22 Vector Algebra

QUICK LOOK

Physical quantities are divided into two categories– scalar quantities and vector quantities. Those quantities which have only magnitude and which are not related to any fixed direction in space are called scalar quantities, or briefly scalars. Examples of scalars are mass, volume, density, work, temperature etc.
A scalar quantity is represented by a real number along with a suitable unit. Second kind of quantities are those which have both magnitude and direction. Such quantities are called vectors. Displacement, velocity, acceleration, momentum, weight, force etc. are examples of vector quantities.

Vectors and Their Representation

- Vector quantities are specified by definite magnitude and definite direction.
- Vector quantities are represented by directed line segments. Vector AB,. i.e., $\overrightarrow{AB}$ is a vector whose magnitude is represented by the length AB and its direction is represented by the direction from A to B along the line segment.

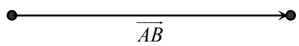


Figure: 22.1

- The magnitude of the vector $\overrightarrow{AB}$ is written as $|\overrightarrow{AB}|$ or simply AB.
- Unit vector $\vec{a}$ is a vector whose magnitude $|\vec{a}|=1$. Zero vector is a vector of 0 magnitude. The direction of $\vec{0}$ (zero vector) is indeterminate.
- If $AB = CD$ and $AB \parallel CD$ then $\overrightarrow{AB}=\overrightarrow{CD}$.
- $\overrightarrow{AB}=\overrightarrow{CD} \Rightarrow AB = CD$ and $AB \parallel CD$

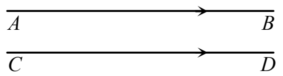


Figure :22.2

Addition of Vectors

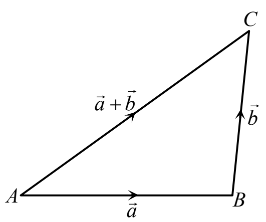


Figure: 22.3

If $\overrightarrow{AB}=\vec{a}$, $\overrightarrow{BC}$ then $\vec{a}+\vec{b}$ is the vector represented by $\overrightarrow{AC}$.

$\therefore \overrightarrow{AB}+\overrightarrow{BC}=\overrightarrow{AC}$

- If A,B,C,D and E are in order then $\overrightarrow{AB}+\overrightarrow{BC}+\overrightarrow{CD}+\overrightarrow{DE}=\overrightarrow{AE}$
- $\overrightarrow{AB}+\overrightarrow{AC}=\overrightarrow{AB}=\overrightarrow{AD}=2\overrightarrow{AE}$
  where $ABDC$ is a parallelogram, E being the point of intersection of the diagonals.
- $\vec{a}+\vec{0}=\vec{a}$
- $-\vec{a}$ is the negative of the vector $\vec{a}$ such that $\vec{a}+(-\vec{a})=\vec{0}$.

Resultant of Two Forces

$$\vec{R}=\vec{P}+\vec{Q} \ |\vec{R}|=R=\sqrt{P^2+Q^2+2PQ\cos\theta}$$

where $|\vec{P}| = P$, $|\vec{Q}|=Q$, $\tan\alpha=\dfrac{Q\sin\theta}{P+Q\cos\theta}$

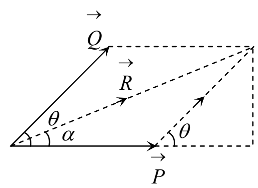


Figure: 22.4

Deduction: When $|\vec{P}|=|\vec{Q}|$,

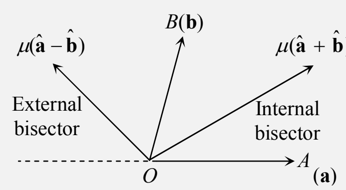
i.e., $P=Q$, $\tan\alpha=\dfrac{P\sin\theta}{P+P\cos\theta}=\dfrac{\sin\theta}{1+\cos\theta}=\tan\dfrac{\theta}{2}$;

$\therefore \quad \alpha=\dfrac{\theta}{2}$

Hence, the angular bisector of two unit vectors $\mathbf{a}$ and $\mathbf{b}$ is along the vector sum $\mathbf{a}+\mathbf{b}$.

Note

- The internal bisector of the angle between any two vectors is along the vector sum of the corresponding unit vectors.
- The external bisector of the angle between two vectors is along the vector difference of the corresponding unit vectors.



Scalar Multiplication of a Vector by a Scalar

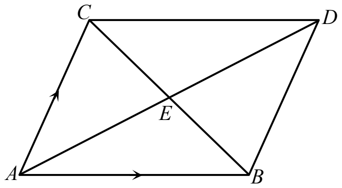


Figure :22.5

- If k is a scalar then $k\vec{a}$ is a vector having the same (or opposite) direction as that of $\vec{a}$ whose magnitude is |k| times that of $\vec{a}$.

$\therefore\ k\vec{a} \parallel \vec{a}$ and $|k\vec{a}| = |k\vec{a}| = |k||\vec{a}|$

- $k(\vec{a}+\vec{b}) = k\vec{a} + k\vec{b}$
- $p(q\vec{a}) = (pq)\vec{a} = q(p\vec{a})$
- Unit vector along $\vec{a} = \hat{a} = \dfrac{\vec{a}}{|\vec{a}|}$

Position Vector of a Point

- If O is the origin and P is any point then the position vector of $P = \vec{O}P$.
- If the position vectors of points A and B are $\vec{a}$ and $\vec{b}$ respectively then the position vector of the point P dividing AB in the ratio $m : n$ is $\dfrac{m\vec{b}+n\vec{a}}{m+n}$.

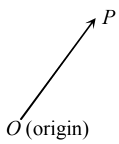


Figure: 22.6

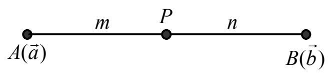


Figure :22.7

Relation between two Collinear Vectors: If $\vec{a}$ and $\vec{b}$ are two collinear (or parallel) vectors then there exists a scalar ë such that $\vec{b} = \lambda\vec{a}$.

Relation between Three Coplanar Vectors

- If $\vec{a}, \vec{b}$ and $\vec{r}$ are three vectors in a plane then there exists two unique scalars x, y such that $\vec{r} = x\vec{a} + y\vec{b}$
- If $\vec{r} = x_1\vec{a} + y_1\vec{b}$ as well as $\vec{r} = x_2\vec{a} + y_2\vec{b}$ then $x_1 = x_2,\ y_1 = y_2$.

Relation between four Vectors in Space

- If $\vec{a}, \vec{b}, \vec{c}$ and $\vec{r}$ are four vectors in space (of which no three are coplanar) then there exists three unique scalars x, y, z such that $\vec{r} = x\vec{a} + y\vec{b} + z\vec{c}$
- If $\vec{r} = x_1\vec{a} + y_1\vec{b} + z_1\vec{c}$ as well as $\vec{r} = x_2\vec{a} + y_2\vec{b} + z_2\vec{c}$ then $x_1 = x_2, y_1 = y_2, z_1 = z_2$.

Vectors

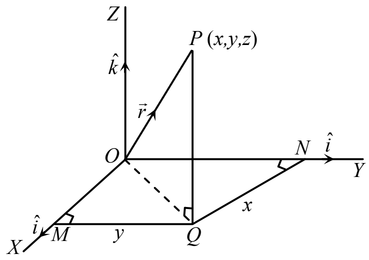


Figure: 22.8

$\vec{i}, \vec{j}, \vec{k}$ $\vec{i}, \vec{j}$ and $\vec{k}$ are three unit vectors along X, Y and Z axes respectively where each pair of axes are mutually perpendicular.

$\therefore\quad |\vec{i}| = 1 = |\vec{j}| = |\vec{k}|$

Let P be a point whose coordinates are (x, y, z).

