Telangana Inter 2nd Year Maths Exam 2025 : Important Questions & Answers

MATHS-1B

2MARKS IMP. QUESTIONS

STRAIGHT LINES

1. If the area of the triangle formed by the straight lines x =0, y =0 and 3x + 4y =1 (a >0) is 6. Find the value of 'a'.
2. Find the area of the triangle formed by the straight line x -4y +2= 0 with the coordinate axes.
3. Find the area of the triangle formed by the straight line $x\cos\alpha + y\sin\alpha = p$ and the coordinate axis
4. Find the equation of the straight line passing through (-4, 5) and cutting off equal nonzero intercepts on the coordinate axes.
5. Find the equation of the straight line passing through the point (3, -4) and making X and Y-intercepts which are in the ratio 2 :3.
6. Find the equation of the straight line passing through (-2, 4) and making non -zero intercepts whose sum is zero.
7. Find the distance between the parallel straight lines 5x-3y-4= 0, 10x-6y-9=0.
8. Find the equation of the straight line passing through the points $\left(at_1^2, 2at_1\right)$ and $\left(at_2^2, 2at_2\right)$.
9. Find the length of the perpendicular drawn from the point (-2, -3) to the straight line 5x-2y +4= 0.
10. Find the value of k, if the straight lines 6x-10y +3= 0 and kx -5y + 8 =0 are parallel.
11. Find the value of p, if the straight lines 3x + 7y -1= 0 and 7x - py + 3=0 are mutually perpendicular.
12. If, a, b, c are in arithmetic progression, then show that the equation ax + by + c = 0 represents a family of concurrent lines and find the point of concurrency.
13. Transofrm the equation (2 +5k)x -3(1+2k)y + (2-k) = 0 into the form $L_1 + \lambda L_2 = 0$ and find the point og concurrency of the family of straight lines.
14. Find the ratio in which the straight line 2x + 3y -20 = 0 divides the join of the points (2,3) and (2, 10).
15. A straight line meets the coordinate axes in A and B. Find the equation of the straight line, when (p,q) bisects $\overline{AB}$.
16. Transform the equation x + y +1= 0 into normal form.
17. If 2x-3y -5= 0 is the perpendicular bisector of the line segment joining (3, -4) and (α, β) and $\alpha + \beta$.
18. Find the values of 't' if the points (t, 2t), (2t, 6t) and (3, 8) are collinear.
19. Find the values of y, if the line joining (3, y) ad (2, 7) is parallel to the line joining the points (-1, 4) and (0,6).
20. Find the equations of the straight lines passing through the origin and making equal angle with the co-ordinate axes.
21. Find the equations of the line passing through (1,2), and having x-intercept 3.

DERIVATIVES (2M)

1. If $x^3 + y^3 - 3axy = 0$, find $\frac{dy}{dx}$
2. Find the derivative of the following functions w.r to x. i) $\cos^{-1}(4x^3-3x)$ (ii) $Tan^{-1}\sqrt{\frac{1-\cos x}{1+\cos x}}$
3. Differentiate f(x) with respect to g(x) if $f(x) = e^x$, $g(x) = \sqrt{x}$
4. Find the derivative of the following functions w.r. to x. i) $Tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ ii) $(\log x)^{\tan x}$

5. If $f(x) = 7^{x^3+3x} (x > 0)$, then find $f^1(x)$.

6. If $y = \sec\left(\sqrt{\tan x}\right)$, find $\frac{dy}{dx}$

7. Find the derivative of the function f(x) = $a^x.e^{x^2}$.

8. If $x = \tan\left(e^{-y}\right)$, then show that $\frac{dy}{dx} = \frac{-e^y}{1+x^2}$

9. If $f(x) = xe^x \sin x$, then find $f^1(x)$

10. If $x = a\cos^3 t,\ y = a\sin^3 t$ find $\frac{dy}{dx}$

11. If $y = ae^{nx} + be^{-nx}$ then prove that $y^{11} = n^2 y$.

12. If $y = x^x (x > 0)$, then find $\frac{dy}{dx}$

13. If $f(x) = 1+x + x^2 + ...+ x^{100}$ then find $f^1(1)$

14. If f(x) = log (secx + tanx), find f1(x)

15. If $y = (\cot^{-1} x^3)^2$, find $\frac{dy}{dx}$

16. Find the derivative of log(sin(log x))

17. Find the derivative of $e^x(x^2+1)$

TANGENTS AND NORMAL (2M)

1. Find the equations of normal to the curve $y = x^2 - 4x + 2$ at (4,2)
2. Find the equations of tangent and normal to the curve xy = 10 at (2,5)
3. Find the equations of tangent and normal to the curve $y = x^3 + 4x^2$ at (-1,3)
4. Find the slope of the normal to the curve $x = a\cos^3\theta,\ y = a\sin^3\theta$ at $\theta = \frac{\pi}{4}$
5. Find the slope of the tangent to the curve $y = \frac{1}{x-1}$ at $\left(3, \frac{1}{2}\right)$.
6. Find the slope of the tangent to the curve $y = x^3 - 3x + 2$ at the point whose x - coordinate is 3.
7. Find the points at which the tangent to the curve $y = x^3 - 3x^2 - 9x + 7$ is parallel to the x-axis.
8. Show that at any point on the curve $y^2 = 4ax$, the length of subnormal is constant.
9. Show that the length of the subnormal at any point on the curve $y^2 = 4ax$ is a constant.
10. Find the lengths of subtangent and subnormal to the curve $y = b\sin\frac{x}{a}$ at any point

