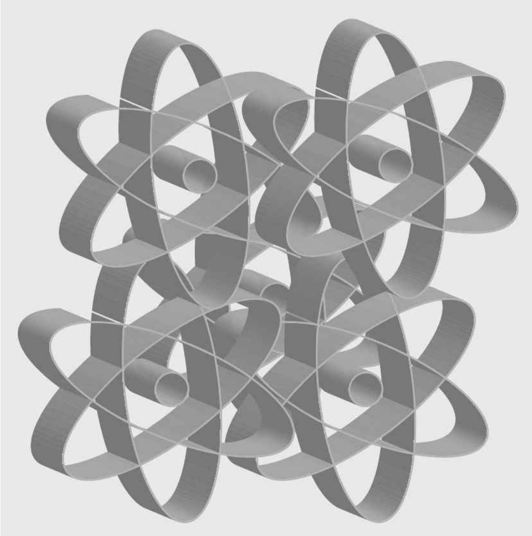


CHAPTER 27

Logarithms

INTRODUCTION

In earlier classes, we have learnt about indices. One of the results we have learnt is that, if $2^x = 2^3$, then $x = 3$ and if $4^x = 4^y$, then $x = y$ i.e., if two powers of the same base are equal and the base is not equal to -1, 0 or 1, then the indices are equal. But when $3^x = 5^2$, just by using the knowledge of indices, we cannot find the numerical value of x. The necessity of the concept of logarithms arises here. Logarithms are useful in long calculations involving multiplication and division.

Definition

The logarithm of any positive number to a given base (a positive number not equal to 1) is the index of the power of the base which is equal to that number. If N and a ($\neq 1$) are any two positive real numbers and for some real x, $a^x = N$, then x is said to be the logarithm of N to the base a. It is written as $\log_a N = x$, i.e., if $a^x = N$, then $x = \log_a N$.

☛ *Examples*

(i) $3^4 = 81 \Rightarrow 4 = \log_3 81$

(ii) $7^3 = 343 \Rightarrow 3 = \log_7 343$

If in a particular relation, all the log expressions are to the same base, we normally do not specify the base.

From the definition of logs, we get the following results

When $a > 0$, $b > 0$ and $b \neq 1$,

1. $\log_a a^n = n$ e.g., $\log_6 6^3$
2. $a^{\log_a b} = b$ e.g., $9^{\log_9 5} = 5$

System of logarithms

Though we can talk of the logarithm of a number to any positive base as not equal to 1, there are two systems of logarithms viz., natural logarithms and common logarithms, which are used most often.

(i) **Natural logarithms:** These were discovered by Napier. They are calculated to the base 'e' which is approximately equal to 2. 7828. These are used in higher mathematics.

(ii) **Common logarithms:** Logarithms to the base 10 are known as common logarithms. This system was introduced by Briggs, a contemporary of Napier. In the rest of this chapter, we shall use the short form 'log' instead of 'logarithm'.

Properties

(i) Logs are defined only for positive real numbers.

(ii) Logs are defined only for positive bases (other then 1).

(iii) In $\mathbf{log_b}$ a neither a is negative nor b is negative but the value of $\mathbf{log_b}$ a can be negative.

☛ *Example* As $10^{-2} = 0.01, \log_{10}0.01 = -2$

(iv) Logs of different numbers to the same base are different i.e., if $a \neq b$, then $\log_m a \neq \log_m b$. In other words, if $\log_m a = \log_m b$, then $a = b$.

☛ *Examples* $\log_{10}2 \neq \log_{10}3$

$\log_{10}2 = \log_{10}y \Rightarrow y = 2$

(v) Logs of the same number to different bases have different values, i.e., if $m \neq n$, then $\log_m a \neq \log_n a$. In other words, if $\log_m a = \log_n a$, then $m = n$.

☛ *Examples* $\log_2 16 \neq \log_4 16$

$= \log_n 16 \Rightarrow n = 2$

(vi) Log of 1 to any base is 0.

☛ *Example* $\log_2 1 = 0$ ($\because 2^0 = 1$)

(vii) Log of a number to the same base is 1.

☛ *Example* $\log_4 4 = 1$.

(viii) Log of 0 is not defined.

Laws

1. $\log_m (ab) = \log_m a + \log_m b$

☛ *Example* $\log 56 = \log(7 \times 8) = \log 7 + \log 8$

2. $\log_m \left(\frac{a}{b}\right) = \log_m a - \log_m b$

☛ *Example* $\log\left(\frac{81}{23}\right) = \log 81 - \log 23$

3. $\log a^m = m \log a$

☛ *Example* $\log 216 = \log 6^3 = 3\log 6$

4. $\log_b a \quad \log_c b = \log_c a$; (Chain Rule)

☛ *Example* $\log_2 3 \times \log_8 2 \times \log_5 8 = \log_8 3 \times \log_5 8 = \log_5 3$

5. $\log_b a = \frac{\log_c a}{\log_c b}$ (Change of base Rule);

☛ *Example* $\log_9 25 = \frac{\log_4 25}{\log_4 9}$

In this relation, if we replace c by a, then we get the following result:

$$\log_b a = \frac{1}{\log_a b}$$

Variation of $\log_a x$ with x

For $1 < a$, and $0 < p < q$, $\log_a p < \log_a q$.
For $0 < a < 1$ and $0 < p < q$, $\log_a p > \log_a q$.