The position vector of P is $\vec{r}$ where

$$\vec{r} = \overrightarrow{OQ} + \overrightarrow{QP} = (\overrightarrow{OM} + \overrightarrow{ON}) + \overrightarrow{QP} = x\vec{i} + y\vec{j} + z\vec{k} = (x, y, z)$$

$$|\vec{r}| = |\overrightarrow{OP}| = \sqrt{OQ^2 + QP^2} = \sqrt{(QN^2 + ON^2) + QP^2}$$

$$= \sqrt{x^2 + y^2 + z^2}$$

$$\cos\angle POZ = \cos(90° - \angle POQ)$$

$$= \sin\angle POQ = \frac{z}{OP} = \frac{z}{|\vec{r}|}$$

Similarly, $\cos\angle POX = \dfrac{x}{|\vec{r}|}, \cos\angle POY = \dfrac{y}{|\vec{r}|}$

The direction of $\vec{r}$ is given by the direction cosines $\dfrac{x}{|\vec{r}|}, \dfrac{y}{|\vec{r}|}, \dfrac{z}{|\vec{r}|}$

- The position vector of the point (x, y, z) is $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$.
- Any vector $\vec{a}$ in space can be written linearly in $\vec{i}, \vec{j}, \vec{k}$ i.e., $\vec{a} = \lambda\vec{i} + \mu\vec{j} + v\vec{k}$
- If $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$ is equally inclined with axes then $\dfrac{x}{|\vec{r}|} = \dfrac{y}{|\vec{r}|} = \dfrac{z}{|\vec{r}|}$ i.e., $x = y = z$.

Addition of Vectors Expressed in Terms of $\vec{i}, \vec{j}, \vec{k}$

If $\vec{a} = a_1\vec{i} + a_2\vec{j} + a_3\vec{k}$ and $\vec{b} = b_1\vec{i} + b_2\vec{j} + b_3\vec{k}$ then

$$\vec{a} + \vec{b} = (a_1 + b_1)\hat{i} + (a_2 + b_2)\vec{j} + (a_3 + b_3)\vec{k}$$

$$\vec{a} - \vec{b} = (a_1 - b_1)\hat{i} + (a_2 - b_2)\vec{j} + (a_3 - b_3)\vec{k}$$

Condition of Collinearity of Three Points: The points A,B and C are collinear if $\overline{AB} = \lambda\,\overline{AC}$

Condition of Coplanarity of four Points of Three Vectors: The points A,B,C and D are coplanar if the three vectors $\overrightarrow{AB}, \overrightarrow{AC}$ and $\overrightarrow{AD}$ are coplanar,i i.e, if two scalars x, y can be found such that $\overrightarrow{AB} = x\overrightarrow{AC} + y\overrightarrow{AD}$

Position Vector of a point of Section

- If $A(\vec{a}), B(\vec{b})$ be two points then the position vector of the middle point P of AB is $\frac{\vec{a}+\vec{b}}{2}$.
- If $A(\vec{a}), B(\vec{b})$ and $C(\vec{c})$ be the vertices of a triangle ABC then the position vector of the centroid G is $\frac{\vec{a}+\vec{b}+\vec{c}}{3}$.

Vector Conditions for Geometrical Results

- Line segments AB and CD are equal $\Leftrightarrow |\overline{AB}| = |\overline{CD}|$
- Lines AB and CD are parallel $\Leftrightarrow \overrightarrow{AB} = k\overrightarrow{CD}$

or $\overrightarrow{AB} \times \overrightarrow{CD} = \vec{0}$.

- Lines AB and CD are perpendicular $\Leftrightarrow \overrightarrow{AB} \cdot \overrightarrow{CD} = 0$
- A, B, C will be collinear $\Leftrightarrow \overrightarrow{AB} = k\overrightarrow{AC}$
- A, B, C, D will be coplanar $\Leftrightarrow \overrightarrow{AB} = \lambda\overrightarrow{AC} + \mu\overrightarrow{AD}$ or $[\overrightarrow{AB}\overrightarrow{AC}\overrightarrow{AD}] = 0$
- A vector perpendicular to the plane passing through the points A, B and C is $\overrightarrow{AB} \times \overrightarrow{AC}$.

Equation of Straight Lines

- The equation of the straight line passing through the point $(\vec{a})$ and parallel to the vector $(\vec{b})$ is $\vec{r} = \vec{a} + t\vec{b}$ where t is an arbitrary scalar and $\vec{r}$ is the position vector of any point on the line.
- The equation of the straight line passing through the point $(\vec{a})$ and $(\vec{b})$ is $\vec{r} = (1-t)\vec{a} + t\vec{b}$ where t is an arbitrary scalar.
- Three points whose position vectors are $\vec{a}, \vec{b}, \vec{c}$ are collinear if $\lambda\vec{a} + \mu\vec{b} + \nu\vec{c} = 0$ where $\lambda + \mu + \nu = 0$

Equations of Bisectors of the Angle between two Lines

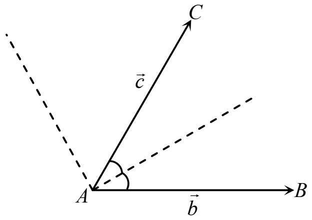


Figure :22.9

Let $A(a)$ be the vertex of the angle BAC where b and c are vectors along AB and AC respectively. The equations of the bisectors of $\angle BAC$ are $\vec{r} = \vec{a} + t\left(\frac{\vec{b}}{|\vec{b}|} \pm \frac{\vec{c}}{|\vec{c}|}\right)$

where t is a scalar parameter.

Shortest Distance between two Non-Coplalar Lines

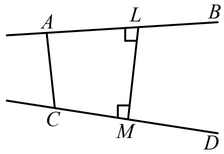


Figure :22.10

- If two lines AB, CD do not intersect, there is always a line cutting both the lines perpendicularly. The intercept on this line made by AB and CD is called the shortest distance between the lines AB and CD. In the figure, the shortest distance = LM where $\angle ALM = \angle CML = 90°$
- In the above figure, shortest distance LM

$= |$ projection of $\overrightarrow{AC}$ along $\overrightarrow{ML}| = \left|\overrightarrow{AC} \cdot \frac{\overrightarrow{ML}}{|\overrightarrow{ML}|}\right|$.

Equation of a Plane

- The equation of the plane passing through the points $A(\vec{a}), B(\vec{b})\, and\, C(\vec{c})$ is $(a)\ \vec{r} = x\vec{a} + y\vec{b} + (1-x-y)\vec{c}$ where $\vec{r}$ is the position vector of any point on the plane and x, y are parameters. $(b)\ [\vec{r}-\vec{a}\ \ \vec{r}-\vec{b}\ \ \vec{r}-\vec{c}] = 0$
- Four point whose position vectors are $\vec{a}, \vec{b}, \vec{c}\ and\ \vec{d}$ will be coplanar if $x\vec{a} + y\vec{b} + z\vec{c} + w\vec{d} = 0$ where $x + y + z + w = 0$.
- The equation of the plane passing through $A(\vec{a})$ and perpendicular to the vector $\vec{p}$ is $(\vec{r} - \vec{a}) \cdot \vec{p} = 0$

Vector Equation of a Sphere

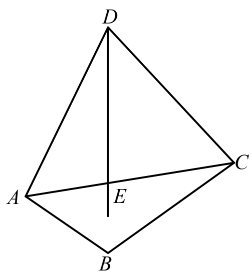


Figure :22.11

The vector equation of a sphere whose centre has the position vector $\vec{a}$ and radius is ρ, is $|\vec{r}-\vec{a}|=\rho$, $\vec{r}$ being the position vectors for any point on the sphere.

Volume of a Tetrahedron

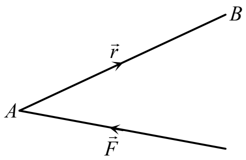


Figure :22.12

- The volume of a tetrahedron

$$=\frac{1}{3}\text{ (area of the base) (corresponding altitute)}$$

$$=\frac{1}{3}\cdot\frac{1}{2}|\overrightarrow{AB}\times\overrightarrow{AC}||\overrightarrow{ED}|$$

$$=\frac{1}{6}|\overrightarrow{AB}\times\overrightarrow{AC}||\overrightarrow{ED}|\cos 0^0 \text{ } for\ \overrightarrow{AB}\times\overrightarrow{AC}\|\overrightarrow{ED}$$

$$=\frac{1}{6}(\overrightarrow{AB}\times\overrightarrow{AC})\cdot\overrightarrow{ED}=\frac{1}{6}[\overrightarrow{AB}\ \overrightarrow{AC}\ \overrightarrow{ED}]$$

$$=\frac{1}{6}[\overrightarrow{AB}\ \overrightarrow{AC}\ \overrightarrow{EA}+\overrightarrow{AD}=\frac{1}{6}[\overrightarrow{AB}\ \overrightarrow{AC}\ \overrightarrow{AD}]$$

$\overrightarrow{AB}$, $\overrightarrow{AC}$, $\overrightarrow{EA}$ are coplanar and so $[\overrightarrow{AB}\ \overrightarrow{AC}\ \overrightarrow{EA}]=0$.