3D -GEOMETRY (2M)

1. The centriod of the triangle whose vertices are (5,4,6), (1, -1,3) and (4,3,2).
2. If (3, 2, -1), (4, 1,1) and (6,2,5) are three vertices and (4, 2,2) is the centriod of a tetrahedron, find the fourth vertex.
3. Find the fourth vertex of the parallelogram whose consecutive vertices are (2,4,-1), (3, 6, -1) and (4, 5,1).
4. Find the ratio in which YZ - palne dives the line joining A (2, 4,5) and B (3, 5, -4). Also find the point of intersection.
5. Find x if the distance between (5, -1, 7) and (x, 5, 1) is 9 units.

6. Find the coordinates of the vertex 'C' of triangle ABC if its centriod is the origin and the vertices A, B are (1,1,1) and (-2, 4,1) respectively.
7. For what value of t, the points (2, -1, 3) , (3, -5, t) and (-1, 11, 9) are collinear?
8. Find the feet of the perpendiculars from (1,4,-3) to the co-ordinate plane.
9. Find the feet of the the perpendiculars from (5,4,3) to the co-ordinate axes.
10. Find the distance of P(1,-2,3) from the co-ordinate axes.
11. Find the ratio in which the xz- plane divides the line joining A(-2,3,4) and B(1,2,3)

THE PLANE (2M)

1. Find the angle between the planes x + 2y + 2z - 5= 0 and 3x + 3y + 2z - 8 = 0 .
2. Reduce the equation x + 2y - 3z - 6= 0 of the plane into the normal form.
3. Find the intercepts of the pame 4x + 3y -2z + 2= 0 on the coordinate axes.
4. Find the direction cosines of the normal to the plane x + 2y +2z -4= 0
5. Find the equation of the plane passing through point (1,1,1) and parallel to the plane x + 2y + 3z - 7 = 0.
6. Find the equation of the plane passing through the point (-2, 1,3) and having (3, -5,4) as direction ratios of its normal.
7. Find the equation to the plane parallel to the ZX- plane passing through (0,4,4).
8. Find the constant k so that the planes x-2y + kz = 0 and 2x + 5y -z= 0 are at right angles.
9. Find the equation of the plane whose intercepts on X, Y, Z axes are 1,2,4 respectively.
10. Find the equation of the plane if the foot of the perpendicular from origin to the plane is (1,3,-5)
11. Find the equation of the plane passing through (2,3,4) and perpendicilar to x-axis
12. Find the equation of the plane passing through the points (2,2,-1), (3,4,2), (7,0,6). (Ans : 5x + 2y - 3z = 17)
13. If H, G, S and I respectively denote orthocentre, centroid, circumcentre and in-centre of a triangle formed by the points (1,2,3), (2,3,1) and (3,1,2), then find H, G, S, I.

Increasing and Decreasing Functions (2M)

1. Find the intervals in which

   (i) $f(x)=-3+12x-9x^2+2x^3$ ii) $x.e^x$ iii) $x^3-3x^2-6x+12$ is increasing and decreasing
2. Determine the intervals in which the functions are increusing and decreasing

   i) $\frac{\ln t}{t}$ ii) $\sqrt{25-4x^2}$ iii) $\ln(\ln x); x>1$
3. Show that $x^3-6x^2+15x=0$ for all $x\geq 0$
4. If $\frac{\pi}{2}\geq x\geq 0$, show that $x\geq \sin x$
5. Show that for all values of 'a' and 'b' $f(x)=x^3+3ax^2+3a^2x+3a^3+b$ is increasing
6. At what points the slope of $y=\frac{x^3}{6}-\frac{3x^2}{2}+\frac{11x}{2}+12$ increases ?

CONTINUITY(2M)

1*. Show that $f(x)=\begin{cases}\frac{\cos ax-\cos bx}{x^2} & if \quad x\neq 0\\ \frac{1}{2}(b^2-a^2) & if \quad x=0\end{cases}$ where a and b are real constants, is continuous at 0.

2*. Is ' f ' defined by $f(x)=\begin{cases}\frac{\sin 2x}{x} & if \quad x\neq 0\\ 1 & if \quad x=0\end{cases}$ continuous at 0?

3. Check the continuity of 'f' given by $f(x)=\begin{cases}(x^2-9)/(x^2-2x-3) & if\quad 0<x<5\quad and\ x\neq 3\\ 1.5 & if\ x=3\end{cases}$

4. If 'f' given by $f(x)=\begin{cases}k^2x-k & if\ x\geq 1\\ 2 & if\ x<1\end{cases}$ is a continuous function on R, then find the values of K.

