CHAPTER 6 Permutations and Combinations

CHAPTER ANALYSIS

	IIT JEE 2010		IIT JEE 2011		IIT JEE 2012		JEE Advanced 2013		JEE Advanced 2014		JEE Advanced 2015		JEE Advanced 2016		JEE Advanced 2017	
	Paper		Paper		Paper		Paper		Paper		Paper		Paper		Paper	
Topic	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2
Introduction																
Fundamental Principle of Counting																
Permutations																
Combinations						1			2		1		1		1	

QUESTIONS

1. Number of divisors of the form $4n + 2 (n \geq 0)$ is

(A) 4 (B) 8
(C) 10 (D) 3

(IIT JEE 1998)

2. An n-digit number is a positive number with exactly n-digits. Nine hundred district n-digit numbers are to be formed using only the three digit 2, 5 and 7. The smallest value of n for which this is possible is

(A) 6 (B) 7
(C) 8 (D) 9

(IIT JEE 1998)

3. For $2 \leq r \leq n$, $\binom{n}{r} + 2\binom{n}{r-1} + \binom{n}{r-2} = ?$

(A) $\binom{n+1}{r-1}$ (B) $2\binom{n+1}{r+1}$
(C) $2\binom{n+2}{r}$ (D) $\binom{n+2}{r}$

(IIT JEE 2000)

4. How many different nine digit numbers can be formed from the number 223355888 by rearranging its digits so that the odd digits occupy even position:

(A) 16 (B) 36
(C) 60 (D) 180

(IIT JEE 2000)

5. Let T_n denote the number of triangles which can be formed using the vertices of a regular polygon of n sides. If $T_n + 1 - T_n = 21$, then n equals

(A) 5 (B) 7
(C) 6 (D) 4

(IIT JEE 2001)

6. The number of arrangements of the letters of the word BANANA in which the two N's do not appear adjacently is

(A) 40 (B) 60
(C) 80 (D) 100

(IIT JEE Screening 2002)

7. Using permutation or otherwise prove that $\frac{n^2!}{(n!)^n}$ is an integer, where n is a positive integer.

(IIT JEE Main 2004)

8. If ${}^{n-1}C_r = (k^2 - 3)\,{}^{n}C_{r+1}$, then $k \in ?$

(A) $(-\infty, -2]$ (B) $[2, \infty)$
(C) $[-\sqrt{3}, \sqrt{3}]$ (D) $(\sqrt{3}, 2]$

(IIT JEE Screening 2004)

9. If r, s, t are prime numbers and p, q are the positive integers such that LCM of p, q is $r^2s^4t^2$, then the number of ordered pairs (p, q) is

(A) 252 (B) 254
(C) 225 (D) 224

(IIT JEE 2006)

10. The letters of the word COCHIN are permuted and all the permutations are arranged in an alphabetical order as in an English dictionary. The number of words that appear before the word COCHIN is

(A) 360 (B) 192
(C) 96 (D) 48

(IIT JEE 2007 Paper-2)

11. Consider all possible permutations of the letters of the word ENDEANOEL.

Match the statements/expressions in **Column I** with the values given in **Column II**:

Column I	Column II
(1) The number of permutations containing the word ENDEA is	**(P)** 5!
(2) The number of permutations in which the letter E occurs in the first and the last positions is	**(Q)** $2\times5!$
(3) The number of permutations in which none of the letters D, L, N occurs in the last five positions is	**(R)** $7\times5!$
(4) The number of permutations in which the letters A, E, O occur only in odd positions is	**(S)** $21\times5!$

(IIT JEE 2008 Paper-2)

12. The number of seven digit integers, with sum of the digits equal to 10 and formed by using the digits 1, 2 and 3 only, is

(A) 55 (B) 66
(C) 77 (D) 88

(IIT JEE 2009 Paper-1)

13. The total number of ways in which 5 balls of different colours can be distributed among 3 persons so that each person gets at least one ball is

(A) 75 (B) 150
(C) 210 (D) 243

(IIT JEE 2012 Paper-2)

14. Let $n \geq 2$ be an integer. Take n distinct points on a circle and join each pair of points by a line segment. Colour the line segment joining every pair of adjacent points by blue and the rest by red. If the number of red and blue line segments is equal, then the value of n is _____.