Work Done by a Force Vector: The work done by the force $\vec{F}$ in shifting a particles from the point A to the point B $=\overrightarrow{AB}\cdot\vec{F}=\vec{r}\cdot\vec{F}$

Vector Moment (Torque) of a Force Vector About a Point

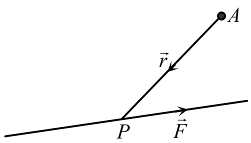


Figure :22.13

If the force $\vec{F}$ acts at the point P then the vector moment of $\vec{F}$ about the point A $=\overrightarrow{AP}\times\vec{F}=\vec{r}\times\vec{F}$

Rotation About an Axis

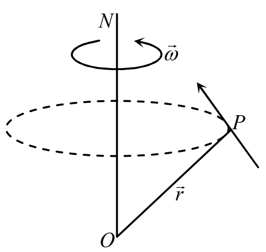


Figure: 22.14

When a rigid body rotates about a fixed axis ON with an angular velocity ω then the velocity ω then the velocity $\vec{v}$ of a particle P is given by $\vec{v}=\vec{w}\times\vec{r}$

where $\vec{r}=\overrightarrow{OP}$ *and* $\vec{w}=w$ (unit vector along ON).

Product of Two Vectors

- **Scalar product of two vectors**
- The scalar product (or dot product) $\vec{a}\cdot\vec{b}$ of two vectors $\vec{a}$ and $\vec{b}$ is a scalar quantity such that $\vec{a}\cdot\vec{b}=|\vec{a}||\vec{b}|\cos\theta$ where θ is the angle between $\vec{a}$ and $\vec{b}$. $\vec{a}\perp\vec{b}\Leftrightarrow\vec{a}\cdot\vec{b}=0$
- The angle θ between two units vectors $\hat{a},\hat{b}$ is given by $\cos\theta=\hat{a}\cdot\hat{b}$.
- The projection of $\hat{b}$ on the vector $\vec{a}=\vec{b}\cdot\frac{\vec{a}}{|\vec{a}|}$, *i.e.*, $\vec{b}\cdot\vec{a}$
- $\vec{i}\cdot\vec{i}=1=\vec{j}\cdot\vec{j}=\vec{k}\cdot\vec{k}$ and $\vec{i}\cdot\vec{j}=0=\vec{j}\cdot\vec{k}=\vec{k}\cdot\vec{i}$.
- $\vec{a}\cdot\vec{b}=\vec{b}\cdot\vec{a}$
- $\vec{a}\cdot(\vec{b}+\vec{c})=\vec{a}\cdot\vec{b}+\vec{a}\cdot\vec{c}$
- If $\vec{a}=a_1\vec{i}+a_2\vec{j}+a_3\vec{k}$ and $\vec{b}=b_1\vec{i}+b_2\vec{j}+b_3\vec{k}$ then $\vec{a}\cdot\vec{b}=a_1b_1+a_2b_2+a_3b_3$
- $(\vec{a}+\vec{b})^2=\vec{a}^2+\vec{b}^2+2\vec{a}\cdot\vec{b}$ where $\vec{a}\cdot\vec{a}=\vec{a}^2$, etc

Vector Product of two Vectors

- The vector product (or cross product) $\vec{a}\times\vec{b}$ of two vectors $\vec{a}$ and $\vec{b}$ is a vector quantity such that $\vec{a}\times\vec{b}=|\vec{a}||\vec{b}|\sin\theta\,\hat{n}$ where $\theta=$ angle between $\vec{a},\vec{b}$ and $\hat{n}$ is a unit vector perpendicular to the plane $\vec{a}$ and $\vec{b}$ such that $\vec{a},\vec{b},\hat{n}$ from a right-handed triad.
- $\vec{a}\|\vec{b}\Leftrightarrow\vec{a}\times\vec{b}=0$
- $\vec{a}\times\vec{b}\perp\vec{a}$ and $\vec{a}\times\vec{b}\perp\vec{b}$
- Unit vector perpendicular to both $\vec{a}$ and $\vec{b}$ is $\frac{\vec{a}\times\vec{b}}{|\vec{a}\times\vec{b}|}$.
- The angle θ between $\vec{a},\vec{b}$ is given by $\sin\theta=|\hat{a}\times\hat{b}|$.
- The vector area $\vec{\Delta}$ of the triangle whose two sides represent vectors $\vec{a}$ and $\vec{b}=\frac{1}{2}(\vec{a}\times\vec{b})$ and area (scalar) $\frac{1}{2}|\vec{a}\times\vec{b}|$.

  The vector area of the parallelogram whose two adjacent sides represent vectors $\vec{a}$ and $\vec{b}=\vec{a}\times\vec{b}$ are area (scalar) $|\vec{a}\times\vec{b}|$.
- $\vec{i}\times\vec{i}=\vec{0}=\vec{j}\times\vec{j}=\vec{k}\times\vec{k}$ $\vec{i}\times\vec{j}=\vec{k}$, $\vec{j}\times\vec{k}=\vec{i}$, $\vec{k}\times\vec{i}=\vec{j}$

- $\vec{a}\times b = -b\times\vec{a}$.

  $\vec{i}\times\vec{k} = -\vec{k}\times\vec{i} = -\vec{j},\ \vec{k}\times\vec{j} = -\vec{j}\times\vec{k} = -\vec{i},\ \vec{j}\times\vec{i} = -\vec{i}\times\vec{j} = -\vec{k}$

  $\vec{a}\times(\vec{b}+\vec{c}) = \vec{a}\times\vec{b}+\vec{a}\times\vec{c}$.

- If $\vec{a} = a_1\vec{i}+a_2\vec{j}+a_3\vec{k}$ and $\vec{b} = b_1\vec{i}+b_2\vec{j}+b_3\vec{k}$ then

$$\vec{a}\times\vec{b} = \begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ a_1 & a_2 & a_3\\ b_1 & b_2 & b_3\end{vmatrix}$$

Product of Three or More Vectors

Scalar triple product

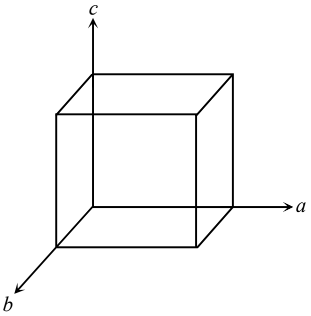


Figure: 22.15

- The dot product of the vectors $\vec{a}\times\vec{b}$ with the vectors $\vec{c}$ is a scalar triple product of the three vectors $\vec{a},\vec{b},\vec{c}$ and it is written as $(\vec{a}\times\vec{b}).\vec{c}$. It is a scalar quantity. The magnitude of $(\vec{a}\times\vec{b}).\vec{c}$ is equal to the volume V of a parallelopiped whose three concurrent edges are represented by $\vec{a},\vec{b}$ and $\vec{c}$ as shown in the figure.

$\therefore$ Volume of the parallelepiped whose three concurrent edges are $\vec{a},\vec{b},\vec{c} = (\vec{a}\times\vec{b})\cdot\vec{c}$

- $(\vec{a}\times\vec{b})\cdot\vec{c} = \vec{a}\cdot(\vec{b}\times\vec{c})$ i.e., dot and cross can be interchanged in a scalar triple product and each scalar triple product is written as $[\vec{a}\,\vec{b}\,\vec{c}]$.
- $[\vec{a}\,\vec{b}\,\vec{c}] = [\vec{b}\,\vec{c}\,\vec{a}] = [\vec{c}\,\vec{a}\,\vec{b}]$
- $[\vec{a}\,\vec{b}\,\vec{c}] = -[\vec{b}\,\vec{a}\,\vec{c}] = -[\vec{a}\,\vec{c}\,\vec{b}] = -[\vec{c}\,\vec{b}\,\vec{a}]$
- $[\vec{a}\,\vec{b}\,\vec{c}] = 0$ if $\vec{a} = \vec{b}$

or $\vec{b} = \vec{c}$

or $\vec{c} = \vec{a}$

or at least one vector $= \vec{0}$ or $\vec{a},\vec{b}$ *and* $\vec{c}$ are coplanar