☛ *Example* $\log_{10} 2 < \log_{10} 3$ and $\log_{0.1} 2 > \log_{0.1} 3$

Bases which are greater than 1 are called **strong bases** and bases which are less than 1 are called **weak bases**. Therefore, for strong bases, the log increases with the number and for weak bases, the log decreases with the number.

Sign of $\log_a x$ for different values of x and a

Strong bases (a > 1)

1. If $x > 1$, $\log_a x$ is positive.
   For example, $\log_2 8$, $\log_4 81$ are positive.
2. If $0 < x < 1$, then $\log_a x$ is negative.

   For example, $\log_4 0.02 = \frac{\log 0.02}{\log 4} = \frac{\log 2 - \log 100}{\log 4}$

   $\log 2 < \log 100$ and $0 < \log 4$ for strong bases

   $\therefore \frac{\log 2 - \log 100}{\log 4} < 0$

   $\Rightarrow \log_4 0.02$ is negative

Weak bases (0 < a < 1)

1. If $x > 1$, then $\log_a x$ is negative.

   For example $\log_{0.3}15$ and $\log_{0.4}16$ are negative.

   Consider $\log_{0.3}15 = \dfrac{\log 15}{\log 0.3} = \dfrac{\log 15}{\log 3 - \log 10}$

   $\log 3 < \log 10$ (for any strong base)

   $\Rightarrow \dfrac{\log 15}{\log 3 - \log 10} < 0$

2. If $0 < x < 1$, then $\log_a x$ is positive.

   For example, $\log_{0.1}0.2$, $\log_{0.4}0.3$ are positive.

Note: Logs of numbers (> 1) to strong bases and numbers (< 1) to weak bases are positive.

To find the log of a number to the base 10

Consider the following numbers:
2, 20, 200, 0.2 and 0.02.
We see that 20 = 10(2) and 200 = 100(2)
$\therefore$ log 20 = 1 + log 2 and log 200 = 2 + log 2
Similarly, log 0.2 = –1 + log 2 and log 0.02 = –2 + log 2
From the tables, we see that log 2 = 0.3010. (Using the tables is explained in greater detail in later examples).
$\therefore$ log 20 = 1.3010, log 200 = 2.3010, log 0.2 = –1 + 0.3010 and log 0.02 = –2 + 0.3010.

Note:

1. Multiplying or dividing by a power of 10 changes only the integral part of the log, not the fractional part.
2. For numbers less than 1, (for example log 0.2) it is more convenient to leave the log value as –1 + 0.3010 instead of changing it to –0.6090. We refer to the first form (in which the fraction is positive) as the standard form and the second form as the normal form. Both the forms represent the same number.

For numbers less than 1, it is convenient to express the log in the standard form. As the negative sign refers only to the integral part, it is written above the integral part, rather than in front. i.e., log 0.2 = $\bar{1}.3010$ and not –1.3010.

The convenience of the standard form will be clear when we learn how to take the anti-log, which will later be explained in detail.
Anti-log (–0.6090) = Anti-log (–1 + 0.3010) = Anti-log $\bar{1}.3010$ = 0.2.

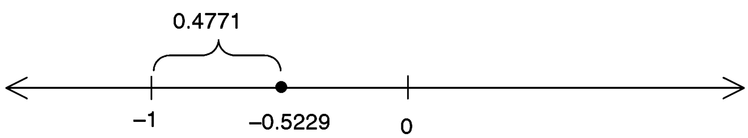
When the logs of numbers are expressed in the standard form (for numbers greater than 1, the standard form of the log is the same as the normal form), the integral part is called the characteristic and the fractional part (which is always positive) is called the mantissa.

Example

Express –0.5229 in the standard form and locate it on the number line.

Solution

–0.5229 = –1 + 1 – 0.5229 = $\bar{1}.4771$



The rule to obtain the characteristic of log x

(A) If $x > 1$ and there are n digits in x, then the characteristic is $n - 1$.

(B) f $x < 1$ and there are m zeroes between the decimal point and the first non-zero digit of x, then the characteristic is $(-m + 1)$ more commonly written as $(\overline{m+1})$.

NOTE: $-4 = \bar{4}$ but $-4.01 \neq \bar{4}.01$

To find the log of a number from the log tables

Example

Find the values of log36, log3600 and log0.0036.