5. Show that funtion t $f(x)=\left[\cos\left(x^{10}+1\right)\right]^{1/3}, x\in R$ is continuous function ?

6. Prove that function sinx , cosx are continuous on R ?

7. Check continuity of the function f defined by $f(x)=\begin{cases}\frac{1}{2}\left(x^2-4\right) & \text{if } 0<x<2\\ 0 & \text{if } x=2\\ 0 & \text{if } x>2\end{cases}$

ERRORS AND APPROXIMATIONS(2M or 4M)

1. Find dy and Δy for the following functions.

   *i) $y=f(x)=x^2+x\ at\ x=10\ when\ \Delta x=0.1$

   *ii) $y=x^2+3x+6, x=10\ and\ \Delta x=0.01$

   iii) $y=e^x+x, x=5\ and\ \Delta x=0.02$

   *iv) $y=\cos(x), x=60^0\ and\ \Delta x=1^0$ [Ans : $\Delta y=-0.0152\ and\ dy=-0.0150$]

   Hint : $1^0=0.0174\,radians$

2. Find the approximate values of following functions.

   *i) $\sqrt[3]{999}$ *ii) $\sqrt{82}$ *iii) $\sqrt[3]{65}$

   *iv) $\sin(62^0)$ *v) $\cos(60^0 5')$

3. If the increase in the side of a square is 2% then find the approximate percentage of increase in its area.

4. The radius of a sphere is measured as 14 cm. Later it was found that there is an error 0.02 cm in measuring the radius. Find the approximate error in surface area of the sphere.

5. The diameter of a sphere is measured to be 40 cm. If an error of 0.02 cm is made in it, then find approximate errors in volume and surface area of the sphere.

6. The time t, of a complete oscillation of a simple pendulum of length l is given by

   $t=2\pi\sqrt{\frac{l}{g}}$ where g is gravitational constant. Find the approximate percentage of error in t when the percentage of error in l is 1%.

Rolle's Theorem and Lagrange's mean value theorem(2M)

1*. Verify Rolle's theorem for the function

   i) $y=f(x)=x^2+4\,in[-3,3]$ ii) $f(x)=x^2-1$ on [-1,1]

2*. Verify Rolle's theorem for the function $f(x)=x(x+3)e^{-x/2}\,in[-3,0]$

3*. Let $f(x)=(x-1)(x-2)(x-3)$. Prove that there is more than one 'c' in (1,3) such that f'(c) = 0.

4. It is given that Rolle's theorem holds for the function $f(x)=x^3+bx^2+ax\, on[1,3]$ with $c=2+\frac{1}{\sqrt{3}}$, Find the values of a and b. **[Ans : a=11,b=-6]**

5. Find a point on the graph of the curve $y=(x-3)^2$, where the tangent is parallel to the chord joining $(3,0)$ and $(4,1)$. **[Ans :** $\left(\frac{7}{2},\frac{1}{4}\right)$**]**

6. Find a point on the graph of the curve $y=x^2$, where the tangent is parallel to the chord joining $(1,1)$ and (0,0). **[Ans :** $\left(\frac{1}{2},\frac{1}{4}\right)$**]**

7. Find 'c', so that $f'(c)=\frac{f(b)-f(a)}{b-a}$ in the following cases :

i) $f(x)=x^2-3x-1; a=\frac{-11}{7}, b=\frac{13}{7}$ **[Ans :** $\frac{1}{7}$**]** ii) $f(x)=e^x; a=0, b=1$.**[Ans :** $\log(e-1)$**]**

8. Verify the Rolle's theorem for the function $(x^2-1)(x-2)\, on[-1,2]$. Find the point in the interval where the derivate vanishes. .**[Ans :** $\frac{2\pm\sqrt{7}}{3}$**]**

9. Verify the conditions of the Lagrange's mean value theorem for the following functions. In each case find a point 'c' in the interval as stated by the theorem.

i) $x^2-1\, on[2,3]$ **[Ans :** $\frac{5}{2}$**]**

ii) $\sin x-\sin 2x\, on[0,\pi]$ **[Ans :** $\cos^{-1}\left(\frac{1\pm\sqrt{33}}{8}\right)$**]**

iii) $\log x\, on[1,2]$ **[Ans :** $\log_2^e$**]**

4MARKS IMP. QUESTIONS

LOCUS

1.* i) Find the equation of locus of a point, the difference of whose distances from (-5, 0) and (5, 0) is 8

ii) Find the equation of locus of P, if A = (4,0), B = (-4, 0) and |PA-PB| = 4

2.* Find the equation of locus of P, if A = (2,3), B = (2, -3) and PA + PB = 8.

3.* i) Find the equation of locus of P, if the line segment joining (2,3) and (-1,5) subtends a right angle at P.

ii) Find the locus of thirdvertex of a right angled triangle the ends of whose hypotenuse are (4,0) and (0,4)

4.* Find the equation of locus of P, if the ratio of the distance from P to (5, -4) and (7,6) is 2 :3.

5.* i) A (5,3) and B (3, -2) are two fixed points. Find the equation of locus of P, so that the area of triangle PAB is 9 sq. units.

ii) A(2,3) and B(-3,4) are two fixed points. Find the equation of locus of P, so that the area of triangle PAB is 8.5 sq. units.

6.* A (1,2), B (2, -3) and C (-2, 3) are three points. A point 'P' moves such that $PA^2+PB^2=2PC^2$. Show that the equation to the locus P is 7x-7y+4= 0.

TRANSFORMATION OF AXES(4M)

1.* When the origin is shofted to (-1, 2) by the translation of axes, find the transformed equation $x^2 +y^2 + 2x - 4y +1 = 0$.

2.* When the origin is shifted to the point (2,3) the transformed equation of a curve is $x^2 + 3xy -2y^2 + 17x-7y-11=0$. Find the original equation of the curve.

