac{h}{3} + \frac{3(k-2)}{3} = 1$

$\Rightarrow \quad 2h + 3k = 9$ and the locus of (h, k) is $2x + 3y = 9$.

Example 34: If the sum of the slopes of the lines given by $x^2 - 2cxy - 7y^2 = 0$ is four times their product, then the value of c is

(a) 2 (b) -1
(c) 1 (d) -2

Ans. (a)

Solution: If m_1, m_2 are the slopes, then $m_1 + m_2 = -2c/7$, $m_1 m_2 = -1/7$

$m_1 + m_2 = 4\, m_1 m_2$

$\Rightarrow \quad c = 2$

Example 35: Locus of mid point of the portion between the axes of $x \cos \alpha + y \sin \alpha = p$, where p is constant is

(a) $x^2 + y^2 = 4/p^2$ (b) $x^2 + y^2 = 4p^2$
(c) $1/x^2 + 1/y^2 = 2/p^2$ (d) $1/x^2 + 1/y^2 = 4/p^2$

Ans. (d)

Solution: If (h, k) is the mid-point, then

$$h = p/2 \cos\alpha,\ k = p/2 \sin\alpha$$

so $\quad (p/2h)^2 + (p/2k)^2 = \cos^2 \alpha + \sin^2 \alpha = 1$

$\Rightarrow \quad 1/h^2 + 1/k^2 = 4/p^2$

Locus of (h, k) is $1/x^2 + 1/y^2 = 4/p^2$

Example 36: The lines $x + y = |a|$ and $ax - y = 1$ intersect each other in the first quadrant. Then the set of all possible values of a is in the interval.

(a) $(0, \infty)$ (b) $(1, \infty)$
(c) $(-1, \infty)$ (d) $(-1, 1)$

Ans.(b)

Solution: Adding $x + y = |a|$ and $ax - y = 1$

We obtain $(1 + a)\, x = |a| + 1 \Rightarrow x = \frac{|a|+1}{a+1}$, $a \neq -1$

Also $y = a\left(\frac{|a|+1}{a+1}\right) - 1 = \frac{a|a|-1}{a+1}$

As $x > 0$, $y > 0$, $|a| + 1 > 0$, we get $a + 1 > 0$

and $a\,|a| - 1 > 0 \Rightarrow a > -1$ and $a\,|a| > 1$

As $|a| \geq 0$, and $a\,|a| > 1$, we get $a > 0$ and thus $a^2 > 1$ or $a > 1$

Example 37: A square of side a lies above the x-axis and has one vertex at the origin. The side passing through the origin makes an angle α $(0 < \alpha < \pi/4)$ with the positive direction of x-axis. The equation of its diagonal not passing through the origin is

(a) $y (\cos \alpha + \sin \alpha) + x (\sin \alpha - \cos \alpha) = a$
(b) $y (\cos \alpha + \sin \alpha) + x (\sin \alpha + \cos \alpha) = a$
(c) $y (\cos \alpha + \sin \alpha) + x (\cos \alpha - \sin \alpha) = a$
(d) $y (\cos \alpha - \sin \alpha) - x (\sin \alpha - \cos \alpha) = a$

Ans. (c)

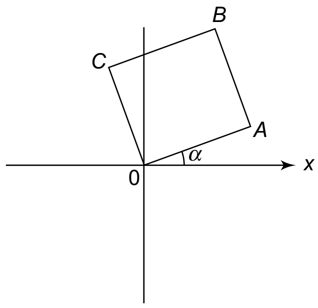


Fig. 16.10

Solution: Coordinates of A are $(a \cos \alpha, a \sin \alpha)$ and of C are

$(a \cos (\alpha + \pi/2), a \sin (\alpha + \pi/2))$

i.e. $(-a \sin \alpha, a \cos \alpha)$

So the equation of the diagonal AC is

$$y - a \sin \alpha = \frac{a(\sin\alpha - \cos\alpha)}{a(\cos\alpha + \sin\alpha)}\,(x - a \cos \alpha)$$

or $y (\cos \alpha + \sin \alpha) + x (\cos \alpha - \sin \alpha)$

$= a (\sin^2 \alpha + \sin \alpha \cos \alpha - \sin \alpha \cos \alpha + \cos^2 \alpha)$

$= a$

Example 38: Locus of the centroid of the triangle whose vertices are $(a \cos t, a \sin t)$, $(b \sin t, -b \cos t)$ and $(1, 0)$; where t is *a* parameter is

(a) $(3x - 1)^2 + (3y)^2 = a^2 + b^2$
(b) $(3x + 1)^2 + (3y)^2 = a^2 + b^2$
(c) $(3x + 1)^2 + (3y)^2 = a^2 - b^2$
(d) $(3x - 1)^2 + (3y)^2 = a^2 - b^2$

Ans. (a)

Solution: If (h, k) is the centroid, then

$$h = \frac{a \cos t + b \sin t + 1}{3},$$

$$k = \frac{a \sin t - b \cos t + 0}{3}$$

$\Rightarrow (3h - 1)^2 + (3k)^2 = (a \cos t + b \sin t)^2 + (a \sin t - b \cos t)^2$

$= a^2 + b^2$

Locus of (h, k) is $(3x - 1)^2 + (3y)^2 = a^2 + b^2$

Example 39: If x_1, x_2, x_3 and y_1, y_2, y_3 are both in G.P. with the same common ratio, then the points (x_1, y_1), (x_2, y_2) and (x_3, y_3)

(a) lie on an ellipse
(b) lie on a circle
(c) are vertices of a triangle
(d) lie on a straight line

Ans. (d)

Solution: Let $x_2 = x_1 r$, $x_3 = x_1 r^2$

$$y_2 = y_1 r, \; y_3 = y_1 r^2$$

The given points lie on the line

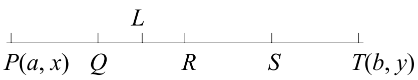
$x_1 y - y_1 x = 0$ for all values of r.

Example 40: Q, R and S are the points on the line joining the points $P(a, x)$ and $T(b, y)$ such that $PQ = QR = RS = ST$, then $\left(\frac{5a+3b}{8}, \frac{5x+3y}{8}\right)$ is the mid point of the segment

(a) PQ (b) QR
(c) RS (d) ST

Ans. (b)

Solution: The point $L\left(\frac{5a+3b}{8}, \frac{5x+3y}{8}\right)$ divides PT in the ratio 3 : 5 and hence is the middle point of QR.



Example 41: If a, b, c form a G.P. with common ratio r, the sum of the ordinates of the points of intersection of the line $ax + by + c = 0$ and the curve $x + 2y^2 = 0$ is

(a) $-r^2/2$ (b) $-r/2$
(c) $r/2$ (d) $r^2/2$

Ans. (c)

Solution: The equation of the given line is

$ax + by + c = 0$

$$\Rightarrow \quad ax + ary + ar^2 = 0 \quad \Rightarrow \quad x + ry + r^2 = 0 \qquad \text{(i)}$$

(i) intersects the curves $x + 2y^2 = 0$ at the points whose ordinates are given by

$$-2y^2 + ry + r^2 = 0 \quad \text{or} \quad 2y^2 - ry - r^2 = 0$$

Therefore required sum of the ordinates $= r/2$.

Example 42: If α, β, γ are the real roots of the equation $x^3 - 3px^2 + 3qx - 1 = 0$, then the centroid of the triangle with vertices $\left(\alpha, \frac{1}{\alpha}\right), \left(\beta, \frac{1}{\beta}\right)$ and $\left(\gamma, \frac{1}{\gamma}\right)$ is at the point

(a) (p, q) (b) $(p/3, q/3)$
(c) $(p + q, p - q)$ (d) $(3p, 3q)$

Ans. (a)

Solution: The centroid of the given triangle is the point

$$\left(\frac{\alpha+\beta+\gamma}{3}, \frac{\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma}}{3}\right) = \left(\frac{3p}{3}, \frac{\alpha\beta+\beta\gamma+\gamma\alpha}{3\alpha\beta\gamma}\right)$$

$= (p, q)$ [$\because$ $\alpha + \beta + \gamma = 3p$, $\alpha\beta + \beta\gamma + \gamma\alpha = 3q$, $\alpha\beta\gamma = 1$]

Example 43: The line L has intercepts a and b on the coordinate axes. The coordinate axes are rotated through a fixed angle, keeping the origin fixed. If p and q are the intercepts of the line L on the new axes, then $\frac{1}{a^2} - \frac{1}{p^2} + \frac{1}{b^2} - \frac{1}{q^2}$ is equal to

(a) -1 (b) 0
(c) 1 (d) none of these

Ans. (b)

Solution: Equation of the line L in the two coordinate systems is $\frac{x}{a} + \frac{y}{b} = 1$, $\frac{X}{p} + \frac{Y}{q} = 1$

where (X, Y) are the new coordinates of a point (x, y) when the axes are rotated through a fixed angle, keeping the origin fixed. As the length of the perpendicular from the origin has not changed.

$$\frac{1}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}}} = \frac{1}{\sqrt{\frac{1}{p^2}+\frac{1}{q^2}}} \quad \Rightarrow \quad \frac{1}{a^2}+\frac{1}{b^2} = \frac{1}{p^2}+\frac{1}{q^2}$$

or $$\frac{1}{a^2} - \frac{1}{p^2} + \frac{1}{b^2} - \frac{1}{q^2} = 0.$$

Example 44: Lines $ax + by + c = 0$ where $3a + 2b + 4c = 0$ $a, b, c \in \mathrm{R}$ are concurrent at the point.

(a) (3, 2) (b) (2, 4)
(c) (3, 4) (d) (3/4, 1/2)

Ans. (d)

Solution: $3a + 2b + 4c = 0$

$$\Rightarrow \quad \frac{3}{4}a + \frac{1}{2}b + c = 0$$

$\Rightarrow$ $ax + by + c$ passes through (3/4, 1/2) for all values of a, b, c.

Example 45: If $P(1, 0)$, $Q(-1, 0)$ and $R(2, 0)$ are three given points. The point S satisfies the relation $SQ^2 + SR^2 = 2SP^2$. The locus of S meets PQ at the point

(a) (0, 0) (b) (2/3, 0)
(c) (– 3/2, 0) (d) (0, – 2/3)

Ans. (c)

Solution: Let S be the point (x, y)

then $\quad (x + 1)^2 + y^2 + (x - 2)^2 + y^2 = 2[(x - 1)^2 + y^2]$

$\Rightarrow 2x + 3 = 0$, the locus of S and equation of PQ is $y = 0$.

So the required points is (– 3/2, 0).

Example 46: If algebraic sum of distances of a variable line from points (2, 0), (0, 2) and (– 2, – 2) is zero, then the line passes through the fixed point

(a) (– 1, – 1) (b) (0, 0)
(c) (1, 1) (d) (2, 2)

Ans. (b)

Solution: Let the equation of the variable line be

$$ax + by + c = 0$$

then according to the given condition

$$\frac{2a+c}{\sqrt{a^2+b^2}} + \frac{2b+c}{\sqrt{a^2+b^2}} + \frac{-2a-2b+c}{\sqrt{a^2+b^2}} = 0$$

$\Rightarrow \quad c = 0$

which shows that the line passes through (0, 0) for all values of a and b.

Example 47: An equation of a straight line passing through the inter-section of the straight lines $3x - 4y + 1 = 0$ and $5x + y - 1 = 0$ and making non-zero, equal intercepts on the axes is

(a) $22x + 22y = 13$ (b) $23x + 23y = 11$
(c) $11x + 11y = 23$ (d) $8x - 3y = 0$

Ans. (b)

Solution: Equation of any line through the point of intersection of the given lines is

$$(3x - 4y + 1) + k(5x + y - 1) = 0 \quad (1)$$

or $\quad (3 + 5k)x + (k - 4)y + 1 - k = 0$

or $\quad \dfrac{x}{(k-1)/(3+5k)} + \dfrac{y}{(k-1)/(k-4)} = 1$

Since $\quad x$-intercept = y-intercept

$\Rightarrow \dfrac{k-1}{3+5k} = \dfrac{k-1}{k-4} \quad \Rightarrow \quad (k - 1)(3 + 5k - k + 4) = 0$

$\Rightarrow \quad k = 1$ or $k = -7/4$

For $k = 1$, (1) becomes $8x - 3y = 0$ which makes zero intercepts on the axes.

$\therefore \quad k = -7/4 \quad \Rightarrow \quad$ The required equation is

$4(3x - 4y + 1) - 7(5x + y - 1) = 0$

$\Rightarrow \quad 23x + 23y = 11.$

Example 48: If the point (3, 4) lies on the locus of the point of intersection of the lines $x \cos \alpha + y \sin \alpha = a$ and $x \sin \alpha - y \cos \alpha = b$ (α is a variable), the point (a, b) lies on the line $3x - 4y = 0$ then $|a + b|$ is equal to

(a) 1 (b) 7
(c) 12 (d) 5

Ans. (b)

Solution: Squaring and adding the given equations of the lines we get $x^2 + y^2 = a^2 + b^2$ as the locus of the point of intersection of these lines.

Since (3, 4) lies on the locus, we get

$9 + 16 = a^2 + b^2 \quad$ i.e. $\quad a^2 + b^2 = 25 \quad$ (i)

Also, (a, b) lies on $3x - 4y = 0$

so $\quad 3a - 4b = 0 \quad \Rightarrow \quad b = (3/4)a \quad$ (ii)

From (i), $\quad a^2 + (9/16)a^2 = 25 \quad \Rightarrow \quad a^2 = 16$

So that $|a + b|^2 = (7/4)^2 a^2 = 49 \quad |a + b| = 7$

Example 49: Equations to the sides of a triangle are $x - 3y = 0$, $4x + 3y = 5$ and $3x + y = 0$. The line $3x - 4y = 0$ passes through the

(a) incentre
(b) centroid
(c) circumcentre
(d) orthocentre of the triangle

Ans. (d)

Solution: Sides $x - 3y = 0$ and $3x + y = 0$ of the triangle being perpendicular to each other, the triangle is right angled at the origin, the point of intersection of these sides. So that origin is the orthocentre of the triangle and the line $3x - 4y = 0$ passes through this orthocentre.

Example 50: Locus of the mid-points of the intercepts between the coordinate axes by the lines passing through $(a, 0)$ does not intersect

(a) x-axis (b) y-axis
(c) $y = x$ (d) $y = a$

Ans. (b)

Solution: Equation of any line through $(a, 0)$ be $\dfrac{x}{a} + \dfrac{y}{b} = 1$, where b is a parameter. This line meets y-axis at $(0, b)$ and if (h, k) denotes the mid-point of the intercept of the line between the coordinate axes, then $h = a/2$, $k = b/2$ and then the locus of (h, k) is $x = a/2$. This clearly does not intersect y-axis.

Example 51: If h denotes the arithmetic mean and k denotes the geometric mean of the intercepts made on the coordinate axes by the lines passing through the point (1, 1), then the point (h, k) lies on

(a) a circle (b) a parabola
(c) a straight line (d) a hyperbola

Ans. (b)

Solution: Let the equation of the line be $\dfrac{x}{a} + \dfrac{y}{b} = 1$ If it passes through (1, 1), then $\dfrac{1}{a} + \dfrac{1}{b} = 1$

$\Rightarrow \quad a + b = ab \quad$ (i)

Since the line makes intercepts of lengths a and b on the coordinate axes, we have

$$h = \frac{a+b}{2} \quad \text{and} \quad k = \sqrt{ab}$$

so that from (i) we get $2h = k^2$ and thus the point (h, k) lies on $2x = y^2$ which is a parabola.

Example 52: If a, b, c are in A.P., a, x, b are in G.P. and b, y, c are in G.P., the point (x, y) lies on

(a) a straight line (b) a circle
(c) an ellipse (d) a hyperbola

Ans. (b)

Solution: We have $2b = a + c$, $x^2 = ab$, $y^2 = bc$ so that $x^2 + y^2 = b(a + c) = 2b^2$ which is a circle.

Example 53: If one of the diagonals of a square is along the line $x = 2y$ and one of its vertices is (3, 0), then its side through this vertex nearer to the origin is given by the equation.

(a) $y - 3x + 9 = 0$ (b) $3y + x - 3 = 0$
(c) $x = 3y - 3 = 0$ (d) $3x + y - 9 = 0$

Ans. (b)

Solution: The point (3, 0) does not lie on the diagonal $x = 2y$. Let the equation of a side through the vertex (3, 0) be

$$y - 0 = m(x - 3)$$

Since the angle between a side and a diagonal of a square is $\pi/4$, we have

$$\pm \tan\frac{\pi}{4} = \frac{m - 1/2}{1 + m(1/2)} = \frac{2m - 1}{2 + m} \qquad \text{(i)}$$

$\Rightarrow$ $m = 3, -1/3$

Thus the equation of a side through (3, 0) is $y = 3(x - 3)$ or $y = \left(-\frac{1}{3}\right)(x - 3)$ and the one nearer to the origin is $3y + x - 3 = 0$

Example 54: A line which is parallel to x-axis and crosses the curve $y = \sqrt{x}$ at angle of 45° is

(a) $x = 1/4$ (b) $y = 1/4$
(c) $y = 1/2$ (d) $y = 1$

Ans. (c)

Solution: Equation of a line parallel to x-axis is $y = k$. Which meets the curve $y = \sqrt{x}$ at the point (k^2, k). Since the line $y = k$ crosses the curve at an angle of 45°, the angle between the tangent to the curve $y = \sqrt{x}$ at (k^2, k) and the line $y = k$ is 45°. The line $y = k$ being parallel to x-axis. We have

$$\left[\frac{dy}{dx}\right]_{(k^2, k)} = \pm 1 \text{ on the curve } y = \sqrt{x}.$$

$$\Rightarrow \quad \left[\frac{1}{2\sqrt{x}}\right]_{(k^2, k)} = \pm 1 \quad \Rightarrow \quad k = \frac{1}{2}$$

$k > 0$, as $y = \sqrt{x}$ lies in the first quadrant

Example 55: A rectangle has two opposite vertices at the points (1, 2) and (5, 5). If the other vertices lie on the line $x = 3$, the coordinates of the vertex nearer the axis of x are

(a) (3, 1) (b) (3, 2)
(c) (3, 4) (d) (3, 6)

Ans. (a)

Solution: Let the coordinates of the other vertices lying on $x = 3$ be $(3, y_1)$ and $(3, y_2)$.

Since the diagonals of a rectangle are equal and bisect each other, we have

$$\frac{y_1 + y_2}{2} = \frac{5 + 2}{2} \Rightarrow y_1 + y_2 = 7$$

and $$|y_2 - y_1| = \sqrt{(5-1)^2 + (5-2)^2} = \sqrt{16 + 9} = 5$$

If $y_2 > y_1$ then $y_2 = 6$, $y_1 = 1$

So that the coordinates of the vertex nearer the axis of x are (3, 1).

Example 56: A straight line through the origin O meets the parallel lines $4x + 2y = 9$ and $2x + y + 6 = 0$ at points P and Q respectively. The point O divides the segment PQ in the ratio

(a) 1 : 2 (b) 3 : 4
(c) 2 : 1 (d) 4 : 3

Ans. (b)

Solution: It is clear that the lines lie on opposite side of the origin O. Let the equation of any line through O be $\frac{x}{\cos\theta} = \frac{y}{\sin\theta}$. If $OP = r_1$ and $OQ = r_2$ then the coordinates of P are $(r_1 \cos\theta, r_1 \sin\theta)$ and that of Q are $(-r_2 \cos\theta, -r_2 \sin\theta)$

Since P lies on $4x + 2y = 9$, $2r_1(2\cos\theta + \sin\theta) = 9$

and Q lies on $2x + y + 6 = 0$, $-r_2(2\cos\theta + \sin\theta) = -6$

so that $\frac{r_1}{r_2} = \frac{9}{12} = \frac{3}{4}$

and the required ratio is thus 3 : 4.

Alternately Let the equation of the line through O be $y = mx$, then coordinates of P and Q are respectively $\left(\frac{9}{4 + 2m}, \frac{9m}{4 + 2m}\right)$ and $\left(\frac{-6}{2 + m}, \frac{-6m}{2 + m}\right)$ so that

$$\frac{OP}{OQ} = \frac{9}{|4 + 2m|} \times \frac{|2 + m|}{6} = \frac{3}{4}$$

Example 57: Let $P = (-1, 0)$, $Q = (0, 0)$ and $R = (3, 3\sqrt{3})$ be three points. Then the equation of the bisector of the angle PQR is

(a) $\frac{\sqrt{3}}{2} + y = 0$ (b) $x + \sqrt{3}y = 0$
(c) $\sqrt{3}x + y = 0$ (d) $x + \frac{\sqrt{3}}{2}y = 0$

Ans. (c)

Solution Let the equation of QS, the bisector of angle PQR be $y = mx$.

Slope of $QR = \sqrt{3} = \tan 60^\circ$

$\Rightarrow \quad \angle PQR = 120^\circ \Rightarrow \angle PQS = 60^\circ$

$\Rightarrow \quad m = -\tan 60^\circ = -\sqrt{3}$ and thus the required equation of the bisector is $\sqrt{3}x + y = 0$.

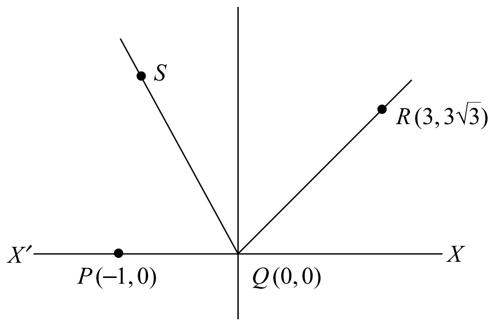


Fig. 16.11

Example 58: The number of integer value of m, for which the x-coordinate of the point of intersection of the lines $3x + 4y = 9$ and $y = mx + 1$ is also an integer is

(a) 2 (b) 0
(c) 4 (d) 1

Ans. (a)

Solution: x-coordinate of the point of intersection is given by

$$3x + 4(mx + 1) = 9$$

$$\Rightarrow \quad x = \frac{5}{3 + 4m}$$

As x is an integer, $3 + 4m$ must be a divisor of 5

so $\quad 3 + 4m = \pm 1$ or ± 5

$\Rightarrow \quad m = -1$ or -2

(considering the integer value only)

Example 59: If the line $2x + y = k$ passes through the point which divides the line segment joining the points (1,1) and (2,4) in the ratio 3:2, then k equals

(a) 6 (b) 11/5
(c) 29/5 (d) 5

Ans. (a)

Solution: Let A (1,1) and B(2,4)

If P (x,y) divides the line segment AB in the ratio 3:2, then

$$x = \frac{3(2) + 2(1)}{5}, \; y = \frac{3(2) + 2(1)}{5}$$

$$P\,(8/5,\ 14/5)$$

As the line $2x + y = k$ passes through P

$2 \times (8/5) + (14/5) = k \Rightarrow k = 6$

Example 60: A line is drawn through the point (1, 2) to meet the coordinate axes at P and Q such that it forms a triangle OPQ, where O in the origin. If the area of the triangle OPQ is least, then the slope of the line PQ is

(a) -2 (b) $-1/2$
(c) $-1/4$ (d) -4

Ans. (a)

Solution: Equation of the line be $y = mx + c$.

Since it passes through (1, 2), $2 = m + c$.

P $(0, c)$, Q $(-c/m, 0)$. If A is the area of the triangle, then

$$A = \frac{1}{2}\ |c|\left|\frac{c}{m}\right| = \frac{c^2}{2m} = \frac{(2-m)^2}{2m} = \frac{m^2 - 4m + 4}{2m}$$

$$= \frac{m}{2} - 2 + \frac{2}{m}$$

$$\frac{dA}{dm} = 0 \Rightarrow \frac{1}{2} - \frac{2}{m^2} = \Rightarrow m^2 = 4 \Rightarrow m = \pm 2$$

Example 61: Let PQR be a right angled isosceles triangle right angled at P (2, 1). If the equation of the line QR is $2x + y = 3$, then the equations representing the pair of lines PQ and PR is

(a) $x + 3y = 5, \quad 3x - y = 5$
(b) $x + y = 3, \quad x - y = 1$
(c) $2x + 3y = 7, \quad 3x - 2y = 4$
(d) $3x + y = 7, \quad x - 3y = -1$

Ans. (a)

Solution: Let the slopes of PQ and PR be m and $-1/m$ respectively. Since PQR is an isosceles triangle $\angle PQR = \angle PRQ$

$$\Rightarrow \quad \left|\frac{m+2}{1-2m}\right| = \left|\frac{-\frac{1}{m}+2}{1+\frac{2}{m}}\right|$$

[$\because$ slope of $QR = -2$]

$\Rightarrow m + 2 = \pm(1 - 2m) \Rightarrow m = 3$ or $-1/3$

So the equations of PQ and PR are

$(y - 1) = 3(x - 2)$ and $y - 1 = (-1/3)(x - 2)$

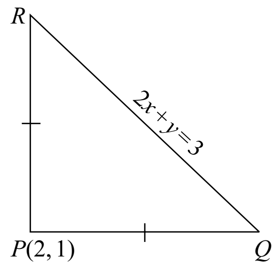


Fig. 16.12

$\Rightarrow \quad 3x - y = 5,\ x + 3y = 5$

Example 62: The orthocentre of the triangle formed by the lines xy = 0 and 2x + 3y – 5 = 0 is

(a) (2, 3) (b) (3, 2)
(c) (0, 0) (d) (5, – 5)

Ans. (c)

Solution: The given triangle is right angled at (0, 0) which is therefore the orthocentre of the triangle.

Example 63: The equations of the sides of a parallelogram are $x = 2$, $x = 3$ and $y = 1$, $y = 5$. Equations to the pair of diagonals are

(a) $4x + y = 7$, $4x - y = 13$
(b) $x + 4y = 13$, $x - 4y = 7$
(c) $x + 4y = 7$, $x - 4y = 13$
(d) $4x + y = 13$, $4x - y = 7$

Ans. (d)

Solution: The vertices of the parallelogram are

$$A(2, 1), B(2, 5), C(3, 5) \text{ and } D(3, 1)$$

Equation of the diagonal AC is

$$y - 1 = \frac{5-1}{3-2}(x - 2) \Rightarrow 4x - y = 7$$

Equation of the diagonal BD is

$$y - 5 = \frac{1-5}{3-2}(x - 2) \Rightarrow 4x + y = 13$$

Example 64: The lines joining the origin to the points of intersection of $3x^2 + \lambda xy - 4x + 1 = 0$ and $2x + y - 1 = 0$ are at right angles for

(a) $\lambda = -4$ (b) $\lambda = 4$
(c) $\lambda = 7$ (d) all values of λ

Ans. (d)

Solution: Equation of the lines joining the origin to the points of intersection of the given lines is

$$3x^2 + \lambda xy - 4x(2x + y) + 1\cdot(2x + y)^2 = 0$$

(Making the equation of the pair of lines homogeneous with the help of the equation of the line)

$\Rightarrow \quad x^2 - \lambda xy - y^2 = 0$

which are perpendicular for all values of λ as the product of the slopes is – 1.