Solution

In log tables we find the number 36 in the first column. In this row in the next column (under zero), we find .5563 (the decimal point is dropped in other columns). This gives 5563 as the mantissa for the log of all numbers whose significant digits are 3 and 6.

∴ Prefixing characteristic, we have log36 = 1.5563

Similarly log3600 = 3.5563 and log0.0036 = $\bar{3}.5563$

Example

Find the values of log3.74, log374000 and log0.3740.

Solution

In the log table we locate 37 in the first column. In this row, in the column under 4, we find 5729. As in the earlier example, the same line as before gives the mantissa of logarithms of all numbers which begin with 37. From this line, we select the mantissa which is located in the column number 4. This gives 5729 as the mantissa for all numbers whose significant digits are 3,7 and 4.

∴ log3.74 = 0.5729

log374000 = 5.5729 and

log0.3740 = $\bar{1}.5729$

Example

Find the values of log5.342 and log 0.05342.

Solution

As found in the above example, we can find the mantissa for the sequence of digits 534 as 7275. Since there are four significant digits in 5342, in the same row where we found 7275 under the column 2 in the mean difference column, we can find the number 2.

∴ The mantissa of the logarithm of 5342 is 7275 + 2 = 7277

Thus log5.342 = 0.7277

Similarly, log0.05342 = $\bar{2}.7277$

Anti-log

As $\log_2 8 = 3$, 8 is the **anti-logarithm** of 3 to the base 2, i.e., **Anti-log** of b to base m is $\mathbf{m^b}$.

In the above example we have seen that log5.342 = 0.7277

∴ Anti-log0.7277 = 5.342

To find the anti-log

☛ *Example*

Find the anti-log of 2.421.

Solution

Step 1: In the anti-log table we find the number.42 in the first column. In that row in the column under 1, we find 2636.

Step 2: As the characteristic is 2, we place the decimal after three digits from the left i.e., anti-log2.421 = 263.6

Note: If the characteristic is n(a non-negative integer), then we would place the decimal after (n + 1) digits from the left.

☛ *Example*

Find the anti-log of 1.4215.

Solution

We have to locate.42 in the first column and scan along the horizontal line and pick out the number in the column headed by 1. We see that the number is 2636. The mean difference for 5 in the same line is 3.

∴ The sum of these numbers is 2636 + 3 = 2639.

As the characteristic is 1, the required anti-log is 26.39.

☛ *Example*

Find the value of $\dfrac{7.211 \times 0.084}{16.52 \times 0.016}$.

Solution

log of numerator = (log of numerator) – (log of denominator)

log of numerator = log7.211 + log0.084 = 0.8580 + $\bar{2}.9243 = \bar{1}.7823$

log of denominator = log16.52 + log0.016 = 1.2180 + $\bar{2}.2041 = \bar{1}.4221$

log of the given fraction = $\bar{1}.7823 - \bar{1}.4221 = 0.3602$

Value of the fraction = Anti-log (0.3602) = 2.292 (As the characteristic is 0, the decimal is kept after one digit from the left)

Example

If $\log_{10}4 = 0.6021$ and $\log_{10}5 = 0.6990$, then find the value of $\log_{10}1600$.

Solution

$$\begin{aligned}\log_{10}1600 &= \log_{10}(64 \times 25) = \log_{10}(4^3 \times 5^2)\\ &= \log_{10} 4^3 + \log_{10} 5^2 \rightarrow\\ &= 3\log_{10} 4 + 2\log_{10} 5\\ &= 3(0.6021) + 2(0.6990)\\ &= 1.8063 + 1.3980\\ \log_{10}1600 &= 3.2043\end{aligned}$$

Example

Find the value of $\sqrt[3]{16.51}$ approximately.

Solution

$$\begin{aligned}\text{Let } P &= \sqrt[3]{16.51}\\ \log P &= \log(16.51)^{1/3}\\ &= \frac{1}{3}\log 16.51\\ &= \frac{1}{3}(1.2178) = 0.4059\\ \log P &= 0.4059\\ P &= \text{anti-log } (0.4059)\\ \therefore\ P &= 2.546\end{aligned}$$

test your concepts

Very short answer type questions

1. $\frac{1}{5}\log_2 32 + 3\log_{64}4 =$ __________.

2. The characteristic of the logarithm of 3.6275 is __________.

3. If $4\log_x 8 = 3$, then $x =$ __________.

4. If $\log x - \frac{2}{3}\log x = 1$, then $x =$ _________.

5. If $a = \log\frac{3}{2}$, $b = \log\frac{4}{25}$ and $c = \log\frac{5}{9}$, then a + b + c = ________.

6. The number of digits in the integral part of the number whose logarithm is 4.8345 is ________.

7. If logx = 32.756, then log10x = _______.

8. The characteristic of the logarithm of 0.0062 is ________.

9. If $\log_a x$ (where a > 1) is positive, then the range of x is ________.

10. If log27.91 = 1.4458, then log2.791 = ________.

11. $\frac{\log 15 - \log 6}{\log 20 - \log 8}$ = ________.

