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Probability

PROBABILITY

INTRODUCTION

1. Factorial notation :

If $n \in N$, then the product $1 \times 2 \times 3 \times ......\times n$ is defined as factorial n which is denoted by $n!$ or $\lfloor n$

i.e., $n! = 1 \times 2 \times 3 \times ......\times n$

We also define $0! = 1$

Note...

$n! = n(n-1)!$

2. Permutation :

If n objects are given and we have to arrange $r(r \leq n)$ out of them such that the order in which we are arranging the objects is important, then such an arrangement is called permutation of n objects taking r at a time. This is denoted

$${}^{n}P_{r} = \frac{n!}{(n-r)!}$$

3. Combination :

If n objects are given and we have to choose $r(r \leq n)$ out of them such that the order in which we are choosing the objects is not important, then such a choice is called combination of n objects taking r at a time. This is denoted by

$${}^{n}C_{r} = \frac{n!}{r!(n-r)!}$$

4. Fundamental Principle of Counting :

If an event can occur in 'm' different ways following which another event can occur in 'n' different ways following which another event can occur in 'p' different ways then the total number of ways of simultaneous occurence of all these events in a definite order is $m \times n \times p$.

DEFINITIONS AND TYPES OF EVENTS

1. Random experiment :

If an act or an experiment has more than one possible results which are known in advance and it is not possible to predict which one is going to occur, then such an experiment is called a **random experiment.**

The following are some random experiments :

(i) Tossing of a coin

(ii) Throwing a six-faced die

(iii) Drawing a card from a well-shuffled pack of cards

(iv) Ten horses run a race

(v) Two persons are selected out of 10 persons to form a committee.

(vi) A ball is drawn from a bag containing 7 balls.

2. Outcome :

The result of a random experiment is called an outcome.

3. Sample space :

The set of all possible outcomes of a random experiment is called a **sample space** and its elements are called **sample points.** A sample space is usually denoted by S.

Illustrations ;

(i) When a fair coin is tossed, then either head or tail will turn up.

Hence S = {H.T.} S contains 2 sample points.

(ii) When a six-faced die is thrown, then only one of 1,2,3,4,5,6 will turn up.

Hence S = {1,2,3,4,5,6}. S contains 6 sample points.

(iii) Suppose a bag contains 7 balls.

Consider the sample points.

(a) The experiment is : one ball is drawn. We can draw one ball out of the 7 balls in ${}^7C_1 = 7$ ways.

$\therefore$ The sample space for this experiment contains 7 sample points.

(b) The experiment is : two balls are drawn. We can draw 2 balls out of the 7 balls in

$${}^7C_2 = \frac{7\times6}{1\times2} = 21 \text{ ways}$$

$\therefore$ the sample space for this experiment contains 21 sample points.

(c) The experiment is : three balls are drawn.

The sample space for this experiment contains ${}^7C_3 =$

$$\frac{7\times6\times5}{1\times2\times3} = 35 \text{ sample points}$$

4. Event :

Any subset of a sample space is called an event.

Example :

In a single throw of a die, the event of getting a prime number is $E \equiv \{2,3,5\}$ The sample space $S \equiv \{1,2,3,4,5,6\}$

$\therefore E \subseteq S \Rightarrow E$ is an event

5. Complementary event :

Let A be an event in a sample space S. Then A is a subset of S We can hence think of the complement of A in S, i..e., S-A. This is also a subset of S and hence an event in S. This event is called the complementary event of A and is denoted by $\overline{A}$ or A'

Now, suppose S contains n sample points, A contains m sample points, Then A' will contain $n - m$ sample points.

6. Impossible event :

Let S be a sample space. Since $\phi \subseteq S$, So ϕ is an event, called an impossible event.

Note..

The event E and not E are such that only one of them can occur in a trial and at least one of them must occur.

7. Union of events :

If A and B are two events of the sample space S then $A \cup B$ or $A + B$ is the event that either A or B (or both) take place.

8. Intersection of events :

If A and B are two events of the sample space S then $A \cap B$ or A . B is the event that both A and B take place.

9. Mutually Exclusive events :

Two events A and B of the sample space S are said to be mutually exclusive if they cannot occur simultaneously. In such case $A \cap B$ is a null set.

10. Exhaustive events :

Two events A and B of the sample space S are said to be exhaustive if $A \cup B = S$ i.e. $A \cup B$ contains all sample points.

11. Probability of an event :

Let A be an event in a sample space S. Then the probability of the event A denoted by P(A) is defined as,

$$P(A) \frac{\text{number of sample points in A}}{\text{number of sample points in S}} = \frac{n(A)}{n(S)}$$

Theorem :

If E is an event of a sample space S, prove that $0 \le P(E) \le 1$ and $P(E') = 1 - P(E)$, where E' is the complementary event of E.

Proof :

Suppose the sample space S contains n sample points and the event E contains m sample points. Then $P(E) = \frac{m}{n}$

Now, $0 \le m \le n \therefore 0 \le \frac{m}{n} \le 1$

$\therefore 0 \le P(E) \le 1$

Further E' contains $n - m$ sample points,

$\therefore P(E') = 1 - P(E)$

12. Equally likely event :

The events are said to be equally likely, if none of them is expected to occur in preference to the other.

Ex :- When a die is thrown, then all the side faces are equally likely to come.

EXPERIMENT NO 1 : TOSSING COINS

Tossing one coin :

Let S be the sample space $S \equiv \{H,T\}$, $n(S) = 2$

Tossing two coin :

Let S be the sample space $S \equiv \{HH, HT, TH, TT\} \Rightarrow n(S) = 4$

Tossing three coin :

Let S be the sample space

$S \equiv \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$

$\Rightarrow n(S) = 8$

Note...

1. No head means all tails.
2. At least one head means one head or two heads or three heads
3. At most two heads means two heads or one head or no head (all tail).

EXPERIMENT NO 2 : THROWING DIE / DICE

One six faced die is thrown :

$S \equiv \{1,2,3,4,5,6\} \Rightarrow n(S) = 6$

Two dice are thrown :

$$S \equiv \begin{Bmatrix} (1,1) & (2,1) & (3,1) & (4,1) & (5,1) & (6,1) \\ (1,2) & (2,2) & (3,2) & (4,2) & (5,2) & (6,2) \\ (1,3) & (2,3) & (3,3) & (4,3) & (5,3) & (6,3) \\ (1,4) & (2,4) & (3,4) & (4,4) & (5,4) & (6,4) \\ (1,5) & (2,5) & (3,5) & (4,5) & (5,5) & (6,5) \\ (1,6) & (2,6) & (3,6) & (4,6) & (5,6) & (6,6) \end{Bmatrix}$$

$\Rightarrow n(S) = 36$

Note...

If one coin is tossed n times or n coins are tossed once the sample space consists of same number of sample points.

i.e. $n(S) = (2)^n$

Prime numbers are 2, 3, 5, 7, 11, ...

1 card is picked $n(S) = {}^{52}C_1 = 52$

2 card are drawn. $n(S) = {}^{52}C_2$

EXPERIMENT NO 3: PACK OF CARDS

1. There are 4 suits (spade, heart, diamond and club) each having 13 cards.
2. There are two colours red (heart and diamond) and black (spade and club) each having 26 cards.
3. Jack, Queen and King are face cards. Therefore face cards are 12 in pack of cards. Face card is also called a picture card.
4. There are four aces. Ace is not a picture card.
5. Face cards and Ace cards are known Coloured Cards.

Theorem 1

If E is an event of sample space S then $0 \le P(E) \le 1$

Proof :

As $E \subseteq S$

$\therefore \quad 0 \le n(E) \le n(S)$

$\therefore \quad \frac{0}{n(S)} \le \frac{n(E)}{n(S)} \le \frac{n(S)}{n(S)}$ $(\because n(S) \ne 0.$

$\therefore \quad 0 \le P(E) \le 1$

Note...

$P(E) = 0$ if and only if E is an impossible event and

$P(E) = 0$ if and only if E is a certain event

Theorem 2

If E is an event of sample space S and E' is the event that E does not happen then

$P(E') = 1 - P(E)$

Proof :

E' is the event that E does not happen.

$\because$ E **and** E' are complements of each other.

$\therefore$ $n(E) + n(E') = n(S)$ Dividing by $n(S)$, ($\because n(S) \neq 0$.

$\therefore$ $\frac{n(E)}{n(S)} + \frac{n(E')}{n(S)} = \frac{n(S)}{n(S)}$

$\therefore$ $P(E) + P(E') = 1$

$\therefore$ $P(E') = 1 - P(E)$

Theorem 3

(i) If A and B are two events of sample sapce S, prove that $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

(ii) $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$

Theorem 4

Show that if A and B are independent events defined on S then

(i) A & B' (ii) A' & B (iii) A' & B' are

independent event where A and B are mutually events of A' and B' respectively.

Ans. Since A and B are known to be independence events.

We have $P(A \cap B) = P(A) . P(B)$

(i) $P(A \cap B') = P(A) - P(A \cap B) = P(A) - P(A). P(B)$

$= P(A)(1 - P(B))$

$= P(A) . P(B')$

$\therefore$ A and B' are independent events

(ii) $P(A' \cap B) = P(B) - P(A \cap B) = P(B) - P(A) . P(B)$

$= P(B)(1 - P(A)) = p(B) . P(A')$

$\therefore$ A' and B are independent events

(iii) $P(A' \cap B') = P(A \cup B)' = 1 - P(A \cup B)$

$= 1 - [P(A) + P(B) - P(A \cap B)]$

$= 1 - [P(A) + P(B) - P(A) . (B)]$

$= 1 - PA - P(B) + P(A) . P(B)$

$= [1 - P(A)] - P(B) [1 - P(A)]$

$= [1 - P(A)] [1 - P(B)] = P(A') . P(B')$

A' and B' are independent events.

IMPORTANT RESULTS

1. A and B are mutually exclusive if $P(A \cap B) = 0$.
2. They are independent if $P(A \cap B) = P(A) . P(B)$
3. Two independent events cannot be mutually exclusive.
4. If A, B and C are independent events with non-zero probabilities then $P(A \cap B \cap B) = P(A) . P(B) . P(C)$
5. If $A_1, A_2, A_3, \ldots A_n$ are independent events with non-zero probabilities, then

   $P(A_1 \cap A_2 \cap A_3 \cap \ldots\ldots \cap A_n) = P(A_1) . P(A_2) \ldots P(A_n)$
6. P [exactly one of A or B occurs]

   $= P(A \cap B') + P(A' \cap B)$

   $= P(A \cup B) - P(A \cap B)$

   $= P(A) + P(B) - 2P(A \cap B)$
7. $P(A' \cap B') = P(A \cup B)' = 1 - P(A \cup B)$
8. $P(A' \cup B') = P(A \cap B)' = 1 - P(A \cap B)$
9. $P(A) = P(A \cap B) + P(A \cap B')$
10. $P(B) = P(A \cap B) + P(A' \cap B)$
11. $P(A \cup B \cup C) = P(A) + P(B) + P(C)$

    $- P(A \cap B) - P(B \cap C) - P(C \cap A)$

    $+ P(A \cap B \cap C)$
12. $P(A \cup B \cup C) = P(A) + P(B) + P(C)$ if A, B and C are mutually exclusive.
13. If A and B are mutually exclusive and exhausive events then $P(A) + P(B) = 1$
14. If A, B and C are mutually exclusive and exhaustive events then

    $P(A) + P(B) + P(C) = 1$

CONDITIONAL PROBABILITY

Let A and B be two events associated with a random experiment. Then, the probability of occurrence of event A under the condition that B has already occurred and P (B) ≠ 0, is called the conditional probability and it is denoted by P (A/B). Thus, we have

P (A/B) = Probability of occurence of A given that B has already occurred.

$$P(A/B) = \frac{\text{Number of elementary events favourable } A \cap B}{\text{Number of elementary events favourable to B}}$$

$$\Rightarrow \quad P(A/B) = \frac{n(A \cap B)}{n(B)}$$

$$\Rightarrow \quad P(A/B) = \frac{P(A \cap B)}{P(B)}$$

Theorem

If A and B are two events associated with a random experiment, then

$P(A \cap B) = P(A)\,P(B/A)$, if $P(A) \neq 0$

Note :

1. $0 \le P(A/B) \le 1$

2. P (A/A) = 1

INDEPENDENT EVENTS

Definition :

Events are said to be independent, if the occurrence or non-occurrence of one does not affect the probability of the occurrence or non-occurrence of the other.

Theorem 1 :

If A and B are independent events associated with a random experiment, then $P(A \cap B) = P(A)\,P(B)$.

Theorem 2 :

If $A_1, A_2, ..., A_n$ are independent events associated with a random experiment, then

$P(A_1 \cap A_2 \cap A_3 ... \cap A_n) = P(A_1)\,P(A_2) ... P(A_n)$

Theorem 3 :

If A and B are independent events associated with a random experiment, then

(i) $\bar{A}$ and B are independent events

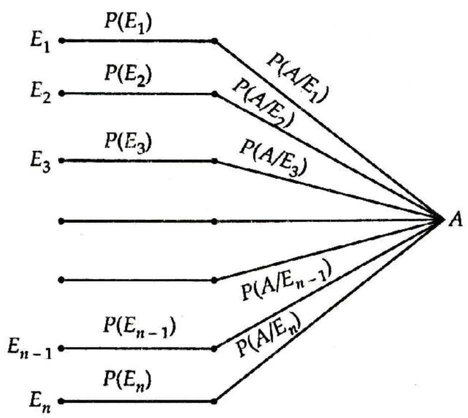
(ii) A and $\bar{B}$ are independent events

(iii) $\bar{A}$ and $\bar{B}$ are also independent events.

THE LAW OF TOTAL PROBABILITY

Theorem : (Law of Total Probability) Let S be the sample space and let $E_1, E_2, ..., E_n$ be n mutually exclusive and exhaustive events associated with a random experiment. If A is any event which occurs with E_1 or E_2 or ... or E_n, then

$P(A) = P(E_1)\,P(A/E_1) + P(E_2)\,P(A/E_2) + ... + P(E_n)\,P(A/E_n)$

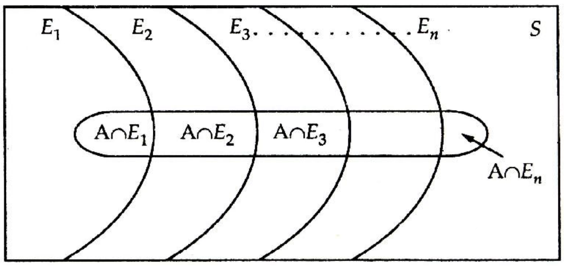


STATEMENT OF BAYES' THEOREM

If $B_1, B_2, B_3, \ldots B_n$ are mutually exclusive and exhaustive events & if A is an event consequent to these B_i's then for each i, where i = 1, 2, 3...., n

$$P\left(\frac{B_i}{A}\right) = \frac{P(B_i)P\left(\frac{A}{B_i}\right)}{P(B_1)P\left(\frac{A}{B_1}\right) + P(B_2)P\left(\frac{A}{B_2}\right) \ldots\ldots + P(B_n)P\left(\frac{A}{B_n}\right)}$$

$$P\left(\frac{B_i}{A}\right) = \frac{P(B_i)P\left(\frac{A}{B_i}\right)}{\sum_{i=1}^{n} P(B_i)P\left(\frac{A}{B_i}\right)}$$



BINOMIAL DISTRIBUTION

(i) Bernoulli Trials : Trials of a random experiment are called Bernoulli trials, if they satisfy the following conditions :

(i) They are finite in number.

(ii) They are independent of each other.

(iii) Each trial has exactly two outcomes : success or failure.

(iv) The probability of success or failure remains same in each trial.

(v) The probability of success is p and failure is q such that p + q = 1

(vi) The probability of r successes in n trails in any order is given by $^nC_r\, p^r\, q^{n-r}$.

(ii) Binomial Distribution : Let X denote the random variable which associates every outcome to the number of successes in it. Then, X assumes values 0, 1, 2, ..., n such that

$P(X = r) = {}^nC_r p^r q^{n-r}$, r = 0, 1, 2, ..., n.

The probability distribution of the random variable X is therefore given by

X :	0	1	2	...	r	...	n
P(X) :	$^nC_0p^0q^{n-0}$	$^nC_1p^1q^{n-1}$	$^nC_2\, p^2q^{n-2}$	...	$^nC_rp^rq^{n-r}$	...	$^nC_np^nq^{n-n}$

(iii) Mean & Variance :

Mean = np

Variance = npq

SOLVED EXAMPLES

Example – 1

From a group of 8 boys and 5 girls a committee of 5 is to be formed. Find the probability that the committee contains

(i) 3 boys and 2 girls

(ii) at least 3 boys

Sol. From a group of 8 boys and 5 girls, a committee of 5 is to be fromed.

$\therefore$ $n(S) = {}^{(8+5)}C_5$

$= {}^{13}C_5$

$= \dfrac{13\times12\times11\times10\times9}{5\times4\times3\times2\times1}$

$= 9\times11\times13 = 1287$

(i) 3 boys & 2 girls

$\therefore$ $n(A) = {}^5C_2 \cdot {}^8C_3$

$= \dfrac{8\times7\times6}{3\times2\times1}\times\dfrac{5\times4}{2\times1}$

$= 7\times8\times10 = 560$

$\therefore$ Probability of the event

$P(A) = \dfrac{n(A)}{n(S)} = \dfrac{7\times8\times10}{9\times11\times13}$

$P(A) = \dfrac{560}{1287}$

(ii) at least 3 boys

Let event B

= To select 5 containing at least 3 boys.

= To select 3 boys, 2 girls or 4 boys, 1 girl or all 5 boys.

