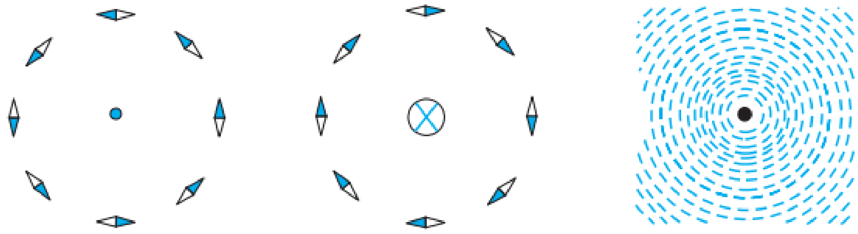
Chapter 4
Moving Charges and Magnetism

Introduction

Christian Oersted discovered that moving charges or currents produce a magnetic field in the surrounding space. The direction of the magnetic field depends on the direction of current.



The magnetic field due to a straight long current-carrying wire. The wire is perpendicular to the plane of the paper. A ring of compass needles surrounds the wire. The orientation of the needles is shown when
(a) the current emerges out of the plane of the paper,
(b) the current moves into the plane of the paper.
(c) The arrangement of iron filings around the wire.
*The darkened ends of the needle represent north poles.
*A current or a field (electric or magnetic) emerging out of the plane of the paper is depicted by a dot (.)
*A current or a field going into the plane of the paper is depicted by a cross (⊗).

Magnetic Force

Sources and fields

A static charge q is the source of electric field(E) .
Moving charges or currents produces a magnetic field (B), in addition to electric field(B).

- Magnetic field is a vector field.
- It obeys the principle of superposition: the magnetic field of several sources is the vector addition of magnetic field of each individual source.

Lorentz Force

The total force acting on a charge q moving with a velocity v in presence of both the electric field E and the magnetic field B is called Lorentz force.

$$F = F_{electric} + F_{magnetic}$$

$$\vec{\mathbf{F}} = \mathbf{q}\vec{\mathbf{E}} + \mathbf{q}(\vec{\mathbf{v}} \times \vec{\mathbf{B}})$$

$$\vec{\mathbf{F}} = \mathbf{q}\,[\,\vec{\mathbf{E}} + (\vec{\mathbf{v}} \times \vec{\mathbf{B}})]$$

Electric Lorentz force

$$\vec{\mathbf{F}} = \mathbf{q}\vec{\mathbf{E}}$$

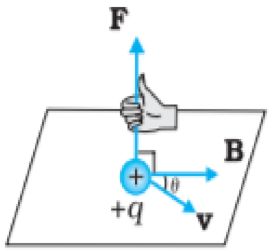
Magnetic Lorentz force

$$q(\vec{v} \times \vec{B})$$

$\vec{F} = qvB\sin\theta$ where θ is the angle between v and B

(i) Magnetic Lorentz force depends on q, v and B (charge of the particle, the velocity and the magnetic field). Force on a negative charge is opposite to that on a positive charge.

(ii) The magnetic force $\vec{F} = qvB\sin\theta$
If velocity and magnetic field are parallel ($\theta = 0$) or anti-parallel($\theta = 180$), $F = 0$.



(iii) The direction of magnetic force is perpendicular to both the velocity and the magnetic field.
Its direction is given by the screw rule or right hand rule.

(iii) The magnetic force is zero if charge is not moving (v= 0).
Only a moving charge feels the magnetic force.

Unit of B

$$F = qvB\sin\theta$$

$$B = \frac{F}{qv}$$

$$\text{Unit of B} = \frac{\text{newton second}}{\text{coulomb metre}} = \text{tesla (T)}$$

tesla is a large unit. A smaller unit (non-SI) called gauss is also often used.

$$1 \text{ gauss} = 10^{-4} \text{ tesla}$$

$$1\text{ G} = 10^{-4}\text{ T}$$

The earth's magnetic field is about 3.6×10^{-5} T

Magnetic force on a current-carrying conductor

Consider a rod of a uniform cross-sectional area A and length l .
The total number of mobile charge carriers in it is nA l
Let e be the charge on each charge carrier .