- $[\vec{i}\,\vec{j}\,\vec{k}] = 1$
- If $\vec{a} = a_1\vec{i}+a_2\vec{j}+a_3\vec{k}, \vec{b} = b_1\hat{i}+b_2\vec{j}+b_3\vec{k}, \vec{c} = c_1\vec{i}+c_2\vec{j}+c_3\vec{k}$

  then $[\vec{a}\,\vec{b}\,\vec{c}] = \begin{vmatrix} a_1 & a_2 & a_3\\ b_1 & b_2 & b_3\\ c_1 & c_2 & c_3\end{vmatrix}$

Condition for Coplanarity of Three Vectors: $\vec{a},\vec{b}$ *and* $\vec{c}$ are coplanar $\Leftrightarrow [\vec{a},\vec{b}\ \vec{c}] = 0$

Vector Triple Product

- The vector product of $\vec{a}\times\vec{b}$ *and* $\vec{c}$ is a vector triple product of the three vectors $\vec{a},\vec{b}$ *and* $\vec{c}$. $(\vec{a}\times\vec{b})\times\vec{c},\ \vec{a}\times(\vec{b}\times\vec{c})$ are vector triple products.
- $\vec{a}\times(\vec{b}\times\vec{c}) = (\vec{a}\cdot\vec{c})\vec{b}-(\vec{a}\cdot\vec{b})\vec{c}$. This is a vector in the plane of $\vec{b}$ *and* $\vec{c}$.
- $(\vec{a}\times\vec{b})\times\vec{c} = (\vec{a}\cdot\vec{c})\vec{b}-(\vec{b}\cdot\vec{c})\vec{a}$. This is a vector in the plane of $\vec{a}$ *and* $\vec{b}$

Scalar Product of Four Vectors

- $(\vec{a}\times\vec{b})\cdot(\vec{c}\times\vec{d})$ is a scalar product of four vectors. It is a dot product of the vectors $\vec{a}\times\vec{b}$ *and* $\vec{c}\times\vec{d}$.

  It is scalar triple product of the vectors $\vec{a},\vec{b}$ *and* $\vec{c}\times\vec{d}$ as well as a scalar triple product of the vectors $\vec{a}\times\vec{b},\vec{c}$ *and* $\vec{d}$.
- $(\vec{a}\times\vec{b})\cdot(\vec{c}\times\vec{d}) = \begin{vmatrix}\vec{a}\cdot\vec{c} & \vec{a}\cdot\vec{d}\\ \vec{b}\cdot\vec{c} & \vec{b}\cdot\vec{d}\end{vmatrix}$

Vector Product of Four Vectors

- $(\vec{a}\times\vec{b})\times(\vec{c}\times\vec{d})$ is a vector product of four vectors.

  If is the cross product of the vectors $\vec{a}\times\vec{b}$ and $\vec{c}\times\vec{d}$.
- $\vec{a}\times\{\vec{b}\times(\vec{c}\times\vec{d})\},\{(\vec{a}\times\vec{b})\times\vec{c}\}\times\vec{d}$ are also different vectors products of four vectors $\vec{a},\vec{b},\vec{c}$ and $\vec{d}$.

To Express a Vector in Terms of Noncoplanar Vectors

- If $\vec{a},\vec{b}$ are two known noncollinear vectors then $\vec{a},\vec{b},\vec{a}\times\vec{b}$ are three noncoplanar vectors. So, any vector $\vec{x}$ can be written as $\vec{x} = \lambda\vec{a}+\mu\vec{b}+\nu\,\vec{a}\times\vec{b}$ where λ,μ,ν are unknown scalars.
- If are three known noncoplalar vectors then any vectors can be written as where λ,μ,ν are unknown scalars.

Note:

The above two ways of expressing $\vec{x}$ is useful in solving vector equations.

MULTIPLE CHOICE QUESTIONS

Rectangular Resolution of a Vector in Two and Three Dimensional Systems

1. If $\mathbf{a}$ is a non-zero vector of modulus a and m is a non-zero scalar, then $m\mathbf{a}$ is a unit vector if :

a. $m = \pm 1$ **b.** $m = |\mathbf{a}|$

c. $m = \dfrac{1}{|\mathbf{a}|}$ **d.** $m = \pm 2$

2. For a non-zero vector $\mathbf{a}$, the set of real numbers, satisfying $|(5-x)\mathbf{a}| < |2\mathbf{a}|$ consists of all x such that:

a. $0 < x < 3$ **b.** $3 < x < 7$

c. $-7 < x < -3$ **d.** $-7 < x < 3$

Properties of Vectors

3. If $ABCDEF$ is a regular hexagon, then $\overrightarrow{AD} + \overrightarrow{EB} + \overrightarrow{FC} = ?$

a. $\vec{O}$ **b.** $2\overrightarrow{AB}$ **c.** $3\overrightarrow{AB}$ **d.** $4\overrightarrow{AB}$

4. The unit vector parallel to the resultant vector of $2\mathbf{i}+4\mathbf{j}-5\mathbf{k}$ and $\mathbf{i}+2\mathbf{j}+3\mathbf{k}$ is:

a. $\frac{1}{7}(3\mathbf{i}+6\mathbf{j}-2\mathbf{k})$ **b.** $\dfrac{\mathbf{i}+\mathbf{j}+\mathbf{k}}{\sqrt{3}}$

c. $\dfrac{\mathbf{i}+\mathbf{j}+2\mathbf{k}}{\sqrt{6}}$ **d.** $\frac{1}{\sqrt{69}}(-\mathbf{i}-\mathbf{j}+8\mathbf{k})$

5. The vector $\mathbf{c}$, directed along the internal bisector of the angle between the vectors $\mathbf{a}=7\mathbf{i}-4\mathbf{j}-4\mathbf{k}$ and $\mathbf{b}=-2\mathbf{i}-\mathbf{j}+2\mathbf{k}$ with $|\mathbf{c}| = 5\sqrt{6}$, is:

a. $\frac{5}{3}(\mathbf{i}-7\mathbf{j}+2\mathbf{k})$ **b.** $\frac{5}{3}(5\mathbf{i}+5\mathbf{j}+2\mathbf{k})$

c. $\frac{5}{3}(\mathbf{i}+7\mathbf{j}+2\mathbf{k})$ **d.** $\frac{5}{3}(-5\mathbf{i}+5\mathbf{j}+2\mathbf{k})$

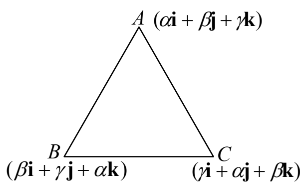
Position Vector

6. If position vector of a point A is $\mathbf{a}+2\mathbf{b}$ and $\mathbf{a}$ divides AB in the ratio 2 : 3, then the position vector of B is:

a. $2\mathbf{a}-\mathbf{b}$ **b.** $\mathbf{b}-2\mathbf{a}$

c. $\mathbf{a}-3\mathbf{b}$ **d.** $\mathbf{b}$

7. Let α, β, γ be distinct real numbers. The points with position vectors $\alpha\mathbf{i}+\beta\mathbf{j}+\gamma\mathbf{k}$, $\beta\mathbf{i}+\gamma\mathbf{j}+\alpha\mathbf{k}$, $\gamma\mathbf{i}+\alpha\mathbf{j}+\beta\mathbf{k}$?