$\therefore$ $n(B) = {}^8C_3 \cdot {}^5C_2 + {}^8C_4 \cdot {}^5C_1 + {}^8C_5 \cdot {}^5C_0$

$= \dfrac{8\times7\times6}{3\times2\times1}\cdot\dfrac{5\times4}{2\times1} + \dfrac{8\times7\times6\times5}{4\times3\times2\times1}\times5 + \dfrac{8\times7\times6}{3\times2\times1}\times1$

$= 560 + 350 + 56$

$= 966$

$\therefore$ Probability of the event

$\therefore$ $P(B) = \dfrac{n(B)}{n(S)} = \dfrac{966}{1287} = \dfrac{322}{429}$

Example – 2

5 letters are to be posted in 5 post boxes. If any number of letters can be posted in 5 post boxes, what is the probability that each box contains only one letter?

Sol. Since any number of letters can be posted in all 5 post boxes, each letter can be posted in 5 different ways.

$\therefore$ $n(S) = 5\times5\times5\times5\times5 = 5^5$

Let $A \equiv$ the event that each box contains only one letter.

The first letter can be posted in 5 post boxes in 5 different ways. Since each box contains only one letter, the second letter can be posted in the remaining 4 post boxes in 4 different ways.

Similarly, the third letter can vbe posted in 3 different ways, the fourth letter can be posted in 2 different ways and the fifth letter can be posted in 1 way.

$\therefore$ $n(A) = 5\times4\times3\times2\times1 = 5!$

$\therefore$ $P(A) = \dfrac{n(A)}{n(S)} = \dfrac{5!}{5^5}$

$= \dfrac{5\times4\times3\times2\times1}{5\times5^4} = \dfrac{24}{625}$

Example – 3

A bag contains 50 tickets, numbered from 1 to 50. One ticket is drawn at random. What is the probability that

(i) number on the ticket is a perfect square or divisible by 4 ?

(ii) number on the ticket is prime number or greater than 30 ?

Sol. One ticket can be drawn out of 50 tickets in ${}^{50}C_1 = 50$ ways

$n(S) = 50$

(i) Let A $\equiv$ the event that the number on the ticket is a perfect square.

$\therefore$ $A = \{1, 4, 9, 16, 25, 36, 49\}$

$\therefore$ $n(A) = 7$

$\therefore \quad P(A)=\frac{n(A)}{n(S)}=\frac{7}{50}$

Let B $\equiv$ the event that the number.

divisible by 4.

$\therefore \quad B=\{4,8,12,16,20,24,28,32,36,40,44,48\}$

$\therefore \quad n(B)=12$

$\therefore \quad P(B)=\frac{n(B)}{n(S)}=\frac{12}{50}$

$A\cap B=\{4,16,36\}$

$\therefore \quad n(A\cap B)=3$

$\therefore \quad P(A\cap B)=\frac{n(A\cap B)}{n(S)}=\frac{3}{50}$

$\therefore$ required probability

$=P(A\cup B)$

$=P(A)+P(B)-P(A\cap B)$

$=\frac{7}{50}+\frac{12}{50}-\frac{3}{50}=\frac{16}{50}=\frac{8}{25}$

(ii) Let C = the event that the number on the ticket is prime numbers or greater than 30.

$\therefore \quad C=\{2,3,5,7,11,13,17,19,23,29,31,32,33,34,35,36,37,38,39,40,41,42,43,44,45,46,47,48,49,50\}$

$\therefore \quad n(C)=30$

$\therefore \quad P(C)=\frac{n(B)}{n(S)}=\frac{30}{50}$

$\therefore \quad P(C)=\frac{3}{5}$

Example – 4

A pair of dice is thown. If sum of the numbers is an even number, what is the probability that it is a perfect square ?

Sol. Let, S be the sample space of throwing two dice.

$\therefore$ S= {(1, 1) (1, 2) (1, 3) (1, 4) (1, 5) (1, 6)
(2, 1) (2, 2) (2, 3) (2, 4) (2, 5) (2, 6)
(3, 1) (3, 2) (3, 3) (3, 4) (3, 5) (3, 6)
(4, 1) (4, 2) (4, 3) (4, 4) (4, 5) (4, 6)
(5, 1) (5, 2) (5, 3) (5, 4) (5, 5) (5, 6)
(6, 1) (6, 2) (6, 3) (6, 4) (6, 5) (6, 6)}

n (S) = 36

$\therefore$ Sum of the no. is even then nos. are 2, 4, 6, 8, 10, 12

Let, event be A and, that has to be a perfect square then these nos. are 1, 4, 9

Let, event be B

$\therefore$ A = {(1, 1) (1, 3) (1, 5) (2, 2) (2, 4) (2, 6)
(3, 1) (3, 3) (3, 5) (4, 2) (4, 4) (4, 6)
(5, 1) (5, 3) (5, 5) (6, 2) (6, 4) (6, 6)}

n (A) = 18

And B = {(1, 3) (2, 2) (3, 1)}

n (B) = 3

$\therefore \quad P\left(\frac{B}{A}\right)=\frac{3}{18}=\frac{1}{6}$

Example – 5

A problem in statistics is given to three students A, B and C. Their chances of solving the problem are 1/3, 1/4 and 1/5 respectively. If all of them try independently, what is the probability that

(i) problem is not solved ?

(ii) problem is solved ?

(iii) exactly two students solve the problems ?

Sol. (i) Let A $\equiv$ the event that A solves the problem

B $\equiv$ the event that B solves the problem

C $\equiv$ the event that C solves the problem

Then $P(A)=\frac{1}{3}, P(B)=\frac{1}{4}, P(C)=\frac{1}{5}$

$\therefore \quad P(A')=1-P(A)=1-\frac{1}{3}=\frac{2}{3}$

$P(B') = 1 - P(B) = 1 - \frac{1}{4} = \frac{3}{4}$

$P(C') = 1 - P(C) = 1 - \frac{1}{5} = \frac{4}{5}$

A, B and C are independent events.

∴ A′, B′ and C′ are independent events.

$A' \cap B' \cap C' \equiv$ event that the problem is not solved.

∴ P (problem is not solved)

= $P(A' \cap B' \cap C')$

= $P(A').P(B').P(C')$

= $\frac{2}{3} \times \frac{3}{4} \times \frac{4}{5} = \frac{2}{5}$

(ii) $A \cup B \cup C \equiv$ the event that the problem is solved.

By De Morgan's law,

$(A \cup B \cup C)' = A' \cap B' \cap C'$

∴ P (problem is solved)

= $P(A \cup B \cup C)$

= 1 – P (problem is not solved)

= $1 - P(A \cup B \cup C)'$

= $1 - P(A' \cap B' \cap C')$

= $1 - \frac{2}{5} = \frac{3}{5}$

(iii) Let E ≡ the event that exactly two students solve the problem.

Then $E = P(A \cap B \cap C') \cup (A \cap B' \cap C) \cup (A' \cap B \cap C)$

where the events in the brackets are mutually exclusive.

Also A, B, C and A′, B′, C′ are independent.

∴ $P(E) = P(A).P(B).P(C') + P(A).P(B').P(C) + P(A').P(B).P(C)$

$= \left(\frac{1}{3} \times \frac{1}{4} \times \frac{4}{5}\right) + \left(\frac{1}{3} \times \frac{3}{4} \times \frac{1}{5}\right) + \left(\frac{2}{3} \times \frac{1}{4} \times \frac{1}{5}\right)$

$= \frac{4}{60} - \frac{3}{60} + \frac{2}{60}$

$= \frac{9}{60}$

$= \frac{3}{20}$

Example – 6

One shot is fired from each of the three guns. Let A,B and C denote the events that the target is hit by the first, second and third gun respectively. Assuming that A, B and C are independent events and that P(A) = 0.5, P(B) = 0.6 and P(C) = 0.8, find the probability that at least one hit is registered.

Sol. $A \equiv$ event that first gun hits the target

$B \equiv$ event that second gun hits the target

$C \equiv$ event that third gun hits the target and

$P(A) = 0.5, P(B) = 0.6, P(C) = 0.8$

∴ $P(A') = 1 - P(A) = 1 - 0.5 = 0.5$

$P(B') = 1 - P(B) = 1 - 0.6 = 0.4$

$P(C') = 1 - P(C) = 1 - 0.8 = 0.2$

Now A, B, C are independent events

∴ A′, B′, C′ are independent events.

∴ $A' \cap B' \cap C' \equiv$ event that the target is not hit by any gun

∴ $P(A' \cap B' \cap C') = P(A').P(B').P(C')$

$= 0.5 \times 0.4 \times 0.2 = 0.040$

$A \cup B \cup C \equiv$ the event that the target is hit by at least one gun

By De Morgan's Law, $(A \cup B \cup C)'$

$= A' \cap B' \cap C'$

∴ P(target is hit by at least one gun)

$= P(A \cup B \cup C)$

$= 1 - P(A \cap B \cap C)'$

$= 1 - P(A' \cap B' \cap C')$

$= 1 - 0.04$

$= 0.96$

Example – 7

A doctor is called to see a sick child. The doctor has prior information that 80% of sick children in that area have the flu, while the other 20% are sick with measles. Assume that there is no other disease in that area. A well-known symptom of measles is a rash. From the past records it is known that, chances of having rashes given that sick child is suffering from measles is 0.95. However, occasionally children with flu also develop rash, whose chances are 0.08. Upon examining the child, the doctor finds a rash. What is the probability that the child has measles?

Sol. Let A $\equiv$ event that the child is sick with flu

B $\equiv$ event that the child is sick with measles

C $\equiv$ event that the child has rash

$\therefore \quad P(A) = 80\% = \frac{80}{100} = \frac{4}{5}$

$P(B) = 20\% = \frac{20}{100} = \frac{1}{5}$

Since the chances of having rashes, if the child is suffering from measles is 0.95 and the chances of having rashes, if the child has flu is 0.08,

$P(C/B) = 0.95 = \frac{95}{100}$

and $P(C/A) = 0.08 = \frac{8}{100}$

By Baye's Theorem, probability that the child has measles provided he has the rashes is given by

$$P(B/C) = \frac{P(B).P(C/B)}{P(A).P(C/A) + P(B).P(C/B)}$$

$$= \frac{\left(\frac{1}{5}\right)\left(\frac{95}{100}\right)}{\left(\frac{4}{5}\right)\left(\frac{8}{100}\right) + \left(\frac{1}{5}\right)\left(\frac{95}{100}\right)}$$

$$= \frac{95}{32+95} = \frac{95}{127}$$

$= 0.748$

Example – 8

Suppose you have a large barrel containing a number of plastic eggs. Some eggs contain pearls, the rest contain nothing. Some eggs are painted blue, the rest are painted red. Suppose that 40% of the eggs are painted blue, $\frac{5}{13}$ of the eggs painted blue contain pearls and 20% of the red eggs are empty. What is the probability that an egg containing pearl is painted blue ?

Sol. Let event A = An egg is painted blue.

event B = An egg is painted red.

The barrel contains egg with blue paint as 40% and red paint as 60%.

$\therefore \quad P(A) = 40\% = \frac{40}{100} = \frac{2}{5}$

$\therefore \quad P(B) = 60\% = \frac{60}{100} = \frac{3}{5}$

Let event C = an egg selected contains a pearl.

Then C/A = A blue painted egg contains pearl. given that P (C/A) = 5/13.

P(C/B) = A red painted egg contains pearl. given that 20% of red eggs are empty.

i.e. 80% of red eggs contain pearls.

$\therefore \quad P(C/B) = 80\% = \frac{80}{100} = \frac{4}{5}$

$\therefore$ Required probability, that an egg containing pearl is painted blue is

$$P(A/C) = \frac{P(A).P(C/A)}{P(A).P(C/A) + P(B).P(C/B)}$$

$$= \frac{\frac{2}{5} \times \frac{5}{13}}{\frac{2}{5} \times \frac{5}{13} + \frac{3}{5} \times \frac{4}{5}}$$

$$= \frac{\frac{2}{13}}{\frac{206}{13 \times 25}} = \frac{50}{206}$$

$$= \frac{25}{103} = 0.243$$

Example – 9

A pair of dice is thrown 7 times. If getting a total of 7 is considered a success, what is the probability of

(i) no success ? (ii) 6 successes ?

(iii) at least 6 successes ? (iv) at most 6 successes ?

Sol. Let p denote the probability of getting a total of 7 in a single throw of a pair of dice. Then,

$$P = \frac{6}{36} = \frac{1}{6} \left[\begin{array}{l}\because \text{The sum can be 7 in any one of the ways :} \\ (1, 6), (6, 1), (2, 5), (5, 2), (3, 4), (4, 3)\end{array}\right]$$

$$\therefore \quad q = 1 - p = 1 - \frac{1}{6} = \frac{5}{6}$$

Let X denote the number of successes in 7 throws of a pair of dice. The X is a binomial variate with parameters n = 7 and p = 1/6 such that

Now,

$$P(X = r) = {}^7C_r \left(\frac{1}{6}\right)^r \left(\frac{5}{6}\right)^{7-r}, \; r = 0, 1, 2, ... 7 \qquad ...(i)$$

(i) Probability of no success

$$= P(X = 0) = {}^7C_0 \left(\frac{1}{6}\right)^0 \left(\frac{5}{6}\right)^{7-0} = \left(\frac{5}{6}\right)^7 \qquad \text{[Using (i)]}$$

(ii) Probability of 6 successes

$$= P(X = 6) = {}^7C_6 \left(\frac{1}{6}\right)^6 \left(\frac{5}{6}\right)^{7-6} \qquad \text{[Using (i)]}$$

$= 35\,(1/6)^7$

(iii) Probability of at least 6 successes = $P(X \geq 6)$

$= P(X = 6) + P(X = 7)$

$$= {}^7C_6 \left(\frac{1}{6}\right)^6 \left(\frac{5}{6}\right)^{7-6} + {}^7C_7 \left(\frac{1}{6}\right)^7 \left(\frac{5}{6}\right)^0 \qquad \text{[Using (i)]}$$

$$= 7.\left(\frac{1}{6}\right)^6 \left(\frac{5}{6}\right) + \left(\frac{1}{6}\right)^7 = \left(\frac{1}{6}\right)^6 \left(\frac{35}{6} + \frac{1}{6}\right) = \left(\frac{1}{6}\right)^5$$

(iv) Probability of at most 6 successes = $P(X \leq 6)$

$= 1 - P(X > 6)$

$= 1 - P(X = 7)$

$$= 1 - {}^7C_7 \left(\frac{1}{6}\right)^7 \left(\frac{5}{6}\right)^0 \qquad \text{[Using (i)]}$$

$$= 1 - \left(\frac{1}{6}\right)^7$$

Example – 10

The mean and variance of a binomial distribution are 4 and 4/3 respectively, find $P(X \geq 1)$.

Sol. Let X be a binomial variate with parameters n and p. Then,

Mean = np and Variance = npq

$$\Rightarrow \quad np = 4 \text{ and } npq = \frac{4}{3}$$

$$\left[\because \text{Mean} = 4, \text{Var}(X) = \frac{4}{3} \text{ (Given)}\right]$$

$$\Rightarrow \quad \frac{npq}{np} = \frac{\frac{4}{3}}{4} \Rightarrow q = \frac{1}{3} \Rightarrow p = 1 - \frac{1}{3} = \frac{2}{3}$$

$[\because = 1 - q]$

Putting p = 2/3 in np = 4, we get

$$n \times \frac{2}{3} = 4 \Rightarrow n = 6$$

Thus, we have

$$n = 6, \; p = \frac{2}{3} \text{ and } q = \frac{1}{3}$$

$$\therefore \quad P(X = r) = {}^nC_r p^r q^{n-r} \Rightarrow P(X = r) = {}^6C_r \left(\frac{2}{3}\right)^r \left(\frac{1}{3}\right)^{6-r},$$

$r = 0, 1, 2, ..., 6$

Now, $P(X \geq 1) = 1 - P(X < 1)$

$\Rightarrow \quad P(X \geq 1) = 1 - P(X = 0)$

$$\Rightarrow \quad P(X \geq 1) = 1 - {}^6C_0 \left(\frac{2}{3}\right)^0 \left(\frac{1}{3}\right)^6 = 1 - \left(\frac{1}{3}\right)^6 = 1 - \frac{1}{729} = \frac{728}{729}$$

EXERCISE - 1 : BASIC OBJECTIVE QUESTIONS

P & C Based Questions

1. A binary number is made up of 8 bits. The probability of an incorrect bit appearing is p and the errors in different bits are independent of one another. The probability of forming an incorrect number is

(a) $\frac{P}{8}$ (b) p^8

(c) $1-(1-p)^8$ (d) $(1-p)^8$

2. Two numbers are chosen from {1, 2, 3, 4, 5, 6} one after the other without replacement. The probability that the smaller value of two is less than 4 is

(a) $\frac{4}{5}$ (b) $\frac{1}{15}$

(c) $\frac{1}{5}$ (d) $\frac{14}{15}$

3. Fifteen coupons are numbered 1 to 15. Seven coupons are selected at random, one at a time with replacement. The probability that the largest number apearing on a selected coupon be 9 is

(a) $\left(\frac{9}{16}\right)^6$ (b) $\left(\frac{8}{15}\right)^7$

(c) $\left(\frac{3}{5}\right)^7$ (d) none of these

4. Three of the six vertices of a regular hexagon are chosen at random. The probability that the triangle with three vertices is equilateral equals

(a) $\frac{1}{2}$ (b) $\frac{1}{5}$

(c) $\frac{1}{10}$ (d) $\frac{1}{20}$

5. $x_1, x_2, x_3, \ldots\ldots x_{50}$ are fifty numbers such that $x_r < x_{r+1}$ for $r = 1, 2, 3, \ldots., 49$. Five numbers out of these are picked up at random. The probability that the five numbers have x_{20} as the middle number is

