1. **(d)** R = {(x, y) : x, $y \in$ N and $x^2 - 4xy + 3y^2 = 0$}

Now, $x^2 - 4xy + 3y^2 = 0 \Rightarrow (x - y)(x - 3y) = 0$

$\therefore$ $x = y$ or $x = 3y$

$\therefore$ R = {(1, 1), (3, 1), (2, 2), (6, 2), (3, 3), (9, 3),......}

Since (1, 1), (2, 2), (3, 3),...... are present in the relation, therefore R is reflexive.

Since (3, 1) is an element of R but (1, 3) is not the element of R, therefore R is not symmetric

Here (3, 1) $\in$ R and (1, 1) $\in$ R

$\Rightarrow$ (3, 1) $\in$ R

(6, 2) $\in$ R and (2, 2) $\in$ R $\Rightarrow$ (6, 2) $\in$ R

For all such (a, b) $\in$ R and (b, c) $\in$ R

$\Rightarrow$ (a, c) $\in$ R

Hence R is transitive.

2. **(b)** Obviously, the relation is not reflexive and transitive but it is symmetric, because $x^2 + y^2 = 1 \Rightarrow y^2 + x^2 = 1$

3. **(c)** Let f (x) ≠ 2 be true and f (y) = 2, f (z) ≠ 1 are false

$\Rightarrow$ f (x) ≠ 2, f (y) ≠ 2, f (z) = 1

$\Rightarrow$ f (x) = 3, f (y) = 3, f (z) = 1 but then function is many one, similarly two other cases.

4. **(a)** f (4) = g (4) $\Rightarrow$ 8 + a = 8 $\Rightarrow$ a = 0

f (–1) = – 2 for a = 0

f (–1) > f (4)

b + 3 > 8 $\Rightarrow$ b > 5

5. **(b)** We have to test the equivalencity of relation R on S.

(1) **Reflexivity :**

In a plane any line be parallel to itself not perpendicular. Hence $a \not R b$,

R is not reflexive.

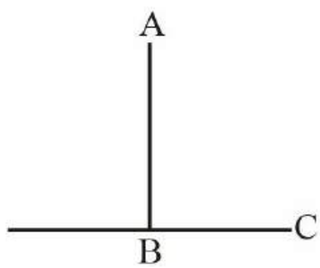
(2) **Symmetry :**

In a plane if a line AB is perpendicular to the other line BC, then BC is also perpendicular to AB, i.e.,

$aRb \Rightarrow AB \perp BC$

And $bRa \Rightarrow BC \perp AB$

Hence R is symmetric.



(3) **Transitivity :**

In a plane, let AB, BC and CA be three lines, such that

$AB \perp BC$ and $BC \perp CD$

$\Rightarrow$ $AB \parallel CD \Rightarrow a \not R b$, R is not transitive.

Hence, R is symmetric but neither reflexive nor transitive.

6. **(a)** Since R is reflexive relation on A, therefore (a,a) ∈ R for all a ∈A.

The minimum number of ordered pairs in R is n.

Hence , $m \geq n$.

7. **(d)** f (2) = $f\left(3^{1/4}\right)$ ⇒ many to one function

and $f(x) \neq -\sqrt{3} \quad \forall\ x \in R$ ⇒ into function

8. **(b)** We have, gof (x) = $g\left(\frac{3x+4}{5x-7}\right) = \frac{7\left(\frac{(3x+4)}{(5x-7)}\right)+4}{5\left(\frac{(3x+4)}{(5x-7)}\right)-3}$

$= \frac{21x+28+20x-28}{15x+20-15x+21} = \frac{41x}{41} = x$

Similarly, fog (x) = $f\left(\frac{7x+4}{5x-3}\right)$

$$= \frac{3\left(\frac{(7x+4)}{(5x-3)}\right)+4}{5\left(\frac{(7x+4)}{(5x-3)}\right)-7}$$

$= \frac{21x+12+20x-12}{35x+20-35x+21} = \frac{41x}{41} = x$

Thus, gof (x) = x, $\forall$ x ∈ B and fog (x) = x, $\forall$ x ∈ A, which implies that gof = I_B and fog = I_A.

9. **(d)** f (x) = $[x]^2 + [x+1] - 3 = \{[x]+2\}\{[x]-1\}$

So, x = 1, 1.1, 1.2, ... ⇒ f (x) = 0

∴ f (x) is many one.

only integral values will be attained.

∴ f (x) is into.