a. Are collinear

b. Form an equilateral triangle

c. Form a scalene triangle

d. Form a right angled triangle

Linear Independence and Dependence of Vectors

8. Let $\mathbf{a}, \mathbf{b}$ and $\mathbf{c}$ be three non-zero vectors such that no two of these are collinear. If the vector $\mathbf{a}+2\mathbf{b}$ is collinear with $\mathbf{c}$ and $\mathbf{b}+3\mathbf{c}$ is collinear with $\mathbf{a}$ (λ being some non-zero scalar) then $\mathbf{a}+2\mathbf{b}+6\mathbf{c}$ equals:

a. $\mathbf{0}$ **b.** $\lambda\mathbf{b}$

c. $\lambda\mathbf{c}$ **d.** $\lambda\mathbf{a}$

9. If the vectors $4\mathbf{i}+11\mathbf{j}+m\mathbf{k}, 7\mathbf{i}+2\mathbf{j}+6\mathbf{k}$ and $\mathbf{i}+5\mathbf{j}+4\mathbf{k}$ are coplanar, then m is:

a. 38 **b.** 0

c. 10 **d.** – 10

10. The perimeter of the triangle whose vertices have the position vectors $(\mathbf{i}+\mathbf{j}+\mathbf{k})$, $(5\mathbf{i}+3\mathbf{j}-3\mathbf{k})$ and $(2\mathbf{i}+5\mathbf{j}+9\mathbf{k})$, is given by:

a. $15+\sqrt{157}$ **b.** $15-\sqrt{157}$

c. $\sqrt{15}-\sqrt{157}$ **d.** $\sqrt{15}+\sqrt{157}$

11. The magnitudes of mutually perpendicular forces **a, b** and **c** are 2, 10 and 11 respectively. Then the magnitude of its resultant is:

a. 12 **b.** 15

c. 9 **d.** None

12. The position vectors of P and Q are $5\mathbf{i}+4\mathbf{j}+a\mathbf{k}$ and $-\mathbf{i}+2\mathbf{j}-2\mathbf{k}$ respectively. If the distance between them is 7, then the value of a will be:

a. – 5, 1 **b.** 5, 1

c. 0, 5 **d.** 1, 0

13. If **a**, **b**, **c**, **d** be the position vectors of the points *A, B, C* and *D* respectively referred to same origin *O* such that no three of these points are collinear and $\mathbf{a}+\mathbf{c}=\mathbf{b}+\mathbf{d}$, then quadrilateral *ABCD* is a:

a. Square **b.** Rhombus

c. Rectangle **d.** Parallelogram

14. If OP = 8 and $\overrightarrow{OP}$ makes angles 45° and 60° with OX-axis and OY-axis respectively, then $\overrightarrow{OP} = ?$

a. $8(\sqrt{2}\mathbf{i}+\mathbf{j}\pm\mathbf{k})$ **b.** $4(\sqrt{2}\mathbf{i}+\mathbf{j}\pm\mathbf{k})$

c. $\frac{1}{4}(\sqrt{2}\mathbf{i}+\mathbf{j}\pm\mathbf{k})$ **d.** $\frac{1}{8}(\sqrt{2}\mathbf{i}+\mathbf{j}\pm\mathbf{k})$

15. The position vectors of A and B are $2\mathbf{i}-9\mathbf{j}-4\mathbf{k}$ and $6\mathbf{i}-3\mathbf{j}+8\mathbf{k}$ respectively, then the magnitude of $\overrightarrow{AB}$ is:

a. 11 **b.** 12
c. 13 **d.** 14

16. ABC is an isosceles triangle right angled at A. Forces of magnitude $2\sqrt{2}, 5$ and 6 act along $\overline{BC}, \overline{CA}$ and $\overline{AB}$ respectively. The magnitude of their resultant force is:

a. 4 **b.** 5
c. $11+2\sqrt{2}$ **d.** 30

17. What should be added in vector $\mathbf{a}=3\mathbf{i}+4\mathbf{j}-2\mathbf{k}$ to get its resultant a unit vector $\mathbf{i}$?

a. $-2\mathbf{i}-4\mathbf{j}+2\mathbf{k}$ **b.** $-2\mathbf{i}+4\mathbf{j}-2\mathbf{k}$
c. $2\mathbf{i}+4\mathbf{j}-2\mathbf{k}$ **d.** None of these

18. In a trapezium, the vector $\overrightarrow{BC}=\lambda\overrightarrow{AD}$. We will then find that $\mathbf{p}=\overrightarrow{AC}+\overrightarrow{BD}$ is collinear with $\overrightarrow{AD}$, If $\mathbf{p}=\mu\overrightarrow{AD}$, then:

a. $\mu=\lambda+1$ **b.** $\lambda=\mu+1$
c. $\lambda+\mu=1$ **d.** $\mu=2+\lambda$

19. If $ABCD$ is a parallelogram and the position vectors of A, B, C are $\mathbf{i}+3\mathbf{j}+5\mathbf{k}$, $\mathbf{i}+\mathbf{j}+\mathbf{k}$ and $7\mathbf{i}+7\mathbf{j}+7\mathbf{k}$, then the position vector of D will be:

a. $7\mathbf{i}+5\mathbf{j}+3\mathbf{k}$ **b.** $7\mathbf{i}+9\mathbf{j}+11\mathbf{k}$
c. $9\mathbf{i}+11\mathbf{j}+13\mathbf{k}$ **d.** $8\mathbf{i}+8\mathbf{j}+8\mathbf{k}$

Product of Two Vectors

20. $(\mathbf{a}.\mathbf{i})\mathbf{i}+(\mathbf{a}.\mathbf{j})\,\mathbf{j}+(\mathbf{a}.\mathbf{k})\mathbf{k}=?$

a. $\mathbf{a}$ **b.** $2\mathbf{a}$
c. $3\mathbf{a}$ **d.** $\mathbf{0}$

21. If $|\mathbf{a}|=3, |\mathbf{b}|=4$ then a value of λ for which $\mathbf{a}+\lambda\mathbf{b}$ is perpendicular to $\mathbf{a}-\lambda\mathbf{b}$ is:

a. 9/16 **b.** 3/4
c. 3/2 **d.** 4/3

22. The vectors $\mathbf{a}=2\lambda^2\mathbf{i}+4\lambda\mathbf{j}+\mathbf{k}$ and $\mathbf{b}=7\mathbf{i}-2\mathbf{j}+\lambda\mathbf{k}$ make an obtuse angle whereas the angle between $\mathbf{b}$ and $\mathbf{k}$ is acute and less than $\pi/6$, then domain of λ is:

a. $0<\lambda<\frac{1}{2}$ **b.** $\lambda>\sqrt{159}$
c. $-\frac{1}{2}<\lambda<0$ **d.** Null set

23. In cartesian co-ordinates the point A is (x_1, y_1) where $x_1=1$ on the curve $y=x^2+x+10$. The tangent at A cuts the x-axis at B. The value of the dot product $\overrightarrow{OA}.\overrightarrow{AB}$ is:

a. $-\frac{520}{3}$ **b.** -148 **c.** 140 **d.** 12

24. The projection of $\mathbf{a}=2\mathbf{i}+3\mathbf{j}-2\mathbf{k}$ on $\mathbf{b}=\mathbf{i}+2\mathbf{j}+3\mathbf{k}$ is:

a. $\frac{1}{\sqrt{14}}$ **b.** $\frac{2}{\sqrt{14}}$ **c.** $\sqrt{14}$ **d.** $\frac{-2}{\sqrt{14}}$

25. Let $\mathbf{u}, \mathbf{v}, \mathbf{w}$ be such that $|\mathbf{u}|=1, |\mathbf{v}|=2, |\mathbf{w}|=3$. If the projection $\mathbf{v}$ along $\mathbf{u}$ is equal to that of $\mathbf{w}$ along $\mathbf{u}$ and $\mathbf{v}$, $\mathbf{w}$ are perpendicular to each other then $|\mathbf{u}-\mathbf{v}+\mathbf{w}|$ equals:

a. 14 **b.** $\sqrt{7}$ **c.** $\sqrt{14}$ **d.** 2

26. A particle is acted upon by constant forces $4\mathbf{i}+\mathbf{j}-3\mathbf{k}$ and $3\mathbf{i}+\mathbf{j}-\mathbf{k}$ which displace it from a point $\mathbf{i}+2\mathbf{j}+3\mathbf{k}$ to the point $5\mathbf{i}+4\mathbf{j}+\mathbf{k}$. The work done in standard units by the force is given by:

a. 15 **b.** 30 **c.** 25 **d.** 40

27. A groove is in the form of a broken line ABC and the position vectors of the three points are respectively $2\mathbf{i}-3\mathbf{j}+2\mathbf{k}$, $3\mathbf{i}+2\mathbf{j}-\mathbf{k}$ and $\mathbf{i}+\mathbf{j}+\mathbf{k}$. A force of magnitude $24\sqrt{3}$ acts on a particle of unit mass kept at the point A and moves it along the groove to the point C. If the line of action of the force is parallel to the vector $\mathbf{i}+2\mathbf{j}+\mathbf{k}$ all along, the number of units of work done by the force is:

a. $144\sqrt{2}$ **b.** $144\sqrt{3}$
c. $72\sqrt{2}$ **d.** $72\sqrt{3}$

28. If $\mathbf{a}, \mathbf{b}, \mathbf{c}$ are non-zero vectors such that $\mathbf{a}.\mathbf{b}=\mathbf{a}.\mathbf{c}$, then which statement is true:

a. $\mathbf{b}=\mathbf{c}$ **b.** $\mathbf{a}\perp(\mathbf{b}-\mathbf{c})$
c. $\mathbf{b}=\mathbf{c}$ or $\mathbf{a}\perp(\mathbf{b}-\mathbf{c})$ **d.** None of these