(JEE Advanced 2014 Paper-1)

15. Six cards and six envelopes are numbered 1, 2, 3, 4, 5, 6 and cards are to be placed in envelopes so that each envelope contains exactly one card and no card is placed in the envelope bearing the same number and moreover the card numbered 1 is always placed in envelope numbered 2. Then the number of ways it can be done is

(A) 264 (B) 265
(C) 53 (D) 67

(JEE Advanced 2014 Paper-1)

16. Let n be the number of ways in which 5 boys and 5 girls can stand in a queue in such a way that all the girls stand consecutively in the queue. Let m be the number of ways in which 5 boys and 5 girls can stand in a queue in such a way that exactly four girls stand consecutively in the queue. Then, the value of $\frac{m}{n}$ is _____.

(JEE Advanced 2015 Paper-1)

17. A debate club consists of 6 girls and 4 boys. A team of 4 members is to be selected from this club including the selection of a captain (from among these 4 members) for the team. If the team has to include at most one boy, then the number of ways of selecting the team is

(A) 380 (B) 320
(C) 260 (D) 95

(JEE Advanced 2016 Paper-1)

18. Words of length 10 are formed using the letters A, B, C, D, E, F, G, H, I, J. Let x be the number of such words where no letter is repeated; and let y be the number of such words where exactly one letter is repeated twice and no other letter is repeated. Then, $\frac{y}{9x}$ = _____.

(JEE Advanced 2017 Paper-1)

ANSWER KEY

1. (A) **2.** (B) **3.** (D) **4.** (C) **5.** (B) **5.** (A) **6.** (A)
8. (D) **9.** (C) **10.** (C) **11.** (1) → (P); (2) → (S); (3) → (Q); (4) → (Q) **12.** (C) **13.** (B) **14.** 5
15. (C) **16.** 5 **17.** (A) **18.** 5

ANSWERS WITH EXPLANATIONS

1. **Topic: Introduction**

To find number of divisors of form $(4n + 2)$ of integer 240 prime factorisation of 240 is $2^4 \times 3^1 \times 5^1$. The divisors of form $(4n + 2)$ are 2, 6, 10, 30.

Thus, there are four divisors of the given form.

Answer (A)

2. **Topic: Combinations**

Distinct n digit numbers can be formed using 2, 5 and 7 that are 3^n. Therefore,

$$3^n \geq 900$$
$$3^{n-2} \geq 100$$
$$\Rightarrow \quad n-2 \geq 5$$
$$\Rightarrow \quad n \geq 7$$

Answer (B)

3. **Topic: Combinations**

Let $r = 2$ and $n = 2$. Therefore,

$${}^2C_2 + 2\,{}^2C_1 + {}^2C_0 = 1+1+1 = 6$$

Now, let us consider option (D):

$$\binom{n+2}{r} = {}^{n+2}C_r = {}^4C_r = \frac{4!}{2!2!} = 6$$

Therefore,

$$\binom{n}{r} + 2\binom{n}{r-1} + \binom{n}{r-2} = \binom{n+2}{r}$$

Answer (D)

4. **Topic: Combinations**

There are four odd numbers; therefore, there are four even places that can be occupied in

$${}^4C_2 \text{ ways} = \frac{4!}{2!2!} = 6$$

There are 5 even numbers therefore there are 5 odd places that can be occupied in

$${}^5C_2 \text{ ways} = \frac{5!}{2!3!} = 10$$

Therefore, the required number of ways is

$$\frac{4!}{2!2!} \times \frac{5!}{2!3!} \Rightarrow 6 \times 10 = 60$$

Answer (C)

5. **Topic: Combinations**

The number of triangles formed by vertices of regular polygon of n sides is

$$T_n = {}^nC_3$$

Now, it is given that

$$T_{n+1} - T_n = 21$$

$$\Rightarrow n + {}^1C_3 - {}^nC_3 = 21$$

Using ${}^{n+1}C_r = {}^nC_{r-1} + {}^nC_r$, we get

$${}^nC_2 + {}^nC_3 - {}^nC_3 = 21$$
$$\Rightarrow {}^nC_2 = 21$$
$$\Rightarrow \frac{n(n-1)}{2} = 2$$
$$\Rightarrow n = -6, 7$$

However, n cannot be negative; therefore, $n = 7$.

