Chapter 1

Electric Charges and Fields

Solutions

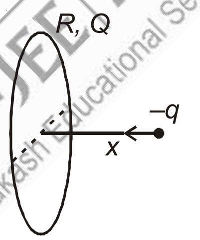
SECTION - A

Objective Type Questions (One option is correct)

1. A fixed ring of radius R is having uniformly distributed charge Q over its circumference. A charge q of opposite nature and mass m is kept on its axis at a distance $a << R$ from the centre of ring. If charge is released from rest then minimum time after which it will pass through centre of the ring will be (Effect of gravity is considered to be negligible) $\left(k = \frac{1}{4\pi\varepsilon_0}\right)$

(1) $\frac{\pi}{2}\sqrt{\frac{mR^3}{kQq}}$ (2) $\pi\sqrt{\frac{mR^3}{kQq}}$ (3) $2\pi\sqrt{\frac{mR^3}{kQq}}$ (4) $\frac{\pi}{4}\sqrt{\frac{mR^3}{kQq}}$

Sol. Answer (1)



$F = E_x q$

$= \frac{kQxq}{(R^2 + x^2)^{3/2}}$

$F = \frac{kQqx}{R^3\left(1 + \frac{x^2}{R^2}\right)^{3/2}}$

$F \approx \frac{kQqx}{R^3} \qquad x << R$

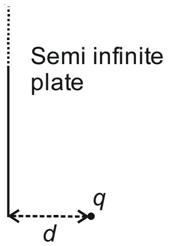
$a = \left(\frac{1}{4\pi\varepsilon_0}\right)\left(\frac{Qq}{mR^3}\right)x$

$a \propto x \qquad$ ...SHM

$T = \left(\frac{2\pi}{\omega}\right) = 0$

$t = \left(\frac{T}{4}\right)$

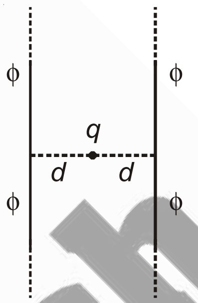
2. A point charge q is placed at a normal distance d from one edge of a semi-infinite large plate as shown in figure. Find net electric flux passing through this plate.



(1) $\left(\frac{q}{2\varepsilon_0}\right)$ (2) $\left(\frac{q}{4\varepsilon_0}\right)$ (3) $\left(\frac{q}{\varepsilon_0}\right)$ (4) $\left(\frac{q}{12\varepsilon_0}\right)$

Sol. Answer (2)

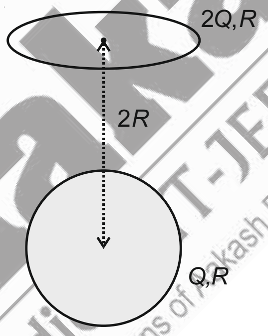
If ϕ is the flux through each plate (Semi-infinite)



$$\Rightarrow \quad 4\phi = \left(\frac{q}{\varepsilon_0}\right)$$

$$\Rightarrow \quad \phi = \left(\frac{1}{4}\right)\left(\frac{q}{\varepsilon_0}\right)$$

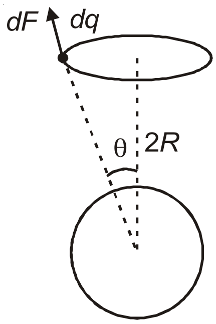
3. A solid non-conducting uniformly charged sphere having charge Q and radius R is placed at a separation of $2R$ from the centre of ring as shown in figure (centre of sphere lies on the axis of ring). Force on sphere due to ring is (Charge on ring is $2Q$ and its radius is R)



(1) $\frac{Q^2}{4\pi\varepsilon_0 R^2}$ (2) $\frac{Q^2}{\sqrt{5}\pi\varepsilon_0 R^2}$ (3) $\frac{Q^2}{5\sqrt{5}\pi\varepsilon_0 R^2}$ (4) $\frac{Q^2}{5\pi\varepsilon_0 R^2}$

Sol. Answer (3)

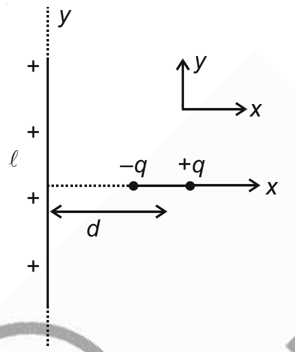
$F = \int dF \cos\theta$



$$dF = \frac{kQ}{\left(R\sqrt{5}\right)^2}(dq)$$

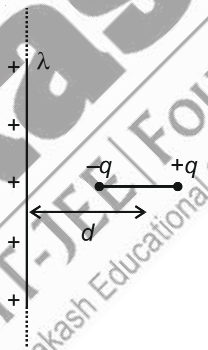
$$F = \int \frac{kQ}{5R^2}(dq)\left(\frac{2R}{R\sqrt{5}}\right)$$

4. A short dipole of dipole moment $\vec{P}$ is placed along x-axis at a distance d from a long wire having uniform linear charge density $+\lambda$ (wire is along Y axis) as shown. Find minimum work required to align this dipole along y axis (so that dipole moment $\vec{P}$ is along +y direction)



(1) $\frac{P\lambda}{\pi\varepsilon_0 d}$ (2) $-\frac{P\lambda}{\pi\varepsilon_0 d}$ (3) $\frac{P\lambda}{2\pi\varepsilon_0 d}$ (4) $-\frac{P\lambda}{2\pi\varepsilon_0 d}$

Sol. Answer (3)



$W = \Delta U$

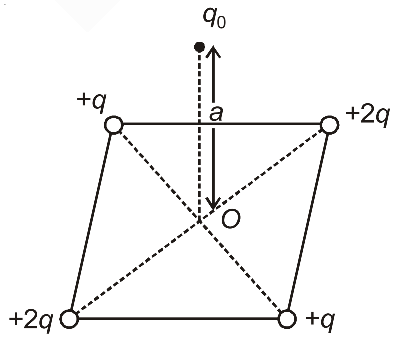
$W = U_f - U_i$

$U_f = -PE \cos 90 = 0$

$U_i = -PE \cos 0$

$W = U_f - U_i$

5. Four point charges are fixed at the corners of a square of side length a. (in horizontal x–y plane). A positive charge q_0 is placed at a distance a from centre of square perpendicular to the plane of square. If point charge q_0 is in equilibrium then its mass m is $\left(a = \sqrt{\frac{3}{2}}\text{ m}\right)$.



(1) $\frac{2qq_0}{9\pi\varepsilon_0 g}\sqrt{\frac{3}{2}}$ (2) $\frac{4qq_0}{9\pi\varepsilon_0 g}\sqrt{\frac{3}{2}}$ (3) $\frac{5qq_0}{9\pi\varepsilon_0 g}\sqrt{\frac{3}{2}}$ (4) $\frac{8}{9}\frac{qq_0}{\pi\varepsilon_0 g}\sqrt{\frac{3}{2}}$

Sol. Answer (2)

$F_{\text{real}} = mg$

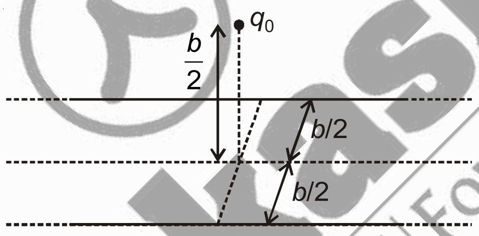
$$\frac{2(k)(2q)q_0}{\left(a^2+\frac{a^2}{2}\right)}\frac{a}{\sqrt{a^2+\frac{a^2}{2}}}+\frac{2kqq_0}{\left(a^2+\frac{a^2}{2}\right)}\frac{a}{\sqrt{a^2+\frac{a^2}{2}}} = mg$$

$$\frac{2kqq_0a}{\left(\frac{3a^2}{2}\right)^{3/2}}(3) = mg$$

$$m = \frac{(2kqq_0)(a)8}{9g} = \left(\frac{2\times 8}{4\pi\varepsilon_0}\right)\frac{(qq_0)}{9g}\sqrt{\frac{3}{2}}$$

$$m = \frac{4qq_0}{9\pi\varepsilon_0 g}\sqrt{\frac{3}{2}}$$

6. An infinitely large plate of width b is placed in horizontal x–y plane. A point charge q_0 is placed symmetrically at a distance of $\frac{b}{2}$ perpendicular to plane of this plate as shown. Find electric flux passing through this plate.

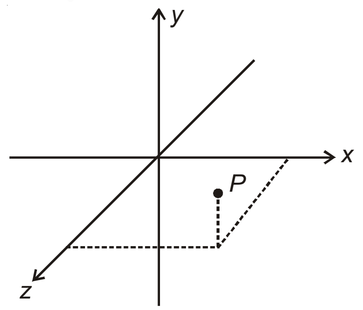


(1) $\left(\frac{q_0}{\varepsilon_0}\right)$ (2) $\left(\frac{1}{2}\right)\left(\frac{q_0}{\varepsilon_0}\right)$ (3) Zero (4) $\left(\frac{1}{4}\right)\left(\frac{q_0}{\varepsilon_0}\right)$

Sol. Answer (4)

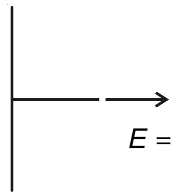
$$\phi = \frac{1}{4}\left(\frac{q_0}{\varepsilon_0}\right)$$

7. Three infinitely long wires having uniform linear charge density λ are placed along x, y, and z-axis respectively. Find electric field at point $P(a, a, a)$.