3.* When the axes are rotated through an angle $\frac{\pi}{4}$, find the transformed equation of $3x^2 + 10xy + 3y^2 = 9$.

4.* When the axes are rotated through an angle $\frac{\pi}{6}$, Find the transformed equation of $x^2+2\sqrt{3}xy-y^2=2a^2$.

5.* When the axes are rotated through an angle α, find the transformed equation of $x\cos\alpha+y\sin\alpha=p$

6.* When the axes are rotated through an angle 45°, the transformed equation of a curve is $17x^2 -16xy+17y^2= 225$. Find the original equation of the curve.

7.* Show that the axes are to be rotated through an angle of $\frac{1}{2}Tan^{-1}\left(\frac{2h}{a-b}\right)$ so as to remove the xy term from the equation $ax^2 + 2hxy + by^2 = 0$, if $a\neq b$ and through the angle $\frac{\pi}{4}$, is a = b.

8. Find the point to which the origin is to be shifted by the translation of axes so as to remove the first degree terms from the equation $4x^2+9y^2-8x+36y+4=0$

RATE MEASURE (4M)

1*. The volume of a cube is increasing at a rate of 9 cubic centimeters per second. How fast is the surface area increasing when the length of the edge is 10 centimeters ?
[Ans : 3.6 cm^2/sec]

2*. A praticle is moving in a straight line so that after t seconds its distance is s (in cms) from a fixed point on the line is given by $s = f(t) = 8t+t^3$. Find (i) the velocity at time t = 2 sec (ii) the initial velocity (iii) acceleration at t = 2 sec. [Ans : 20 cm/sec, 8 cm/sec, 12 cm/sec^2]

3*. A container in the shape of an inverted cone has height 12 cm an radius 6 cm at the top. If it is filled with water at the rate of 12 cm^3/sec., what is the rate of change in the height of water level when the tank is filled 8 cm ? [Ans : $3/4p\ cm/\sec$]

4*. A particle is moving along a line according to $s = f(t) = 4t^3-3t^2+5t-1$ where s is measured in meters and t is measured in seconds. Find the velocity and acceleration at time t. At what time the acceleration is zero [Ans : t = 1/4 sec]

5*. A stone is dropped into a quiet lake and ripples move in circles at the speed of 5 cm/sec. At the instant when the radius of circular ripple is 8 cm., how fast is the enclosed area increases ? [Ans : $80p\ cm^2/\sec$]

6*. A balloon, which always remains spherical on inflation, is being inflated by pumping in 900 cubic centimeters of gas per second. Find the rate at which the radius of ballon increases when the radius is 15 cm.[Ans : $1/p\ cm/\sec$]

7*. A point P is moving on the curve $y = 2x^2$. The x coordinate of P is increasing at the rate of 4 units per second. Find the rate at which the y coordinate is increasing when the point is at (2,8). [Ans : 32 units/sec]

8*. A particle moving along a straight line has the relation $s = t^3 + 2t + 3$, connecting the distance s described by the particle in time t. Find the velocity and acceleration of the particle at t = 4 sec

9*. The distance -time formula for the motion of a particle along a straight line is $s = t^3 - 9t^2 + 24t - 18$ .Find when and where the velocity is zero.

10. The displacement s of a particle travelling in a straight line t seconds is given by $s = 45t + 11t^2 - t^3$. Find the time when the particle comes to rest.

11*. Suppose we have a rectangular aquarium with dimensions of length 8m, width 4m and height 3m. Suppose we are filling the tank with water at the rate of 0.4 m^3/sec. How fast is the height of water changing when the water level is 2.5m ?

DERIVATIVES (4M)

1.* Find the derivative of the following functions from the first principlex w.r to x.
(i) $\cos^2 x$(ii) tan2x (iii) $\sqrt{x+1}$ (iv) sec3x (v) cos (ax) (vi) sin2x vii) x sinx

2.* If $x^y = e^{x-y}$ then show that $\frac{dy}{dx} = \frac{\log x}{(1+\log x)^2}$.

3.* If $y = x^y$ then show that $\frac{dy}{dx} = \frac{y^2}{x(1-\log y)} = \frac{y^2}{x(1-y\log x)}$

4.* Find $\frac{dy}{dx}$ for the functions x = a(cost + t sint), y =a(sint - t cost).

5.* If f(x) = $Tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$, $g(x) = Tan^{-1}x$ then. Differentiate f (x) with respect to g(x).

6.* If $ax^2+2hxy+by^2=1$ then prove that $\frac{d^2y}{dx^2} = \frac{h^2-ab}{(hx+by)^3}$.

7.* If siny = x sin (a + y), then show that $\frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}$ (a is not a multiple of π)

8.* If $x = a(t - \sin t), y = a(1+\cos t)$ then find $\frac{d^2y}{dx^2}$.

9.* If y = a cosx + (b +2x) sinx, then show that $y^{11} + y = 4\cos x$

10.* If $y = ax^{n+1} + bx^{-n}$ then Prove that $x^2y = n(n+1)y$

STRAIGHT LINES (4M)

1.* Transform the equation $\frac{x}{a}+\frac{y}{b}=1$ into the normal form when a > 0 and b >0. If the perpendicular distance of straight line from the origin is p, deduce that $\frac{1}{p^2}=\frac{1}{a^2}+\frac{1}{b^2}$.