Answer (B)

6. **Topic: Permutations**

The given word is BANANA.

Let us assume that two N occur adjacently and keeping them together and permuting with the remaining letters, the number of ways of arrangements "BA(NN)AA" is

$$\frac{5!}{3!} = \frac{120}{6} = 20 \text{ ways}$$

and the total number of ways of arrangement of the letters of word BANANA is

$$\frac{6!}{3! \cdot 2!} = \frac{720}{12} = 60 \text{ ways}$$

Therefore, the number of ways in which two N are not together is

$$(60 - 20) = 40 \text{ ways}$$

Answer (A)

7. **Topic: Permutations**

We have

$$\frac{n^2!}{(n!)^n} = \frac{1\cdot 2\cdot 3\cdots n\cdot (x+1)\cdots n^2}{(1\cdot 2\cdot 3\cdots n)^n} \quad (n \in \mathbb{N})$$

Let n^2 objects be divided equally among n groups. This can be done in

$$\frac{(n^2)!}{\underbrace{n!\cdot n!\cdot n!\cdots n!}_{n-\text{times}}} = \frac{(n^2)!}{(n!)^n}$$

and hence it (being a number of ways) must be an integer rather a positive integer.

Hence, it is proved that $\frac{(n^2)!}{(n!)^n} \in \mathbb{N}$.

8. **Topic: Permutations**

It is given that

$${}^{n-1}C_r = (k^2 - 3) \cdot {}^nC_{r+1}$$

$$\Rightarrow \quad {}^{n-1}C_r = (k^2 - 3) \cdot \frac{n}{r+1} \cdot {}^{n-1}C_r$$

$$\Rightarrow k^2 - 3 = \frac{r+1}{n} \quad (1)$$

Now, $0 \le r \le n - 1$; therefore,

$$1 \le r + 1 \le n$$

$$\Rightarrow \frac{1}{n} \le \frac{r+1}{n} \le 1 \Rightarrow 0 < \frac{r+1}{n} \le 1 \quad (\text{as } n \to \infty)$$

$$\Rightarrow k^2 - 3 \in (0,1]$$

$$\Rightarrow k^2 \in (3,4]$$

$$\Rightarrow k \in [-2, -\sqrt{3}) \cup (\sqrt{3}, 2]$$

Answer (D)

9. Topic: Permutations

We have LCM of $(p, q) = r^2s^4t^2$. Now, the distribution of r in p and q is listed as follows:

p	q
r^2	r
r^2	r^1
r^2	r^2
r^0	r^2
r^1	r^2
r^2	r^2

That is, the total cases $6 - 1 = 5$ and the repeated case is 1.

The same distribution of 5 and t in p and q is done by 9 and 5 ways.

The total number of ordered pairs (p, q) is

$$5 \times 9 \times 5 = 25 \times 9 = 225$$

Answer (C)

10. Topic: Permutations

The given word is COCHIN. That is,

C, C, H, I, N, O

The number of words starting with $CC = 4! = 24$.

The number of words starting with $CH = 4! = 24$.

The number of words starting with $CI = 4! = 24$.

The number of words starting with $CN = 4! = 24$.

Now, the first word of the series CO is COCHIN.

Therefore, the number of words that appear before the word COCHIN is 96.

Answer (C)

11. Topic: Permutations

We have the word as follows:

ENDEANOEL

(A)→(P)

ENDEA, N, O, E, L are five different letters, then permutation is = 5!