(1) $\frac{\lambda}{2\pi\varepsilon_0 a}(\hat{i}-\hat{j}+\hat{k})$ (2) $\frac{\lambda}{2\pi\varepsilon_0 a}(\hat{i}+\hat{j}+\hat{k})$ (3) $\frac{\lambda}{4\pi\varepsilon_0 a}(\hat{i}-\hat{j}+\hat{k})$ (4) $\frac{\lambda}{4\pi\varepsilon_0 a}(\hat{i}+\hat{j}+\hat{k})$

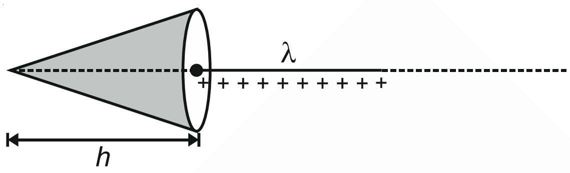
Sol. Answer (2)



$$E = \frac{\lambda}{2\pi\varepsilon_0 r}$$

$$\vec{E} = \frac{\lambda\vec{r}}{2\pi\varepsilon_0 r^2}$$

8. A semi-infinite wire of uniform linear charge density λ is placed as shown in figure. The conical surface is of radius R and height h. The electric flux through the curved surface of the cone is

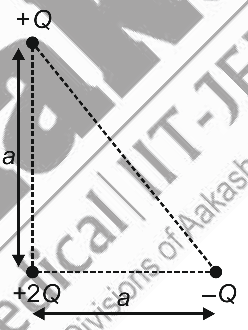


(1) $\frac{\lambda R}{\varepsilon_0}$ (2) $\frac{\lambda R}{2\varepsilon_0}$ (3) $\frac{\lambda R}{4\varepsilon_0}$ (4) $\frac{\sqrt{2}\,\lambda R}{3\varepsilon_0}$

Sol. Answer (2)

$$\phi = \int_0^R \frac{\lambda}{4\pi\varepsilon_0 x} \times 2\pi x\, dx = \frac{\lambda R}{2\varepsilon_0}$$

9. Three point charges $+Q$, $-Q$ and $+2Q$ are placed at the vertices of an isosceles right angled triangle as shown in figure. The magnitude of electric field intensity produced by these charges at the middle point of hypotenuse is



(1) $\frac{(2+\sqrt{2})Q}{4\pi\varepsilon_0 a^2}$ (2) $\frac{2Q}{\pi\varepsilon_0 a^2}$ (3) $\frac{\sqrt{2}\,Q}{\pi\varepsilon_0 a^2}$ (4) $\frac{Q}{8\pi\varepsilon_0 a^2}$

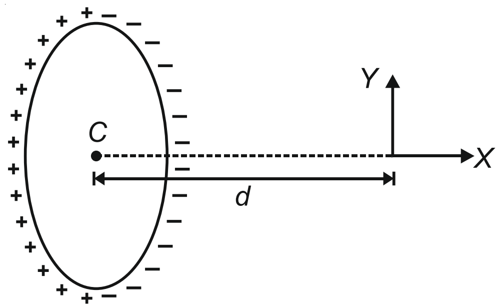
Sol. Answer (3)

$$E_1 = E_2 = \frac{Q}{4\pi\varepsilon_0\left(\frac{9}{\sqrt{2}}\right)^2}$$

$$E_3 = \frac{2Q}{4\pi\varepsilon_0\left(\frac{9}{\sqrt{2}}\right)^2}$$

$$E_{net} = \frac{\sqrt{2}Q}{\pi\varepsilon_0 a^2}$$

10. On a non-conducting ring of radius R, $+q$ and $-q$ charges are uniformly distributed in two halves as shown in figure. The electric field intensity on its axis at a distance d from its centre is given by $\vec{E}$, then



(1) $\vec{E}.\hat{i} = 0$

(2) $\vec{E}.\hat{j} = 0$

(3) $\vec{E} = \dfrac{qd\hat{i}}{4\pi\varepsilon_0(R^2+d^2)^{\frac{3}{2}}}$

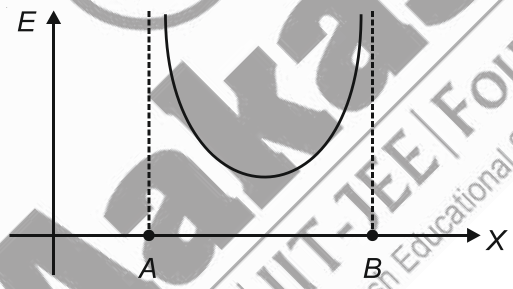
(4) $\vec{E} = \dfrac{qd\hat{j}}{4\pi\varepsilon_0(R^2+d^2)^{\frac{3}{2}}}$

Sol. Answer (1)

Along X axis change in potential is zero.

So, $\vec{E}.\hat{i} = 0$

11. Two point charges A and B are placed on X-axis. Considering electric field E is positive if directed in positive X-direction. Variation of electric field strength with distance x is shown in figure for x lying between the charges. Then the nature of charges A and B are respectively



(1) Positive, positive

(2) Negative, negative

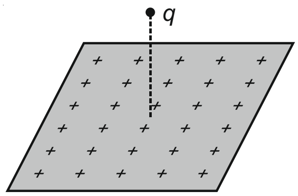
(3) Positive, negative

(4) Negative, positive

Sol. Answer (3)

$\vec{E}_{net} = \vec{E}_1 + \vec{E}_2$

12. A non-conducting square plate of side length $2a$ is uniformly charged with charge Q and a point charge q is placed at perpendicular distance a as shown in figure. The electric force on the point charge due to plate is



(1) $\dfrac{Qq}{8\varepsilon_0 a^2}$

(2) $\dfrac{Qq}{4\pi\varepsilon_0 a^2}$

(3) $\dfrac{Qq}{12\pi\varepsilon_0 a^2}$

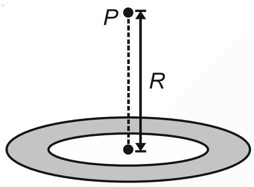
(4) $\dfrac{Qq}{24\varepsilon_0 a^2}$

Sol. Answer (4)

$$\sigma = \frac{Q}{4a^2}$$

$$\phi = \frac{q}{6\varepsilon_0}, \ \therefore \ F = \sigma\phi = \frac{Qq}{24\varepsilon_0 a^2}$$

13. An annular disc of inner radius R and outer radius $2R$ has uniformly distributed charge Q. Electric field strength at point P is



(1) $\frac{Q}{3\pi\varepsilon_0 R^2}$ (2) $\frac{Q}{2\pi\varepsilon_0 R^2}(\sqrt{2}-1)$

(3) $\frac{Q}{6\pi\varepsilon_0 R^2}\left(\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{5}}\right)$ (4) $\frac{Q}{6\pi\varepsilon_0 R^2}(\sqrt{5}-\sqrt{2})$

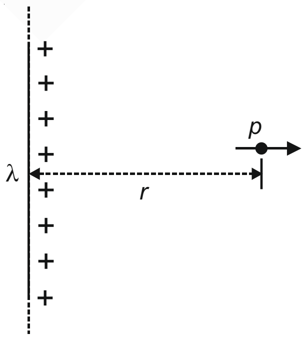
Sol. Answer (3)

$$E_1 = \frac{\sigma}{2\varepsilon_0}\left(1-\frac{1}{\sqrt{5}}\right)$$

$$E_2 = \frac{\sigma}{2\varepsilon_0}\left(1-\frac{1}{\sqrt{2}}\right)$$

$$E_{net} = E_1 - E_2, \ \sigma = \frac{Q}{3\pi R^2}$$

14. A small electric dipole of dipole moment p is placed perpendicular to an infinitely long wire of linear charge density λ. The centre of dipole is at a distance r from the wire as shown. The electric force on the dipole is



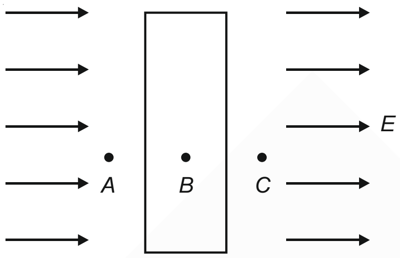
(1) $\frac{\lambda p}{6\pi\varepsilon_0 r^2}$ (2) $\frac{\lambda p}{4\pi\varepsilon_0 r^2}$ (3) $\frac{\lambda p}{2\pi\varepsilon_0 r^2}$ (4) $\frac{\lambda p}{\varepsilon_0 r^2}$

Sol. Answer (3)

$$E = \frac{\lambda}{2\pi\varepsilon_0 r} \quad \therefore \quad dE = \frac{\lambda}{2\pi\varepsilon_0 r^2} dr$$

$$\therefore \quad F = q \times dE = \frac{\lambda p}{2\pi\varepsilon_0 r^2}$$

15. A dielectric slab is placed in uniform electric field E as shown in figure. Then



(1) Electric field strength at A and C remains unchanged

(2) Electric field strength at A increases whereas at C decreases

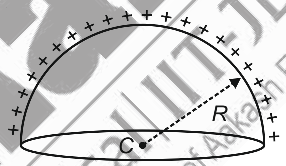
(3) Electric field strength at A and B decreases

(4) Electric field strength at B decreases whereas at A and C increases

Sol. Answer (4)

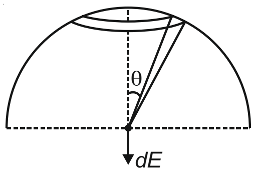
Electric field strength in dielectric decreases.