29. If $\mathbf{a}$, $\mathbf{b}$, $\mathbf{c}$ are mutually perpendicular unit vectors, then $|\mathbf{a}+\mathbf{b}+\mathbf{c}|=?$

a. $\sqrt{3}$ **b.** 3 **c.** 1 **d.** 0

30. If θ be the angle between the unit vectors $\mathbf{a}$ and $\mathbf{b}$, then $\cos\frac{\theta}{2}=?$

a. $\frac{1}{2}|\mathbf{a}-\mathbf{b}|$ **b.** $\frac{1}{2}|\mathbf{a}+\mathbf{b}|$
c. $\frac{|\mathbf{a}-\mathbf{b}|}{|\mathbf{a}+\mathbf{b}|}$ **d.** $\frac{|\mathbf{a}+\mathbf{b}|}{|\mathbf{a}-\mathbf{b}|}$

31. If the vector $\mathbf{i}+\mathbf{j}+\mathbf{k}$ makes angles α, β, γ with vectors $\mathbf{i}, \mathbf{j}, \mathbf{k}$ respectively, then:

a. $\alpha = \beta \neq \gamma$ **b.** $\alpha = \gamma \neq \beta$

c. $\beta = \gamma \neq \alpha$ **d.** $\alpha = \beta = \gamma$

32. If a unit vector lies in *yz*–plane and makes angles of 30° and 60° with the positive *y*-axis and *z*-axis respectively, then its components along the co-ordinate axes will be:

a. $\frac{\sqrt{3}}{2}, \frac{1}{2}, 0$ **b.** $0, \frac{\sqrt{3}}{2}, \frac{1}{2}$

c. $\frac{\sqrt{3}}{2}, 0, \frac{1}{2}$ **d.** $0, \frac{1}{2}, \frac{\sqrt{3}}{2}$

33. A vector whose modulus is $\sqrt{51}$ and makes the same angle with $\mathbf{a} = \frac{\mathbf{i}-2\mathbf{j}+2\mathbf{k}}{3}$, $\mathbf{b} = \frac{-4\mathbf{i}-3\mathbf{k}}{5}$ and $\mathbf{c} = \mathbf{j}$, will be:

a. $5\mathbf{i}+5\mathbf{j}+\mathbf{k}$ **b.** $5\mathbf{i}+\mathbf{j}-5\mathbf{k}$

c. $5\mathbf{i}+\mathbf{j}+5\mathbf{k}$ **d.** $\pm(5\mathbf{i}-\mathbf{j}-5\mathbf{k})$

34. If $\mathbf{d} = \lambda(\mathbf{a}\times\mathbf{b}) + \mu(\mathbf{b}\times\mathbf{c}) + \nu(\mathbf{c}\times\mathbf{a})$ and $[\mathbf{a}\,\mathbf{b}\,\mathbf{c}] = \frac{1}{8}$, then $\lambda+\mu+\nu$ is equal to:

a. $8\mathbf{d}.(\mathbf{a}+\mathbf{b}+\mathbf{c})$ **b.** $8\mathbf{d}\times(\mathbf{a}+\mathbf{b}+\mathbf{c})$

c. $\frac{\mathbf{d}}{8}.(\mathbf{a}+\mathbf{b}+\mathbf{c})$ **d.** $\frac{\mathbf{d}}{8}\times(\mathbf{a}+\mathbf{b}+\mathbf{c})$

35. If $|\mathbf{a}|=3, |\mathbf{b}|=1, |\mathbf{c}|=4$ and $\mathbf{a}+\mathbf{b}+\mathbf{c}=\mathbf{0}$, then $\mathbf{a}.\mathbf{b}+\mathbf{b}.\mathbf{c}+\mathbf{c}.\mathbf{a}=?$

a. – 13 **b.** – 10

c. 13 **d.** 10

36. If **a** is any vector in space, then:

a. $\mathbf{a}=(\mathbf{a}.\mathbf{i})\mathbf{i}+(\mathbf{a}.\mathbf{j})\mathbf{j}+(\mathbf{a}.\mathbf{k})\mathbf{k}$

b. $\mathbf{a}=(\mathbf{a}\times\mathbf{i})+(\mathbf{a}\times\mathbf{j})+(\mathbf{a}\times\mathbf{k})$

c. $\mathbf{a}=\mathbf{j}(\mathbf{a}.\mathbf{i})+\mathbf{k}(\mathbf{a}.\mathbf{j})+\mathbf{i}(\mathbf{a}.\mathbf{k})$

d. $\mathbf{a}=(\mathbf{a}\times\mathbf{i})\times\mathbf{i}+(\mathbf{a}\times\mathbf{j})\times\mathbf{j}+(\mathbf{a}\times\mathbf{k})\times\mathbf{k}$

37. If $\mathbf{a}=(1,-1,2), \mathbf{b}=(-2,3,5), \mathbf{c}=(2,-2,4)$ and $\mathbf{i}$ is the unit vector in the *x*-direction, then $(\mathbf{a}-2\mathbf{b}+3\mathbf{c}).\mathbf{i}=?$

a. 11 **b.** 15

c. 18 **d.** 36

Vector or Cross Product of Two Vectors

38. The sine of the angle between the vectors $\mathbf{a}=3\mathbf{i}+\mathbf{j}+\mathbf{k}$, $\mathbf{b}=2\mathbf{i}-2\mathbf{j}+\mathbf{k}$ is:

a. $\sqrt{\frac{74}{99}}$ **b.** $\sqrt{\frac{25}{99}}$ **c.** $\sqrt{\frac{37}{99}}$ **d.** $\frac{5}{\sqrt{41}}$

39. The vectors $\mathbf{c}$, $\mathbf{a}=x\mathbf{i}+y\mathbf{j}+z\mathbf{k}$ and $\mathbf{b}=\mathbf{j}$ are such that **a, c, b** form a right handed system, then **c** is:

a. $z\mathbf{i}-x\mathbf{k}$ **b.** 0 **c.** $y\mathbf{j}$ **d.** $-z\mathbf{i}+x\mathbf{k}$

40. The area of a triangle whose vertices are $A(1,-1,2)$, $B(2,1,-1)$ and $C(3,-1,2)$ is:

a. 13 **b.** $\sqrt{13}$ **c.** 6 **d.** $\sqrt{6}$

41. If $\mathbf{a}=\mathbf{i}+\mathbf{j}+\mathbf{k}, \mathbf{b}=\mathbf{i}+3\mathbf{j}+5\mathbf{k}$ and $\mathbf{c}=7\mathbf{i}+9\mathbf{j}+11\mathbf{k}$, then the area of the parallelogram having diagonals $\mathbf{a}+\mathbf{b}$ and $\mathbf{b}+\mathbf{c}$ is:

a. $4\sqrt{6}$ **b.** $\frac{1}{2}\sqrt{21}$ **c.** $\frac{\sqrt{6}}{2}$ **d.** $\sqrt{6}$

42. If **a** and **b** are unit vectors such that $\mathbf{a}\times\mathbf{b}$ is also a unit vector, then the angle between **a** and **b** is:

a. 0 **b.** $\frac{\pi}{3}$ **c.** $\frac{\pi}{2}$ **d.** π

43. If $\mathbf{a}+\mathbf{b}+\mathbf{c}=\mathbf{0}$, then which relation is correct:

a. $\mathbf{a}=\mathbf{b}=\mathbf{c}=\mathbf{0}$ **b.** $\mathbf{a}.\mathbf{b}=\mathbf{b}.\mathbf{c}=\mathbf{c}.\mathbf{a}$

c. $\mathbf{a}\times\mathbf{b}=\mathbf{b}\times\mathbf{c}=\mathbf{c}\times\mathbf{a}$ **d.** None of these