12. If log2 = 0.3010, then log5 = _______.

13. The value of $\log_{16} \sqrt[5]{64}$ = ________.

14. $\frac{\log 216}{\log 6}$ = _________.

15. If $\log_4 3 = x$, then $\log_{\sqrt[4]{3}} \sqrt[4]{64}$ = ________.

16. If $\log_x \left(\frac{1}{243}\right) = -5$, then find the value of x.

17. $7^{\log_{343} 27}$ = _________.

18. If $3^{\log_9 x} = 2$, then x = ________.

19. If $\log_{xyz}x + \log_{xyz}y + \log_{xyz}z = \log_{10}p$, then p = ________.

20. If $\log_{10}4 + \log_{10}m = 2$, then m = _________.

21. Simplify: $3\log_3 5 + \log_3 10 - \log_3 625$.

22. If log(a + 1) + log(a – 1) = log15, then a = _________.

23. The value of log10 + log100 + log1000 + ...+ log10000000000 = ___________.

24. If the number of zeroes between the decimal point and the first non-zero digit of a number is 2, then the characteristic of logarithm of that number is _________.

25. The value of log(tan10°) + log(tan20°) + log(tan45°) + log(tan70°) + log(tan80°) = ________.

Short answer type questions

26. Simplify: $\log\left(\frac{3}{18}\right) + \log\left(\frac{45}{8}\right) - \log\left(\frac{15}{16}\right)$.

27. Show that $\frac{1}{\log_a abc} + \frac{1}{\log_b abc} + \frac{1}{\log_c abc} = 1$.

28. Solve for real value of x: $\log(x-1) + \log(x^2+x+1) = \log 999$.

29. If $\frac{1}{1+\log_a 10} = \frac{3}{2}$, then find the value of a.

30. If $x^2 + y^2 = 23xy$, then show that $2\log(x+y) = 2\log 5 + \log x + \log y$.

31. If $\log_{10}2 = 0.3010$ and $\log_{10}3 = 0.4771$, then find the value $\log_{10}135$.

32. If $\log_{10}2 = x$ and $\log_{10}3 = y$, then find $\log_{10}21.6$.

33. If $\log_{10}2 = 0.3010$, then find the number of digits in $(64)^{10}$.

34. Simplify $\frac{1}{\log_2 \log_2 \log_2 256}$.

35. Prove that $\log_3 810 = 4 + \log_3 10$.

Essay type questions

36. Solve: $x^{\log_4 3} + 3^{\log_4 x} = 18$.

37. If $p^2 + q^2 = 14pq$, then prove that $\log\left(\frac{p+q}{4}\right) = \frac{1}{2}[\log p + \log q]$.

38. Without using tables, find the value of $4\log_{10}5 + 5\log_{10}2 - \frac{1}{2}\log_{10}4$.

39. If $\frac{\log a}{b-c} = \frac{\log b}{c-a} = \frac{\log c}{a-b}$, then prove that $a^a\, b^b\, c^c = 1$.

40. Arrange the following numbers in the increasing order of their magnitude.

$\log_7 9, \log_{18} 16, \log_6 41, \log_2 10$.

CONCEPT APPLICATION

Concept Application Level—1

1. If $\log_{16}x = 2.5$, then x =

(1) 40 (2) 256 (3) 1024 (4) None of these

2. If $\log 5 = 0.699$ and $(1000)^x = 5$, then find the value of x.

(1) 0.0699 (2) 0.0233 (3) 0.233 (4) 10

3. The value of $\log\left(\frac{18}{14}\right) + \log\left(\frac{35}{48}\right) - \log\left(\frac{15}{16}\right) =$

(1) 0　　(2) 1　　(3) 2　　(4) $\log_{16} 15$

4. If $\log_3 a + \log_9 a + \log_{81} a = \frac{35}{4}$, then a =

(1) 27　　(2) 243　　(3) 81　　(4) None of these

5. If $\log_9 (\log_8 x) < 0$, then x belongs to

(1) (1, 8)　　(2) $(-\infty, 8)$　　(3) $(8, \infty)$　　(4) None of these

6. If $\log_3 \frac{x^3}{3} - 2\log_3 3x^3 = a - b\log_3 x$, then find the value of a + b.

(1) 6　　(2) −6　　(3) 0　　(4) −3

7. The value of $\log_{40} 5$ lies between

(1) $\frac{1}{3}$ and $\frac{1}{2}$　　(2) $\frac{1}{4}$ and $\frac{1}{3}$　　(3) $\frac{1}{2}$ and 1　　(4) None of these