16. A hemisphere of radius R as shown in figure is uniformly charged with a surface charge density σ. The electric field at the centre C is



(1) $\frac{\sigma}{\varepsilon_0}$ (2) $\frac{\sigma}{2\varepsilon_0}$ (3) $\frac{\sigma}{4\varepsilon_0}$ (4) $\frac{\sqrt{2}\,\sigma}{\varepsilon_0}$

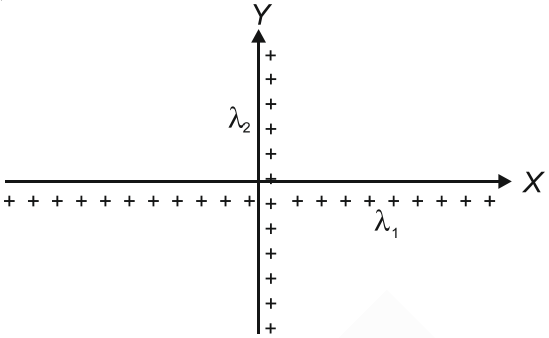
Sol. Answer (3)



$$\int dE = \int_{\theta=0}^{\frac{\pi}{2}} \frac{1}{4\pi\varepsilon_0} \times \frac{(\sigma \times 2\pi R \sin\theta\, R d\theta) \times (R\cos\theta)}{R^3}$$

$$= \frac{\sigma}{4\varepsilon_0}$$

17. Two non-conducting infinitely long straight wires with uniform linear charge densities λ_1 and λ_2 are arranged along X and Y-axis respectively. Their point of intersection is at origin. One of the field lines due to these wires in X-Y plane is a straight line passing through the origin. The angle made by this line with X-axis is



(1) $\tan^{-1}\left(\frac{\lambda_1}{\lambda_2}\right)$ (2) $\tan^{-1}\left(\frac{\lambda_2}{\lambda_1}\right)$ (3) $\tan^{-1}\left(\sqrt{\frac{\lambda_1}{\lambda_2}}\right)$ (4) $\tan^{-1}\left(\sqrt{\frac{\lambda_2}{\lambda_1}}\right)$

Sol. Answer (3)

$$\tan\theta = \frac{y}{x} = \frac{E_y}{E_x} = \frac{\left(\frac{\lambda_1}{2\pi\varepsilon_0 y}\right)}{\left(\frac{\lambda_2}{2\pi\varepsilon_0 x}\right)}$$

$$\Rightarrow \quad \theta = \tan^{-1}\left(\sqrt{\frac{\lambda_1}{\lambda_2}}\right)$$

SECTION - B

Objective Type Questions (More than one options are correct)

1. Two charged metallic spheres of same size repel each other by a force F. They are now touched with each other and are then separated to same initial distance. Now the force of repulsion is F'. Which of the following are possible?

(1) $F' = F$ (2) $F' > F$ (3) $F' \neq F$ (4) $F' < F$

Sol. Answer (1, 2, 3)

If the two spheres have same charge initially then $F' = F$. Since charge will not flow as the size and charge is same. If they have unequal charges, say, q and $Q - q$, where Q is the total charge on two spheres.

$F = \frac{Kq(Q-q)}{r^2}$. The force will be maximum when

$$\frac{dF}{dq} = 0 \Rightarrow q = \frac{Q}{2}$$

and $\frac{d^2F}{dq^2} < 0$ at $q = \frac{Q}{2}$

So, the force in maximum when the two charges are equal.

When the two spheres are touched with each other, their charges will be equal owing to their size.

$\therefore \quad F' > F$ and in this case, $F' \neq F$

2. Two fixed charges $+4Q$ and $-Q$ are located at A and B. Select the correct statements

(1) A point (P) where a third charged particle will not experience a force lies outside AB

(2) A point (P) where a third charged particle will not experience a force lies inside AB

(3) A positive charge is kept at the neutral point (P), can oscillate along horizontal

(4) If a negative charge is kept at the neutral point (P), it can oscillate along horizontal

Sol. Answer (1, 4)

Charge has to be kept closer to charge B.

3. Two point charges (Q each) are placed at $(0, y)$ and $(0, -y)$. A point charge q of same polarity is constrained to move along x-axis. Select the correct alternatives

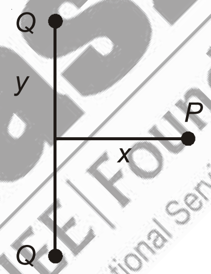
(1) The force on q is maximum at $x = \pm\dfrac{y}{\sqrt{2}}$

(2) The charge q is in equilibrium at origin

(3) The charge q performs an oscillatory motion about origin

(4) For any position of q other than origin, the force is directed away from origin

Sol. Answer (1, 2, 4)



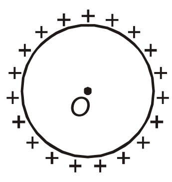
$$E = \frac{Qx}{2\pi\varepsilon_0\left(x^2 + y^2\right)^{3/2}}$$

For E to be maximum,

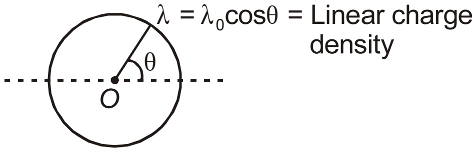
$$\frac{dE}{dx} = 0$$

$$\Rightarrow \quad x = \pm\frac{y}{\sqrt{2}}$$

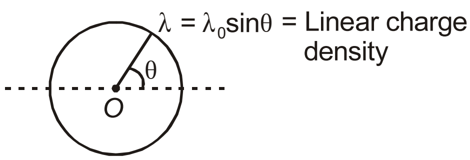
4. In which of the following figures, electric field at point O is non-zero?

(1) 

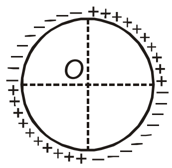
Uniform charge/length

(2) 

Charge per unit length varies with angular position

(3) 

Charge per unit length varies with angular position

(4) 

Uniform charge/length for different quadrants

Sol. Answer (2, 3)

Symmetrical configuration will lead to zero field.

5. A short electric dipole is placed along y-axis at the origin. The electric field vector at a point P on x-axis is $\vec{E}_1$. The dipole is rotated by $\frac{\pi}{2}$ at its position. Now, the electric field vector at point P is $\vec{E}_2$. Select the correct alternative(s).

(1) $\vec{E}_2 = -\vec{E}_1$ (2) $\vec{E}_2 = -2\vec{E}_1$

(3) $|\vec{E}_2| = 2|\vec{E}_1|$ (4) $\vec{E}_2 . \vec{E}_1 = 0$

Sol. Answer (3, 4)

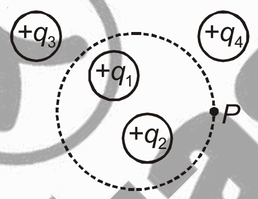
$$\vec{E}_1 = \frac{-\vec{p}}{4\pi\varepsilon_0 .r^3} = \frac{p}{4\pi\varepsilon_0 .r^3}(-\hat{j})$$

$$\vec{E}_2 = \frac{2\vec{p}}{4\pi\varepsilon_0 .r^3} = \frac{2p}{4\pi\varepsilon_0 .r^3}.\hat{i}$$

$\Rightarrow \ |\vec{E}_2| = 2|\vec{E}_2|$

and $\vec{E}_2 \cdot \vec{E}_1 = 0$

6. Which of the four particles contribute to the electric field at point P on the surface?

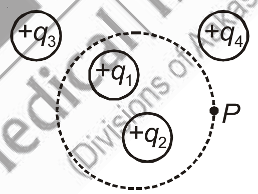


(1) q_1 (2) q_2 (3) q_3 (4) q_4

Sol. Answer (1, 2, 3, 4)

Electric field will be contributed by all the charges, irrespective of their location.

7. Which of the four particles contribute to the net electric flux through the closed surface?



(1) q_1 (2) q_2 (3) q_3 (4) q_4

Sol. Answer (1, 2)

For flux only charges inside surface will contribute.