(a) $\frac{{}^{20}C_2 \times {}^{30}C_2}{{}^{50}C_2}$ (b) $\frac{{}^{30}C_2 \times {}^{19}C_2}{{}^{50}C_2}$

(c) $\frac{{}^{19}C_2 \times {}^{31}C_3}{{}^{50}C_5}$ (d) none the these

6. A determinant is chosen at random from the set of all determinants of order 2 with elements 0 or 1 only. The probability that value of the determinant chosen is positive is

(a) $\frac{16}{81}$ (b) $\frac{7}{16}$

(c) $\frac{3}{16}$ (d) none of these

7. Four tickets marked 00, 01, 10 and 11 respectively are placed in bag. A ticket is drawn at random five times, being replaced each time. The probability that the sum of the numbers on the ticket is 15, is

(a) $\frac{3}{1024}$ (b) $\frac{5}{1024}$

(c) $\frac{7}{1024}$ (d) none of these

8. If the integers m and n belongs to set of first hundred natural numbers then the probability that a number of the form $7^m + 7^n$ is divisible by 5 is

(a) $\frac{1}{5}$ (b) $\frac{1}{7}$

(c) $\frac{1}{4}$ (d) $\frac{1}{49}$

9. 5 grils and 10 boys sit at random in a row having 15 chairs numbered as 1 to 15. Find the probability that end seats are occupied by the girls and between any two girls odd number of boys sit, is

(a) $\frac{20\times 10!\times 5!}{15!}$ (b) $\frac{20\times 10!}{15!}$

(c) $\frac{20\times 5!}{15!}$ (d) none of these

10. If the letters of the word ATTEMPT are written down at random, the chance that all Ts are consecutive is

(a) $\frac{1}{42}$ (b) $\frac{6}{7}$

(c) $\frac{1}{7}$ (d) none of these

11. 2n boys are randomly divided into two subgroups containing n boys each. The probability that the two tallest boys are in different groups is

(a) $\frac{n}{2n-1}$ (b) $\frac{n-1}{2n-1}$

(c) $\frac{2n-1}{4n^2}$ (d) none of these

12. A square is inscribed in a circle. If p_1 is the probability that a randomly chosen point of the circle lies within the square and p_2 is the probability that the point lies outside the square then

(a) $p_1 = p_2$

(b) $p_1 > p_2$ and $p_1^2 - p_2^2 < \frac{1}{3}$

(c) $p_1 < p_2$

(d) none of these

Set Theory Based Questions

13. A and B are events such that $P(A) = 0.4$, $P(B) = 0.3$ and $P(A\cup B) = 0.5$, then $P(B'\cap A)$ equals

(a) $\frac{2}{3}$ (b) $\frac{1}{2}$

(c) $\frac{3}{10}$ (d) $\frac{1}{5}$

14. Let X denote the number of hours you study during a randomly selected school day. The probability that X can take value x has the following form, where k is some constant.

$$P(X=x)=\begin{cases} 0.1 & \text{if } x=0 \\ kx & \text{if } x=1 \text{ or } 2 \\ k(5-x) & \text{if } x=3 \text{ or } 4 \\ 0 & \text{otherwise} \end{cases}$$

The probability that you study for atleast two hours is

(a) 2k (b) 0.55

(c) 0.75 (d) none of these

15. A, B, C are three events for which $P(A) = 0.6, P(B) = 0.4$, $P(C) = 0.5$, $P(A\cup B) = 0.8$, $P(A\cap C) = 0.3$ and $P(A\cap B\cap C) = 0.2$.

If $P(A\cup B\cup C) \geq 0.85$ then the interval of values of $P(B\cap C)$ is

(a) [0.2, 0.35] (b) [0.55, 0.7]

(c) [0.2, 0.55] (d) none of these

16. The probability that at least one of the event A and B occurs is $\frac{3}{5}$. If A and B occur simultaneously with probability $\frac{1}{5}$ then $P(A') + P(B')$ is

(a) $\frac{2}{5}$ (b) $\frac{4}{5}$

(c) $\frac{6}{5}$ (d) $\frac{7}{5}$

17. A coin is tossed again and again until a head appears or it has been tossed 3 times. Given that head does not appear on the first toss, the probability that the coin is tossed three times is

(a) $\frac{1}{8}$ (b) $\frac{1}{4}$

(c) $\frac{1}{2}$ (d) none of these

18. A and B are two events. Odds against A are 2 : 1. Odds in favour of $A \cup B$ are 3 : 1. If $x \le P(B) \le y$, then the ordered pair (x, y) is

(a) $\left(\frac{5}{12}, \frac{3}{4}\right)$ (b) $\left(\frac{2}{3}, \frac{3}{4}\right)$

(c) $\left(\frac{1}{3}, \frac{3}{4}\right)$ (d) none of these

Condition Probability

19. If $P(B) = \frac{3}{5}, P(A/B) = \frac{1}{2}$ and $P(A \cup B) = \frac{4}{5}$, then $P((A \cup B)') + P(A' \cup B) =$

(a) $\frac{1}{5}$ (b) $\frac{4}{5}$

(c) $\frac{1}{2}$ (d) 1

20. If A and B are two events such that P (A) > 0 and $P(B) \neq 1$, then $P(A'/B')$ equals

(a) $1 - P(A/B)$ (b) $1 - P(A'/B)$

(c) $\frac{1 - P(A \cup B)}{P(B')}$ (d) $P(A')/P(B')$

21. A coin is tossed again and again until a head appears or it has been tossed 3 times. Given that head does not appear on the first toss, the probability that the coin is tossed three times is

(a) $\frac{1}{8}$ (b) $\frac{1}{4}$

(c) $\frac{1}{2}$ (d) none of these

22. A bag contains 5 red and 3 blue balls. If 3 balls are drawn at random without replacement, the probability of getting exactly one red ball

(a) $\frac{45}{196}$ (b) $\frac{135}{392}$

(c) $\frac{15}{56}$ (d) $\frac{15}{29}$

23. In reference to Q. 22 above, the probability that exactly two of the three balls are red, the first ball being red, is

(a) $\frac{1}{3}$ (b) $\frac{4}{7}$

(c) $\frac{15}{28}$ (d) $\frac{5}{28}$

24. Assume that in a family, each child is equally likely to be a boy or a girl. A family with three children is chosen at random. The probability that the eldest child is a girl given that the family has atleast one girl is

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$

(c) $\frac{2}{3}$ (d) $\frac{4}{7}$

25. Two dice are thrown. If it is known that the sum of the numbers on the dice is less than 6, the probability of a getting a sum 3 is

(a) $\frac{1}{18}$ (b) $\frac{2}{5}$

(c) $\frac{1}{5}$ (d) $\frac{5}{18}$

26. In a college, 30% students fail in Physics, 25% fail in Mathematics and 10% fail in both. One student is chosen at random. The probability that the student fails in Physics if he/she has failed in mathematics is

(a) $\frac{1}{10}$ (b) $\frac{2}{5}$

(c) $\frac{9}{20}$ (d) $\frac{1}{3}$

27. Three distinguishable balls are distributed in three cells. The probability that all three occupy the same cell, given that atleast two of them are in the same cell, is

(a) $\frac{1}{7}$ (b) $\frac{1}{9}$

(c) $\frac{1}{6}$ (d) none of these

28. Let $A = \{2, 3, 4, ..., 20, 21\}$. A number is chosen at random from the set A and it is found to be a prime number. The probability that it is more than 10 is

(a) $\frac{9}{10}$ (b) $\frac{1}{10}$

(c) $\frac{1}{2}$ (d) none of these

Multiplication Theorem & Independent Event

29. If A and B are independent events such that $0 < P(A) < 1$ and $0 < P(B) < 1$, then which of the following is not correct ?

(a) A and B are mutually exclusive

(b) A and B' are independent

(c) A' and B are independent

(d) A' and B' are independent

30. Let A and B be two events such that $P(A) = \frac{3}{8}$, $P(B) = \frac{5}{8}$ and $P(A \cup B) = \frac{3}{4}$, then $P(A/B)\, P(A'/B)$ is equal to

(a) $\frac{2}{5}$ (b) $\frac{3}{8}$

(c) $\frac{3}{20}$ (d) $\frac{6}{25}$

31. Three persons A, B and C fire at a target in turn, starting with A. Their probabilities of hitting the target are 0.4, 0.3 and 0.2 respectively. The probability of two hits is

(a) 0.024 (b) 0.188

(c) 0.336 (d) 0.452

32. A die is thrown and a card is selected at random from a deck of 52 playing cards. The probability of getting an even number on the die and a spade card is

(a) $\frac{1}{8}$ (b) $\frac{1}{4}$

(c) $\frac{1}{2}$ (d) $\frac{3}{4}$

33. A box contains 3 orange balls, 3 green balls and 2 blue balls. Three balls are drawn at random from the box without replacement. The probability of drawing 2 green balls and one blue ball is

(a) $\frac{2}{21}$ (b) $\frac{3}{28}$

(c) $\frac{1}{28}$ (d) $\frac{167}{168}$

34. Let $0 < P(A) < 1, 0 < P(B) < 1$ and $P(A \cup B) = P(A) + P(B) - P(A)\, P(B)$ then

(a) $P(B/A) = P(B) - P(A)$

(b) $P(A/B) = P(A) - P(B)$

(c) $P(A/B) = P(A)$

(d) $P((A \cup B)^c) \neq P(A^c)\, P(B^c)$

35. A pair of dice is rolled again and again till a total of 5 or a total of 7 is obtained. The chance that a total of 5 comes before a total of 7 is

(a) $\frac{2}{5}$ (b) $\frac{3}{7}$

(c) $\frac{3}{13}$ (d) none of these

36. An unbiased die is tossed until a number greater than 4 appears. The probability that an even number of tosses is needed is

(a) $\frac{1}{2}$ (b) $\frac{2}{5}$

(c) $\frac{1}{5}$ (d) $\frac{2}{3}$

37. Ram and Shyam throw with one dice for a prize of Rs 88 which is to be won by the player who throws 1 first. If Ram starts, then mathematical expectation for Shyam is

(a) Rs 32 (b) Rs 40

(c) Rs 48 (d) none of these

Total Probability & Baye's Theorem

38. One bag contains 5 white and 4 black balls. Another bag contains 7 white and 9 black balls. A ball is transferred from the first bag to the second and then a ball is drawn from second. The probability that the ball is white, is

(a) 8/17 (b) 40/153

(c) 5/9 (d) 4/9

39. Three groups A, B, C are competing for positions on the Board of Directors of a company. The probabilities of their winning are 0.5, 0.3, 0.2 respectively. If the group A wins, the probability of introducing a new product is 0.7 and the corresponding probabilities for group B and C are 0.6 and 0.5 respectively. The probability that the new product will be introduced, is

(a) 0.18 (b) 0.35

(c) 0.10 (d) 0.63

40. A survey of people in a given region showed that 20% were smokers. The probability of death due to lung cancer, given that a person smoked, was 10 times the probability of death due to lung cancer, given that a person did not smoke. If the probability of death due to lung cancer in the region is 0.006, what is the probability of death due to lung cancer given that a person is a smoker

(a) 1/140 (b) 1/70

(c) 3/140 (d) 1/10

41. A letter is known to have come either from LONDON or CLIFTON; on the postmark only the two consecutive letters ON are legible. The probability that it came from LONDON is

(a) $\frac{5}{17}$ (b) $\frac{12}{17}$

(c) $\frac{17}{30}$ (d) $\frac{3}{5}$

42. In a certain town, 40% of the people have brown hair, 25% have brown eyes and 15% have both brown hair and brown eyes. If a person selected at random from the town, has brown hair, the probability that he also has brown eyes, is

(a) $\frac{1}{5}$ (b) $\frac{3}{8}$

(c) $\frac{1}{3}$ (d) $\frac{2}{3}$

43. For k = 1, 2,3 the box B_k contains k red balls and (k + 1) white balls. Let $P(B_1) = \frac{1}{2}$, $P(B_2) = \frac{1}{3}$ and $P(B_3) = \frac{1}{6}$. A box is selected at random and a ball is drawn from it. If a red ball is drawn, then the probability that it has come from box B_2, is

(a) $\frac{35}{78}$ (b) $\frac{14}{39}$

(c) $\frac{10}{13}$ (d) $\frac{12}{13}$

44. In an entrance test there are multiple choice questions. There are four possible answers to each question of which one is correct. The probability that a student knows the answer to a question is 90%. If he gets the correct answer to a question, then the probability that he was guessing, is

(a) $\frac{37}{40}$ (b) $\frac{1}{37}$

(c) $\frac{36}{37}$ (d) $\frac{1}{9}$

45. Two coins are available, one fair and the other two headed. Choose a coin and toss it once assume that the unbiased coin is chosen with probability $\frac{3}{4}$. Given that the outcome is head, the probability that the two-headed coin was chosen is

(a) $\frac{3}{5}$ (b) $\frac{2}{5}$

(c) $\frac{1}{5}$ (d) $\frac{2}{7}$

46. The probabilities of four cricketers A, B, C and D scoring more than 50 runs in a match are $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$ and $\frac{1}{10}$. It is known that exactly two of the players scored more than 50 runs in a particular match. The probability that players were A and B is

(a) $\frac{27}{65}$ (b) $\frac{5}{6}$

(c) $\frac{1}{6}$ (d) none of these

47. A, B and C are contesting the election for the post of secretary of a club which does not allow ladies to become members. The probabilities of A, B and C winning the election are $\frac{1}{3}, \frac{2}{9}$ and $\frac{4}{9}$ respectively. The probabilities of introducing the clause of admitting lady members to the club by A, B and C are 0.6, 0.7 and 0.5 respectively. The probability that ladies will be taken as members in the club after the election is

(a) $\frac{26}{45}$ (b) $\frac{5}{9}$

(c) $\frac{19}{45}$ (d) none of these

48. A certain player, say X, is know to win with probability 0.3 if the track is fast and 0.4 if the track is slow. For Monday, there is a 0.7 probability of a fast-track and 0.3 probability of slow track. The probability that player X will win on Monday, is

(a) 0.22 (b) 0.11
(c) 0.33 (d) none of these

49. The probability that certain electronic component fails when first used is 0.10. If it does not fail immediately, the probability that it lasts for one year is 0.99. The probability that a new component will last for one year is

(a) 0.891 (b) 0.692
(c) 0.92 (d) none of these

50. A man is know to speak truth 3 out 4 times. He throws a dice and reports that it is six. The probability that it is actually six is

(a) $\frac{3}{8}$ (b) $\frac{1}{5}$
(c) $\frac{3}{5}$ (d) none of these

51. The probability that the birthdays of six different people will fall in exactly two calendar months is

(a) $\frac{1}{6}$ (b) ${}^{12}C_2 \times \frac{2^6}{12^6}$
(c) ${}^{12}C_2 \times \frac{2^6 - 1}{12^6}$ (d) $\frac{341}{12^5}$

Random Variable

52. Which of the following can serve as a probability function of a discrete random variable for the range $\{1, 2, 3, 4\}$?

(a) $P(x) = \frac{x+2}{18}$ (b) $P(x) = \frac{x-2}{2}$
(c) $P(x) = \frac{x^2}{15}$ (d) none of these

53. The probability of guessing correctly atleast 8 out of 10 answers on a true-false type examination is

(a) $\frac{7}{64}$ (b) $\frac{7}{128}$
(c) $\frac{45}{1024}$ (d) $\frac{7}{41}$

54. The mean and variance of a random variable X having a binomial distribution are 4 and 2 respectively, then $P(X = 1)$ is

(a) $\frac{1}{4}$ (b) $\frac{1}{32}$
(c) $\frac{1}{16}$ (d) $\frac{1}{8}$

55. Suppose that a random variable X follows Binomial distribution with parameters n and p, where $0 < p < 1$. If $P(X = r)/P(X = n - r)$ is independent of n and r, then p is equal to

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$
(c) $\frac{1}{5}$ (d) $\frac{1}{7}$

56. How many times must a man toss a fair coin so that the probability of having atleast one head is more than 90%?

(a) 2 (b) 3
(c) 4 (d) 5

57. A coin is tossed n times. The probability of getting at least one head is greater than that of getting at least two tails by $\frac{5}{32}$. Then n is

(a) 5 (b) 10
(c) 15 (d) none of these

58. 6 ordinary dice are rolled. The probability that at least half of them will show at least 3 is

(a) $41 \times \frac{2^4}{3^6}$ (b) $\frac{2^4}{3^6}$
(c) $20 \times \frac{2^4}{3^6}$ (d) none of these

59. A coin is tossed 7 times. Each time a man calls head. The probability that he wins the toss on more occasions is

(a) $\frac{1}{4}$ (b) $\frac{5}{8}$
(c) $\frac{1}{2}$ (d) none of these

60. If X and Y are the independent random variables B $\left(5, \frac{1}{2}\right)$ and B $\left(7, \frac{1}{2}\right)$, then $P(X + Y \geq 1) =$

(a) $\frac{4095}{4096}$ (b) $\frac{309}{4096}$
(c) $\frac{4032}{4096}$ (d) none of these

EXERCISE - 2 : PREVIOUS YEAR JEE MAINS QUESTIONS

1. A problem in mathematics is given to three student A, B, C and their respective probability of solving the problem is 1/2, 1/3 and 1/4. Probability that the problem is solved, is **(2002)**

(a) 3/4 (b) 1/2

(c) 2/3 (d) 1/3

2. A and B are events such that $P(A \cup B) = 3/4$, $P(A \cap B) = 1/4$, $P(\bar{A}) = 2/3$ then $P(\bar{A} \cap B)$ is **(2002)**

(a) 5/12 (b) 3/8

(c) 5/8 (d) 1/4

3. A die is tossed 5 times. Getting an odd number is considered a success. Then the variance of distribution of success is **(2002)**

