CHAPTER

3

TRIGONOMETRIC FUNCTIONS

CONCEPT TYPE QUESTIONS

Directions : This section contains multiple choice questions. Each question has four choices (a), (b), (c) and (d), out of which only one is correct.

1. The value of $\tan^2\theta\ \sec^2\theta\ (\cot^2\theta - \cos^2\theta)$ is

(a) 0 (b) 1 (c) –1 (d) $\frac{1}{2}$

2. Value of cot 5° cot 10° ..... cot 85° is

(a) 0 (b) –1 (c) 1 (d) 2

3. Value of sin 10° + sin 20° + sin 30° +......+sin360° is

(a) 1 (b) 0 (c) 2 (d) $\frac{1}{2}$

4. If $\tan A = \frac{1}{2}$ and $\tan B = \frac{1}{3}$, then value of A + B is

(a) π (b) $\frac{\pi}{6}$ (c) $\frac{\pi}{2}$ (d) $\frac{\pi}{4}$

5. If $\sin 2\theta + \sin 2\phi = 1/2$, $\cos 2\theta + \cos 2\phi = 3/2$, then value of $\cos^2(\theta - \phi)$ is

(a) $\frac{5}{8}$ (b) $\frac{3}{8}$ (c) $-\frac{5}{8}$ (d) $\frac{3}{5}$

6. If $0 < \theta < 360°$, then solutions of $\cos\theta = -1/2$ are

(a) 120º, 360º (b) 240º, 90º
(c) 60º, 270º (d) 120º, 240º

7. If $\tan\theta = -\frac{1}{\sqrt{3}}$, then general solution of the equation is

(a) $2n\pi + \frac{\pi}{6}, n \in I$ (b) $n\pi + \frac{\pi}{6}, n \in I$

(c) $2n\pi - \frac{\pi}{6}, n \in I$ (d) $n\pi - \frac{\pi}{6}, n \in I$

8. If $2\tan^2\theta = \sec^2\theta$, then general value of θ are

(a) $n\pi \pm \frac{\pi}{4}, n \in I$ (b) $n\pi \pm \frac{\pi}{6}, n \in I$

(c) $2n\pi + \frac{\pi}{4}, n \in I$ (d) $2n\pi \pm \frac{\pi}{6}, n \in I$

9. If sin 5x + sin 3x + sinx = 0 and $0 \le x \le \pi/2$, then value of x is

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{6}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{4}$

10. If $y = \frac{2\sin\alpha}{1 + \cos\alpha + \sin\alpha}$, then value of $\frac{1 - \cos\alpha + \sin\alpha}{1 + \sin\alpha}$ is

(a) $\frac{y}{3}$ (b) y (c) 2y (d) $\frac{3}{2}y$

11. The number of solution of $\tan x + \sec x = 2\cos x$ in $(0, 2\pi)$ is

(a) 2 (b) 3 (c) 0 (d) 1

12. If $\sin A = \frac{3}{5}, 0 < A < \frac{\pi}{2}$ and $\cos B = \frac{-12}{13}, \pi < B < \frac{3\pi}{2}$, then value of sin (A – B) is

(a) $-\frac{13}{82}$ (b) $-\frac{15}{65}$ (c) $-\frac{13}{75}$ (d) $-\frac{16}{65}$

13. Value of tan15°. tan45° tan75° is

(a) 0 (b) 1 (c) $\frac{\sqrt{3}}{2}$ (d) –1

14. Value of

$\left(1+\cos\frac{\pi}{8}\right)\left(1+\cos\frac{3\pi}{8}\right)\left(1+\cos\frac{5\pi}{8}\right)\left(1+\cos\frac{7\pi}{8}\right)$ is

(a) $\frac{1}{8}$ (b) $\frac{3}{4}$ (c) $\frac{2}{3}$ (d) $\frac{5}{8}$

15. The large hand of a clock is 42 cm long. How much distance does its extremity move in 20 minutes?

(a) 88 cm (b) 80 cm (c) 75 cm (d) 77 cm

16. The angle in radian through which a pendulum swings and its length is 75 cm and tip describes an arc of length 21 cm, is

(a) $\frac{7}{25}$ (b) $\frac{6}{25}$ (c) $\frac{8}{25}$ (d) $\frac{3}{25}$

17. The length of an arc of a circle of radius 3 cm, if the angle subtended at the centre is 30° is (π = 3.14)

(a) 1.50 cm (b) 1.35 cm (c) 1.57 cm (d) 1.20 cm

18. A circular wire of radius 7 cm is cut and bent again into an arc of a circle of radius 12 cm. The angle subtended by the arc at the centre is

(a) 50° (b) 210° (c) 100° (d) 60°

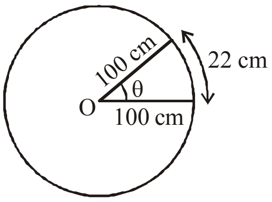
19. A circular wire of radius 3 cm is cut and bent so as to lie along the circumference of a hoop whose radius is 48 cm. The angle in degrees which is subtended at the centre of hoop is

(a) 21.5° (b) 23.5° (c) 22.5° (d) 24.5°

20. The radius of the circle in which a central angle of 60° intercepts an arc of length 37.4 cm is $\left(\text{Use } \pi = \frac{22}{7}\right)$

(a) 37.5 cm (b) 32.8 cm (c) 35.7 cm (d) 34.5 cm

21. The degree measure of the angle subtended at the centre of a circle of radius 100 cm by an arc of length 22 cm as shown in figure, is $\left[\text{Use } \pi = \frac{22}{7}\right]$



(a) $12° 30'$
(b) $12° 36'$
(c) $11° 36'$
(d) $11° 12'$

22. If $\tan \theta = 3$ and θ lies in III^{rd} quadrant, then the value of $\sin \theta$ is
(a) $\frac{1}{\sqrt{10}}$ (b) $\frac{2}{\sqrt{10}}$ (c) $\frac{-3}{\sqrt{10}}$ (d) $\frac{-5}{\sqrt{10}}$

23. If $\frac{\sin x}{\cos x} \times \frac{\sec x}{\operatorname{cosec} x} \times \frac{\tan x}{\cot x} = 9$, where $x \in \left(0, \frac{\pi}{2}\right)$, then the value of x is equal to
(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{2}$ (d) π

24. Find x from the equation:
$\operatorname{cosec}(90° + \theta) + x \cos \theta \cot(90° + \theta) = \sin(90° + \theta)$.
(a) $\cot \theta$ (b) $\tan \theta$ (c) $-\tan \theta$ (d) $-\cot \theta$

25. If A + B = 45°, then (cot A – 1) (cot B – 1) is equal to
(a) 1 (b) $\frac{1}{2}$ (c) –1 (d) 2

26. If $\sin A = \frac{3}{5}$ and A is in first quadrant, then the values of sin 2A, cos 2A and tan 2A are
(a) $\frac{24}{25}, \frac{7}{25}, \frac{24}{7}$ (b) $\frac{1}{25}, \frac{7}{25}, \frac{1}{7}$
(c) $\frac{24}{25}, \frac{1}{25}, \frac{24}{7}$ (d) $\frac{1}{25}, \frac{24}{25}, \frac{1}{24}$

27. The value of $\tan(\alpha + \beta)$, given that $\cot \alpha = \frac{1}{2}$, $\alpha \in \left(\pi, \frac{3\pi}{2}\right)$ and $\sec \beta = \frac{-5}{3}$, $\beta \in \left(\frac{\pi}{2}, \pi\right)$ is
(a) $\frac{1}{11}$ (b) $\frac{2}{11}$ (c) $\frac{5}{11}$ (d) $\frac{3}{11}$

28. The value of tan 75° – cot 75° is equal to
(a) $2\sqrt{3}$ (b) $2+\sqrt{3}$
(c) $2-\sqrt{3}$ (d) 1

29. The value of tan 3A – tan 2A – tan A is equal to
(a) tan 3A tan 2A tan A
(b) –tan 3A tan 2A tan A
(c) tan A tan 2A – tan 2A tan 3A – tan 3A tan A
(d) None of these

30. If $\tan A = \frac{1}{2}$, $\tan B = \frac{1}{3}$, then tan(2A + B) is equal to
(a) 1 (b) 2 (c) 3 (d) 4

31. If $\tan \theta = \frac{a}{b}$, then $b \cos 2\theta + a \sin 2\theta$ is equal to
(a) a (b) b (c) $\frac{a}{b}$ (d) None of these

32. Number of solutions of the equation $\tan x + \sec x = 2 \cos x$ lying in the interval $[0, \pi]$ is
(a) 0 (b) 1 (c) 2 (d) 3

33. If $\cos A = \frac{4}{5}$, $\cos B = \frac{12}{13}$, $\frac{3\pi}{2} < A, B < 2\pi$, the value of the $\cos(A+B)$ is
(a) $\frac{65}{33}$ (b) $\frac{33}{65}$ (c) $\frac{30}{65}$ (d) $\frac{65}{30}$

34. What is the value of radian measures corresponding to the 25° measures ?
(a) $\frac{5\pi}{36}$ (b) $\frac{2\pi}{36}$ (c) $\frac{3\pi}{36}$ (d) $\frac{4\pi}{36}$

35. If $\tan \theta = \frac{-4}{3}$, then $\sin \theta$ is
(a) $\frac{-4}{5}$ but not $\frac{4}{5}$ (b) $\frac{-4}{5}$ or $\frac{4}{5}$
(c) $\frac{4}{5}$ but not $-\frac{4}{5}$ (d) None of these

36. cos (A + B). cos (A – B) is given by:
(a) $\cos^2 A - \cos^2 B$ (b) $\cos(A^2 - B^2)$
(c) $\cos^2 A - \sin^2 B$ (d) $\sin^2 A - \cos^2 B$

37. If $\sin \theta = \frac{24}{25}$ and $0° < \theta < 90°$ then what is the value of $\sin\left(\frac{\theta}{2}\right)$?
(a) $\frac{12}{25}$ (b) $\frac{7}{25}$ (c) $\frac{3}{5}$ (d) $\frac{4}{5}$

38. What is the value of $\sin\left(\frac{5\pi}{12}\right)$?
(a) $\frac{\sqrt{3}+1}{2}$ (b) $\frac{\sqrt{6}+\sqrt{2}}{4}$
(c) $\frac{\sqrt{3}+\sqrt{2}}{4}$ (d) $\frac{\sqrt{6}+1}{2}$

39. If $x + \frac{1}{x} = 2\cos\theta$, then $x^3 + \frac{1}{x^3}$ is:
(a) $\frac{1}{2}\cos 3\theta$ (b) $2\cos 3\theta$
(c) $\cos 3\theta$ (d) $\frac{1}{3}\cos 3\theta$

40. If $1 + \cot \theta = \operatorname{cosec} \theta$, then the general value of θ is
(a) $n\pi + \frac{\pi}{2}$ (b) $2n\pi - \frac{\pi}{2}$
(c) $2n\pi + \frac{\pi}{2}$ (d) $2n\pi \pm \frac{\pi}{2}$

41. If $\sin 3\alpha = 4 \sin \alpha \sin(x + \alpha) \sin(x - \alpha)$, then $x =$
(a) $n\pi \pm \frac{\pi}{6}$ (b) $n\pi \pm \frac{\pi}{3}$
(c) $n\pi \pm \frac{\pi}{4}$ (d) $n\pi \pm \frac{\pi}{2}$

42. The general value of θ satisfying the equation $\tan\theta + \tan\left(\frac{\pi}{2} - \theta\right) = 2$, is
(a) $n\pi \pm \frac{\pi}{4}$ (b) $n\pi + \frac{\pi}{4}$
(c) $2n\pi \pm \frac{\pi}{4}$ (d) $n\pi + (-1)^n \frac{\pi}{4}$

43. The general solution of $\sin^2\theta \sec\theta + \sqrt{3}\tan\theta = 0$ is
(a) $\theta = n\pi + (-1)^{n+1}\frac{\pi}{3}$, $\theta = n\pi$; $n \in I$
(b) $\theta = n\pi$; $n \in I$
(c) $\theta = \frac{n\pi}{2}$, $n \in I$
(d) $\theta = n\pi + (-1)^{n+1}\frac{\pi}{2}$, $\theta = n\pi$; $n \in I$

44. If $\tan\theta + \tan 2\theta + \sqrt{3}\tan\theta\tan 2\theta = \sqrt{3}$, then
(a) $\theta = (6n+1)\frac{\pi}{18}$, $\forall n \in I$
(b) $\theta = (6n+1)\frac{\pi}{9}$, $\forall n \in I$
(c) $\theta = (3n+1)\frac{\pi}{9}$, $\forall n \in I$
(d) $\theta = (3n+1)\frac{\pi}{18}$

45. The most general value of θ satisfying the equations $\sin\theta = \sin\alpha$ and $\cos\theta = \cos\alpha$ is
(a) $2n\pi + \alpha$ (b) $2n\pi - \alpha$
(c) $n\pi + \alpha$ (d) $n\pi - \alpha$