Then q=neA l

Let each mobile carrier has an average drift velocity v_d.

$$\vec{F} = q\,(\vec{v} \times \vec{B})$$

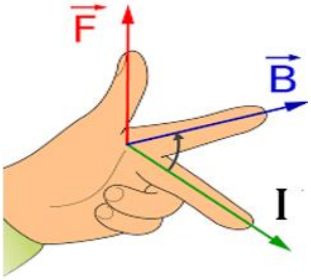
$$\vec{F} = neAl\,(\vec{v_d} \times \vec{B})$$

$$\vec{F} = (ne\,A\,v_d)\,\vec{l} \times \vec{B} \qquad (neAv_d = I)$$

$$\vec{F} = I\,(\vec{l} \times \vec{B})$$

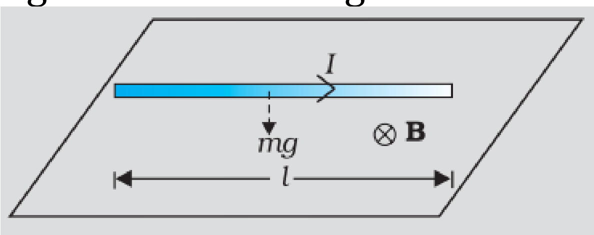
Fleming's left hand rule

Stretch the fore finger , middle finger and thumb of left hand in three mutually perpendicular directions, such that fore finger in the direction of magnetic fileld, the middle finger in the direction of current ,then the thumb gives the direction of force.



Example

1 A straight wire of mass 200 g and length 1.5 m carries a current of 2 A. It is suspended in mid-air by a uniform horizontal magnetic field B . What is the magnitude of the magnetic field?



There is an upward force F, of magnitude I l B,. For mid-air suspension, this must be balanced by the force due to gravity:

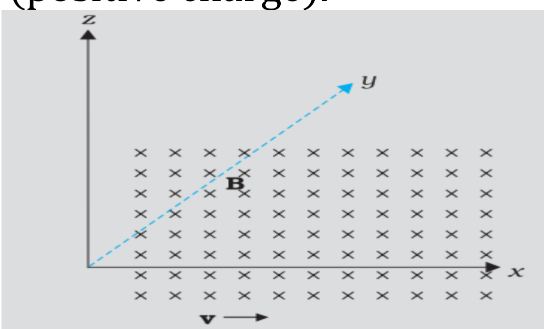
$$m\,g = I\,lB$$

$$B = \frac{mg}{Il} = \frac{2 \times 9.8}{2 \times 1.5} = 0.65\ \text{T}$$

Example

The magnetic field is parallel to the positive y-axis and the charged particle is moving along the positive x-axis (which way would the Lorentz force be for (a) an electron (negative charge),

(b) a proton (positive charge).



The velocity v of particle is along the x-axis, while B, the magnetic field is along the y-axis, so $\vec{\mathbf{v}} \times \vec{\mathbf{B}}$ is along the z-axis (screw rule or right-hand thumb rule).

(a) for electron it will be along –z axis.

(b) for a positive charge (proton) the force is along +z axis.

Motion of a charged particle in a Magnetic field

Case 1 - When $\theta = 0^0$ or $\theta = 180^0$

i.e. the charge is moving in the same direction or opposite direction of magnetic field (parallel or antiparallel)

$$F = qvB \sin 0 = 0$$
$$F = qvB \sin 180 = 0$$

Thus there is no magnetic force on the charge and **the charge moves undeflected.**

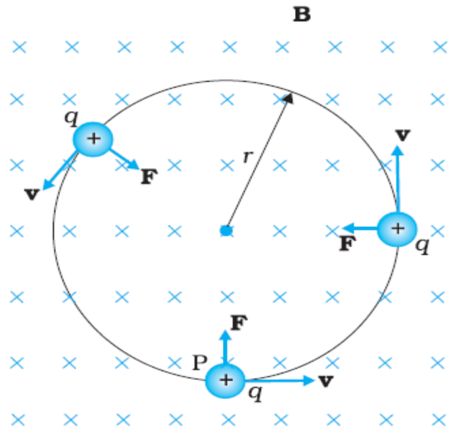
Case 2 - When $\theta = 90^0$

i.e. the charged particle entering perpendicular to a magnetic field.

$$F = qvB \sin 90$$
$$F = qvB$$

The perpendicular force, F=q v B, acts as a centripetal force and produces a circular motion perpendicular to the magnetic field.