Example 65: The lines $p(p^2 + 1)x - y + q = 0$ and $(p^2 + 1)^2x + (p^2 + 1)y + 2q = 0$ are perpendicular to a common line for

(a) exactly two values of p
(b) more than two values of p
(c) no value of p
(d) exactly one value of p

Ans. (d)

Solution: Two lines will be perpendicular to a common line if these two are parallel.

$$\therefore \quad p(p^2 + 1) = -\frac{(p^2+1)^2}{p^2+1}$$

$\Rightarrow \quad (p + 1)(p^2 + 1) = 0 \Rightarrow p = -1$

Example 66: The line L is given by $\frac{x}{5} + \frac{y}{b} = 1$ passes through the point (13, 32). The line K is parallel to L and has the equation $\frac{x}{c} + \frac{y}{3} = 1$. Then the distance between L and K is

(a) $17/\sqrt{15}$ (b) $23/\sqrt{17}$
(c) $23/\sqrt{15}$ (d) $\sqrt{15}$

Ans. (b)

Solution: As $L : \frac{x}{5} + \frac{y}{b} = 1$ passes through (13, 32), we get

$$\frac{13}{5} + \frac{32}{b} = 1 \Rightarrow b = -20$$

As $K : \frac{x}{c} + \frac{y}{3} = 1$ is parallel to L

$$-\frac{b}{5} = -\frac{3}{c} \Rightarrow c = -\frac{3}{4}.$$

Now equation of L and K are respectively $4x - y - 20 = 0$ and $4x - y + 3 = 0$ and the distance between them is

$$\frac{|-20-3|}{\sqrt{16+1}} = \frac{23}{\sqrt{17}}.$$

Example 67: Consider three points $P(\cos\alpha, \sin\beta)$, $Q(\sin\alpha, \cos\beta)$ and $R(0, 0)$. Where $0 < \alpha, \beta < \pi/4$, Then

(a) P lies on the line segment RQ
(b) Q lies on the line segment PR
(c) R lies on the line segment QP
(d) P, Q, R are non-collinear.

Ans. (d)

Solution: Consider $\Delta = \begin{vmatrix} \cos\alpha & \sin\beta & 1 \\ \sin\alpha & \cos\beta & 1 \\ 0 & 0 & 1 \end{vmatrix}$

$$= \cos\alpha\cos\beta - \sin\alpha\sin\beta$$
$$= \cos(\alpha + \beta) \neq 0 \text{ as } 0 < \alpha + \beta < \pi/2$$

Hence the given points are non-collinear.

Example 68: Consider the lines given by $L_1 : x + 3y - 5 = 0$, $L_2 : 3x - ky - 1 = 0$ $L_3 : 5x + 2y - 12 = 0$. If one of L_1, L_2, L_3 is parallel to at least one of the other two, then k satisfies

(a) $k^2 + 4k - 45 = 0$
(b) $5k^2 + 51k + 45 = 0$
(c) $5k^2 - 19k - 6 = 0$
(d) $5k^2 + 51k + 54 = 0$

Ans. (*d*)

Solution: L_1 and L_3 are not parallel.

L_2 is parallel to L_1 if $\dfrac{-1}{3} = \dfrac{3}{k} \Rightarrow k = -9$

L_2 is parallel to L_3 if $\dfrac{-5}{2} = \dfrac{3}{k} \Rightarrow k = \dfrac{-6}{5}$.

So the required values of k are given by $(k + 9)(k + 6/5) = 0$

or $\qquad 5k^2 + 51k + 54 = 0$

Example 69: Consider the lines

$L_1 : x + y = 10, \quad L_2 : x + y = 60$

$L_3 : x = 40, \qquad L_4 : y = 40.$

L_1 meets x-axis and y-axis at A and B respectively.

L_4 meets y-axis at C and L_2 at D

L_3 meets L_2 at E and x-axis at F.

Perimeter of the hexagon $ABCDEF$ is

(a) $100 + 30\sqrt{2}$ (b) $50 + 40\sqrt{2}$

(c) 100 (d) none of these

Ans. (*a*)

Solution:

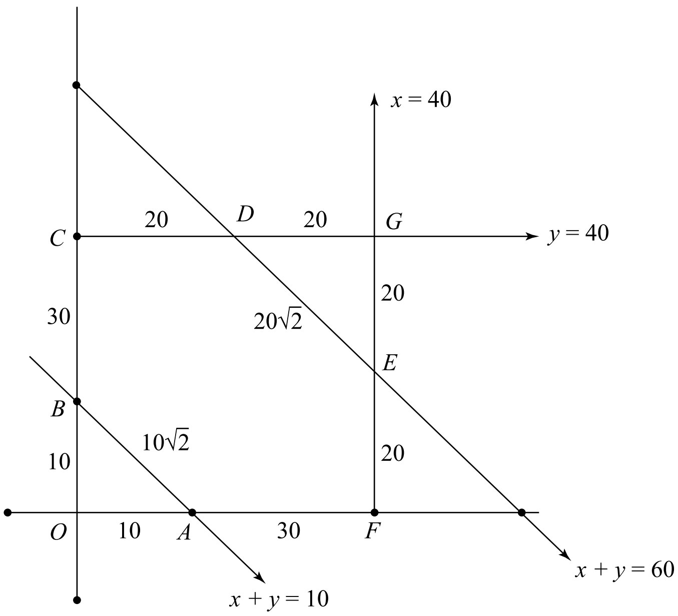


Fig. 16.13

From the figure it is clear that $OF = 40$, $OA = 10 \Rightarrow AF = 30$

Similarly $BC = 30$.

Coordinates of D are (20, 40)

so $\qquad CD = DG = 20$

Similarly $\quad GE = EF = 20$

$$AB = \sqrt{10^2 + 10^2} = 10\sqrt{2}$$

$$ED = \sqrt{(20)^2 + (20)^2} = 20\sqrt{2}$$

Hence the required perimeter is $AB + BC + CD + DE + EF + FA$

$$= 10\sqrt{2} + 30 + 20 + 20\sqrt{2} + 20 + 30$$

$$= 100 + 30\sqrt{2}$$

Example 70: If a line through $P(a, 2)$ making an angle $\pi/4$ with the positive direction of x-axis, meets the curve $4x^2 + 9y^2 = 36$ at A and D and meets the axes at B and C, so that PA, PB, PC, PD are in G.P, then a is equal to

(a) $\dfrac{2}{13}$ (b) $\dfrac{13}{2}$

(c) $\dfrac{13}{5}$ (d) $\dfrac{5}{13}$

Ans. (*b*)

Solution: Equation of any line through $P(a, 2)$ making an angle $\theta = \pi/4$ with the +ve direction of x-axis is $\dfrac{x-a}{1/\sqrt{2}} = \dfrac{y-2}{1/\sqrt{2}}$. Any point on this line at a distance r from P is $(a + r/\sqrt{2},\ 2 + r/\sqrt{2})$. If it lies on the curve, then $4\left(a + r/\sqrt{2}\right)^2 + 9\left(2 + r/\sqrt{2}\right)^2 = 36$

This gives two values of r representing PA and PD

So that $\qquad PA \cdot PD = \dfrac{4a^2 \times 2}{4+9} = \dfrac{8a^2}{13}$.

Since it meets the axes at B and C

$$PB = -a\sqrt{2},\ PC = -2\sqrt{2}$$

So $\qquad PB \cdot PC = 4a$

As PA, PB, PC, PD are in G.P

$$PA \cdot PD = PB \cdot BC$$

$$\Rightarrow \qquad \frac{8a^2}{13} = 4a \Rightarrow a = \frac{13}{2}.$$

Example 71: The quadrilateral formed by the lines $y = ax + c$, $y = ax + d$ and $y = bx + c$ and $y = bx + d$ has area 18. The quadrilateral formed by the lines $y = ax + c$, $y = ax - d$, $y = bx + c$ and $y = bx - d$ has area 72. If a, b, c, d are positive integers then the minimum value of the sum $a + b + c + d$ is

(a) 13 (b) 14

(c) 15 (d) 16.

Ans. (d)

Solution: Coordinates of A are $(0, c)$ and of C are $(0, d)$

So Length of $AC = |c - d|$

Length of the perpendicular from B on AC ($x = 0$) is equal to the x-coordinate $\dfrac{c-d}{a-b}$ of B. Thus area of this quadrilateral is equal to $\dfrac{(c-d)^2}{|a-b|} = 18$

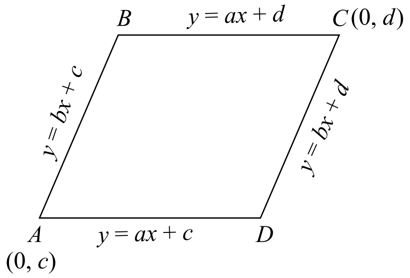


Fig. 16.14

Similarly area of the other quadrilateral is $\frac{(c+d)^2}{|a-b|} = 72$

As a, b, c, d, are positive integers minimum value of $|a - b| = 2$

$$\frac{c+d}{c-d} = \pm 2$$

$\Rightarrow$ $c = 3d$ or $d = 3c$.

We can take $a = 3$ or $b = 1$ or $a = 1$, $b = 3$

and $c = 3$, $d = 9$ or $d = 3$, $c = 9$.

In all the cases

$$a + b + c + d = 3 + 1 + 3 + 9 = 16.$$

Example 72: A straight line L through the point $(3, -2)$ is inclined at an angle 60° to the line $\sqrt{3}x + y = 1$. If L also intersects the x-axis, then the equation of L is

(a) $y + \sqrt{3}x + 2 - 3\sqrt{3} = 0$
(b) $y - \sqrt{3}x + 2 + 3\sqrt{3} = 0$
(c) $\sqrt{3}y - x + 3 + 2\sqrt{3} = 0$
(d) $\sqrt{3}y + x - 3 + 2\sqrt{3} = 0$

Ans. (b)

Solution: Let the slope of L be m

then $\left|\frac{m+\sqrt{3}}{1-\sqrt{3}m}\right| = \tan 60° = \sqrt{3}$

$\Rightarrow$ $m + \sqrt{3} = \sqrt{3}(1 - \sqrt{3}m)$

or $m + \sqrt{3} = -\sqrt{3}(1 - \sqrt{3}m)$

$\Rightarrow$ $m = 0$ *or* $m = \sqrt{3}$

but $m \neq 0$ as L intersects x- axis

Hence equation of L is

$$y + 2 = \sqrt{3}(x - 3)$$

or $y - \sqrt{3}x + 2 + 3\sqrt{3} = 0$

Example 73: Given four lines with equations $x + 2y - 3 = 0$, $3x + 4y - 7 = 0$, $2x + 3y - 4 = 0$ $4x + 5y - 6 = 0$

(a) they are all concurrent
(b) they are sides of a qudrilateral
(c) only three lines are concurrent
(d) none of these

Ans. (c)

Solution: $x + 2y - 3 = 0$, $2x + 3y - 4 = 0$ intersect at $(-1, 2)$. $4x + 5y - 6 = 0$ passes through this point but $3x + 4y - 7 = 0$ does not pass through it, so only three of them are concurrent

Example 74: The point (4, 1) undergoes the following three transformations successively.

(i) reflection about the line $y = x$
(ii) transformation through a distance 2 units along the positive direction of x- axis
(iii) rotation through an angle $\pi/4$ about the origin in the counter clockwise direction. Then the final position of the point is given by the coordinates

(a) $\left(\frac{1}{\sqrt{2}}, \frac{7}{\sqrt{2}}\right)$ (b) $(-\sqrt{2}, 7\sqrt{2})$

(c) $\left(-\frac{1}{\sqrt{2}}, \frac{7}{\sqrt{2}}\right)$ (d) $(\sqrt{2}, 7\sqrt{2})$

Ans. (c)

Solution: Reflection of the point (4, 1) about the line $y = x$ is (1,4) which is transfered to the point (3,4) at a distance 2 units along the positive direction of x- axis.

Finally a rotation through an angle $\pi/4$ takes it to

$$(3\cos(\pi/4) - 4\sin(\pi/4)), (3\sin(\pi/4) + 4\cos(\pi/4))$$

$$= \left(\frac{-1}{\sqrt{2}}, \frac{7}{\sqrt{2}}\right)$$

Example 75: If the sum of the distances of a point from two perpendicular lines is 1 then its locus is

(a) square
(b) circle
(c) straight line
(d) two intersecting lines.

Ans. (a)

Solution: Let us take the two perpendicular line as the axes of coordinates. If (h, k) is any point of the locus, then $|h| + |k| = 1$

Locus is $|x| + |y| = 1$

This consists of four line segment

$$x + y = \pm 1, x - y = \pm 1$$

which form a square

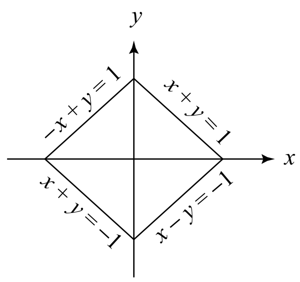


Fig. 16.15

Example 76: The diagonals of a parallelgram $PQRS$ are along the lines $x + 3y = 4$ and $6x – 2y = 7$. Then $PQRS$ must be a

(a) rectangle (b) square
(c) cyclic quadrilateral (d) rhombus

Ans.(d)

Solution: Since the diagonals of the parallelogram are at right angles, it is a rhombus.

Example 77: Let $A_0A_1A_2A_3A_4A_5$ be a regular hexagon inscribed in a circle of unit radius then the product of the lengths of the line segments A_0A_1, A_0A_2 and A_0A_4 is

(a) 3/4 (b) $3\sqrt{3}$
(c) 3 (d) $3\sqrt{3}/2$

Ans. (c)

Solution: The vertices of the hexagon be as shown in the figure

$$(A_0A_1)^2 = \frac{1}{4}+\frac{3}{4}=1$$

$$(A_0A_2)^2 = \frac{9}{4}+\frac{3}{4}=3$$

$$(A_0A_4)^2 = (A_0A_2)^2 = 3$$

So $(A_0A_1)(A_0A_2)(A_0A_4) = \sqrt{1\times3\times3} = 3$

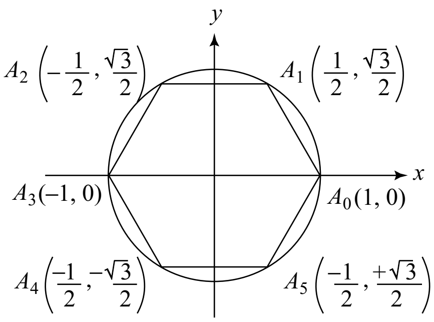


Fig. 16.16

Example 78: The incentre of the triangle with vertices $(1, \sqrt{3})$ (0,0) and (2,0) is

(a) $(1,\sqrt{3}/2)$ (b) $(2/3, 1/\sqrt{3})$
(c) $(2/3,\sqrt{3}/2)$ (d) $(1, 1/\sqrt{3})$

Ans. (d)

Solution: Length of each side of the triangle is 2 so it is an equilaltral triangle and hence it incentre is the centroid $(1, 1/\sqrt{3})$ of the triangle.

Example 79: A ray of light travels along the line $2x – 3y + 5 = 0$ and strikes a plane mirror lying along the line $x + y = 2$. The equation of the straight line containing the reflected ray is

(a) $2x – 3y + 3 = 0$ (b) $3x – 2y + 3 = 0$
(c) $21x – 7y + 1 = 0$ (d) $21x + 7y – 1= 0$

Ans. (b)

Solution: Let θ be the angle which the ray $2x – 3y + 5 = 0$ makes with the line $x + y = 2$ then $\tan\theta = \left|\frac{2/3+1}{1-2/3\,(1)}\right|$ $= |5| = 5$

Reffected ray makes angle $-\theta$ with $x + y = 2$, so if the slope of the reflected ray is m then $\frac{m+1}{1-m} = \tan(-\theta) = -5$

$\Rightarrow$ $m = 3/2$.

Also, the reflected ray passes through (1/5, 9/5), the point of intersection of the ray and the line

Hence equation of the reflected ray is

$$(y – 9/5) = 3/2\,(x – 1/5)$$

$\Rightarrow 3x – 2y + 3 = 0$

Example 80: Let $0 < \alpha < \pi/2$ be a fixed angle. If $P = (\cos\theta, \sin\theta)$ and $Q = (\cos(\alpha – \theta), \sin(\alpha – \theta))$

Then Q is obtained from P by

(a) clockwise rotation around origin through an angle α
(b) anti clockwise rotation around origin through an angle α
(c) reflection in the line through the origin with slope $\tan\alpha$
(d) reflection in the line through the origin with slope $\tan\alpha/2$

Ans. (d)

Solution: Clockwise rotation of P through an angle α takes it to the point

$(\cos(\theta – \alpha), \sin(\theta – \alpha))$ and anticlockwise takes it to $(\cos(\alpha + \theta), \sin(\alpha + \theta))$

Now slope of PQ

$$= \frac{\sin\theta-\sin(\alpha-\theta)}{\cos\theta-\cos(\alpha-\theta)}$$

$$= \frac{2\cos(\alpha/2)\sin(\theta-\alpha/2)}{-2\sin(\alpha/2)\sin(\theta-\alpha/2)}$$

$$= -\cot(\alpha/2)$$

$\Rightarrow PQ$ is perpendicular to the line with slope $\tan(\alpha/2)$

Hence Q is the reflection of P in the line through the origin with slope $\tan(\alpha/2)$.

Assertion-Reason Type Questions

Example 81: Consider the lines

$$L_1 : p^2x + py - 1 = 0$$
$$L_2 : q^2x + qy + 6 = 0$$

L_1 passes through the point (3, 2) and L_2 passes through the point (2, 7)

Statement-1: If the product of the slopes of L_1 and L_2 is 2, then they intersect at the point (– 4, – 5)

Statement-2: L_1 and L_2 are neither parallel nor perpendi cular.

Ans. (b)

Solution: we have $3p^2 + 2p - 1 = 0$ and

$$2q^2 + 7q + 6 = 0$$

$$\Rightarrow \quad p = -1 \text{ or } \frac{1}{3} \text{ and } q = -2 \text{ or } \frac{-3}{2}$$

Slope of $L_1 = -p$, slope of $L_2 = -q$, product of the slopes = $pq \neq -1$ for any values of p and q. So L_1 and L_2 are not perpendicular. Also as $p \neq q$ for any value, they are not parallel.

Thus the statement-2 is true. Next, if the product of the slope is 2 then $p = -1$, $q = -2$ and the equations of L_1 and L_2 are respectively

$$x - y - 1 = 0 \text{ and } 4x - 2y + 6 = 0$$

which intersect at (– 4, – 5) so the statement-1 is also true but does not follow from statement-1.

Example 82: **Statement-1:** Consider the point $A(0, 1)$ and $B(2, 0)$ and P be a point on the line $4x + 3y + 9 = 0$, then the coordinates of P such that $|PA - PB|$ is maximum is (– 24/5, 17/5)

Statement-2: If A and B are two fixed points and P is any point in a plane, then $|PA - PB| \leq AB$.

Ans. (a)

Solution: From geometry $|PA - PB| < AB$ and $|PA - PB| = AB$ if P lies on extended line segment AB so statement-2 is true. Using in statement-1, $|PA - PB|$ is maximum if P lies on AB whose equation is $x + 2y = 2$

The given line is $4x + 3y + 9 = 0$, on solving we get the required coordinates of P as (– 24/5, 17/5) and hence statement-1 is also true.

Example 83: **Statement-1:** The points $A(3, 4)$, $B(2, 7)$, $C(4, 4)$ and $D(3, 5)$ are such that one of them lies inside the triangle formed by the other points.

Statement-2: Centroid of a triangle lies inside the triangle.

Ans. (a)

Solution: $\frac{3+2+4}{3} = 3, \ \frac{4+7+4}{3} = 5$

$\Rightarrow$ $D(3, 5)$ is the centroid of the triangle ABC.

Using statement-2 which is true, statement-1 is also true.

Example 84: **Statement-1:** If the circumcentre of a triangle lies at the origin and centroid is the mid point of the line joining the points (2, 3) and (4, 7), then its orthocentre lies on the line $5x - 3y = 0$

Statement-2: Circumcentre, centroid and orthocentre of a triangle lie on the same line

Ans. (a)

Solution: From geometry, statement-2 is True. Using it in statement-1, orthocentre lies on the line joining (0, 0) and $\left(\frac{4+2}{2}, \frac{7+3}{2}\right)$ i.e. (3, 5) which is $5x - 3y = 0$ and so the statement-1 is also true.

Example 85: **Statement-1:** The points $A(-2, 2)$, $B(2, -2)$ and $C(1, 1)$ are the vertices of an obtuse angled isosceles triangle.

Statement-2: Every obtuse angled triangle is isosceles

Ans. (c)

Solution: Equation of AB is $y + x = 0$ in statement-1 and $y = x$ is the perpendicular bisector of AB. $C(1, 1)$ lies on it so the triangle ABC is isosceles, $AC = BC$ slope of $AC = -1/3$, slope of $AB = -1$

$$\tan A = \frac{-1/3 + 1}{1 + 1/3} = \frac{1}{2} \quad \Rightarrow \quad A < \pi/4.$$

(A cannot be obtuse as $A = B$)

$\Rightarrow A + B < \pi/2 \quad \Rightarrow \quad C > \pi/2$ and the triangle is obtuse angled. So statement-1 is True.

Statement-2 is false as a triangle with angles 120°, 40°, 20° is obtuse angled but not isosceles.

Example 86: **Statement-1:** The lines

$(a + b)x + 2(a - b)y = 2a$ are concurrent at the point (1, 1/2)

Statement-2: The lines $x + 2y - 2 = 0$ and $x - 2y = 0$ intersect at the point(1, 1/2)

Ans. (a)

Solution: Statement-2 is true. Lines in statement-1 can be written as $a(x + 2y - 2) + b(x - 2y) = 0$ which passes through the point of intersection(1, 1/2) of the lines $x + 2y - 2 = 0$, $x - 2y = 0$, follows from statement-1

Example 87: **Statement-1:** Lines $L_1 : y - x = 0$ and $L_2 : 2x + y = 0$ intersect the line $L_3 : y + 2 = 0$ at P and Q respectively. The bisector of the acute angle between L_1 and L_2 intersect L_3 at R.

Statement-1: The ratio $PR : RQ$ equals $2\sqrt{2} : \sqrt{5}$.

Statement-2: The bisector of an angle of a triangle divides the triangle into two similar triangles.

Ans. (c)

◎ **Solution:** Vertices of the triangle in statement-1 are $O(0, 0)$, $P(-2, -2)$, $Q(1, -2)$. Since the bisector of an angle of a triangle divides the opposite side. In the ratio of the side containing the angle.

$PR : RQ = OP : OQ = 2\sqrt{2} : \sqrt{5}$. So statement-1 is true

Statement-2 is false, because for example if $A = 50°$, $B = 60°$, $C = 70°$ then bisector of angle B does not divide the triangle into two equiangular and hence similar triangles.

Example 88: Statement-1: If the perpendicular bisector of the line segment joining $P(1, 4)$ and $Q(k, 3)$ has y-intercept equal to -4, then $k^2 - 16 = 0$

Statement-2: Centroid of an isoceles triangle ABC lies on the perpendicular bisector of the base BC where $B = C$.

Ans. (b)

◎ **Solution:** Any point $L(x, y)$ on the perpendicular bisector in statement-1 is equidistant from P and Q. Locus of L is $(x-1)^2 + (y-4)^2 = (x-k)^2 + (y-3)^2$

$$\Rightarrow \quad 2(k+1)x - 2y = k^2 - 8$$

$$y\text{-intercept} = -\frac{k^2 - 8}{2} = -4 \quad \Rightarrow \quad k^2 - 16 = 0$$

So statement-1 is true but does not follow from statement-2 which is also true, as the perpendicular bisector of BC is also the median through A.

Example 89: Statement-1: Circumcentre of the triangle formed by the lines $x + y = 0$, $x - y = 0$ and $x - 7 = 0$ is $(7, 0)$

Statement-2: Circumcentre of a triangle lies inside the triangle.

Ans. (c)

◎ **Solution:** In statement-1, the triangle is right angled with hypotenuse $x - 7 = 0$ and the vertices of the hypotenuse are $(7, 7)$ and $(7, -7)$, circumcentre is the mid-point $(7, 0)$ of the hypotenuse. So statement-1 is True. Statement-2 is false as the circumcentre of an obtuse angled triangle lies outside the triangle and that of the right angled is on the hypotenuse.

Example 90: Statement-1: Equation of the pair of lines through the origin perpendicular to the pair of lines $7x^2 - 55xy - 8y^2 = 0$ is $8x^2 - 55xy - 7y^2 = 0$

Statement-2: Equation of the pair of lines through the origin perpendicular to the pair to lines $ax^2 + 2hxy + by^2 = 0$ is $bx^2 - 2hxy + ay^2 = 0$

Ans. (a)

◎ **Solution:** Statement-2 is True because if m_1 and m_2 are the slopes of $ax^2 + 2hxy + by^2 = 0$, then $m_1 + m_2 = -\frac{2h}{b}$, $m_1 m_2 = \frac{a}{b}$.

$\Rightarrow \quad \frac{1}{m_1} + \frac{1}{m_2} = -\frac{2h}{a}$ and $\frac{1}{m_1 m_2} = \frac{b}{a}$. Equation of the lines with slope $-\frac{1}{m_1}$ and $-\frac{1}{m_2}$ is $y^2 + \left(\frac{1}{m_1} + \frac{1}{m_2}\right)xy + \frac{1}{m_1 m_2}x^2 = 0$

$$\Rightarrow y^2 - \frac{2h}{a}xy + \frac{b}{a}x^2 = 0 \quad \Rightarrow \quad bx^2 - 2hxy + ay^2 = 0$$

Using it, statement-1 is also true.

Example 91: Statement-1: The area enclosed with in the curve $|x| + |y| = 1$ is 2 square units.

Statement-2: The curve $|x| + |y| = 1$ is a square of each side $\sqrt{2}$ units

Ans. (a)

◎ **Solution:** The statement-2 is true as it represents the sides $x + y = \pm 1$ and $x - y = \pm 1$, of a square of each side $\sqrt{2}$ units which shows that the statement-1 is also true

Example 92:

Statement-1 If $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \begin{vmatrix} a_1 & b_1 & 1 \\ a_2 & b_2 & 1 \\ a_3 & b_3 & 1 \end{vmatrix}$

then the two triangle with vertices (x_1,y_1), (x_2,y_2), (x_3,y_3) and (a_1,b_1), (a_2,b_2), (a_3,b_3) must be congruent

Statement-2 Two congruent triangles have the same area

Ans. (d)

◎ **Solution:** Statement-2 is true, but two triangles having the same area may not be congruent for example, the triangles with vertices $(0, 0)$, $(4, 0)$, $(4, 2)$ and $(0, 0)$, $(2, 0)$ and $(1, 4)$ have same area but are not congruent and thus statement-1 is false.

Example 93: Let θ_1 be the angle between two lines $2x + 3y + c_1 = 0$ and $-x + 5y + c_2 = 0$ and θ_2 be the angle between two lines $2x + 3y + c_1 = 0$ and $-x + 5y + c_3 = 0$ where c_1, c_2, c_3 are any real number.

Statement 1: If c_1 and c_2 are proportional then $\theta_1 = \theta_2$

Statement 2: $\theta_1 = \theta_2$ for all c_2 and c_3.

Ans. (a)

◎ **Solution:** Lines $-x + 5y + c_2 = 0$ and $-x + 5y + c_3 = 0$ are parallel for all values of c_2 and c_3 so both will make the same angle with the line $2x + 3y + c_1 = 0$ and $\theta_1 = \theta_2$ for all c_2 and c_3 showing that statement-2 is true and hence statement-1 is also true.