2.* Transform the equation i) $\sqrt{3}x+y=4$ ii) 3x + 4y + 12 = 0 into (a) slope - intercept form (b) intercept form and (c) normal form.

3.* i) Find the value of k, if the lines 2x -3y + k =0, 3x -4y -13 = 0 and 8x -11y -33= 0 are concurrent.
ii) Find the value of p if the lines 4x - 3y - 7 = 0, 2x + py + 2 = 0, 6x + 5y - 1 = 0 are concurrent. (Ans : p = 4)

4.* i) If the straight lines ax + by +c= 0, bx +cy +a= 0 and cx +ay + b =0 are concurrent, then prove that $a^3 + b^3 + c^3 = 3abc$.
ii) If the straight lines ax + hy + g = 0, hx +by +f= 0 and gx +fy + c =0 are concurrent, then find the condition.

5.* i) A straight line through $Q\left(\sqrt{3},2\right)$ makes an angle $\frac{\pi}{6}$ with the positive direction of the X-axis. If the straight line intersects the line $\sqrt{3}x-4y+8=0$ at P, find the distance PQ.
ii) A st. line parallell to the line $y=\sqrt{3x}$ passes through Q(2,3) and cuts the line 2x + 4y - 27 = 0 at P. Find the length of PQ.

6.* i) Find the equation of straight line making non-zero equal intersepts on the coordinate axes passing through the point of intersection of lines 2x-5y +1= 0 and x- 3y -4= 0.
ii) Find the equation of the straight line parallel to the line 3x + 4y = 7 and passing through the point of intersection of the lines x -2y -3 = 0 and x + 3y - 6 =0.
iii) Find the equation of the straight line perpendicular to the line 2x + 3y = 0 and passing through the point of intersection of the lines x + 3y-1=0 and x -2y +4= 0.
iv) The line $\frac{x}{a}-\frac{y}{b}=1$ meets the x-axis at P. Find the equation of the line perpendicular to this line at P. (Ans : $\frac{x}{b}+\frac{y}{a}=\frac{a}{b}$)

7.* i) Find the equations of the straight lines passing through the point (-3, 2) and making an angle of 45° with the straight line 3x-y +4= 0
ii) Find the equations of the straight lines passing through the point (1, 2) and making an angle of 60° with the straight line $\sqrt{3x}+y+2=0$
iii) Find the equations of the stright lines passing through the point (-10,4) and making an angle θ with the line x - 2y = 10 such that $\tan\theta = 2$.

8.* i) Find the points on the line 4x -3y -10=0 which are at a distance of 5 units from the point (1, -2).
ii) A st. line passing through A(1,-2) makes an angle of $\tan^{-1}\left(\frac{4}{3}\right)$ with +ve direction of x-axis in anticlock wise sense. Find the pts on the line whose distance from A is ± 5 units

9.* Find the value of k, if the angle between the straight lines 4x -y +7 =0 and kx -5y-9 =0 is 45°.

10. A variable st. line drawn through the points of intersection of st. lines $\frac{x}{a}+\frac{y}{b}=1$ and $\frac{x}{b}+\frac{y}{a}=1$ meets the co-ordinate axes at A and B. Show that the locus of the mid point of $\overline{AB}$ is 2(a + b) xy = ab (x + y).

11. Find the point on the straight line 3x + y + 4 = 0 which is equidistant from the points (-5,6) and (3,2)

LIMITS (2M)or(4M)

1. Compute $\lim_{x\to 2}\frac{(2x^2-7x-4)}{(2x-1)(\sqrt{x}-2)}$

2. Compute $\lim_{x\to 2}\left(\frac{1}{x-2}-\frac{4}{x^2-4}\right)$

3. Compute $\lim_{x\to 2}\frac{x-2}{x^3-8}$

4. Find $\lim_{x\to 0}\left(\frac{\sqrt{1+x}-1}{x}\right)$.

5*. $\lim_{x\to 0}\left(\frac{x}{\sqrt{1+x}-\sqrt{1-x}}\right)$

6*. i) Find $\lim_{x\to 0}\left(\frac{3^x-1}{\sqrt{1+x}-1}\right)$ ii) Compute $\lim_{x\to 0}\frac{a^x-1}{b^x-1}(a>0,\ b>0\ b\neq 1)$

7*. Compute $\lim_{x\to 0}\left(\frac{e^x-1}{\sqrt{1+x}-1}\right)$

8. i) Evaluate $\underset{x\to 1}{lt}\frac{\sin(x-1)}{x^2-1}$ ii) Evaluate $\lim_{x\to 0}\frac{\sin ax}{x\cos x}$