The particle will describe a circle if v and B are perpendicular to each other



$$\frac{mv^2}{r} = qvB$$

$$v = \frac{qBr}{m}$$

$$\text{angular frequency, } \omega = \frac{v}{r} = \frac{qB}{m}$$

$$\text{Period } T = \frac{2\pi}{\omega} = \frac{2\pi}{\frac{qB}{m}}$$

$$T = \frac{2\pi m}{qB}$$

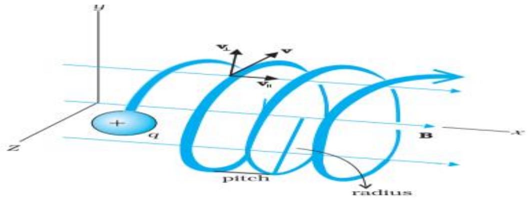
$$\text{Frequency } \nu = \frac{1}{T}$$

$$\nu = \frac{qB}{2\pi m}$$

Case 3- When θ between 0^0 and 90^0

i.e. when the charged particle moves at an arbitrary angle θ with the field direction, **it undergoes helical path.**

Here velocity has one component along B, and the other perpendicular to B. The motion in the direction of field is unaffected by magnetic field, as the magnetic force is zero. The motion in a plane perpendicular to B is circular , thereby producing a helical motion.

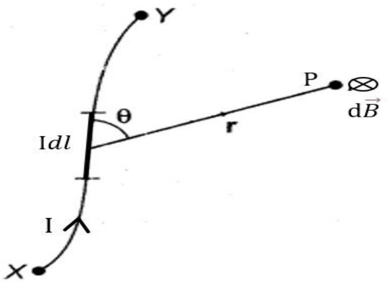


The distance moved along the magnetic field in one rotation is called pitch p.

$$p = v_{parallel} \times T$$

$$\mathbf{P = \frac{qB}{2\pi m} v_{parallel}}$$

Magnetic Field due to a Current Element – Biot-Savart Law



The magnetic field due to a small element of a current carrying conductor is directly proportional to the current (I) ,the length of the element dl , sine of the angle between dl and r and inversely proportional to the square of the distance r.

$$\mathbf{dB = \frac{\mu_0}{4\pi}\frac{I\,dl\sin\theta}{r^2}}$$

μ_0 = permeability of free space

$\mu_0 = 4\pi \times 10^{-7}\ \mathrm{Tm/A}$

$\frac{\mu_0}{4\pi} = 10^{-7}\ \mathrm{Tm/A}$

In vector form Biot – Savart law can be written as

$$\mathbf{d\vec{B} = \frac{\mu_0}{4\pi}\frac{Id\bar{l}\times\bar{r}}{r^3}}$$

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{Idl\sin\theta}{r^2}$$

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{Id\bar{l}\times\hat{r}}{r^2}$$

$$\hat{r} = \frac{\bar{r}}{r}$$

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{Id\bar{l}\times\bar{r}}{r^3}$$

Comparison between Coulomb's law and Biot -Savart's law

(i)Coulombs law $F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}$. Biot-Savart law $dB = \frac{\mu_0}{4\pi}\frac{I\,dl\sin\theta}{r^2}$

(ii) Both are long range. The principle of superposition applies to both fields.

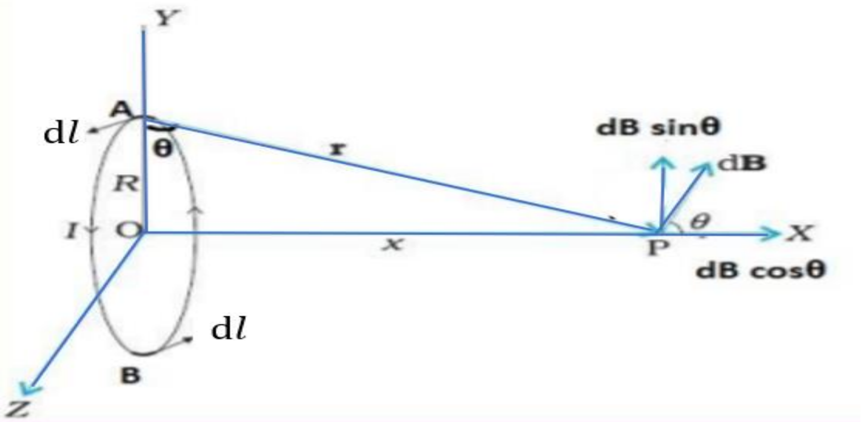
(iii) The electrostatic field is produced by a scalar source, i.e., the electric charge. The magnetic field is produced by a vector source i.e., current element Idl.

(iv) The electrostatic field is along the displacement vector joining the source and the field point. The magnetic field is perpendicular to the plane containing the displacement vector r and the current element Idl.