Example 94: $L_1 : x - y + 1 + \lambda_1 (2x - y - 2) = 0$ and L_2 : $5x + 3y - 2 + \lambda_2 (3x - y - 4) = 0$ where λ_1, λ_2 are arbitrary numbers.

Statement 1: $5x - 2y - 7 = 0$ is the equation of a line belonging to both the family of lines L_1 and L_2.

Statement 2: There is only one line belonging to both the family of lines L_1 and L_2

Ans. (a)

Solution: L_1 represent the lines passing through the point (3, 4), the point of intersection of $x - y + 1 = 0$ and $2x - y - 2 = 0$ and L_2 represents the lines passing through the point (1, – 1), the point of intersection of the lines $5x + 3y - 2 = 0$ and $3x - y - 4 = 0$. So the line belonging to both the families will be the only one line joining (3, 4) and (1, – 1) which is

$$y + 1 = \frac{4+1}{3-1}(x - 1)$$

or $\qquad 5x - 2y - 7 = 0.$

So statement-2 is true and is a correct explanation for statement-1 to be true.

Example 95: Statement-1: The line $2x + y + 6 = 0$ is perpendicular to the line $x - 2y + 5 = 0$ and second line passes through (1, 3)

Statement-2: Product of the slopes of any two parallel lines is equal to – 1

Ans. (c)

Solution: Statement-2 is false as the product of the slopes of two perpendicular line is – 1. Product of the slopes of the lines in statement-1 is $-2 \times \frac{1}{2} = -1$ so they are perpendicular and the second line passes through (1, 3) so the statement-1 is true.

LEVEL 2

Straight Objective Type Questions

Example 96: If a, b, c are unequal and different from 1 such that the points $\left(\frac{a^3}{a-1}, \frac{a^2-3}{a-1}\right), \left(\frac{b^3}{b-1}, \frac{b^2-3}{b-1}\right)$ and $\left(\frac{c^3}{c-1}, \frac{c^2-3}{c-1}\right)$, are collinear, then

(a) $bc + ca + ab + abc = 0$
(b) $a + b + c = abc$
(c) $bc + ca + ab = abc$
(d) $bc + ca + ab - abc = 3(a + b + c)$

Ans. (d)

Solution: Suppose the given points lie on the line

$$lx + my + n = 0$$

then a, b, c are the roots of the equation.

$$lt^3 + m(t^2 - 3) + n(t - 1) = 0$$

or $\qquad lt^3 + mt^2 + nt - (3m + n) = 0$

$\Rightarrow \qquad a + b + c = -m/l$

$$bc + ca + ab = n/l$$

$$abc = (3m + n)/l$$

Eliminating l, m, n, we get

$$abc = -3(a + b + c) + bc + ca + ab$$

$\Rightarrow \quad bc + ca + ab - abc = 3(a + b + c)$

Example 97: If two vertices of a triangle are (–2, 3) and (5, –1), orthocentre lies at the origin and centroid on the line $x + y = 7$, then the third vertex lies at

(a) (7, 4) (b) (8, 14)
(c) (12, 21) (d) none of these

Ans. (d)

Solution: Let $O(0, 0)$ be the orthocentre; $A(h, k)$ the third vertex; and $B(-2, 3)$ and $C(5, -1)$ the other two vertices. Then the slope of the line through A and O is k/h, while the line through B and C has the slope – 4/7. By the property of the orthocentre, these two lines must be perpendicular, so we have

$$\left(\frac{k}{h}\right)\left(-\frac{4}{7}\right) = -1 \Rightarrow \frac{k}{h} = \frac{7}{4} \qquad \text{(i)}$$

Also $\qquad \frac{5-2+h}{3} + \frac{-1+3+k}{3} = 7$

$\Rightarrow \qquad h + k = 16 \qquad$ (ii)

Which is not satisfied by the points given in (a), (b) or (c).

Example 98: The points A (2, 3); B (3, 5), C (7, 7) and D (4, 7) are such that

(a) $ABCD$ is a parallelogram
(b) A,B,C and D are collinear
(c) D lies inside the triangle ABC
(d) D lies on the boundary of the triangle ABC

Ans. (d)

◎ **Solution:** A, B, C are not collinear

as slope of $BA \neq$ slope of BC slope of AD = slope of BD
$\Rightarrow$ D lies on the side AB of the triangle ABC

Example 99: If $p, x_1, x_2 \ldots x_i, \ldots$ and $q, y_1, y_2 \ldots, y_i, \ldots$ are in A.P., with common difference a and b respectively, then the centre of mean position of the points A_i (x_i, y_i), $i = 1, 2 \ldots n$ lies on the line

(a) $ax - by = aq - bp$
(b) $bx - ay = ap - bq$
(c) $bx - ay = bp - aq$
(d) $ax - by = bq - ap$

$$\left[\textbf{Note.}\text{ Centre of Mean Position } \left(\frac{\Sigma xi}{n}, \frac{\Sigma yi}{n}\right)\right]$$

Ans. (c)

◎ **Solution:** Let the coordinates of the centre of mean position of the points

A_i, $i = 1, 2, \ldots n$ be (x, y), then

$$x = \frac{x_1 + x_2 + \cdots + x_n}{n}, \; y = \frac{y_1 + y_2 + \cdots + y_n}{n}$$

$$\Rightarrow \quad x = \frac{np + a(1 + 2 + \cdots + n)}{n},$$

$$y = \frac{nq + b(1 + 2 + \cdots + n)}{n}$$

$$\Rightarrow \quad x = p + \frac{n(n+1)}{2n}\, a, \; y = q + \frac{n(n+1)}{2n}\, b$$

$$\Rightarrow \quad x = p + \frac{n+1}{2}\, a, \; y = q + \frac{n+1}{2}\, b$$

$$\Rightarrow \quad 2\frac{(x-p)}{a} = 2\frac{(y-q)}{b} \quad \Rightarrow \quad bx - ay = bp - aq.$$

Example 100: If P is a point (x, y) on the line. $y = -3x$ such that P and the point (3, 4) are on the opposite sides of the line $3x - 4y = 8$, then

(a) $x > 8/15$, $y < -8/5$ (b) $x > 8/5$, $y < -8/15$
(c) $x = 8/15$, $y = -8/5$ (d) none of these

Ans. (a)

◎ **Solution:** Let $k = 3x - 4y - 8$

then the value of k at (3, 4) $= 3 \times 3 - 4 \times 4 - 8 = -15 < 0$

$\therefore$ For the point P (x, y) we should have $k > 0$

$\Rightarrow \quad 3x - 4y - 8 > 0$

$\Rightarrow \quad 3x - 4(-3x) - 8 > 0$

[$\because$ P (x, y) lies on $y = -3x$]

$\Rightarrow \quad x > 8/15$

$\Rightarrow \quad y < -8/5.$

Example 101: The area enclosed by $2|x| + 3|y| \leq 6$ is

(a) 3 sq. units (b) 4 sq. units
(c) 12 sq. units (d) 24 sq. units

Ans. (c)

◎ **Solution:** The given inequality is equivalent to the following system of inequalities.

$2x + 3y \leq 6$, when $x \geq 0$, $y \geq 0$
$2x - 3y \leq 6$, when $x \geq 0$, $y \leq 0$
$-2x + 3y \leq 6$, when $x \leq 0$, $y \geq 0$
$-2x - 3y \leq 6$, when $x \leq 0$, $y \leq 0$

which represents a rhombus with sides

$$2x \pm 3y = 6 \text{ and } 2x \pm 3y = -6$$

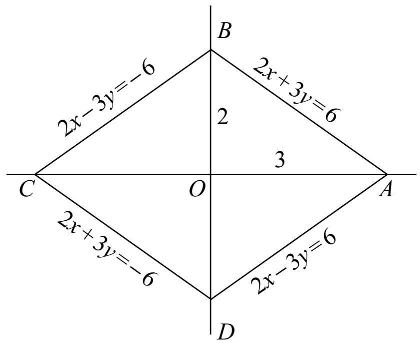


Fig. 16.17

Length of the diagonals is 6 and 4 units along x-axis and y-axis.

$\therefore$ The required area

$$= \frac{1}{2} \times 6 \times 4 = 12 \text{ sq. units.}$$

Example 102: Let O be the origin, A (1, 0) and B (0, 1) and P (x, y) are points such that $xy > 0$ and $x + y < 1$, then

(a) P lies either inside the triangle OAB or in the third quadrant
(b) P cannot lie inside the triangle OAB
(c) P lies inside the triangle OAB
(d) P lies in the first quadrant only

Ans. (a)

◎ **Solution:** Since $xy > 0$, P either lies in the first quadrant or in the third quadrant. The inequality $x + y < 1$ represents all points below the line $x + y = 1$. So that $xy > 0$ and $x + y < 1$ imply that either P lies inside the triangle OAB or in the third quadrant.

Example 103: A line has intercepts a and b on the coordinate axes. When the axes are rotated through an angle α, in the anti clockwise direction keeping the origin fixed, the line makes equal intercepts on the coordinate axes, then $\tan\alpha =$

(a) $\dfrac{a+b}{a-b}$ (b) $\dfrac{a-b}{a+b}$

(c) $a^2 - b^2$ (d) none of these

Ans. (b)

Solution: Let the equation of the line in the original coordinate system be $\dfrac{x}{a}+\dfrac{y}{b}=1$. If (X, Y) denote the coordinates of any point $P(x, y)$ in the new coordinate system obtained by rotation of the axes through an angle α, then

$$x = X\cos\alpha - Y\sin\alpha,\ y = X\sin\alpha + Y\cos\alpha$$

So that the equation of the line with reference to new system of coordinates is

$$\frac{X\cos\alpha - Y\sin\alpha}{a}+\frac{X\sin\alpha + Y\cos\alpha}{b}=1$$

or $$X\left(\frac{\cos\alpha}{a}+\frac{\sin\alpha}{b}\right)+Y\left(\frac{\cos\alpha}{b}-\frac{\sin\alpha}{a}\right)=1$$

Since it makes equal intercepts on the coordinates axes.

$b\cos\alpha + a\sin\alpha = a\cos\alpha - b\sin\alpha$

$\Rightarrow (a-b)\cos\alpha = (a+b)\sin\alpha$

$\Rightarrow \quad \tan\alpha = \dfrac{a-b}{a+b}$

Example 104: The equations to the sides of a triangle are $x - 3y = 0$, $4x + 3y = 5$ and $3x + y = 0$. The line $3x - 4y = 0$ passes through the

(a) incentre
(b) centroid
(c) circumcentre
(d) orthocentre of the triangle

Ans. (d)

Solution: Two sides $x - 3y = 0$ and $3x + y = 0$ of the triangle being perpendicular to each other, the triangle is right angled at the origin, the point of intersection of these sides. So that origin is the orthocentre of the triangle and the line $3x - 4y = 0$ passes through this orthocentre.

Example 105: A line through $P(1, 2)$ is such that it makes unequal intercepts on the axes, and the intercept between the axes is trisected at P, an equation of the line is

(a) $x + y = 3$ (b) $4x + y = 6$

(c) $4x - y = 6$ (d) $3x + 2y = 1$

Ans. (b)

Solution: Let the equation of the line be $\dfrac{x}{a}+\dfrac{y}{b}=1$, which meets x-axis at $A(a, 0)$ and y-axis at $B(0, b)$. Then the coordinates of the points which divide AB in the ratio 1 : 2 and 2 : 1 are respectively $\left(\dfrac{2a}{3},\dfrac{b}{3}\right)$ and $\left(\dfrac{a}{3},\dfrac{2b}{3}\right)$

Now if $\left(\dfrac{2a}{3},\dfrac{b}{3}\right)$ represents $P(1, 2)$ then $a = 3/2$, $b = 6$ and the required equation of the line is $\dfrac{2x}{3}+\dfrac{y}{6}=1 \Rightarrow 4x + y = 6$ which is given in (b)

If $\left(\dfrac{a}{3},\dfrac{2b}{3}\right)$ represents $P(1, 2)$ then $a = b = 3$ and the equation of the line is $x + y = 3$ which makes equal intercepts on the axes and so we do not consider this.

Example 106: The point (4, 1) undergoes the following successive transformations:

(i) reflection about the line $y = x$
(ii) translation through a distance 2 units along the positive x-axis.
then, the final coordinates of the point are

(a) (4, 3) (b) (3, 4)

(c) (1, 4) (d) (4, 4)

Ans. (b)

Solution: Let $Q(x, y)$ be the reflection of $P(4, 1)$ about the line $y = x$, then mid-point of PQ lies on this line and PQ is perpendicular to it. So we have $\dfrac{y+1}{2}=\dfrac{x+4}{2}$ and $\dfrac{y-1}{x-4} = -1$.

$\Rightarrow \quad x - y = -3$ and $x + y = 5$

$\Rightarrow \quad x = 1, y = 4$

Therefore reflection of (4, 1) about $y = x$ is (1, 4). Next, this point is shifted 2 units along the positive x-axis, the new coordinates are $(1 + 2, 4 + 0) = (3, 4)$

Example 107: Two adjacent sides of a parallelogram are $4x + 5y = 0$ and $7x + 2y = 0$. If an equation to one of the diagonals is $11x + 7y - 9 = 0$, then an equation of the other diagonal is

(a) $x + y = 0$ (b) $7x - 11y = 0$

(c) $x - y = 0$ (d) none of these

Ans. (c)

Solution: Since the given lines intersect at the origin O, one of the vertex is $O(0, 0)$. Let A and B be the points of intersection of the sides $4x + 5y = 0$ and $7x + 2y = 0$ respectively with the diagonal. $11x + 7y - 9 = 0$, then the coordinates of A and B are respectively $\left(\dfrac{5}{3},\dfrac{-4}{3}\right)$ and $\left(-\dfrac{2}{3},\dfrac{7}{3}\right)$ the coordinates of the mid point of AB are $\left(\dfrac{1}{2},\dfrac{1}{2}\right)$. Since the other diagonal passes through the vertex (0, 0) and the mid-point $\left(\dfrac{1}{2},\dfrac{1}{2}\right)$ of AB, its equation is $y = x$.

Example 108: If a line joining two points A (2, 0) and B (3, 1) is rotated about A in anticlockwise direction through an angle 15º, then equation of the line in the new position is

(a) $\sqrt{3}\,x + y = 2\sqrt{3}$ (b) $\sqrt{3}\,x - y = 2\sqrt{3}$

(c) $x + \sqrt{3}y = 2\sqrt{3}$ (d) $x - \sqrt{3}y = 2\sqrt{3}$

Ans. (b)

Solution: Slope of $AB = \dfrac{1-0}{3-2} = 1$

$\Rightarrow \quad \angle BAX = 45º$ (Ref. Fig. 16.18)

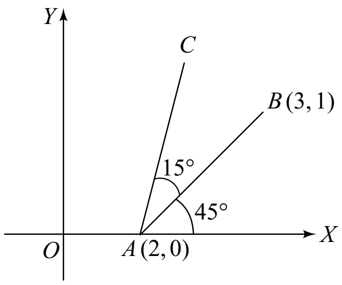


Fig. 16.18

If AC is the new position of the line AB then $\angle CAX = 45° + 15º = 60º$.
and thus its equation is

$$y = \tan 60º\,(x - 2)$$

$$\Rightarrow \quad y = \sqrt{3}\,(x - 2) \quad \Rightarrow \quad \sqrt{3}x - y = 2\sqrt{3}$$

Example 109: If the medians AD and BE of the triangle with vertices $A(0, b)$, B (0, 0) and C $(a, 0)$ are perpendicular, then an equation of the line through (a, b) perpendicular to AC is

(a) $y = \sqrt{2}x + b$ (b) $y = -\sqrt{2}x + b$

(c) $y = \sqrt{2}x - b$ (d) $x = \sqrt{2}y - a$

Ans. (c)

Solution: Slope of $AD = \dfrac{b-0}{0-a/2} = -\dfrac{2b}{a}$

Slope of $BE = \dfrac{b/2}{a/2} = \dfrac{b}{a}$

Since AD and BE are perpendicular

$$\frac{-2b}{a}.\frac{b}{a} = -1$$

$$\Rightarrow \quad a^2 = 2b^2 \qquad \text{(i)}$$

Slope of AC is $- b/a$

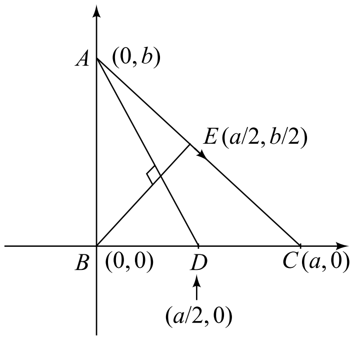


Fig. 16.19

$\therefore$ Equation of the line through (a, b) perpendicular to AC is

$$y - b = \frac{a}{b}\,(x - a)$$

$$\Rightarrow \quad ax - by = a^2 - b^2$$

$$\Rightarrow \pm\sqrt{2}bx - by = b^2 \text{ from (i)}$$

$$\pm\sqrt{2}x - y = b \quad \Rightarrow \quad y = \pm\sqrt{2}x - b$$

So an equation of required line is $y = \sqrt{2}x - b$

Example 110: $A_1, A_2 \ldots A_n$ are points on the line $y = x$ lying in the positive quadrant such that $OA_n = nOA_{n-1}$, O being the origin. If $OA_1 = 1$ and the coordinates of A_n are $(2520\sqrt{2}, 2520\sqrt{2})$, then $n =$

(a) 5 (b) 6
(c) 7 (d) 8

Ans. (c)

Solution: We have

$$OA_n = n.\,OA_{n-1} = n\,(n-1)\,OA_{n-2} \ldots$$
$$= n\,(n-1) \ldots 2(OA_1) = n!\,OA_1 = n!$$

$$\Rightarrow \quad \sqrt{2}\,(2520\sqrt{2}) = n!$$

$$\Rightarrow \quad 2520 \times 2 = n!$$

$$\Rightarrow \quad 7 \times 6 \times 5 \times 4 \times 3 \times 2 = n!$$

$$\Rightarrow \quad n = 7$$

Example 111: If x_1, x_2, x_3 are the abcissa of the points A_1, A_2, A_3 respectively where the lines $y = m_1 x$, $y = m_2 x$, $y = m_3 x$ meet the line $2x - y + 3 = 0$ such that m_1, m_2, m_3 are in A.P., then x_1, x_2, x_3 are in

(a) A.P. (b) G.P.
(c) H.P. (d) none of these

Ans. (c)

Solution: We have $x_1 = \dfrac{3}{m_1 - 2}, x_2 = \dfrac{3}{m_2 - 2}, x_3 = \dfrac{3}{m_3 - 2}$

$$\Rightarrow \quad \frac{1}{x_1} + \frac{1}{x_3} = \frac{m_1 - 2 + m_3 - 2}{3} = \frac{m_1 + m_3 - 4}{3}$$

$$= \frac{2m_2 - 4}{3} = \frac{2}{x_2}$$

(as m_1, m_2, m_3 are in A.P.)

$\Rightarrow$ x_1, x_2, x_3 are in H.P.

Example 112: If the lines $x = k$; $k = 1, 2 \ldots, n$ meet the line $y = 3x + 4$ at the points $A_k(x_k, y_k)$, $k = 1, 2, \ldots, n$ then the ordinate of the centre of Mean position of the points A_k, $k = 1, 2, \ldots, n$ is

(a) $\frac{n+1}{2}$ (b) $\frac{3n+11}{2}$

(c) $\frac{3(n+1)}{2}$ (d) none of these

Ans. (b)

Solution: We have $y_k = 3k + 4$, the ordinate of A_k, the point of intersection of $x = k$ and $y = 3x + 4$. So the ordinate of the centre of Mean position of the points A_k, $k = 1, 2, \ldots n$ is

$$\frac{1}{n}\sum_{k=1}^{n} y_k = \frac{1}{n}\sum_{k=1}^{n} (3k+4) = \frac{3}{n}\sum_{k=1}^{n} k + 4$$

$$= \frac{3n(n+1)}{n \cdot 2} + 4 = \frac{3n+11}{2}.$$

Example 113: If $y = ax$ is one of the lines belonging to the family of lines representing the sides of an equilateral triangle with one vertex at the origin, then the product of the slopes of all the lines of this family is

(a) a^3 (b) $a(a^2 - 3)$

(c) $a(1 - 3a^2)$ (d) $\frac{a(a^2-3)}{1-3a^2}$

Ans. (d)

Solution: Let $a = \tan\theta$, then the slopes of the lines making an equilateral triangle with one vertex at the origin are:

$\tan\theta$, $\tan(\theta + 60°)$, $\tan(\theta + 120°)$, $\tan(\theta + 180°) = \tan\theta$

So the product of the slopes is

$$\tan\theta \tan(\theta + 60°) \tan(\theta + 120°)$$

$$= \tan\theta \tan(\theta + 60°) \tan(\theta - 60°)$$

$$= a \cdot \frac{a+\sqrt{3}}{1-a\sqrt{3}} \cdot \frac{a-\sqrt{3}}{1+a\sqrt{3}} = \frac{a(a^2-3)}{1-3a^2}.$$

Example 114: The sum of the intercepts cut off by the axes on the lines $x + y = a$, $x + y = ar$, $x + y = ar^2$, … where $a \neq 0$ and $r = 1/2$ is

(a) $2a$ (b) $a\sqrt{2}$

(c) $2\sqrt{2}a$ (d) $a/\sqrt{2}$

Ans. (c)

Solution: The intercepts between the axes made by the given lines are $a\sqrt{2}$, $ar\sqrt{2}$, $ar^2\sqrt{2}$, …

So the required sum $= \sqrt{2}\,[a + ar + ar^2 + \ldots]$

$$= \sqrt{2}\frac{a}{1-r} = \frac{\sqrt{2}a}{1-(1/2)} = 2\sqrt{2}a.$$

Example 115: The distance between the parallel lines given by $(x + 7y)^2 + 4\sqrt{2}\,(x + 7y) - 42 = 0$ is

(a) 4/5 (b) $4\sqrt{2}$

(c) 2 (d) $10\sqrt{2}$

Ans. (c)

Solution: The lines given by the equation are

$$(x + 7y - 3\sqrt{2})(x + 7y + 7\sqrt{2}) = 0$$

$\Rightarrow$ $x + 7y - 3\sqrt{2} = 0$ and $x + 7y + 7\sqrt{2} = 0$

distance between these lines $= \left|\frac{7\sqrt{2} - (-3\sqrt{2})}{\sqrt{1^2+7^2}}\right|$

$$= \frac{10\sqrt{2}}{5\sqrt{2}} = 2.$$

Example 116: If the lines joining the origin to the intersection of the line $y = mx + 2$ and the curve $x^2 + y^2 = 1$ are at right angles, then

(a) $m^2 = 1$ (b) $m^2 = 3$

(c) $m^2 = 7$ (d) $2m^2 = 1$

Ans. (c)

Solution: Joint equation of the lines joining the origin and the point of intersection of the line $y = mx + 2$ and the curve $x^2 + y^2 = 1$ is

$$x^2 + y^2 = \left(\frac{y - mx}{2}\right)^2$$

$\Rightarrow$ $x^2(4 - m^2) + 2mxy + 3y^2 = 0$

Since these lines are at right angles

$4 - m^2 + 3 = 0$ $\Rightarrow m^2 = 7.$

Example 117: If one of the lines given by the equation $2x^2 + axy + 3y^2 = 0$ coincide with one of those given by $2x^2 + bxy - 3y^2 = 0$ and the other lines represented by them be perpendicular, then

(a) $a = -5, b = 1$ (b) $a = 5, b = -1$

(c) $a = 5, b = 1$ (d) none of these

Ans. (c)

Solution: Let $\frac{2}{3}x^2 + \frac{a}{3}xy + y^2 = (y - mx)(y - m'x)$

and $$\frac{2}{-3}x^2 + \frac{b}{-3}xy + y^2 = \left(y + \frac{1}{m}x\right)(y - m'x)$$

then $$m + m' = -\frac{a}{3},\ mm' = \frac{2}{3} \quad \text{(i)}$$

$$\frac{1}{m} - m' = \frac{-b}{3},\ -\frac{m'}{m} = -\frac{2}{3} \quad \text{(ii)}$$

$\Rightarrow$ $$m^2 = 1 \Rightarrow m = \pm 1$$

$$\text{If } m = 1,\ m' = \frac{2}{3} \Rightarrow a = -5,\ b = -1$$

$$\text{If } m = -1,\ m' = -\frac{2}{3} \Rightarrow a = 5,\ b = 1.$$

Example 118: If the equation $ax^3 + 3bx^2y + 3cxy^2 + dy^3 = 0$ ($a, b, c, d, \neq 0$) represents three coincident lines then

(a) $a = d$ (b) $b = c$

(c) $\frac{a}{b} = \frac{b}{c} = \frac{c}{d}$ (d) $ac = bd$

Ans. (c)

Solution: Let the given equation represent three coincident lines $y - mx = 0$, then

$$y^3 + \frac{3c}{d}y^2x + \frac{3b}{d}yx^2 + \frac{a}{d}x^3 = (y - mx)^3$$

$$= y^3 - 3y^2(mx) + 3y(mx)^2 - m^3x^3$$

$\Rightarrow$ $$m^3 = -\frac{a}{d},\ m^2 = \frac{b}{d},\ m = -\frac{c}{d}$$

$\Rightarrow$ $$m = -\frac{a}{b} = -\frac{b}{c} = -\frac{c}{d}$$

$\Rightarrow$ $$\frac{a}{b} = \frac{b}{c} = \frac{c}{d}.$$

Example 119: Two of the lines represented by $x^3 - 6x^2y + 3xy^2 + dy^3 = 0$ are perpendicular for

(a) all real values of d
(b) two real values of d
(c) three real values of d
(d) no real value of d

Ans. (b)

Solution: Let m_1, m_2, m_3 be the slopes of the three lines represented by the given equation such that

$$m_1 m_2 = -1$$

We have $m_1 m_2 m_3 = -\frac{1}{d}$ so that $m_3 = \frac{1}{d}$

Since $y = m_3 x$ i.e. $x = dy$ satisfies the given equation, we get

$$d^3 - 6d^2 + 3d + d = 0$$

$\Rightarrow$ $$d(d^2 - 6d + 4) = 0$$

If $d = 0$, the given equation represents the lines $x = 0$ and $x^2 - 6xy + 3y^2 = 0$ which are not perpendicular.

$\therefore$ $d \neq 0$ and $d^2 - 6d + 4 = 0$

$\Rightarrow$ $$d = \frac{6 \pm \sqrt{36-16}}{2} = 3 \pm \sqrt{5}$$

which gives two real values of d.

Example 120: The line $x + y = 1$ meets the lines represented by the equation $y^3 - xy^2 - 14x^2y + 24x^3 = 0$ at the points A, B, C. If O is the point of intersection of the lines represented by the given equation then $OA^2 + OB^2 + OC^2$ is equal to

(a) 22/9 (b) 85/72
(c) 181/72 (d) 221/72

Ans. (d)

Solution: The given cubic can be written as

$$(y - 2x)(y - 3x)(y + 4x) = 0$$

$\therefore$ The three lines given by this equation are

$y = 2x$, $y = 3x$ and $y = -4x$, they intersect at O (0,0) and meet the line $x + y = 1$ at the points

A (1/3, 2/3), B (1/4, 3/4) and C (–1/3, 4/3)

$\therefore$ $$OA^2 + OB^2 + OC^2 = \frac{5}{9} + \frac{10}{16} + \frac{17}{9} = \frac{221}{72}$$

Example 121: A line passing through the point P (2, 3) meets the lines represented by $x^2 - 2xy - y^2 = 0$ at the points A and B such that $PA \cdot PB = 17$, the equation of the line is

(a) $x = 2$ (b) $y = 3$
(c) $3x - 2y = 0$ (d) none of these

Ans. (b)

Solution: Let the equation of the line through P(2, 3) making an angle θ with the positive direction of x-axis be

$$\frac{x-2}{\cos\theta} = \frac{y-3}{\sin\theta}.$$

Then the coordinates of any point on this line at a distance r from P are $(2 + r\cos\theta, 3 + r\sin\theta)$. If $PA = r_1$ and $PB = r_2$, then r_1, r_2 are the roots of the equation.