8. If $x = \log_{\frac{1}{2}} \frac{4}{3} \, . \, \log_2 \frac{1}{3} \, . \, \log_{\frac{2}{3}} 0.8$, then

(1) $x > 0$　　(2) $x < 0$　　(3) $x = 0$　　(4) $x \geq 0$

9. If $\log_{144} 729 = x$, then the value of $\log_{36} 256$ is

(1) $\frac{4(3-x)}{(3+x)}$　　(2) $\frac{4(3+x)}{(3-x)}$　　(3) $\frac{(3+x)}{4(3-x)}$　　(4) $\frac{(3-x)}{4(3+x)}$

10. The solution set of the equation log(2x − 5) − log3 = log4 − log(x + 9) is

(1) $\left\{\frac{-19}{2}, 3\right\}$　　(2) $\left\{-3, \frac{19}{2}\right\}$　　(3) $\left\{3, \frac{19}{2}\right\}$　　(4) {3 }

11. If $\log_{10}\tan 19° + \log_{10}\tan 21° + \log_{10}\tan 37° + \log_{10}\tan 45° + \text{long}_{10}\tan 69° + \log_{10}\tan 71° + \log_{10}\tan 53° = \log_{10}\frac{x}{2}$, then x =

(1) 0　　(2) 1　　(3) 2　　(4) 4

12. The solution set of the equation log(x + 6) − log8 = log9 − log(x + 7) is

(1) {−15, 2}　　(2) {2}　　(3) {−15, 0, 2}　　(4) {0, 2}

13. If $\log_{40} 4 = x$ and $\log_{40} 5 = y$, then express $\log_{40} 32$ in terms of x and y.

(1) 5(1 + x + y)　　(2) 5(1 − x + y)　　(3) 5(1 − x − y)　　(4) 5(1 + x − y)

14. If $\log_{10} 11 = p$, then $\log_{10}\left(\frac{1}{110}\right) =$

(1) $(1 + p)^{-1}$ (2) $-(1 + p)$ (3) $1 - p$ (4) $\frac{1}{10p}$

15. If $\log_4 \frac{x^4}{4} + 3\log_4 4x^4 = p + q\log_4 x$, then the value of $\log_p(q)$ is ________.

(1) 4 (2) −4 (3) 3 (4) 2

16. If $\log_4 x + \log_8 x^2 + \log_{16} x^3 = \frac{23}{2}$, then $\log_x 8 =$

(1) 2 (2) $\frac{1}{2}$ (3) 3 (4) $\frac{3}{4}$

17. If $\log_{(x+y)}(x - y) = 7$, then the value of $\log_{(x^2-y^2)}(x^2 + 2xy + y^2)$ is ________.

(1) 14 (2) $\frac{2}{7}$ (3) $\frac{7}{2}$ (4) $\frac{1}{4}$

18. The value of $\log_{35} 3$ lies between

(1) $\frac{1}{4}$ and $\frac{1}{3}$ (2) $\frac{1}{3}$ and $\frac{1}{2}$ (3) $\frac{1}{2}$ and 1 (4) None of these

19. If $\log\left(\frac{a+b}{6}\right) = \frac{1}{2}(\log a + \log b)$, then $\frac{a}{b} + \frac{b}{a} =$

(1) 30 (2) 31 (3) 32 (4) 34

20. If $\log_p q = x$, then $\log_{\frac{1}{p}}\left(\frac{1}{q}\right) =$

(1) $\frac{1}{x}$ (2) $-x$ (3) x (4) x^2

21. If $\log_{(x-y)}(x + y) = 5$, then what is the value of $\log_{x^2-y^2}(x^2 - 2xy + y^2)$?

(1) 1 (2) $\frac{\sqrt{5}}{3}$ (3) $\frac{1}{3}$ (4) 0

22. The value of $\log_a 1 + \log_2 2^2 + \log_3 3^3$ $\log_a 1 + \log_2 2^2 + \log_3 3^3$ (where a is a positive number and $a \neq 1$) is

(1) 210 (2) 209 (3) 145 (4) 89

23. $\log_{\frac{1}{2}} \frac{2}{3}$ _____ $\log_{\frac{2}{3}} \frac{1}{2}$. The appropriate symbol in the blank is

(1) > (2) <

(3) = (4) Cannot be determined

24. The value of $\log_3[\log_2\{\log_4(\log_5 625^4)\}]$ is

(1) 0 (2) 1 (3) 2 (4) $\log_3 4$

25. If $\log(x - 3) + \log(x + 2) = \log(x^2 + x - 6)$, then the real value of x, which satisfies the above equation is

(1) is any value of x (2) is any value of x except x = 0

(3) is any values of x except x = 3 (4) does not exist

Concept Application Level—2

26. If $\log_{\left(\sqrt{b\sqrt{b\sqrt{b\sqrt{b}}}}\right)}\left(\sqrt{a\sqrt{a\sqrt{a\sqrt{a\sqrt{a}}}}}\right) = x\log_b a$, then x =