(v)There is an angle dependence in the Biot-Savart law which is not present in the electrostatic case.

Applications of Biot-Savart law

Magnetic Field on the Axis of a Circular Current Loop



$$dB = \frac{\mu_0}{4\pi}\frac{Idl\sin 90}{r^2}$$

$$dB = \frac{\mu_0}{4\pi}\frac{Idl}{r^2}$$

$$r^2 = x^2 + R^2$$

$$dB = \frac{\mu_0}{4\pi}\frac{Idl}{x^2+R^2} \quad \text{----------------(1)}$$

Total field $B = \int dB\cos\theta$ ----------------(2)

$$\cos\theta = \frac{R}{r} = \frac{R}{(x^2+R^2)^{1/2}}$$

Substituting for dB and $\cos\theta$

$$B = \int \frac{\mu_0}{4\pi}\frac{Idl}{x^2+R^2}\frac{R}{(x^2+R^2)^{1/2}} \quad \text{----------(3)}$$

$$B = \frac{\mu_0}{4\pi}\frac{IR}{(x^2+R^2)^{3/2}}\int dl$$

$$B = \frac{\mu_0}{4\pi}\frac{IR}{(x^2+R^2)^{3/2}} \times 2\pi R$$

$$B = \frac{\mu_0 IR^2}{2(x^2+R^2)^{3/2}}$$

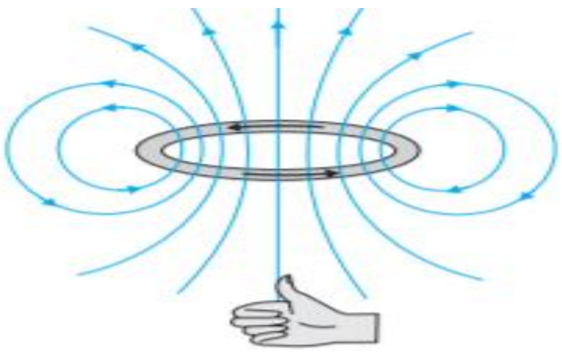
Magnetic field at the centre of the loop

At the centre x=0

$$B = \frac{\mu_0 I R^2}{2R^3}$$

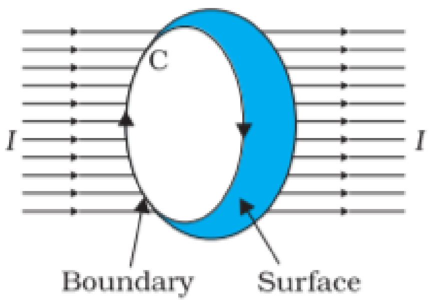
$$\mathbf{B = \frac{\mu_0 I}{2R}}$$

The direction of the magnetic field is given by **right-hand thumb rule .** Curl the palm of your right hand around the circular wire with the fingers pointing in the direction of the current. The right-hand thumb gives the direction of the magnetic field.



The upper side of the loop(current is anticlockwise) may be thought of as the north pole and the lower side(current is clockwise) as the south pole of a magnet.

Ampere's Circuital Law



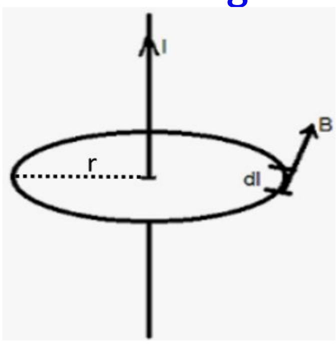
The line integral of magnetic field over a closed loop is equal to μ_0 times the total current passing through the surface.

The closed loop is called Amperian Loop.