46. If $\sec 4\theta - \sec 2\theta = 2$, then the general value of θ is
(a) $(2n+1)\frac{\pi}{4}$ (b) $(2n+1)\frac{\pi}{10}$
(c) $n\pi + \frac{\pi}{2}$ or $\frac{n\pi}{5} + \frac{\pi}{10}$ (d) $(2n+1)\frac{\pi}{2}$

47. General solution of the equation $\tan\theta\tan 2\theta = 1$ is given by
(a) $(2n+1)\frac{\pi}{4}$, $n \in I$ (b) $n\pi + \frac{\pi}{6}$, $n \in I$
(c) $n\pi - \frac{\pi}{6}$, $n \in I$ (d) $n\pi \pm \frac{\pi}{6}$, $n \in I$

48. If $\cot\theta + \cot\left(\frac{\pi}{4} + \theta\right) = 2$, then the general value of θ is
(a) $2n\pi \pm \frac{\pi}{6}$ (b) $2n\pi \pm \frac{\pi}{3}$
(c) $n\pi \pm \frac{\pi}{3}$ (d) $n\pi \pm \frac{\pi}{6}$

49. If $2\cos^2 x + 3\sin x - 3 = 0$, $0 \le x \le 180°$, then $x =$
(a) 30°, 90°, 150° (b) 60°, 120°, 180°
(c) 0°, 30°, 150° (d) 45°, 90°, 135°

50. If $\cot\theta + \tan\theta = 2\,\text{cosec}\,\theta$, the general value of θ is
(a) $n\pi \pm \frac{\pi}{3}$ (b) $n\pi \pm \frac{\pi}{6}$
(c) $2n\pi \pm \frac{\pi}{3}$ (d) $2n\pi \pm \frac{\pi}{6}$

51. If $\sqrt{3}\tan 2\theta + \sqrt{3}\tan 3\theta + \tan 2\theta\tan 3\theta = 1$, then the general value of θ is
(a) $n\pi \pm \frac{\pi}{5}$ (b) $\left(n + \frac{1}{6}\right)\frac{\pi}{5}$
(c) $\left(2n \pm \frac{1}{6}\right)\frac{\pi}{5}$ (d) $\left(n + \frac{1}{3}\right)\frac{\pi}{5}$

52. If $\cos 7\theta = \cos\theta - \sin 4\theta$, then the general value of θ is
(a) $\frac{n\pi}{4}, \frac{n\pi}{3} + \frac{\pi}{18}$ (b) $\frac{n\pi}{3}, \frac{n\pi}{3} + (-1)^n\frac{\pi}{18}$
(c) $\frac{n\pi}{4}, \frac{n\pi}{3} + (-1)^n\frac{\pi}{18}$ (d) $\frac{n\pi}{6}, \frac{n\pi}{3} + (-1)^n\frac{\pi}{18}$

53. Which among the following is/are correct?
(a) The angle is called negative, if the rotation is clockwise
(b) The angle is called positive, if the rotation is anti-clockwise
(c) The amount of rotation performed to get the terminal side from the initial side is called the measure of an angle
(d) All the above are correct

54. Angle subtended at the centre by an arc of length 1 unit in a unit circle is said to have a measure of
(a) 1 degree (b) 1 grade (c) 1 radian (d) 1 arc

55. Radian measure of $40°\,20'$ is equal to
(a) $\frac{120\pi}{504}$ radian (b) $\frac{121\pi}{540}$ radian
(c) $\frac{121\pi}{3}$ radian (d) None of these

56. π radian in degree measure is equal to
(a) 18° (b) 180° (c) 200° (d) 360°

57. The value of $\sin\frac{31\pi}{3}$ is
(a) $\frac{\sqrt{3}}{2}$ (b) $-\frac{\sqrt{3}}{2}$ (c) $-\frac{1}{\sqrt{2}}$ (d) $\frac{1}{\sqrt{2}}$

58. The value of $\cot\left(\frac{-15\pi}{4}\right)$ is
(a) $\frac{-1}{\sqrt{3}}$ (b) 1 (c) $\sqrt{3}$ (d) $-\sqrt{3}$

59. If $\cos\theta = \frac{-3}{5}$ and $\pi < \theta < \frac{3\pi}{2}$, then the value of $\left(\frac{\text{cosec}\,\theta + \cot\theta}{\sec\theta - \tan\theta}\right)$ is equal to
(a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) $\frac{13}{2}$ (d) None of these

60. $\cos\left(\frac{3\pi}{4} + x\right) - \cos\left(\frac{3\pi}{4} - x\right)$ is equal to
(a) $\sqrt{2}\sin x$ (b) $-2\sin x$
(c) $-\sqrt{2}\sin x$ (d) None of these

61. $\frac{\sin x - \sin 3x}{\sin^2 x - \cos^2 x}$ is equal to
(a) $\sin 2x$ (b) $\cos 2x$ (c) $\tan 2x$ (d) None of these

62. The solution of $\sin x = -\frac{\sqrt{3}}{2}$ is
(a) $x = n\pi + (-1)^n\frac{4\pi}{3}$, where $n \in Z$
(b) $x = n\pi + (-1)^n\frac{2\pi}{3}$, where $n \in Z$
(c) $x = n\pi + (-1)^n\frac{3\pi}{3}$, where $n \in Z$
(d) None of the above

63. If $x = \sec\theta + \tan\theta$, then $x + \frac{1}{x} =$
(a) 1 (b) $2\sec\theta$ (c) 2 (d) $2\tan\theta$

64. The value of $\frac{\tan 70° - \tan 20°}{\tan 50°} =$
(a) 1 (b) 2 (c) 3 (d) 0

65. $\frac{1}{\sin 10°} - \frac{\sqrt{3}}{\cos 10°} =$
(a) 0 (b) 1 (c) 2 (d) 4

66. The value of $\cos^2\frac{\pi}{12} + \cos^2\frac{\pi}{4} + \cos^2\frac{5\pi}{12}$ is
(a) $\frac{3}{2}$ (b) $\frac{2}{3}$ (c) $\frac{3+\sqrt{3}}{2}$ (d) $\frac{2}{3+\sqrt{3}}$

67. $1 + \cos 2x + \cos 4x + \cos 6x =$
(a) $2\cos x \cos 2x \cos 3x$
(b) $4\sin x \cos 2x \cos 3x$
(c) $4\cos x \cos 2x \cos 3x$
(d) None of these

68. cosec A – 2 cot 2A cos A =
(a) 2 sin A (b) sec A
(c) 2 cos A cot A (d) None of these

69. If $\sin x + \cos x = \frac{1}{5}$, then $\tan 2x$ is
(a) $\frac{25}{17}$ (b) $\frac{7}{25}$ (c) $\frac{25}{7}$ (d) $\frac{24}{7}$

70. If $\sqrt{3}\tan 2\theta + \sqrt{3}\tan 3\theta + \tan 2\theta \tan 3\theta = 1$, then the general value of θ is
(a) $n\pi + \frac{\pi}{5}$ (b) $\left(n + \frac{1}{6}\right)\frac{\pi}{5}$
(c) $\left(2n \pm \frac{1}{6}\right)\frac{\pi}{5}$ (d) $\left(n + \frac{1}{3}\right)\frac{\pi}{5}$

71. If $\tan\theta - \sqrt{2}\sec\theta = \sqrt{3}$, then the general value of θ is
(a) $n\pi + (-1)^n\frac{\pi}{4} - \frac{\pi}{3}$ (b) $n\pi + (-1)^n\frac{\pi}{3} - \frac{\pi}{4}$
(c) $n\pi + (-1)^n\frac{\pi}{3} + \frac{\pi}{4}$ (d) $n\pi + (-1)^n\frac{\pi}{4} + \frac{\pi}{3}$

72. The general solution of $\sin^2\theta \sec\theta + \sqrt{3}\tan\theta = 0$ is
(a) $\theta = n\pi + (-1)^{n+1}\frac{\pi}{3}$, $\theta = n\pi$, $n \in Z$
(b) $\theta = n\pi$, $n \in Z$
(c) $\theta = n\pi + (-1)^{n+1}\frac{\pi}{3}$, $n \in Z$
(d) $\theta = \frac{n\pi}{2}$, $n \in Z$

73. The value of $\frac{\cot 54°}{\tan 36°} + \frac{\tan 20°}{\cot 70°}$ is
(a) 2 (b) 3 (c) 1 (d) 0

STATEMENT TYPE QUESTIONS

Directions : Read the following statements and choose the correct option from the given below four options.

74. I : $\cos\alpha + \cos\beta + \cos\gamma = 0$
II : $\sin\alpha + \sin\beta + \sin\gamma = 0$
If $\cos(\beta - \gamma) + \cos(\gamma - \alpha) + \cos(\alpha - \beta) = -\frac{3}{2}$, then
(a) I is false and II is true (b) I and II both are true
(c) I and II both are false (d) I is true and II is false

75. Consider the statements given below:
I. $\sin x$ is positive in first and second quadrants.
II. cosec x is negative in third and fourth quadrants.
III. tan x and cot x are negative in second and fourth quadrants.
IV. cos x and sec x are positive in first and fourth quadrants.
Choose the correct option.
(a) All are correct
(b) Only I and IV are correct
(c) Only III and IV are correct
(d) None is correct

76. Which among the following is/are true?
I. The values of cosec x repeat after an interval of 2π.
II. The values of sec x repeat after an interval of 2π.
III. The values of cot x repeat after an interval of π.
(a) I is true (b) II is true
(c) III is true (d) All are true

77. Consider the following statements.
I. cot x decreases from 0 to $-\infty$ in first quadrant and increases from 0 to ∞ in third quadrant.
II. sec x increases from $-\infty$ to –1 in second quadrant and decreases from ∞ to 1 in fourth quadrant.
III. cosec x increases from 1 to ∞ in second quadrant and decreases from –1 to $-\infty$ in fourth quadrant.
Choose the correct option.
(a) I is incorrect (b) II is incorrect
(c) III is incorrect (d) IV is incorrect

78. Consider the statements given below:
I. $2\cos x \cdot \cos y = \cos(x + y) - \cos(x - y)$.
II. $-2\sin x \cdot \sin y = \cos(x + y) - \cos(x - y)$.
III. $2\sin x \cdot \cos y = \sin(x + y) - \sin(x - y)$.
IV. $2\cos x \cdot \sin y = \sin(x + y) + \sin(x - y)$.
Choose the correct statements.
(a) I is correct
(b) II is correct
(c) Both I and II are correct
(d) III is correct

79. If $\sin 2x + \cos x = 0$, then which among the following is/are true?
I. $\cos x = 0$
II. $\sin x = -\frac{1}{2}$
III. $x = (2n + 1)\frac{\pi}{2}$, $n \in Z$
IV. $x = n\pi + (-1)^n\frac{7\pi}{6}$, $n \in Z$
(a) I is true (b) I and II are true
(c) I, II and III are true (d) All are true

MATCHING TYPE QUESTIONS

Directions : Match the terms given in column-I with the terms given in column-II and choose the correct option from the codes given below.

80.

Column-I (Degree Measure)	Column-II (Radian Measure)
A. 25°	1. $\frac{26\pi}{9}$
B. – 47° 30′	2. $\frac{4\pi}{3}$
C. 240°	3. $\frac{-19\pi}{72}$
D. 520°	4. $\frac{5\pi}{36}$

Codes:

	A	B	C	D
(a)	4	1	2	3
(b)	4	3	2	1
(c)	1	3	2	4
(d)	1	4	3	2

81. $\left[\text{Use } \pi = \frac{22}{7}\right]$.

Column-I (Radian Measure)	Column-II (Degree Measure)
A. $\frac{11}{16}$	1. 300°
B. -4	2. 210°
C. $\frac{5\pi}{3}$	3. 39° 22′ 30″
D. $\frac{7\pi}{6}$	4. $-229°\ 5'\ 27''$

Codes:

	A	B	C	D
(a)	3	4	2	1
(b)	1	4	2	3
(c)	3	4	1	2
(d)	2	4	1	3

82.

Column-I	Column-II
A. $\sin\frac{25\pi}{3}$	1. $-\sqrt{3}$
B. $\cos\frac{41\pi}{4}$	2. $\frac{\sqrt{3}}{2}$
C. $\tan\left(\frac{-16\pi}{3}\right)$	3. 1
D. $\cot\frac{29\pi}{4}$	4. $\frac{1}{\sqrt{2}}$

Codes:

	A	B	C	D
(a)	2	4	1	3
(b)	2	1	4	3
(c)	3	1	4	2
(d)	3	4	1	2

83.

Column-I	Column-II
A. $\cos(\pi - x)$	1. $-\cos x$
B. $\sin(\pi - x)$	2. $-\sin x$
C. $\sin(\pi + x)$	3. $\cos x$
D. $\cos(\pi + x)$	4. $\sin x$
E. $\cos(2\pi - x)$	
F. $\sin(2\pi - x)$	

Codes:

	A	B	C	D	E	F
(a)	1	4	2	1	3	2
(b)	1	2	4	1	3	1
(c)	2	4	1	2	3	2
(d)	1	2	2	4	3	1

84.