8. In Gauss theorem $\oint \vec{E}.d\vec{s} = \frac{q}{\varepsilon_0}$. The surface integral is evaluated by choosing a closed surface, called the Gaussian surface. Here the correct statement is/are

(1) The closed surface can have any shape or size

(2) 'q' is the net charge enclosed by the Gaussian surface

(3) $\vec{E}$, must be the electric field due to all the charges inside the surface only

(4) The exact location of the charges inside the surface does not affect the value of the integral

Sol. Answer (1, 2, 4)

Basic principle of Gauss Law.

9. Consider Gauss' law $\oint \vec{E}.\overrightarrow{dA} = \frac{q}{\varepsilon_0}$. Which of the following is not true?

(1) $\vec{E}$ must be the electric field due to enclosed charge only

(2) If net charge inside the Gaussian surface = 0, $\vec{E}$ must be zero everywhere over the Gaussian surface

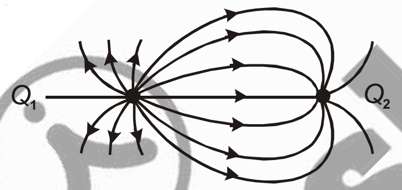
(3) If the only charge inside the Gaussian surface is an electric dipole, then the integral is zero

(4) $\vec{E}$ is parallel to $\overrightarrow{dA}$ everywhere over the Gaussian surface

Sol. Answer (1, 2, 4)

$\oint \vec{E}.\overrightarrow{dS} = 0$ does not imply electric field is zero everywhere.

10. A few electric field lines for a system of two charges Q_1 and Q_2 fixed at two different points on the *x*-axis are shown in the figure. These lines suggest that



(1) $|Q_1| > |Q_2|$

(2) $|Q_1| < |Q_2|$

(3) At a finite distance to the left of Q_1 the electric field is zero

(4) At a finite distance to the right of Q_2 the electric field is zero

Sol. Answer (1, 4)

Lines are denser around Q_1

$\Rightarrow |Q_1| > |Q_2|$

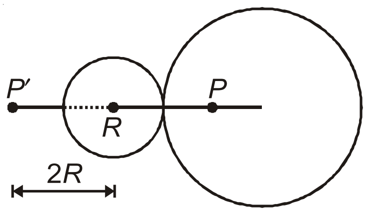
Since $|Q_1| > |Q_2|$, then electric field will be zero at some distance to right of Q_2

11. Two non-conducting solid spheres of radii R and $2R$, having uniform volume charge densities ρ_1 and ρ_2 respectively, touch each other. The net electric field at a distance $2R$ from the centre of the smaller sphere, along the line joining the centres of the spheres, is zero. The ratio $\frac{\rho_1}{\rho_2}$ can be

(1) –4 (2) $-\frac{32}{25}$ (3) $\frac{32}{25}$ (4) 4

Sol. Answer (2, 4)

$E_P = 0$



$$\Rightarrow \frac{\rho_1 \frac{4}{3}\pi R^3}{4\pi\varepsilon_0 (2R)^2} = \frac{\rho_2 (R)}{3\varepsilon_0}$$

$$\Rightarrow \frac{\rho_1}{\rho_2} = 4$$

$E_{P'} = 0$

$$\Rightarrow \quad \frac{\rho_1 \frac{4}{3}\pi R^3}{4\pi\varepsilon_0 (2R)^2} = \frac{-\rho_2 \frac{4}{3}\pi (2R)^3}{4\pi\varepsilon_0 (5R)^2}$$

$$\Rightarrow \quad \frac{\rho_1}{\rho_2} = -\frac{32}{25}$$

12. Let $E_1(r)$, $E_2(r)$ and $E_3(r)$ be the respective electric fields at a distance r from a point charge Q, an infinitely long wire with constant linear charge density λ, and an infinite plane with uniform surface charge density σ. If $E_1(r_0) = E_2(r_0) = E_3(r_0)$ at a given distance r_0, then

(1) $Q = 4\sigma\pi r_0^2$

(2) $r_0 = \frac{\lambda}{2\pi\sigma}$

(3) $E_1(r_0/2) = 2E_2(r_0/2)$

(4) $E_2(r_0/2) = 4E_3(r_0/2)$

Sol. Answer (3)

$$E_1(r_0) = \frac{Q}{4\pi\varepsilon_0 r^2}, E_1\left(\frac{r_0}{2}\right) = \frac{4Q}{4\pi\varepsilon_0 r^2}$$

$$E_2(r_0) = \frac{\lambda}{2\pi\varepsilon_0 r}, E_2\left(\frac{r_0}{2}\right) = \frac{2\lambda}{2\pi\varepsilon_0 r}$$

$$E_3(r_0) = E_3\left(\frac{r_0}{2}\right) = \frac{\sigma}{2\varepsilon_0}$$

Now, $E_1(r_0) = E_2(r_0) = E_3(r_0)$

$$\Rightarrow \quad \frac{Q}{4\pi\varepsilon_0 r^2} = \frac{\lambda}{2\pi\varepsilon_0 r} = \frac{\sigma}{2\varepsilon_0}$$

$$\Rightarrow \quad Q = 2\pi r^2 \sigma$$

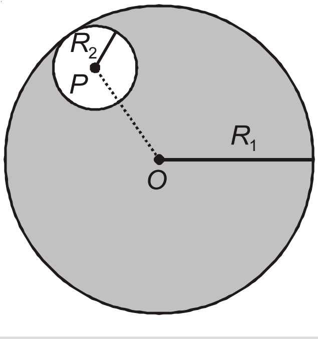
$$\Rightarrow \quad \frac{\lambda}{\pi r} = \sigma \quad \text{or} \quad r = \frac{\lambda}{\sigma\pi}$$

$$E_1\left(\frac{r_0}{2}\right) = 4E_1(r_0) = 2E_2\left(\frac{r_0}{8}\right)$$

Also, $E_2\left(\frac{r_0}{2}\right) = 2E_3(r_0) = 2E_3\left(\frac{r_0}{2}\right)$

Only (3) is correct.

13. Consider a uniform spherical charge distribution of radius R_1 centred at the origin O. In this distribution, a spherical cavity of radius R_2, centred at P with distance $OP = a = R_1 - R_2$ (see figure) is made. If the electric field inside the cavity at position $\vec{r}$ is $\vec{E}(\vec{r})$, then the correct statement(s) is(are)



(1) $\vec{E}$ is uniform, its magnitude is independent of R_2 but its direction depends on $\vec{r}$

(2) $\vec{E}$ is uniform, its magnitude depends on R_2 and its direction depends on $\vec{r}$

(3) $\vec{E}$ is uniform, its magnitude is independent of a but its direction depends on $\vec{a}$

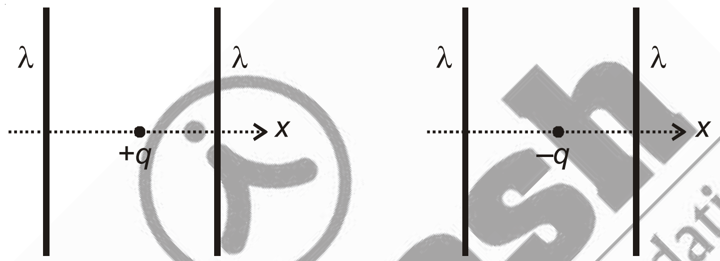
(4) $\vec{E}$ is uniform and both its magnitude and direction depend on $\vec{a}$

Sol. Answer (4)

The field in the cavity is uniform, $\vec{E} = \left(\rho \dfrac{a}{3\varepsilon_0}\right)\widehat{OP}$

$a = R_1 - R_2$

14. The figures below depict two situations in which two infinitely long static line charges of constant positive line charge density λ are kept parallel to each other. In their resulting electric field, point charges q and $-q$ are kept in equilibrium between them. The point charges are confined to move in the x direction only. If they are given a small displacement about their equilibrium positions, then the correct statement(s) is(are)



(1) Both charges execute simple harmonic motion

(2) Both charges will continue moving in the direction of their displacement

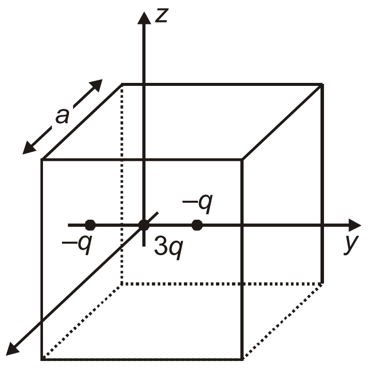
(3) Charge $+q$ executes simple harmonic motion while charge $-q$ continues moving in the direction of its displacement

(4) Charge $-q$ executes simple harmonic motion while charge $+q$ continues moving in the direction of its displacement

Sol. Answer (3)

As the '+' charge is moved in $+x$-direction, it will experience a greater repulsion in x-direction while '–' charge will be attracted.

15. A cubical region of side a has its centre at the origin. It encloses three fixed point charges, $-q$ at $\left(0, -\dfrac{a}{4}, 0\right)$, $+3q$ at (0, 0, 0) and $-q$ at $\left(0, +\dfrac{a}{4}, 0\right)$. Choose the correct option(s).