$$\oint \vec{\mathbf{B}}.\,\vec{dl} = \mu_0 \mathbf{I}$$

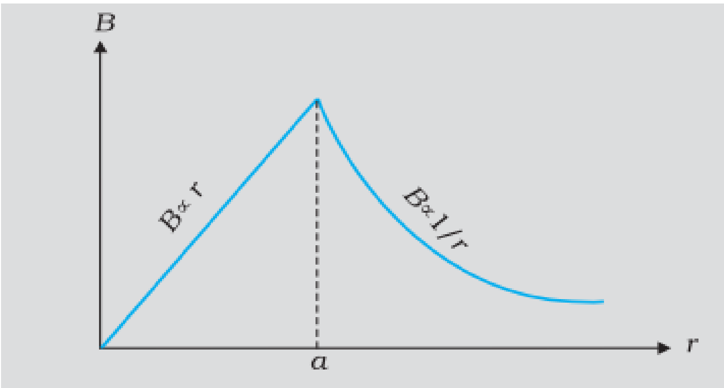
Applications of Ampere's Circuital Law

1.Magnetic field due to a straight infinite current-carrying wire



By Ampere's Circuital Law

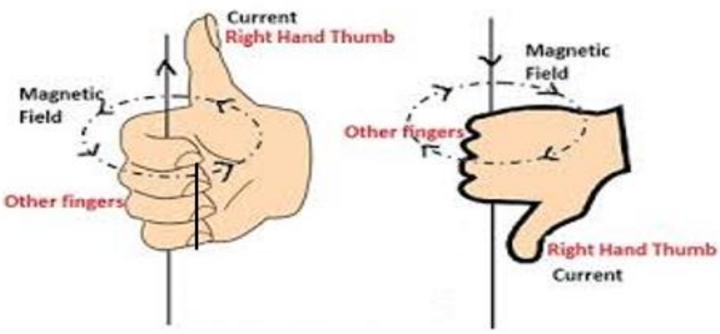
$$\oint \vec{B}.d\vec{l} = \mu_0 I$$
$$\oint Bdl \cos 0 = \mu_0 I$$
$$B\oint dl = \mu_0 I$$
$$B \times 2\pi r = \mu_0 I$$
$$\mathbf{B = \frac{\mu_0 I}{2\pi r}}$$



A plot of the magnitude of B with distance r from the centre of the wire having radius a

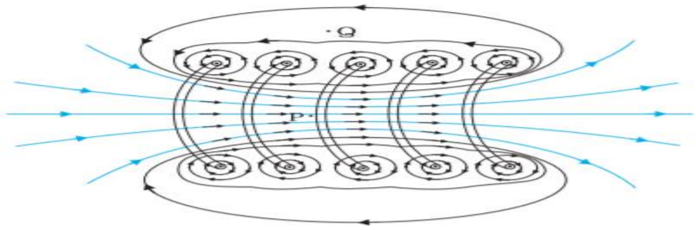
Right-hand rule

There exists a simple rule to determine the direction of the magnetic field due to a long wire ,called the right-hand rule. Grasp the wire in your right hand with your extended thumb pointing in the direction of the current. Your fingers will curl around in the direction of the magnetic field.

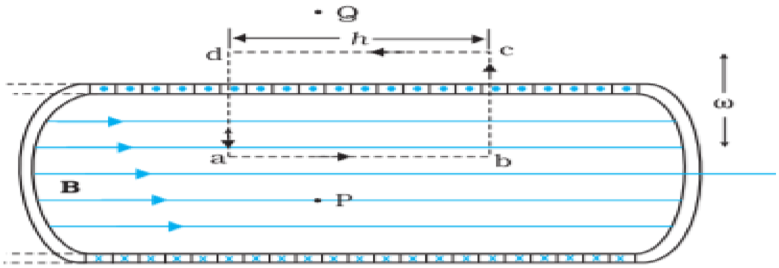


Solenoid

A solenoid consists of a long wire wound in the form of a helix where the neighbouring turns are closely spaced. The field between two neighbouring turns vanishes and the field at the interior mid-point P is uniform. The field outside the solenoid approaches zero.



2.Mangnetic field due to a Solenoid



$$\oint_{abcd}\vec{B}.d\vec{l} = \oint_{ab}\vec{B}.d\vec{l} + \oint_{bc}\vec{B}.d\vec{l} + \oint_{cd}\vec{B}.d\vec{l} + \oint_{da}\vec{B}.d\vec{l} \text{ ------------- } \mathbf{(1)}$$

$$\oint_{ab}\vec{B}.d\vec{l} = \oint_{ab} B\, dl \cos 0 = \oint_{ab} B\, dl = B\oint_{ab} dl = Bl$$

$$\oint_{bc}\vec{B}.d\vec{l} = \oint_{ab} B\, dl \cos 90 = 0$$

$$\oint_{cd}\vec{B}.d\vec{l} = 0 \qquad \text{(since B = 0 outside)}$$

$$\oint_{da}\vec{B}.d\vec{l} = \oint_{ab} B\, dl \cos 90 = 0$$

Substituting in eqn (1)

$$\oint_{abcd}\vec{B}.d\vec{l} = Bl + 0 + 0 + 0$$

$$\oint_{abcd}\vec{B}.d\vec{l} = Bl \text{ -------------(2)}$$

By Ampere's Circuital Law for N turns of solenoid

$$\oint B.d\vec{l} = \mu_0 NI \text{ ---------------(3)}$$

From eqns (2) and (3)

$$Bl = \mu_0 NI$$

$$B = \frac{\mu_0 NI}{l}$$

$$\mathbf{B = \mu_0 nI} \qquad \textbf{where } \mathbf{n} = \frac{\mathbf{N}}{l}$$