$(2 + r\cos\theta)^2 - 2(2 + r\cos\theta)(3 + r\sin\theta) - (3 + r\sin\theta)^2 = 0$

$\Rightarrow r^2(\cos 2\theta - \sin 2\theta) - 2r(\cos\theta + 5\sin\theta) - 17 = 0$

$$\Rightarrow 17 = \mathbf{PA \cdot PB} = r_1 r_2 = \frac{17}{\cos 2\theta - \sin 2\theta}$$

$\Rightarrow \cos 2\theta - \sin 2\theta = 1$ which is satisfied by $\theta = 0$ and thus the equation of the line is $y = 3$.

Example 122: If (λ, 2) is an interior point of the triangle formed by the lines $x + y = 4$, $3x - 7y = 8$ and $4x - y = 31$, then

(a) $2 < \lambda < 22/3$ (b) $2 < \lambda < 33/4$
(c) $22/3 < \lambda < 33/4$ (d) $\lambda > 9$

Ans. (c)

Solution: The point (λ, 2) lies on the line $y = 2$ which meets the line $x + y = 4$ at a point outside the triangle and the line $3x - 7y = 8$ at D = (22/3, 2) and the line $4x - 3y = 31$ at E = (33/4, 2).

So x-coordinate of the point on the line $y = 2$, lying inside the triangle lies between 22/3 and 33/4.

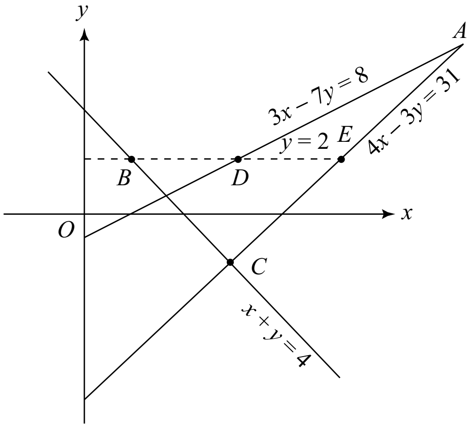


Fig. 16.20

Example 123: The lines given by L_1: $x + 3y - 5 = 0$, L_2: $3x - k y - 1 = 0$ L_3: $5x + 2y - 12 = 0$
form a triangle if k is

(a) 5 (b) 5/6
(c) – 6/5 (d) – 9

Ans. (b)

Solution: L_1 and L_3 intersect at (2,1)
The lines will form a triangle if no two of them are parallel and all of them are not concurrent.
Lines are concurrent at (2,1) if k = 5
L_2 is parallel to L_1 if $k = -9$
L_3 is parallel to L_1 if $k = -6/5$
So the lines form a triangle for any value of k other than 5, – 9, – 6/5

Example 124: Let the algebraic sum of the perpendicular distances from the points (2, 0) (0, 2) and (1, 1) to a variable straight line be zero, then the line passes through a fixed point whose coordinates are

(a) (1, – 1) (b) (– 1, 1)
(c) (1, 1) (d) (– 1, – 1)

Ans. (c)

Solution: Let the equation of the line be $ax + by + c = 0$
then $\dfrac{2a+c}{\sqrt{a^2+b^2}}+\dfrac{2b+c}{\sqrt{a^2+b^2}}+\dfrac{a+b+c}{\sqrt{a^2+b^2}}=0$
$\Rightarrow \quad 3a + 3b + 3c = 0$
$\Rightarrow \quad a + b + c = 0$
So the equation of the line can be $ax + by - (a + b) = 0$
$\Rightarrow \quad a(x - 1) + b(y - 1) = 0$
which passes through (1,1)

Example 125: The locus of the point P(h, k), when the area of the triangle formed by the lines $y = x$, $x + y = 2$ and the line through P (h,k) and parallel to the x-axis is $4h^2$ is

(a) $x + 2y - 1 = 0$ (b) $2x + y - 1 = 0$
(c) $2x - y - 1 = 0$ (d) $x - 2y + 1 = 0$

Ans. (b)

Solution: Let the line through P (h, k) be $y = k$
Vertices of the triangle are (1,1), (k,k), ($2 - k$, k)

$$4h^2 = \frac{1}{2}\begin{vmatrix}1 & 1 & 1\\ k & k & 1\\ 2 & -k & 1\end{vmatrix} \Rightarrow 4h^2 = (k-1)^2$$

$= 2h = \pm (k - 1)$
Hence locus of P is $2x \pm (y - 1) = 0$
i.e $\quad 2x + y - 1 = 0$ or $2x - y + 1 = 0$

EXERCISE

Concept-based Straight Objective Type Questions

1. A is a point on the positive x-axis at a distance 3 units from the origin and B is a point on the positive y-axis at a distance 4 units from the origin. If P divides AB in the ratio 1 : 2, the coordinates of P are
   (a) (1, 8/3) (b) (2, 4/3)
   (c) 8/3, 1) (d) (4/3, 2)
2. The slopes of the line which passes through the origin, and the mid-point of the line segment joining the points. P(3, – 4) and Q (– 5, – 2) is
   (a) 3 (b) – 3
   (c) – 1 (d) 1
3. Distance between P (x_1, y_1) and Q (x_2, y_2) when PQ is parallel to y-axis is
   (a) $x_1 - x_2$ (b) $|x_1 - x_2|$
   (c) $y_1 - y_2$ (d) $|y_1 - y_2|$
4. The lines parallel to the axes and passing through the point (4, – 5) are
   (a) $x = -5, y = 4$ (b) $x = 5, y = -4$
   (c) $x = 4, y = -5$ (d) $x = -4, y = 5$
5. The equation of the line whose perpendicular distance from the origin is 3 units and the angle which the normal makes with the positive direction of x-axis is 30° is

(a) $x + \sqrt{3}\,y = 3$ (b) $\sqrt{3}\,x + y = 6$
(c) $\sqrt{3}\,x + y = 1$ (d) $x + \sqrt{3}\,y = 6$

6. Points (8, 2), (– 2, – 2) and (3, 0) are the vertices of
(a) an equilateral triangle (b) an isosceles triangle
(c) right angled triangle (d) none of these

7. If the angle between the lines $\sqrt{3}\,y - x + 4 = 0$ and $x + y - 6 = 0$ is θ, then $\tan\theta$ is equal to.
(a) $\sqrt{3} + 1$ (b) $\sqrt{3} - 1$
(c) $2 + \sqrt{3}$ (d) $3 + \sqrt{2}$

8. Equation of the line passing through the point $(a - 1, a + 1)$ and making zero intercept on both axes is
(a) $ax + ay - 1 = 0$ (b) $(a + 1)x + (a - 1)y = 0$
(c) $(a - 1)x - (a + 1)y = 0$ (d) $(a + 1)x - (a - 1)y = 0$

9. The angle which the normal to the line $x - \sqrt{3}\,y + 8 = 0$ passing through the origin, makes with the positive x-axis is
(a) 30° (b) 60°
(c) 120° (d) 150°

10. If the line through the points $(h, 7)$ and (2, 3) intersects the line $3x - 4y - 5 = 0$ at right angles, then the value of h is
(a) – 1 (b) 1
(c) 5 (d) – 5

11. Equation of a line passing through the intersection of the lines $7x - y + 2 = 0$ and $x - 3y + 6 = 0$ parallel to x-axis is
(a) $y = 2$ (b) $y = -2$
(c) $y = 3$ (d) $y = -3$

12. The value of p for which the lines $2x + y - 3 = 0$, $3x - y - 2 = 0$ and $x - py + 5 = 0$ may intersect at a point is
(a) 2 (b) 3
(c) 5 (d) 6

13. Equation of the line equidistant from the parallel lines $9x + 6y - 7 = 0$ and $3x + 2y + 6 = 0$ is
(a) $6x + 4y + 5 = 0$ (b) $18x + 12y + 11 = 0$
(c) $18x + 12y - 11 = 0$ (d) $12x + 8y + 7 = 0$

14. Area of the triangle formed by the lines $y - x = 0$, $x + y = 0$ and $y = k$ in square units is
(a) $2k$ (b) k^2
(c) $2k^2$ (d) $k^2/2$.

15. The distance of the line $2x + 3y - 5 = 0$ from the point (3, 5) along the line $5x - 3y = 0$ in units is
(a) $\frac{2\sqrt{34}}{21}$ (b) $\frac{16}{21}$
(c) $\frac{4\sqrt{34}}{21}$ (d) $\frac{3\sqrt{43}}{21}$

LEVEL 1

Straight Objective Type Questions

16. The points $(k - 1, k + 2)$, $(k, k + 1)$, $(k + 1, k)$ are collinear for
(a) any value of k (b) $k = -1/2$ only
(c) no value of k (d) integral values of k only

17. The area of the triangle formed by the points $(k, 2 - 2k)$, $(-k + 1, 2k)$ and $(-4 - k, 6 - 2k)$ is 70 units. For
(a) four real values of k
(b) no integral value of k
(c) two integral values of k
(d) only one integral value of k

18. The quadrilateral $ABCD$ formed by the points A (0, 0); B (3, 4), C (7, 7) and D (4, 3) is a
(a) rectangle (b) square
(c) rhombus (d) parallelogram

19. The triangle with vertices A (2, 7), B $(4, y)$ and C (–2, 6) is right angled at A if
(a) $y = -1$ (b) $y = 0$
(c) $y = 1$ (d) none of these

20. The join of the points (–3, –4) and (1, –2) is divided by y-axis in the ratio.
(a) 1 : 3 (b) 2 : 3
(c) 3 : 1 (d) 3 : 2

21. The straight lines $x + y - 4 = 0$, $3x + y - 4 = 0$ and $x + 3y - 4 = 0$ form a triangle which is
(a) isosceles (b) right angled
(c) equilateral (d) none of these

22. The points P $(a, b + c)$, Q $(b, c + a)$ and R $(c, a + b)$ are such that $PQ = QR$ if
(a) a, b, c are in A.P. (b) a, b, c are in G.P.
(c) a, b, c are in H.P. (d) none of these

23. If a, b, c are in A.P. then the points (a, x), (b, y) and (c, z) are collinear if
(a) $x^2 = y$ (b) $x = z^2$
(c) $y^2 = z$ (d) x, y, z are in A.P.

24. The centroid of a triangle lies at the origin and the coordinates of its two vertices are (–8, 7) and (9, 4). The area of the triangle is
 (a) 95/6 (b) 285/2
 (c) 190/3 (d) 285
25. The mid points of the sides AB and AC of a triangle ABC are (2, –1) and (–4, 7) respectively, then the length of BC is
 (a) 10 (b) 20
 (c) 25 (d) 30
26. If the vertices of a triangle ABC are A (–4, –1), B (1, 2) and C (4, –3), then the coordinates of the circumcentre of the triangle are,
 (a) (1/3, – 2/3) (b) (0, –4)
 (c) (0, –2) (d) (– 3/2, 1/2)
27. The extremities of a diagonal of a parallelogram are the points (3, –4) and (–6, 5). If third vertex is (–2, 1) then the coordinates of the fourth vertex are
 (a) (1, 0) (b) (0, 0)
 (c) (1, 1) (d) none of these
28. If the vertices A and B of a triangle ABC are given by (2, 5) and (4, –11) and C moves along the line $L_1 : 9x + 7y + 4 = 0$, the locus of the centroid of the triangle ABC is a straight line parallel to
 (a) AB (b) BC
 (c) CA (d) L_1
29. The number of lines that can be drawn through the point (4, –5) at a distance 12 from the point (–2, 3) is
 (a) 0 (b) 1
 (c) 2 (d) infinite
30. If O is the origin and the coordinates of A and B are (x_1, y_1) and (x_2, y_2) respectively then $OA \times OB \cos \angle AOB$ is equal to
 (a) $x_1 y_1 + x_2 y_2$ (b) $x_1 x_2 + y_1 y_2$
 (c) $x_1 y_2 + x_2 y_1$ (d) $x_1 x_2 - y_1 y_2$
31. If the lines $x + ay + a = 0$, $bx + y + b = 0$ and $cx + cy + 1 = 0$ (a, b, c being distinct and $\neq 1$) are concurrent, then the value of $\frac{a}{a-1}+\frac{b}{b-1}+\frac{c}{c-1}$ is
 (a) –1 (b) 0
 (c) 1 (d) none of these
32. The line $\frac{x}{3}+\frac{y}{4}=1$ meets the axis of y and axis of x at A and B respectively. A square $ABCD$ is constructed on the line segment AB away from the origin, the coordinates of the vertex of the square farthest from the origin are
 (a) (7, 3) (b) (4, 7)
 (c) (6, 4) (d) (3, 8)
33. A line $2x + 3y - 1 = 0$ intersects the three sides BC, CA and AB of a triangle ABC in P, Q, R respectively then $\frac{BP}{PC}\times\frac{CQ}{QA}\times\frac{AR}{RB}=$
 (a) – 1 (b) 1
 (c) 2 (d) 3
34. If the vertices P, Q, R of a triangle are rational points, which of the following points of the triangle PQR is (are) always rational point (s)?
 (a) centroid (b) incentre
 (c) circumcentre (d) orthocentre
 (A rational point is a point both of whose coordinates are rational number)
35. Let $A_0\, A_1\, A_2\, A_3\, A_4\, A_5$ be a regular hexagon with vertex A_0 and A_3 at (1, 0) and (– 1, 0) respectively. The equations representing $A_1 A_4$ and $A_2 A_5$ are
 (a) $y = \pm\sqrt{3}x$ (b) $x = \pm\sqrt{3}y$
 (c) $y = \pm x$ (d) none of these
36. Let PS be the median of the triangle with vertices P (2, 2), Q (6, – 1) and R (7, 3). The equation of the line passing through (1, – 1) and parallel to PS is
 (a) $2x - 9y - 7 = 0$ (b) $2x - 9y - 11 = 0$
 (c) $2x + 9y - 11 = 0$ (d) $2x + 9y + 7 = 0$
37. Two equal sides of an isosceles triangle are given by $7x - y + 3 = 0$ and $x + y - 3 = 0$, the slope m of the third side is given by
 (a) $m^2 - 1 = 0$ (b) $m^2 - 3 = 0$
 (c) $3m^2 - 1 = 0$ (d) $3m^2 + 8m - 3 = 0$
38. Two sides of a rhombus $ABCD$ are parallel to the line $y = x + 2$ and $y = 7x + 3$. If the diagonals of the rhombus intersect at the point (1, 2) and the vertex A is on Y-axis at a distance a from the origin, then a is equal to
 (a) 2 (b) 5
 (c) 2/5 (d) 5/2
39. A line which makes an acute angle θ with the positive direction of x-axis is drawn through the point P (3, 4) to meet the lines $x = 6$ and $y = 8$ at R and S respectively then $RS = 8 - 2\sqrt{3}$, if
 (a) $\theta = \pi/3$ (b) $\theta = \pi/4$
 (c) $\pi/6$ (d) $\pi/12$
40. In a right angled triangle ABC right angled at C: $CA = a$, $CB = b$. If the angular points A and B slide along x-axis and y-axis respectively then C lies on
 (a) $bx \pm ay = 0$ (b) $ax \pm by = 0$
 (c) $\frac{x}{a}\pm\frac{y}{b}=1$ (d) $\frac{x}{b}+\frac{y}{a}=1$

41. $ABCD$ is a quadrilateral. P (3, 7) and Q (7, 3) are the middle points of the diagonals AC and BD respectively. The coordinates of the mean point (or the centre of mean position) of the vertices of the quadrilateral are
    (a) (0, 0) (b) (3, 3)
    (c) (5, 5) (d) (7, 7)

42. A line meets x-axis at A and y-axis at B such that in-centre of the triangle OAB is (1, 1). Equation of AB is
    (a) $2x + y = 2$ (b) $3x + 4y = 12$
    (c) $2x + y = 6$ (d) $2x + y = 4$

43. $ABCD$ is a rectangle in the clockwise direction. The coordinates of A are (1, 3) and of C are (5, 1), vertices B and D lie on the line $y = 2x + c$, then the coordinates of D are
    (a) (2, 0) (b) (4, 4)
    (c) (0, 2) (d) (2, 4)

44. The area of a triangle, two of whose vertices are (2, 1) and (3, – 2) is 5. The coordinates of the third vertex can not be
    (a) (6, – 1) (b) (4, 5)
    (c) (– 1, 20) (d) (2, 9)

45. The diagonals of a parallelogram $PQRS$ are along the lines $x + 3y = 4$ and $6x - 2y = 7$, the $PQRS$ must be a
    (a) rectangle (b) square
    (c) cyclic quadrilateral (d) rhombus

46. If a, b, c are in A.P., then $ax + by + c = 0$ represents
    (a) a single line
    (b) a family of concurrent lines
    (c) a family of parallel lines
    (d) none of these

47. If area of the triangle formed by the line L perpendicular to $5x - y = 1$ and the coordinate axes is 5, then the distance of L from the origin is
    (a) $5\sqrt{2}$ (b) $5/\sqrt{13}$
    (c) $5\sqrt{13}$ (d) none of these

48. The area enclosed by the lines $|x| + |y| = 2$ is
    (a) 1 sq unit (b) 2 sq units
    (c) 4 sq units (d) none of these

49. If the circumcentre of a triangle lies at the point (a, a) and the centroid is the mid-point of the line joining the points $(2a + 3, a + 4)$ and$(a - 4, 2a - 3)$; then the orthocentre of the triangle lies on the line
    (a) $y = x$
    (b) $(a - 1)x + (a + 1)y = 0$
    (c) $(a - 1)x - (a + 1)y = 0$
    (d) $(a + 1)x - (a - 1)y = 2a$

50. If A $(at^2, 2at)$, $B(a/t^2, -2a/t)$ and S $(a, 0)$ are three points, then $\dfrac{1}{SA} + \dfrac{1}{SB}$ is independent of
    (a) a (b) t
    (c) both a and t (d) none of these

51. $qx + py + (p + q - r) = 0$ is the reflection of the line $px + qy + r = 0$ in the line
    (a) $x - y = p + q$ (b) $x - y = p - q$
    (c) $x + y + 1 = 0$ (d) $x + y - 1 = 0$

52. Coordinates of the vertices B and C of the base of a triangle ABC are $(-a, 0)$ and $(a, 0)$ respectively. If $C - B = \pi/3$, the vertex A lies on the curve
    (a) $x^2 - y^2 + 2\sqrt{3}\ xy - a^2 = 0$
    (b) $x^2 + y^2 + 2\sqrt{3}\ xy - a^2 = 0$
    (c) $\sqrt{3}\ (x^2 - y^2) + 2xy - \sqrt{3}\, a^2 = 0$
    (d) $\sqrt{3}\ (x^2 + y^2) - 2xy + \sqrt{3}\, a^2 = 0$

53. If the line $\sqrt{5}x = y$ meets the lines $x = 1$, $x = 2$, … $x = n$ at points $A_1, A_2, \ldots, A_n$ respectively, then $(0\,A_1)^2 + (0\,A_2)^2 + \ldots + (0\,A_n)^2$ is equal to
    (a) $3n^2 + 3n$ (b) $2n^3 + 3n^2 + n$
    (c) $3n^3 + 3n^2 + 2$ (d) $(3/2)(n^4 + 2n^3 + n^3)$

54. One diagonal of a square is the portion of the line $3x + 2y = 12$ intercepted between the axes. The coordinates of the extremity of the other diagonals not lying in the first quadrant are
    (a) (1, – 1) (b) (– 1, – 1)
    (c) (– 1, 1) (d) none of these

55. If $y = m_i x + \dfrac{1}{m_i}$ $(i = 1, 2, 3)$ represents three straight lines whose slopes are the roots of the equation. $2m^3 - 3m^2 - 3m + 2 = 0$, A and B are the algebraic sum of the intercepts made by the lines on x-axis and y-axis respectively, then $\alpha A + \beta B = 0$ if (α, β) is
    (a) (4, 7) (b) (2, 7)
    (c) (7, 2) (d) (– 1,– 7)

56. If $u = a_1x + b_1y + c_1$, $v = a_2x + b_2y + c_2 = 0$ and $\dfrac{a_1}{a_2} = \dfrac{b_1}{b_2} \neq \dfrac{c_1}{c_2}$ then $u + kv = 0$ represents
    (a) a family of concurrent lines
    (b) a family of parallel lines
    (c) $u = 0$ or $v = 0$
    (d) none of these

57. Reflection of the line $x + y + 1 = 0$ in the line $lx + my + n = 0$ is
    (a) $(x + y + 1)(l + m) - 2(l^2 + m^2)(lx + my + n) = 0$
    (b) $(x + y + 1)(l^2 + m^2) - 2(l + m)(lx + my + n) = 0$
    (c) $(l + m + 1)(x + y) - 2(lx + my)(l^2 + m^2) = 0$
    (d) none of these

58. $ABCD$ is a square in which A lies on the positive y-axis and B lies on the positive x-axis. If D is the point (12, 17), the coordinates of C are
(a) (17, 12) (b) (17, 5)
(c) (14, 16) (d) (15, 3)

59. $ABCD$ is a rhombus. Its diagonals AC and BD intersect at the point M and satisfy $BD = 2AC$. If the coordinates of D and M are (1, 1) and (2, – 1) respectively, the coordinates of A are
(a) (3, 1/2) (b) (1, – 3/2)
(c) (3/2, 1) (d) (1/2, 3)

60. A straight line passes through the point (1, 1) and the portion of the line intercepted between x and y axes is divided at the point in the ratio 3 : 4. An equation of the line is
(a) $3x + 4y = 7$ (b) $4x + 3y = 7$
(c) $4x - 3y = 1$ (d) $3x - 4y + 1 = 0$

61. The equation $ax^2 + 2hxy + by^2 = 0$ represents a pair of perpendicular lines if
(a) $a = 3, b = 4$ (b) $a = 4, b = -3$
(c) $h = -1$ (d) $a = 11, b = -11$

62. The equation $ax^2 + 2hxy + ay^2 = 0$ represent a pair of coincident lines through the origin if
(a) $h = 2a$ (b) $2h = a$
(c) $h^2 = a$ (d) $h^2 = a^2$

63. The equation $ax^2 + by^2 + cx + cy = 0$ represents a pair of straight lines if
(a) $a = 0$ (b) $b = 0$
(c) $a + b = 0$ (d) none of these

64. The equation $x^3 - 6x^2y + 11xy^2 - 6y^3 = 0$ represent three straight lines passing through the origin, the slops of which form
(a) an A.P. (b) a G.P.
(c) an H.P. (d) none of these

65. If the slopes of the lines given by $8x^3 + ax^2y + bxy^2 + y^3 = 0$ are in G.P., then
(a) $a = b$ (b) $2a = b$
(c) $a = 2b$ (d) $a + b = 0$

66. If the slope of one of the lines given by $6x^2 + axy + y^2 = 0$ exceeds the slope of the other by one, then a is equal to
(a) ± 2 (b) 5
(c) – 5 (d) ± 5

67. If the slope of one of the lines represented by $ax^2 + (3a + 1)\,xy + 3y^2 = 0$ be reciprocal of the slope of the other, then the slopes of the lines are
(a) 3/2, 2/3 (b) 1/2, 2/1
(c) 1/3, 3 (d) – 1/3, – 3

68. The equations of the pairs of opposite sides of a rectangle are $x^2 - 7x + 6 = 0$ and $y^2 - 14y + 40 = 0$, the equation of the diagonal nearer the origin is
(a) $5x - 6y + 14 = 0$ (b) $6x - 5y + 14 = 0$
(c) $6x + 5y - 56 = 0$ (d) $6x + 5y - 14 = 0$

69. The equation of a line bisecting the join of (2010, 1600) and (– 1340, 1080) and having intercept on the axes in the ratio 1 : 2 is
(a) $2x + y = 1680$ (b) $x + 2y = 1680$
(c) $2x + y = 2010$ (d) none of these

70. Let the coordinates of P be (x, y) and of Q be (α, β) where α is the geometric and β is the arithmetic mean of the coordinates of P. If the mid point of PQ is (42, 31) the coordinates of P are
(a) (61, 21) (b) (49, 25)
(c) (31, 31) (d) none of these

71. Image of the point (–1, 3) with respect to the line $y = 2x$ is
(a) (7/5, 14/5) (b) (1, 2)
(c) (3, 1) (d) (5, 1)

72. The point (a^2, a) lies between the straight lines $x + y = 6$ and $x + y = 2$ for
(a) all values of a (b) no value of a
(c) $|2a - 3| < 1$ (d) $|2a + 5| > 1$

73. Perimeter of the quadrilateral bounded by the coordinate axis and the lines $x + y = 50$ and $3x + y = 90$ is
(a) $80 + 20\sqrt{2}$
(b) $80 + 10\sqrt{10}$
(c) $80 + 20\sqrt{2} + 10\sqrt{10}$
(d) 110.

74. If the sum of the slopes of the lines given by $3x^2 - 2cxy - 5y^2 = 0$ is twice their product, then the value of c is
(a) 2 (b) 3
(c) 6 (d) none of these

75. If one of the lines given by
$2cx^2 + 2xy - (c^2 - 1)y^2 = 0$ is $2x + 3y = 0$, then the integral value of c is
(a) 2 (b) 3
(c) 4 (d) 8

Assertion-Reason Type Questions

76. Let $A(2, -3)$ and $B(-2, 1)$ be the vertices of a triangle ABC.
Statement-1: If the centroid of the triangle moves on the line $x + y = 5$, the vertex moves on the line $x + y = 17$.
Statement-2: If the centroid of the triangle moves the line $x - y + 1 = 0$ $(x \neq 0)$, the triangle is either isosceles or equilateral.

77. **Statement-1:** $x^2y - 3xy - 2x^2 + 6x - 4y + 8 = 0$ represents three straight lines two of which are parallel and the third is perpendicular to the other two
Statement-2: $xy - 2x + y - 2 = 0$ represents a pair of straight lines one of which is common to the pair of straight lines $xy + 2x - y - 2 = 0$

78. **Statement-1:** The locus represented by the equation $(2x + 3y - 1)^2 + (x + y + 1)^2 = 0$ is a pair of straight lines $2x + 3y - 1 = 0$ and $x + y + 1 = 0$.
Statement-2: The equation $(x + 4y - 2)^2 + (4x + y - 8)^2 + (x \pm y - k)^2 = 0$ represents a point if $k = 2$.

79. **Statement-1:** Points $P = (-\sin(\beta - \alpha), -\cos\beta)$, $Q = (\cos(\beta - \alpha), \sin\beta)$ and $R = (\cos(\beta - \alpha + \theta), \sin(\beta - \theta))$ where $0 < \alpha, \beta, \theta < \pi/4$ are non collinear
Statement-2: If the slopes of the lines PQ and QR are not equal the points P, Q and R are non collinear.

80. **Statement-1:** If $x + ky = 1$ and $x = a$ are the equations of the hypotenuse and a side of a right angled isosceles triangle then $k = \pm a$.
Statement-2: Each side of a right angled isosceles triangle makes an angle $\pi/4$ with the hypotenuse.