(a) 8/3 (b) 3/8

(c) 4/5 (d) 5/4

4. Events A, B, C are mutually exclusive events such that $P(A) = \frac{3x+1}{3}$, $P(B) = \frac{1-x}{4}$ and $P(C) = \frac{1-2x}{2}$. Then set of possible values of x are in the interval **(2003)**

(a) $\left[\frac{1}{3}, \frac{2}{3}\right]$ (b) $\left[\frac{1}{3}, \frac{13}{3}\right]$

(c) [0, 1] (d) $\left[\frac{1}{3}, \frac{1}{2}\right]$

5. Five horses are in a race. Mr. A selects two of the horses at random and bets on them. The probability that Mr. A selected the winning horse, is **(2003)**

(a) 3/5 (b) 1/5

(c) 2/5 (d) 4/5

6. The mean and variance of a random variable X having a binomial distribution are 4 and 2 respectively, then $P(X = 1)$ is **(2003)**

(a) 1/16 (b) 1/8

(c) 1/4 (d) 1/32

7. The probability that A speaks truth is 4/5, while this probability for B is 3/4. The probability that they contradict each other when asked to speak on a fact, is **(2004)**

(a) 7/20 (b) 1/5

(c) 3/20 (d) 4/5

8. A random variable X has the probability distribution

X	1	2	3	4	5	6	7	8
P(X)	0.15	0.23	0.12	0.10	0.20	0.08	0.07	0.05

For the events E = {X is a prime number} and $F = \{X < 4\}$, the probability $P(E \cup F)$ is **(2004)**

(a) 0.35 (b) 0.77

(c) 0.87 (d) 0.50

9. The mean and the variance of a binomial distribution are 4 and 2 respectively. Then the probability of 2 successes is **(2004)**

(a) 128/256 (b) 219/256

(c) 37/256 (d) 28/256

10. Let A and B be two events such that $P(\overline{A \cup B}) = \frac{1}{6}$, $P(A \cap B) = \frac{1}{4}$ and $P(\bar{A}) = \frac{1}{4}$, where $\bar{A}$ stands for complement of event A. Then events A and B are **(2005)**

(a) equally likely but not independent

(b) equally likely and mutually exclusive

(c) mutually exclusive and independent

(d) independent but not equally likely

11. Three houses are available in a locality. Three persons apply for the houses. Each applies for one house without consulting others. The probability that all the three apply for the same house, is **(2005)**

(a) $\frac{1}{9}$ (b) $\frac{2}{9}$

(c) $\frac{7}{9}$ (d) $\frac{8}{9}$

12. Two aeroplanes I and II bomb a target in succession. The probabilities of I and II scoring a hit correctly are 0.3 and 0.2, respectively. The second plane will bomb if the first misses the target. The probability that the target is hit by the second plane, is **(2007)**

(a) 0.2 (b) 0.7

(c) 0.06 (d) 0.14

13. A pair of fair dice is thrown independently three times. The probability of getting a score of exactly 9 twice, is **(2007)**

(a) 8/729 (b) 8/243

(c) 1/729 (d) 8/9

14. It is given that the event A and B are such that $P(A)=\frac{1}{4}, P\left(\frac{A}{B}\right)=\frac{1}{2}$ and $P\left(\frac{B}{A}\right)=\frac{2}{3}$. Then P (B) is **(2008)**

(a) $\frac{1}{2}$ (b) $\frac{1}{6}$

(c) $\frac{1}{3}$ (d) $\frac{2}{3}$

15. A die is thrown. Let A be the event that the number obtained is greater than 3, Let B be the event that the number obtained is less than 5. Then $P(A \cup B)$ is **(2008)**

(a) $\frac{2}{5}$ (b) $\frac{3}{5}$

(c) 0 (d) 1

16. Let A and B be two events such that $P(\overline{A \cup B})=\frac{1}{6}, P(A \cap B)=\frac{1}{4}$ and $P(\bar{A})=\frac{1}{4}$, where $\bar{A}$ stands for the complement of the event A. Then the events A and B are : **(2014)**

(a) independent and equally likely

(b) mutually exclusive and independent

(c) equally likely but not independent

(d) independent but not equally likely

17. A set S contains 7 elements. A non-empty subset A of S and an element x of S are chosen at random. Then the probability that $x \in A$ is **(2014/Online Set–2)**

(a) $\frac{1}{2}$ (b) $\frac{64}{127}$

(c) $\frac{63}{128}$ (d) $\frac{31}{128}$

18. A number x is chosen at random from the set {1, 2, 3, 4..... 100}. Define the event: A = the chosen number x satisfies $\frac{(x-10)(x-50)}{(x-30)} \geq 0$. Then P(A) is: **(2014/Online Set–3)**

(a) 0.71 (b) 0.70

(c) 0.51 (d) 0.20

19. Let A and E by any two events with positive probabilities: **(2014/Online Set–4)**

Statement 1 : $P(E/A) \geq P(A/E)\,P(E)$

Statement 2 : $P(A/E) \geq P(A \cap E)$

(a) Both the statements are true

(c) Statement–1 is true, Statement–2 is false

(d) Statement–1 is false, Statement–2 is true

20. If 12 identical balls are to be placed in 3 identical boxes, then the probability that one of the boxes contains exactly 3 balls is: **(2015)**

(a) $220\left(\frac{1}{3}\right)^{12}$ (b) $22\left(\frac{1}{3}\right)^{11}$

(c) $\frac{55}{3}\left(\frac{2}{3}\right)^{11}$ (d) $55\left(\frac{2}{3}\right)^{10}$

21. Let X be a set containing 10 elements and P(X) be its power set. If A and B are picked up at random from P(X), with replacement, then the probability that A and B have equal number of elements, is : **(2015/Online Set–1)**

(a) $\frac{\left(2^{10}-1\right)}{2^{20}}$ (b) $\frac{{}^{20}C_{10}}{2^{10}}$

(c) $\frac{\left(2^{10}-1\right)}{2^{10}}$ (d) $\frac{{}^{20}C_{10}}{2^{20}}$

22. If the lengths of the sides of a triangle are decided by the three throws of a single fair die, then the probability that the triangle is of maximum area given that it is an isosceles triangle, is : **(2015/Online Set–2)**

(a) $\frac{1}{21}$ (b) $\frac{1}{27}$

(c) $\frac{1}{15}$ (d) $\frac{1}{26}$

23. Let two fair six-faced dice A and B be thrown simultaneously. If E_1 is the event that die A shows up four, E_2 is the event that die B shows up two and E_3 is the event that the sum of numbers on both dice is odd, then which of the following statements is NOT true ? **(2016)**

(a) E_2 and E_3 are independent.

(b) E_1 and E_3 are independent.

(c) E_1, E_2 and E_3 are independent.

(d) E_1 and E_2 are independent.

24. If A and B are any two events such that $P(A) = \frac{2}{5}$ and $P(A \cap B) = \frac{3}{20}$, then the conditional, probability, $P(A | (A' \cup B'))$, where A' denotes the complement of A, is equal to : **(2016/Online Set–1)**

(a) $\frac{1}{4}$ (b) $\frac{5}{17}$

(c) $\frac{8}{17}$ (d) $\frac{11}{20}$

25. An experiment succeeds twice as often as it fails. The probability of at least 5 successes in the six trials of this experiment is : **(2016/Online Set–2)**

(a) $\frac{240}{729}$ (b) $\frac{192}{729}$

(c) $\frac{256}{729}$ (d) $\frac{496}{729}$

26. For three events A, B and C, P(Exactly one of A or B occurs)

= P(Exactly one of B or C occurs)

= P(Exactly one of C or A occurs) $= \frac{1}{4}$ and P (All the three events occur simultaneously) $= \frac{1}{16}$.

Then the probability that at least one of the events occurs, is: **(2017)**

(a) $\frac{7}{32}$ (b) $\frac{7}{16}$

(c) $\frac{7}{64}$ (d) $\frac{3}{16}$

27. If two different numbers are taken from the set $\{0, 1, 2, 3, \ldots\ldots, 10\}$; then the probability that their sum as well as absolute difference are both multiple of 4, **(2017)**

(a) $\frac{6}{55}$ (b) $\frac{12}{55}$

(c) $\frac{14}{45}$ (d) $\frac{7}{55}$

28. Three persons P, Q and R independently try to hit a target. If the probabilities of their hitting the target are $\frac{3}{4}$, $\frac{1}{2}$ and $\frac{5}{8}$ respectively, then the probability that the target is hit by P or Q but not by R is : **(2017)**

(a) $\frac{21}{64}$ (b) $\frac{9}{64}$

(c) $\frac{15}{64}$ (d) $\frac{39}{64}$

29. An unbiased coin is tossed eight times. The probability of obtaining at least one head and at least one tail is : **(2017)**

(a) $\frac{255}{256}$ (b) $\frac{127}{128}$

(c) $\frac{63}{64}$ (d) $\frac{1}{2}$

30. From a group of 10 men and 5 women, four member committees are to be formed each of which must contain at least one woman. Then the probability for these committees to have more women than men, is **(2017)**

(a) $\frac{21}{220}$ (b) $\frac{3}{11}$

(c) $\frac{1}{11}$ (d) $\frac{2}{23}$

31. Let E and F be two independent events. The probability that both E and F happen is $\frac{1}{12}$ and the probability that neither E nor F happens is $\frac{1}{2}$, then a value of $\frac{P(E)}{P(F)}$ is: **(2017)**

(a) $\frac{4}{3}$ (b) $\frac{3}{2}$

(c) $\frac{1}{3}$ (d) $\frac{5}{12}$

32. Three persons P, Q and R independently try to hit a target. If the probabilities of their hitting the target are $\frac{3}{4}$, $\frac{1}{2}$ and $\frac{5}{8}$ respectively, then the probability that the target is hit by P or Q but not by R is : **(2017/Online Set–1)**

(a) $\frac{21}{64}$ (b) $\frac{9}{64}$

(c) $\frac{15}{64}$ (d) $\frac{39}{64}$

33. An unbiased coin is tossed eight times. The probability of obtaining at least one head and at least one tail is : **(2017/Online Set–1)**

(a) $\frac{255}{256}$ (b) $\frac{127}{128}$

(c) $\frac{63}{64}$ (d) $\frac{1}{2}$

34. From a group of 10 men and 5 women, four member committees are to be formed each of which must contain at least one woman. Then the probability for these committees to have more women than men, is **(2017/Online Set–2)**

(a) $\frac{21}{220}$ (b) $\frac{3}{11}$

(c) $\frac{1}{11}$ (d) $\frac{2}{23}$

35. Let E and F be two independent events. The probability that both E and F happen is $\frac{1}{12}$ and the probability that neither E nor F happens is $\frac{1}{2}$, then a value of $\frac{P(E)}{P(F)}$ is : **(2017/Online Set–2)**

(a) $\frac{4}{3}$ (b) $\frac{3}{2}$

(c) $\frac{1}{3}$ (d) $\frac{5}{12}$

36. A bag contains 4 red and 6 black balls. A ball is drawn at random from the bag, its colour is observed and this ball along with two additional balls of the same colour are returned to the bag. If now a ball is drawn at random from the bag, then the probability that this drawn ball is red, is: **(2018)**

(a) $\frac{3}{4}$ (b) $\frac{3}{10}$

(c) $\frac{2}{5}$ (d) $\frac{1}{5}$

37. A box 'A' contains 2 white, 3 red and 2 black balls. Another box 'B' contains 4 white, 2 red and 3 black balls. If two balls are drawn at random, without replacement, from a randomly selected box and one ball turns out to be white while the other ball turns out to be red, then the probability that both balls are drawn from box 'B' is : **(2018/Online Set–1)**

(a) $\frac{9}{16}$ (b) $\frac{7}{16}$

(c) $\frac{9}{32}$ (d) $\frac{7}{8}$

38. A player X has a biased coin whose probability of showing heads is p and a player Y has a fair coin. They start playing a game with their own coins and play alternately. The player who throws a head first is a winner. If X starts the game, and the probability of winning the game by both the players is equal, then the value of 'p' is: **(2018/Online Set–2)**

(a) $\frac{1}{5}$ (b) $\frac{1}{3}$

(c) $\frac{2}{5}$ (d) $\frac{1}{4}$

39. Two different families A and B are blessed with equal number of children. There are 3 tickets to be distributed amongst the children of these families so that no child gets more than one ticket. If the probability that all the tickets go to the children of the family B is $\frac{1}{12}$, then the number of children in each family is : **(2018/Online Set–3)**

(a) 3 (b) 4

(c) 5 (d) 6

EXERCISE - 3 : ADVANCED OBJECTIVE QUESTIONS

Single Choice Questions

1. Whenever horses a, b, c race together, their respective probabilities of winning the race are 0.3, 0.5 and 0.2 respectively. If they race three times the probability that "the same horse wins all the three races" and the probablity that a, b, c each wins one race, are respectively

(a) $\frac{8}{50};\frac{9}{50}$ (b) $\frac{16}{100},\frac{3}{100}$

(c) $\frac{12}{50};\frac{15}{50}$ (d) $\frac{10}{50};\frac{8}{50}$

2. Let A & B be two events. Suppose $P(A)=0.4$, $P(B)=p$ & $P(A \cup B)=0.7$. The value of p for which A & B are independent is :

(a) 1/3 (b) 1/4
(c) 1/2 (d) 1/5

3. A & B are two independent events such that $P(\overline{A})=0.7, P(\overline{B})=a$ & $P(A \cup B)=0.8$, then, a =

(a) 5/7 (b) 2/7
(c) 1 (d) none

4. A pair of numbers is picked up randomly (without replacement) from the set {1, 2, 3, 5, 7, 11, 12, 13, 17, 19}. The probability that the number 11 was picked given that the sum of the numbers was even, is nearly :

(a) 0.1 (b) 0.125
(c) 0.24 (d) 0.18

5. For a biased die the probabilities for the diffferent faces to turn up are given below :

Faces :	1	2	3	4	5	6
Probabilities :	0.10	0.32	0.21	0.15	0.05	0.17

The die is tossed & you are told that either face one or face two has turned up. Then the probability that it is face one is :

(a) 1/6 (b) 1/10
(c) 5/49 (d) 5/21

6. A determinant is chosen at random from the set of all determinants of order 2 with elements 0 or 1 only. The probability that the determinant chosen has the value non negative is :

(a) 3/16 (b) 6/16
(c) 10/16 (d) 13/16

7. 15 coupons are numbered 1, 2, 3,..... , 15 respectively. 7 coupons are selected at random one at a time with replacement. The probability that the largest number appearing on a selected coupon is 9 is :

(a) $\left(\frac{9}{16}\right)^6$ (b) $\left(\frac{8}{15}\right)^7$

(c) $\left(\frac{3}{5}\right)^7$ (d) $\frac{9^7-8^7}{15^7}$

8. Two cubes have their faces painted either red or blue. The first cube has five red faces and one blue face. When the two cubes are rolled simultaneously, the probability that the two top faces show the same colour is 1/2. Number of red faces on the second cube, is

(a) 1 (b) 2
(c) 3 (d) 4

9. The probability that an automobile will be stolen and found withing one week is 0.0006. The probability that an automobile will be stolen is 0.0015. The probability that a stolen automobile will be found in one week is

(a) 0.3 (b) 0.4
(c) 0.5 (d) 0.6

10. One bag contains 3 white & 2 black balls, and another contains 2 white & 3 black balls. A ball is drawn from the second bag & placed in the first, then a ball is drawn from the first bag & placed in the second. When the pair of the operations is repeated, the probability that the first bag will contain 5 white balls is:

(a) 1/25 (b) 1/125
(c) 1/225 (d) 2/15

11. A person draws a card from a pack of 52 cards, replaces it & shuffles the pack. He continues doing this till he draws a spade. The probability that he will fail exactly the first two times is :

(a) 1/64 (b) 9/64
(c) 36/64 (d) 60/64

12. An unbaised cubic die marked with 1, 2, 2, 3, 3, 3 is rolled 3 times. The probability of getting a total score of 4 or 6 is

(a) $\frac{16}{216}$ (b) $\frac{50}{216}$

(c) $\frac{60}{216}$ (d) none

13. A bag contains 3 R & 3 G balls and a person draws out 3 at random. He then drops 3 blue balls into the bag & again draws out 3 at random. The chance that the 3 later balls being all of different colours is

(a) 15% (b) 20%
(c) 27% (d) 40%

14. A biased coin with probability P, $0 < P < 1$, of heads is tossed until a head appears for the first time. If the probability that the number of tosses required is even is 2/5 then the value of P is

(a) 1/4 (b) 1/6
(c) 1/3 (d) 1/2

15. If $a, b \in N$ then the probability that $a^2 + b^2$ is divisible by 5 is

(a) $\frac{9}{25}$ (b) $\frac{7}{18}$

(c) $\frac{11}{36}$ (d) $\frac{17}{81}$

16. In an examination, one hundred candidates took paper in Physics and Chemistry. Twenty five candidates failed in Physics only. Twenty candidates failed in chemistry only. Fifteen failed in both Physics and Chemistry. A candidate is selected at random. The probability that he failed either in Physics or in Chemistry but not in both is

(a) $\frac{9}{20}$ (b) $\frac{3}{5}$

(c) $\frac{2}{5}$ (d) $\frac{11}{20}$

17. A fair die is tossed repeatidly. A wins if it is 1 or 2 on two consecutive tosses and B wins if it is 3, 4, 5 or 6 on two consecutive tosses. The probability that A wins if the die is tossed indefinitely, is

(a) $\frac{1}{3}$ (b) $\frac{5}{21}$

(c) $\frac{1}{4}$ (d) $\frac{2}{5}$

18. A purse contains 2 six sided dice. One is a normal fair die, while the other has 2 ones, 2 threes, and 2 fives. A die is picked up and rolled. Because of some secret magnetic attraction of the unfair die, there is 75% chance of picking the unfair die and a 25% chance of picking a fair die. The die is rolled and shows up the face 3. The probability that a fair die was picked up, is

