Construct a Square Root Spiral

OBJECTIVE

To construct a square root spiral.

Materials Required

1. Adhesive
2. Geometry box
3. Marker
4. A piece of plywood

Prerequisite Knowledge

1. Concept of number line.
2. Concept of irrational numbers.
3. Pythagoras theorem.

Theory

1. A number line is a imaginary line whose each point represents a real number.
2. The numbers which cannot be expressed in the form p/q where $q \neq 0$ and both p and q are integers, are called irrational numbers, e.g. $\sqrt{3}$, π, etc.
3. According to Pythagoras theorem, in a right angled triangle, the square of the hypotenuse is equal to the sum of the squares of other two sides containing right angle. ΔABC is a right angled triangle having right angle at B. (see Fig. 1.1)

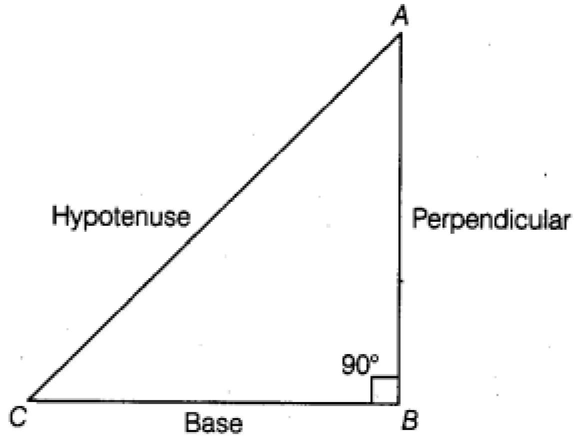

Fig. 1.1

Therefore, $AC^2 = AB^2 + BC^2$
where, AC = hypotenuse, AB = perpendicular and BC = base

Procedure

1. Take a piece of plywood having the dimensions 30 cm x 30 cm.
2. Draw a line segment PQ of length 1 unit by taking 2 cm as 1 unit, (see Fig. 1.2)

Fig. 1.2

3. Construct a line QX perpendicular to the line segment PQ, by using compasses or a set square, (see Fig. 1.3)

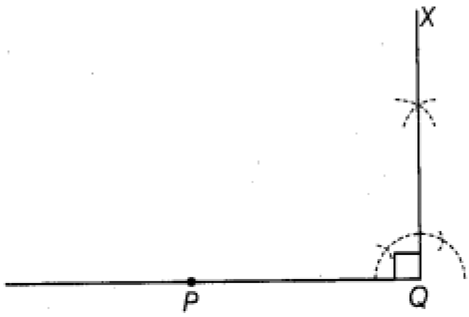

Fig. 1.3

4. From Q, draw an arc of 1 unit, which cut QX at C(say). (see Fig. 1.4)

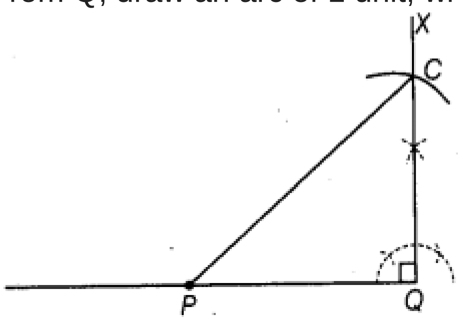

Fig. 1.4

5. Join PC.
6. Taking PC as base, draw a perpendicular CY to PC, by using compasses or a set square.
7. From C, draw an arc of 1 unit, which cut CY at D (say).