N=number of turns of solenoid
l= length of solenoid
n=number of turns per unit length of solenoid

Example

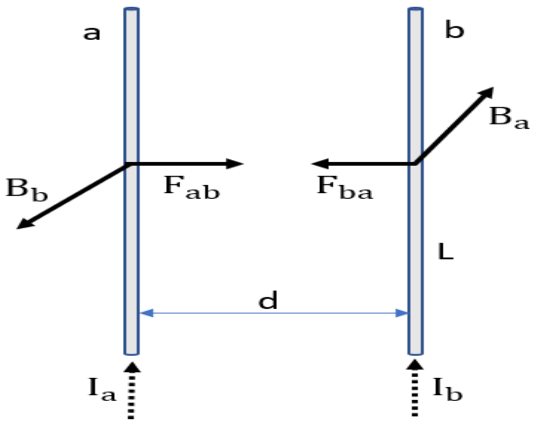
A solenoid of length 0.5 m has a radius of 1 cm and is made up of 500 turns. It carries a current of 5 A. What is the magnitude of the magnetic field inside the solenoid?

The number of turns per unit length , $n=\frac{N}{l}$

$$=\frac{500}{0.5}=1000$$

$$\begin{aligned} B &= \mu_0 n I \\ &= 4\pi \times 10^{-7} x1000x5 \\ &= 6.28 \times 10^{-3}\ T \end{aligned}$$

Force between Two Parallel Current Carrying Condutors



Two long parallel conductors a and b separated by a distance d and carrying (parallel) currents I_a and I_b, respectively.

Magnetic field produced by conductor a along the conductor 'b'

$$B_a = \frac{\mu_0 I_a}{2\pi d}$$

Force acting on conductor b due to this field B_a,

$$\boxed{\vec{F} = I(\vec{l} \times \vec{B})}$$

$$F_{ba} = I_b L B_a$$

$$F_{ba} = I_b L \frac{\mu_0 I_a}{2\pi d}$$

$$F_{ba} = \frac{\mu_0 I_a I_b L}{2\pi d}$$

The force F_{ba} per unit length,

$$\mathbf{f_{ba} = \frac{\mu_0 I_a I_b}{2\pi d}}$$

Similarly the force on 'a' due to 'b'

$$F_{ab} = -F_{ba}$$

- Biot-Savart law and the Lorentz force yield results in accordance with Newton's third Law.
- Parallel currents attract, and antiparallel currents repel.

Definition of ampere

$$f_{ba} = \frac{\mu_0 I_a I_b}{2\pi d}$$

$$\text{If } I_a = I_b = 1A$$

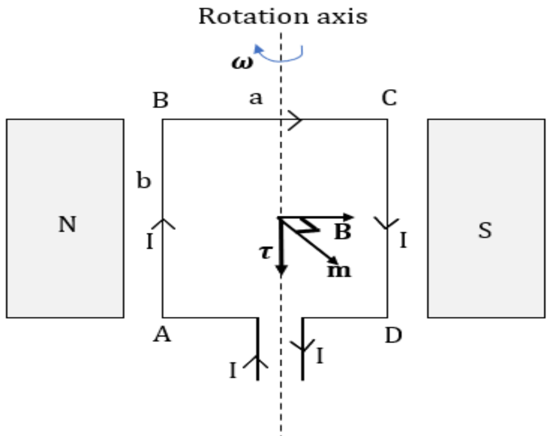
$$\text{and}, \quad d = 1m$$

$$f_{ba} = \frac{\mu_0}{2\pi} = \frac{4\pi \times 10^{-7}}{2\pi} = 2 \times 10^{-7} N/m$$

The ampere is that current which, when flaws through two very long, straight, parallel conductors placed one metre apart in vacuum, would produce a force equal to 2×10^{-7} N/m on each other.

Torque on Current Loop, Magnetic Dipole

Torque on a rectangular current loop in a uniform magnetic field



A rectangular loop carrying a steady current I is placed in a uniform magnetic field B,which is applied in the plane of the loop.