9. $\lim_{x\to \pi/2}\frac{cosx}{\left(x-\frac{\pi}{2}\right)}$

10*. Find $\lim_{x\to 0}\left(\frac{\cos ax-\cos bx}{x^2}\right)$

11*. Find $\lim_{x\to 0}\frac{\sin(a+bx)-\sin(a-bx)}{x}$

12*. Find $\lim_{x\to a}\frac{\sin(x-a)\tan^2(x-a)}{(x^2-a^2)^2}$

13. Find $\lim_{x\to a}\frac{\tan(x-a)}{(x-a)^2}$

14. Show that $\lim_{x\to 0^+}\left(\frac{2|x|}{x}+x+1\right)=3$

15. Show $\lim_{x\to 2^-}\frac{|x-2|}{x-2}=-1$

16. Compute $\lim_{x\to 2+}([x]+x)$ and $\lim_{x\to 2-}([x]+x)$

17*. Find $\lim_{x\to 0}\frac{\sqrt[3]{1+x}-\sqrt[3]{1-x}}{x}$

18. $\lim_{x\to\infty}\frac{11x^3-3x+4}{13x^3-5x^2-7}$

19. Find $\lim_{x\to\infty}\left(\sqrt{x^2+x}-x\right)$

20. Compute $\lim_{x\to\infty}\left(\sqrt{x+1}-\sqrt{x}\right)$

21. Find $\lim_{x\to -\infty}\left(\frac{2x+3}{\sqrt{x^2-1}}\right)$

22. Find $\lim_{x\to\infty}\frac{8|x|+3x}{3|x|-2x}$

23*. Find $\lim_{x\to a}\frac{x\sin a-a\sin x}{x-a}$ (Hinit : Adding & Subtracting **a sin a**)

7MARKS IMP. QUESTIONS

DIRECTION COSINES AND DIRECTION RATIOS

1.* i) If a ray makes the angles α, β, γ and δ with four diagonals of a cube then find $\cos^2\alpha+\cos^2\beta+\cos^2\gamma+\cos^2\delta$.

ii) Find the angle between two diagonals of a cube.

2.* i)Find the angle between the lines whose direction cosines satisfy the equations $l+m+n=0,\ l^2+m^2-n^2=0$

ii) Find the angle between the lines whose direction cosines are given by the equations $3l+m+5n=0$ and $6mn-2nl+5lm=0$.

iii) Find the direction cosines of two lines which are connected by the relations $l+m+n=0$ and $mn-2nl-2lm=0$.

iv) Find the direction cosines of two lines which are connected by the relations
l - 5m + 3n = 0 and $7l^2 + 5m^2 - 3n^2 = 0$.

3. Show that the lines whose d.c.'s are given by $l + m + n = 0$, $2mn + 3nl - 5lm = 0$ are perpendicular to each other.

4.* If a variable line in two adjacent positions has direction cosines (l,m,n) and $(l+\delta l, m+\delta m, n+\delta n)$, show that the small angle $\delta\theta$ between the two positions is given by $(\delta\theta)^2 = (\delta l)^2 + (\delta m)^2 + (\delta n)^2$.

PAIR OF STRAIGHT LINES (7M)

1.* Show that the product of the perpendicular distances from a point (α,β) to the pair of straight lines $ax^2 + 2hxy + by^2 = 0$ is $\frac{|a\alpha^2 + 2h\alpha\beta + b\beta^2|}{\sqrt{(a-b)^2 + 4h^2}}$.

2.* If the equation $ax^2 + 2hxy + by^2 = 0$ represents a pair of distinct (i.e., intersecting) lines, then the combined equation of the pair of bisectors of the angles between these lines is $h(x^2-y^2) = (a-b)xy$.

3.* If the equation $S = ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represents a pair of parallel straight lines, then show that (i) $h^2 = ab$ (ii) $af^2 = bg^2$ and (iii) the distance between the parallel lines is $2\sqrt{\frac{g^2-ac}{a(a+b)}} = 2\sqrt{\frac{f^2-bc}{b(a+b)}}$.

4.* Show that the area of the triangle formed by the lines $ax^2 + 2hxy + by^2 = 0$ and $lx + my + n = 0$ is $\frac{n^2\sqrt{h^2-ab}}{|am^2 - 2hlm + bl^2|}$ sq. units

5.* Let the equation $ax^2 + 2hxy + by^2 = 0$ repsresents a pair of straight lines. Then the angle θ between the lines is given by $\cos\theta = \frac{a+b}{\sqrt{(a-b)^2 + 4h^2}}$

6.* Find the values of k, if the lines joining the origin to the points of intersection of the curve $2x^2 - 2xy + 3y^2 + 2x - y - 1 = 0$ and the line $x + 2y = k$ are mutually perpendicular.

7.* i) Show that the lines joining the origin to the points of intersection of the curve $x^2 - xy + y^2 + 3x + 3y - 2 = 0$ and the straight line $x - y - \sqrt{2} = 0$ are mutually perpendicular.
ii) Find the lines joining the origin to the points of intersection of the curve $7x^2 - 4xy + 8y^2 + 2x - 4y - 8 = 0$ with the line $3x - y = 2$ and angle between them.

8.* Find the angle between the lines joining the origin to the points of intersection of the curve $x^2 + 2xy + y^2 + 2x + 2y - 5 = 0$ and the line $3x - y + 1 = 0$.

9. Find the conditions for the lines joining the origin to the points of intersection of the circle $x^2 + y^2 = a^2$ and the line $lx + my = 1$ i) to subtend a right angle at the origin ii) to coincide.

10.* Write down the equation of the pair of straight lines joining the origin to the points of intersection of the line $6x - y + 8 = 0$ with the pair of straight lines $3x^2 + 4xy - 4y^2 - 11x + 2y + 6 = 0$ Show that the lines so obtained make equal angles with the coordinate axes

11.* i) Show that the pair of straight lines $6x^2 - 5xy - 6y^2 = 0$ and $6x^2 - 5xy - 6y^2 + x + 5y - 1 = 0$ forms a square.

ii) Show that the two pairs of lines $3x^2+8xy-3y^2=0$ and $3x^2+8xy-3y^2+2x-4y-1=0$ form a square.