(1) $\frac{32}{16}$ (2) $\frac{31}{15}$ (3) $\frac{31}{30}$ (4) $\frac{1}{2}$

27. If $7^{\log x} + x^{\log 7} = 98$, then $\log_{10}\sqrt{x}$ then $\frac{a}{b} + \frac{b}{a} =$

(1) 47 (2) 51 (3) 14 (4) 49

28. If $7^{\log x} + x^{\log 7} = 98$, then $\log_{10}\sqrt{x} =$

(1) 1 (2) $\frac{1}{2}$

(3) 2 (4) Cannot be determined

29. The value of $\log_b a + \log_{b^2} a^2 + \log_{b^3} a^3 + \ldots.. + \log_{b^n} a^n$ is

(1) n (2) $\log_b a$ (3) $\frac{n(n+1)}{2}\log_b a$ (4) $\log_b a^n$

30. If $(\log_2 x) + \log_2(\log_4 x) = 2$, then find $\log_x 4$.

(1) 2 (2) $\frac{1}{2}$

(3) 1 (4) Cannot be determined $\log_4$

31. If pqr = 1 then find the value of $\log_{rq} p + \log_{rp} q + \log_{pq} r$

(1) 0 (2) −1 (3) −3 (4) 1

32. If $\log_3[\log_2\{\log_x(\log_6 216^3)\}] = 0$ then $\log_3(3x) =$

(1) $\log_3 12$ (2) 1 (3) 2 (4) $\log_3 6$

33. If a^x, b^x and c^x are in G.P., then which of the following is/are true?

(a) a, b, c are in G.P. (b) loga, logb, logc are in G.P.

(c) loga, logb, logc are in A.P. (d) a, b, c are in A.P.

(1) a and b (2) a and c (3) b and d (4) only a

34. The value of $\frac{1}{\log_3 n} + \frac{1}{\log_4 n} + \frac{1}{\log_5 n} + ... + \frac{1}{\log_8 n}$ is ________.

(1) $\log_n 8!$ (2) $\log_{n!} 8$ (3) $\log_n\left(\frac{8!}{2}\right)$ (4) $\log_{n!} 8!$

35. If $\frac{\log a}{y-z} = \frac{\log b}{z-x} = \frac{\log c}{x-y}$, then abc =

(1) $a^x b^y c^z$ (2) $a^{y+z}\, b^{z+x}\, c^{x+y}$ (3) 1 (4) All the above

Concept Application Level—3

36. The solution set for $|1-x|^{\log_{10}(x^2-5x+5)} = 1$, is

(1) {0, 1, 4} (2) {1, 4} (3) {0, 4} (4) {0, 2, 4}

37. The value of $\log\sqrt{2\sqrt{2\sqrt{2......\infty \text{ times}}}} + \log\sqrt{3\sqrt{3\sqrt{3\ ......\infty \text{ times}}}}$ is

(1) 1 (2) 2 (3) log5 (4) log6

38. The least positive integral value of the expression $\frac{1}{2}\log_{10} m - \log_{m^{-2}} 10$ is

(1) 0 (2) 1 (3) 2 (4) −1

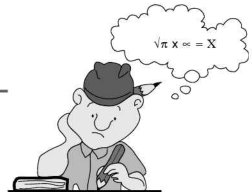
39. The domain of log(3 − 5x) is

(1) $\left(\frac{3}{5}, \infty\right)$ (2) $\left(0, \frac{3}{5}\right)$ (3) $\left(-\infty, \frac{3}{5}\right)$ (4) $\left(-\frac{3}{5}, 0\right)$

40. If $\log_7 x + \log_7 y \geq 2$, then the smallest possible integral value of x + y (given $x \neq y$) is

(1) 7 (2) 14 (3) 15 (4) 20

KEY



Very short answer type questions

1. 2
2. 0
3. 16
4. 10^3
5. $\log\frac{2}{15}$
6. 5
7. 33.756
8. $\overline{3}$
9. $(1, \infty)$
10. 0.4458
11. 1
12. 0.6990
13. $\frac{3}{10}$
14. 3
15. $\frac{3}{x}$
16. 3

17. 3

18. 4

19. 10

20. 25

21. $\log_3 2$

22. 4

23. 55

24. $\bar{3}$

25. 0

Short answer type questions

26. 0.

28. $x = 10$

29. 10^{-3}

31. 2.1303

32. $3(x + y) - 1$.

33. 19.

34. (i) $\log_3 2$

Essay type questions

36. 16.

37.

38. 4

40. $\log_{18}16, \log_7 9, \log_6 41, \log_2 10$.

key points for selected questions

Very short answer type questions

16. (i) Express $\left(\frac{1}{243}\right)$ as the power of 3.