(a) $\frac{1}{7}$ (b) $\frac{1}{4}$

(c) $\frac{1}{6}$ (d) $\frac{1}{24}$

19. The first 12 letters of the english alphabets are written down at random. The probability that there are 4 letters between A & B is :

(a) 7/33 (b) 12/33
(c) 14/33 (d) 7/66

20. Events A and C are independent. If the probabilities relating A, B and C are P (A) = 1/5; $P(B) = 1/6$; $P(A \cap C) = 1/20$; $P(B \cup C) = 3/8$ then

(a) events B and C are independent

(b) events B and C are mutually exclusive

(c) events B and C are neither independent nor mutually exclusive

(d) events B and C are equiprobable

21. Assume that the birth of a boy or girl to a couple to be equally likely, mutually exclusive, exhaustive and independent of the other children in the family. For a couple having 6 children, the probability that their "three oldest are boys" is

(a) $\frac{20}{64}$ (b) $\frac{1}{64}$

(c) $\frac{2}{64}$ (d) $\frac{8}{64}$

22. Box A contains 3 red and 2 blue marbles while box B contains 2 red and 8 blue marbles. A fair coin is tossed. If the coin turns up heads, a marble is drawn from A, if it turns up tails, a marble is drawn from bag B. The probability that a red marble is chosen, is

(a) $\frac{1}{5}$ (b) $\frac{2}{5}$

(c) $\frac{3}{5}$ (d) $\frac{1}{2}$

23. A examination consists of 8 questions in each of which one of the 5 alternatives is the correct one. On the assumption that a candidate who has done no preparatory work chooses for each question any one of the five alternatives with equal probability, the probability that he gets more than one correct answer is equal to :

(a) $(0.8)^8$ (b) $3(0.8)^8$

(c) $1-(0.8)^8$ (d) $1-3(0.8)^8$

24. The germination of seeds is estimated by a probability of 0.6. The probability that out of 11 sown seeds exactly 5 or 6 will spring is :

(a) $\frac{^{11}C_5 . 6^5}{5^{10}}$ (b) $\frac{^{11}C_6 \left(3^5 2^5\right)}{5^{11}}$

(c) $^{11}C_5 \left(\frac{5}{6}\right)^{11}$ (d) none of these

25. The probability of obtaining more tails than heads in 6 tosses of a fair coins is :

(a) 2/64 (b) 22/64

(c) 21/64 (d) none

26. An instrument consists of two units. Each unit must function for the instrument to operate. The reliability of the first unit is 0.9 & that of the second unit is 0.8. The instrument is tested & fails. The probability that "only the first unit failed & the second unit is sound" is :

(a) 1/7 (b) 2/7

(c) 3/7 (d) 4/7

27. Lot A consists of 3G and 2D articles. Lot B consists of 4G and 1D article. A new lot C is formed by taking 3 articles from A and 2 from B. The probability that an article chosen at random from C is defective, is

(a) $\frac{1}{3}$ (b) $\frac{2}{5}$

(c) $\frac{8}{25}$ (d) none

28. A die is weighted so that the probability of different faces to turn up is as given :

Number	1	2	3	4	5	6
Probability	0.2	0.1	0.1	0.3	0.1	0.2

If $P(A/B)=p_1$ and $P(B/C)=p_2$ and $P(C/A)=p_3$ then the values of p_1, p_2, p_3 respectively are

Take the events A, B & C as $A=\{1,2,3\}$, $B=\{2,3,5\}$ and $C=\{2,4,6\}$

(a) $\frac{2}{3}, \frac{1}{3}, \frac{1}{4}$ (b) $\frac{1}{3}, \frac{1}{3}, \frac{1}{6}$

(c) $\frac{1}{4}, \frac{1}{3}, \frac{1}{6}$ (d) $\frac{2}{3}, \frac{1}{6}, \frac{1}{4}$

29. A box has four dice in it. Three of them are fair dice but the fourth one has the number five on all of its faces. A die is chosen at random from the box and is rolled three times and shows up the face five on all the three occassions. The chance that the die chosen was a rigged die, is

(a) $\frac{216}{217}$ (b) $\frac{215}{219}$

(c) $\frac{216}{219}$ (d) none

30. On a Saturday night 20% of all drivers in U.S.A. are under the influence of alcohol. The probability that a driver under the influence of alcohol will have an accident is 0.001. The probability that a sober driver will have an accident is 0.0001. If a car on a saturday night smashed into a tree, the probability that the driver was under the influence of alcohol, is

(a) 3/7 (b) 4/7

(c) 5/7 (d) 6/7

31. A bowl has 6 red marbles and 3 green marbles. The probability that a blind folded person will draw a red marble on the second draw from the bowl without replacing the marble from the first draw, is

(a) $\frac{2}{3}$ (b) $\frac{1}{4}$

(c) $\frac{1}{2}$ (d) $\frac{5}{8}$

32. 5 out of 6 persons who usually work in an office prefer coffee in the mid morning, the other always drink tea. This morning of the usual 6, only 3 are present. The probability that one of them drinks tea is :

(a) 1/2 (b) 1/12

(c) 25/72 (d) 5/72

33. The probability that a radar will detect an object in one cycle is p. The probability that the object will be detected in n cycles is :

(a) $1-p^n$ (b) $1-(1-p)^n$

(c) p^n (d) $p(1-p)^{n-1}$

34. Nine cards are labelled 0, 1, 2, 3, 4, 5, 6, 7, 8. Two cards are drawn at random and put on a table in a successive order, and then the resulting number is read, say, 07 (seven), 14 (fourteen) and so on. The probability that the number is even, is

(a) $\frac{5}{9}$ (b) $\frac{4}{9}$

(c) $\frac{1}{2}$ (d) $\frac{2}{3}$

35. Two cards are drawn from a well shuffled pack of 52 playing cards one by one. If

A : the event that the second card drawn is an ace and

B : the event that the first card drawn is an ace card.

then which of the following is true?

(a) $P(A)=\frac{4}{17}$; $P(B)=\frac{1}{13}$

(b) $P(A)=\frac{1}{13}$; $P(B)=\frac{1}{13}$

(c) $P(A)=\frac{1}{13}$; $P(B)=\frac{1}{17}$

(d) $P(A)=\frac{16}{221}$; $P(B)=\frac{4}{51}$

36. If $\frac{(1+3p)}{3}, \frac{(1-p)}{4}$ & $\frac{(1-2p)}{2}$ are the probabilities of three mutually exclusive events defined on a sample space S, then the true set of all values of p is

(a) $\left[\frac{1}{3},\frac{1}{2}\right]$ (b) $\left[\frac{1}{3},1\right]$

(c) $\left[\frac{1}{4},\frac{1}{3}\right]$ (d) $\left[\frac{1}{4},\frac{1}{2}\right]$

37. An Urn contains 'm' white and 'n' black balls. All the balls except for one ball, are drawn from it. The probability that the last ball remaining in the Urn is white, is

(a) $\frac{m}{m+n}$ (b) $\frac{n}{m+n}$

(c) $\frac{1}{(m+n)!}$ (d) $\frac{mn}{(m+n)!}$

38. A Urn contains 'm' white and 'n' black balls. Balls are drawn one by one till all the balls are drawn. Probability that the second drawn ball is white, is

(a) $\frac{m}{m+n}$

(b) $\frac{n(m+n-1)}{(m+n)(m+n-1)}$

(c) $\frac{m(m-1)}{(m+n)(m+n-1)}$

(d) $\frac{mn}{(m+n)(m+n-1)}$

39. Mr. Dupont is a professional wine taster. When given a French wine, he will identify it with probability 0.9 correctly as French, and will mistake it for a Californian wine with probability 0.1. When given a Californian wine, he will identify it with probability 0.8 correctly as Californian, and will mistake it for a French wine with probability 0.2. Suppose that Mr. Dupont is given ten unlabelled glasses of wine, three with French and seven with Californian wines. He randomly picks a glass, tries the wine, and solemnly says : "French". The probability that the wine he tasted was Californian, is nearly equal to

(a) 0.14 (b) 0.24
(c) 0.34 (d) 0.44

40. Let A, B & C be 3 arbitrary events defined on a sample space 'S' and if, $P(A)+P(B)+P(C)=p_1$, $P(A\cap B)+P(B\cap C)+P(C\cap A)=p_2$ & $P(A\cap B\cap C)=p_3$, then the probability that exactly one of the three events occurs is given by :

(a) $p_1-p_2+p_3$ (b) $p_1-p_2+2p_3$
(c) $p_1-2p_2+p_3$ (d) $p_1-2p_2+3p_3$

41. Three numbers are chosen at random without replacement from $\{1, 2, 3,......, 10\}$. The probability that the minimum of the chosen numbers is 3 or their maximum is 7 is

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$

(c) $\frac{1}{4}$ (d) $\frac{11}{40}$

42. If at least one child in a family with 3 children is a boy then the probability that 2 of the children are boys, is

(a) $\frac{3}{7}$ (b) $\frac{1}{4}$

(c) $\frac{1}{3}$ (d) $\frac{3}{8}$

43. The probabilities of events, $A\cap B$, A, B & $A\cup B$ are respectively in A.P. with probability of second term equal to the common difference. Therefore the events A and B are

(a) compatible
(b) independent
(c) such that one of them must occur
(d) such that one is twice as likely as the other

44. From an urn containing six balls, 3 white and 3 black ones, a person selects at random an even number of balls (all the different ways of drawing an even number of balls are considered equally probable, irrespective of their number). Then the probability that there will be the same number of black and white balls among them

(a) $\frac{4}{5}$ (b) $\frac{11}{15}$

(c) $\frac{11}{30}$ (d) $\frac{2}{5}$

45. One purse contains 6 copper coins and 1 silver coin ; a second purse contains 4 copper coins. Five coins are drawn from the first purse and put into the second, and then 2 coins are drawn from the second and put into the first. The probability that the silver coin is in the second purse is

(a) $\frac{1}{2}$ (b) $\frac{4}{9}$

(c) $\frac{5}{9}$ (d) $\frac{2}{3}$

46. A box contains a normal coin and a doubly headed coin. A coin selected at random and tossed twice, fell headwise on both the occasions. The probability that the drawn coin is a doubly headed coin is

(a) $\frac{2}{3}$ (b) $\frac{5}{8}$

(c) $\frac{3}{4}$ (d) $\frac{4}{5}$

47. A box contains 5 red and 4 white marbles. Two marbles are drawn successively from the box without replacement and the second drawn marble drawn is found to be white. Probability that the first marble is also white is

(a) $\frac{3}{8}$ (b) $\frac{1}{2}$

(c) $\frac{1}{3}$ (d) $\frac{1}{4}$

48. A and B in order draw a marble from bag containing 5 white and 1 red marbles with the condition that whosoever draws the red marble first, wins the game. Marble once drawn by them are not replaced into the bag. Then their respective chances of winning are

(a) $\frac{2}{3}$ & $\frac{1}{3}$ (b) $\frac{3}{5}$ & $\frac{2}{5}$

(c) $\frac{2}{5}$ & $\frac{3}{5}$ (d) $\frac{1}{2}$ & $\frac{1}{2}$

49. In a maths paper there are 3 sections A, B & C. Section A is compulsory. Out of sections B & C a student has to attempt any one. Passing in the paper means passing in A & passing in B or C. The probability of the student passing in A, B & C are p, q & 1/2 respectively. If the probability that the student is successful is 1/2 then :

(a) $p = q = 1$ (b) $p = q = \frac{1}{2}$

(c) $p = 1, q = 0$ (d) $p = 1, q = \frac{1}{2}$

50. A box contains 100 tickets numbered 1, 2, 3,.... ,100. Two tickets are chosen at random. It is given that the maximum number on the two chosen tickets is not more than 10. The minimum number on them is 5, with probability

(a) $\frac{1}{9}$ (b) $\frac{2}{11}$

(c) $\frac{3}{19}$ (d) none

51. Sixteen players $s_1, s_2, \ldots, s_{16}$ play in a tournament. They are divided into eight pairs at random. From each pair a winner is decided on the basis of a game played between the two players of the pair. Assume that all the players are of equal strength. The probability that "exactly one of the two players s_1 & s_2 is among the eight winners" is

(a) $\frac{4}{15}$ (b) $\frac{7}{15}$

(c) $\frac{8}{15}$ (d) $\frac{9}{15}$

52. The number 'a' is randomly selected from the set $\{0, 1, 2, 3, \ldots\ 98, 99\}$. The number 'b' is selected from the same set. Probability that the number $3^a + 7^b$ has a digit equal to 8 at the units place, is

(a) $\frac{1}{16}$ (b) $\frac{2}{16}$

(c) $\frac{4}{16}$ (d) $\frac{3}{16}$

53. Two boys A and B find the jumble of n ropes lying on the floor. Each takes hold of one loose end. If the probability that they are both holding the same rope is $\frac{1}{101}$ then the number of ropes is equal to

(a) 101 (b) 100

(c) 51 (d) 50

54. A student appears for tests, I, II and III. The student is successful if he passes either in tests I and II or test I and III. The probabilities of the student passing in tests I, II and III are p, q and $\frac{1}{2}$, respectively. If the probability that the student is successful is $\frac{1}{2}$, then

(a) $p = q = 1$ (b) $p = q = \frac{1}{2}$

(c) $p = 1, q = 0$ (d) $p = 1, q = \frac{1}{2}$

55. The probability that at least one of events A and B occur is 0.6. If A and B occur simultaneously with probability 0.2, then $P(\bar{A}) + P(\bar{B})$ is

(a) 0.4 (b) 0.8

(c) 1.2 (d) 1.4

56. One hundred identical coins, each with probability, p, of showing up heads are tosses once. If $0 < p < 1$ and the probability of heads showing on fifty coins is equal to that of heads showing on 51 coins, then the value of p is :

(a) 1/2 (b) 49/101

(c) 50/101 (d) 51/101

57. If P (A/B) = P (B/A). A and B are two non-mutually exclusive events then

(a) A and B are necessarily same events

(b) P (A) = P (B)

(c) $P(A \cap B) = P(A)P(B)$

(d) all the above

58. A and B are two events such that P(A) = 0.2 and $P(A \cup B) = 0.7$. If A and B are independent events then P(B) equals

(a) 2/7 (b) 7/9

(c) 5/8 (d) none of these

59. A die is thrown a fixed number of times. If probability of getting even number 3 times is same as the probability of getting even number 4 times, then probability of getting even number exactly once is

(a) $\frac{1}{4}$ (b) $\frac{3}{128}$

(c) $\frac{5}{64}$ (d) $\frac{7}{128}$

60. A number is chosen at random from the numbers 10 to 99. By seeing the number a man will laugh if product of the digits is 12. If he choose three numbers with replacement then the probability that he will laugh at least once is

(a) $1-\left(\frac{3}{5}\right)^3$ (b) $\left(\frac{43}{45}\right)^3$

(c) $1-\left(\frac{4}{25}\right)^3$ (d) $1-\left(\frac{43}{45}\right)^3$

61. If two events A and B are such that P (A) > 0 and P (B) $\neq$ 1, then $P(\bar{A}/\bar{B})$ is equal to

(a) $1 - P(A/B)$ (b) $1 - P(\bar{A}/B)$

(c) $\frac{1-P(A \cup B)}{P(\bar{B})}$ (d) $\frac{P(A)}{P(\bar{B})}$

62. One boy can solve 60% of the problems in a book and another can solve 80%. The probability that at least one of the two can solve a problem chosen at random from the book is

(a) 2/25 (b) 23/25

(c) 4/5 (d) 9/10

63. Two small squares on a chess board are chosen at random. Probability that they have a common side is,

(a) 1/3 (b) 1/9

(c) 1/18 (d) none of these

64. A and B play a game of tennis. The situation of the game is as follows; if one scores two consecutive points after a deuce he wins; if loss of a point is followed by win of a point, it is deuce. The chance of a server to win a point is 2/3. The game is at deuce and A is serving. Probability that A will win the match is, (serves are changed after each pt)

(a) 3/5 (b) 2/5

(c) 1/2 (d) 4/5

65. Fifteen coupons are numbered 1, 2, 3, ... 15. Seven coupons are selected at random one at a time with replacement. The probability that the largest number appearing on the selected coupon is 9, is

(a) $\left(\frac{9}{16}\right)^6$ (b) $\left(\frac{8}{15}\right)^7$

(c) $\left(\frac{3}{5}\right)^7$ (d) none of these

66. A fair die is thrown until a score of less than 5 points is obtained. The probability of obtaining not less than 2 points on the last throw is

(a) 3/4 (b) 5/6

(c) 4/5 (d) 1/3

67. A fair coin is tossed a fixed number of times. If the probability of getting 7 heads is equal to getting 9 heads, then the probability of getting 2 heads is,

(a) $15/2^8$ (b) 2/15

(c) $15/2^{13}$ (d) none of these

68. A fair die is tossed eight times. Probability that on the eighth throw a third six is observed is,

(a) ${}^8C_3 \frac{5^5}{6^8}$ (b) $\frac{{}^7C_2.5^5}{6^8}$

(c) $\frac{{}^7C_2.5^5}{6^7}$ (d) none of these

69. If the papers of 4 students can be checked by any one of the 7 teachers, then the probability that all the 4 papers are checked by exactly 2 teachers is;

(a) 2/7 (b) 32/343

(c) 12/49 (d) none of these

70. Find the probability that a leap year selected at random will contain 53 Mondays.

(a) $\frac{1}{7}$ (b) $\frac{2}{7}$

(c) $\frac{3}{4}$ (d) none of these

71. There are n different gift coupons, each of which can occupy N(N > n) different envelopes, with the same probability 1/N

P_1: The probability that there will be one gift coupon in each of n definite envelopes out of N given envelopes

P_2: The probability that there will be one gift coupon in each of n arbitrary envelopes out of N given envelopes

Consider the following statements

(i) $P_1 = P_2$ (ii) $P_1 = \frac{n!}{N^n}$

(iii) $P_2 = \frac{N!}{N^n (N-n)!}$

(iv) $P_2 = \frac{n!}{N^n (N-n)!}$ (v) $P_1 = \frac{N!}{N^n}$

Now, which of the following is true

(a) Only (i) (b) (ii) and (iii)

(c) (ii) and (iv) (d) (iii) and (v)

72. A child throws 2 fair dice. If the numbers showing are unequal, he adds them together to get his final score. On the other hand, if the numbers showing are equal, he throws 2 more dice & adds all 4 numbers showing to get his final score. The probability that his final score is 6 is:

(a) $\frac{145}{1296}$ (b) $\frac{146}{1296}$

(c) $\frac{147}{1296}$ (d) $\frac{148}{1296}$

73. Indicate the correct order sequence in respect of the following :

I. If the probability that a computer will fail during the first hour of operation is 0.01, then if we turn on 100 computers, exactly one will fail in the first hour of operation.