12.* If the pairs of lines represented by $ax^2+2hxy+by^2=0$ and $ax^2+2hxy+by^2+2gx+2fy+c=0$ form a rhombus, prove that $(a-b)fg+h(f^2-g^2)=0$

13. Show that the lines represented by $(lx+my)^2-3(mx-ly)^2=0$ and lx + my + n =0 forms an equilateral triangle with area $\frac{n^2}{\sqrt{3}(l^2+m^2)}$ square units.

14. Show that the straight lines represented by $3x^2+48xy+23y^2=0$ and $3x-2y+13=0$ froms an equilateral triangle of area $\frac{13}{\sqrt{3}}$ sq. units.

15.* Show that the equation $2x^2-13xy-7y^2+x+23y-6=0$ represents a pair of straight lines. Also find the angle between them and the coordinates of the point of intersection of the lines.

16. If the second degree equation $S=ax^2+2hxy+by^2+2gx+2fy+c=0$ in two variables x and y represents a pair of straight lines, then
(i) $abc+2fgh-af^2-bg^2-ch^2=0$ and (ii) $h^2\ge ab,\ g^2\ge ac$ and $f^2\ge bc$

17.* Show the following lines form an equilateral triangle and find the area of the triangle
i) $x^2-4xy+y^2=0, x+y=3$ ii) $(x+2a)^2-3y^2=0, x=a$

18.* Find the centroid and area of the triangle formed by the lines
i) $12x^2-20xy+7y^2=0, 2x-3y+4=0$ ii) $2y^2-xy-6x^2=0, x+y+4=0$

19.* Show that the product of the perpendicular distances from the origin to the pair of straight lines represented by $ax^2+2hxy+by^2+2gx+2fy+c=0$ is $\frac{|c|}{\sqrt{(a-b)^2+4h^2}}$

20. If (α,β) is the centriod of the triangle formed by the lines $ax^2+2hxy+by^2=0$ and lx + my =1, prove that $\frac{\alpha}{bl-hm}=\frac{\beta}{am-hl}=\frac{2}{3(bl^2-2hlm+am^2)}$

21.* Show that the straight lines $y^2-4y+3=0$ and $x^2+4xy+4y^2+5x+10y+4=0$ form a parallelogram and find the lengths of its sides.

STRAIGHT LINES (7M)

1.* Find the circumcentre of the triangle whose sides are
i) 3x-y-5=0, x +2y -4= 0 and 5x +3y +1= 0 Ans : (-6/7, 2/7)
ii) x + y + 2 = 0, 5x - y - 2 = 0, x -2y + 5 = 0 Ans : (-1/3, 2/3)

2.* Find the circumcentre of the triangle formed by the points (1,3), (0,–2), (–3, 1)

3.* Find the orthocentre of the triangle whose sides are
i)7x +y -10 =0, x-2y+5= 0 and x + y +2=0 Ans : (-2/3, 4/3)
ii) x + 2y = 0, 4x + 3y - 5 = 0, 3x + y = 0 Ans : (-4, -3)

4.* Find the orthocentre of the triangle with the vertices (–5, –7), (13,2) & (–5, 6)

5.* If Q(h,k) is the image of the point P (x_1, y_1) w.r.t the straight line ax + by + c = 0. Then $\frac{h-x_1}{a}=\frac{k-y_1}{b}=\frac{-2(ax_1+by_1+c)}{a^2+b^2}$ and find the image of (1,-2) w.r.t. The straight line 2x-3y+5=0.

6.* If Q(h,k) is the foot of the perpendicular from P (x_1, y_1) on the line ax + by + c= 0, then prove that $\frac{h-x_1}{a}=\frac{k-y_1}{b}=\frac{-(ax_1+by_1+c)}{a^2+b^2}$. Also find the foot of the perpendicular from

(-1,3) on the line 5x-y -18 = 0.

7.* If p and q are the lengths of the perpendiculars from the origin to the straight lines $x\sec\alpha + y\,\mathrm{cosec}\,\alpha = a$ and $x\cos\alpha - y\sin\alpha = a\cos 2\alpha$, prove that $4p^2 + q^2 = a^2$.

8.* i) Find the equations of the straight lines passing through the point of intersection of the lines 3x + 2y +4= 0, 2x + 5y =1 and whose distance from (2, -1) is 2.
ii) Find the equations of the straight lines passing through (1,1) and which is at a distance of 3units from (-2,3)

9.* Find the incentre of the triangle formed by the straight lines $y = \sqrt{3}x, y = -\sqrt{3}x$ & $y = 3$. Ans:(0,2)

DIFFERENTIATION (7M)

1.* If $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$ then show that $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$

2.* i) If $y = x^{\tan x} + (\sin x)^{\cos x}$, find $\frac{dy}{dx}$. ii) $y = (\sin x)^{\log x} + x^{\sin x}$ then find $\frac{dy}{dx}$

3.* If $x^y + y^x = a^b$ then show that $\frac{dy}{dx} = -\left[\frac{yx^{y-1} + y^x \log y}{x^y \log x + xy^{x-1}}\right]$

4.* If $y = x\sqrt{a^2+x^2} + a^2 \log\left(x + \sqrt{a^2+x^2}\right)$ then prove that $\frac{dy}{dx} = 2\sqrt{a^2+x^2}$.