(ii) Use, $\log_n x = a \Rightarrow x = n^a$ and find x.

21. Use, $\log a + \log b - \log c = \log\left(\frac{a \times b}{c}\right)$ and $m\log a = \log a^m$.

Short answer type questions

26. Use, $\log a + \log b - \log c = \log\left(\frac{ab}{c}\right)$ and simplify.

27. (i) $\log_b a = \frac{1}{\log_a b}$

(ii) Then, use $\log_x a + \log_x b + \log_x c = \log_x abc$ and simplify.

28. (i) Express the LHS as a single logarithm by using, $\log a + \log b = \log ab$.

(ii) Eliminate logs on both sides and solve for x.

29. (i) Take 1 as $\log_3 3$ and use $\log_x a + \log_x b = \log_x ab$

(ii) Cross multiply and proceed.

30. (i) Add 2xy on both sides of the given equation.

(ii) Express LHS as a perfect square.

(iii) Apply logs on both sides and use the relavent laws to prove.

31. (i) Express 135 as multiples of 3 and 5 and log 5 as $\log\left(\frac{10}{2}\right)$.

(ii) Use the laws of logarithms and simplify.

33. (i) Assume $(64)^{10}$ as x and apply log on both sides.

(ii) Express 64 as power of 2 and evaluate the characteristic of logx and there by number of digits of x.

34. (i) Simplify from the extreme right logarithm i.e., express 256 as the power of 2 and use $\log a^m = m \log a$.

35. Take 4 as $4 \log_3 3$ and use $\log ab = \log a + \log b$..

Essay type questions

36. Use, $a^{\log_c b} = b^{\log_c a}$ and solve for x.

37. (i) Add 2pq on both sides of the equation given and express LHS as a perfect square.

(ii) Use the laws of logarithms and prove the required result.

38. (i) Use, m loga = log a^m.

(ii) Then, use log a + log b – log c = log $\left(\frac{ab}{c}\right)$ and simplify.

39. (i) Let the given equation to k and equate each term to k and evaluate a^a, b^b and c^c.

(ii) Take the product of $a^a.b^b.c^c$.

40. $7^1 < 9 < 7^2 \Rightarrow 1 < \log_7 9 < 2$

KEY

Concept Application Level—1,2,3

1. 3	**2.** 3
3. 1	**4.** 2
5. 1	**6.** 3
7. 1	**8.** 1
9. 1	**10.** 4
11. 3	**12.** 2
13. 3	**14.** 2
15. 1	**16.** 2
17. 4	**18.** 1
19. 4	**20.** 3
21. 3	**22.** 2
23. 2	**24.** 1
25. 4	**26.** 3
27. 2	**28.** 1
29. 4	**30.** 2
31. 3	**32.** 3
33. 2	**34.** 3
35. 4	**36.** 3
37. 4	**38.** 2
39. 3	**40.** 3

Concept Application Level—1,2,3

Key points for select questions

1. $\log_b a = n \Rightarrow a = b^n$.

2. Taking logarithms for $10^{3x} = 5$ and substituting in the given equation we get the value of x.

3. loga + logb – logc = log $\left(\frac{ab}{c}\right)$ and log 1 = 0.

4. $\log_{b^n} a = \frac{1}{n}\log_b a$

5. If $\log_a y < 0$,then $0 < y < 1$ for $a > 1$.

6. use loga – logb = log $\frac{a}{b}$ and loga + logb = logab.

7. consider $5^2 < 40 < 5^3$ take logarithm with base 5.

8. $\log_b a > 0$; when $a > 1$ and $b < 1$
$\log_b a < 0$ when $a < 1$ and $b > 1$
$\log_b a > 0$ when $a < 1$ and $b < 1$

9. Find the values of $\log_{12} 4$, $\log_{12} 36$ and using $\log_{12} 27 = x$.

10. $\log a - \log b = \log\left(\frac{a}{b}\right)$

11. $\tan\theta . \tan(90-\theta) = 1$

12. $\log a - \log b = \log \frac{a}{b}$.

13. Express $\log_{40}32$ interms of x and y.

14. $\log \frac{1}{a} = \log a^{-1}$; $\log mn =$ $\log m + \log n$. and $\log_a a = 1$

15. $\log a + \log b = \log ab$
$\log a - \log b = \log a/b$.

16. $\log_{b^n} a^m = \frac{m}{n}\log_b a$

17. Adding '1' on both sides and the '1' on the left side is expressed as $\log_{x+y} x + y$.

18. Consider the inequality and $3^3 < 35 < 3^4$ taking logarithm with base 3.

19. (i) Use, $\log a + \log b = \log ab$. remove the logarithms on both sides and evaluate $(a + b)^2$.
(ii) Use $m(\log a + \log b) = \log(ab)^m$.
(iii) Eliminate logarithms on both sides and obtain equations in terms of a and b.
(iv) Divide both sides of the equation with ab and obtain the required answer.