Force on AD and BC is zero

Force on BC , F=IaBsin0=0
Force on AD , F=IaBsin 180=0

Force on AB =Force on CD = IbB sin 90=IbB

Forces on AB and CD are equal and oppsite. So the coil does not experience a net force, but it experiences a torque.

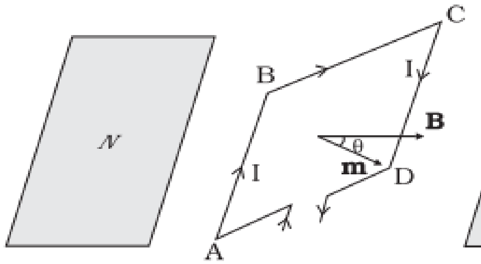
Torque , τ =Force x perpendicular distance

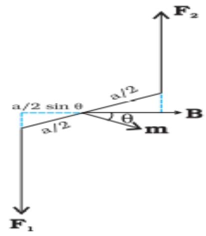
τ =IbB x a =IabB

$\tau = IAB$

where A = ab is the area of the rectangle.

When the plane of the loop, makes an angle with the magnetic field. We take the angle between the field and the normal to the coil to be angle θ.





$$\tau = IbB \times a\sin\theta$$
$$\tau = IAB\sin\theta$$

For N turns of the coil

$$\tau = NIAB\sin\theta$$

We define the magnetic moment of the current loop as, $m = IA$

For N turns, $m = NIA$

Unit of magnetic moment is Am^2 and dimensions are AL^2

$$\vec{\tau} = mB\sin\theta$$
$$\vec{\tau} = \vec{m} \times \vec{B}$$

Circular current loop as a magnetic dipole

Magnetic field on the axis of circular loop

$$B = \frac{\mu_0 I R^2}{2(x^2+R^2)^{3/2}}$$

For $x >> R$,

$$B = \frac{\mu_0 I R^2}{2x^3}$$

$$A = \pi R^2$$

$$B = \frac{\mu_0 I A}{2\pi x^3}$$

$$\mathbf{B} = \frac{\mu_0}{4\pi} \frac{2\mathbf{m}}{x^3}$$

Comparing with electric field along the axial line of a an electric dipole

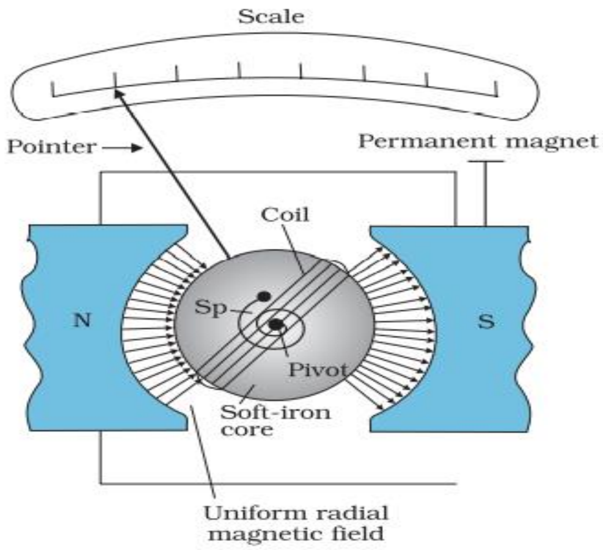
$$\mathbf{E} = \frac{1}{4\pi\varepsilon_0} \frac{2\mathbf{p}}{x^3}$$

$$\mu_0 \to \frac{1}{\varepsilon_0}$$

$\mathbf{m} \to \mathbf{p}$ (electrostatic dipole moment)

$\mathbf{B} \to \mathbf{E}$ (electrostatic field)

The Moving Coil Galvanometer



The moving coil galvanometer(MCG) consists of a coil, with many turns, free to rotate about a fixed axis , in a uniform radial magnetic field. There is a cylindrical soft iron core which not only makes the field radial but also increases the strength of the magnetic field.

When a current flows through the coil, a torque acts on it.

$$\tau = NI\,AB \text{--------------}(1)$$

The magnetic torque NIAB tends to rotate the coil. A spring Sp provides a counter torque.

$$\tau = k\phi \text{-----------------}(2)$$

where k is the torsional constant of the spring; i.e. the restoring torque per unit twist.

ϕ is the deflection is indicated on the scale by a pointer attached to the spring.