81. **Statement-1:** Locus of the centroid of the triangle whose vertices are $(a\cos t, a\sin t)$, $(b\sin t, -b\cos t)$ and $(1, 0)$ where t is a parameter is $(3x - 1)^2 + (3y)^2 = a^2 + b^2$
Statement-2: The centroid of a triangle is equidistance from the vertices of the triangle.

82. **Statement-1:** If non zero numbers a, b, c are in Harmonic progression, then the straight line $\frac{x}{a} + \frac{y}{b} + \frac{1}{c} = 0$ always passes through the fixed point $(1, 1)$
Statement-2: If a, b, c are in Harmonic progression. then $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in arithmetic progression.

83. **Statement-1:** The straight line. $(\sin\theta + \sqrt{3}\cos\theta)x + (\sqrt{3}\sin\theta - \cos\theta)y + (5\sin\theta - 7\cos\theta) = 0$ for all values of θ except $\theta = n\pi/2$, n is an integer; passes through the point of intersection of the lines $x + \sqrt{3}\,y + 5 = 0$ and $\sqrt{3}\,x - y - 7 = 0$.
Statement-2: $L_1 + \lambda L_2 = 0$ represents a line through the point of intersection of the lines $L_1 = 0$, $L_2 = 0$ for all non-zero finite values of λ.

84. **Statement-1:** If the area of the triangle formed by the lines $y = x$, $x + y = 2$ and the lines through $P(h, k)$ parallel to x-axis is $4h^2$, then P lies on the lines $4x^2 + 2y - y^2 - 1 = 0$.
Statement-2: Area of the triangle formed by the lines $y = x$, $x + y = 2$ and the axis of x is equal to half the area of the triangle formed by the line $x + y = 2$ and the coordinate axes.

85. **Statement-1:** Reflexion of the point $(6, 1)$ in the line $x + y = 0$ is $(-1, -6)$.
Statement-2: Reflexion of a point $P(\alpha, \beta)$ in the line $ax + by + c = 0$ is $Q(\alpha', \beta')$ if $R\left(\frac{\alpha + \alpha'}{2}, \frac{\beta + \beta'}{2}\right)$ lies on the line.

86. **Statement-1:** $4x^2 + 12xy + 9y^2 = 0$ represents a pair of perpendicular lines through the origin.
Statement-2: $ax^2 + 2hxy + by^2 = 0$ represents a pair of coincident lines if $h^2 = ab$.

87. **Statement-1:** $x^2y^2 - x^2 - y^2 + 1 = 0$ represents the sides of a square of area 4 square units.
Statement-2: $3x^2 + \lambda xy - 3y^2 = 0$ represents a pair of perpendicular lines for all values of λ

LEVEL 2

Straight Objective Type Questions

88. If $(0, 1)$, $(1, 1)$ and $(1, 0)$ are the mid points of the sides of a triangle, the coordinates of its incentre are
(a) $(2 + \sqrt{2}, 2 + \sqrt{2})$
(b) $((2 + \sqrt{2}), -(2 + \sqrt{2}))$
(c) $((2 - \sqrt{2}), (2 - \sqrt{2}))$
(d) $((2 - \sqrt{2}), -(2 - \sqrt{2}))$

89. The vertices of the triangle ABC are $A(1, 2)$, $B(0, 0)$ and $C(2, 3)$, then the greatest angle of the triangle is
(a) 75° (b) 105°
(c) 120° (d) None of these

90. The points $(0, 8/3)$, $(1, 3)$ and $(82, 30)$ are the vertices of
(a) obtuse angled triangle
(b) acute angled triangle
(c) right angled triangle
(d) none of these

91. Area of the rhombus enclosed by the lines $ax \pm by \pm c = 0$ is
(a) $2a^2/bc$ (b) $2b^2/ca$
(c) $2c^2/ab$ (d) none of these

92. If $x \cos \alpha + y \sin \alpha = -\sin \alpha \tan \alpha$ be the equation of a line, then the length of the perpendiculars on the line from the points $(a^2, 2a)$, $(ab, a + b)$ and $(b^2, 2b)$ are in
(a) A.P. (b) G.P.
(c) H.P. (d) none of these

93. The coordinates of the points A and B are respectively $(-3, 2)$ and $(2, 3)$. P and Q are points on the line joining A and B such that $AP = PQ = QB$. A square $PQRS$ is constructed on PQ as one side, the coordinates of R are
(a) $(-4/3, 7/3)$ (b) $(0, 13/3)$
(c) $(1/3, 8/3)$ (d) $(2/3, 1)$

94. If $A(x_1, y_1)$, $B(x_2, y_2)$, $C(x_3, y_3)$ are the vertices of a triangle, then the equation

$$\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} + \begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$$

represents
(a) the median through A
(b) the altitude through A
(c) the perpendicular bisector of BC
(d) the line joining the centroid with a vertex

95. Given four lines whose equations are $x + 2y - 3 = 0$, $2x + 3y - 4 = 0$, $3x + 4y - 7 = 0$ and $4x + 5y - 6 = 0$, then the lines are
(a) concurrent
(b) the sides of a quadrilateral with one vertex at $(3, 0)$
(c) the sides of a cyclic quadrilateral
(d) none of these

96. A ray of light coming from the point $(1, 2)$ is reflected at a point A on the axis of x and then passes through the point $(5, 3)$. The coordinates of the point A are
(a) $(5/13, 0)$ (b) $(-7, 0)$
(c) $(13/5, 0)$ (d) $(15, 0)$

97. If Δ_1, Δ_2, Δ_3 are the areas of the triangles with vertices $(0, 0)$, $(a \tan \alpha, b \cot \alpha)$, $(a \sin \alpha, b \cos \alpha)$; (a, b), $(a \sec^2 \alpha, b \operatorname{cosec}^2 \alpha)$, $(a + a \sin^2 \alpha, b + b \cos^2 \alpha)$ and $(0, 0)$, $(a \tan \alpha, -b \cot \alpha)$, $(a \sin \alpha, b \cos \alpha)$, then Δ_1, Δ_2, Δ_3 are in G.P. for
(a) all values of α
(b) only one value of α
(c) finite number of values of α
(d) no value of α

98. The orthocentre of the triangle formed by the lines $y = 0$, $(1 + t)x - ty + t(1 + t) = 0$ and $(1 + u)x - uy + u(1 + u) = 0$ $(t \neq u)$ for all values of t and u lies on the line
(a) $x - y = 0$ (b) $x + y = 0$
(c) $x - y + 1 = 0$ (d) $x + y + 1 = 0$

99. If $A(3, 0)$ and $B(6, 0)$ are two fixed points and $U(x_1, y_1)$ is a variable point in the plane. AU and BU meet y-axis at C and D respectively and AD meets OU at V, then for all positions of U in the plane, CV passes through the point
(a) $(0, 0)$ (b) $(0, 2)$
(c) $(2, 0)$ (d) none of these

100. The incentre of the triangle with vertices $(1, \sqrt{3})$, $(0, 0)$ and $(2, 0)$ is
(a) $(1, \sqrt{3}/2)$ (b) $(2/3, 1/\sqrt{3})$
(c) $(2/3, \sqrt{3}/2)$ (d) $(1, 1/\sqrt{3})$

101. If $x + 2y + 3 = 0$, $x + 2y - 7 = 0$ and $2x - y - 4 = 0$ form three sides of a square, the equation of the fourth side nearer the point $(1, -1)$ is
(a) $2x - y - 6 = 0$ (b) $2x - y + 6 = 0$
(c) $2x - y - 14 = 0$ (d) $2x - y + 14 = 0$

102. The distance between the orthocentre and the circumcentre of the triangle with vertices $(0, 0)$, $(0, a)$ and $(b, 0)$ is
(a) $\sqrt{a^2 - b^2}/2$ (b) $a + b$
(c) $a - b$ (d) $\sqrt{a^2 + b^2}/2$

103. The centroid of a triangle lies at the origin and the coordinates of its two vertices are $(-8, 0)$ and $(9, 11)$, the area of the triangle in sq. units is
(a) 11/8 (b) 8/11
(c) 88 (d) none of these

104. The line $3x + 2y = 24$ meets the y-axis at A and the x-axis at B; C is a point on the perpendicular bisector of AB such that the area of the triangle ABC is 91 sq. units. The coordinates of C can be

(a) (29/2, – 1) (b) (29/2, 13)
(c) (– 13/2, 1) (d) (– 13/2, 13)

105. The straight lines $4x - 3y - 5 = 0$, $x - 2y = 0$, $7x + y - 40 = 0$ and $x + 3y + 10 = 0$ form
(a) a rectangle
(b) a parallelogram
(c) a cyclic quadrilateral
(d) none of these

106. If the straight lines $x + 2x - 9 = 0$, $3x + 5y - 5 = 0$ and $ax + by + 1 = 0$ are concurrent, then the straight line $35x - 22y - 1 = 0$ passes through
(a) (a, b) (b) (b, a)
(c) $(a, -b)$ (d) $(-a, b)$

107. If the slope of one of the lines represented by $ax^2 + 2hxy + by^2 = 0$ be the square of the other, then $\dfrac{a+b}{h}+\dfrac{8h^2}{ab}$ is equal to
(a) 0 (b) 1
(c) 6 (d) 8

108. The locus represented by the equation $(x - y + c)^2 + (x + y - c)^2 = 0$ is
(a) a line parallel to x-axis
(b) a point
(c) a pair of straight lines
(d) a line parallel of y-axis

Previous Years' AIEEE/JEE Main Questions

1. If the pair of lines $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ intersect on y-axis then
(a) $2fgh = bg^2 + ch^2$ (b) $bg^2 \neq ch^2$
(c) $abc = 2fgh$ (d) none of these **[2002]**

2. Locus of mid point of the portion between the axes of $x \cos \alpha + y \sin \alpha = p$ where p is constant is
(a) $x^2 + y^2 = 4/p^2$
(b) $x^2 + y^2 = 4p^2$
(c) $1/x^2 + 1/y^2 = 2/p^2$
(d) $1/x^2 + 1/y^2 = 4/p^2$ **[2002]**

3. A triangle with vertices (4, 0), (– 1, – 1), (3, 5) is
(a) isosceles and right angled
(b) isosceles but not right angled
(c) right angled but not isosceles
(d) neither right angled nor isosceles. **[2002]**

4. The sides of a triangle are $3x + 4y = 0$, $4x - 3y = 0$ and $5x + 5y = 1$ then the triangle is
(a) right angled (b) obtuse angled
(c) equilateral (d) none of these **[2002]**

5. A square of side a lies above the x-axis and has one vertex at the origin. The side passing through the origin makes an angle α $(0 < \alpha < \pi/4)$ with the positive direction of x-axis. The equation of the diagonal not passing through the origin is
(a) $y (\cos \alpha + \sin \alpha) + x (\sin \alpha - \cos \alpha) = a$
(b) $y (\cos \alpha + \sin \alpha) + x (\sin \alpha + \cos \alpha) = a$
(c) $y (\cos \alpha + \sin \alpha) + x (\cos \alpha - \sin \alpha) = a$
(d) $y (\cos \alpha - \sin \alpha) - x (\sin \alpha - \cos \alpha) = a$ **[2003]**

6. If the pair of straight lines $x^2 - 2pxy - y^2 = 0$ and $x^2 - 2qxy - y^2 = 0$ be such that each pair bisects the angle between the other pair, then
(a) $p = -q$ (b) $pq = 1$
(c) $pq = -1$ (d) $p = q$ **[2003]**

7. Locus of centroid of the triangle whose vertices are $(a \cos t, a \sin t)$,$(b \sin t, -b \cos t)$ and (1, 0), where t is a parameter is
(a) $(3x - 1)^2 + (3y)^2 = a^2 + b^2$
(b) $(3x + 1)^2 + (3y)^2 = a^2 + b^2$
(c) $(3x + 1)^2 + (3y)^2 = a^2 - b^2$
(d) $(3x - 1)^2 + (3y)^2 = a^2 - b^2$ **[2003]**

8. If x_1, x_2, x_3 and y_1, y_2, y_3 are both in G.P. with the same common ratio, then the points (x_1, y_1), (x_2, y_2) and (x_3, y_3)
(a) lie on an ellipse
(b) lie on a circle
(c) are vertices of a triangle
(d) lie on a straight line **[2003]**

9. If the equation of the locus of point equidistant from the points (a_1, b_1) and (a_2, b_2) is $(a_1 - a_2) x + (b_1 - b_2) y + c = 0$, then $c =$
(a) $a_1^2 - a_2^2 + b_1^2 - b_2^2$
(b) (1/2) $(a_1^2 + a_2^2 + b_1^2 + b_2^2)$
(c) $\sqrt{a_1^2 + b_1^2 - a_2^2 - b_2^2}$
(d) (1/2) $(a_2^2 + b_2^2 - a_1^2 - b_1^2)$ **[2003]**

10. Let $A(2, -3)$ and $B(-2, 1)$ be vertices of a triangle ABC. If the centroid of this triangle moves on the line $2x + 3y = 1$, then the locus of the vertex C is the line
(a) $3x + 2y = 5$ (b) $2x - 3y = 7$
(c) $2x + 3y = 9$ (d) $3x - 2y = 3$ **[2004]**

11. The equation of the straight line passing through the point (4, 3) and making intercepts on the coordinate axes whose sum is – 1 is

(a) $\frac{x}{2}+\frac{y}{3}=1$ and $\frac{x}{2}+\frac{y}{1}=1$

(b) $\frac{x}{2}-\frac{y}{3}=-1$ and $\frac{x}{-2}-\frac{y}{1}=-1$

(c) $\frac{x}{2}+\frac{y}{3}=-1$ and $\frac{x}{-2}-\frac{y}{1}=-1$

(d) $\frac{x}{2}-\frac{y}{3}=1$ and $\frac{x}{-2}+\frac{y}{1}=1$ **[2004]**

12. If the sum of the slopes of the lines given by $x^2-2cxy-7y^2=0$ is four times their product, then c has the value

(a) 2 (b) – 1
(c) 1 (d) – 2 **[2004]**

13. If one of the lines given by $6x^2-xy+4cy^2=0$ is $3x+4y=0$, then c equals

(a) 3 (b) – 1
(c) 1 (d) – 3 **[2004]**

14. The line parallel to x-axis passing through the intersection of the lines $ax+2by+3b=0$ and $bx-2ay-3a=0$ where $(a, b)\neq(0, 0)$ is

(a) above x-axis at a distance 3/2 from it
(b) above x-axis at a distance 2/3 from it
(c) below x-axis at a distance 3/2 from it
(d) below x-axis at a distance 2/3 from it. **[2005]**

15. If a vertex of a triangle is (1, 1) and the mid points of two sides through this vertex are (– 1, 2) and (3, 2), then the centroid of the triangle is

(a) (1, 7/3) (b) (1/3, 7/3)
(c) (– 1, 7/3) (d) (– 1/3, 7/3) **[2005]**

16. If non zero numbers a, b, c are in H.P., then the straight line $\frac{x}{a}+\frac{y}{b}+\frac{1}{c}=0$ always passes through a fixed point. That point is

(a) (1, – 2) (b) (1, – 1/2)
(c) (– 1, 2) (d) (– 1, – 2) **[2005]**

17. If the pair of lines $ax^2+2(a+b)xy+by^2=0$ lie along diameters of a circle and divide the circle into four sectors such that the area of one of the sector is thrice the area of the another sector, then

(a) $3a^2+10ab+3b^2=0$
(b) $3a^2+2ab+3b^2=0$
(c) $3a^2-10ab+3b^2=0$
(d) $3a^2-2ab+3b^2=0$ **[2005]**

18. A straight line through the point $A(3, 4)$ is such that its intercept between the axes is bisected at A. Its equation is

(a) $3x+4y=25$ (b) $x+y=7$
(c) $3x-4y+7=0$ (d) $4x+3y=24$ **[2006]**

19. Let $A(h, k)$, $B(1, 1)$ and $C(2, 1)$ be the vertices of a right angled with AC as its hypotenuse. If the area of the triangle is 1, then the set of values which 'k' can take is given by

(a) {1, 3} (b) {0, 2}
(c) {– 1, 3} (d) {– 3, – 2} **[2007]**

20. Let $P=(-1, 0)$, $Q=(0, 0)$ and $R=(\sqrt{3}, 3)$, be three points. The equation of the bisector of angle PQR is

(a) $\sqrt{3}\,x+y=0$ (b) $x+(\sqrt{3}/2)\,y=0$
(c) $(\sqrt{3}/2)\,x+y=0$ (d) $x=\sqrt{3}\,y=0$ **[2007]**

21. If one of the lines of $my^2+(1-m^2)xy-mx^2=0$ is a bisector of the angle between the line $xy=0$ then m is

(a) – 1/2 (b) – 2
(c) 1 (d) 2 **[2007]**

22. The perpendicular bisector of the lines segment joining $P(1, 4)$ and $Q(k, 3)$ has y-intercept-4. Then a possible value of k is

(a) – 4 (b) 1
(c) 2 (d) – 2 **[2008]**

23. The lines $p(p^2+1)x-y+q=0$ and $(p^2+1)^2x+(p^2+1)y+2q=0$ are perpendicular to a common line for

(a) exactly two values of p
(b) more than two values of p
(c) no value of p
(d) exactly one value of p **[2009]**

24. The line L is given by $\frac{x}{5}+\frac{y}{b}=1$ passes through the point (13, 32). The line K is parallel to L and has the equation $\frac{x}{c}+\frac{y}{3}=1$. Then the distance between L and K is

(a) $\frac{17}{\sqrt{15}}$ (b) $\frac{23}{\sqrt{17}}$
(c) $\frac{23}{\sqrt{15}}$ (d) $\sqrt{15}$ **[2010]**

25. The line $L_1: y-x=0$ and $L_2: 2x+y=0$ intersect the line L_3: $y+2=0$ at P and Q respectively The bisector of the acute angle between L_1 and L_2 intersect L_3 at **R**.

Statement-1 : The ratio $PR:RQ$ equals $2\sqrt{2}\ :\ \sqrt{5}$

Statement-2 : In any triangle, bisector of an angle divides the triangle into two similar triangles **[2011]**

26. The lines $x + y = |a|$ and $ax - y = 1$ intersect each other in the first quadrant then the set of all possible values of a is in the interval.

(a) $(0, \infty)$ (b) $(1, \infty)$
(c) $(-1, \infty)$ (d) $(-1, 1)$ **[2011]**

27. If A (2, – 3), B (–2, 1) are two vertices of a triangle and the third vertex moves on the line $2x + 3y = 9$, then the locus of the centroid of the triangle is

(a) $x - y = 1$ (b) $2x + 3y = 1$
(c) $2x + 3y = 3$ (d) $2x - 3y = 1$ **[2011]**

28. If the line $2x + y = k$ passes through the point which divides the segment joining the points (1, 1) and (2, 4) in the ratio 3:2, then k equals

(a) 6 (b) 11/5
(c) 29/5 (d) 5 **[2012]**

29. A line is drawn through the point (1, 2) to meet the coordinate axes at P and Q such that it forms a triangle OPQ, where O is the origin. If the area of the triangle OPQ is least, then the slope of the line PQ is

(a) – 2 (b) –1/2
(c) –1/4 (d) – 4 **[2012]**

30. A ray of light along $x + \sqrt{3}\, y = \sqrt{3}$ gets reflected upon reaching the x-axis, an equation of the reflected ray is

(a) $\sqrt{3}y = x - \sqrt{3}$ (b) $y = \sqrt{3}x - \sqrt{3}$
(c) $\sqrt{3}y = x - 1$ (d) $y = x + \sqrt{3}$ **[2013]**

31. Equation of the line passing through the points of intersection of the parabola $x^2 = 8y$ and the ellipse $\frac{x^2}{3} + y^2 = 1$ is:

(a) $y - 3 = 0$ (b) $y + 3 = 0$
(c) $3y + 1 = 0$ (d) $3y - 1 = 0$ **[2013, online]**

32. A light ray emerging from the point source placed at $P(1, 3)$ is reflected at a point Q in the axis of x. If the reflected ray passes through the point $R(6, 7)$, then the abscissa of Q is:

(a) 1 (b) 3
(c) 7/2 (d) 5/2 **[2013, online]**

33. If three lines $x - 3y = p$, $ax + 2y = q$ and $ax + y = r$ form a right-angled triangle, then:

(a) $a^2 - 9a + 18 = 0$ (b) $a^2 - 6a - 12 = 0$
(c) $a^2 - 6a - 18 = 0$ (d) $a^2 - 9a + 12 = 0$

[2013, online]

34. The x-coordinates of the incentre of the triangle that has the coordinates of mid points of its sides as (0, 1), (1, 1) and (1, 0) is

(a) $2 - \sqrt{2}$ (b) $1 + \sqrt{2}$
(c) $1 - \sqrt{2}$ (d) $2 + \sqrt{2}$ **[2013]**

35. If x-intercept of some line L is double as that of the line $3x + 4y = 12$ and the y-intercept of L is half as that of the same line, then the slope of L is:

(a) – 3 (b) – 3/8
(c) – 3/2 (d) – 3/16 **[2013, online]**

36. If the extremities of the base of an isosceles triangle are the points $(2a, 0)$ and $(0, a)$ and the equation of one of the sides is $x = 2a$, then the area of the triangle, in square units, is:

(a) $\frac{5}{4}a^2$ (b) $\frac{5}{2}a^2$
(c) $\frac{25}{4}a^2$ (d) $5a^2$ **[2013, online]**

37. Let θ_1, be the angle between two lines $2x + 3y + c_1 = 0$ and $-x + 5y + c_2 = 0$ and θ_2 be the angle between two lines $2x + 3y + c_1 = 0$ and $-x + 5y + c_3 = 0$, where c_1, c_2, c_3 are any real numbers:

Statement 1: If c_2 and c_3 are proportional, then $\theta_1 = \theta_2$

Statement 2: $\theta_1 = \theta_2$ for all c_2 and c_3.

(a) Statement-1 is true, statement-2 is true and statement-2 is a correct explanation for statement-1.
(b) Statement-1 is true, statement-2 is true but statement-2 is Not a correct explanation for statement-1
(c) Statement-1 is false, Statement-2 is true.
(d) Statement-1 is true, Statement-2 is false.

[2013, online]

38. If the image of point P (2, 3) in a line L is Q (4, 5) then the image of point R(0, 0) in the same line is:

(a) (2, 2) (b) (4, 5)
(c) (3, 4) (d) (7, 7) **[2013, online]**

39. Let A(– 3, 2) and B(– 2, 1) be the vertices of a triangle ABC. If the centroid of this triangle lies on the line $3x + 4y + 2 = 0$, then vertex C lies on the line.

(a) $4x + 3y + 5 = 0$ (b) $3x + 4y + 3 = 0$
(c) $4x + 3y + 3 = 0$ (d) $3x + 4y + 5 = 0$

[2013, online]

40. Let a, b, c and d be non zero numbers. If the point of intersection of the lines $4ax + 2ay + c = 0$ and $5bx + 2by + d = 0$ lies in the fourth quadrant and is equidistant from the two axes then:

(a) $2bc - 3ad = 0$ (b) $2bc + 3ad = 0$
(c) $3bc - 2ad = 0$ (d) $3bc + 2ad = 0$ **[2014]**

41. Let PS be the median of the triangle with vertices $P(2, 2)$, $Q(6, -1)$ and $R(7, 3)$. The equation of the line passing $(1, -1)$ and parallel to PS is:
(a) $4x - 7y - 11 = 0$ (b) $2x + 9y + 7 = 0$
(c) $4x + 7y + 3 = 0$ (d) $2x - 9y - 11 = 0$
[2014]

42. Let a and b be any two numbers satisfying $\frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{4}$. Then, the foot of perpendicular from the origin on the variable line $\frac{x}{a} + \frac{y}{b} = 1$, lies on:
(a) a hyperbola with each semi-axis = $\sqrt{2}$
(b) a hyperbola with each semi-axis = 2
(c) a circle of radius = 2
(d) a circle of radius = $\sqrt{2}$ **[2014, online]**

43. Given three points P, Q, R with $P(5, 3)$ and R lies on the x-axis. If equation of RQ is $x - 2y = 2$ and PQ is parallel to x-axis, then centroid of ΔPQR lies on the line
(a) $2x + y - 9 = 0$ (b) $x - 2y + 1 = 0$
(c) $5x - 2y = 0$ (d) $2x - 5y = 0$
[2014, online]

44. The base of an equilateral triangle is along the line given by $3x + 4y = 9$. If a vertex of the triangle is $(1, 2)$, then length of a side of the triangle is:
(a) $\frac{2\sqrt{3}}{15}$ (b) $\frac{4\sqrt{3}}{15}$
(c) $\frac{4\sqrt{3}}{5}$ (d) $\frac{2\sqrt{3}}{5}$ **[2014, online]**

45. If a line intercepted between the coordinates axes is trisected at a point A $(4, 3)$, which is nearer to the x-axis, then its equation is
(a) $4x - 3y = 7$ (b) $3x + 2y = 18$
(c) $3x + 8y = 36$ (d) $x + 3y = 13$
[2014, online]

46. If three distinct lines $x + 2ay + a = 0$, $x + 3by + b = 0$ and $x + 4ay + a = 0$ are concurrent, then the point (a, b) lies on a:
(a) circle (b) hyperbola
(c) straight-line (d) parabola
[2014, online]

47. The circumcentre of a triangle lies at the origin and its centroid is the mid point of the line segment joining the points $(a^2 + 1, a + 1)$ and $(2a, -2a)$, $a \neq 0$. Then for any a, the orthocentre of this triangle lies on the line:
(a) $y - 2ax = 0$
(b) $y - (a^2 + 1)x = 0$
(c) $y + x = 0$
(d) $(a - 1)^2 x - (a + 1)^2 y = 0$ **[2014, online]**

48. If a line L is perpendicular to the line $5x - y = 1$, and the area of the triangle formed by the line L and the coordinate axes, is 5, then the distance of L from the line $x + 5y = 0$ is
(a) $7/\sqrt{5}$ (b) $5/\sqrt{13}$
(c) $7/\sqrt{13}$ (d) $5/\sqrt{7}$ **[2014, online]**

49. The number of points having both coordinates as integer, that lie in the interior of the triangle with vertices $(0, 0)$, $(0, 41)$ and $(41, 0)$ is
(a) 901 (b) 861
(c) 820 (d) 780 **[2015]**

50. Locus of the image of the point $(2, 3)$ in the line $(2x - 3y + 4) + k(x - 2y + 3) = 0$, $k \in \mathbf{R}$, is a:
(a) straight line parallel to x-axis
(b) straight line parallel to y-axis
(c) circle of radius $\sqrt{2}$
(d) circle of radius $\sqrt{3}$ **[2015]**

51. A straight line L through the point $(3, -2)$ is inclined at an angle of 60° to the line $\sqrt{3}x + y = 1$. If L also intersects x-axis, then equation of L is:
(a) $y + \sqrt{3}x + 2 - 3\sqrt{3} = 0$
(b) $y - \sqrt{3}x + 2 + 3\sqrt{3} = 0$
(c) $\sqrt{3}y - x + 3 + 2\sqrt{3} = 0$
(d) $\sqrt{3}y + x - 3 + 2\sqrt{3} = 0$ **[2015, online]**

52. The points $\left(0, \frac{8}{3}\right)$, $(1, 3)$ and $(82, 30)$
(a) form an obtuse angled triangle
(b) form an acute angled triangle
(c) form a right angled triangle
(d) lie on a straight line. **[2015, online]**