8. Join PD. (see Fig. 1.5)

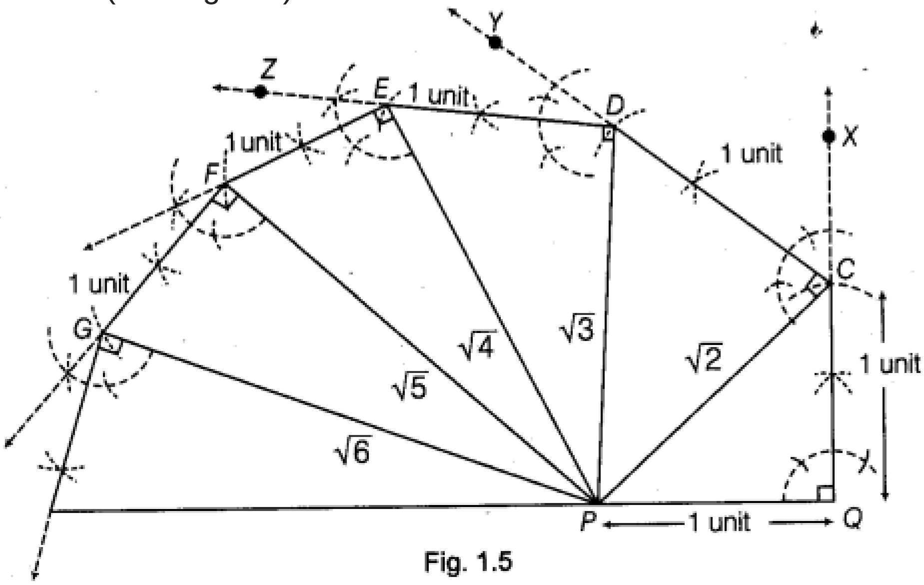

Fig. 1.5

9. Taking PD as base, draw a perpendicular DZ to PD, by using compasses or a set square.
10. From D, draw an arc of 1 unit, which cut DZ at E (say).
11. Join PE. (see Fig. 1.5)

Keep repeating the above process for sufficient number of times. Then, the figure so obtained is called a 'square root spiral'.

Demonstration

1. In the Fig. 1.5, ΔPQC is a right angled triangle.
   So, from Pythagoras theorem,
   we have $PC^2 = PQ^2 + QC^2$
   [∴ $(\text{Hypotenuse})^2 = (\text{Perpendicular})^2 + (\text{Base})^2$]
   $= 1^2 + 1^2 = 2$
   $\Rightarrow PC = \sqrt{2}$
   Again, ΔPCD is also a right angled triangle.
   So, from Pythagoras theorem,
   $PD^2 = PC^2 + CD^2$
   $= (\sqrt{2})^2 + (1)^2 = 2+1 = 3$
   $\Rightarrow PD = \sqrt{3}$
2. Similarly, we will have
   $PE = \sqrt{4}$
   $\Rightarrow PF = \sqrt{5}$
   $\Rightarrow PG = \sqrt{6}$ and so on.

Observations

On actual measurement, we get
PC = …….. ,

PD = …….. ,
PE = …….. ,
PF = …….. ,
PG = …….. ,
$\sqrt{2}$ = PC = …. (approx.)
$\sqrt{3}$ = PD = …. (approx.)
$\sqrt{4}$ = PE = …. (approx.)
$\sqrt{5}$ = PF = …. (approx.)

Result
A square root spiral has been constructed.

Application
With the help of explained activity, existence of irrational numbers can be illustrated.

Viva Voce
Question 1:
Define a rational number.
Answer:
A number which can be expressed in the form of p/q, where $q \neq 0$ and p, q are integers, is called a rational number.

Question 2:
Define an irrational number.
Answer:
A number which cannot be expressed in the form of p/q, where $q \neq 0$ and p, q are integers, is called an irrational number.

Question 3:
Define a real number.
Answer:
A number which may be either rational or irrational is called a real number.

Question 4:
How many rational and irrational numbers lie between any two real numbers?
Answer:
There are infinite rational and irrational numbers lie between any two real numbers.

Question 5:
Is it possible to represent irrational numbers on the number line?
Answer:
Yes, as we know that each point on the number line represent a real number (i.e. both rational and irrational), so irrational number can be represented on number line.

Question 6:
In which triangle, Pythagoras theorem is applicable?
Answer:
Right angled triangle

Question 7:
Give some examples of irrational numbers.
Answer:
Some examples of irrational numbers are $\sqrt{5}$, $3 - \sqrt{7}$,2π, etc.

Question 8:
Can we represent the reciprocal of zero on the number line.
Answer:
No, because reciprocal of zero is undefined term, so we cannot represent on number line.

Question 9:
In a square root spiral, is it true that in each square root of natural number is equal to the square root of the sum of 1 and previous natural number (> 1)?
Answer:
Yes

Question 10:
Is it possible that we make a square root spiral of negative nymbers?
Answer:
No

Suggested Activity
Represent square root of 7 and 9 by constructing a square root spiral.