II. A man has ten keys only one of which fits the lock. He tries them in a door one by one discarding the one he has tried. The probability that fifth key fits the lock is 1/10.

III. Given the events A and B in a sample space. If P(A) = 1, then A and B are independent.

IV. When a fair six sided die is tossed on a table top, the bottom face can not be seen. The probability that the product of the numbers on the five faces that can be seen is divisible by 6 is one.

(a) FTFT (b) FTTT

(c) TFTF (d) TFFF

74. Two buses A and B are scheduled to arrive at a town central bus station at noon. The probability that bus A will be late is 1/5. The probability that bus B will be late is 7/25. The probability that the bus B is late given that bus A is late is 9/10. Then the probabilities

(i) neither bus will be late on a particular day

and

(ii) bus A is late given that bus B is late, are respectively

(a) 2/25 and 12/28 (b) 18/25 and 22/28

(c) 7/10 and 18/28 (d) 12/25 and 2/28

75. Sixteen players $s_1, s_2, ..., s_{16}$ play in a tournament. They are divided into eight pairs at random. From each pair a winner is decided on the basis of a game played between the two players of the pair. Assume that all the players are of equal strength. The probability that "exactly one of the two players s_1 & s_2 is among the eight winners" is

(a) $\frac{4}{15}$ (b) $\frac{7}{15}$

(c) $\frac{8}{15}$ (d) $\frac{9}{15}$

Multiple Type

76. If A & B are two events such that $P(B) \neq 1$, B^C denotes the event complementry to B, then

(a) $P\left(A/B^C\right) = \frac{P(A) - P(A \cap B)}{1 - P(B)}$

(b) $P(A \cap B) \geq P(A) + P(B) - 1$

(c) $P(A) > P\left(A/B\right)$ if $P\left(A/B^C\right) > P(A)$

(d) $P\left(A/B^C\right) + P\left(A^C/B^C\right) = 1$

77. A bag initially contains one red & two blue balls. An experiment consisting of selecting a ball at random, noting its colour & replacing it together with an additional ball of the same colour. If three such trials are made, then :

(a) probability that atleast one blue ball is drawn is 0.9

(b) probability that exactly one blue ball is drawn is 0.2

(c) probability that all the drawn balls are red given that all the drawn balls are of same colour is 0.2

(d) probability that atleast one red ball is drawn is 0.6.

78. Two real numbers, x & y are selected at random. Given that $0 \leq x \leq 1$; $0 \leq y \leq 1$. Let A be the event that $y^2 \leq x$; B be the event that $x^2 \leq y$, then :

(a) $P(A \cap B) = \frac{1}{3}$

(b) A & B are exhaustive events

(c) A & B are mutually exclusive

(d) A & B are independent events.

79. For any two events A & B defined on a sample space ,

(a) $P\left(A/B\right) \geq \frac{P(A) + P(B) - 1}{P(B)}$, $P(B) \neq 0$ is always true

(b) $P\left(A \cap \overline{B}\right) = P(A) - P(A \cap B)$

(c) $P(A \cup B) = 1 - P(A^c).P(B^c)$, if A & B are independent

(d) $P(A \cup B) = 1 - P(A^c).P(B^c)$, if A & B are disjoint

80. If E_1 and E_2 are two events such that $P(E_1) = 1/4$, $P(E_2/E_1) = 1/2$ and $P(E_1/E_2) = 1/4$

(a) then E_1 and E_2 are independent

(b) E_1 and E_2 are exhaustive

(c) E_2 is twice as likely to occur as E_1

(d) Probabilities of the events $E_1 \cap E_2$, E_1 and E_2 are in G.P.

81. Let $0 < P(A) < 1$, $0 < P(B) < 1$ & $P(A \cup B) = P(A) + P(B) - P(A).P(B)$, then :

(a) $P(B/A) = P(B) - P(A)$

(b) $P(A^C \cup B^C) = P(A^C) + P(B^C)$

(c) $P((A \cup B)^C) = P(A^C).P(B^C)$

(d) $P(A/B) = P(A)$

82. If M & N are independent events such that $0 < P(M) < 1$ & $0 < P(N) < 1$, then :

(a) M & N are mutually exclusive

(b) M & $\overline{N}$ are independent

(c) $\overline{M}$ & $\overline{N}$ are independent

(d) $P\left(M/N\right) + P\left(\overline{M}/N\right) = 1$

83. For two given events A & B, $P(A \cap B)$ is :

(a) not less than $P(A) + P(B) - 1$

(b) not greater than $P(A) + P(B)$

(c) equal to $P(A) + P(B) - P(A \cup B)$

(d) equal to $P(A) + P(B) + P(A \cup B)$

Passage Type

Direction for Q.84 to Q.86

Let S and T are two events defined on a sample space with probabilities

$P(S) = 0.5, P(T) = 0.69, P(S/T) = 0.5$

84. Events S and T are:

(a) mutually exclusive
(b) independent
(c) mutually exclusive and independent
(d) neither mutually exclusive nor independent

85. The value of P(S and T)

(a) 0.3450 (b) 0.2500
(c) 0.6900 (d) 0.350

86. The value of P(S or T)

(a) 0.6900 (b) 1.19
(c) 0.8450 (d) 0

Direction for Q.87 to Q.89 (3 Questions)

A JEE aspirant estimates that she will be successful with an 80 percent chance if she studies 10 hours per day, with a 60 percent chance if she studies 7 hours per day and with a 40 percent chance if she studies 4 hours per day. She further believes that she will study 10 hours, 7 hours and 4 hours per day with probabilities 0.1, 0.2 and 0.7, respectively

87. The chance she will be successful, is

(a) 0.28 (b) 0.38
(c) 0.48 (d) 0.58

88. Given that she is successful, the chance she studied for 4 hours, is

(a) $\frac{6}{12}$ (b) $\frac{7}{12}$

(c) $\frac{8}{12}$ (d) $\frac{9}{12}$

89. Given that she does not achieve success, the chance she studied for 4 hour, is

(a) $\frac{18}{26}$ (b) $\frac{19}{26}$

(c) $\frac{20}{26}$ (d) $\frac{21}{26}$

Direction for Q.90 to Q.93 (4 Questions)

Read the passage given below carefully before attempting these questions.

A standard deck of playing cards has 52 cards. There are four suit (clubs, diamonds, hearts and spades), each of which has thirteen numbered cards (2, ..., 9, 10, Jack, Queen, King, Ace)

In a game of card, each card is worth an amount of points. Each numbered card is worth its number (e.g. a 5 is worth 5 points) ; the Jack, Queen and King are each worth 10 points ; and the Ace is worth your choice of either 1 point or 11 points. The object of the game is to have more points in your set of cards than your opponent without going over 21. Any set of cards with sum greater than 21 automatically loses.

Here's how the game played. You and your opponent are each dealt two cards. Usually the first card for each player is dealt face down, and the second card for each player is dealt face up. After the initial cards are dealt, the first player has the option of asking for another card or not taking any cards. The first player can keep asking for more cards until either he or she goes over 21, in which case the player loses, or stops at some number less than or equal to 21. When the first player stops at some number less than or equal to 21, the second player then can take more cards until matching or exceeding the first player's number without going over 21, in which case the second player wins, or until going over 21, in which case the first player wins.

We are going to simplify the game a little and assume that all cards are dealt face up, so that all cards are visible. Assume your opponent is dealt cards and plays first.

90. The chance that the second card will be a heart and a Jack, is

(a) $\frac{4}{52}$ (b) $\frac{13}{52}$

(c) $\frac{17}{52}$ (d) $\frac{1}{52}$

91. The chance that the first card will be a heart or a Jack, is

(a) $\frac{13}{52}$ (b) $\frac{16}{52}$

(c) $\frac{17}{52}$ (d) none

92. Given that the first card is a Jack, the chance that it will be the heart, is

(a) $\frac{1}{13}$ (b) $\frac{4}{13}$

(c) $\frac{1}{4}$ (d) $\frac{1}{3}$

93. Your opponent is dealt a King and a 10, and you are dealt a Queen and a 9. Being smart, your opponent does not take any more cards and stays at 20. The chance that you will win if you are allowed to take as many cards as you need, is

(a) $\frac{97}{564}$ (b) $\frac{25}{282}$

(c) $\frac{15}{188}$ (d) $\frac{1}{6}$

Match the Column

94. A determinant Δ is chosen at random from the set of all determinant of order two with elements 0 and 1 only.

Value of Δ	Probability
(a) 1	(p) 5/8
(b) 0	(q) 3/16
(c) 2	(r) 3/8
(d) non zero	(s) 0

95. A ten digit number N is formed by using the digits 0 to 9 exactly once. The probability that N is divisible by

(a) 4	(p) 1
(b) 5	(q) 20/81
(c) 45	(r) 17/81
(d) 12	(s) 2/81

Assertion Reason

(A) If both assertion and reason are correct and reason is the correct explanation of assertion.

(B) If both assertion and reason are true but reason is not the correct explanation of assertion.

(C) If assertion is true but reason is false.

(D) If assertion is false but reason is true.

96. From an urn containing a white and b black balls, k (< a, b) are drawn and laid aside, their colour unnoted. Then another ball, that is, $(k + 1)^{th}$ ball is drawn.

Assertion : Probability that $(k + 1)^{th}$ ball drawn is white is $\frac{a}{a+b}$.

Reason : Probability that $(k + 1)^{th}$ ball drawn is black is $\frac{a}{a+b}$

97. Let A and B are two events such that P(A) > 0.

Assertion : If P (A) + P (B) > 1, then P (B/A) ≥ 1 – P (B′)/P(A)

Reason : If P (A/B′) ≥ P (A), then P(A) ≥ P(A/B).

Integral Type

98. 7 persons are stopped on the road at random and asked about their birthdays. If the probability that 3 of them are born on Wednesday, 2 on Thursday and the remaining 2 on Sunday is $\frac{K}{7^6}$, then K is equal to

99. A bag contains n + 1 coins. It is known that one of these coins has heads on both sides, whereas the other coins are fair. One coin is selected at random and tossed. If the probability that the toss results in heads is 7/12, find n.

100. Two integers r and s are chosen one at a time without replacement from the numbers 1, 2, 3,... 100. Let p be the probability that r ≤ 25 given that s ≤ 25. Find the value of 33p.

EXERCISE - 4 : PREVIOUS YEAR JEE ADVANCED QUESTIONS

SINGLE ANSWER CORRECT

1. Two events A and B have probabilities 0.25 and 0.50 respectively. The probability that both A and B occur simultaneously is 0.14. Then, the probability that neither A nor B occurs is **(1980)**

(a) 0.39 (b) 0.25

(c) 0.11 (d) None of these

2. The probability that an event A happens in one trial of an experiment is 0.4. Three independent trials of the experiments are performed. The probability that the event A happens at least ones is **(1980)**

(a) 0.936 (b) 0.784

(c) 0.904 (d) None of these

3. If A and B are two independent events such that $P(A) > 0$, and $P(B) \neq 1$, then $P(\bar{A}/\bar{B})$ is equal to **(1982)**

(a) $1-P\left(\bar{A}/B\right)$ (b) $1-P(A/\bar{B})$

(c) $\dfrac{1-P(A\cup B)}{P(B)}$ (d) $\dfrac{P(\bar{A})}{P(\bar{B})}$

4. Fifteen coupons are numbered 1, 2, ..., 15, respectively. Seven coupons are selected at random one at a time with replacement. The probability that the largest number appearing on a selected coupon is 9, is **(1983)**

(a) $\left(\dfrac{9}{16}\right)^6$ (b) $\left(\dfrac{8}{15}\right)^7$

(c) $\left(\dfrac{3}{5}\right)^7$ (d) None of these

5. Three identical dice are rolled. The probability that the same number will appear on each of them, is **(1984)**

(a) $\dfrac{1}{6}$ (b) $\dfrac{1}{36}$

(c) $\dfrac{1}{18}$ (d) $\dfrac{3}{28}$

6. A student appears for test I, II and III. The student is successful if he passes either in tests I and II or tests I and III. The probabilities of the student passing in tests I, II and III are p, q and $\dfrac{1}{2}$ respectively. If the probability that the student is successful, is $\dfrac{1}{2}$, then **(1986)**

(a) $p=q=1$ (b) $p=q=\dfrac{1}{2}$

(c) $p=1, q=0$ (d) $p=1, q=\dfrac{1}{2}$

7. The probability that at least one of the events A and B occurs is 0.6. If A and B occur simultaneously with probability 0.2, then $P(\bar{A})+P(\bar{B})$ is **(1987)**

(a) 0.4 (b) 0.8

(c) 1.2 (d) 1.4

8. One hundred identical coins, each with probability p, of showing up heads are tossed once. If $0 < p < 1$ and the probability of heads showing on 50 coins is equal to that of heads showing on 51 coins, then the value of p is **(1988)**

(a) 1/2 (b) 49/101

(c) 50/101 (d) 51/101

9. India plays two matches each with West Indies and Australia. In any match the probabilities of India getting points 0, 1 and 2 are 0.45, 0.05 and 0.50 respectively. Assuming that the outcomes are independent, the probability of India getting at least 7 points, is **(1992)**

(a) 0.8750 (b) 0.0875

(c) 0.0625 (d) 0.0250

10. An unbiased die with faces marked 1, 2, 3, 4, 5 and 6 is rolled four times. Out of four face values obtained, the probability that the minimum face value is not less than 2 and the maximum face value is not greater than 5, is **(1993)**

(a) 16/81 (b) 1/81

(c) 80/81 (d) 65/81

11. Let $0 < P(A) < 1, 0 < P(B) < 1$
and $P(A \cup B) = P(A) + P(B) - P(A)P(B)$, then **(1995)**

(a) $P(B/A) = P(B) - P(A)$

(b) $P(A' - B') = P(A') - P(B')$

(c) $P(A \cup B)' = P(A)'P(B)'$

(d) $P(A/B) = P(A) - P(B)$

12. The probability of India winning a test match against West Indies is 1/2. Assuming independence from match to match the probability that in a 5 match series India's second win occurs at third test is **(1995)**

(a) 1/8 (b) 1/4

(c) 1/2 (d) 2/3

13. Three of the six vertices of a regular hexagon are chosen at rondom. The probability that the triangle with three vertices is equilateral, equals **(1995)**

(a) 1/2 (b) 1/5

(c) 1/10 (d) 1/20

14. For the three events A, B and C,
P(exactly one of the events A or B occurs) = P (exactly one of the events B or C occurs) = P(exactly one of the events C or A occurs) = p and P (all the three events occurs simultaneously) = p^2, where $0 < p < \frac{1}{2}$. Then, the probability of at least one of the three events A, B and C occurring is **(1996)**

(a) $\frac{3p+2p^2}{2}$ (b) $\frac{p+3p^2}{4}$

(c) $\frac{p+3p^2}{2}$ (d) $\frac{3p+2p^2}{4}$

15. If from each of the three boxes containing 3 white and 1 black, 2 white and 2 black, 1 white and 3 black balls, one ball is drawn at random, then the probability that 2 white and 1 black balls will be drawn, is **(1998)**

(a) $\frac{13}{32}$ (b) $\frac{1}{4}$

(c) $\frac{1}{32}$ (d) $\frac{3}{16}$

16. There are four machines and it is known that exactly two of them are faulty. They are tested, one by one, in a random order till both the faulty machines are identified. Then, the probability that only two tests are needed, is **(1998)**

(a) $\frac{1}{3}$ (b) $\frac{1}{6}$

(c) $\frac{1}{2}$ (d) $\frac{1}{4}$

17. A fair coin is tossed repeatedly. If tail appears on first four tosses, then the probability of head appearing on fifth toss equals **(1998)**

(a) $\frac{1}{2}$ (b) $\frac{1}{32}$

(c) $\frac{31}{32}$ (d) $\frac{1}{5}$

18. Seven white balls and three black balls are randomly placed in a row. The probability that no two black balls are placed adjacently, equals **(1998)**

(a) $\frac{1}{2}$ (b) $\frac{7}{15}$

(c) $\frac{2}{15}$ (d) $\frac{1}{3}$

19. If E and F are events with $P(E) \le P(F)$ and $P(E \cap F) > 0$, then **(1998)**

(a) occurrence of E $\Rightarrow$ occurrence of F

(b) occurrence of F $\Rightarrow$ occurrence of E

(c) non-occurrence of E $\Rightarrow$ non-occurrence of F

(d) None of the above implication holds

20. If the integers m and n are chosen at random between 1 and 100, then the probability that a number of the form $7^m + 7^n$ is divisible by 5, equals **(1999)**