Column-I	Column-II
A. 1 radian is equal to	1. 0.01746 radian
B. 1° is equal to	2. 57° 16′ (approx.)
C. 3° 45′ is equal to	3. $\frac{9\pi}{32}$ radian
D. 50° 37′ 30″ is equal to	4. $\frac{\pi}{48}$ radian

Codes:

	A	B	C	D
(a)	1	4	3	2
(b)	2	4	1	3
(c)	2	1	4	3
(d)	3	1	4	2

85.

Column-I (Degree measure)	Column-II (Radian measure)
(A) 25°	1. $\frac{-19\pi}{72}$
(B) –47° 30′	2. $\frac{4\pi}{3}$
(C) 240°	3. $\frac{26\pi}{9}$
(D) 520°	4. $\frac{5\pi}{36}$

Codes:

	A	B	C	D
(a)	4	2	1	3
(b)	3	1	2	4
(c)	4	1	2	3
(d)	3	2	1	4

86.

Column-I	Column-II
(A) $\sin x =$	1. $\frac{1}{\sqrt{3}}$
(B) $\tan x =$	2. -2
(C) $\cot x =$	3. $\frac{-\sqrt{3}}{2}$
(D) $\sec x =$	4. $\frac{-2}{\sqrt{3}}$
(E) $\text{cosec } x =$	5. $\sqrt{3}$

Codes:

	A	B	C	D	E
(a)	3	5	1	2	4
(b)	1	5	3	2	4
(c)	3	5	1	4	2
(d)	3	1	5	4	2

87.

Column-I (Trigonometric Equation)	Column-II (General Solution)
(A) $\cos 4x = \cos 2x$	1. $x = n\pi \pm \frac{\pi}{3}$
(B) $\cos 3x + \cos x - \cos 2x = 0$	2. $x = \frac{n\pi}{2} + \frac{3\pi}{8}$
(C) $\sin 2x + \cos x = 0$	3. $x = 2n\pi \pm \frac{\pi}{3}$
(D) $\sec^2 2x = 1 - \tan 2x$	4. $x = \frac{n\pi}{3}$ or $x = n\pi,\ n \in Z$
(E) $\sin x + \sin 3x + \sin 5x = 0$	5. $x = (2n+1)\frac{\pi}{2}$ or $x = n\pi + (-1)^n . \frac{7\pi}{6}$, $n \in Z$

Codes:

	A	B	C	D	E
(a)	4	3	5	1	2
(b)	4	3	5	2	1
(c)	3	4	5	1	2
(d)	1	3	5	2	4

88.

Column-I	Column-II
(A) $\frac{\cos 9x - \cos 5x}{\sin 17x - \sin 3x} =$	1. $\tan 2x$
(B) $\frac{\sin 5x + \sin 3x}{\cos 5x + \cos 3x} =$	2. $\tan \frac{x-y}{2}$
(C) $\frac{\sin x + \sin 3x}{\cos x + \cos 3x} =$	3. $\frac{-\sin 2x}{\cos 10x}$
(D) $\frac{\sin x - \sin y}{\cos x + \cos y} =$	4. $2 \sin x$
(E) $\frac{\sin x - \sin 3x}{\sin^2 x - \cos^2 x} =$	5. $\tan 4x$

Codes:

	A	B	C	D	E
(a)	3	5	2	1	4
(b)	3	5	1	4	2
(c)	3	1	2	5	4
(d)	3	5	1	2	4

89. Let $\sin x = \frac{3}{5}$, x lies in second quadrant.

Column-I (Trigonometric Function)	Column-II (Value)
(A) $\cos x =$	1. $-4/3$
(B) $\sec x =$	2. $-3/4$
(C) $\tan x =$	3. $-4/5$
(D) $\text{cosec } x =$	4. $-5/4$
(E) $\cot x =$	5. $5/3$

Codes:

	A	B	C	D	E
(a)	3	4	2	5	1
(b)	3	4	1	5	2
(c)	3	2	4	5	1
(d)	1	2	5	4	3

INTEGER TYPE QUESTIONS

Directions : This section contains integer type questions. The answer to each of the question is a single digit integer, ranging from 0 to 9. Choose the correct option.

90. The value of cosec $(-1410)°$ is equal to

(a) 1 (b) $\frac{1}{2}$ (c) 2 (d) None of these

91. The expression $\cos\frac{10\pi}{13} + \cos\frac{8\pi}{13} + \cos\frac{3\pi}{13} + \cos\frac{5\pi}{13}$ is equal to

(a) -1 (b) 0 (c) 1 (d) None of these

92. If $\sin\theta + \cos\theta = 1$, then $\sin\theta\cos\theta =$

(a) 0 (b) 1 (c) 2 (d) $\frac{1}{2}$

93. If $\frac{\cos A}{3} = \frac{\cos B}{4} = \frac{1}{5}$, $\frac{-\pi}{2} < A < 0$ $\frac{-\pi}{2} < B < 0$, then value of $2 \sin A + 4 \sin B$ is $-$ a. The value of 'a' is

(a) 4 (b) 2 (c) 3 (d) 0

94. The value of $\sin 765°$ is $\frac{1}{\sqrt{n}}$. Value of n is

(a) 2 (b) 3 (c) 4 (d) 0

95. The value of cosec $(-1410)°$ is equal to

(a) 1 (b) 2 (c) $\frac{1}{2}$ (d) None of these

96. The value of $\tan\frac{19\pi}{3}$ is $\sqrt{n}$. Value of 'n' is

(a) 1 (b) 2 (c) 3 (d) 5

97. The value of $\sin\left(\frac{-11\pi}{3}\right)$ is $\frac{\sqrt{3}}{m}$. Value of 'm' is

(a) 1 (b) 2 (c) 3 (d) 5

98. The value of $\left(1+\cos\frac{\pi}{6}\right)\left(1+\cos\frac{\pi}{3}\right)\left(1+\cos\frac{2\pi}{3}\right)\left(1+\cos\frac{7\pi}{6}\right)$ is $\frac{m}{16}$. Value of m is

(a) 1 (b) 2 (c) 3 (d) 8

99. If $\tan\theta = \frac{1}{\sqrt{7}}$, then $\left(\frac{\text{cosec}^2\theta - \sec^2\theta}{\text{cosec}^2\theta + \sec^2\theta}\right)$ is equal to $\frac{m}{m+1}$. The value of m is

(a) 1 (b) 2 (c) 3 (d) 4

100. If $\sin x = \frac{-2\sqrt{6}}{5}$ and x lies in III quadrant, then the value of $\cot x$ is $\frac{1}{m\sqrt{6}}$. Value of m is

(a) 1 (b) 2 (c) 3 (d) 5

101. If $\cos\theta = \frac{-3}{5}$ and $\pi < \theta < \frac{3\pi}{2}$, then the value of $\left(\frac{\text{cosec}\,\theta + \cot\theta}{\sec\theta - \tan\theta}\right)$ is equal to $\frac{1}{m}$. Value of m is

(a) 2 (b) 4 (c) 5 (d) 6

102. The value of

$3\sin\frac{\pi}{6}\sec\frac{\pi}{3} - 4\sin\frac{5\pi}{6}\cot\frac{\pi}{4}$ is equal to

(a) 2 (b) 1 (c) 3 (d) 4

103. Value of $2\sin^2\frac{\pi}{6} + \text{cosec}^2\frac{7\pi}{6}\cdot\cos^2\frac{\pi}{3}$ is $\frac{m}{m-1}$. The value of 'm' is

(a) 3 (b) 2 (c) 4 (d) None of these

104. $\cot^2\frac{\pi}{6} + \text{cosec}\frac{5\pi}{6} + 3\tan^2\frac{\pi}{6}$ is equal to

(a) 1 (b) 5 (c) 3 (d) 6

105. Value of

$\cos\left(\frac{3\pi}{2}+x\right)\cos(2\pi+x)\left[\cot\left(\frac{3\pi}{2}-x\right)+\cot(2\pi+x)\right]$ is

(a) 0 (b) 1 (c) 2 (d) 3

106. $\cot x\cot 2x - \cot 2x\cot 3x - \cot 3x\cot x$ is equal to

(a) 0 (b) 1 (c) 2 (d) 3

ASSERTION - REASON TYPE QUESTIONS

Directions : Each of these questions contains two statements, Assertion and Reason. Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.

(a) Assertion is correct, reason is correct; reason is a correct explanation for assertion.

(b) Assertion is correct, reason is correct; reason is not a correct explanation for assertion

(c) Assertion is correct, reason is incorrect

(d) Assertion is incorrect, reason is correct.

107. Assertion : The ratio of the radii of two circles at the centres of which two equal arcs subtend angles of 30° and 70° is 21 : 10.

Reason : Number of radians in an angle subtended at the centre of a circle by an arc is equal to the ratio of the length of the arc to the radius of the circle.

108. Assertion : If $\tan\left(\frac{\pi}{2}\sin\theta\right) = \cot\left(\frac{\pi}{2}\cos\theta\right)$, then $\sin\theta + \cos\theta = \pm\sqrt{2}$.

Reason : $-\sqrt{2} \le \sin\theta + \cos\theta \le \sqrt{2}$.

109. Assertion : The solution of the equation

$\tan\theta + \tan\left(\theta+\frac{\pi}{3}\right) + \tan\left(\theta+\frac{2\pi}{3}\right) = 3$

is $\theta = \frac{n\pi}{3} + \frac{\pi}{12}$, $n \in I$.

Reason : If $\tan\theta = \tan\alpha$, then $\theta = n\pi + \alpha$, $n \in I$.

110. Assertion : The degree measure corresponding to (–2) radian is –114° 19 min.

Reason : The degree measure of a given radian measure $= \frac{180}{\pi} \times$ Radian measure.

111. Assertion : $\frac{\cos(\pi+x)\cdot\cos(-x)}{\sin(\pi-x)\cdot\cos\left(\frac{\pi}{2}+x\right)} = \cot^2 x$

Reason : $\cos(\pi+\theta) = -\cos\theta$ and $\cos(-\theta) = \cos\theta$. Also, $\sin(\pi-\theta) = \sin\theta$ and $\sin(-\theta) = -\sin\theta$.

112. Assertion : If $\tan 2x = -\cot\left(x+\frac{\pi}{3}\right)$, then $x = n\pi + \frac{5\pi}{6}$, $n \in Z$.

Reason : $\tan x = \tan y \Rightarrow x = n\pi + y$, where $n \in Z$.

113. Assertion : The measure of rotation of a given ray about its initial point is called an angle.

Reason : The point of rotation is called a vertex.

114. Assertion : In a unit circle, radius of circle is 1 unit.

Reason : 1 min (or 1′) is divided into 60s.

115. Assertion : Area of unit circle is π unit^2.

Reason : Radian measure of 40° 20′ is equal to $\frac{|2|\pi}{540}$ radian.

116. Assertion : The second hand rotates through an angle of 180° in a minute.

Reason : The unit of measurement is degree in sexagesimal system.

117. Assertion: cosec x is negative in third and fourth quadrants.

Reason : $\cot x$ decreases from 0 to $-\infty$ in first quadrant and increases from 0 to ∞ in third quadrant.

CRITICALTHINKING TYPE QUESTIONS

Directions : This section contains multiple choice questions. Each question has four choices (a), (b), (c) and (d), out of which only one is correct.

118. The value of $\tan 20° + 2\tan 50° - \tan 70°$ is equal to

(a) 1 (b) 0

(c) tan 50° (d) None of these

119. If α and β lies between 0 and $\frac{\pi}{2}$ and if $\cos(\alpha+\beta) = \frac{12}{13}$ and $\sin(\alpha-\beta) = \frac{3}{5}$, then value of $\sin 2\alpha$ is

(a) $\frac{55}{56}$ (b) $\frac{13}{58}$ (c) 0 (d) $\frac{56}{65}$

120. The most general value of θ satisfying the equation $\cos\theta = \frac{1}{\sqrt{2}}$ and $\tan\theta = -1$ is

(a) $2n\pi - 7\frac{\pi}{4}$ (b) $n\pi - \frac{\pi}{4}$

(c) $n\pi + \frac{\pi}{2}$ (d) $2n\pi + \frac{7\pi}{4}$

121. Value of $\sqrt{3}$ cosec 20° – sec 20° is

(a) 3 (b) $\frac{3}{2}$ (c) 1 (d) 4

122. The solution of the equation $\cos^2\theta + \sin\theta + 1 = 0$, lies in the interval

(a) $\left(-\frac{\pi}{4}, \frac{\pi}{4}\right)$ (b) $\left(\frac{\pi}{4}, \frac{3\pi}{4}\right)$

(c) $\left(\frac{3\pi}{4}, \frac{5\pi}{4}\right)$ (d) $\left(\frac{5\pi}{4}, \frac{7\pi}{4}\right)$

123. The number of values of x in the interval $[0, 3\pi]$ satisfying the equation $2\sin^2 x + 5\sin x - 3 = 0$ is

(a) 4 (b) 6 (c) 1 (d) 2

124. Value of $2\cos\frac{\pi}{13}\cos\frac{9\pi}{13} + \cos\frac{3\pi}{13} + \cos\frac{5\pi}{13}$ is

(a) $-\frac{1}{2}$ (b) 0 (c) 1 (d) $\frac{\sqrt{3}}{2}$

125. Value of sin 47° + sin 61° – sin 11° – sin 25° is

(a) cos 7° (b) sin 7° (c) sin 61° (d) –sin 25°

126. The value of expression $\sin\theta + \cos\theta$ lies between

(a) –2 and 2 both inclusive

(b) 0 and $\sqrt{2}$ both inclusive

(c) $-\sqrt{2}$ and $\sqrt{2}$ both inclusive

(d) 0 and 2 both inclusive

127. The solution of $\tan 2\theta \tan\theta = 1$ is

(a) $2n\pi + \frac{\pi}{3}$ (b) $n\pi + \frac{\pi}{4}$

(c) $2n\pi - \frac{\pi}{6}$ (d) $(2n+1)\frac{\pi}{6}$

128. Number of solutions of equation, $\sin 5x \cos 3x = \sin 6x \cos 2x$, in the interval $[0, \pi]$ is

(a) 4 (b) 5 (c) 3 (d) 2

129. If tan (cot x) = cot (tan x), then

(a) $\sin 2x = \frac{2}{(2n+1)\pi}$ (b) $\sin x = \frac{4}{(2n+1)\pi}$

(c) $\sin 2x = \frac{4}{(2n+1)\pi}$ (d) None of these

130. Find the distance from the eye at which a coin of a diameter 1 cm be placed so as to hide the full moon, it is being given that the diameter of the moon subtends an angle of 31′ at the eye of the observer.