5.* If $f(x) = \sin^{-1}\sqrt{\frac{x-\beta}{\alpha-\beta}}$ and $g(x) = Tan^{-1}\sqrt{\frac{x-\beta}{\alpha-x}}$ then show that $f^1(x) = g^1(x)$ $(\beta < x < \alpha)$.

6.* Find the derivative of $f(x) = Tan^{-1}\left(\frac{2x}{1-x^2}\right)$ w.r to $g(x) = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$.

7.* If $y = Tan^{-1}\left(\frac{2x}{1+x^2}\right) + Tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right) - Tan^{-1}\left(\frac{4x-4x^3}{1-6x^2+x^4}\right)$ then show that $\frac{dy}{dx} = \frac{1}{1+x^2}$.

8.* If $x^y = y^x$ then show that $\frac{dy}{dx} = \frac{y(x\log y - y)}{x(y\log x - x)}$.

9.* If $x^{\log y} = \log x$ then $\frac{dy}{dx} = \frac{y}{x}\left[\frac{1-\log x \log y}{\log^2 x}\right]$

10.* If $y = Tan^{-1}\left[\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right]$ for $0 < |x| < 1$ find $\frac{dy}{dx}$.

TANGENTS AND NORMALS (7M)

1.* If the tangent at any point P on the curve $x^m y^n = a^{m+n}$ $(mn \neq 0)$ meets the coordinate axes in A and B then show that AP : BP is a constant.

2.* If the tangent at any point on the curve $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$ intersects the coordinate axes in A and B, then show that the length AB is a constant.

3.* If p,q denote the lengths of perpendiculars drawn from the origin in the tangent and normal at a point respectively on the curve $x^{2/3} + y^{2/3} = a^{2/3}$, then show that $4p^2 + q^2 = a^2$.

4.* Show that the equation of tangent to the curve $\sqrt{x} + \sqrt{y} = \sqrt{a}$ at the point (x_1, y_1) is

$xx_1^{-\frac{1}{2}} + yy_1^{-\frac{1}{2}} = a^{\frac{1}{2}}$

5.* Show that the curves $y^2 = 4(x+1)$ and $y^2 = 36(9-x)$ intersect orthogonally.

6.* Show that the curves $6x^2 - 5x + 2y = 0$ and $4x^2 + 8y^2 = 3$ touch each other at $\left(\frac{1}{2}, \frac{1}{2}\right)$

7.* Show that the condition for the orthogonality of the curves $ax^2 + by^2 = 1$ and $a_1x^2 + b_1y^2 = 1$ is $\frac{1}{a} - \frac{1}{b} = \frac{1}{a_1} - \frac{1}{b_1}$.

8. Find the angle between the curves i) $2y^2 - 9x = 0; 3x^2 + 4y = 0$ (in the 4th quadrant)
ii) $y^2 = 4x$, $x^2 + y^2 = 5$ [March 2012]

9.* i) At any point 't' on the curve x =a(t + sint), y =a (1-cost), find the lengths of tangent, normal, subtangent an subnormal.
ii) Find the length of subtangent and subnormal at a point t on the curve
x = a(cos t + t sin t), y = a(sin t - t cos t).

MAXIMA AND MINIMA (7M)

1.* Show that when the curved surface of right circular cylinder inscribed in a sphere of radus 'r' is maximum, then the height of the cylinder is $\sqrt{2}$ r.

2.* From a rectangular sheet of dimensions 30 cm x 80 cm. four equal squares of side x cm. are removed at the corners, and the sides are then turned up so as to form an open rectangular box. Find the value of x, so that the volume of the box is the greatest.

3.* A window is in the shape of a rectangle surmounted by a semicircle. If the perimeter of the window be 20ft., find the maximum area.

4.* A wire of length *l* is cut into two parts which are bent respectively in the form of a square and a circle. What are the lengths of pieces of wire so that the sum of the areas is least ?

5*. Find the point on the graph $y^2 = x$ which is the nearest to the point (4,0).

6*. *i) Find two positive numbers whose sum is 15 so that the sum of their squares is minimum. [Ans : 15/2 & 15/2]
ii) Find two positive integers x and y such that x + y = 60 and xy^3 is maximum.
[Ans : 15, 45]

7*. Find the maximum area of the rectangle that can be formed with fixed perimeter 20.
[Ans : 25]

8*. Prove that the radius of the right circular cylinder of greatest curved surface area which can be inscribed in a given cone is half of that of the cone.

9. i) The profit function P(x) of a company, selling x items per day is given by
P(x) = (150 – x)x – 1600. Find the number of items that the company should sell to get maximum profit. Also find the maximum profit. [Ans : x = 75, 4025]
ii) The profit function P(x) of a company selling x items per day is given by
P(x) = (150 – x)x – 1000. Find the number of items that company should manufacture to get maximum profit. Also find the maximum profit.
[No. of items = 75, Maximum profit = 4625]

10. A manufacturer can sell x items at a price of rupees (5 – x/100) each. The cost price of x items is Rs. (x/5 + 500). Find the number of items that the manufacturer should sell to earn maximum profits. [Ans : 240 items]