(a) $\frac{1}{4}$ (b) $\frac{1}{7}$

(c) $\frac{1}{8}$ (d) $\frac{1}{49}$

21. If $P(B) = \frac{3}{4}$, $P(A \cap B \cap \bar{C}) = \frac{1}{3}$ and $P(\bar{A} \cap B \cap \bar{C}) = \frac{1}{3}$, then $P(B \cap C)$ is **(2002)**

(a) $\frac{1}{12}$ (b) $\frac{1}{6}$

(c) $\frac{1}{15}$ (d) $\frac{1}{9}$

22. Two numbers are selected randomly from the set $S = \{1, 2, 3, 4, 5, 6\}$ without replacement one by one. The probability that minimum of the two numbers is less than 4, is **(2003)**

(a) 1/15 (b) 14/15

(c) 1/5 (d) 4/5

23. If three distinct numbers are chosen randomly from the first 100 natural numbers, then the probability that all three of them are divisible by both 2 and 3, is **(2004)**

(a) $\frac{4}{55}$ (b) $\frac{4}{35}$

(c) $\frac{4}{33}$ (d) $\frac{4}{1155}$

24. A fair die is rolled. The probability that the first time 1 occurs at the even throw, is **(2005)**

(a) 1/6 (b) 5/11

(c) 6/11 (d) 5/36

25. One Indian and four American men and their wives are to be seated randomly around a circular table. Then, the conditional probability that the Indian man is seated adjacent to his wife given that each American man is seated adjacent to his wife, is **(2007)**

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$

(c) $\frac{2}{5}$ (d) $\frac{1}{5}$

26. Let E^c denotes the complement of an event E. Let E, F, G be pairwise independent events with $P(G) > 0$ and $P(E \cap F \cap G) = 0$. Then, $P(E^c \cap F^c | G)$ equals **(2007)**

(a) $P(E^c) + P(F^c)$ (b) $P(E^c) - P(F^c)$

(c) $P(E^c) - P(F)$ (d) $P(E) - P(F^c)$

27. An experiment has 10 equally likely outcomes. Let A and B be two non-empty events of the experiment. If A consists of 4 outcomes, the number of outcomes that B must have so that A and B are independent, is **(2008)**

(a) 2, 4, or 8 (b) 3, 6 or 9

(c) 4 or 8 (d) 5 or 10

28. Let ω be a complex cube root of unity with $\omega \neq 1$. A fair die is thrown three times, If r_1, r_2 and r_3 are the numbers obtained on the die, then the probability that $\omega^{r_1} + \omega^{r_2} + \omega^{r_3} = 0$ is **(2010)**

(a) 1/18 (b) 1/9

(c) 2/9 (d) 1/36

29. A signal which can be green or red with probability $\frac{4}{5}$ and $\frac{1}{5}$ respectively, is received by station A and then transmitted to station B. The probability of each station receiving the singal correctly is $\frac{3}{4}$. If the signal received at station B is green, then the probability that the original signal green is **(2010)**

(a) $\frac{3}{5}$ (b) $\frac{6}{7}$

(c) $\frac{20}{23}$ (d) $\frac{9}{20}$

30. Three boys and two girls stand in a queue. The probability, that the number of boys ahead of every girl is at least one more than the number of girls ahead of her, is **(2014)**

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$

(c) $\frac{2}{3}$ (d) $\frac{3}{4}$

31. A computer producing factory has only two plants T_1 and T_2. Plant T_1 produces 20% and plant T_2 produces 80% of the total computers produced. 7% of computers produced in the factory turn out to be defective. It is known that P (computer turns out to be defective given that it is produced in plant T_1) = 10P (computer turns out to be defective given that it is produced in plant T_2), where P(E) denotes the probability of an event E. A computer produced in the factory is randomly selected and it does not turn out to be defective. Then the probability that it is produced in plants T_2 is **(2016)**

(a) $\frac{36}{73}$ (b) $\frac{47}{79}$

(c) $\frac{78}{93}$ (d) $\frac{75}{83}$

32. Three randomly chosen nonnegative integers x, y and z are found to satisfy the equation $x + y + z = 10$. Then the probability that z is even, is **(2017)**

(a) $\frac{1}{2}$ (b) $\frac{36}{55}$

(c) $\frac{6}{11}$ (d) $\frac{5}{11}$

MULTIPLE ANSWER CORRECT

33. If M and N are any two events, then the probability that exactly one of them occurs is **(1984)**

(a) $P(M) + P(N) - 2P(M \cap N)$

(b) $P(M) + P(N) - P(\overline{M \cup N})$

(c) $P(\bar{M}) + P(\bar{N}) - 2P(\bar{M} \cap \bar{N})$

(d) $P(M \cap \bar{N}) - P(\bar{M} \cap N)$

34. For two given events A and B, $P(A \cap B)$ is **(1988)**

(a) not less than $P(A) + P(B) - 1$

(b) not greater than $P(A) + P(B)$

(c) equal to $P(A) + P(B) - P(A \cup B)$

(d) equal to $P(A) + P(B) + P(A \cup B)$

35. If E and F are independent events such that $0 < P(E) < 1$ and $0 < P(F) < 1$, then **(1989)**

(a) E and F are mutually exclusive

(b) E and F^c (the complement of the event F) are independent

(c) E^c and F^c are independent

(d) $P(E/F) + P(E^c/F) = 1$

36. For any two events A and B in a sample space **(1991)**

(a) $P\left(\frac{A}{B}\right) \geq \frac{P(A) + P(B) - 1}{P(B)}$, $P(B) \neq 0$ is always true

(b) $P(A \cap \bar{B}) = P(A) - P(A \cap B)$ does not hold

(c) $P(A \cup B) = 1 - P(\bar{A})P(\bar{B})$, if A and B are independent

(d) $P(A \cup B) = 1 - P(\bar{A})P(\bar{B})$, if A and B are disjoint

37. Let E and F be two independent events. The probability that both E and F happen is 1/12 and the probability that neither E nor F happen is 1/2. Then, **(1993)**

(a) $P(E) = 1/3, P(F) = 1/4$

(b) $P(E) = 1/2, P(F) = 1/6$

(c) $P(E) = 1/6, P(F) = 1/2$

(d) $P(E) = 1/4, P(F) = 1/3$

38. If $\bar{E}$ and $\bar{F}$ are the complementary events of E and F respectively and if $0 < P(F) < 1$, then **(1998)**

(a) $P(E/F) + P(\bar{E}/F) = 1$

(b) $P(E/F) + P(E/\bar{F}) = 1$

(c) $P(\bar{E}/F) + P(E/\bar{F}) = 1$

(d) $P(E/\bar{F}) + P(\bar{E}/\bar{F}) = 1$

39. The probabilities that a student passes in Mathematics, Physics and Chemistry are m, p and c respectively. Of these subjects, the students has a 75% chance of passing in at least one, a 50% chance of passing in at least two, and a 40% chance of passing in exactly two. Which of the following relations are true? **(1999)**

(a) $p + m + c = \frac{19}{20}$ (b) $p + m + c = \frac{27}{20}$

(c) $pmc = \frac{1}{10}$ (d) $pmc = \frac{1}{4}$

40. Let E and F be two independent events. The probability that exactly one of them occurs is $\frac{11}{25}$ and the probability of none of them occuring is $\frac{2}{25}$. If P (T) denotes the probability of occurence of the event T, then **(2011)**

(a) $P(E) = \frac{4}{5}, P(F) = \frac{3}{5}$ (b) $P(E) = \frac{1}{5}, P(F) = \frac{2}{5}$

(c) $P(E) = \frac{2}{5}, P(F) = \frac{1}{5}$ (d) $P(E) = \frac{3}{5}, P(F) = \frac{4}{5}$

41. Let X and Y be two events such that $P(X)=\frac{1}{3}$, $P(X|Y)=\frac{1}{2}$ and $P(Y|X)=\frac{2}{5}$. Then **(2017)**

(a) $P(Y)=\frac{4}{15}$ (b) $P(X'|Y)=\frac{1}{2}$

(c) $P(X\cup Y)=\frac{2}{5}$ (d) $P(X\cap Y)=\frac{1}{5}$

Comprehension Type

Passage – 1

There are n urns each containing $n+1$ balls such that the i^{th} urn contains i white balls and $(n+1-i)$ red balls. Let u_i be the event of selecting i^{th} urn, $i = 1, 2, 3, ..., n$ and w denotes the event of getting a white ball. **(2006)**

42. If $P(u_i) \propto i$ where $i = 1, 2, 3,....., n$ then $\lim_{n\to\infty} P(w)$ is equal to

(a) 1 (b) 2/3
(c) 3/4 (d) 1/4

43. If $P(u_i) = c$, where c is a constant then $P(u_n/w)$ is equal to

(a) $\frac{2}{n+1}$ (b) $\frac{1}{n+1}$

(c) $\frac{n}{n+1}$ (d) $\frac{1}{2}$

44. If n is even and E denotes the event of choosing even numbered urn ($P(u_i)=\frac{1}{n}$), then the value of $P(w/E)$, is

(a) $\frac{n+2}{2n+1}$ (b) $\frac{n+2}{2(n+1)}$

(c) $\frac{n}{n+1}$ (d) $\frac{1}{n+1}$

Passage – 2

A fair die is tossed repeatedly until a six is obtained. Let X denote the number of tosses required. **(2009)**

45. The probability that X = 3 equals

(a) $\frac{25}{216}$ (b) $\frac{25}{36}$

(c) $\frac{5}{36}$ (d) $\frac{125}{216}$

46. The probability that $X \geq 3$ equals

(a) $\frac{125}{216}$ (b) $\frac{25}{36}$

(c) $\frac{5}{36}$ (d) $\frac{25}{216}$

47. The conditional probability that $X \geq 6$ given $X > 3$ equals

(a) $\frac{125}{216}$ (b) $\frac{25}{216}$

(c) $\frac{5}{36}$ (d) $\frac{25}{36}$

Passage – 3

Let U_1 and U_2 be two urns such that U_1 contains 3 white and 2 red balls and U_2 contains only 1 white ball. A fair coin is tossed. If head appears then 1 ball is drawn at random from U_1 and put into U_2. However, if tail appears then 2 balls are drawn at random from U_1 and put into U_2. Now, 1 ball is drawn at random from U_2. **(2011)**

48. The probability of the drawn ball from U_2 being white is

(a) $\frac{13}{30}$ (b) $\frac{23}{30}$

(c) $\frac{19}{30}$ (d) $\frac{11}{30}$

49. Given that the drawn ball from U_2 is white, the probability that head appeared on the coin is

(a) $\frac{17}{23}$ (b) $\frac{11}{23}$

(c) $\frac{15}{23}$ (d) $\frac{12}{23}$

Passage – 4

Box 1 contains three cards bearing numbers 1,2,3 ; box 2 contains five cards bearing numbers 1,2,3,4,5 and box 3 contains seven cards bearing numbers 1,2,3,4,5,6,7. A card is drawn from each of the boxes. Let x_i be the number on the card drawn from the i^{th} box, $i = 1,2,3$. **(2014)**

50. The probability that $x_1 + x_2 + x_3$ is odd, is

(a) $\frac{29}{105}$ (b) $\frac{53}{105}$

(c) $\frac{57}{105}$ (d) $\frac{1}{2}$

51. The probability that x_1, x_2, x_3 are in an arithmetic progression, is

(a) $\frac{9}{105}$ (b) $\frac{10}{105}$

(c) $\frac{11}{105}$ (d) $\frac{7}{105}$

Passage – 5

Let n_1 and n_2 be the number of red and black balls, respectively, in box I. Let n_3 and n4 be the number of red and black balls, respectively, in box II. **(2015)**

52. One of the two boxes, box I and box II, was selected at random and a ball was drawn randomly out of this box. The ball was found to be red. If the probability that this red ball was drawn from box II is $\frac{1}{3}$, then the correct option(s) with the possible values of n_1, n_2, n_3 and n_4 is (are).

(a) $n_1 = 3, n_2 = 3, n_3 = 5, n_4 = 15$

(b) $n_1 = 3, n_2 = 6, n_3 = 10, n_4 = 50$

(c) $n_1 = 8, n_2 = 6, n_3 = 5, n_4 = 20$

(d) $n_1 = 6, n_2 = 12, n_3 = 5, n_4 = 20$

53. A ball is drawn at random from box I and transferred to box II. If the probability of drawing a red ball from box I, after this transfer, is $\frac{1}{3}$, then the correct option(s) with the possible values of n_1 and n_2 is(are)

(a) $n_1 = 4$ and $n_2 = 6$ (b) $n_1 = 2$ and $n_2 = 3$

(c) $n_1 = 10$ and $n_2 = 20$ (d) $n_1 = 3$ and $n_2 = 6$

Passage – 6

Football teams T_1 and T_2 have to play two games against each other. It is assumed that the outcomes of the two games are independent. The probabilities of T_1 winning, drawing and losing a game against T_2 are $\frac{1}{2}, \frac{1}{6}$ and $\frac{1}{3}$, respectively. Each team gets 3 points for a win, 1point for a draw and 0 point for a loss in a game. Let X and Y denote the total points scored by teams T_1 and T_2, respectively, after two games. **(2016)**

54. $P(X > Y)$ is

(a) $\frac{1}{4}$ (b) $\frac{5}{12}$

(c) $\frac{1}{2}$ (d) $\frac{7}{12}$

55. $P(X = Y)$ is

(a) $\frac{11}{36}$ (b) $\frac{1}{3}$

(c) $\frac{13}{36}$ (d) $\frac{1}{2}$

Passage – 7

There are five students S_1, S_2, S_3, S_4 and S_5 in a music class and for them there are five seats R_1, R_2, R_3, R_4 and R_5 arranged in a row, where initially the seat R_i is allotted to the student S_i, i = 1, 2, 3, 4, 5. But, on the examination day, the five students are randomly allotted the five seats. **(2018)**

56. The probability that, on the examination day, the student S_1 gets the previously allotted seat R_1, and **NONE** of the remaining students gets the seat previously allotted to him/her is

(a) $\frac{3}{40}$ (b) $\frac{1}{8}$

(c) $\frac{7}{40}$ (d) $\frac{1}{5}$

57. For i = 1, 2, 3, 4, let T_i denote the event that the students S_i and S_{i+1} do **NOT** sit adjacent to each other on the day of the examination. Then, the probability of the event $T_1 \cap T_2 \cap T_3 \cap T_4$ is

(a) $\frac{1}{15}$ (b) $\frac{1}{10}$

(c) $\frac{7}{60}$ (d) $\frac{1}{5}$

Assertion Reason

(A) If both assertion and reason are correct and reason is the correct explanation of assertion.

(B) If both assertion and reason are true but reason is not the correct explanation of assertion.

(C) If assertion is true but reason is false.

(D) If assertion is false but reason is true.

58. Let $H_1, H_2, \ldots, H_n$ be mutually exclusive and exhaustive events with $P(H_i) > 0$, i = 1, 2,...., n. Let E be any other event with $0 < P(E) < 1$. **(2007)**

Assertion : $P(H_i/E) > P(E/H_i) \cdot P(H_i)$ for $i = 1, 2, \ldots, n$

Reason : $\sum_{i=1}^{n} P(H_i) = 1$

59. Consider the system of equations

$ax + by = 0, cx + dy = 0,$

where $a, b, c, d \in \{0, 1\}$.

Assertion : The probability that the system of equations has a unique solution, is 3/8.

Reason : The probability that the system of equations has a solution, is 1. **(2008)**

SUBJECTIVE QUESTIONS

60. If p & q are chosen randomly from the set {1, 2, 3, 4, 5, 6, 7, 8, 9, 10} with replacement. Determine the probability that the roots of the equation $x^2 + px + q = 0$ are real. **(1997)**

61. There is 30% chance that it rains on any particular day. What is the probability that there is at least one rainy day within a period of 7 – days ? Given that there is at least one rainy day, what is the probability that there are at least two rainy days ? **(1997)**

62. 3 players A, B & C toss a coin cyclically in that order (that is A, B, C, A, B, C, A, B, ...) till a head shows. Let p be the probability that the coin shows a head. Let α, β & γ be respectively the probabilities that A, B and C gets the first head . Prove that $\beta = (1-p)\alpha$. Determine α, β & γ (in terms of p). **(1998)**

63. Eight players $P_1, P_2, P_3, \ldots\ldots P_8$ play a knock-out tournament. It is known that whenever the players P_i and P_j play, the player P_i will win if $i < j$. Assuming that the players are paired at random in each round, what is the probability that the player P_4 reaches the final. **(1999)**

64. Four cards are drawn from a pack of 52 playing cards. Find the probability (correct upto two places of decimals) of drawing exactly one pair. **(1999)**

65. A coin has probability 'p' of showing head when tossed. It is tossed 'n' times. Let p_n denote the probability that no two (or more) consecutive heads occur. Prove that, $p_1 = 1$, $p_2 = 1 - p^2$ & $p_n = (1-p)\,p_{n-1} + p\,(1-p)\,p_{n-2}$, for all $n \geq 3$. **(2000)**

66. A and B are two independent events. The probability that both occur simultaneously is 1/6 and the probability that neither occurs is 1/3. Find the probabilities of occurance of the events A and B separately. **(2000)**

67. Two cards are drawn at random from a pack of playing cards. Find the probability that one card is a heart and the other is an ace. **(2001)**

68. (a) An urn contains 'm' white and 'n' black balls. A ball is drawn at random and is put back into the urn along with K additional balls of the same colour as that of the ball drawn. A ball is again drawn at random. What is the probability that the ball drawn now is white.

(b) An unbiased die, with faces numbered 1, 2, 3, 4, 5, 6 is thrown n times and the list of n numbers showing up is noted. What is the probability that among the numbers 1, 2, 3, 4, 5, 6, only three numbers appear in the list. **(2001)**

69. A box contains N coins, m of which are fair and the rest are biased. The probability of getting a head when a fair coin is tossed is 1/2, while it is 2/3 when a biased coin is tossed. A coin is drawn from the box at random and is tossed twice. The first time it shows head and the second time it shows tail. What is the probability that the coin drawn is fair? **(2002)**

70. (a) A person takes three tests in succession. The probability of his passing the first test is p, that of his passing each successive test is p or p/2 according as he passes or fails in the preceding one. He gets selected provided he passes at least two tests. Determine the probability that the person is selected.