(1) The net electric flux crossing the plane $x = +\frac{a}{2}$ is equal to the net electric flux crossing the plane $x = -\frac{a}{2}$

(2) The net electric flux crossing the plane $y = +\frac{a}{2}$ is more than the net electric flux crossing the plane $y = -\frac{a}{2}$

(3) The net electric flux crossing the entire region is $\frac{q}{\varepsilon_0}$

(4) The net electric flux crossing the plane $z = +\frac{a}{2}$ is equal to the net electric flux crossing the plane $x = +\frac{a}{2}$

Sol. Answer (1, 3, 4)

Option (1) is correct due to symmetry.

For option (2), $\phi\left(y = \frac{a}{2}\right) = \phi\left(y = -\frac{a}{2}\right)$

For option (3), net charge enclosed, is Q.

16. In terms of potential difference V, electric current I, permittivity ε_0, permeability μ_0 and speed of light c, the dimensionally correct equation(s) is(are)

(1) $\mu_0 I^2 = \varepsilon_0 V^2$ (2) $\varepsilon_0 I = \mu_0 V$

(3) $I = \varepsilon_0 cV$ (4) $\mu_0 cI = \varepsilon_0 V$

Sol. Answer (1, 3)

Using $V = Bv\ell$, we have

$$V = (\mu_0\, nI)\, v\ell$$

or $$V = \mu_0\left(\frac{1}{\ell}\right) I \times c \times \ell$$

$$V = \mu_0 Ic$$

or $$Vc = \mu_0 Ic^2$$

$\Rightarrow$ $$Vc = \frac{\mu_0 I}{\mu_0 \varepsilon_0}$$

$\Rightarrow$ $$V\varepsilon_0 c = I \quad \text{... (i)}$$

Also, $V = \mu_0 Ic$

Squaring $V^2 = \mu_0^2 I^2 c^2 = \frac{\mu_0^2 I^2}{\mu_0 \varepsilon_0} = \frac{\mu_0}{\varepsilon_0} I^2$

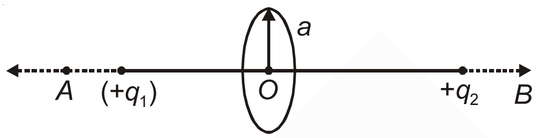
$\Rightarrow$ $$V^2 \varepsilon_0 = \mu_0 I^2 \quad \text{... (ii)}$$

SECTION - C

Linked Comprehension Type Questions

Comprehension-I

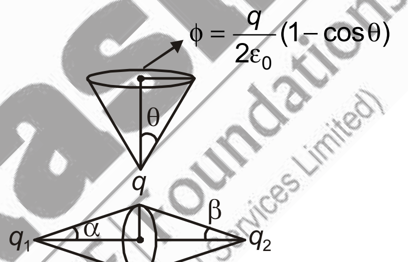
A fixed circle of radius a and centre O is drawn and a charge $+q_1$ is placed at a distance $\left(\frac{3a}{4}\right)$ from O on a line through O and perpendicular to plane of circle. And a second charge q_2 is similarly placed at a distance $\frac{5a}{12}$ on the opposite side of circle



1. If the net flux through circle is zero, then $\frac{q_1}{q_2}$ would be

(1) 1 (2) $\frac{13}{20}$ (3) $\frac{20}{13}$ (4) $\frac{2}{9}$

Sol. Answer (3)



$$\frac{q_1}{2\varepsilon_0}(1-\cos\alpha)=\frac{q_2}{2\varepsilon_0}(1-\cos\beta)$$

$$\frac{q_1}{q_2}=\frac{1-\cos\beta}{1-\cos\alpha}=\frac{1-\frac{5}{13}}{1-\frac{3}{5}}=\frac{8\times5}{13\times2}=\frac{20}{13}$$

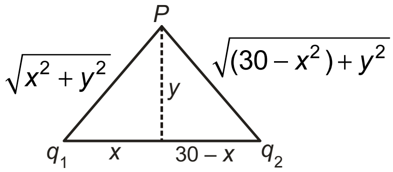
2. If the value of q_1 and q_2 are 100 μC and –400 μC respectively and they are separated by a distance of 30 cm. The locus of all points in the plane where electric potential is zero would be

(1) Straight line (2) Parabola whose axis is line joining the charges

(3) Circle of radius 8 cm (4) Circle of radius 4 cm

Sol. Answer (3)

$$\frac{q_1}{4\pi\varepsilon\sqrt{x^2+y^2}}+\frac{q_2}{4\pi\varepsilon_0\sqrt{(30-x)^2+y^2}}=0$$



$$\frac{100}{\sqrt{x^2+y^2}}-\frac{400}{\sqrt{(30-x)^2+y^2}}=0$$

$$\sqrt{(30-x)^2+y^2}=4\sqrt{x^2+y^2}$$

$(30 - x)^2 + y^2 = 16x^2 + 16y^2$

$x^2 + y^2 - 60x - 16x^2 - 6y^2 + 900 = 0$

$15x^2 + 15y^2 + 60x - 900 = 0$

$x^2 + y^2 + 4x - 60 = 0$

radius $= \sqrt{g^2 + f^2 - c}$

$= \sqrt{4 + 60}$

$= \sqrt{64} = 8$ cm

3. If the values of q_1 and q_2 are $+q$, $-2q$ respectively, now a third charge $-q$ has to be placed on line AB so that it can be in stable equilibrium. The distance of point where third charge is placed from charge q, would be (take a = 12 cm)

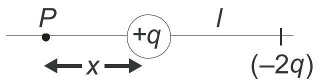
(1) $\dfrac{14}{\sqrt{2}-1}$ cm

(2) $\dfrac{14}{\sqrt{2}+1}$ cm

(3) $\dfrac{14}{2\sqrt{2}-1}$ cm

(4) $-q$ cannot be in stable equilibrium at any point.

Sol. Answer (4)



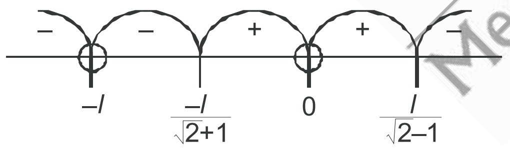
$$EP = Kq\left[\frac{1}{x^2} - \frac{2}{(l+x)^2}\right]$$

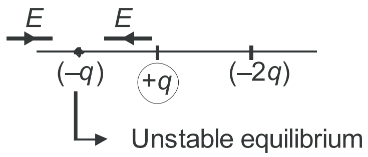
$$= Kq\left[\frac{(l+n)^2 - 2x^2}{x^2(l+n)^2}\right]$$

$$= Kq\frac{\left(l + x + \sqrt{2}x\right)\left(l + x - \sqrt{2}x\right)}{x^2(l+x)^2}$$

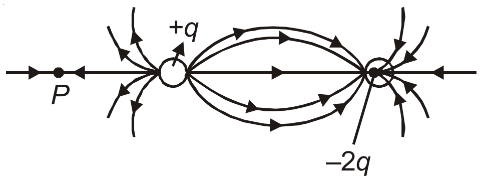
$$= Kq\frac{\left[l + x\left(\sqrt{2}+1\right)\right]\left[l + x\left(1-\sqrt{2}\right)\right]}{x^2\ (l+n)^2}$$

$$= Kq\left(l+\sqrt{2}\right)\left(l-\sqrt{2}\right)\frac{\left[\left(x + \frac{l}{\sqrt{2}+1}\right)\left(x - \frac{l}{\sqrt{2}-1}\right)\right]}{x^2(l+n)^2}$$



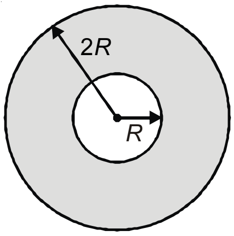


Alternating



Comprehension-II

A spherical shell of inner radius R and outer radius $2R$, has a uniform charge distribution and total charge Q.