(a) 110 cm (b) 108 cm

(c) 110.9 cm (d) 112 cm

131. A wheel rotates making 20 revolutions per second. If the radius of the wheel is 35 cm, what linear distance does a point of its rim travel in three minutes? $\left(\text{Take } \pi = \frac{22}{7}\right)$

(a) 7.92 km (b) 7.70 km

(c) 7.80 km (d) 7.85 km

132. The minute hand of a watch is 1.5 cm long. How far does its tip move in 40 minutes? (Use π = 3.14)

(a) 2.68 cm (b) 6.28 cm

(c) 6.82 cm (d) 7.42 cm

133. If the arcs of the same lengths in two circles subtend angles 65° and 110° at the centre, the ratio of their radii is

(a) 12 : 13 (b) 22 : 31 (c) 22 : 13 (d) 21 : 13

134. If tan A + cot A = 4, then $\tan^4 A + \cot^4 A$ is equal to

(a) 110 (b) 191 (c) 80 (d) 194

135. If $\frac{\sin A}{\sin B} = m$ and $\frac{\cos A}{\cos B} = n$, then the value of tan B; $n^2 < 1 < m^2$, is

(a) n^2 (b) $\pm\sqrt{\frac{1-n^2}{m^2-1}}$

(c) $\frac{n^2}{(m^2-1)}$ (d) m^2

136. If $\tan(A - B) = 1$, $\sec(A + B) = \frac{2}{\sqrt{3}}$, the smallest positive value of B is

(a) $\frac{25\pi}{24}$ (b) $\frac{19\pi}{24}$ (c) $\frac{13\pi}{24}$ (d) $\frac{7\pi}{24}$

137. The value of $4 \sin \alpha \sin\left(\alpha+\frac{\pi}{3}\right) \sin\left(\alpha+\frac{2\pi}{3}\right) =$

(a) $\sin 3\alpha$ (b) $\sin 2\alpha$ (c) $\sin \alpha$ (d) $\sin^2 \alpha$

138. The solution of the equation

$[\sin x + \cos x]^{1+\sin 2x} = 2, -\pi \le x \le \pi$ is

(a) $\frac{\pi}{2}$ (b) π

(c) $\frac{\pi}{4}$ (d) $\frac{3\pi}{4}$

139. If $\tan \theta + \sec \theta = p$, then what is the value of $\sec \theta$?

(a) $\frac{p^2+1}{p^2}$ (b) $\frac{p^2+1}{\sqrt{p}}$

(c) $\frac{p^2+1}{2p}$ (d) $\frac{p+1}{2p}$

140. The number of solutions of the given equation

$\tan \theta + \sec \theta = \sqrt{3}$, where $0 \le \theta \le 2\pi$ is

(a) 0 (b) 1 (c) 2 (d) 3

141. If n is any integer, then the general solution of the equation $\cos x - \sin x = \frac{1}{\sqrt{2}}$ is

(a) $x = 2n\pi - \frac{\pi}{12}$ or $x = 2n\pi + \frac{7\pi}{12}$

(b) $x = n\pi \pm \frac{\pi}{12}$

(c) $x = 2n\pi + \frac{\pi}{12}$ or $x = 2n\pi - \frac{7\pi}{12}$

(d) $x = n\pi + \frac{\pi}{12}$ or $x = n\pi - \frac{7\pi}{12}$

142. If $4 \sin^2 \theta + 2\left(\sqrt{3}+1\right) \cos \theta = 4 + \sqrt{3}$, then the general value of θ is

(a) $2n\pi \pm \frac{\pi}{3}$ (b) $2n\pi + \frac{\pi}{4}$

(c) $n\pi \pm \frac{\pi}{3}$ (d) $n\pi - \frac{\pi}{3}$

143. The number of values of x in the interval $[0, 3\pi]$ satisfying the equation

$2 \sin^2 x + 5 \sin x - 3 = 0$ is

(a) 4 (b) 6

(c) 1 (d) 2

144. If $\sin \theta + \cos \theta = 1$, then the general value of θ is

(a) $2n\pi$ (b) $n\pi + (-1)^n \frac{\pi}{4} - \frac{\pi}{4}$

(c) $2n\pi + \frac{\pi}{2}$ (d) $(2n - 1) + \frac{\pi}{4}$

145. $\sin 6\theta + \sin 4\theta + \sin 2\theta = 0$, then $\theta =$

(a) $\frac{n\pi}{4}$ or $n\pi \pm \frac{\pi}{3}$ (b) $\frac{n\pi}{4}$ or $n\pi \pm \frac{\pi}{6}$

(c) $\frac{n\pi}{4}$ or $2n\pi \pm \frac{\pi}{6}$ (d) None of these

146. If $\sqrt{2} \sec \theta + \tan \theta = 1$, then the general value of θ is

(a) $n\pi + \frac{3\pi}{4}$ (b) $2n\pi + \frac{\pi}{4}$

(c) $2n\pi - \frac{\pi}{4}$ (d) $2n\pi \pm \frac{\pi}{4}$

147. If $12 \cot^2 \theta - 31 \operatorname{cosec} \theta + 32 = 0$, then the value of $\sin \theta$ is

(a) $\frac{3}{5}$ or 1 (b) $\frac{2}{3}$ or $\frac{-2}{3}$

(c) $\frac{4}{5}$ or $\frac{3}{4}$ (d) $\pm\frac{1}{2}$

148. If $\sec^2 \theta = \frac{4}{3}$, then the general value of θ is

(a) $2n\pi \pm \frac{\pi}{6}$ (b) $n\pi \pm \frac{\pi}{6}$

(c) $2n\pi \pm \frac{\pi}{3}$ (d) $n\pi \pm \frac{\pi}{3}$

149. General solution of $\tan 5\theta = \cot 2\theta$ is

(a) $\theta = \frac{n\pi}{7} + \frac{\pi}{14}$ (b) $\theta = \frac{n\pi}{7} + \frac{\pi}{5}$

(c) $\theta = \frac{n\pi}{7} + \frac{\pi}{2}$ (d) $\theta = \frac{n\pi}{7} + \frac{\pi}{3}$

150. If none of the angles x, y and $(x + y)$ is a multiple of π, then

(a) $\cot(x+y) = \dfrac{\cot x \cdot \cot y - 1}{\cot y + \cot x}$

(b) $\cot(x-y) = \dfrac{\cot x \cdot \cot y + 1}{\cot y - \cot x}$

(c) (a) and (b) are true

(d) (a) and (b) are not true

151. Solution of the equation $3\tan(\theta - 15) = \tan(\theta + 15)$ is

(a) $\theta = \dfrac{n\pi}{2} + (-1)^n \dfrac{\pi}{4}$ (b) $\theta = n\pi + (-1)^n \dfrac{\pi}{3}$

(c) $\theta = n\pi - \dfrac{\pi}{3}$ (d) $\theta = n\pi - \dfrac{\pi}{4}$

152. If angle θ is divided into two parts such that the tangent of one part is K times the tangent to other and ϕ is their difference, then $\sin\theta$ is equal to

(a) $\dfrac{K+1}{K-1}\sin\dfrac{\theta}{2}$ (b) $\dfrac{K+1}{K-1}\sin\dfrac{\phi}{2}$

(c) $\dfrac{K+1}{K-1}\sin\phi$ (d) $\dfrac{K-1}{K+1}\sin\phi$

153. If $m\sin\theta = n\sin(\theta + 2\alpha)$, then $\tan(\theta+\alpha)\cdot\cot\alpha$ is equal to

(a) $\dfrac{m+n}{m-n}$ (b) $\dfrac{m-n}{m+n}$

(c) $\dfrac{m+n}{mn}$ (d) $\dfrac{m-n}{mn}$

154. If $5\tan\theta = 4$, then $\dfrac{5\sin\theta - 3\cos\theta}{5\sin\theta + 2\cos\theta} =$

(a) 0 (b) 1 (c) $\dfrac{1}{6}$ (d) 6

155. $\dfrac{1+\sin A - \cos A}{1 + \sin A + \cos A} =$

(a) $\sin\dfrac{A}{2}$ (b) $\cos\dfrac{A}{2}$ (c) $\tan\dfrac{A}{2}$ (d) $\cot\dfrac{A}{2}$

156. $\dfrac{1}{4}\left[\sqrt{3}\cos 23° - \sin 23°\right] =$

(a) $\cos 43°$ (b) $\cos 7°$ (c) $\cos 53°$ (d) None of these

157. If $\cos x + \cos y + \cos\alpha = 0$ and $\sin x + \sin y + \sin\alpha = 0$, then $\cot\left(\dfrac{x+y}{2}\right) =$

(a) $\sin\alpha$ (b) $\cos\alpha$ (c) $\cot\alpha$ (d) $\sin\left(\dfrac{x+y}{2}\right)$

158. $\sin 12° \sin 24° \sin 48° \sin 84° =$

(a) $\cos 20° \cos 40° \cos 60° \cos 80°$

(b) $\sin 20° \sin 40° \sin 60° \sin 80°$

(c) $\dfrac{3}{15}$

(d) None of these

159. If $\dfrac{2\sin\alpha}{\{1+\cos\alpha+\sin\alpha\}} = y$, then $\dfrac{\{1-\cos\alpha+\sin\alpha\}}{1+\sin\alpha} =$

(a) $\dfrac{1}{y}$ (b) y (c) $1-y$ (d) $1+y$

160. If $\sin 2\theta + \sin 2\phi = \dfrac{1}{2}$ and $\cos 2\theta + \cos 2\phi = \dfrac{3}{2}$, then $\cos^2(\theta - \phi) =$

(a) $\dfrac{3}{8}$ (b) $\dfrac{5}{8}$ (c) $\dfrac{3}{4}$ (d) $\dfrac{5}{4}$

HINTS AND SOLUTIONS

CONCEPT TYPE QUESTIONS

1. **(b)** $\tan^2\theta\sec^2\theta\,(\cot^2\theta-\cos^2\theta)$
$=\sec^2\theta\,(\tan^2\theta\cot^2\theta-\tan^2\theta\cos^2\theta)$

$=\sec^2\theta\left(1-\frac{\sin^2\theta}{\cos^2\theta}\cos^2\theta\right)=\sec^2\theta(1-\sin^2\theta)$

$=\sec^2\theta.\cos^2\theta=1$

2. **(c)** $\cot 5^\circ\cot 10^\circ$ .... $\cot 85^\circ$
$=\cot 5^\circ\cot 10^\circ....\cot(90^\circ-10^\circ)\cot(90^\circ-5^\circ)$
$=\cot 5^\circ\cot 10^\circ......\tan 10^\circ\tan 5^\circ$
$=(\tan 5^\circ\cot 5^\circ)(\tan 10^\circ\cot 10^\circ)$........
$=(1)(1)(1).......=1$

3. **(b)** $\because\sin 190^\circ=\sin(180^\circ+10^\circ)=-\sin 10^\circ$
$\sin 200^\circ=-\sin 20^\circ$
$\sin 210^\circ=-\sin 30^\circ$
....
$\sin 360^\circ=\sin 180^\circ=0$
$\therefore$ given expression $=0$

4. **(d)** $\tan(A+B)=\frac{\tan A+\tan B}{1-\tan A\tan B}=\frac{\frac{1}{2}+\frac{1}{3}}{1-\frac{1}{2}.\frac{1}{3}}=\frac{5/6}{5/6}=1$