(B)→(S)

If E is at the first and at the last position, we get

$$\frac{(9-2)!}{2!} = 7 \times 3 \times 5! = 21 \times 5!$$

(C)→(Q)

The arrangements of last five letters:

$$\frac{5!}{3!} = 20$$

Therefore, the number of permutation is

$$12 \times 20 = 240 = 2 \times 5!$$

(D)→(Q)

The arrangements of O, E and A are $\frac{5!}{3!}$ and that of other letters are $\frac{4!}{2!}$. Therefore, the number of permutation is

$$\frac{5!}{3!} \times \frac{4!}{2!} = 2 \times 5!$$

Answer (1)→(P); (2)→(S); (3)→(Q); (4)→(Q)

12. Topic: Combinations

The two possible cases are as follows:

Case 1: There are five 1's; one 2; one 3. Therefore, the number of numbers is $7!/5! = 42$.

Case 2: There are four 1's; three 2's. Therefore, the number of numbers is $7!/4!\ 3! = 35$. Hence, the total number of numbers is $42 + 35 = 77$.

Answer (C)

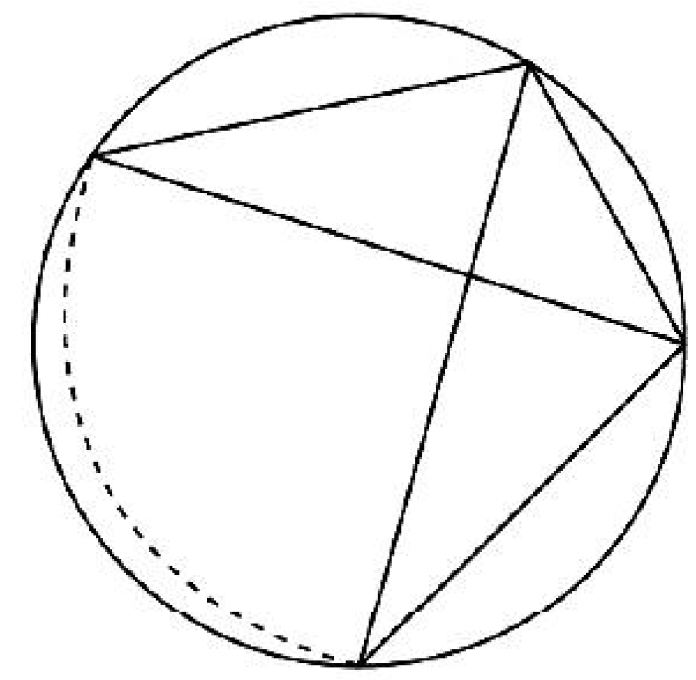
13. Topic: Combinations

Number of ways is obtained as follows:

$$3^5 - {}^3C_1 \cdot 2^5 + {}^3C_2 \cdot 1^5 = 243 - 96 + 3 = 150$$

Answer (B)

14. Topic: Combinations



The number of blue lines = n = number of sides of polygon so formed.

The number of red lines = $^nC_2 - n$.

Thus, by joining n points (not more than 2 on a line), there are $^nC_2$ lines formed because for each line two points are required.

Also, red lines come after excluding sides of polygon.

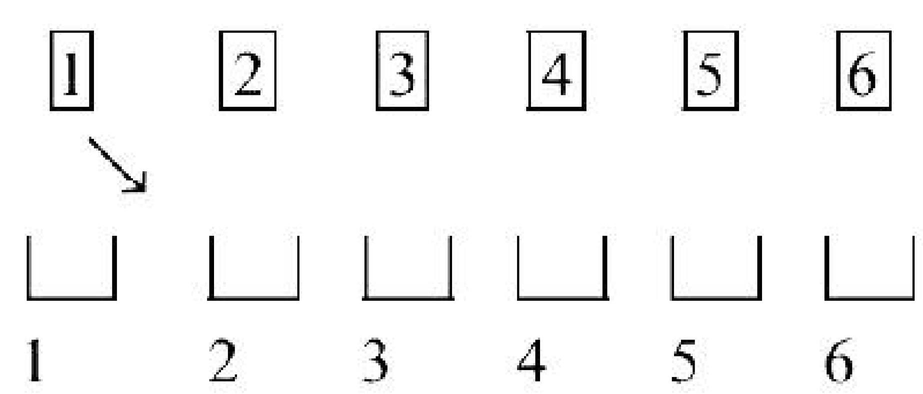
Therefore, $\quad n = {}^nC_2 - n$ or ${}^nC_2 = 2n$ or

$$\frac{n(n-1)}{2} = 2n \text{ or } n - 1 = 4 \qquad (\text{as } n \neq 0)$$

Hence, $\quad n = 5.$

Answer (5)

15. Topic: Combinations



This is a problem of derangement.