10. **(b)** $f(x) = |x-1| = \begin{cases} 1-x, & 0 < x < 1 \\ x-1, & x \geq 1 \end{cases}$

g (x) = e^x, $x \geq -1$

$$(fog)(x) = \begin{cases} 1-g(x), & 0 < g(x) < 1 \text{ i.e. } -1 \leq x < 0 \\ g(x)-1, & g(x) \geq 1 \text{ i.e. } 0 \leq x \end{cases}$$

$$= \begin{cases} 1-e^x, & -1 \le x < 0 \\ e^x - 1, & x \ge 0 \end{cases}$$

$\therefore$ domain = [–1, ∞)

fog is decreasing in [–1, 0) and increasing in [0, ∞)

$$fog(-1) = 1 - \frac{1}{e} \quad \text{and} \quad fog(0) = 0$$

As $x \to \infty$, fog (x) $\to \infty$,

$\therefore$ range = [0, ∞)

$$\therefore \quad x = \frac{1}{2}\log_e\left(\frac{y}{2-y}\right)$$

11. **(b)** (a) Non-reflexive because $(x_3, x_3) \notin R_1$

(b) Reflexive

(c) Non-Reflexive

(d) Non-reflexive because $x_4 \notin X$

12. **(c)** Here R = {(1, 3), (2, 2); (3, 2)}, S = {(2, 1); (3, 2); (2, 3)}

Then RoS = {(2, 3), (3, 2); (2, 2)}

13. **(a)** g(f(x)) = |sin x| indicates that possibly f(x) = sinx, g(x) = |x|

Assuming it correct, f(g(x)) = f(|x|) = sin |x|, which is not correct.

$f(g(x)) = \left(\sin\sqrt{x}\right)^2$ indicates that possibly

$g(x) = \sqrt{x}$ $f(x) = \sin^2 x$ or

$g(x) = \sin\sqrt{x}$, $f(x) = x^2$

Then $g(f(x)) = g(\sin^2 x) = \sqrt{\sin x} = |\sin x|$

(for the first combination), which is given.

Hence $f(x) = \sin^2 x$, $g(x) = \sqrt{x}$

[Students may try by checking the options one by one]

14. **(b)** Let $f : R \to R$ be a function defined by

$$f(x) = \frac{x-m}{x-n}$$

For any $(x, y) \in R$

Let f (x) = f (y)

$$\frac{x-m}{x-n}=\frac{y-m}{y-n} \qquad x = y$$

f is one – one

Let $\alpha \in R$ such that f (x) = α

$$\alpha=\frac{x-m}{x-n}$$

$(x – n)\alpha = x – m$

$x\alpha - n\alpha = x - m$

$x\alpha - x = n\alpha - m$

$x(\alpha - 1) = n\alpha - m$

$x = \frac{n\alpha - m}{\alpha - 1}$. for $\alpha = 1$, $x \notin R$

So, f is not onto.

15. (a) Given $f(x) = \frac{x}{x-1}$

$$(f \circ f)(x) = f\{f(x)\} = f\left(\frac{x}{x-1}\right)$$

$$= \frac{\frac{x}{x-1}}{\frac{x}{x-1}-1} = \frac{\frac{x}{x-1}}{\frac{x-x+1}{x-1}} = \frac{\frac{x}{x-1}}{\frac{1}{x-1}} = x.$$

$$\Rightarrow (f \circ f \circ f)(x) = f(f \circ f)(x) = f(x) = \frac{x}{x-1}$$

$$\underbrace{(f \circ f \circ f \ldots\ldots \circ f)}_{19 \text{ times}}(x) = f(f \circ f)(x) = f(x) = \frac{x}{x-1}$$

16. (b) By definition only $f(x) = x^2 + 4x - 5$ with domain $[0, \infty)$ is one to one.

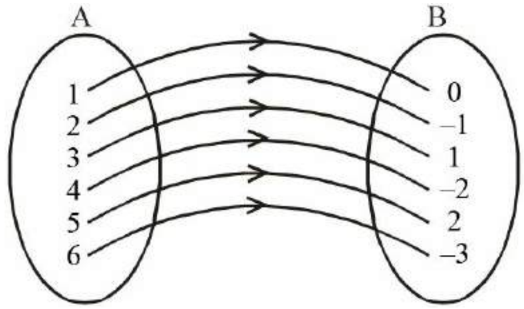
17. (b) If $y = \frac{2}{3}\frac{10^x - 10^{-x}}{10^x + 10^{-x}}$, $10^{2x} = \frac{3y+2}{2-3y}$

or $x = \frac{1}{2}\log_{10}\frac{2+3y}{2-3y}$ $\therefore f^{-1}(x) = \frac{1}{2}\log_{10}\frac{2+3x}{2-3x}$.

$f : N \to I$

18. (d) $f(1) = 0, f(2) = -1, f(3) = 1, f(4) = -2,$

$f(5) = 2, \text{and } f(6) = -3$ so on.