$\therefore A+B=45^\circ=\frac{\pi}{4}$

5. **(a)** Using cosine formula
$\sin 2\theta+\sin 2\phi=2\sin(\theta+\phi)\cos(\theta-\phi)=1/2$ .....(i)
$\cos 2\theta+\cos 2\phi=2\cos(\theta+\phi)\cos(\theta-\phi)=3/2$ .....(ii)
Squaring (i) and (ii) and then adding

$4\cos^2(\theta-\phi)=\frac{1}{4}+\frac{9}{4}=\frac{5}{2}$

$\Rightarrow\cos^2(\theta-\phi)=\frac{5}{8}$

6. **(d)** $\cos\theta=-1/2=\cos 120^\circ$ or $\cos 240^\circ$ $[0<\theta<360^\circ]$
$\therefore\theta=120^\circ,240^\circ$

7. **(d)** $\tan\theta=-\frac{1}{\sqrt{3}}=\tan\left(-\frac{\pi}{6}\right)$

$\therefore\theta=n\pi-\frac{\pi}{6}$

8. **(a)** $2\tan^2\theta=\sec^2\theta=1+\tan^2\theta$

$\tan^2\theta=1=(1)^2=\tan^2\frac{\pi}{4}$

$\theta=n\pi\pm\frac{\pi}{4},n\in I$

9. **(c)** $\sin 5x+\sin x=-\sin 3x$
$\Rightarrow 2\sin 3x\cos 2x+\sin 3x=0$
$\Rightarrow\sin 3x(2\cos 2x+1)=0$
$\Rightarrow\sin 3x=0,\cos 2x=-1/2$
$\Rightarrow x=n\pi,x=n\pi\pm(\pi/3)$
So, $x=\pi/3$

10. **(b)** $\frac{1-\cos\alpha+\sin\alpha}{1+\sin\alpha}=\frac{1-\cos\alpha+\sin\alpha}{1+\sin\alpha}.\frac{1+\cos\alpha+\sin\alpha}{1+\cos\alpha+\sin\alpha}$

$=\frac{(1+\sin\alpha)^2-\cos^2\alpha}{(1+\sin\alpha)(1+\cos\alpha+\sin\alpha)}$

$=\frac{(1+\sin^2\alpha+2\sin\alpha)-(1-\sin^2\alpha)}{(1+\sin\alpha)(1+\cos\alpha+\sin\alpha)}$

$=\frac{2\sin\alpha(1+\sin\alpha)}{(1+\sin\alpha)(1+\cos\alpha+\sin\alpha)}=\frac{2\sin\alpha}{1+\cos\alpha+\sin\alpha}=y$

11. **(b)** The given equation is $\tan x+\sec x=2\cos x$;
$\Rightarrow\sin x+1=2\cos^2 x\Rightarrow\sin x+1=2(1-\sin^2 x)$;
$\Rightarrow 2\sin^2 x+\sin x-1=0$;

$\Rightarrow(2\sin x-1)(\sin x+1)=0\Rightarrow\sin x=\frac{1}{2},-1$

$\Rightarrow x=30^\circ,150^\circ,270^\circ$.

12. **(d)** We have : $\sin A=\frac{3}{5}$, where $0<A<\frac{\pi}{2}$

$\therefore\quad\cos A=\pm\sqrt{1-\sin^2 A}$

$\Rightarrow\quad\cos A=+\sqrt{1-\sin^2 A}=\sqrt{1-\frac{9}{25}}=\frac{4}{5}$

[$\because$ cos is positive in first quadrant]

It is given that : $\cos B=\frac{-12}{13}$ and $\pi<B<\frac{3\pi}{2}$

$\therefore\quad\sin B=\pm\sqrt{1-\cos^2 B}$

$\Rightarrow\quad\sin B=-\sqrt{1-\cos^2 B}$

[$\because$ Sine is negative in the third quadrant]

$\Rightarrow\quad\sin B=-\sqrt{1-\left(\frac{-12}{13}\right)^2}=-\frac{5}{13}$

Now, $\sin(A-B)=\sin A\cos B-\cos A\sin B$

$=\frac{3}{5}\times\frac{-12}{13}-\frac{4}{5}\times\frac{-5}{13}=-\frac{16}{65}$

13. **(b)** $\tan 15^\circ.\tan 45^\circ\tan 75^\circ$
$=\tan 15^\circ.\tan(60^\circ-15^\circ).\tan(60^\circ+15^\circ)$
$=\tan(3\times 15^\circ)=\tan 45^\circ=1$

14. **(a)** $\left(1+\cos\frac{\pi}{8}\right)\left(1+\cos\frac{3\pi}{8}\right)\left(1+\cos\left(\pi-\frac{3\pi}{8}\right)\right)\left(1+\cos\left(\pi-\frac{\pi}{8}\right)\right)$

$=\left(1+\cos\frac{\pi}{8}\right)\left(1+\cos\frac{3\pi}{8}\right)\left(1-\cos\frac{3\pi}{8}\right)\left(1-\cos\frac{\pi}{8}\right)$

$=\left(1-\cos^2\frac{\pi}{8}\right)\left(1-\cos^2\frac{3\pi}{8}\right)$

$= \frac{1}{4}\left(2-1-\cos\frac{\pi}{4}\right)\left(2-1-\cos\frac{3\pi}{4}\right)$

$= \frac{1}{4}\left(1-\cos\frac{\pi}{4}\right)\left(1-\cos\frac{3\pi}{4}\right)$

$= \frac{1}{4}\left(1-\frac{1}{\sqrt{2}}\right)\left(1+\frac{1}{\sqrt{2}}\right). = \frac{1}{4}\left(1-\frac{1}{2}\right) = \frac{1}{8}$

15. (a) The large hand of the clock makes a complete revolution in 60 minutes.

$\therefore$ Angle traced out by the large hand in 20 minutes (of time)

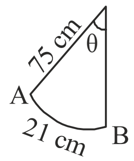
$= \frac{360° \times 20}{60} = 120° = \frac{120\pi}{180}$ radian $= \frac{2\pi}{3}$ radian

Hence, the distance moved by the extremity of the large hand $= (42) \times \frac{2\pi}{3} = 88$ cm. $(\because l = r\theta)$

16. (a) Given, length of pendulum = 75 cm

Radius (r) = length of pendulum = 75 cm

Length of arc (l) = 21 cm



Now, $\theta = \frac{l}{r} = \frac{21}{75} = \frac{7}{25}$ radian.

17. (c) Let l be the length of the arc. We know that,

Angle $\theta = \frac{l}{r}$, where θ is in radian.

Given, r = 3 cm

$\theta = 30° = 30 \times \frac{\pi}{180} = \frac{\pi}{6}$ rad

On putting the values of r and θ, we get

$\frac{\pi}{6} = \frac{l}{3} \Rightarrow l = \frac{\pi}{2} = \frac{3.14}{2} = 1.57$ cm.

18. (b) Circumference of a circular wire of radius 7 cm is $= 2\pi \times 7 = 14\pi$

As we know, $\theta = \frac{l}{r}$

$\Rightarrow \theta = \frac{14\pi}{12} = \frac{7\pi \times 180°}{6\pi} = 210°$.

19. (c) Length of wire $= 2\pi \times 3 = 6\pi$ cm and r = 48 cm is the radius of the circle. Therefore, the angle θ (in radian) subtended at the centre of the circle is given by

$\theta = \frac{\text{Arc}}{\text{Radius}} = \frac{6\pi}{48} = \frac{\pi}{8} = 22.5°$.

20. (c) Here, $l = 37.4$ cm and $\theta = 60° = \frac{60\pi}{180}$ radian $= \frac{\pi}{3}$

Hence, by $r = \frac{l}{\theta}$, we have

$r = \frac{37.4 \times 3}{\pi} = \frac{37.4 \times 3 \times 7}{22} = 35.7$ cm.

21. (b) Given radius, r = 100 cm and arc length, $l = 22$ cm

We know that, $l = r\theta$

$\theta = \frac{l}{r} = \frac{\text{Arc length}}{\text{Radius}}$

$= \frac{22}{100} = 0.22$ rad $= 0.22 \times \frac{180}{\pi}$ degree

$= 0.22 \times \frac{180 \times 7}{22} = \frac{22}{100} \times \frac{180 \times 7}{22}$

$= \frac{126}{10} = 12\frac{6°}{10} = 12° + \frac{6}{10} \times 60' \quad [\because 1° = 60']$

$= 12° + 36' = 12°\ 36'$

Hence, the degree measure of the required angle is 12°36′.

22. (c) Given, $\tan\theta = \frac{3}{1}$ and θ lies in III quadrant.

We know that $\sec^2\theta = 1 + \tan^2\theta = 1 + \left(\frac{3}{1}\right)^2 = 10$

$\Rightarrow \sec\theta = \pm\sqrt{10}$

Since, θ lies in III quadrant, so $\sec\theta = -\sqrt{10}$

$\Rightarrow \cos\theta = \frac{1}{\sec\theta} = \frac{1}{-\sqrt{10}}$

Also,

$\sin^2\theta = 1 - \cos^2\theta = 1 - \left(-\frac{1}{\sqrt{10}}\right)^2$

$= 1 - \frac{1}{10} = \frac{9}{10}$

$\Rightarrow \sin\theta = \pm\sqrt{\frac{9}{10}}$

Since, θ lies in III quadrant so $\sin\theta = -\sqrt{\frac{9}{10}} = \frac{-3}{\sqrt{10}}$.

23. (b) $\frac{\sin x}{\cos x} \times \frac{\sec x}{\text{cosec}\, x} \times \frac{\tan x}{\cot x} = 9$

$\Rightarrow \tan x \times \tan x \times \frac{\tan x}{\cot x} = 9$

$\Rightarrow \tan^4 x = 9$

$\Rightarrow \tan x = \pm\sqrt{3}$

$\Rightarrow x = \frac{\pi}{3} \in \left(0, \frac{\pi}{2}\right)$.

24. (b) The given equation is

$\text{cosec}(90° + \theta) + x\cos\theta\cot(90° + \theta) = \sin(90° + \theta)$

$\Rightarrow \sec\theta + x\cos\theta(-\tan\theta) = \cos\theta$

$\Rightarrow \sec\theta - x\cos\theta\left(\frac{\sin\theta}{\cos\theta}\right) = \cos\theta$

$\Rightarrow \sec\theta - x\sin\theta = \cos\theta$

$\Rightarrow x\sin\theta = \sec\theta - \cos\theta = \frac{1}{\cos\theta} - \cos\theta$

$\Rightarrow x\sin\theta = \frac{1-\cos^2\theta}{\cos\theta} = \frac{\sin^2\theta}{\cos\theta}$

$\Rightarrow x = \tan\theta$.

25. (d) We have $A + B = \frac{\pi}{4}$

$\Rightarrow \cot(A+B) = \cot\frac{\pi}{4} \Rightarrow \frac{\cot A\cot B - 1}{\cot A + \cot B} = 1$

$\Rightarrow \cot A\cot B - 1 = \cot A + \cot B$

$\Rightarrow$ cot A cot B – cot A – cot B – 1 = 0
$\Rightarrow$ cot A cot B – cot A – cot B + 1 = 2
$\Rightarrow$ cot A(cot B – 1) – 1(cot B – 1) = 2
$\Rightarrow$ (cot A – 1) (cot B – 1) = 2.

26. (a) We have, $\sin A = \frac{3}{5}$

$$\Rightarrow \cos A = \sqrt{1-\sin^2 A}$$
$$= \sqrt{1-\frac{9}{25}} = \sqrt{\frac{16}{25}} = \frac{4}{5}$$

and $\tan A = \frac{\sin A}{\cos A} = \frac{\frac{3}{5}}{\frac{4}{5}} = \frac{3}{4}$

Now, $\sin 2A = 2\sin A\cdot\cos A = 2\times\frac{3}{5}\times\frac{4}{5} = \frac{24}{25}$

$\cos 2A = 1-2\sin^2 A = 1-2\times\frac{9}{25} = 1-\frac{18}{25} = \frac{7}{25}$

and $\tan 2A = \frac{24}{7}$.