53. Let L be the line passing through the point $P(1, 2)$ such that its intercepted segment between the coordinate axes is bisected at P. If L_1 is the line perpendicular to L and passing through the point $(-2, 1)$, then the point of intersection of L and L_1 is:
(a) $\left(\frac{4}{5}, \frac{12}{5}\right)$ (b) $\left(\frac{11}{20}, \frac{29}{10}\right)$
(c) $\left(\frac{3}{10}, \frac{17}{5}\right)$ (d) $\left(\frac{3}{5}, \frac{23}{10}\right)$ **[2015,online]**

54. Two sides of a rhombus are along the lines, $x - y + 1 = 0$ and $7x - y - 5 = 0$. If its diagonals intersect at $(-1, -2)$, then which one of the following is a vertex of this rhombus?
 (a) $(-3, -9)$ (b) $(-3, -8)$
 (c) $\left(\frac{1}{3} - \frac{8}{3}\right)$ (d) $\left(-\frac{10}{3}, -\frac{7}{3}\right)$ **[2016]**

55. If a variable line drawn through the intersection on the lines $\frac{x}{3}+\frac{y}{4}=1$ and $\frac{x}{4}+\frac{y}{3}=1$, meets the coordinate axes at A and B, $(A \neq B)$, then the locus of the midpoint of AB is:
 (a) $7xy = 6(x + y)$
 (b) $4(x + y)^2 - 28(x + y) + 49 = 0$
 (c) $6xy = 7(x + y)$
 (d) $14(x + y)^2 - 97(x + y) + 168 = 0$ **[2016, online]**

56. The point (2, 1) is translated parallel to the line $L : x - y - 4 = 0$ by $2\sqrt{3}$ units. If the new point Q lies in the third quadrant, then the equation of the line passing through Q and perpendicular to L is:
 (a) $x + y = 2 - \sqrt{6}$ (b) $2x + 2y = 1 - \sqrt{6}$
 (c) $x + y = 3 - 3\sqrt{6}$ (d) $x + y = 3 - \sqrt{6}$
 [2016, online]

57. A straight line through origin O meets the lines $3y = 10 - 4x$ and $8x + 6y + 5 = 0$ at points A and B respectively. Then O divides the segment AB in the ratio:
 (a) 2 : 3 (b) 1 : 2
 (c) 4 : 1 (d) 3 : 4 **[2016, online]**

58. A ray of light is incident along a line which meets another line $7x - y + 1 = 0$, at the point (0, 1). The ray is then reflected from this point along the line, $y + 2x = 1$. Then the equation of the line of incidence of the ray of light is:
 (a) $41x - 25y + 25 = 0$ (b) $41x + 25y - 25 = 0$
 (c) $41x - 38y + 38 = 0$ (d) $41x + 38y - 38 = 0$
 [2016, online]

Previous Years' B-Architecture Entrance Examination Questions

1. The y-axis and the lines $(a^5 - 2a^3)x + (a + 2)y + 3a = 0$ and $(a^5 - 3a^2)x + 4y + a - 2 = 0$ are concurrent for
 (a) Two values of a (b) Three values of a
 (c) Five values of a (d) no value of a **[2007]**

2. If the point of intersection of the lines $2px + 3qy + r = 0$ and $px - 2qy - 2r = 0$ lies strictly in the fourth quadrant and is equidistant from the two axes, then
 (a) $5p + 4q = 0$ (b) $4p - 5q = 0$
 (c) $4p + 5q = 0$ (d) $5p - 4q = 0$ **[2009]**

3. Consider a triangle ABC with vertices at $(0, -3)$, $(-2\sqrt{3}, 3)$ and $(2\sqrt{3}, 3)$ respectively. The incentre of the triangle with vertices at the mid points of the sides of triangle ABC is
 (a) (0, 0) (b) (0, 1)
 (c) $(-\sqrt{3}, -\sqrt{3})$ (d) $(\sqrt{3}, \sqrt{3})$ **[2009]**

4. The equation of a straight line belonging to both the families of lines $x - y + 1 + \lambda_1 (2x - y - 2) = 0$ and $5x + 3y - 2 + \lambda_2 (3x - y - 4) = 0$, where λ_1, λ_2 are arbitrary numbers is:
 (a) $5x - 2y - 7 = 0$ (b) $2x + 5y - 7 = 0$
 (c) $5x + 2y - 7 = 0$ (d) $2x - 5y - 7 = 0$ **[2010]**

5. **Statement-1:** The equation $|x| + |y| = 2$ represents a parallelogram.
 Statement-2: Lines $x + y = 2$ and $x + y = -2$ are parallel. Also lines $x - y = 2$ and $-x + y = 2$ are parallel.
 (a) Statement-1 is false, Statement-2 is true;
 (b) Statement-1 is true, Statement-2 is true; statement-2 is not correct explanation for statement-1.
 (c) Statement-1 is true, Statement-2 is true; statement-2 is **not** *a* correct explanation for statement-1.
 (d) Statement-1 is true, Statement-2 is false. **[2011]**

6. If the line joining the points $A(2, 0)$ and $B(3, 1)$ is rotated about A in anti-clockwise direction through an angle of 15°, then the equation of the line in new position is
 (a) $\sqrt{3}\,x - y = 2\sqrt{3}$ (b) $\sqrt{3}\,x + y = 2\sqrt{3}$
 (c) $x + \sqrt{3}\,y = 2$ (d) $x - \sqrt{3}\,y = 2$ **[2012]**

7. If m_1 and m_2 are the roots of the equation $x^2 + (\sqrt{3} + 2)x + (\sqrt{3} - 1) = 0$, then the area of the triangle formed by the lines $y = m_1 x$, $y = -m_2 x$ and $y = 1$ is
 (a) $\frac{1}{2}\left(\frac{\sqrt{3}+2}{\sqrt{3}-1}\right)$ (b) $\frac{1}{2}\left(\frac{\sqrt{3}+2}{\sqrt{3}+1}\right)$
 (c) $\frac{1}{2}\left(\frac{-\sqrt{3}+2}{\sqrt{3}-1}\right)$ (d) $\frac{1}{2}\left(\frac{-\sqrt{3}+2}{\sqrt{3}+1}\right)$ **[2012]**

8. **Statement-1:** The line $2x + y + 6 = 0$ is perpendicular to the line $x - 2y + 5 = 0$ and second line passes through (1, 3).

   Statement-2: Product of the slopes of any two parallel lines is equal to – 1.

   (a) Statement-1 is true, statement-2 is true; statement-2 is a *correct* explanation for statement-1
   (b) Statement-1 is true, statement-2 is true; statement-2 is *not* correct explanation for statement-1.
   (c) Statement-1 is true, statement-2 is false.
   (d) Statement-1 is false, statsement-2 is true **[2013]**

9. If a variable line, passing through the point of intersection of the lines $x + 2y - 1 = 0$ and $2x - y - 1 = 0$, meet the coordinate axes in A and B, then the locus of the mid-point of AB is

   (a) $x + 3y = 0$ (b) $x + 3y = 10$
   (c) $x + 3y = 10xy$ (d) $x + 3y + 10xy = 0$ **[2013]**

10. If the point $(p, 5)$ lies on the line parallel to the y-axis and passing through the intersection of the lines $2(a^2 + 1)x + by + 4(a^3 + a) = 0$ and $(a^2 + 1)x - 3by + 2(a^3 + a) = 0$, then p is equal to

    (a) $-2a$ (b) $-3a$
    (c) $2a$ (d) $3a$ **[2014]**

11. If the points $(x, -3x)$ and (3, 4) lie on the opposite side of the line $3x - 4y = 8$, then

    (a) $x > \frac{8}{15}, y < \frac{-8}{5}$ (b) $x > \frac{8}{5}, y > \frac{-8}{15}$
    (c) $x < \frac{8}{15}, y > \frac{-8}{5}$ (d) $x = \frac{8}{15}, y = \frac{-8}{5}$ **[2015]**

12. Two vertices of a triangle are (3, – 2) and (–2, 3) and its orthocentre is (–6, 1). Then the third vertex of this triangle can NOT lie on the line:

    (a) $4x + y = 2$ (b) $5x + y = 2$
    (c) $3x + y = 3$ (d) $6x + y = 0$ **[2016]**

13. A line passing through the point $P(1, 2)$ meets the line $x + y = 7$ at the distance of 3 units from P. Then the slope of this line satisfies the equation

    (a) $7x^2 - 18x + 7 = 0$ (b) $16x^2 - 39x + 16 = 0$
    (c) $7x^2 - 6x - 7 = 0$ (d) $8x^2 - 9x + 1 = 0$ **[2016]**

Answers

Concept-based

1. (b)	**2.** (a)	**3.** (d)	**4.** (c)
5. (b)	**6.** (d)	**7.** (c)	**8.** (d)
9. (c)	**10.** (a)	**11.** (a)	**12.** (d)
13. (b)	**14.** (b)	**15.** (c)	

Level 1

16. (a)	**17.** (d)	**18.** (c)	**19.** (a)
20. (c)	**21.** (a)	**22.** (a)	**23.** (d)
24. (b)	**25.** (b)	**26.** (c)	**27.** (d)
28. (d)	**29.** (a)	**30.** (b)	**31.** (c)
32. (b)	**33.** (a)	**34.** (a)	**35.** (a)
36. (d)	**37.** (d)	**38.** (d)	**39.** (c)
40. (b)	**41.** (c)	**42.** (b)	**43.** (a)
44. (d)	**45.** (d)	**46.** (b)	**47.** (b)
48. (d)	**49.** (d)	**50.** (b)	**51.** (c)
52. (c)	**53.** (b)	**54.** (c)	**55.** (b)
56. (b)	**57.** (b)	**58.** (b)	**59.** (b)
60. (b)	**61.** (d)	**62.** (d)	**63.** (c)
64. (c)	**65.** (c)	**66.** (d)	**67.** (d)
68. (b)	**69.** (c)	**70.** (b)	**71.** (c)
72. (c)	**73.** (c)	**74.** (b)	**75.** (c)
76. (b)	**77.** (c)	**78.** (d)	**79.** (a)
80. (d)	**81.** (c)	**82.** (d)	**83.** (a)
84. (b)	**85.** (c)	**86.** (d)	**87.** (b)

Level 2

88. (c)	**89.** (d)	**90.** (d)	**91.** (c)
92. (b)	**93.** (d)	**94.** (a)	**95.** (d)
96. (c)	**97.** (d)	**98.** (b)	**99.** (c)
100. (d)	**101.** (b)	**102.** (d)	**103.** (d)
104. (b)	**105.** (c)	**106.** (a)	**107.** (c)
108. (b)			

Previous Years' AIEEE/JEE Main Questions

1. (a)	**2.** (d)	**3.** (a)	**4.** (a)
5. (c)	**6.** (c)	**7.** (a)	**8.** (d)
9. (d)	**10.** (c)	**11.** (d)	**12.** (a)
13. (d)	**14.** (c)	**15.** (a)	**16.** (a)
17. (b)	**18.** (d)	**19.** (c)	**20.** (a)
21. (c)	**22.** (a)	**23.** (d)	**24.** (b)
25. (c)	**26.** (b)	**27.** (b)	**28.** (a)
29. (a)	**30.** (a)	**31.** (d)	**32.** (d)
33. (a)	**34.** (a)	**35.** (d)	**36.** (b)
37. (a)	**38.** (d)	**39.** (b)	**40.** (c)
41. (b)	**42.** (c)	**43.** (d)	**44.** (b)
45. (b)	**46.** (c)	**47.** (d)	**48.** (b)

49. (d)	**50.** (c)	**51.** (a)	**52.** (d)
53. (a)	**54.** (c)	**55.** (a)	**56.** (d)
57. (c)	**58.** (c)		

Previous Years' B-Architecture Entrance Examination Questions

1. (a)	**2.** (d)	**3.** (b)	**4.** (a)
5. (b)	**6.** (a)	**7.** (a)	**8.** (c)
9. (c)	**10.** (a)	**11.** (a)	**12.** (b)
13. (a)			

Hints and Solutions

Concept-based

1. Coordinate of A are (3, 0) and of B are (0, 4) so coordinate of P are $\left(\dfrac{1\times 0+2\times 3}{3}, \dfrac{1\times 4+2\times 0}{3}\right) = \left(2, \dfrac{4}{3}\right)$.

2. Coordinate of the mid-point of the line segment joining P and Q are $\left(\dfrac{3-5}{2}, \dfrac{-4-2}{2}\right) = (-1, -3)$ so slope of the required line is $\dfrac{0-(-3)}{0-(-1)} = 3$

3. When PQ is parallel to y-axis $x_1 = x_2$ and the required distance is $|y_1 - y_2|$

4. Line parallel to x-axis is $y = -5$ and parallel to y-axis is $x = 4$.

5. Equation of the line is $x\cos\alpha + y\sin\alpha = p$ where $p = 3$, $\alpha = 30°$. So the required equation is $\left(\dfrac{\sqrt{3}}{2}\right)x + \left(\dfrac{1}{2}\right)y = 3$ or $\sqrt{3}\,x + y = 6$

6. Slope of the line joining (8, 2), (– 2, – 2) is $\dfrac{2}{5}$ and the slope of the line joining (8, 2), (3, 0) is also $\dfrac{2}{5}$, so the points are collinear.

7. Slopes of the given lines are $\dfrac{1}{\sqrt{3}}$ and – 1, so

$$\tan\theta = \left[\frac{\frac{1}{\sqrt{3}}+1}{1-\frac{1}{\sqrt{3}}}\right] = \left[\frac{\sqrt{3}+1}{\sqrt{3}-1}\right] = 2+\sqrt{3}.$$

8. Since the line makes zero intercept on both the axes, it must pass through the origin. So equation of the line is $y = \dfrac{a+1}{a-1}x$ as the slope of the line is $\dfrac{a+1-0}{a-1-0}$ or $(a+1)x - (a-1)y = 0$

9. We write the equation of the given line in the normal form as $-\dfrac{1}{2}x + \dfrac{\sqrt{3}}{2}y = \dfrac{8}{2}$ or $x\cos\alpha + y\sin\alpha = p$ where $\cos\alpha = \dfrac{-1}{2}$, $\sin\alpha = \dfrac{\sqrt{3}}{2}$ and $p = 4$, α being the required angle.

$\Rightarrow \alpha = 120°$

10. Product of the slopes of the given lines is – 1.

$\Rightarrow \dfrac{3}{4} \times \dfrac{7-3}{h-2} = -1$

$h = -1$

11. Equation of any line through the intersection of given line is

$7x - y + 2 + \lambda(x - 3y + 6) = 0$

It will be parallel to x-axis if

$7 + \lambda = 0 \Rightarrow \lambda = -7$ and the required equation is $y = 2$.

12. First two lines intersect at (1, 1) and the third line passes through (1, 1) if $p = 6$

Alternatively $\begin{bmatrix} 1 & -p & 5 \\ 2 & 1 & -3 \\ 3 & -1 & -2 \end{bmatrix} = 0.$

13. The given lines are $3x + 2y + 6 = 0$ and $3x + 2y - \dfrac{7}{3} = 0$.

Let the required line be $3x + 2y + k = 0$. Since it is equidistant from the given lines

$$\frac{k+\frac{7}{3}}{\sqrt{9+4}} = \frac{6-k}{\sqrt{9+4}} \quad \Rightarrow \quad k = \frac{11}{6}.$$

14. Vertices of the triangle are (0, 0), (k, k), (– k, k)

$$\text{Area} = \left(\frac{1}{2}\begin{bmatrix} 0 & 0 & 1 \\ k & k & 1 \\ -k & k & 1 \end{bmatrix}\right) = k^2$$

15. Two lines intersect at the point $\left(\dfrac{5}{7}, \dfrac{25}{21}\right)$. Distance of this point from the given point (3, 5) is

$$\sqrt{\left(3-\frac{5}{7}\right)^2 + \left(5-\frac{25}{21}\right)^2} = \frac{4\sqrt{34}}{21}$$

Level 1

16. Slope of the line joining any two points is independent of k

17. $\begin{vmatrix} k & 2-2k & 1 \\ -k+1 & 2k & 1 \\ -4-k & 6-2k & 1 \end{vmatrix} = 140$
which is true for only one integral value of k.
18. Opposite sides are parallel, all sides are equal. AB is not perpendicular to AD.
19. Product of the slopes of AB and AC is -1
20. $\frac{\lambda-3}{\lambda+1} = 0 \Rightarrow \lambda = 3$, ratio is 3 : 1
21. Vertices are (0, 4), (4, 0), (1, 1) and the sides are $\sqrt{10}, \sqrt{10}$ and $\sqrt{32}$.
22. $PQ = \sqrt{2}\ |a-b|$, $QR = \sqrt{2}\ |b-c|$ so $PQ = QR$ if $a - b = b - c$ *i.e.* a, b, c are in A.P.
23. $\frac{y-x}{b-a} = \frac{z-y}{c-b} \Rightarrow y - x = z - y$
$(\because\ b - a = c - b)$
$\Rightarrow$ x, y, z are in A.P.
24. Area is thrice the area of the triangle with third vertex at the centroid.
25. Length of BC is twice the length of the line joining the mid-point of AB and AC.
26. Triangle is right angled at B so mid-point of AC is the circumcentre.
27. Diagonals of a parallelogram bisect each other.
28. Let C (h, k), then Centroid is $\left(x = \frac{h+2+4}{3}, y = \frac{k+5-1}{3}\right)$ and $9h + 7k + 4 = 0 \Rightarrow 9(3x - 6) + 7(3y - 4) + 4 = 0$ is the locus of the centroid parallel to L_1.
29. The distance between the given points is less than 12.
30. $2OA \times OB \cos \underline{|AOB} = (OA)^2 + (OB)^2 - (AB)^2$
31. $\frac{x}{a} + y + 1 = 0, \quad x + \frac{y}{b} + 1 = 0,$

$x + y + \frac{1}{c} = 0$

$x = \frac{c-1}{a-1} \cdot \frac{a}{c}, \quad y = \frac{c-1}{b-1} \cdot \frac{b}{c}$

$x + y + \frac{1}{c} = 0 \Rightarrow \frac{a}{a-1} + \frac{b}{b-1} + \frac{1}{c-1} = 0$

$\Rightarrow \frac{a}{a-1} + \frac{b}{b-1} + \frac{c}{c-1} = 1$
32. Coordinates of D are $(5 \cos \alpha, 4 + 5 \sin \alpha)$
$\cos \alpha = 4/5$, $\sin \alpha = 3/5$
check the coordinates of C.

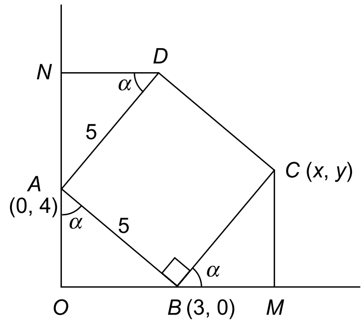


Fig. 16.21

33. Let P divide BC in the ratio $\lambda : 1$. As P lies on the line
$2\left(\frac{\lambda x_3 + x_2}{\lambda+1}\right) + 3\left(\frac{\lambda y_3 + y_2}{\lambda+1}\right) - 1 = 0$

$\Rightarrow \lambda = -\frac{2x_2 + 3y_2 - 1}{2x_3 + 3y_3 - 1} = \frac{BP}{PC}$

Similarly $\frac{CQ}{QA} = -\frac{2x_3 + 3y_3 - 1}{2x_1 + 3y_1 - 1}$

and $\frac{AR}{RB} = -\frac{2x_1 + 3y_1 - 1}{2x_2 + 3y_2 - 1}$
34. Sum of rational numbers is a rational number and a rational number divided by 3 is again a rational number.
35. $A_1 A_4$ and $A_2 A_5$ pass through the mid point of $A_0 A_3$ *i.e.* (0, 0) and make an angle of 60° with $A_0 A_3$ which is x-axis.
36. Slope of the line joining $P(2, 2)$ and $S(13/2, 1)$ is $-2/9$.
37. $\frac{m-7}{1+7m} = -\frac{m-(-1)}{1-m}$
38. The diagonals of the rhombus are parallel to the angle bisectors of the given lines. Slope of the diagonal through $A(0, a)$ is $2 - a$.
39. $\frac{x-3}{\cos\theta} = \frac{y-4}{\sin\theta} = r$
$PR = 3/\cos\theta$, $PS = 4/\sin\theta$
$RS = PS - PR$
40. Let $A(p, 0)$, B$(0, q)$ and $C(h, k)$, then
$(h-p)^2 + k^2 = a^2$, $h^2 + (k-q)^2 = b^2$
and $p^2 + q^2 = a^2 + b^2$
Eliminate p, q
41. Centre of mean position is the mid point of PQ.
42. Length of the perpendicular from (1, 1) to the line is equal to 1.
43. Mid point (3, 2) of AC lies on BD, $c = -4$. Let the coordinates of D be $(x, 2x - 4)$. $AD \perp CD \Rightarrow x = 4, 2$. The required coordinates are (4, 4) or (2, 0). As D lies below A, its coordinates are (2, 0)

44. $\begin{vmatrix} x & y & 1 \\ 2 & 1 & 1 \\ 3 & -2 & 1 \end{vmatrix} = 10 \Rightarrow 3x + y - 7 = 10$

45. Diagonals of a rhombus are at right angles.

46. $ax + \dfrac{(a+c)}{2} y + c = 0$

$a(2x + y) + c(y + 2) = 0$

which represents a family of lines passing through the intersection of $2x + y = 0$ and $y + 2 = 0$

47. L: $y = -(1/5)x + c$, meets axes at $(0, c)$ and $(5c, 0)$, $(1/2) \times 5c \times c = 5 \Rightarrow c^2 = 2$ Distance of L from the origin $= \dfrac{c}{\sqrt{1+1/25}} = \dfrac{5\sqrt{2}}{\sqrt{26}}$

48. The lines enclose a square, the length of each side being $2\sqrt{2}$.

49. Circumcentre, centroid and the orthocentre lie on the same line.

50. $SA = a(1 + t^2)$, $SB = a(1 + t^2)/t^2$

51. Equation of the bisectors of the angle between the given lines are

$$\frac{px + qy + r}{\sqrt{p^2 + q^2}} = \pm \frac{qx + py + (p + q - r)}{\sqrt{p^2 + q^2}}$$

Taking the negative sign we get $x + y + 1 = 0$ so one is the reflection of the other line, in the line $x + y + 1 = 0$.

52. Let the coordinates of A be (x, y), then

$\tan C = \dfrac{y}{a-x}$, $\tan B = \dfrac{y}{a+x}$

$\tan (C - B) = \tan (\pi/3) = \sqrt{3}$

53. A_n $(n, n\sqrt{5})$, $(0 A_n)^2 = 6n^2$, required

Sum $= 6\Sigma n^2 = \dfrac{6n(n+1)(2n+1)}{6}$.

54. Line meets the coordinate axes at $(4, 0)$ and $(0, 6)$. If (x, y) are the coordinates of an extremity of the other diagonal, then $(x - 4)^2 + y^2 = x^2 + (y - 6)^2$

$\Rightarrow$ $2x - 3y + 5 = 0$ which is satisfied by the coordinates in (c).

55. $(m + 1)(2m - 1)(m - 2) = 0$

$\Rightarrow m_1 = -1, m_2 = 1/2, m_3 = 2$

$$A = -\sum \frac{1}{m_i^2} = -\frac{21}{4}$$

$$B = \sum \frac{1}{m_i} = \frac{3}{2}$$

$$-\frac{21}{4}\alpha + \frac{3}{2}\beta = 0$$

$$\Rightarrow \quad -21\alpha + 6\beta = 0$$

which is satisfied by (b)

56. $u = 0$, $v = 0$ are parallel lines but not identical. $u + kv = 0$ represents a family of lines parallel to these lines.

57. Required line is

$(x + y + 1) + \lambda(lx + my + n) = 0$ such that $lx + my + n = 0$ is equally inclined to $x + y + 1 = 0$ and this line.

58. Triangles NDA, MBC and OAB are congruent.

$\Rightarrow$ $MB = OA = ND = 12$

and $CM = OB = NA = ON - OA = 5$

$OM = OB + BM = 5 + 12 = 17$

coordinates of $C = (OM, CM) = (17, 5)$

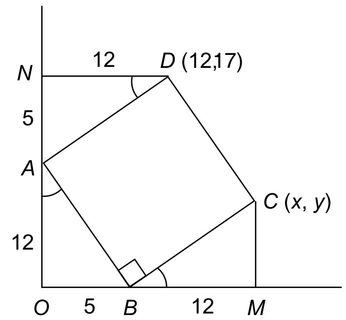


Fig. 16.22

59. Let the coordinates of A be (x, y)

AM is perpendicular to MD and $AM = (1/2)\ MD$.

then $\dfrac{y+1}{x-2} \times \dfrac{1+1}{1-2} = -1 \Rightarrow x - 2y - 4 = 0$,

$MD = \sqrt{5}$.

which is satisfied by (b) only.

60. Let the equation of the line be $\dfrac{x}{a} + \dfrac{y}{b} = 1$. Meets the coordinate axes at $A(a, 0)$ and $B(0, b)$ $(1, 1)$ divides AB in the ratio $3 : 4$.

61. Lines are perpendicular if $a + b = 0$

62. Lines are coincident if $(h)^2 = a\ .\ a$

63. If $a + b = 0$, the lines are

$(x + y)(ax - ay + c) = 0$

64. We can write $(x - y)(x - 2y)(x - 3y) = 0$

65. Let the slopes be m/r, m, mr, then product $= m^3 = -8 \Rightarrow m = -2$

and m satisfies $8 + am + bm^2 + m^3 = 0$

66. Let the slopes be m and $m + 1$, then

$2m + 1 = -a$, $m(m + 1) = 6$.

67. Product of the slopes is $1 = a/3 \Rightarrow a = 3$

68. Vertices of the rectangle are $A(1, 4)$ $B(6, 4)$, $C(6, 10)$ and $D(1, 10)$

Equation of AC is $6x - 5y + 14 = 0$

Equation of BD is $6x + 5y - 56 = 0$

AC is nearer the origin.

69. Mid point is $P(335, 1340)$. Let the required equation be $\frac{x}{k}+\frac{y}{2k} = 1$, it passes through P if $k = 1005$.

70. $Q = (\alpha, \beta) = \left(\sqrt{xy}, \frac{x+y}{2}\right)$,

mid-point of $PQ = \left(\frac{x+\sqrt{xy}}{2}, \frac{\frac{x+y}{2}+y}{2}\right)$

$\Rightarrow x + \sqrt{xy} = 84$, $x + 3y = 124$. both are satisfied by (b) (49, 25)

71. Equation of the line through $(-1, 3)$ perpendicular to $y = 2x$ is $x + 2y = 5$ which meets $y = 2x$ at $(1, 2)$. Required point is (α, β) such that $\frac{\alpha-1}{2} = 1$, $\frac{\beta+3}{2} = 2$.

72. (a^2, a) lies between the given lines
if $a^2 + a - 6 > 0$ and $a^2 + a - 2 < 0$ ... (1)
or $a^2 + a - 6 < 0$ and $a^2 + a - 2 > 0$ ... (2)
(1) does not hold for any value of a
From (2) $-3 < a < 2$ and $[a < -2$ or $a > 1]$
$\Rightarrow -3 < a < -2$ or $1 < a < 2$.
$\Rightarrow |2a + 5| < 1$ or $|2a - 3| < 1$

73. Coordinates of the vertices are (0, 0), (30, 0), (20, 30), (0, 50). Lengths of the sides are 30, $10\sqrt{10}$, $20\sqrt{2}$, 50.