1. The charges inside the "Gaussian sphere" for the given three regions are respectively

   (i) $0 < a < R$

   (ii) $R < a < 2R$

   (iii) $2R < a$

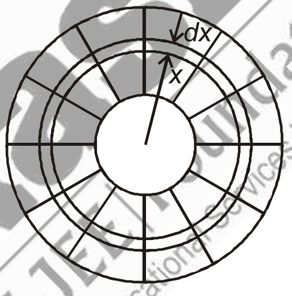
   (1) $0, \frac{Q}{7}\left[\left(\frac{a}{R}\right)^3 - 1\right], Q$ (2) $0, \frac{Q}{9}\left[\left(\frac{a}{R}\right)^2 - 1\right], 2Q$

   (3) $0, Q\left[\left(\frac{a}{R}\right)^2 - 1\right], Q$ (4) $0, Q\left[\left(\frac{a}{R}\right)^2 - 1\right], 4Q$

Sol. Answer (1)

$$Q = \rho\int_0^x 4\pi x^2 dx$$

$$\rho(x) = \begin{cases} 0 & 0 < x < R \\ \frac{3Q}{28\pi R^3} & R < x < 2R \\ 0 & x > 2R \end{cases}$$



2. Use Gauss's Law, the electric fields at the surface of the "Gaussian surface" when

   (i) $0 < a < R$ (ii) $R < a < 2R$

   (iii) $2R < a$

   are respectively

   (1) $0, \frac{Q}{28\pi\varepsilon_0 a^2}\left[\left(\frac{a}{R}\right)^3 - 1\right], \frac{Q}{4\pi\varepsilon_0 a^2}$ (2) $0, \frac{Q}{25\pi\varepsilon_0 a^2}\left[\left(\frac{a}{R}\right)^3 - 2\right], \frac{Q}{2\pi\varepsilon_0 a^2}$

   (3) $0, \frac{Q}{32\pi\varepsilon_0 a^2}\left[\left(\frac{a}{R}\right)^3 - 3\right], \frac{Q}{4\pi\varepsilon_0 a^2}$ (4) $0, \frac{Q}{30\pi\varepsilon_0 a^2}\left[\left(\frac{a}{R}\right)^3 - 4\right], \frac{Q}{4\pi\varepsilon_0 a^2}$

Sol. Answer (1)

3. The electric field in a region of space is $\vec{E} = E_0 x\hat{i}$. The flux of electric field through a sphere of radius r, centred at origin is

   (1) $\frac{4\pi E_0 r^3}{3}$ (2) $E_0\pi r^3$ (3) $E_0 r^3$ (4) Zero

Sol. Answer (1)

Comprehension-III

For a uniformly charged disc of radius R and surface charge density σ centred at origin and lying in y-z plane, the electric field strength at (x, 0) is $\frac{\sigma}{2\varepsilon_0}\left[1-\frac{x}{\sqrt{R^2+x^2}}\right]$.

1. Consider an annular disc of inner radius r and outer radius $2r$, having uniform surface charge density σ. The electric field on the axis of this disc at a distance r from the centre will be

(1) $\frac{\sigma}{2\sqrt{10}\varepsilon_0}$ (2) $\frac{\sigma}{2\sqrt{10}\varepsilon_0}(\sqrt{5}-\sqrt{2})$ (3) $\frac{\sigma}{2\sqrt{10}\varepsilon_0}(\sqrt{3}-\sqrt{2})$ (4) $\frac{\sigma}{2\varepsilon_0}\left(1-\frac{1}{\sqrt{2}}\right)$

Sol. Answer (2)

Use principle of superposition resultant. Field will be due to disc of radius $2R$ and charge density σ, disc of radius R, charge density $-\sigma$.

2. The electric field strength on the axis of the cylinder (length L, radius R) at a point P, whose distance from nearer face of the cylinder is r, is

(1) $\frac{\rho}{2\varepsilon_0}\left[L-\sqrt{R^2+(r+L)^2}+\sqrt{R^2+r^2}\right]$ (2) $\frac{\rho}{2\varepsilon_0}\left[L-\sqrt{R^2+r^2}-\sqrt{R^2+(R+L)^2}\right]$

(3) $\frac{\rho}{2\varepsilon_0}\left[\sqrt{R^2+(r+L)^2}-\sqrt{R^2+r^2}\right]$ (4) $\frac{\rho}{2\varepsilon_0}\frac{R^2}{L}$

3. Consider a cylinder of radius R and length L having a charge distributed uniformly over its volume. The cylinder can be assumed to be made up of a large number of discs, each of thickness dx. The surface charge density of one such disc will be

(1) ρdx (2) $\frac{\rho L dx}{R}$ (3) $\frac{\rho R dx}{L}$ (4) $\frac{\rho\pi R^2}{L}$

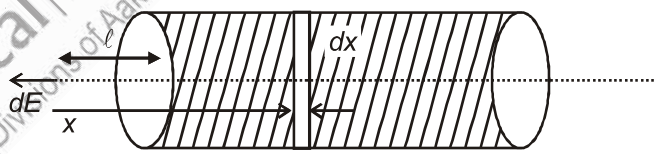
2. Answer (1)

3. Answer (1)

Solution of Q. No. 2 and 3

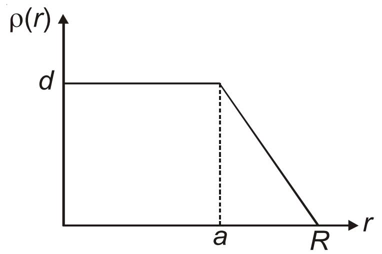
$\sigma = \rho dx$

$$E = \int dE = \int_r^{r+L}\frac{\rho dx}{2\varepsilon_0}\left[1-\frac{x}{\sqrt{R^2+x^2}}\right]$$



Comprehension-IV

The nuclear charge (Ze) is non-uniformly distributed within a nucleus of radius R. The charge density $\rho(r)$ [charge per unit volume] is dependent only on the radial distance r from the centre of the nucleus as shown in figure. The electric field is only along the radial direction



1. The electric field at $r = R$ is

(1) Independent of a
(2) Directly proportional to a
(3) Directly proportional to a^2
(4) Inversely proportional to a

Sol. Answer (1)

The electric field at R (at the surface) is

$$E = \frac{1}{4\pi\varepsilon_0}\frac{(Ze)}{R^2}$$

It is independent of a.

2. For $a = 0$, the value of d (maximum value of ρ as shown in the figure) is

(1) $\frac{3Ze}{4\pi R^3}$ (2) $\frac{3Ze}{\pi R^3}$ (3) $\frac{4Ze}{3\pi R^3}$ (4) $\frac{Ze}{3\pi R^3}$

Sol. Answer (2)

When $a = 0$, $\rho = d\left(1 - \frac{r}{R}\right)$

Now, Ze = $\int_0^R \rho \times 4\pi r^2 dr = 4\pi d\int_0^R \left(r^2 - \frac{r^3}{R}\right) dr$

$\Rightarrow \quad d = \frac{3Ze}{\pi R^3}$

3. The electric field within the nucleus is generally observed to be linearly dependent on r. This implies

(1) $a = 0$ (2) $a = \frac{R}{2}$ (3) $a = R$ (4) $a = \frac{2R}{3}$

Sol. Answer (3)

When $E \propto \rho \Rightarrow$ charge density is uniform (like the case of a uniformly charged sphere).

So, $a = R$

SECTION - D

Matrix-Match Type Questions

1. Column I shows charge distribution and column II shows electrostatic field created by that charge at a point r distance from its centre.

Column I		Column II
(A) A stationary point charge	(p)	$E \propto r^0$
(B) A stationary uniformly long charge rod	(q)	$E \propto r^{-1}$
(C) A stationary electric dipole	(r)	$E \propto r^{+1}$
(D) A uniform charged solid sphere	(s)	$E \propto r^{-2}$
	(t)	$E \propto r^{-3}$

Sol. Answer A(s), B(q), C(t), D(r, s)

For a point charge, $E = \frac{kq}{r^2}$

For a line charge, $E = \frac{2k\lambda}{r}$

For an electric dipole, $E = \frac{2kp}{r^3}$

For a uniformly charged sphere, $E \propto r$, inside;

$E \propto \frac{1}{r^2}$, outside.

2. Column I shows distribution of electric field lines and column II shows charge distribution. Match the columns.

Column I	Column II
(A) Straight lines but not parallel	(p) Uniformly distributed charged shell
(B) Parallel straight lines	(q) Uniformly distributed charged long rod
(C) Curved lines	(r) Uniformly distributed infinite charged sheet
(D) Some regions where no lines are present	(s) Electric dipole
	(t) Any conducting material

Sol. Answer A(p, q), B(r), C(s), D(p, t)

For a uniformly charged shell, inside field is zero, and outside, field lines are straight and radial.

For a long rod, field lines are straight, but radial.

For an infinite charged sheet, field lines are parallel

For a dipole, field lines are in general curved.

For a conductor, field is zero in some region. They may be straight or curved.

3. $\oint \vec{E} \cdot d\vec{s}$ = Electric flux through a closed surface '*s*'

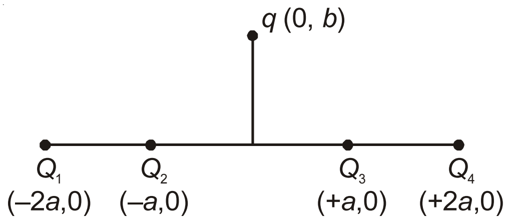
Match the columns :

Column I	Column II
(A) The value of $\oint \vec{E}.d\vec{s}$ is affected by	(p) +ve charge inside '*s*'
(B) Electric field at a point inside '*s*' is affected by	(q) –ve charge inside '*s*'
(C) Electric field at a point outside '*s*' is affected by	(r) +ve charge outside '*s*'
(D) Electric field at a point at the surface of '*s*' is affected by	(s) –ve charge outside '*s*'
	(t) Outside charges but very far away from '*s*'

Sol. Answer A(p, q), B(p, q, r, s), C(p, q, r, s), D(p, q, r, s)

Electric flux through a closed surface is affected only by charged inside the surface, while the electric field is affected by all charges weather inside or outside.