27. (b) Given, $\cot\alpha = \frac{1}{2} \Rightarrow \tan\alpha = 2$ and $\sec\beta = \frac{-5}{3}$

Then, $\tan\beta = \sqrt{\sec^2\beta - 1}$

$$\Rightarrow \tan\beta = \pm\sqrt{\frac{25}{9}-1} = \pm\sqrt{\frac{16}{9}}$$

$$\Rightarrow \tan\beta = \pm\frac{4}{3}$$

But, $\tan\beta = \frac{-4}{3}$

[$\because$ $\tan\beta$ is negative in IInd quadrant]

$$\therefore \tan(\alpha+\beta) = \frac{\tan\alpha+\tan\beta}{1-\tan\alpha\cdot\tan\beta} = \frac{2+\left(-\frac{4}{3}\right)}{1-(2)\left(\frac{-4}{3}\right)}$$

$$= \frac{\left(2-\frac{4}{3}\right)}{\left(1+\frac{8}{3}\right)} = \frac{2}{11}.$$

28. (a) tan 75° – cot 75° = tan(45° + 30°) – cot(45° + 30°)

$$= \frac{\tan 45° + \tan 30°}{1-\tan 45°\tan 30°} - \frac{\cot 45°\cot 30° - 1}{\cot 45° + \cot 30°}$$

$$= \frac{1+\frac{1}{\sqrt{3}}}{1-\frac{1}{\sqrt{3}}} - \frac{\sqrt{3}-1}{1+\sqrt{3}} = \frac{(\sqrt{3}+1)}{(\sqrt{3}-1)} - \frac{\sqrt{3}-1}{\sqrt{3}+1}$$

$$= \frac{(3+1+2\sqrt{3})}{3-1} - \frac{(3+1-2\sqrt{3})}{3-1} = \frac{4\sqrt{3}}{2} = 2\sqrt{3}.$$

29. (a) tan 3A = tan(2A + A)

$$\Rightarrow \tan 3A = \frac{\tan 2A + \tan A}{1-\tan 2A\tan A}$$

$\Rightarrow$ tan 3A – tan 3A tan 2A tan A = tan 2A + tan A
$\Rightarrow$ tan 3A – tan 2A – tan A = tan 3A tan 2A tan A

30. (c) Given, $\tan A = \frac{1}{2}$, $\tan B = \frac{1}{3}$ … (i)

Now, tan(2A + B)

$$= \frac{\tan 2A + \tan B}{1-\tan 2A\tan B} = \frac{\frac{2\tan A}{1-\tan^2 A} + \tan B}{1-\frac{2\tan A}{1-\tan^2 A}\times\tan B}$$

$$= \frac{\left(\frac{2\times\frac{1}{2}}{1-\frac{1}{4}}\right)+\frac{1}{3}}{1-\left(\frac{2\times\frac{1}{2}}{1-\frac{1}{4}}\right)\times\frac{1}{3}} = \frac{\frac{4}{3}+\frac{1}{3}}{1-\frac{4}{3}\times\frac{1}{3}} = \frac{\frac{5}{3}}{\frac{5}{9}} = 3.$$

31. (b) We have, $\tan\theta = \frac{a}{b}$

Now, $b\cos 2\theta + a\sin 2\theta$

$$= b\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right) + a\left(\frac{2\tan\theta}{1+\tan^2\theta}\right)$$

$$= b\left(\frac{1-\frac{a^2}{b^2}}{1+\frac{a^2}{b^2}}\right) + a\left(\frac{2\times\frac{a}{b}}{1+\frac{a^2}{b^2}}\right)$$

$$= b\left(\frac{b^2-a^2}{b^2+a^2}\right) + \left(\frac{2\frac{a^2}{b}\times b^2}{b^2+a^2}\right)$$

$$= \frac{1}{b^2+a^2}[b^3 - a^2 b + 2a^2 b]$$

$$= \frac{1}{(b^2+a^2)}\times b(a^2+b^2) = b.$$

32. (c) Given, equation is $\tan x + \sec x = 2\cos x$

$$\Rightarrow \frac{\sin x}{\cos x} + \frac{1}{\cos x} = 2\cos x$$

$$\Rightarrow 1 + \sin x = 2\cos^2 x$$

$\Rightarrow 1 + \sin x = 2(1 - \sin^2 x)$
$\Rightarrow 2\sin^2 x + 2\sin x - \sin x - 1 = 0$
$\Rightarrow 2\sin x(\sin x + 1) - 1(\sin x + 1) = 0$
$\Rightarrow (2\sin x - 1)(\sin x + 1) = 0$

$\Rightarrow$ either $\sin x = \frac{1}{2}$ or $\sin x = -1$

$\Rightarrow$ either $x = \frac{\pi}{6}, \frac{5\pi}{6} \in [0, \pi]$ or $x = \frac{3\pi}{2}$

But, $x = \frac{3\pi}{2}$ can not be possible.

$\therefore$ Number of solutions are 2.

33. **(b)** Since A and B both lie in the IV quadrant, it follows that $\sin A$ and $\sin B$ are negative. Therefore,

$\sin A = -\sqrt{1-\cos^2 A}$

$\Rightarrow \sin A = -\sqrt{1-\frac{16}{25}} = -\frac{3}{5}$

and, $\sin B = -\sqrt{1-\cos^2 B}$

$\Rightarrow \sin B = -\sqrt{1-\frac{144}{169}} = -\frac{5}{13}$

Now, $\cos(A+B) = \cos A\cos B - \sin A\sin B$

$= \frac{4}{5}\times\frac{12}{13} - \left(\frac{-3}{5}\right)\left(\frac{-5}{13}\right) = \frac{33}{65}$

34. **(a)** π radians $= 180°$

$1° = \frac{\pi}{180}$ radians

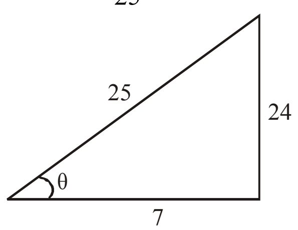
$\therefore\ 25° = 25\times\frac{\pi}{180} = \frac{5\pi}{36}$

35. **(b)** Since $\tan\theta = -\frac{4}{3}$ is negative, θ lies either in second quadrant or in fourth quadrant. Thus $\sin\theta = \frac{4}{5}$ if θ lies in the second quadrant

or $\sin\theta = -\frac{4}{5}$, if θ lies in the fourth quadrant.

36. **(c)** $\cos(A+B).\cos(A-B) = (\cos A\cos B - \sin A\sin B)(\cos A\cos B + \sin A\sin B)$
$= \cos^2 A\cos^2 B - \sin^2 A\sin^2 B.$
$= \cos^2 A(1-\sin^2 B) - \sin^2 A\sin^2 B$
$= \cos^2 A - \cos^2 A\sin^2 B - \sin^2 A\sin^2 B$
$= \cos^2 A - \sin^2 B(\cos^2 A + \sin^2 A)$
$= \cos^2 A - \sin^2 B$

37. **(c)** We have, $\sin\theta = \frac{24}{25}, 0° < \theta < 90°$



$\cos^2\theta = 1-\sin^2\theta = 1-\left(\frac{24}{25}\right)^2$

Since θ lies in first quadrant $\Rightarrow \cos\theta = \frac{7}{25}$

$\cos\theta = 1-2\sin^2\frac{\theta}{2}$

$2\sin^2\frac{\theta}{2} = 1-\cos\theta = 1-\frac{7}{25}$

$2\sin^2\frac{\theta}{2} = \frac{18}{25}$

$\sin^2\frac{\theta}{2} = \frac{9}{25} \Rightarrow \sin\frac{\theta}{2} = \pm\frac{3}{5}$

$\Rightarrow \sin\frac{\theta}{2} = \frac{3}{5}$

[Negative sign discarded since θ is in first quadrant]

38. **(b)** $\sin\frac{5\pi}{12} = \sin 75°$
$= \sin(45°+30°) = \sin 45°\cos 30° + \cos 45°\sin 35°$

$= \frac{1}{\sqrt{2}}.\frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}}.\frac{1}{2} = \frac{1}{\sqrt{2}}\left(\frac{\sqrt{3}+1}{2}\right)$

$= \frac{\sqrt{3}+1}{2\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{6}+\sqrt{2}}{4}$

39. **(b)** Given : $x + \frac{1}{x} = 2\cos\theta$ ...(i)

Cubic both sides in eq^n (i) we get

$x^3 + \frac{1}{x^3} + 3\left(x + \frac{1}{x}\right) = 8\cos^3\theta$

$\Rightarrow x^3 + \frac{1}{x^3} + 3(2\cos\theta) = 8\cos^3\theta$

$\Rightarrow x^3 + \frac{1}{x^3} = 8\cos^3\theta - 6\cos\theta$

$\Rightarrow x^3 + \frac{1}{x^3} = 2(4\cos^3\theta - 3\cos\theta) = 2\cos 3\theta$

40. **(c)** $1 + \cot\theta = \mathrm{cosec}\,\theta$

$\Rightarrow \frac{1}{\sin\theta} = 1 + \frac{\cos\theta}{\sin\theta} \Rightarrow \sin\theta + \cos\theta = 1$

$\sin\theta\sin\frac{\pi}{4} + \cos\theta\cos\frac{\pi}{4} = \cos\frac{\pi}{4}$

$\Rightarrow \cos\left(\theta - \frac{\pi}{4}\right) = \cos\frac{\pi}{4} \Rightarrow \theta - \frac{\pi}{4} = 2n\pi \pm \frac{\pi}{4}$

Hence, $\theta = 2n\pi$ or $\theta = 2n\pi + \frac{\pi}{2}$

But, $\theta = 2n\pi$ is ruled out

41. **(b)** $\sin 3\alpha = 4\sin\alpha\sin(x+\alpha)\sin(x-\alpha)$
$\therefore\ \sin 3\alpha = 4\sin\alpha(\sin^2 x\cos^2\alpha - \cos^2 x\sin^2\alpha)$
$\therefore\ 3\sin\alpha - 4\sin^3\alpha = 4\sin\alpha(\sin^2 x - \sin^2\alpha)$

$\therefore\ \sin^2 x = \left(\frac{3}{4}\right) \Rightarrow \sin^2 x = \sin^2\frac{\pi}{3}$

$\therefore\ x = n\pi \pm \frac{\pi}{3}.$

42. **(b)** $\tan\theta + \tan\left(\frac{\pi}{2} - \theta\right) = 2$

$\Rightarrow \tan\theta + \frac{1}{\tan\theta} = 2$

$\Rightarrow \tan^2\theta - 2\tan\theta + 1 = 0$

$\Rightarrow \tan\theta = 1 = \tan\frac{\pi}{4} \Rightarrow \theta = n\pi + \frac{\pi}{4}.$

43. **(b)** $\sin^2\theta \sec\theta + \sqrt{3}\tan\theta = 0$

$\therefore \left(\sin^2\theta + \sqrt{3}\sin\theta\right)\sec\theta = 0$

$\therefore \sin\theta\left(\sin\theta + \sqrt{3}\right)\sec\theta = 0$

$\Rightarrow \sin\theta = 0$

$\therefore \theta = n\pi,\ n \in I \quad \left[\because \sin\theta \neq -\sqrt{3},\ \sec\theta \neq 0\right]$

44. **(c)** $\tan\theta + \tan 2\theta + \sqrt{3}\tan\theta\tan 2\theta = \sqrt{3}$

$\therefore \tan\theta + \tan 2\theta = \sqrt{3}(1 - \tan\theta\tan 2\theta)$

$\therefore \frac{\tan\theta + \tan 2\theta}{1 - \tan\theta\tan 2\theta} = \sqrt{3} \Rightarrow \tan 3\theta = \tan\frac{\pi}{3}$

$\therefore 3\theta = n\pi + \frac{\pi}{3} \Rightarrow \theta = (3n+1)\frac{\pi}{9}.$

45. **(a)**

46. **(c)** $\sec 4\theta - \sec 2\theta = 2$

$\therefore \cos 2\theta - \cos 4\theta = 2\cos 4\theta\cos 2\theta$

$\therefore -\cos 4\theta = \cos 6\theta \Rightarrow 2\cos 5\theta\cos\theta = 0$

$\therefore \theta = n\pi + \frac{\pi}{2}$ or $\frac{n\pi}{5} + \frac{\pi}{10}.$

47. **(d)** $\tan\theta\tan 2\theta = 1$

$\therefore \tan\theta\frac{2\tan\theta}{1-\tan^2\theta} = 1$

$\therefore 2\tan^2\theta = 1 - \tan^2\theta$

$\therefore 3\tan^2\theta = 1$

$\therefore \tan^2\theta = \frac{1}{3} = \tan^2\left(\frac{\pi}{6}\right)$

$\therefore \theta = n\pi \pm \frac{\pi}{6}.$

48. **(d)** $\cot\theta + \cot\left(\frac{\pi}{4}+\theta\right) = 2$

$\therefore \frac{\cos\theta}{\sin\theta} + \frac{\cos\left(\frac{\pi}{4}+\theta\right)}{\sin\left(\frac{\pi}{4}+\theta\right)} = 2$

$\therefore \sin\left(\frac{\pi}{4}+2\theta\right) = 2\sin\theta\sin\left(\frac{\pi}{4}+\theta\right)$

$= \cos\left(\theta - \frac{\pi}{4} - \theta\right) - \cos\left(\theta + \frac{\pi}{4} + \theta\right)$

$\therefore \sin\left(\frac{\pi}{4}+2\theta\right) = \cos\left(\frac{-\pi}{4}\right) - \cos\left(2\theta + \frac{\pi}{4}\right)$

$\Rightarrow \sin\left(\frac{\pi}{4}+2\theta\right) + \cos\left(\frac{\pi}{4}+2\theta\right) = \frac{1}{\sqrt{2}}$

$\Rightarrow \left(\frac{1}{\sqrt{2}}\cos 2\theta + \frac{1}{\sqrt{2}}\sin 2\theta\right)$

$+ \left(\frac{1}{\sqrt{2}}\cos 2\theta - \frac{1}{\sqrt{2}}\sin 2\theta\right) = \frac{1}{\sqrt{2}}$

$\Rightarrow \frac{2}{\sqrt{2}}\cos 2\theta = \frac{1}{\sqrt{2}}$

$\Rightarrow \cos 2\theta = \frac{1}{2} = \cos\left(\frac{\pi}{3}\right)$

$\Rightarrow 2\theta = 2n\pi \pm \frac{\pi}{3} \Rightarrow \theta = n\pi \pm \frac{\pi}{6}.$

49. **(a)** $2\cos^2 x + 3\sin x - 3 = 0$

$2 - 2\sin^2 x + 3\sin x - 3 = 0$

$\Rightarrow (2\sin x - 1)(\sin x - 1) = 0$

$\Rightarrow \sin x = \frac{1}{2}$ or $\sin x = 1$

$\Rightarrow x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{\pi}{2}$, i.e. 30°, 150°, 90°.