The number of derangements of n things is

$$\underline{|n}\sum_{k=0}^{n}\frac{(-1)^k}{\underline{|k}} - \underline{|n}\left[1 - \frac{1}{\underline{|1}} + \frac{1}{\underline{|2}} - \frac{1}{\underline{|3}} + \cdots + \frac{(-1)^n}{\underline{|n}}\right]$$

In the equation, two possibilities are there:

- **When card 1 goes to envelope 1**

  Derangement of 3, 4, 5, 6 cards is

$$\underline{|4}\left(1 - \frac{1}{\underline{|1}} + \frac{1}{\underline{|2}} - \frac{1}{\underline{|3}} + \frac{1}{\underline{|4}}\right)$$

$$= 24\left(\frac{1}{2} - \frac{1}{6} + \frac{1}{24}\right) = 12 - 4 + 1 = 9 \text{ ways}$$

- **When card 2 does not go to envelope 1**

  Now, it is derangement of 2, 3, 4, 5, 6 in envelopes 1, 3, 4, 5, 6, in

$$\underline{|5}\left(1 - \frac{1}{\underline{|1}} + \frac{1}{\underline{|2}} - \frac{1}{\underline{|3}} + \frac{1}{\underline{|4}} - \frac{1}{\underline{|5}}\right)$$

$$= 120\left(\not{1} - \frac{\not{1}}{\not{1}} + \frac{1}{2} - \frac{1}{6} + \frac{1}{24} - \frac{1}{120}\right)$$

$$= 60 - 20 + 5 - 1 = 44 \text{ ways}$$

  Hence, the total number of ways is 9 + 44 = 53.

Answer (C)

16. Topic: Combinations

We have

n = (5 girls are consecutive) = $6! \cdot 5!$

m = (4 girls are consecutive)

= (Arrange 5 boys in a queue in 5! ways, arrange 4 girls out of 5 together in $({}^5C_4 \cdot 4!)$ ways

Put a girl and group of 4 girls (together) in any two places out of 6 between the 5 boys in ${}^6P_2$ ways. Therefore,

$$m = 5!\,({}^5C_4 \cdot 4!) \cdot {}^6P_2$$

$$\frac{m}{n} = \frac{5! \cdot {}^5C_4 \cdot 4! \cdot {}^6P_2}{6! \cdot 5!} = 5$$

Answer (5)

17. Topic: Combinations

The club consists of 6 girls and 4 boys. If a team of 4 members to be selected which consists at most 1 boy (including 1 captain), then the number of ways of selecting the team is obtained as follows:

$${}^4C_1({}^4C_1 \cdot {}^6C_3 + {}^6C_4) = 4(80 + 15) = 380 \text{ ways}$$

Answer (A)

18. Topic: Combinations

The given, formed word is of length 10.

It is given that x is the number of words where no letter is repeated.

Also, it is given that y is the number of words where exactly one letter is repeated twice and no other letter is repeated. Therefore,

$$x = 10!$$

and $\quad y = {}^{10}C_1 \times {}^{10}C_2 \times {}^9C_8 \times 8!$

Thus, $\quad \dfrac{y}{9x} = \dfrac{{}^{10}C_1 \times {}^{10}C_2 \times {}^9C_8 \times 8!}{9 \times 10!}$

Using ${}^nC_r = \dfrac{n!}{r!(n-r)!}$, we get

$$\frac{y}{9x} = \frac{\left[\frac{10!}{1!(10-1)!} \times \frac{10!}{2!(10-2)!} \times \frac{9!}{8!(9-8)!} \times 8!\right]}{9 \times 10!}$$

$$= \frac{10!}{9 \times 2! \times 8!} = \frac{10 \times 9 \times 8!}{9 \times 2 \times 8!} = \frac{10}{2} = 5$$

[Using $n! = n(n-1)(n-2)\ldots.1!$]

Answer (5)