44. $(2\mathbf{a}+3\mathbf{b})\times(5\mathbf{a}+7\mathbf{b})=?$

a. $\mathbf{a}\times\mathbf{b}$ **b.** $\mathbf{b}\times\mathbf{a}$

c. $\mathbf{a}+\mathbf{b}$ **d.** $7\mathbf{a}+10\mathbf{b}$

45. $|(\mathbf{a}\times\mathbf{b}).\mathbf{c}|=|\mathbf{a}||\mathbf{b}||\mathbf{c}|$, if :

a. $\mathbf{a}.\mathbf{b}=\mathbf{b}.\mathbf{c}=0$ **b.** $\mathbf{b}.\mathbf{c}=\mathbf{c}.\mathbf{a}=0$

c. $\mathbf{c}.\mathbf{a}=\mathbf{a}.\mathbf{b}=0$ **d.** $\mathbf{a}.\mathbf{b}=\mathbf{b}.\mathbf{c}=\mathbf{c}.\mathbf{a}=0$

46. If $\mathbf{a}\times\mathbf{b}=\mathbf{b}\times\mathbf{c}\neq 0$, where **a, b** and **c** are coplanar vectors, then for some scalar *k*:

a. $\mathbf{a}+\mathbf{c}=k\,\mathbf{b}$ **b.** $\mathbf{a}+\mathbf{b}=k\,\mathbf{c}$

c. $\mathbf{b}+\mathbf{c}=k\,\mathbf{a}$ **d.** None of these

Moment of a Force and Couple

47. Three forces $\mathbf{i}+2\mathbf{j}-3\mathbf{k}, 2\mathbf{i}+3\mathbf{j}+4\mathbf{k}$ and $\mathbf{i}-\mathbf{j}+\mathbf{k}$ are acting on a particle at the point (0, 1, 2). The magnitude of the moment of the forces about the point (1, – 2, 0) is:

a. $2\sqrt{35}$ **b.** $6\sqrt{10}$

c. $4\sqrt{17}$ **d.** None of these

48. The moment of the couple formed by the forces $5\mathbf{i}+\mathbf{k}$ and $-5\mathbf{i}-\mathbf{k}$ acting at the points $(9,-1,2)$ and $(3,-2,1)$ respectively is:

a. $-\mathbf{i}+\mathbf{j}+5\mathbf{k}$ **b.** $\mathbf{i}-\mathbf{j}-5\mathbf{k}$

c. $2\mathbf{i}-2\mathbf{j}-10\mathbf{k}$ **d.** $-2\mathbf{i}+2\mathbf{j}+10\mathbf{k}$

Scalar Triple Product

49. If **u**, **v** and **w** are three non-coplanar vectors, then $(\mathbf{u}+\mathbf{v}-\mathbf{w}).[(\mathbf{u}-\mathbf{v})\times(\mathbf{v}-\mathbf{w})]$ equals:

a. 0 **b.** $\mathbf{u}.(\mathbf{v}\times\mathbf{w})$

c. $\mathbf{u}.(\mathbf{w}\times\mathbf{v})$ **d.** $3\mathbf{u}.(\mathbf{v}\times\mathbf{w})$

50. If $\mathbf{a},\mathbf{b},\mathbf{c}$ are non-coplanar vectors and λ is a real number, then the vectors $\mathbf{a}+2\mathbf{b}+3\mathbf{c}, \lambda\mathbf{b}+4\mathbf{c}$ and $(2\lambda-1)\mathbf{c}$ are non-coplanar for:

a. No value of λ **b.** All except one value of λ

c. All except two values of λ **d.** All values of λ

51. $\mathbf{x},\mathbf{y},\mathbf{z}$ are distinct scalars such that $[x\mathbf{a}+y\mathbf{b}+z\mathbf{c}, x\mathbf{b}+y\mathbf{c}+z\mathbf{a}, x\mathbf{c}+y\mathbf{a}+z\mathbf{b}]=0$ where $\mathbf{a},\mathbf{b},\mathbf{c}$ are non-coplanar vectors then:

a. $x+y+z=0$ **b.** $x+y+z=0$

c. $x^3+y^3+z^3=0$ **d.** $x^2+y^2+z^2=0$

52. If **a**, **b**, **c** are three non-coplanar vector, then $\frac{\mathbf{a}.\mathbf{b}\times\mathbf{c}}{\mathbf{c}\times\mathbf{a}.\mathbf{b}}+\frac{\mathbf{b}.\mathbf{a}\times\mathbf{c}}{\mathbf{c}.\mathbf{a}\times\mathbf{b}}=?$

a. 0 **b.** 2

c. – 2 **d.** None of these

53. If the vectors $2\mathbf{i}-3\mathbf{j}$, $\mathbf{i}+\mathbf{j}-\mathbf{k}$ and $3\mathbf{i}-\mathbf{k}$ form three concurrent edges of a parallelopiped, then the volume of the parallelopiped is:

a. 8 **b.** 10

c. 4 **d.** 14

54. If the vectors $2\mathbf{i}-\mathbf{j}+\mathbf{k}$, $\mathbf{i}+2\mathbf{j}-3\mathbf{k}$ and $3\mathbf{i}+\lambda\mathbf{j}+5\mathbf{k}$ be coplanar, then $\lambda=?$

a. – 1 **b.** – 2

c. – 3 **d.** – 4

55. If $\mathbf{a}=2\mathbf{i}+\mathbf{j}-\mathbf{k}$, $\mathbf{b}=\mathbf{i}+2\mathbf{j}+\mathbf{k}$ and $\mathbf{c}=\mathbf{i}-\mathbf{j}+2\mathbf{k}$, then $\mathbf{a}.(\mathbf{b}\times\mathbf{c})=?$

a. 6 **b.** 10

c. 12 **d.** 24

Vector Triple Product

56. Let $\mathbf{a},\mathbf{b}$ and $\mathbf{c}$ be non-zero vectors such that $(\mathbf{a}\times\mathbf{b})\times\mathbf{c}=\frac{1}{3}|\mathbf{b}||\mathbf{c}|\mathbf{a}$. If θ is the acute angle between the vectors $\mathbf{b}$ and $\mathbf{c}$, then $\sin\theta$ equals:

a. $\frac{2\sqrt{2}}{3}$ **b.** $\frac{\sqrt{2}}{3}$ **c.** $\frac{2}{3}$ **d.** $\frac{1}{3}$

Scalar and Vector product of Four Vectors

57. $\mathbf{a}\times[\mathbf{a}\times(\mathbf{a}\times\mathbf{b})]$ is equal to:

a. $(\mathbf{a}\times\mathbf{a}).(\mathbf{b}\times\mathbf{a})$ **b.** $\mathbf{a}.(\mathbf{b}\times\mathbf{a})-\mathbf{b}.(\mathbf{a}\times\mathbf{b})$

c. $[\mathbf{a}.(\mathbf{a}\times\mathbf{b})]\mathbf{a}$ **d.** $(\mathbf{a}.\mathbf{a})(\mathbf{b}\times\mathbf{a})$

58. $[\mathbf{b}\times\mathbf{c}\;\mathbf{c}\times\mathbf{a}\;\mathbf{a}\times\mathbf{b}]$ is equal to:

a. $\mathbf{a}\times(\mathbf{b}\times\mathbf{c})$ **b.** $2[\mathbf{a}\,\mathbf{b}\,\mathbf{c}]$

c. $[\mathbf{a}\,\mathbf{b}\,\mathbf{c}]^2$ **d.** $[\mathbf{a}\,\mathbf{b}\,\mathbf{c}]$

59. Let the vectors $\mathbf{a},\mathbf{b},\mathbf{c}$ and $\mathbf{d}$ be such that $(\mathbf{a}\times\mathbf{b})\times(\mathbf{c}\times\mathbf{d})=\mathbf{0}$. Let P_1 and P_2 be planes determined by pair of vectors $\mathbf{a},\mathbf{b}$ and $\mathbf{c},\mathbf{d}$ respectively. Then the angle between P_1 and P_2 is:

a. $0°$ **b.** $\frac{\pi}{4}$

c. $\frac{\pi}{3}$ **d.** $\frac{\pi}{2}$

Vector Equations

60. If $\mathbf{a}=\mathbf{i}+\mathbf{j}+\mathbf{k}$, $\mathbf{a}.\mathbf{b}=1$ and $\mathbf{a}\times\mathbf{b}=\mathbf{j}-\mathbf{k}$, then $\mathbf{b}=?$

a. $\mathbf{i}$ **b.** $\mathbf{i}-\mathbf{j}+\mathbf{k}$ **c.** $2\mathbf{j}-\mathbf{k}$ **d.** $2\mathbf{i}$