(b) In a combat, A targets B, and both B and C target A. The probabilities of A, B, C hitting their targets are 2/3 , 1/2 and 1/3 respectively. They shoot simultaneously and A is hit. Find the probability that B hits his target whereas C does not. **(2003)**

71. (a) If A and B are independent events, prove that $P(A \cup B) \cdot P(A' \cap B') \le P(C)$, where C is an event defined that exactly one of A or B occurs.

(b) A bag contains 12 red balls and 6 white balls. Six balls are drawn one by one without replacement of which atleast 4 balls are white. Find the probability that in the next two draws exactly one white ball is drawn (leave the answer in terms of ${}^nC_r$). **(2004)**

72. A person goes to office either by car, scooter, bus or train probability of which being $\frac{1}{7}, \frac{3}{7}, \frac{2}{7}$ and $\frac{1}{7}$ respectively. Probability that he reaches office late, if he takes car, scooter, bus or train is $\frac{2}{9}, \frac{1}{9}, \frac{4}{9}$ and $\frac{1}{9}$ respectively. Given that he reached office in time, then what is the probability that he travelled by a car. **(2005)**

73. Six boys and six girls sit in a row at random. Find the probability that

(a) the six girls sit together

(b) the boys and girls sit alternatively **(1978)**

74. A box contains 2 black, 4 white and 3 red balls. One ball is drawn at random from the box and kept aside. From the remaining balls in the box, another ball is drawn at random and kept beside the first. This process is repeated till all the balls are drawn from the box. Find the probability that the balls drawn are in the sequence of 2 black, 4 white and 3 red. **(1979)**

75. An anti-aircraft gun can take a maximum of four shots at an enemy plane moving away from it. The probabilities of hitting the plane at the first, second, third and fourth shot are 0.4, 0.3, 0.2, and 0.1 respectively. What is the probability that the gun hits the plane ? **(1981)**

76. A and B are two candidates seeking admission in IIT. The probability that A is selected is 0.5 and the probability that both A and B selected is atmost 0.3. Is it possible that the probability of B getting selected is 0.9 ? **(1982)**

77. Cards are drawn one by one at random from a well shuffled full pack of 52 playing cards until 2 aces are obtained for the first time. If N is the number of cards required to be drawn, then show that

$$P_r\{N = n\} = \frac{(n-1)(52-n)(51-n)}{50 \times 49 \times 17 \times 13}$$

where $2 < n \le 50$. **(1983)**

78. A, B, C are events such that

$P_r(A) = 0.3, P_r(B) = 0.4, P_r(C) = 0.8,$

$P_r(AB) = 0.08, P_r(AC) = 0.28, P_r(ABC) = 0.09.$

If $P_r(A \cup B \cup C) \ge 0.75$, then show that $P_r(BC)$ lies in the interval $0.23 \le x \le 0.48$. **(1983)**

79. A and B are two independent events. The probability that both A and B occur is $\frac{1}{6}$ and the probability that neither of them occurs is $\frac{1}{3}$. Find the probability of the occurrence of A. **(1984)**

80. In a certain city only two newspapers A and B are published, it is known that 25% of the city population reads A and 20% reads B, while 8% reads both A and B. It is also known that 30% of those who read A but not B look into advertisements and 40% of those who read B but not A look into advertisements while 50% of those who read both A and B look into advertisements. What is the percentage of the population reads an advertisement ? **(1984)**

81. In a multiple-choice question there are four alternative answers, of which one or more are correct. A candidate will get marks in the question only if he ticks the correct answers. The candidate decides to tick the answers at random. If he is allowed upto three chances to answer the questions, find the probability that he will get marks in the question. **(1985)**

82. A lot contains 20 articles. The probability that the lot contains exactly 2 defective articles is 0.4 and the probability that the lot contains exactly 3 defective articles is 0.6. Articles are drawn from the lot at random one by one without replacement and are tested till all defective articles are found. What is the probability that the testing procedure ends at the twelth testing ?

(1986)

83. A man takes a step forward with probability 0.4 and backwards with probability 0.6. Find the probability that at the end of eleven steps he is one step away from the starting point. **(1987)**

84. An urn contains 2 white and 2 blacks balls. A ball is drawn at random. If it is white it is not replaced into the urn. Otherwise it is replaced along with another ball of the same colour. The process is repeated. Find the probability that the third ball drawn is black. **(1987)**

85. A box contains 2 fifty paise coins, 5 twenty five paise coins and a certain fixed number n (≥ 2) of ten and five paise coins. Five coins are taken out of the box at random. Find the probability that the total value of these 5 coins is less than one rupee and fifty paise. **(1988)**

86. Suppose the probability for A to win a game against B is 0.4. If A has an option of playing either a "best of 3 games" or a "best of 5 games" match against B, which option should choose so that the probability of his winning the match is higher ? (No game ends in a draw). **(1989)**

87. A is set containing n elements. A subset P of A is chosen at random. The set A is reconstructed by replacing the elements of P. A subset Q of A is again chosen at random. Find the probability that P and Q have no common elements. **(1991)**

88. In a test an examinee either guesses or copies or knows the answer to a multiple choice question with four choices. The probability that he make a guess is $\frac{1}{3}$ and the probability that he copies the answer is $\frac{1}{6}$. The probability that his answer is correct given that he copied it, is $\frac{1}{8}$. Find the probability that he knew the answer to the question given that he correctly answered it. **(1991)**

89. A lot contains 50 defective and 50 non-defective bulbs. Two bulbs are drawn at random, one at a time, with replacement. The events A, B, C are defined as :
A = (the first bulb is defective)
B = (the second bulb is non-defective)
C = (the two bulbs are both defective or both non-defective).
Determine whether
(a) A, B, C are pairwise independent,
(b) A, B, C are independent. **(1992)**

90. Numbers are selected at random, one at a time, from the two-digit numbers 00, 01, 02, ..., 99 with replacement. An event E occurs if and only if the product of the two digits of a selected number is 18. If four numbers are selected, find probability that the event E occurs at least 3 times. **(1993)**

91. An unbiased coin is tossed. If the result is a head, a pair of unbiased dice is rolled and the number obtained by adding the numbers on the two faces is noted. If the result is a tail, a card from a well shuffled pack of eleven cards numbered 2, 3, 4, ..., 12 is picked and the number on the card is noted. What is the probability that the noted number is either 7 or 8 ? **(1994)**

92. In how many ways three girls and nine boys can be seated in two vans, each having numbered seats, 3 in the front and 4 at the back ? How many seating arrangements are possible if 3 girls should sit together in a back row on adjacent seats ? Now, if all the seating arrangements are equally likely, what is the probability of 3 girls sitting together in a back row on adjacent seats ? **(1996)**

93. Sixteen players $S_1, S_2, \ldots, S_{16}$ play in a tournament. They are divided into eight pairs at random. From each pair a winner is decided on the basis of a game played between the two players of the pair. Assume that all the players are of equal strength.
(a) Find the probability that the players S_1 is among the eight winners.
(b) Find the probability that exactly one of the two players S_1 and S_2 is among the eight winners. **(1997)**

94. A person goes to office either by car, scooter, bus or train probability of which being $\frac{1}{7}, \frac{3}{7}, \frac{2}{7}$ and $\frac{1}{7}$ respectively. Probability that he reaches offices late, if he takes car, scooter, bus or train is $\frac{2}{9}, \frac{1}{9}, \frac{4}{9}$ and $\frac{1}{9}$ respectively. Given that he reached office in time, then what is the probability that he travelled by a car ? **(2005)**

95. The minimum number of times a fair coin needs to be tossed, so that the probability of getting at least two heads is at least 0.96, is. **(2015)**

Fill in the Blanks

96. For a biased die the probabilities for the different faces to turn up are given below

Face	1	2	3	4	5	6
Probability	0.1	0.32	0.21	0.15	0.05	0.17

This die is tossed and you are told that either face 1 or face 2 has turned up. Then, the probability that it is face 1, is.... **(1981)**

97. A determinant is chosen at random from the set of all determinants of order 2 with elements 0 or 1 only. The probability that the value of the determinant chosen is positive, is.... **(1982)**

98. $P(A \cup B) = P(A \cap B)$ if and only if the relation between P (A) and P (B) is.... **(1985)**

99. A box contains 100 tickets numbered 1, 2, ..., 100. Two tickets are chosen at random. It is given that the maximum number on the two chosen tickets is not more than 10. The minimum number on them is 5 with probability.... **(1985)**

100. If $\frac{1+3p}{3}, \frac{1-p}{4}$ and $\frac{1-2p}{2}$ are the probabilities of three mutually exclusive events, then the set of all values of p is... **(1986)**

101. Urn A contains 6 red and 4 black balls and urn B contains 4 red and 6 black balls. One ball is drawn at random from urn A and placed in urn B. Then, one ball is drawn at random from urn B and placed in urn A. If one ball is drawn at random from urn A, the probability that it is found to be red, is..... **(1988)**

102. A pair of fair dice is rolled together till a sum of either 5 or 7 is obtained. Then, the probability that 5 comes before 7, is... **(1989)**

103. Let A and B be two events such that P(A) = 0.3 and $P(A \cup B) = 0.8$. If A and B are independent events, then P (B) = ... **(1990)**

104. If the mean and the variance of a binomial variate X are 2 and 1 respectively, then the probability that X takes a value greater than one is equal to... **(1991)**

105. Three faces of a fair die are yellow, two faces red and one blue. The die is tossed three times. The probability that the colours, yellow, red and blue, appear in the first, second and the third tosses respectively, is.... **(1992)**

106. If two events A and B are such that $P(A^c) = 0.3$, $P(B) = 0.4$ and $P(A \cap B^c) = 0.5$ then $P[B / (A \cup B^c)] =$.... **(1994)**

107. Three numbers are chosen at random without replacement from {1, 2,..., 10}. The probability that the minimum of the chosen number is 3, or their maximum is 7, is.... **(1997)**

True/False

108. If the letters of the word "ASSASSIN" are written down at random in a row, the probability that no two S's occur together is 1/35. **(1983)**

109. If the probability for A to fail in an examination is 0.2 and that of B is 0.3, then the probability that either A or B fails is 0.5. **(1989)**

ANSWER KEY

EXERCISE - 1 : BASIC OBJECTIVE QUESTIONS

1. (c)	2. (a)	3. (d)	4. (c)	5. (b)	6. (c)	7. (b)	8. (c)
9. (a)	10. (c)	11. (a)	12. (b)	13. (d)	14. (c)	15. (a)	16. (c)
17. (c)	18. (a)	19. (d)	20. (c)	21. (c)	22. (c)	23. (b)	24. (d)
25. (c)	26. (b)	27. (a)	28. (c)	29. (a)	30. (d)	31. (b)	32. (a)
33. (b)	34. (c)	35. (a)	36. (b)	37. (b)	38. (d)	39. (d)	40. (c)
41. (b)	42. (b)	43. (b)	44. (b)	45. (b)	46. (a)	47. (a)	48. (c)
49. (a)	50. (a)	51. (d)	52. (a)	53. (b)	54. (b)	55. (a)	56. (c)
57. (a)	58. (a)	59. (c)	60. (a)				

EXERCISE - 2 : PREVIOUS YEAR JEE MAINS QUESTIONS

1. (a)	2. (a)	3. (d)	4. (d)	5. (c)	6. (d)	7. (a)	8. (b)
9. (d)	10. (d)	11. (a)	12. (d)	13. (b)	14. (c)	15. (d)	16. (d)
17. (b)	18. (a)	19. (b)	20. (d)	21. (d)	22. (b)	23. (c)	24. (b)
25. (c)	26. (b)	27. (a)	28. (a)	29. (b)	30. (c)	31. (a)	32. (a)
33. (b)	34. (c)	35. (a)	36. (c)	37. (b)	38. (b)	39. (c)	

EXERCISE - 3 : ADVANCED OBJECTIVE QUESTIONS

1. (a)	2. (c)	3. (b)	4. (c)	5. (d)	6. (d)	7. (d)	8. (c)
9. (b)	10. (c)	11. (b)	12. (b)	13. (c)	14. (c)	15. (a)	16. (a)
17. (b)	18. (a)	19. (d)	20. (a)	21. (d)	22. (b)	23. (d)	24. (a)
25. (b)	26. (b)	27. (c)	28. (d)	29. (c)	30. (c)	31. (a)	32. (a)
33. (b)	34. (a)	35. (b)	36. (a)	37. (a)	38. (a)	39. (c)	40. (d)
41. (d)	42. (a)	43. (d)	44. (b)	45. (c)	46. (d)	47. (a)	48. (d)
49. (d)	50. (a)	51. (c)	52. (d)	53. (c)	54. (c)	55. (c)	56. (d)
57. (b)	58. (c)	59. (d)	60. (d)	61. (c)	62. (b)	63. (c)	64. (c)
65. (d)	66. (a)	67. (c)	68. (b)	69. (d)	70. (b)	71. (b)	72. (d)
73. (b)	74. (c)	75. (c)	76. (a,b,c,d)	77. (a,b,c,d)	78. (a,b)	79. (a,b,c)	
80. (a,c,d)	81. (c,d)	82. (b,c,d)	83. (a, b, c)	84. (b)	85. (a)	86. (c)	87. (c)
88. (b)	89. (d)	90. (d)	91. (b)	92. (c)	93. (d)	94. a–q; b–p; c–s; d–r	
95. a–q; b–r; c–r; d–q		96. (c)	97. (b)	98. (30)	99. 5	100. 8	

EXERCISE - 4 : PREVIOUS YEAR JEE ADVANCED QUESTIONS

1. (a)	2. (b)	3. (b)	4. (d)	5. (b)	6. (c)	7. (c)	8. (d)
9. (b)	10. (a)	11. (c)	12. (b)	13. (c)	14. (a)	15. (a)	16. (a)
17. (a)	18. (b)	19. (d)	20. (a)	21. (a)	22. (d)	23. (d)	24. (b)
25. (c)	26. (c)	27. (d)	28. (c)	29. (c)	30. (a)	31. (c)	32. (c)
33. (a,c)	34. (a,b,c)	35. (b,c,d)	36. (a,c)	37. (a,d)	38. (a,d)	39. (b,c)	40. (a,d)
41. (a,b)	42. (b)	43. (a)	44. (b)	45. (a)	46. (b)	47. (d)	48. (b)
49. (d)	50. (b)	51. (c)	52. (a, b)	53. (c,d)	54. (b)	55. (c)	56. (a)

57. (c) 58. (d) 59. (b) 60. 31/50 61. $1-(0.7)^7$, $\dfrac{\left[1-(0.7)^7-{}^7C_1(0.3)(0.7)^6\right]}{1-(0.7)^7}$

62. $\alpha=\dfrac{p}{1-(1-p)^3},\beta=\dfrac{(1-p)\,p}{1-(1-p)^3},\gamma=\dfrac{(1-p)^2p}{1-(1-p)^3}$ 63. 4/35 64. 0.3 66. $\dfrac{1}{2}$ & $\dfrac{1}{3}$ or $\dfrac{1}{3}$ & $\dfrac{1}{2}$

67. $\dfrac{1}{26}$ 68. (a) $\dfrac{m}{m+n}$ (b) $\dfrac{{}^6C_3\,(3^n-3.2^n+3)}{6^n}$ 69. $\dfrac{9m}{m+8N}$ 70. (a) $p^2(2-p)$ (b) 1/2

71. (b) $\dfrac{{}^{12}C_2\,{}^6C_4\,{}^{10}C_1\,{}^2C_1+{}^{12}C_1\,{}^6C_5\,{}^{11}C_1\,{}^1C_1}{{}^{12}C_2\left({}^{12}C_2\,{}^6C_4+{}^{12}C_1\,{}^6C_5+{}^{12}C_0\,{}^6C_6\right)}$ 72. $\dfrac{1}{7}$ 73. (a) $\dfrac{1}{132}$ (b) $\dfrac{1}{462}$ 74. $\dfrac{1}{1260}$

75. 0.6976 76. No 79. $\dfrac{1}{3}$ or $\dfrac{1}{2}$ 80. 13.9% 81. $\dfrac{1}{5}$ 82. $\dfrac{99}{1900}$ 83. ${}^{11}C_6(0.24)^5$

84. $\dfrac{23}{30}$ 85. $1-\dfrac{10\,(n+2)}{{}^{n+7}C_5}$ 86. best of 3 games 87. $\left(\dfrac{3}{4}\right)^n$ 88. $\dfrac{24}{29}$

89. (a) A, B and C are pairwise independent (b) A, B and C are dependent 90. $\dfrac{97}{25^4}$ 91. $\dfrac{193}{792}$

92. $\dfrac{14!}{2!}, 2\times2\times3!\times{}^{11}P_9, \dfrac{1}{91}$ 93. (a) $\dfrac{1}{2}$ (b) $\dfrac{8}{15}$ 94. $\dfrac{1}{7}$ 95. (8) 96. $\dfrac{5}{21}$ 97. $\dfrac{3}{16}$

98. $P(A\cap B)$ 99. $\dfrac{1}{9}$ 100. $\dfrac{1}{3}\le p\le\dfrac{1}{2}$ 101. $\dfrac{32}{55}$ 102. $\dfrac{2}{5}$ 103. $\dfrac{5}{7}$

104. $\dfrac{11}{16}$ 105. $\dfrac{1}{36}$ 106. $\dfrac{1}{4}$ 107. $\dfrac{11}{40}$ 108. False 109. False

Dream on !!