74. $\frac{m_1+m_2}{m_1m_2} = 2 \Rightarrow \frac{2c}{3} = 2 \Rightarrow c = 3$

75. Let the slope of the other line be m

then $-\frac{2}{3} \times m = \frac{2c}{-(c^2-1)}$

and $-\frac{2}{3} + m = \frac{-2}{-(c^2-1)}$

$\Rightarrow -\frac{2}{3} + \frac{3c}{c^2-1} = \frac{2}{c^2-1}$

$\Rightarrow 2c^2 - 9c + 4 = 0$

76. In statement-1, let the vertex C be (α, β), then centroid is $\left(\frac{\alpha+2-2}{3}, \frac{\beta-3+1}{3}\right)$ which lies on $x + y = 5 \Rightarrow \alpha + \beta = 17$
In statement-2, $x - y + 1 = 0$ is the equation of the perpendicular bisector of AB which is also the median through C.

77. Statement -1 is true, as the lines are $x + 1 = 0$, $y - 2 = 0$, $x - 4 = 0$
Statement -2 is false as the two sets of lines are $x + 1 = 0$, $y - 2 = 0$ and $x - 1 = 0$, $y + 2 = 0$

78. Statement -2 is true, the given equation represents points which satisfy $x + 4y - 2 = 0$, $4x + y - 8 = 0$ and $x \pm y - k = 0$. Point of intersection of first two is (2, 0) which also satisfies the third equation if $k = 2$.
Statement -1 is false, as the locus represents a point, the point of intersection of the lines given.

79. Statement-2 is true and using it statement-1 is also true as the slope of $PQ = \frac{\sin\beta+\cos\beta}{\cos(\beta-\alpha)+\sin(\beta-\alpha)}$,

Slope of $QR = \frac{\sin\beta-\sin(\beta-\theta)}{\cos(\beta-\alpha)-\cos(\beta-\alpha+\theta)}$

$= \frac{\cos(\beta-(\theta/2))}{\sin(\beta-\alpha+(\theta/2))}$

80. Statement-2 is true, using it in statement-1 hypotenuse makes an angle $\pi/4$ with $x = a$ which is parallel to y-axis, so it makes an angle $\pi/4$ with x-axis also and then the slope of the hypotenuse is ± 1
$\Rightarrow$ $k = \pm 1$, statement-1 is false

81. Statement-2 is false, In statement-1
$x = \frac{a\cos t+b\sin t-1}{3}$, $y = \frac{a\sin t-b\cos t}{3}$
$\Rightarrow$ $(3x - 1)^2 + (3y)^2 = a^2 + b^2$, statement-1 is true.

82. Statement-2 is true, using it $\frac{2}{b}=\frac{1}{a}+\frac{1}{c}$ $\Rightarrow$ the line $\frac{x}{a}+\frac{y}{b}+\frac{1}{c} = 0$ passes through the point $(1, -2)$, so the statement-1 is false.

83. Statement-2 is true, equation in statement-1 is
$\left(x+\sqrt{3}.y+5\right)$ $\sin\theta$ + $\left(\sqrt{3}x-y-7\right)$ $\cos\theta = 0$

or $\left(x+\sqrt{3}y+5\right)$ + $\tan\theta$ $\left(\sqrt{3}x-y-7\right) = 0$

$(\cos\theta \neq 0)$
Using statement-2, statement-1 is true.

84. Statement-1, equation of the third line is $y = k$, vertices are $(1, 1)$,(k, k) and $(2 - k, k)$

$\Rightarrow \quad 4h^2 = \frac{1}{2}\begin{vmatrix} 1 & 1 & 1 \\ k & k & 1 \\ 2-k & k & 1 \end{vmatrix} \Rightarrow (k-1)^2$

So statement-1 is true. Area of the first triangle in statement-2 is $\frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ 1 & 1 & 1 \\ 2 & 0 & 1 \end{vmatrix}$ = 1 and the area of the second is $\left(\frac{1}{2}\right)$ (2) (2) = 2 and statement-2 is also true but does not lead to statement-1

85. Statement-2 is false, PQ should be perpendicular to the line and mid point of PQ lies on the line. Using this correct statement, statement-1 is true.
86. Statement-1 is false, the lines are coincident as the statement-2 is true and statement-1 satisfies the condition in statement-2.
87. In statement-1, $(x^2 - 1)(y^2 - 1) = 0 \Rightarrow x = \pm 1$, $y = \pm 1$ which enclose a square of each side equal 2, so statement-1 is true. Statement-2 is also true as the sum of the coefficients of x^2 and y^2 is zero but does not imply statement-1

Level 2

88. The given triangle is right-angled with vertices $A(0, 0)$, $B(2, 0)$ and $C(0, 2)$.
Coordinates of the incentre are

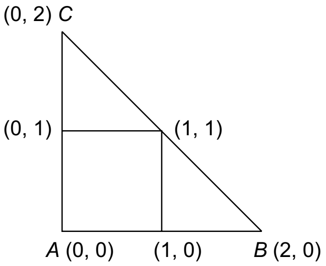


Fig. 16.23

$$\left(\frac{a\,x_1 + b\,x_2 + c\,x_3}{a+b+c}, \frac{a\,y_1 + b\,y_2 + c\,y_3}{a+b+c}\right)$$

$$= \left(\frac{2\sqrt{2}\times 0 + 2\times 2 + 2\times 0}{2\sqrt{2}+2+2}, \frac{2\sqrt{2}\times 0 + 2\times 0 + 2\times 2}{2\sqrt{2}+2+2}\right)$$

$$= \left(2-\sqrt{2}, 2-\sqrt{2}\right)$$

89. Sides of the triangle are
$a = BC = \sqrt{4+9} = \sqrt{13}$, $b = \sqrt{2}$, $c = \sqrt{5}$
The greatest angle A is given by

$$\cos A = \frac{b^2 + c^2 - a^2}{2\,bc} = \frac{2+5-13}{2\times\sqrt{2}\times\sqrt{5}} = \frac{-3}{\sqrt{10}}$$

90. The given points are collinear.
91. Area of the rhombus
$= \frac{1}{2} \times$ product of the diagonals

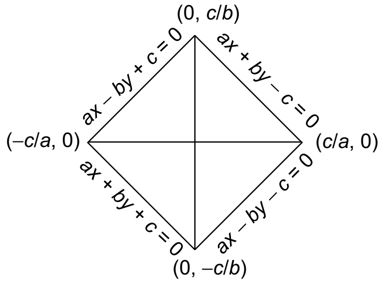


Fig. 16.24

$$= \frac{1}{2}\times\frac{2\,c}{a}\times\frac{2\,c}{b} = \frac{2\,c^2}{ab}$$

92. $l_1 = a^2 \cos\alpha + 2a \sin\alpha + \sin\alpha \tan\alpha$
$l_2 = ab \cos\alpha + (a + b) \sin\alpha + \sin\alpha \tan\alpha$
$l_3 = b^2 \cos\alpha + 2b \sin\alpha + \sin\alpha \tan\alpha$
$\Rightarrow l_1 l_3 = l_2^2 \Rightarrow l_1, l_2, l_3$ are in G.P.
93. Length of $AB = \sqrt{26}$

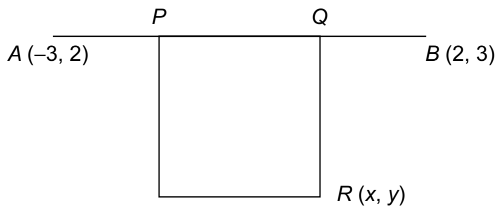


Fig. 16.24

Length of $PQ = \sqrt{26}/3$

Coordinates of Q are $\left(\frac{2\times 2-3}{3}, \frac{2\times 3+2}{3}\right)$

$= \left(\frac{1}{3}, \frac{8}{3}\right)$

R (x, y) lies on the line through Q perpendicular to $AB \Rightarrow y - (8/3) = -5\,(x - (1/3))$

$\Rightarrow 5x + y = 13/3$ (1)

Equation of AB is $x - 5y + 13 = 0$

Distance of R from AB is equal to PQ.

$$\Rightarrow \left|\frac{x-5y+13}{\sqrt{1+25}}\right| = \frac{\sqrt{26}}{3}$$

$\Rightarrow x - 5y + 13 = \pm\, 26/3$ (2)

Solving (1) & (2) we get R $(x, y) = (0, 13/3)$ or $(2/3, 1)$

94. The given equation can be written as
$x(y_1 - y_2 + y_1 - y_3) - y(x_1 - x_2 + x_1 - x_3)$
$+ x_1y_2 - x_2y_1 + x_1y_3 - y_1x_3 = 0$
$\Rightarrow x\,(2y_1 - y_2 - y_3) - y\,(2x_1 - x_2 - x_3)$
$+ y_1\,(2x_1 - x_2 - x_3) - x_1(2y_1 - y_2 - y_3) = 0$

$$\Rightarrow y - y_1 = \frac{2y_1 - y_2 - y_3}{2x_1 - x_2 - x_3}(x - x_1)$$

which represents the median through A

95. Three lines $x + 2y - 3 = 0$, $2x + 3y - 4 = 0$ and $4x + 5y - 6 = 0$ are concurrent at $(-1, 2)$. But the fourth line $3x + 4y - 7 = 0$ does not pass through this point.

96. Slope of the incident ray PA is $2/(1 - x)$ and that of the reflected ray QA is $3/(5 - x)$

so $\dfrac{2}{1-x} = \dfrac{-3}{5-x} \Rightarrow x = \dfrac{13}{5} \Rightarrow A\left(\dfrac{13}{5}, 0\right)$

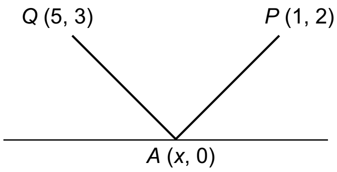


Fig. 16.26

97. $$\Delta_1 = \frac{1}{2}\left|\begin{vmatrix} 0 & 0 & 1 \\ a\tan\alpha & b\cot\alpha & 1 \\ a\sin\alpha & b\cos\alpha & 1 \end{vmatrix}\right|$$

$$= \frac{1}{2}ab\,|(\sin\alpha - \cos\alpha)|$$

$$\Delta_2 = \frac{1}{2}\left|\begin{vmatrix} a & b & 1 \\ a\sec^2\alpha & b\cos es^2\alpha & 1 \\ a + a\sin^2\alpha & b + b\cos^2\alpha & 1 \end{vmatrix}\right|$$

$$= \frac{1}{2}ab\ [\text{cosee}^2\ \alpha - 1 - \cos^2\ \alpha + 1 + \sin^2\ \alpha -$$
$$\sec^2\ \alpha + \sec^2\alpha + 1 - \text{cosec}^2\alpha - 1]$$

$$= \frac{1}{2}\ ab\ |(\sin^2\ \alpha - \cos^2\ \alpha)|$$

and $$\Delta_3 = \frac{1}{2}\left|\begin{vmatrix} 0 & 0 & 1 \\ a\tan\alpha & -b\cot\alpha & 1 \\ a\sin\alpha & b\cos\alpha & 1 \end{vmatrix}\right|$$

$$= \frac{1}{2}ab\,|(\sin\alpha + \cos\alpha)|$$

Now $\Delta_1\,\Delta_3 = (1/4)\ a^2\ b^2\ |\sin^2\alpha - \cos^2\alpha| = \Delta_2^2$

$\Rightarrow |\sin^2\ \alpha - \cos^2\ \alpha| = 0$ or 1

$\Rightarrow \alpha = \pm\ \pi/4$ or $\alpha = \pm\ \pi/2$

But for these values of α, the triangles are not defined.

Hence Δ_1, Δ_2, Δ_3 are in G.P. for no value of α.

98. The orthocentre (x, y) lies on the lines.

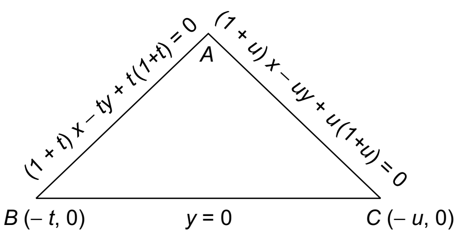


Fig. 16.27

$$y = \frac{-u}{1+u}(x+t)$$

and $y = \dfrac{-t}{1+t}(x+u)$

$\Rightarrow [(1 + u) - (1 + t)]\ y + (u - t)\ x = 0$

$\Rightarrow (u - t)\ (x + y) = 0$

$\Rightarrow x + y = 0$ as $u \neq t$

99. Equation of AU is

$$y - y_1 = \frac{0 - y_1}{3 - x_1}(x - x_1)$$

so coordinates of C are $(0,\ 3y_1/(3 - x_1))$

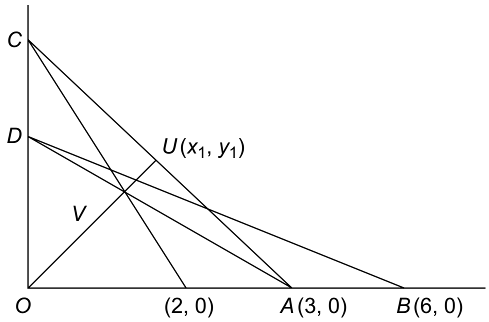


Fig. 16.28

Similarly coordinates of D are $(0,\ 6y_1/(6 - x_1))$

Equation of AD is $\dfrac{x}{3} + \dfrac{y(6 - x_1)}{6y_1} = 1$

Equation of OU is $y_1x = x_1y$

Solving, we get the coordinates of

$$V = \left(\frac{6x_1}{6+x_1}, \frac{6y_1}{6+x_1}\right)$$

Equation of CV is

$$y = \frac{3y_1}{3-x_1} - \frac{3y_1}{2(3-x_1)}\,x = \frac{3y_1}{3-x_1}\left(1 - \frac{x}{2}\right)$$

which passes through (2, 0) for all values of (x_1, y_1)

100. The triangle is equilateral, so the incentre is the centroid $(1,\ 1/\sqrt{3})$ of the triangle

101. Equation of the fourth side is $2x - y - k = 0$

such that $\dfrac{4-k}{\sqrt{5}} = \pm\dfrac{3+7}{\sqrt{5}} \Rightarrow k = -\,6$ or 14.

So the required equation is $2x - y + 6 = 0$, as it is nearer to $(1, -1)$

102. The triangle is right angled at (0, 0) so the orthocentre is (0, 0) and circumcentre is $(a/2,\ b/2)$, the mid point of the hypotenuse.

103. Third vertex of the triangle is (–1, –11)

104. $A(0, 12)$, $B(8, 0)$, $C(x, y)$ lies on the line

$$2x - 3y + 10 = 0 \text{ and } \frac{1}{2}\begin{vmatrix} x & y & 1 \\ 0 & 12 & 1 \\ 8 & 0 & 1 \end{vmatrix} = \pm 91$$

Solving we get $(x, y) = (-13/2, -1)$ or $(29/2, 13)$

105. $$\tan A = \frac{\frac{1}{2}+7}{1-\frac{7}{2}} = -3$$

$$\tan C = \frac{\frac{4}{3}+\frac{1}{3}}{1-\frac{4}{3}\times\frac{1}{3}} = 3$$

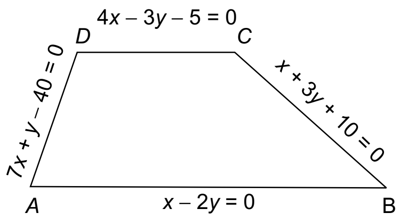


Fig. 16.29

$\Rightarrow \tan(A + C) = 0$

$\Rightarrow A + C = 180°$

$\Rightarrow ABCD$ is a cyclic quadrilateral

Note: $ABCD$ cannot be a rectangle or a parallelogram, so verify for cyclic quadrilateral.

106. If given lines are concurrent at (–35, 22), then

$35a - 22b - 1 = 0$

$\Rightarrow$ The line $35x - 22y - 1 = 0$ passes through (a, b).

107. We can write $$\left(\frac{y}{x}\right)^2 + \frac{2h}{b}\left(\frac{y}{x}\right) + \frac{a}{b} = 0$$

If the slopes of the lines are m and m^2 then

$$m + m^2 = -\frac{2h}{b} \text{ and } mm^2 = \frac{a}{b}$$

$$\Rightarrow \left(\frac{a}{b}\right)^{1/3} + \left(\frac{a}{b}\right)^{2/3} = -\frac{2h}{b}$$

$$\Rightarrow \frac{a}{b} + \left(\frac{a}{b}\right)^2 + 3\left(\frac{a}{b}\right)\left(-\frac{2h}{b}\right) = -\frac{8h^3}{b^3}$$

$$\Rightarrow \frac{a}{b^2}[b + a - 6h] + \frac{8h^3}{b^3} = 0$$

$$\Rightarrow a + b - 6h + \frac{8h^3}{ab} = 0$$

$$\Rightarrow \frac{a+b}{h} + \frac{8h^2}{ab} = 6.$$

108. From the given equation we have

$x - y + c = 0$ and $x + y - c = 0$ which represents the point $(0, c.)$

Previous Years' AIEEE/JEE Main Questions

1. Since the given equation represents a point lines we have

$$abc + 2fgha - af^2 - bg^2 - ch^2 = 0 \quad (1)$$

as they intersect on y-axis,

$$by^2 + 2fy + c = 0 \quad (2)$$

(2) gives a unique value of y if $4f^2 = 4bc$

$\Rightarrow af^2 = abc$ and from (1) we have

$$2fgh = bg^2 + ch^2.$$

2. Let (h, k) be mid-point of the portion, then

$$h = \frac{p}{2\cos\alpha},\ k = \frac{p}{2\sin\alpha}$$

$$\Rightarrow 1 = \cos^2\alpha + \sin^2\alpha = \frac{p^2}{4h^2} + \frac{p^2}{4k^2}$$

$$\Rightarrow \frac{1}{h^2} + \frac{1}{k^2} = \frac{4}{p^2}$$

Locus of (h, k) is $\frac{1}{x^2} + \frac{1}{y^2} = \frac{4}{p^2}$

3. Let A (4, 0), B (–1, – 1), C (3, 5)

$(AB)^2 = 26$, $(BC)^2 = 52$, $(AC)^2 = 26$

so the triangle is right angled.

4. $3x + 4y = 0$ and $4x - 3y = 0$ are at right angle so the triangle is right angled.

5. Coordinates of A are $(a\cos\alpha, a\sin\alpha)$ and of C are $(a\cos(\alpha + \pi/2)a\sin(\alpha + \pi/2))$

i.e. $(-a\sin\alpha, a\cos\alpha)$

So the equation of the diagonal AC is

$$y - a\sin\alpha = \frac{a(\sin\alpha - \cos\alpha)}{a(\cos\alpha - \sin\alpha)}(x - a\cos\alpha)$$

$\Rightarrow y(\cos\alpha + \sin\alpha + x(\sin\alpha - \cos\alpha) = a$

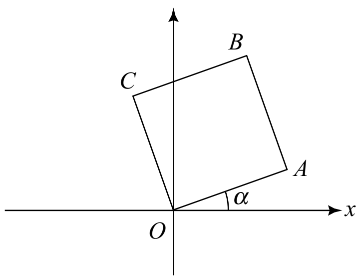


Fig. 16.30

6. Equation of the bisectors of $x^2 - 2pxy - y^2 = 0$ is

$$\frac{x^2 - y^2}{2} = \frac{xy}{-p}$$

or $x^2 + \frac{2}{p}xy - y^2 = 0$

Comparing with $x^2 - 2qxy - y^2 = 0$ we get $pq = -1$

7. Let (h, k) be the centroid of the triangle, then

$$h = \frac{a\cos t + b\sin t + 1}{3},\ k = \frac{a\sin t - b\cos t}{3}$$

$\Rightarrow (3h - 1)^2 + (3k)^2 = a^2 + b^2$

Locus of (h, k) is

$(3x - 1)^2 + (3y)^2 = a^2 + b^2$

8. Let $x_2 = x_1r$, $x_3 = x_1r^2$

$y_2 = y_1r$, $y_3 = y_1r^2$

then

$$\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \begin{vmatrix} x_1 & y_1 & 1 \\ x_1r & y_1r & 1 \\ x_1r^2 & y_1r^2 & 1 \end{vmatrix} = 0$$

So the given points lie on a straight line.

9. Mid-point $\left(\frac{a_1 + a_2}{2}, \frac{b_1 + b_2}{2}\right)$ of the given points lies on the locus, so

$$(a_1 - a_2)\frac{(a_1 + a_2)}{2} + (b_1 - b_2)\frac{(b_1 + b_2)}{2} + c = 0$$

$$\Rightarrow \quad c = \frac{1}{2}\left(a_2^2 + b_2^2 - a_1^2 - b_1^2\right)$$

10. Let the third vertex be $C(h, k)$. Centroid of ΔABC is

$$\frac{1}{2}\left(a_2^2 + b_2^2 - a_1^2 - b_1^2\right) = G\left(\frac{h}{3}, \frac{k-2}{3}\right)$$

which lies on $2x + 3y = 1$.

$$\Rightarrow \quad \frac{2h}{3} + 3\left(\frac{k-2}{3}\right) = 1 \Rightarrow 2h + 3k = 9$$

11. Let the intercepts on the axes be a and $-a - 1$. Equation of such a straight line

$$\frac{x}{a} - \frac{y}{a+1} = 1$$

As it pass through (4, 3) we get

$$\frac{4}{a} - \frac{3}{a+1} = 1 \quad \Rightarrow 4a + 4 - 3a = a(a + 1)$$

$\Rightarrow \quad a^2 = 4 \Rightarrow a = \pm 2$

Thus, required lines are

$$\frac{x}{2} - \frac{y}{3} = 1 \text{ and } \frac{x}{-2} + \frac{y}{1} = 1$$

12. Slopes m_1, m_2 of two lines are given by

$$1 - 2cm - 7m^2 = 0$$

We are given $\quad m_1 + m_2 = 4m_1m_2$

$$\Rightarrow \quad \frac{-2c}{7} = 4\left(\frac{-1}{7}\right) \Rightarrow c = 2$$

13. Slopes of the two lines are connected by

$$6 - m + 4cm^2 = 2$$

As $y = -\frac{3}{4}x$ is one of the lines

$$6 + \frac{3}{4} + 4c\left(-\frac{3}{4}\right)^2 = 0$$

$$\Rightarrow \quad \frac{9}{4}c = \frac{-27}{4} \Rightarrow c = -3$$

14. A straight line passing through the point of intersection of given lines is

$ax + 2by + 3b + k(bx - 2ay - 3a) = 0$

or $\quad (a + kb)x + 2(b - ka)y + 3(b - ka) = 0$

This line will be parallel to the x-axis if $a + kb = 0$ or $\quad k = -a/b$

Thus, the line parallel to the x-axis is

$\frac{2}{b}(a^2 + a^2)y + \frac{3}{b}(b^2 + a^2) = 0 \quad$ or $2y + 3 = 0$

$\Rightarrow \quad y = -3/2$

This line is below the x-axis at a distance of (3/2) from it.

15. Let coordinates of B be (x, y). Then

$$\frac{x+1}{2} = -1, \frac{y+1}{2} = 2$$

$\Rightarrow \quad x = -3, y = 3$

$\therefore$ Coordinates of $B(-3, 3)$

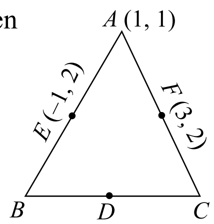


Fig. 16.31

Similarly coordinates of C are (5, 3)

Thus, centroid of ΔABC is

$$\left(\frac{1-3+5}{3}, \frac{1+3+3}{3}\right) = \left(1, \frac{7}{3}\right)$$

16. a, b, c are in H.P.

$$\Rightarrow \quad \frac{1}{a}, \frac{1}{b}, \frac{1}{c} \text{ are A.P.}$$

$$\Rightarrow \quad \frac{1}{a} - \frac{2}{b} + \frac{1}{c} = 0$$

Thus, $\frac{x}{a}+\frac{y}{b}+\frac{1}{c}=0$ passes through (1, –2).

17. As area of one of the sectors thrice that of the other, $\pi - \theta = 3\theta$ or $\theta = \pi/4$.

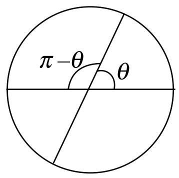


Fig. 16.32

Thus, angle between two lines is $\pi/4$ or $3\pi/4$

$\therefore \quad \pm 1 = \frac{2\sqrt{(a+b)^2 - ab}}{a+b}$

$\Rightarrow \quad (a + b)^2 = 4(a^2 + b^2 + ab)$

$\Rightarrow \quad 3a^2 + 2ab + 3b^2 = 0$

18. Let equation of straight line be

$\frac{x}{a}+\frac{y}{b}=1$

It meets the axes in $P(a, 0)$ and $Q(0, b)$. As the mid-point of PQ is $A(3, 4)$, we get

$$\left(\frac{a}{2},\frac{b}{2}\right)=(3, 4)$$

$\Rightarrow \quad a = 6,\ b = 8$

Thus, required line is

$$\frac{x}{6}+\frac{y}{8}=1$$

or $\quad 4x + 3y = 24$

19. $AB^2 + BC^2 = AC^2$

$\Rightarrow (h - 1)^2 + (k - 1)^2 + 1$
$= (h - 2)^2 + (k - 1)^2$

$\Rightarrow \quad h = 1$

Area of $\quad \Delta ABC = 1$

$\Rightarrow \quad \frac{1}{2}(1)\,|k-1|=1$

$\Rightarrow \quad |k - 1| = 2 \quad \Rightarrow \quad k - 1 = \pm 2$

$\Rightarrow \quad k = 3, -1$

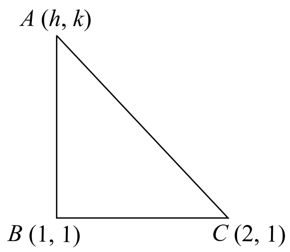


Fig. 16.33

20. $\underline{|RQX} = 60°$

Let QS be the bisector of $\underline{|PQR}$, then

$\underline{|RQS} = 60°$

$\Rightarrow \quad \underline{|SQX} = 120°$

$\Rightarrow$ Slope of $QS = -\sqrt{3}$

$\therefore$ Equation of QS is

$y = -\sqrt{3}\,x$ or $\quad \sqrt{3}\,x + y = 0$

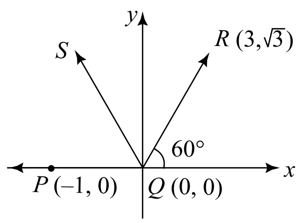


Fig. 16.34

21. Bisectors of lines $xy = 0$ i.e. $x = 0$, $y = 0$ are $y = \pm x$.

For $m = 1$, the given equation reduces to $y^2 - x^2 = 0$

22. Mid-point of PQ is $\left(\frac{k+1}{2},\frac{7}{2}\right)$

Equation of the perpendicular bisector of PQ in

$$y-\frac{7}{2}=-\frac{k-1}{3-4}\left(x-\frac{k-1}{2}\right)$$

y-intercept $= \frac{(k-1)(k+1)}{-2}+\frac{7}{2}=4$

$\Rightarrow \quad k^2 = 16 \quad \Rightarrow k = \pm 4$

23. Slope of two lines are

$m_1 = \frac{1}{p(p^2+1)}, m_2 = -\frac{1}{p\left(p^2+1\right)} = -\frac{1}{p^2+1}$

Two lines will be perpendicular to a common line if these two lines are parallel.