4. Four charges Q_1, Q_2, Q_3 and Q_4 of same magnitude are fixed along the x axis at $x = -2a, -a, +a$ and $+2a$, respectively. A positive charge q is placed on the positive y axis at a distance $b > 0$. Four options of the signs of these charges are given in Column-I. The direction of the forces on the charge q is given in Column-II. Match Column-I with Column-II and select the correct answer using the code given below the lists.



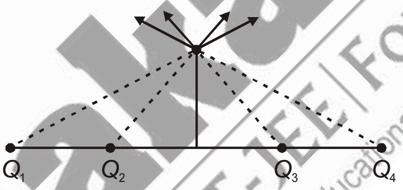
	Column-I		**Column-II**
(A)	Q_1, Q_2, Q_3, Q_4 all positive	(p)	$+x$
(B)	Q_1, Q_2 positive; Q_3, Q_4 negative	(q)	$-x$
(C)	Q_1, Q_4 positive; Q_2, Q_3 negative	(r)	$+y$
(D)	Q_1, Q_3 positive; Q_2, Q_4 negative	(s)	$-y$

Code :

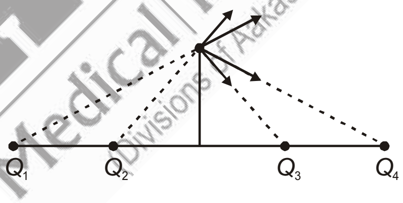
	A	**B**	**C**	**D**
(1)	r	p	s	q
(2)	s	q	r	p
(3)	r	p	q	s
(4)	s	q	p	r

Sol. Answer (1)

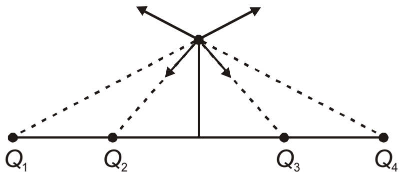
A. Not along $+y$



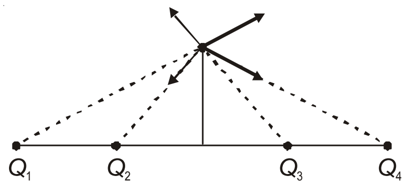
B. Not along $+x$



C. Not along $-y$



D. Not along $-x$



SECTION - E

Assertion-Reason Type Questions

1. A positively charged ball is brought close to a neutral conducting isolated sphere. The isolated sphere is grounded for an instant and then the ground connection is removed. Consider the following statements.

   STATEMENT-1 : The sphere will acquire a negative charge after this process.

   and

   STATEMENT-2 : If the sphere is kept grounded and positively charged ball is moved away, the sphere will again become neutral.

Sol. Answer (2)

Negative charge flows to the grounded sphere (placed near positively charged ball) to make its positive potential zero. If the charged ball is moved away, then sphere is no longer at positive potential. To keep the sphere at zero potential, negative charge flows back to ground.

2. A Gaussian surface encloses a proton *p*. The electric field at any point on the surface is $\vec{E}$. The flux linked with the Gaussian surface is ϕ.

   STATEMENT-1 : When an electron is kept close to this system outside the Gaussian surface, the flux linked with the surface would change.

   and

   STATEMENT-2 : The presence of electron will alter the electric field on the gaussian surface.

Sol. Answer (4)

Outside charge will alter the value of electric field at different points of Gaussian surface but its net contribution in the flux through the Gaussian surface is zero.

3. STATEMENT-1 : An isolated neutral conductor is given some charge. As it is being charged, an electric field appears in the conductor.

   and

   STATEMENT-2 : Under static conditions, the net electric field inside a conductor is zero.

Sol. Answer (2)

During charging of the conductor distribution of charge on the conductor takes place due to existence of electric field. Charge will keep on moving if there is electric field in the conductor. But after under electrostatic condition is reached, net electric field inside the conductor is zero.

4. STATEMENT-1 : When a charged particle is released from rest in a region of electric field, its path will represent the electric field lines in the region.

   and

   STATEMENT-2 : The force experienced by the charged particle will be along a tangent drawn to the electric field line at a point.

Sol. Answer (4)

At any point in the field, a charge moves tangent to the field, but not along the field. Only in a straight line, the field and tangent coincides.

5. STATEMENT-1 : A point charge be rotated in a circle around a fixed charge, the work done by electric field of fixed charge will be zero.

   and

   STATEMENT-2 : Work done is the dot product of force and displacement.

Sol. Answer (2)

Here, force and displacement are perpendicular.

6. STATEMENT-1 : Two positively charged objects may attract each other.

and

STATEMENT-2 : A charge of large magnitude produces more induced charge on a body of small charge.

Sol. Answer (1)

The opposite charge induced may be more so as to produce attraction.

7. STATEMENT-1 : If a charge particle enters into a uniform electric field, then its path must be straight line.

and

STATEMENT-2 : Electrostatic force on a positive charge particle is always in the direction of electric field.

Sol. Answer (4)

In a uniform electric field, path may or may not be a straight line.

8. STATEMENT-1 : If potential energy of a dipole in stable equilibrium position is zero, its potential energy in unstable equilibrium position will be $2pE$, where p and E represent the dipole moment and electric field respectively.

and

STATEMENT-2 : The potential energy of a dipole is minimum in stable equilibrium position and maximum in unstable equilibrium position.

Sol. Answer (2)

Difference in potential energy is independent of the reference level.

$\theta = 0°$ is the position of stable equilibrium and $\theta = 180°$ is the position of unstable equilibrium.

If $U = 0$ at $\theta = 90°$

ΔU (between $\theta = 0°$ and $\theta = 180°$) $= +pE - (-pE) = 2pE$

If reference level is taken at $\theta = 0°$, then

$\Delta U = U_2$ (at $\theta = 180°$) $- U_1$ (at $q = 0°$)

$2pE = U_2 - 0$

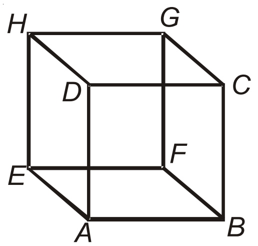
$\Rightarrow U_2 = 2pE$

Two statements are independently true.

SECTION - F

Integer Answer Type Questions

1. Consider an imaginary cube with different vertices of the cube marked as shown. A point charge is placed at the vertex H and flux through the face $ABCD$ is ϕ in this case. Now, the charge is shifted to centroid of the cube. Flux through the face $AEHD$ in this case is $k\phi$. What is the value of k ?



Sol. Answer (4)

Case I : $\phi = \dfrac{q}{24\varepsilon_0}$

Case II : $k\phi = \dfrac{q}{6\varepsilon_0} \Rightarrow k = 4$

2. The maximum electric field upon the axis of a circular ring (q, R) is given by $E = \dfrac{q}{\pi\varepsilon_0 R^2} \times \dfrac{1}{6\sqrt{n}}$. Find n.

Sol. Answer (3)

$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q.x}{(x^2 + R^2)^{3/2}}$$

Putting $\dfrac{dE}{dx} = 0$

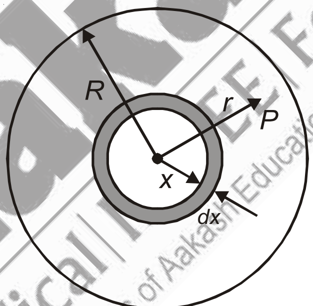
$$\Rightarrow \quad x = \pm \frac{R}{\sqrt{2}}$$

Find E_{max}.

3. A solid sphere of radius R has a charge Q distributed in its volume with a charge density $\rho = kr^a$, where k and a are constants and r is the distance from its centre. If the electric field at $r = \dfrac{R}{2}$ is $\dfrac{1}{8}$ times that at $r = R$, find the value of a.