20. $\log_{b^n} a^m = \frac{m}{n}\log_b a$.

21. Adding '1' on both sides, the '1' on the left side is expressed as $\log_{x-y} x - y$.

22. (i) $\log_b a^m = m \log_b a$ and $\sum n = \frac{n(n+1)}{2}$.
(ii) $\log_a 1 = 0, \log_2 2^2 = 2, \log_3 3^3 = 3$, and so on.
(iii) The required answer is the sum of first 20 natural numbers except 1.

23. (i) Use $a > b \Rightarrow \log_b a > 1$ and $a < b \Rightarrow \log_b a < 1$.

24. (i) $\log a^m = m\log a$
(ii) Express 625 in terms of base 5 and simplify from the extreme right logarithm.

25. $\log a + \log b = \log ab$

26. (i) Use, $\sqrt{a\sqrt{a\sqrt{a.....n \text{ terms}}}} = a^{\frac{2^n - 1}{2^n}}$ and then $\log_{b^n} a^m = \frac{m}{n}\log_b a$ and simplify LHS.
(ii) Compare LHS and RHS and find the value of x.

27. (i) Use $\log a + \log b = \log ab$ and remove logarithms on both the sides and evaluate $(a - b)^2$.
(ii) Express RHS into single logarithm with coefficient 1.
(iii) Apply anti-log and cancel the logarithms on both sides.
(iv) Divide LHS and RHS by ab and obtain the required value.

28. (i) $x^{\log y} = y^{\log x}$
(ii) $7^{\log x} = x^{\log 7}$
(iii) Convert LHS into $7^{\log x}$ (or) $x^{\log 7}$ and solve for x.

29. (i) $\log_{b^n} a^m = \frac{m}{n}\log_b a$
$\log a + \log b = \log ab$.
(ii) Each term of the given expression is equal to $\log_b a$.
(iii) There are n terms in the expression.
(iv) Use the above information and find the required sum.

30. (i) Assume $\log_2 x = a$ then $\log_4 x = a/2$
(ii) Take $\log_2(\log_4 x)$ as $2\log_4(\log_4 x)$ and convert LHS into single logarithm.
(iii) Express the result in terms of $\log_2 x$ and solve for $\log_2 x$.
(iv) Find $\log_x 2$ and then $\log_x 4$.

31. (i) Use $\log_a a = 1$.
(ii) Replace $rq = p^{-1}, rp = q^{-1}$ and $pq = r^{-1}$ in the given expression.
(iii) Simplify and eliminate logarithms.

32. (i) Remove logarithm one by one by using $\log_a^x = b \Rightarrow x = a^b$.
(ii) Now substitute the value of x in $\log_3 3x$ and simplify.

33. (i) Verify from options.
(ii) Use if a, b and c are in G.P., then $b^2 = ac$.
(iii) Substitute a^x, b^x and c^x in the above equation and simplify.
(iv) Apply logarithm for the above result and proceed.

34. (i) $\text{Log}\log_b a = \frac{1}{\log_a b}$
(ii) Take all the logarithms to the numerators by using the formula $\frac{1}{\log_b a} = \log_a b$.
(iii) Use, loga + logb + ….. + logn = log(abc …..n) and simplify.

35. (i) Equate the given ratios to k and get the values of loga, logb and logc.
(ii) Add loga, logb and logc and solve for abc.

36. (i) Use $a^0 = 1$ and $1^m = 1$
(ii) Consider RHS i.e., 1 as $|1 - x|^0$ and equate the exponents.
(iii) Convert the logarithm in the exponential form by using $\log_b a = N \Rightarrow a = b^N$.
(iv) Solve the quadratic equation for x.

37. (i) loga + logb = logab and $\log\sqrt{x\sqrt{x\ldots\infty}} = x$.
(ii) Use, $\sqrt{a\sqrt{a\sqrt{a\sqrt{a\ldots\ldots\infty}}}} = a$ and then use loga + logb = logab.

38. (i) The least positive integral value of $x + \frac{1}{x} = 2$.
(ii) Let $\log_m 10 = x$ then $\log_m 10 = \frac{1}{x}$.
(iii) The given expression becomes $\frac{1}{2}\left(x + \frac{1}{x}\right)$
(iv) Now the least positive value is obtained if $x + \frac{1}{x}$ is minimum.

39. (i) log f(x) is defined only when f (x) > 0.
(ii) logarithms take only positive values. i.e., $(3 - 5x) > 0$.
(iii) Solve the above inequation for x

40. (i) Refer to hint 12 in ex 2(a).
(ii) For the smallest value of a + b, $\log_{10}a + \log_{10}b = 2$.
(iii) Find the least value of a such that ab = 100