$\therefore \quad -\frac{1}{p\left(p^2+1\right)} = -\frac{1}{p^2+1}$

$\Rightarrow \quad p = -1$

For $p = -1$, two lines become

$-2x - y + q = 0$

and $4x + 2y + 2q = 0$ which are parallel.

24. As L: $\frac{x}{5}+\frac{y}{b}=1$ passes through (13, 32), we get

$\frac{13}{5}+\frac{32}{b}=1 \;\Rightarrow\; \frac{32}{b}=1-\frac{13}{5}=-\frac{8}{5}$

$\Rightarrow \quad b = -20$

$\therefore$ equation of line L is $\frac{x}{5}-\frac{y}{20}=1$

As K: $\frac{x}{c}+\frac{y}{3}=1$ is parallel to L.

$$\frac{1/5}{1/c}=\frac{-1/20}{1/3}\Rightarrow\frac{c}{5}=\frac{-3}{20}\Rightarrow c=\frac{-3}{4}$$

Thus, equation of K is $\frac{-4x}{3}+\frac{y}{3}=1$.

$\therefore$ Equation of L and K are

$4x - y - 20 = 0$ and $4x - y + 3 = 0$

Distance between L and K is

$$\frac{|-20-3|}{\sqrt{16+1}}=\frac{23}{\sqrt{17}}$$

25. We have $OP = 2\sqrt{2}$ and $OQ = \sqrt{5}$.

R divides PQ is the ratio $2\sqrt{2}:\sqrt{5}$, that is,

PR: RQ = $2\sqrt{2}:\sqrt{5}$

However, statement-2 is false.

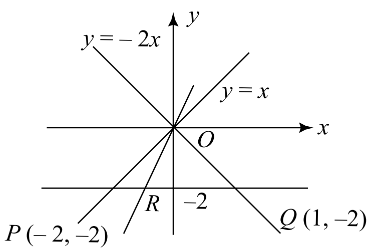


Fig. 16.35

26. If $a > 0$, the given lines intersect at the point $(1, a - 1)$. Since this point lies in the first quadrant, $a - 1 > 0 \Rightarrow a > 1$. If $a < 0$, then the equation $ax - y = 1$ gives $y = ax - 1$ which is negative for all positive values of x. Hence $a < 0$ is not possible and the required values of a are in the interval $(1, \infty)$.

27. Let $G(h, k)$ be centroid of $\Delta\ ABC$, then coordinates of C are $(3h - 2 + 2, 3k + 3 - 1) = (3h, 3k + 2)$

As C lies on $2x + 3y = 9$, we get

$$2(3h) + 3\,(3k + 2) = 9$$

$$\Rightarrow \qquad 2h + 3k = 1$$

Thus, locus of centroid G is $2x + 3y = 1$

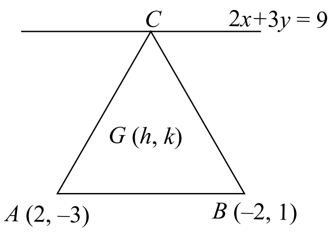


Fig. 16.36

28. Coordinates of the point dividing the join of the points (1, 1), (2, 4) in the ratio 3 : 2 are $\left(\frac{3\times2+2\times1}{3+2},\frac{3\times4+2\times1}{3+2}\right)=\left(\frac{8}{5},\frac{14}{5}\right)$

$2x + y = k$ passes through this points.

if $2 \times (8/5) + (14/5) = k \quad \Rightarrow \quad k = 6$

29. Let equation of line through (1, 2) be

$$y - 2 = m(x - 1).$$

It meets the coordinate axes in $P\left(\frac{m-2}{m},0\right)$ and $Q(0, 2-m)$.

Let A = Area of ΔOPQ = $\frac{1}{2}\left|\frac{m-2}{m}\right||m-2|$

$$=\frac{1}{2}\frac{(m-2)^2}{|m|}$$

$$=\frac{1}{2}\left||m|+\frac{4}{|m|}-\frac{4m}{|m|}\right|$$

$$=\frac{1}{2}\left[\left(\sqrt{|m|}-\frac{2}{\sqrt{|m|}}\right)^2+4\left(1-\frac{m}{|m|}\right)\right]$$

Note A is least if $|m| = 2$ or $m = \pm 2$.

For $m = 2$, the line passes through the origin, and P and Q coincide.

$\therefore$ required value of m is -2.

30. Ray of light $x+\sqrt{3}y=\sqrt{3}$ meets the x-axis at $(\sqrt{3}, 0)$. The reflect ray makes an angle of 30° with the x-axis. Its equation is

$$y-0=\frac{1}{\sqrt{3}}(x-\sqrt{3})\Rightarrow\sqrt{3}y=x-\sqrt{3}$$

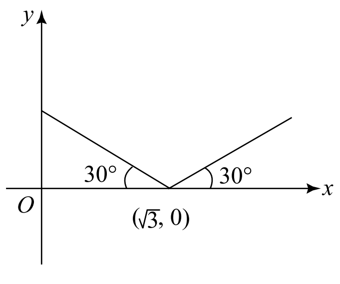


Fig. 16.37

31. Solving the given equation we get

$\frac{8y}{3}+y^2=1 \quad \Rightarrow \quad 3y^2 + 8y - 3 = 0$

$\Rightarrow \quad y = \frac{1}{3}$ or $y = -3$ but $y \neq -3$.

so $y = \frac{1}{3}$ and the line $3y - 1 = 0$ passes through the point of intersection of the the two curves.

32. Slope of PQ = – (Slope of RQ)

$$\Rightarrow \quad \frac{3}{1-x} = \frac{-7}{6-x}$$

$$\Rightarrow \quad x = \frac{5}{2}$$

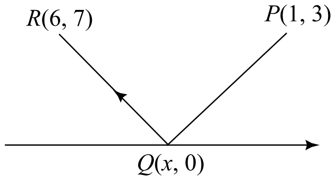


Fig. 16.38

33. Slopes of the given lines are

$$\frac{1}{3}, -\frac{a}{2}, -a$$

Two of the lines are perpendicular.

$$\text{if} \quad \frac{1}{3}\left(-\frac{a}{2}\right) = -1 \quad \text{or} \quad \frac{1}{3}(-a) = -1$$

$\Rightarrow \quad a = 6 \quad$ or $\quad a = 3$

$\Rightarrow \quad a$ satisfies the equation

$a^2 - 9a + 18 = 0$

34. Vertices of the triangle are

$A(0, 0)$, $B(2, 0)$, $C(0, 2)$.

$AB = AC = 2$ and $BC = 2\sqrt{2}$

So x-coordinates of the incentre is

$$\frac{2 \times 0 + 2 \times 2 + 2\sqrt{2} \times 0}{2 + 2 + 2\sqrt{2}}$$

$$= \frac{4}{4 + 2\sqrt{2}} = 2 - \sqrt{2}$$

35. $3x + 4y = 12$

$$\Rightarrow \quad \frac{x}{4} + \frac{y}{3} = 1$$

Equation of L is $\frac{x}{8} + \frac{4}{3/2} = 1$

Slope of L = $-\frac{3}{2} \times \frac{1}{8} = -\frac{3}{16}$

36. Let $A(2a, 0)$, $B(0, a)$ and $C(2a, y)$

$BC = AC \Rightarrow (2a)^2 + (y - a)^2 = y^2$

$$\Rightarrow \quad y = \frac{5a}{2}$$

Area of the triangle

$$= \frac{1}{2} AC \times 2a = \frac{1}{2} y \times 2a = ay$$

$$= \frac{5a^2}{2} \text{ sq. units}$$

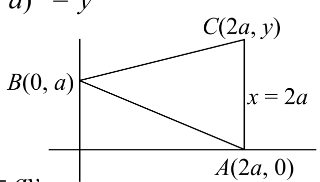


Fig. 16.39

37. $2x + 3y + c_1 = 0$ makes the same angle with the lines $-x + 5y + c_2 = 0$ and $-x + 5y + c_1 = 0$ as they are parallel. Hence $\theta_1 = \theta_2$ for all c_1 and c_2. So statement-2 is true and thus it follows that statement-1 in also true.

38. Slope of $PQ = 1$.
Mid point of PQ is (3, 4). So equation of L is

$$y - 4 = -(x - 3)$$

$$\Rightarrow \quad x + y - 7 = 0$$

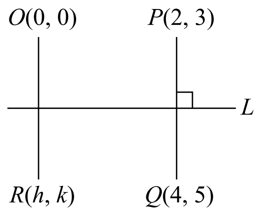


Fig. 16.40

Let the image of θ (0, 0) be R (h, k) the slope of OR = slope of PQ

$$\Rightarrow \quad \frac{k}{h} = 1 \quad \Rightarrow \quad k = h$$

Also mid-point $\left(\frac{h}{2}, \frac{k}{2}\right)$ of OR lies on L

$$\therefore \quad \frac{h}{2} + \frac{k}{2} - 7 = 0 \quad \Rightarrow \quad h = k = 7$$

and the required point is (7, 7)

39. See Question No. 10 (2004)

40. Let the point of intersection be (h, – h) then

$4ah - 2ah + c = 0$ and $5bh - 2bh + d = 0$

$$\Rightarrow \quad h = \frac{-c}{2a} = \frac{-d}{3b} \quad \Rightarrow \quad 3bc - 2ad = 0$$

41. Coordinates of S are $\left(\frac{13}{2}, 1\right)$.

Slope of PS is $\dfrac{2-1}{2-\frac{13}{2}} = -\dfrac{2}{9}$

Required equation of the live is

$$y + 1 = -\frac{2}{9}(x - 1)$$

$$\Rightarrow \quad 2x + 9y + 7 = 0$$

42. Let (h, k) be the foot of perpendicular from (0, 0) to the line $\frac{x}{a} + \frac{4}{b} = 1$. If this length of perpendicular is p, then

$$p = \frac{\left|\frac{0}{a} + \frac{0}{b} - 1\right|}{\sqrt{(1+a)^2 + (1/b)^2}} = 2$$

Now,

$$\sqrt{h^2 + k^2} = p = 2$$

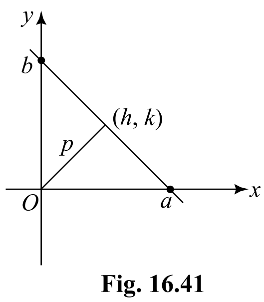


Fig. 16.41

$\Rightarrow h^2 + k^2 = 4$

Thus (h, k) likes on a circle of radius 2.

43. Coordinates of R are (2, 0). As PQ is parallel to the x-axis, and meets $x - 2y = 2$ in Q, coordinates of Q are (8, 3). Centroid of ΔPQR is (5, 2) and it lies on $2x - 5y = 0$

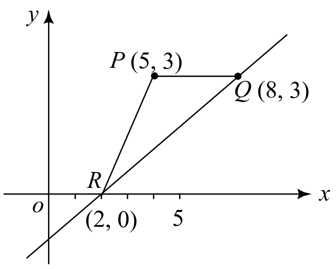


Fig. 16.42

44. If a is the side of the equilateral triangle, then

$$\frac{\sqrt{3}}{2}a = \text{AM}$$

$$= \frac{|3(1)+4(2)-9|}{\sqrt{3^2+4^2}}$$

$$\Rightarrow \quad a = \frac{2}{5}\cdot\frac{2}{\sqrt{3}} = \frac{4}{15}\sqrt{3}$$

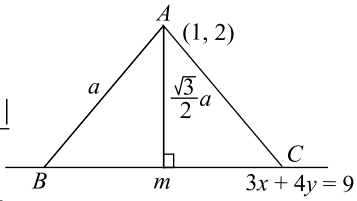


Fig. 16.43

45. We have

$$\left(\frac{2h}{3}, \frac{k}{3}\right) = (4, 3)$$

$\Rightarrow h = 6,\ k = 9$

Equation of line is

$$\frac{x}{6} + \frac{y}{9} = 1 \quad \text{or } 3x + 2x = 18$$

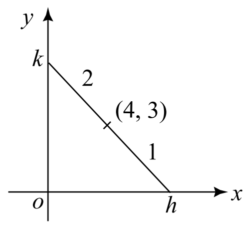


Fig. 16.44

46. As the lines are distinct $a \neq 0$. As the lines are concurrent.

$$\begin{vmatrix} 1 & 2a & a \\ 1 & 3b & b \\ 1 & 4a & a \end{vmatrix} = 0$$

$R_3 \to R_3 - R_1$, we get

$$\begin{vmatrix} 1 & 2a & a \\ 1 & 3b & b \\ 0 & 2a & 0 \end{vmatrix} = 0$$

$\Rightarrow \quad -2a(b - a) = 0 \Rightarrow a = b$

Thus, (a, b) lies on a straight line.

47. We know that circumcentre. centroid and othocentre of a triangle lie on a straight line, say L.
centroid of triangle is

$$\left(\frac{a^2+1+2a}{2}, \frac{a^2+1-2a}{2}\right) = \left(\frac{1}{2}(a+1)^2, \frac{1}{2}(a-1)^2\right)$$

Equation of line L is

$$\frac{x-0}{(a+1)^2/2-0} = \frac{y-0}{(a-1)^2/2-0}$$

$$\Rightarrow \quad (a-1)^2 x - (a+1)^2 y = 0$$

48. An equation of line perpendicular to $5x - y = 1$ is $x + 5y = c$

Now, 5 = Area of ΔOAB

$$= \frac{1}{2}|c|\left|\frac{c}{5}\right|$$

$\Rightarrow \quad c^2 = 50$

$\Rightarrow \quad |c| = 5\sqrt{2}$

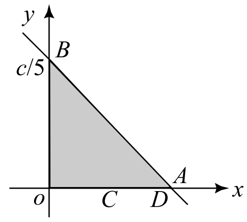


Fig. 16.45

Distance between $x + 5y = c$ and $x + 5y = 0$ is

$$\frac{|c|}{\sqrt{1+5^2}} = \frac{5\sqrt{2}}{\sqrt{26}} = \frac{5}{\sqrt{13}}$$

49. For $1 \le k \le 40$, the number of integral points on the line $x = k$ and lying in the interior of triangles is $k - 1$.

Thus, required number of points $= \sum_{k=1}^{40}(k-1)$

$$\frac{1}{2}(39)(40) = 780$$

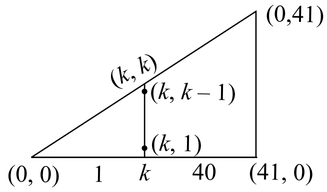


Fig. 16.46

50. $(2x - 3y + 4) + k(x - 2y + 3) = 0,\ k \in R$, represents a family of lines passing through the intersection of the lines $2x - 3y + 4 = 0$ and $x - 2y + 3 = 0$, that is, through (1, 2).

If (x, y) is image of (2, 3) in (1), then both (x, y) and (2, 3) are equidistant from (1, 2), thus,

$$(x-1)^2 + (y-2)^2 = (2-1)^2 + (3-2)^2 = 2.$$

Thus, (x, y) lies on a circle of radius $\sqrt{2}$

51. Let slope of required line L be m, then

$$\pm \tan 60^\circ = \frac{m-(-\sqrt{3})}{1+m(-\sqrt{3})}$$

$$\Rightarrow \pm\sqrt{3} = m+\sqrt{3}$$

$$\Rightarrow m = 0 \text{ or } m = -\sqrt{3}$$

As the desired line intersects the x-axis, $m = -\sqrt{3}$.

Thus, equation of required line is

$$y + 2 = -\sqrt{3}(x-3)$$

or $\quad y+\sqrt{3}x+2-3\sqrt{3}=0$

52. Slope of line joining $A(0, 8/3)$ and $B(1, 3)$ is

$$m_1 = \frac{3-8/3}{1-0} = \frac{1}{3}$$

and slope of line joining $B(1, 3)$ and $C(82, 30)$ is

$$m_2 = \frac{30-3}{82-1} = \frac{1}{3}$$

As $m_1 = m_2$, $AB \parallel AC$. But B is common, therefore A, B, C are collinear.

53. Suppose L meets the axes at $A(a, 0)$ and $B(0, b)$. As $P(1, 2)$ is the mid-point of AB,

$$\frac{a}{2}=1 \text{ and } \frac{b}{2}=2$$

$$\Rightarrow \quad a = 2,\ b = 4.$$

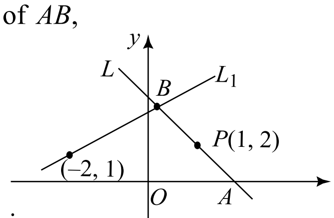


Fig. 16.47

Thus, equation of line L is

$$\frac{x}{2}+\frac{y}{4}=1 \quad \text{or } 2x+y=4 \qquad (1)$$

and the equation of L_1 is $(x + 2) - 2(y - 1) = 0$

or $\qquad x - 2y + 4 = 0 \qquad (2)$

solving (1) and (2) we get the required point as $\left(\frac{4}{5}, \frac{12}{5}\right)$

54. A vertex of the rhombus is (1, 2), the point of intersection of $x - y + 1 = 0$ and $7x - y - 5 = 0$

Another vertex of the rhombus is

$(2(-1) - 1, 2(-2) - 2) = (-3, -6)$

Other two sides to the rhombus are

$(x + 3) - (y + 6) = 0$ and $7(x + 3) - (y + 6) = 0$

or $x - y - 3 = 0$ and $7x - y + 15 = 0$.

A vertex can be obtained by solving $7x - y - 5 = 0$ and $x - y - 3 = 0$

Thus a vertex of rhombus is $\left(\frac{1}{3}, -\frac{8}{3}\right)$

55. An equation of line through intersection of $\frac{1}{3}x+\frac{1}{4}y-1=0$ and $\frac{1}{4}x+\frac{1}{3}y-1=0$ is

$$\left(\frac{1}{3}x+\frac{1}{4}y-1\right)+k\left(\frac{1}{4}x+\frac{1}{3}y-1\right)=0$$

or $\left(\frac{1}{3}+\frac{k}{4}\right)x+\left(\frac{1}{4}+\frac{k}{3}\right)y-(1+k)=0$

It meets the x-axis is in $A\left(\frac{12(k+1)}{4+3k}, 0\right)$ and the y-axis in $B\left(0, \frac{12(k+1)}{3+4k}\right)$

Let (α, β) be the mid-point of AB, therefore

$$2\alpha = \frac{12(k+1)}{4+3k},\ 2\beta = \frac{12(k+1)}{3+4k}$$

$$\Rightarrow \frac{1}{2\alpha}+\frac{1}{2\beta} = \frac{7(k+1)}{12(k+1)} = \frac{7}{12}$$

$\Rightarrow 6(\alpha + \beta) = 7\alpha\beta$

Thus, locus of mid-point AB is $6(x + y) = 7xy$

56. An equation of line through (2, 1) and parallel to $x - y = 4$ is $(x - 2) - (y - 1) = 0$ or $x - y - 1 = 0$.

A point on this line is $(t, t - 1)$.

Its distance from (2, 1) is

$$d = \sqrt{(t-2)^2+(t-1-1)^2} = \sqrt{2}|t-2|$$

Now $\sqrt{2}|t-2| = 2\sqrt{3}$

$\Rightarrow t - 2 = \pm\sqrt{6} \Rightarrow t = 2 \pm \sqrt{6}$

As the point lies in the third quadrant, we take $t = 2 - \sqrt{6}$

$\therefore$ point Q is $(2 - \sqrt{6}, 1 - \sqrt{6})$

An equation of required line through Q and perpendicular to L is

$(x - (2 - \sqrt{6})) + (y - (1 - \sqrt{6})) = 0$

or $x + y = 3 - 2\sqrt{6}$

57. Two given lines are

$4x + 3y - 10 = 0$

and $8x + 6y + 5 = 0$

These lines are parallel.

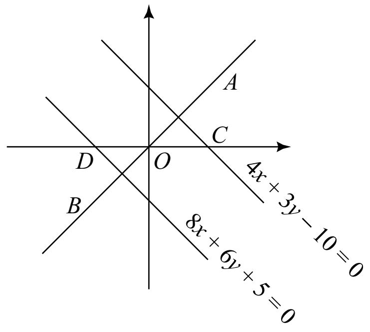


Fig. 16.48

Thus $OA : OB = OC : OD$

$= \frac{10}{4} : \frac{5}{8} = 4 : 1$

58. An equation of normal AN is $(x - 0) + 7(y - 1) = 0$

or $x + 7y - 7 = 0$

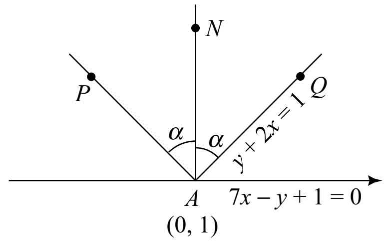


Fig. 16.49

Let an equation of PA be $y = mx + 1$,

then $\angle PAN = \angle NAQ$

$$\frac{-1/7 - m}{1 - m/7} = \frac{-2 - (-1/7)}{1 + (2/7)}$$

$$\Rightarrow \left(\frac{1}{7} + m\right)(9) = 13\left(1 - \frac{m}{7}\right)$$

$$\Rightarrow \left(9 + \frac{13}{7}\right)m = 13 - \frac{9}{7}$$

$\Rightarrow 76m = 82 \Rightarrow m = 41/38$

Thus, required equation is

$y = \frac{41}{38}x + 1$

or $41x - 38y + 38 = 0$

Previous Years' B-Architecture Entrance Examination Questions

1. Let the lines intersect at the point k.

then $(a + 2)k + 3a = 0$ and $4k + a - 2 = 0$

$$\Rightarrow \frac{3a}{a+2} = \frac{a-2}{4} \Rightarrow a^2 - 12a + 4 = 0$$

which gives us two values of a.

2. Let the point of intersection be $(h, -h)$; $h > 0$ then $2ph - 3qh + r = 0$ and $ph + 2qh - 2r = 0$,

$$\Rightarrow \frac{r}{2p - 3q} = \frac{-2r}{p + 2q}$$

$\Rightarrow p + 2q + 2(2p - 3q) = 0$

$\Rightarrow 5p - 4q = 0$

3. Mid-points of the sides of the triangle ABC are $P(0, 3)$, $Q\left(-\sqrt{3}, 0\right)$, $R\left(\sqrt{3}, 0\right)$

PQR is an equilateral triangle with each side of length $2\sqrt{3}$. So its in-centre is same as its centroid and hence its coordinates are (0, 1).

4. $(1 + 2\lambda_1)x - (1 + \lambda_1)y + (1 - 2\lambda_1) = 0$

$$(5 + 3\lambda_2)x + (3 - \lambda_2)y - (2 + 4\lambda_2) = 0$$

They represent the same line.

if $\frac{5 + 3\lambda_2}{1 + 2\lambda_1} = \frac{3 - \lambda_2}{-1 - \lambda_1} = \frac{-2 - 4\lambda_2}{1 - 2\lambda_1}$

Solving, we get $\lambda_1 = -3$, $\lambda_2 = -25$ and the required equation of the straight line is

$$5x - 2y - 7 = 0$$

5. Statement-2 is true and the statement-1 represents the lines given in statement so statement-1 is also true.

6. See Example 108. Page 15.26.

7. $m_1 + m_2 = -\left(\sqrt{3} + 2\right)$, $m_1 m_2 = \sqrt{3} - 1$

vertices the triangle are (0, 0), $\left(\frac{1}{m_1}, 1\right)$, $\left(\frac{-1}{m_2}, 1\right)$

so the area of the triangle is

$$\frac{1}{2}\left|\begin{vmatrix} 0 & 0 & 1 \\ \frac{1}{m_1} & 1 & 1 \\ \frac{-1}{m_2} & 1 & 1 \end{vmatrix}\right| = \frac{1}{2}\left[\frac{1}{m_1} + \frac{1}{m_2}\right]$$

$$= \frac{1}{2}\left|\frac{m_1 + m_2}{m_1 m_2}\right| = \frac{1}{2}\left|\frac{-(\sqrt{3} + 2)}{\sqrt{3} - 1}\right| = \frac{1}{2}\left[\frac{\sqrt{3} + 2}{\sqrt{3} - 1}\right]$$

8. Statement-1 is true but the statement-2 is false as the slopes of parallel lines are same.

9. Equation of a line through the point of intersection of the given lines is

$$(1 + 2\lambda)x + (2 - \lambda)y - (1 + \lambda) = 0$$

coordinates of A are $\left(\frac{1 + \lambda}{1 + 2\lambda}, 0\right)$ and of B are

$\left(0, \dfrac{1+\lambda}{2-\lambda}\right)$

If (h, k) is the mid-point of AB, then

$$h = \frac{1+\lambda}{2(1+2\lambda)}, \quad \frac{1+\lambda}{2(2-\lambda)}$$

$$\Rightarrow \quad \lambda = \frac{1+2h}{4h-1} = \frac{4k-1}{2k+1}$$

$$\Rightarrow \quad (1 - 2h)(2k + 1) = (4h - 1)(4k - 1)$$

$$\Rightarrow \quad 2h + 6k - 20hk = 0$$

Locus of (h, k) is $x + 3y = 10xy$.

10. The point of intersection of the given lines is $(-2a, 0)$ and the equation of the line parallel to y-axis passing through this point is $x = -2a$, $(p, 5)$ lies on it so $p = -2a$.

11. $[3x - 4(-3x) - 8]\,[3(3) - 4(4) - 8] < 0$

$\Rightarrow (15x - 8)(-17) < 0 \Rightarrow 15x - 8 > 0$

$$\Rightarrow \quad x > \frac{8}{15}.$$

Also, $y = -3x < -8/5$.

12. Let m_1 = slope of

$$BH = \frac{1-(-2)}{-6-3} = \frac{-1}{3}$$

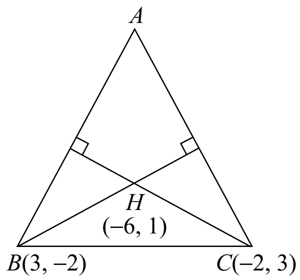


Fig. 16.50

$\Rightarrow$ Slope of $AC = 3$

$\therefore$ Equation of AC is

$y - 3 = 3(x + 2)$ or $3x - y + 9 = 0$ (1)

Let m_2 = slope of $CH = \dfrac{1-3}{-6+2} = \dfrac{1}{2}$

$\Rightarrow$ Slope of $AB = -2$

$\therefore$ Equation of AB is $y + 2 = -2(x - 3)$

or $2x + y - 4 = 0$ (2)

As A lies on (1) and (2), coordinates of A are $(-1, 6)$ and it does not lie on $5x + y = 2$.

13. An equation of line through $(1, 2)$ is $y - 2 = m(x - 1)$

A point on this line is of the form $Q(x, mx - m + 2)$.

This point will lie on the line $x + y = 7$ if

$$x + mx - m + 2 = 7 \Rightarrow x = \frac{m+5}{m+1}$$

As $PQ = 3$, $PQ^2 = 9$

$\Rightarrow (x - 1)^2 + (mx - m)^2 = 9$

$\Rightarrow (x - 1)^2\, 1 + m^2 = 9$

$\Rightarrow 16m^2 + 16 = 9m^2 + 18m + 9$

$\Rightarrow 7m^2 - 18m + 7 = 0$

$\Rightarrow m$ satisfies the equation $7x^2 - 18x + 7 = 0$