61. Let $\mathbf{p},\mathbf{q},\mathbf{r}$ be three mutually perpendicular vectors of the same magnitude. If a vector $\mathbf{x}$ satisfies equation $\mathbf{p}\times|(\mathbf{x}-\mathbf{q})\times\mathbf{p}|+\mathbf{q}\times|(\mathbf{x}-\mathbf{r})\times\mathbf{q}|+\mathbf{r}\times|(\mathbf{x}-\mathbf{p})\times\mathbf{r}|=0$, then $\mathbf{x}$ is given by:

a. $\frac{1}{2}(\mathbf{p}+\mathbf{q}-2\mathbf{r})$ **b.** $\frac{1}{2}(\mathbf{p}+\mathbf{q}+\mathbf{r})$

c. $\frac{1}{3}(\mathbf{p}+\mathbf{q}+\mathbf{r})$ **d.** $\frac{1}{3}(2\mathbf{p}+\mathbf{q}-\mathbf{r})$

62. Let the unit vectors **a** and **b** be perpendicular and the unit vector **c** be inclined at an angle θ to both **a** and **b**. If $\mathbf{c}=\alpha\mathbf{a}+\beta\mathbf{b}+\gamma(\mathbf{a}\times\mathbf{b})$, then:

a. $\alpha=\beta=\cos\theta,\ \gamma^2=\cos 2\theta$

b. $\alpha=\beta=\cos\theta, \gamma^2=-\cos 2\theta$

c. $\alpha=\cos\theta, \beta=\sin\theta, \gamma^2=\cos 2\theta$

d. None of these

63. The locus of a point equidistant from two given points whose position vectors are **a** and **b** is equal to:

a. $\left[\mathbf{r}-\frac{1}{2}(\mathbf{a}+\mathbf{b})\right].(\mathbf{a}+\mathbf{b})=0$ **b.** $\left[\mathbf{r}-\frac{1}{2}(\mathbf{a}+\mathbf{b})\right].(\mathbf{a}-\mathbf{b})=0$

c. $\left[\mathbf{r}-\frac{1}{2}(\mathbf{a}+\mathbf{b})\right].\mathbf{a}=0$ **d.** $[\mathbf{r}-(\mathbf{a}+\mathbf{b})].\mathbf{b}=0$

NCERT EXEMPLAR PROBLEMS

More than One Answer

64. Let $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}, \vec{b} = \hat{i} + 2j - \hat{k}$ and $\vec{c} = \hat{i} + \hat{j} - 2\hat{k}$ be three vectors. A vector in the plane of $\vec{b}$ and $\vec{c}$ whose projection on $\vec{a}$ is of magnitude $\sqrt{2/3}$, is:

a. $2\hat{i} + 3\hat{j} - 3\hat{k}$ **b.** $2\hat{i} + 3\hat{j} + 3\hat{k}$

c. $-2\hat{i} - \hat{j} + 5\hat{k}$ **d.** $2\hat{i} + \hat{j} + 5\hat{k}$

65. Which of the following expressions are meaningful question?

a. $\vec{u} \cdot (\vec{v} \times \vec{w})$ **b.** $(\vec{u} \cdot \vec{v}) \cdot \vec{w}$

c. $(\vec{u} \cdot \vec{v})\vec{w}$ **d.** $\vec{u} \times (\vec{v} \cdot \vec{w})$

66. Let $\vec{a}$ and $\vec{b}$ be two non-collinear unit vectors. If $\vec{u} = \vec{a} - (\vec{a} \cdot \vec{b})\vec{b}$ and $\vec{v} = \vec{a} \times \vec{b}$, then $|\vec{v}|$ is:

a. $|\vec{u}|$ **b.** $|\vec{u}| + |\vec{u} \cdot \vec{a}|$

c. $|\vec{u}| + |\vec{u} \cdot \vec{b}|$ **d.** $|\vec{u}| + \vec{u} \cdot (\vec{a} + \vec{b})$

67. Let $\vec{A}$ be vector parallel to line of intersection of planes P_1 and P_2 through origin. P_1 is parallel to the vectors $2\hat{j} + 3\hat{k}$ and $4\hat{j} - 3\hat{k}$ and P_2 is parallel to $\hat{j} - \hat{k}$ and $3\hat{i} + 3\hat{j}$, then the angle between vector $\vec{A}$ and $2\hat{i} + \hat{j} - 2\hat{k}$ is:

a. $\frac{\pi}{2}$ **b.** $\frac{\pi}{4}$

c. $\frac{\pi}{6}$ **d.** $\frac{3\pi}{4}$

68. The vector(s) which is/are coplanar with vectors $\hat{i} + \hat{j} + 2\hat{k}$ and $\hat{i} + 2\hat{j} + \hat{k}$, are perpendicular to the vector $\hat{i} + \hat{j} + \hat{k}$ is/are:

a. $\hat{j} - \hat{k}$ **b.** $-\hat{i} + \hat{j}$

c. $\hat{i} - \hat{j}$ **d.** $-\hat{j} + \hat{k}$

69. If $\vec{a}, \vec{b}, \vec{c}$ $|\vec{a}| = 4, |\vec{b}| = 2$ and the angle between $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{6}$ then $(\vec{a} \times \vec{b})$ is:

a. 48 **b.** $(\vec{a})^2$ **c.** 16 **d.** 32

70. If the unit vectors $\vec{a}$ and $\vec{b}$ are inclined at an angle 2θ such that $\left|\vec{a} - \vec{b}\right| <$ and $0 \le \theta \le \pi$, then θ lies in the interval:

a. $\left[0, \frac{\pi}{6}\right]$ **b.** $\left[\frac{5\pi}{6}, \pi\right]$ **c.** $\left[\frac{\pi}{6}, \frac{\pi}{2}\right]$ **d.** $\left[\frac{\pi}{2}, \frac{5\pi}{6}\right]$

71. The vectors $2\hat{i} - \lambda\hat{j} + 3\lambda\hat{k}$ and $(1+\lambda)\hat{i} - 2\lambda\hat{j} + \hat{k}$ include an acute angle for:

a. all values of m **b.** $\lambda < -2$

c. $\lambda > -\frac{1}{2}$ **d.** $\lambda \in [-2, -1/2]$

72. Let $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}, b = \hat{i} + 2\hat{j} - \hat{k}$ and $c = \hat{i} + \hat{j} - 2\hat{k}$ be three vectors. A vector in the plane of $\vec{b}$ and $\vec{c}$ whose projection at $\vec{a}$ is of magnitude $\sqrt{(2/3)}$?

a. $2i + 3j - 3k$ **b.** $2i + 3j + 3k$

c. $-2i - j + 5k$ **d.** $2i + j + 5k$

73. The vector $(x, x+1, x+2), (x+3, x+4, x+5)$ and $(x+6, x+7, x+8)$ are coplanar for:

a. all values of x **b.** $x < 0$

c. $x > 0$ **d.** None of these

Assertion and Reason

Note: Read the Assertion (A) and Reason (R) carefully to mark the correct option out of the options given below:

a. If both assertion and reason are true and the reason is the correct explanation of the assertion.

b. If both assertion and reason are true but reason is not the correct explanation of the assertion.

c. If assertion is true but reason is false.

d. If the assertion and reason both are false.

e. If assertion is false but reason is true.

74. **Assertion:** If vectors $\vec{a}$ and $\vec{c}$ are non-collinear then the lines $\vec{r} = 6\vec{a} - \vec{c} + \lambda(2\vec{c} - \vec{a})$; $\vec{r} = \vec{a} - \vec{c} + \mu(\vec{a} + \vec{c})$ are coplanar

Reason: There exist λ and μ such that the two values of $\vec{r}$ become same

75. Given that $\vec{a}, \vec{b}, \vec{c}$ are the position vectors of the vertices of ΔABC

Assertion: The area of ΔABC is $\frac{1}{2}[\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}]$

Reason: Cross product is distributive over addition of vectors

76. Let A, B, C be three point with position vectors $\hat{i} + 2\hat{j} - \hat{k}$, $2\hat{i} + 3\hat{k}, 3\hat{i} - \hat{j} + 2\hat{k}$

Assertion: The angle between $\overrightarrow{AB}$ and $\overrightarrow{AC}$ is acute

Reason: If θ is the angel between $\overrightarrow{AB}$ and $\overrightarrow{AC}$ then $\cos\theta = \frac{17}{\sqrt{21}\sqrt{22}}$