50. **(c)** $\cot\theta + \tan\theta = 2\,\mathrm{cosec}\,\theta$

$\Rightarrow \frac{1}{\sin\theta\cos\theta} = \frac{2}{\sin\theta}$

$\Rightarrow \cos\theta = \frac{1}{2} = \cos\left(\frac{\pi}{3}\right)$

$\Rightarrow \theta = 2n\pi \pm \frac{\pi}{3}.$

51. **(b)** $\sqrt{3}\tan 2\theta + \sqrt{3}\tan 3\theta + \tan 2\theta\tan 3\theta = 1$

$\Rightarrow \frac{\tan 2\theta + \tan 3\theta}{1 - \tan 2\theta\tan 3\theta} = \frac{1}{\sqrt{3}}$

$\Rightarrow \tan 5\theta = \tan\frac{\pi}{6}$

$\Rightarrow 5\theta = n\pi + \frac{\pi}{6} \Rightarrow \theta = \left(n + \frac{1}{6}\right)\frac{\pi}{5}.$

52. **(c)** $\cos 7\theta = \cos\theta - \sin 4\theta$

$\sin 4\theta = \cos\theta - \cos 7\theta$

$\Rightarrow \sin 4\theta = 2\sin(4\theta)\sin(3\theta)$

$\Rightarrow \sin 4\theta = 0 \Rightarrow 4\theta = n\pi$ or

$\sin 3\theta = \frac{1}{2} = \sin\left(\frac{\pi}{6}\right) \Rightarrow 3\theta = n\pi + (-1)^n\frac{\pi}{6}$

$\therefore \theta = \frac{n\pi}{4} = \frac{n\pi}{3} + (-1)^n\frac{\pi}{18}.$

53. **(d)** In anti-clockwise rotation, the angle is said to be positive.
In clockwise rotation, the angle is said to be negative.
The measure of an angle is the amount of rotation performed to get the terminal side from the initial side.
So, all the statements are correct.

54. **(c)** Angle subtended at the centre by an arc of length 1 unit in a unit circle is said to have a measure of 1 radian.

55. **(b)** We know that, 180° = π radian

Hence, $40°\,20' = 40\frac{1}{3}$ degree $[\because\ 1° = 60']$

$= \frac{\pi}{180} \times \frac{121}{3}$ radian $= \frac{121\pi}{540}$ radian

Therefore, $40°\,20' = \frac{121\pi}{540}$ radian.

56. **(b)** π radian = 180°.

57. **(a)** We know that, values of sin x repeats after an interval of 2π. Therefore,

$\sin\frac{31\pi}{3} = \sin\left(10\pi + \frac{\pi}{3}\right) = \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$

58. (b) $\cot\left(-\frac{15\pi}{4}\right) = -\cot\left(\frac{15\pi}{4}\right)$ $[\because \cot(-\theta) = -\cot\theta]$

$= -\cot\left(4\pi - \frac{\pi}{4}\right) = -\cot\left(2\pi \times 2 - \frac{\pi}{4}\right)$

$= -\left(-\cot\frac{\pi}{4}\right) = \cot\frac{\pi}{4} = 1$

$[\because \cot(2n\pi - \theta) = -\cot\theta]$

59. (d) In III quadrant, only tan θ and cot θ are positive.

$\sin^2\theta = (1 - \cos^2\theta) = \left(1 - \frac{9}{25}\right) = \frac{16}{25}$

$\Rightarrow \quad \sin\theta = \pm\sqrt{\frac{16}{25}} = \pm\frac{4}{5}$

$\Rightarrow \quad \sin\theta = \frac{-4}{5}$ (as sin θ is negative in 3rd quadrant)

$\therefore \quad \tan\theta = \left(\frac{-4}{5} \times \frac{5}{-3}\right) = \frac{4}{3}$

and $\cot\theta = \frac{3}{4} \Rightarrow \operatorname{cosec}\theta = \frac{-5}{4}$

and $\sec\theta = \frac{-5}{3}$

$\therefore \quad \frac{(\operatorname{cosec}\theta + \cot\theta)}{(\sec\theta - \tan\theta)} = \frac{\left(\frac{-5}{4} + \frac{3}{4}\right)}{\left(\frac{-5}{3} - \frac{4}{3}\right)} = \frac{\left(\frac{-2}{4}\right)}{\left(\frac{-9}{3}\right)}$

$= \frac{-2}{4} \times \frac{3}{-9} = \frac{1}{6}.$

60. (c) $\cos\left(\frac{3\pi}{4} + x\right) - \cos\left(\frac{3\pi}{4} - x\right)$

$= -2\sin\frac{\frac{3\pi}{4} + x + \frac{3\pi}{4} - x}{2}\sin\frac{\frac{3\pi}{4} + x - \frac{3\pi}{4} + x}{2}$

$= -2\sin\frac{3\pi}{4}\sin x = -2\sin\left(\pi - \frac{\pi}{4}\right)\sin x$

$= -2\sin\frac{\pi}{4}\sin x = -2 \times \frac{1}{\sqrt{2}}\sin x = -\sqrt{2}\sin x$

61. (d) $\frac{\sin x - \sin 3x}{\sin^2 x - \cos^2 x} = \frac{\sin 3x - \sin x}{\cos^2 x - \sin^2 x}$

$= \frac{2\cos\frac{3x + x}{2} \cdot \sin\frac{3x - x}{2}}{\cos 2x} = \frac{2\cos 2x \cdot \sin x}{\cos 2x}$

$= 2\sin x$

62. (a) We have,

$\sin x = -\frac{\sqrt{3}}{2} = -\sin\frac{\pi}{3} = \sin\left(\pi + \frac{\pi}{3}\right) = \sin\frac{4\pi}{3}$

Hence, $\sin x = \sin\frac{4\pi}{3}$, which gives

$x = n\pi + (-1)^n \frac{4\pi}{3}$, where $n \in Z$.

Note: $\frac{4\pi}{3}$ is one such value of x for which $\sin x = -\frac{\sqrt{3}}{2}$. One may take any other value of x for which $\sin x = -\frac{\sqrt{3}}{2}$. The solutions obtained will be the same although these may apparently look different.

63. (b) Given that, $x = \sec\theta + \tan\theta$

$\Rightarrow x + \frac{1}{x} = \sec\theta + \tan\theta + \frac{1}{\sec\theta + \tan\theta}$

$= \sec\theta + \tan\theta + \sec\theta - \tan\theta = 2\sec\theta$

Aliter: $x = \frac{1 + \sin\theta}{\cos\theta}$

$\therefore \quad x + \frac{1}{x} = \frac{1 + \sin\theta}{\cos\theta} + \frac{\cos\theta}{1 + \sin\theta}$

$\Rightarrow \quad \frac{2(1 + \sin\theta)}{\cos\theta(1 + \sin\theta)} = 2\sec\theta.$

64. (b) $\frac{\tan 70° - \tan 20°}{\tan 50°} = \frac{\frac{\sin 70°}{\cos 70°} - \frac{\sin 20°}{\cos 20°}}{\frac{\sin 50°}{\cos 50°}}$

$= \frac{\frac{\sin 70° \cos 20° - \cos 70° \sin 20°}{\cos 70° \cos 20°}}{\frac{\sin 50°}{\cos 50°}}$

$= \frac{2}{2} \times \frac{\sin(70° - 20°)\cos 50°}{\cos 70° \cos 20° \sin 50°}$

$= \frac{2\sin 50° \cos 50°}{2\cos 70° \cos 20° \sin 50°}$

$= \frac{2\cos 50°}{\cos 90° + \cos 50°} = \frac{2\cos 50°}{0 + \cos 50°} = 2.$

65. (d) $\frac{1}{\sin 10°} - \frac{\sqrt{3}}{\cos 10°} = \frac{\left[\cos 10° - \sqrt{3}\sin 10°\right]}{\sin 10° \cos 10°}$

$= \frac{2\left[\frac{1}{2}\cos 10° - \frac{\sqrt{3}}{2}\sin 10°\right]}{\sin 10° \cos 10°}$

$= \frac{2\left[\sin 30° \cos 10° - \cos 30° \sin 10°\right]}{\sin 10° \cos 10°}$

$= \frac{2[\sin(30° - 10°)]}{\sin 10° \cos 10°}$

$= \frac{2 \cdot 2\sin(30° - 10°)}{2\sin 10° \cos 10°} = \frac{4 \sin 20°}{\sin 20°} = 4.$

66. **(a)** $\cos^2\frac{\pi}{12} + \cos^2\frac{\pi}{4} + \cos^2\frac{5\pi}{12}$

$= 1 - \sin^2\left(\frac{\pi}{12}\right) + \left(\frac{1}{\sqrt{2}}\right)^2 + \cos^2\left(\frac{5\pi}{12}\right)$

$= 1 + \frac{1}{2} + \left(\cos^2\frac{5\pi}{12} - \sin^2\frac{\pi}{12}\right)$

$= \frac{3}{2} + \cos\left(\frac{5\pi}{12} + \frac{\pi}{12}\right)\cos\left(\frac{5\pi}{12} - \frac{\pi}{12}\right)$

$= \frac{3}{2} + \cos\frac{\pi}{2}\cos\frac{\pi}{3} = \frac{3}{2} + 0 \cdot \frac{1}{2} = \frac{3}{2}.$

67. **(c)** $1 + \cos 2x + \cos 4x + \cos 6x$
$= (1 + \cos 6x) + (\cos 2x + \cos 4x)$
$= 2\cos^2 3x + 2\cos 3x \cos x$
$= 2\cos 3x(\cos 3x + \cos x)$
$= 4\cos x \cos 2x \cos 3x.$

68. **(a)** $\text{cosec } A - 2\cot 2A \cos A = \frac{1}{\sin A} - \frac{2\cos A \cos 2A}{\sin 2A}$

$= \frac{1}{\sin A} - \frac{2\cos A\cos 2A}{2\sin A\cos A} = \frac{1 - \cos 2A}{\sin A} = \frac{2\sin^2 A}{\sin A}$
$= 2\sin A.$

69. **(d)** $\sin x + \cos x = \frac{1}{5}$

$\Rightarrow \sin^2 x + \cos^2 x + 2\sin x\cos x = \frac{1}{25}$

$\sin 2x = \frac{-24}{25} \Rightarrow \cos 2x = \frac{-7}{25} \Rightarrow \tan 2x = \frac{24}{7}.$

70. **(b)** $\sqrt{3}\tan 2\theta + \sqrt{3}\tan 3\theta + \tan 2\theta\tan 3\theta = 1$

$\Rightarrow \sqrt{3}(\tan 2\theta + \tan 3\theta) = 1 - \tan 2\theta \tan 3\theta$

$\Rightarrow \frac{\tan 2\theta + \tan 3\theta}{1 - \tan 2\theta\tan 3\theta} = \frac{1}{\sqrt{3}}$

$\Rightarrow \tan 5\theta = \tan\frac{\pi}{6}$

$\Rightarrow 5\theta = n\pi + \frac{\pi}{6} \Rightarrow \theta = \left(n + \frac{1}{6}\right)\frac{\pi}{5}.$

71. **(d)** $\tan\theta - \sqrt{2}\sec\theta = \sqrt{3}$

$\Rightarrow \frac{\sin\theta}{\cos\theta} - \frac{\sqrt{2}}{\cos\theta} = \sqrt{3}$

$\Rightarrow \sin\theta - \sqrt{2} = \sqrt{3}\cos\theta$

$\Rightarrow \sin\theta - \sqrt{3}\cos\theta = \sqrt{2}$

$\Rightarrow \sin\left(\theta - \frac{\pi}{3}\right) = \sin\frac{\pi}{4}$

$\Rightarrow \theta = n\pi + (-1)^n\frac{\pi}{4} + \frac{\pi}{3}.$

72. **(b)** The given equation can be written as

$\frac{\sin^2\theta}{\cos\theta} + \sqrt{3}\tan\theta = 0$

$\Rightarrow \tan\theta\sin\theta + \sqrt{3}\tan\theta = 0$

$\tan\theta(\sin\theta + \sqrt{3}) = 0$ as $\sin\theta \neq -\sqrt{3}$

Hence, $\tan\theta = 0 \Rightarrow \theta = n\pi, n \in Z.$

73. **(a)** $\tan(90° - \theta) = \cot\theta$, $\cot(90° - \theta) = \tan\theta$

Therefore, $\frac{\cot 54°}{\tan 36°} + \frac{\tan 20°}{\cot 70°}$

$= \frac{\cot 54°}{\tan(90° - 54°)} + \frac{\tan 20°}{\cot(90° - 20°)}$

$= \frac{\cot 54°}{\cot 54°} + \frac{\tan 20°}{\tan 20°} = 1 + 1 = 2.$

STATEMENT TYPE QUESTIONS

74. **(b)** We have

$\cos(\beta - \gamma) + \cos(\gamma - \alpha) + \cos(\alpha - \beta) = -\frac{3}{2}$