In equilibrium,

$$k\phi = NI\,AB \text{-------------}(3)$$

$$\boldsymbol{\phi = \left(\frac{N\,AB}{k}\right)\ I}$$

The quantity in brackets is a constant for a given galvanometer.

$$\boldsymbol{\phi \propto I}$$

Thus the deflection produced in the coil is directly proportional to the current through the coil.

Current Sensitivity of the Galvanometer

Current sensitivity of the galvanometer is defined as the deflection per unit current.

$$\frac{\phi}{I} = \left(\frac{N\,AB}{k}\right)$$

A convenient way for the manufacturer to increase the sensitivity is to increase the number of turns N.

Voltage sensitivity of the galvanometer

Voltage sensitivity of the galvanometer is defined as the deflection per unit voltage.

$$\frac{\phi}{V} = \left(\frac{N\,AB}{k}\right)\frac{I}{V} = \left(\frac{N\,AB}{k}\right)\frac{1}{R}$$

$$\frac{\phi}{V} = \left(\frac{N\,AB}{k}\right)\frac{1}{R}$$

Increasing the current sensitivity may not necessarily increase the voltage sensitivity.

If $N \rightarrow 2N$, i.e., we double the number of turns, then current sensitivity,

$$\frac{\phi}{I} = \left(\frac{2N\,AB}{k}\right) \rightarrow 2\frac{\phi}{I}$$

Thus, the current sensitivity doubles.

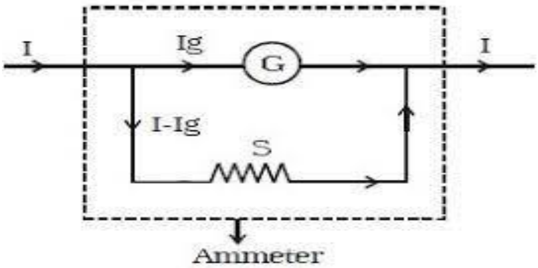
If $N \rightarrow 2N$, then $R \rightarrow 2R$ then the voltage sensitivity,

$$\frac{\phi}{V} = \left(\frac{2N\,AB}{k}\right)\frac{1}{2R} = \left(\frac{N\,AB}{k}\right)\frac{1}{R} = \frac{\phi}{V}$$

Thus, the voltage sensitivity remains unchanged..

Conversion of Galvanometer to Ammeter

To convert a Galvanometer to an Ammeter a small resistance , called shunt resistance S ,is connected in parallel with the galvanometer coil.

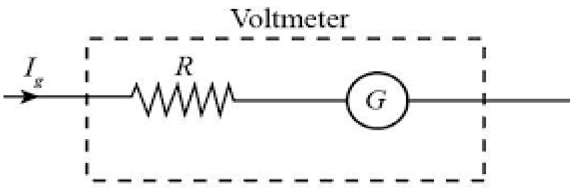


$$I_g G = (I - I_g)S$$

$$S = \frac{I_g G}{I - I_g}$$

Conversion of Galvanometer to Voltmeter

To convert a Galvanometer to a volteter a high resistance , R is connected in series with the galvanometer coil.



$$V = I_g(R + G)$$

$$R + G = \frac{V}{I_g}$$

$$\mathbf{R = \frac{V}{I_g} - G}$$

Example

A galvanometer with coil resistance 12Ω shows full scale deflection for a current of 2.5mA. How will you convert it into an ammeter of range 0 – 7.5 A?

$$S = \frac{I_g G}{I - I_g}$$

$$S = \frac{2.5 \text{ x } 10^{-3} \text{ x } 12}{7.5 - 2.5 \text{ x } 10^{-3}} = \frac{2.5 \text{ x } 10^{-3} \text{ x } 12}{7.5 - 0.0025} = 4 \text{ x } 10^{-3} \ \Omega$$

A resistance of 4 x 10^{-3} Ω is to be connected in parallel to the galvanometer coil to convert it into an ammeter.

Example

A galvanometer with coil resistance 12Ω shows full scale deflection for a current of 3mA. How will you convert it into a voltmeter of range 0 – 18V?

$$R = \frac{V}{I_g} - G$$

$$R = \frac{18}{3 \text{ x } 10^{-3}} - 12 = 6\text{x } 10^{3} - 12 = 6000 - 12 = 5988 \ \Omega$$

A resistance of 5988 Ω is to be connected in seriesl to the galvanometer coil to convert it into a voltmeter.