In this type of function every element of set A has unique image in set B and there is no element left in set B.

Hence f is one-one and onto function.

19. (a) We have

$$f(x) = \sin^2 x + \sin^2(x + \pi/3) + \cos x \cos(x + \pi/3)$$

$$= \frac{1 - \cos 2x}{2} + \frac{1 - \cos(2x + 2\pi/3)}{2}$$

$$+ \frac{1}{2}\{2\cos x \cos(x + \pi/3)\}$$

$$= \frac{1}{2}\left[\frac{5}{2} - \left\{\cos 2x + \cos\left(2x + \frac{2\pi}{3}\right)\right\} + \cos\left(2x + \frac{\pi}{3}\right)\right]$$

$$= \frac{1}{2}\left[\frac{5}{2} - 2\cos\left(2x + \frac{\pi}{3}\right)\cos\frac{\pi}{3} + \cos\left(2x + \frac{\pi}{3}\right)\right]$$

$= \dfrac{5}{4}$ for all x.

gof (x) = g (f(x)) = g $\left(\dfrac{5}{4}\right)$ = 1 [$\because$ g$\left(\dfrac{5}{4}\right)$ = 1 (given)] Hence, gof(x) = 1, for all x.

20. (d) We have, If x < 0 |x| = –x

$\therefore f(x) = \dfrac{e^{-x} - e^{-x}}{e^{x} + e^{-x}} = 0$ $\qquad \therefore f(x) = 0 \ \forall \ x < 0$

$\therefore$ f (x) is not one-one

Next if x ≥ 0, |x| = x $\therefore f(x)=\dfrac{e^{x}-e^{-x}}{e^{x}+e^{-x}}$

Let $y=\dfrac{e^{x}-e^{-x}}{e^{x}+e^{-x}} \Rightarrow y=\dfrac{e^{2x}-1}{e^{2x}+1}$ $\therefore e^{2x}=\dfrac{1+y}{1-y}$

For $x \ge 0,\ e^{2x}\ge 1$ $\therefore \dfrac{1+y}{1-y}\ge 1 \Rightarrow \dfrac{2y}{1-y}\ge 0$

$\Rightarrow y(y-1)\le 0,\ y\ne 1 \Rightarrow 0\le y<1$

∴ Range of f(x) = [0 1) ∴ f(x) is not onto

21. **(4)** We have, R = {(1, 3); (1, 5); (2, 3); (2, 5); (3, 5); (4,5)}

R^{-1} = {(3, 1); (5, 1); (3, 2); (5, 2); (5, 3); (5,4)}

Hence RoR^{-1} = {(3, 3); (3, 5); (5, 3); (5, 5)}

22. **(2)** Here, $f(2)=\dfrac{5}{4}$

$$\Rightarrow (fof)(2)=f(f(2))=f\left(\frac{5}{4}\right)=\frac{2\times\frac{5}{4}+1}{3\times\frac{5}{4}-2}=2.$$

23. **(1)** ∵ $fog\left(-\dfrac{1}{4}\right)=f\left[g\left(-\dfrac{1}{4}\right)\right]=f(-1)=1$

and $gof\left(-\dfrac{1}{4}\right)=g\left[f\left(-\dfrac{1}{4}\right)\right]=g\left(\dfrac{1}{4}\right)=[1/4]=0$

∴ required value = 1 + 0 = 1

24. **(7)** *R* is reflexive if it contains $(1,1),(2,2),(3,3)$

$\because (1,2)\in R,(2,3)\in R$

$\because$ *R* is symmetric if $(2,1),(3,2)\in R.$

Now,

$R=\{(1,1),(2,2),(3,3),(2,1),(3,2),(2,3),(1,2)\}$

R will be transitive if $(3,1),\ (1,3)\in R.$

Thus, *R* becomes an equivalence relation by adding (1, 1) (2, 2) (3, 3) (2, 1) (3, 2) (1, 3) (3,1).

Hence, the total number of ordered pairs is 7.

25. **(14)** If set A has m elements and set B has n elements then number of onto functions from A to B is

$$\sum_{r=1}^{n}(-1)^{n-r}\ {}^{n}C_{r}\ r^{m} \text{ where } 1 \le n \le m$$

Here $E = \{1,2,3,4\}$, $F = \{1,2\}$; $m = 4, n = 2$

no. of onto functions from E to F

$$= \sum_{r=1}^{2}(-1)^{2-r}\ {}^{2}C_{r}(r)^{4} = (-1)\ {}^{2}C_{1} + {}^{2}C_{2}(2)^{4}$$

= – 2 +16 = 14