Sol. Answer (2)

$$\oint \bar{E}.d\bar{s} = \frac{q_{in}}{\varepsilon_0}$$



$$E.4\pi r^2 = \frac{1}{\varepsilon_0}\int_0^r kx^a \, 4\pi x^2 dx$$

$$E.4\pi r^2 = \frac{1}{\varepsilon_0}\int_0^r kx^a \, 4\pi x^2 dx$$

$$E = \frac{k}{\varepsilon_0 r^2}\left[\frac{r^{a+3}}{a+3}\right]$$

$$E = \frac{kr^{a+1}}{\varepsilon_0(a+3)}$$

$$E = \frac{kr^{a+1}}{\varepsilon_0(a+3)}$$

$$E_{R/2} = \frac{1}{8}E_R$$

$$\Rightarrow \quad \frac{k}{\varepsilon_0(a+3)}\left[\frac{R}{2}\right]^{a+1} = \frac{1}{8}\frac{k}{\varepsilon_0(a+3)}[R]^{a+1}$$

$$\Rightarrow \quad a = 2$$

4. Four point charges, each of $+q$, are rigidly fixed at the four corners of a square planar soap film of side 'a'. The surface tension of the soap film is γ. The system of charges and planar film are in equilibrium, and $a = k\left[\frac{q^2}{\gamma}\right]^{1/N}$, where '$k$' is a constant. Then N is

Sol. Answer (3)

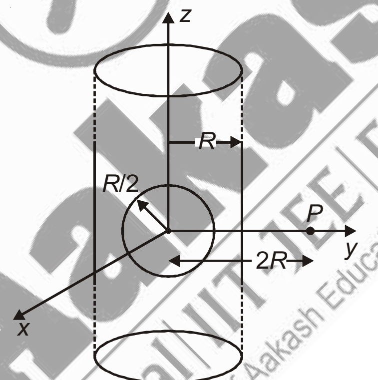
$$(2 \times a \times g) \times c = 2\times\left\{\frac{kq^2}{a^2}+\frac{Kq^2}{2a^2}\times\frac{1}{\sqrt{2}}\right\}$$

$$\Rightarrow \quad a^3 = k\left\{\frac{q^2}{\gamma}\right\}\left\{1+\frac{1}{2\sqrt{2}}\right\}$$

$$\Rightarrow \quad a^3 = K'\left\{\frac{q^2}{\gamma}\right\}$$

$$\Rightarrow \quad a = K''\left\{\frac{q^2}{\gamma}\right\}^{\frac{1}{3}}$$

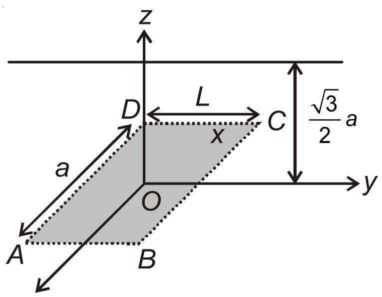
5. An infinitely long solid cylinder of radius R has a uniform volume charge density ρ. It has a spherical cavity of radius $R/2$ with its centre on the axis of the cylinder, as shown in the figure. The magnitude of the electric field at the point P, which is at a distance $2R$ from the axis of the cylinder, is given by the expression $\frac{23\rho R}{16k\varepsilon_0}$. The value of k is



Sol. Answer (6)

$$E_P = E_{\text{cylinder}} - E_{\text{sphere}} = \frac{\rho R^2}{2\varepsilon_0(2R)} - \frac{\rho(R^3/8)}{3\varepsilon_0(2R)^2} = \frac{\rho R}{4\varepsilon_0} - \frac{\rho R}{96\varepsilon_0} = \frac{23\rho R}{96\varepsilon_0} \Rightarrow k = 6$$

6. An infinitely long uniform line charge distribution of charge per unit length λ lies parallel to the y-axis in the y-z plane at $z = \frac{\sqrt{3}}{2}a$ (see figure). If the magnitude of the flux of the electric field through the rectangular surface $ABCD$ lying in the x-y plane with its centre at the origin is $\frac{\lambda L}{n\varepsilon_0}$ (ε_0 = permittivity of free space), then the value of n is

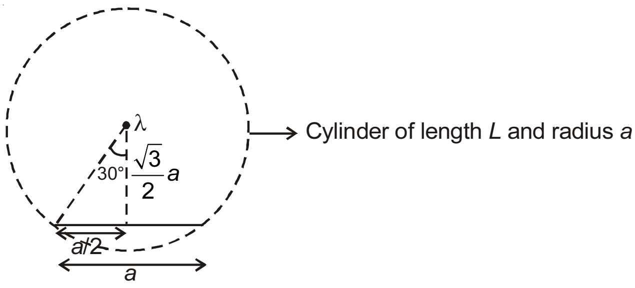


Sol. Answer (6)

The flux through the surface is $\frac{1}{6}\times\frac{q}{\varepsilon_0}$

$$q = \lambda L \Rightarrow \phi = \frac{q}{6\varepsilon_0} = \frac{\lambda L}{6\varepsilon_0}$$

$\Rightarrow n = 6$



7. Two inifinite rods parallel to z-axis with linear charge density λ passes through points ($-a$, 0, 0) and (a, 0, 0) respectively. A charge particle having charge ($-q$) and mass m placed at origin is given slight displacement along y-axis. Time period of resulting motion is $\sqrt{\frac{x\pi^3\varepsilon_0 ma^2}{8\lambda q}}$, value of x is

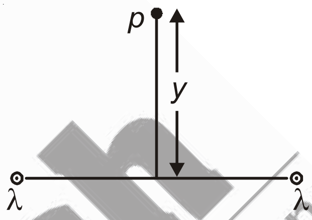
Sol. Answer (32)

$$F = \frac{2\lambda q}{2\pi\varepsilon_0}\frac{y}{\left(a^2 + y^2\right)}$$

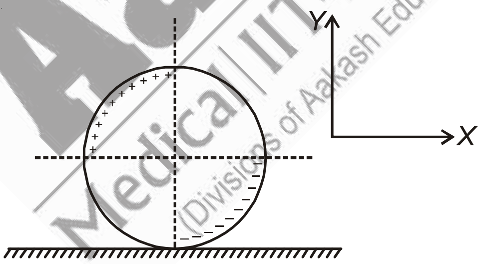
For $y << a$

$$F = \frac{\lambda qy}{\pi\varepsilon_0 a^2}$$

$$T = 2\pi\sqrt{\frac{\pi\varepsilon_0 ma^2}{\lambda q}}$$



8. A non-conducting ring of mass m and radius R is charged as shown. The linear charge density for corresponding region shown is λ. It is placed on tough horizontal surface. A uniform electric field $E_0\hat{i}$ is switched on and the ring starts rolling without sliding. Angular velocity acquired by the ring when it turns through an angle of $\frac{\pi}{2}$ is 16 rad/s. Value of $\frac{\lambda E_0}{m}$ (in SI unit) is



Sol. Answer (64)

Since there is pure rolling

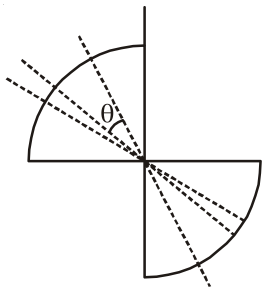
$W_{\text{friction}} = 0$

$\Delta K = -\Delta U$

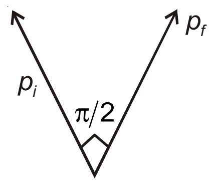
$$p_i = \int_{-\pi/4}^{\pi/4} \lambda R d\theta 2R\cos\theta$$

$$p_i = 2\lambda R^2\sqrt{2}$$

$$p_f = 2\lambda R^2\sqrt{2}$$



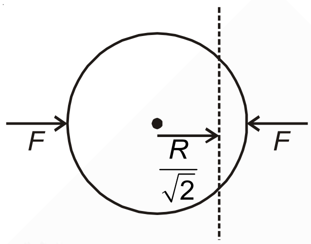
$\Delta U = pE_0\cos 45° + pE_0\cos 45° = -2\lambda R^2\sqrt{2}\,E_0 \times \sqrt{2} = -4\lambda R^2 E_0$



$$\frac{1}{2}\left(2MR^2\right)\omega^2 = 4\lambda R^2 E_0$$

$$\omega = 2\sqrt{\frac{\lambda E_0}{m}}$$

9. A thin spherical shell of radius R carries uniform surface charge density σ. It is cut into two parts by a plane at a distance $\frac{R}{\sqrt{2}}$ from centre as shown. To hold two parts together force F from both side has to be applied. The minimum force F is $\frac{3\pi\sigma^2 R^2}{n\varepsilon_0}\cdot$ Value of n, is

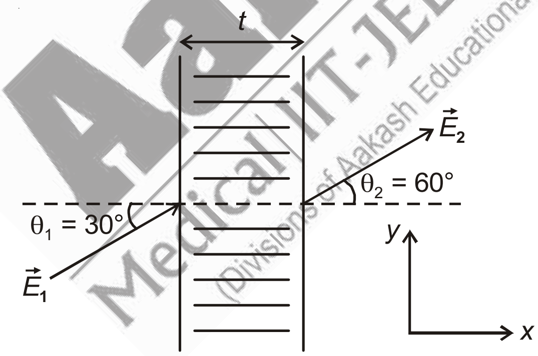


Sol. Answer (12)

$$P = \frac{\sigma^2}{2\varepsilon_0}$$

$$F = P.\ \text{(Projected Area)} = \frac{\pi R^2}{2}\left(\frac{\sigma^2}{2\varepsilon_0}\right)$$

10. An infinitely large layer of charge of uniform thickness t is placed in existing uniform electric field. Presence of this charged layer so alters the electric field that it remains uniform on both the sides and assumes values E_1 and E_2 as shown in figure. If, $\left|\vec{E}_2\right| = 20$ N/ C, then component of external field in x direction (in N/C) is



Sol. Answer (20)

$$E_2\sin\theta_2 = E_1\sin\theta_1$$

$$E_0 + \frac{\sigma}{2\varepsilon_0} = E_2\cos\theta_2$$

$$E_0 - \frac{\sigma}{2\varepsilon_0} = E_1\cos\theta_1$$

$$E_0 = \frac{E_2}{2\sin\theta_1}$$

❑ ❑ ❑