$\Rightarrow 2[\cos(\beta - \gamma) + \cos(\gamma - \alpha) + \cos(\alpha - \beta)] + 3 = 0$

$\Rightarrow 2[\cos(\beta - \gamma) + \cos(\gamma - \alpha) + \cos(\alpha - \beta)]$
$+ \sin^2\alpha + \cos^2\alpha + \sin^2\beta + \cos^2\beta + \sin^2\gamma + \cos^2\alpha = 0$

$\Rightarrow [\sin^2\alpha + \sin^2\beta + \sin^2\gamma + 2\sin\alpha\sin\beta + 2\sin\beta\sin\gamma$
$+ 2\sin\gamma\sin\alpha] + [\cos^2\alpha + \cos^2\beta + \cos^2\gamma + 2\cos\alpha\cos\beta$
$+ 2\cos\beta\cos\gamma + 2\cos\gamma\cos\alpha] = 0$

$\Rightarrow [\sin\alpha + \sin\beta + \sin\gamma]^2 + (\cos\alpha + \cos\beta + \cos\gamma)^2 = 0$

$\Rightarrow \sin\alpha + \sin\beta + \sin\gamma = 0$ and $\cos\alpha + \cos\beta + \cos\gamma = 0$

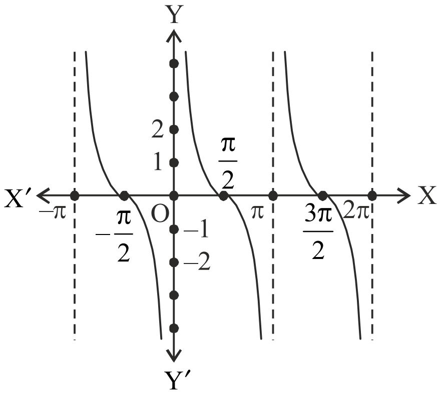
$\therefore$ I and II both are true.

75. **(a)** The signs of trigonometric functions in different quadrants are shown below

	I	II	III	IV
sin x	+	+	–	–
cos x	+	–	–	+
tan x	+	–	+	–
cosec x	+	+	–	–
sec x	+	–	–	+
cot x	+	–	+	–

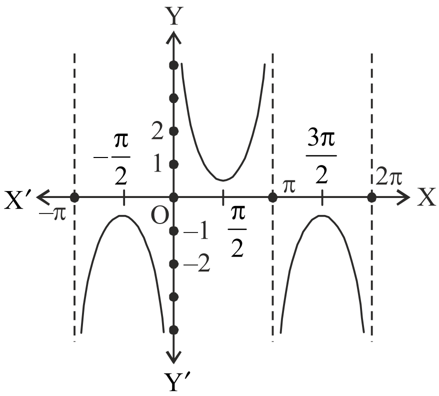
According to the above table, option (a) is correct.

76. **(d)** Using behaviour of trigonometric functions we can draw the graphs of $y = \cot x$, $y = \text{cosec } x$ and $y = \sec x$ as shown below.



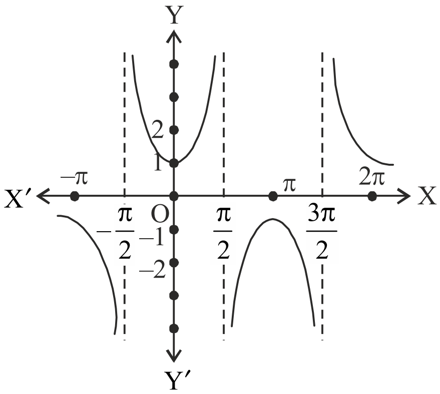
y = cot x

So, we see that the values of cot x repeat after an interval of π.



y = cosec x

Also, we can see that the values of cosec x repeat after an interval of 2π by using above graph of y = cosec x. Similarly, we can say that the values of sec x repeat after an interval of 2π by using the graph of y = sec x as shown below.



y = sec x

Hence, it is concluded that all the given statements are true.

77. (a) Only option (a) is incorrect.

78. (b) As a part of identities from above, we can also show that

I. $2\cos x \cos y = \cos(x+y) + \cos(x-y)$

II. $-2\sin x \sin y = \cos(x+y) - \cos(x-y)$

III. $2\sin x \cos y = \sin(x+y) + \sin(x-y)$

IV. $2\cos x \sin y = \sin(x+y) - \sin(x-y)$

Hence, option (b) is correct.

79. (d) $\sin 2x + \cos x = 0$

$\Rightarrow 2\sin x\cos x + \cos x = 0$

[$\because \sin 2x = 2\sin x \cos x$]

$\Rightarrow \cos x(2\sin x + 1) = 0$

$\Rightarrow \cos x = 0$ or $\sin x = -\frac{1}{2}$

When $\cos x = 0$,

Then, $x = (2n+1)\frac{\pi}{2}$

When $\sin x = -\frac{1}{2}$,

Then, $\sin x = -\sin\frac{\pi}{6}$

$\sin x = \sin\left(\pi + \frac{\pi}{6}\right)$ [$\because \sin(\pi+\theta) = -\sin\theta$]

$\sin x = \sin\frac{7\pi}{6}$

$\Rightarrow x = n\pi + (-1)^n\frac{7\pi}{6}$. [$n \in Z$]

MATCHING TYPE QUESTIONS

80. (b) We know that,

Radian measure $= \frac{\pi}{180} \times$ Degree measure

A. Radian measure of $25° = \frac{\pi}{180} \times 25° = \frac{5\pi}{36}$.

B. We know that, $30' = \left(\frac{1}{2}\right)^o$ [$\because 60' = 1°$]

$\therefore\ -47°\,30' = -\left(47\frac{1}{2}\right)^o = \left(\frac{-95}{2}\right)^o$

$\therefore$ Radian measure of $(-47°\,30') = \frac{\pi}{180} \times \left(\frac{-95}{2}\right)$

$= \frac{-19\pi}{72}$.

C. Radian measure of $240° = \frac{\pi}{180} \times 240 = \frac{4\pi}{3}$.

D. Radian measure of $520° = \frac{\pi}{180} \times 520 = \frac{26\pi}{9}$

81. (c) We know that

Degree measure $= \frac{180}{\pi} \times$ Radian measure

A. Degree measure of $\frac{11}{16} = \left(\frac{180}{\pi} \times \frac{11}{16}\right)^o$

$= \left(\frac{180}{22} \times \frac{11}{16} \times 7\right)^o$ $\left[\because \pi = \frac{22}{7}\right]$

$= \left(\frac{90 \times 7}{16}\right)^o = \left(\frac{315}{8}\right)^o$

$= \left(39\frac{3}{8}\right)^o = 39°\left(\frac{3}{8}\times 60\right)'$ $[\because 1° = 60']$

$= 39°\left(22\frac{1}{2}\right)' = 39°\,22'\left(\frac{1}{2}\times 60\right)''$ $[\because 1' = 60'']$

$= 39°\,22'\,30''$.

B. Degree measure of –4 $= \left(\frac{180}{\pi}\times -4\right)^o$

$= \left(\frac{180}{22}\times -4\times 7\right)^o$ $\left[\because \pi = \frac{22}{7}\right]$

$= \left(\frac{90\times(-28)}{11}\right)^o = -\left(\frac{2520}{11}\right)^o = -\left(229\frac{1}{11}\right)^o$

$= -229°\left(\frac{1}{11}\times 60'\right)$ $[\because 1° = 60']$

$= -229°\left(5\frac{5}{11}\right)'$

$= -229°\,5'\left(\frac{5}{11}\times 60\right)''$ $[\because 1' = 60'']$

$\approx -229°\,5'\,27.3'' \approx -229°\,5'\,27''$ (approx.)

C. Degree measure of $\frac{5\pi}{3} = \left(\frac{180}{\pi}\times\frac{5\pi}{3}\right)^o = 300°$.

D. Degree measure of $\frac{7\pi}{6} = \left(\frac{180}{\pi}\times\frac{7\pi}{6}\right)^o = 210°$

82. (a) A. $\sin\frac{25\pi}{3} = \sin\left(8\pi + \frac{\pi}{3}\right) = \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$

$[\because \sin(2n\pi + \theta) = \sin\theta]$

B. $\cos\frac{41\pi}{4} = \cos\left(10\pi + \frac{\pi}{4}\right) = \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$

$[\because \cos(2n\pi + \theta) = \cos\theta]$

C. $\tan\left(\frac{-16\pi}{3}\right) = -\tan\frac{16\pi}{3}$ $[\because \tan(-\theta) = -\tan\theta]$

$= -\tan\left(5\pi + \frac{\pi}{3}\right) = -\tan\frac{\pi}{3} = -\sqrt{3}$

$[\because \tan(n\pi + \theta) = \tan\theta]$

D. $\cot\frac{29\pi}{4} = \cot\left(7\pi + \frac{\pi}{4}\right) = \cot\frac{\pi}{4} = 1$

$[\because \cot(n\pi + \theta) = \cot\theta]$

83. (a) By taking suitable values of x and y in the identities, we get the following results:

$\cos(\pi - x) = -\cos x$; $\sin(\pi - x) = \sin x$

$\cos(\pi + x) = -\cos x$; $\sin(\pi + x) = -\sin x$

$\cos(2\pi - x) = \cos x$; $\sin(2\pi - x) = -\sin x$

84. (c) A. 1 radian $= \frac{180°}{\pi} = 57°\,16'$ (approx.)

B. $1° = \left(\frac{\pi}{180}\right)^c = 0.01746$ radian (approx.)

C. $3°\,45' = \left(3\frac{45}{60}\right)^o = \left(3\frac{3}{4}\right)^o = \left(\frac{15}{4}\right)^o$

Also, $180° = \pi$ radian

$\Rightarrow 1° = \frac{\pi}{180}$ radian

$\Rightarrow \left(\frac{15}{4}\right)^o = \frac{\pi}{180}\times\frac{15}{4} = \frac{\pi}{48}$ radian

D. $50°\,37'\,30'' = 50° + \left(37\frac{30}{60}\right)'$

$= 50° + \left(\frac{75}{2}\right)' = 50° + \left(\frac{75}{2\times 60}\right)^o$

$= \left(\frac{405}{8}\right)^o = \left(\frac{\pi}{180}\times\frac{405}{8}\right)^c = \frac{9\pi}{32}$ radian

85. (c) π radians $= 180°$

$\Rightarrow 1° = \frac{\pi}{180}$ radians

(A) $25° = 25\times\frac{\pi}{180} = \frac{5\pi}{36}$

(B) $60' = 1°$ $\therefore 30' = \frac{30°}{60} = \frac{1}{2}^o$

$\therefore$ $47°30' = \left(47 + \frac{1}{2}\right)^o = \left(\frac{95}{2}\right)^o$

$\therefore$ $180° = \pi$ radian

$-\frac{95°}{2} = \frac{-\pi}{180}\times\frac{95}{2}$ radians $= \frac{-19\pi}{72}$ radians

(C) $240° = 240\times\frac{\pi}{180} = \frac{4\pi}{3}$ radians.

(D) $180° = \pi$ radians

$520° = \frac{\pi}{180}\times 520$ radians $= \frac{26\pi}{9}$ radians

86. (a) Since x lies in the 3rd quadrant

$\cos x = -\frac{1}{2}$

$\therefore$ $\sin x = -\sqrt{1-\cos^2 x}$ ($\because x$ lies in III rd quadrant)

$= -\sqrt{1-\frac{1}{4}} = -\sqrt{3}/2$

$\tan x = \sqrt{3}$, $\cot x = \frac{1}{\sqrt{3}}$

$\sec x = \left(\frac{1}{\cos x}\right) = -2$, $\text{cosec}\, x = \frac{1}{\sin x} = -\frac{2}{\sqrt{3}}$

87. (b) A. $\cos 4x = \cos 2x$

$\Rightarrow 4x = 2n\pi \pm 2x$

Taking + ve sign, we get

$4x = 2n\pi + 2x$

$\Rightarrow 4x - 2x = 2n\pi$

$\Rightarrow \quad x = n\pi, n \in Z$

Taking – ve sign

$4x = 2n\pi - 2x$

$\Rightarrow \quad 4x + 2x = 2n\pi$

$\Rightarrow \quad 6x = 2n\pi$

$\Rightarrow \quad x = \frac{n\pi}{3}, n \in Z$

$\therefore$ General solution is $x = \frac{n\pi}{3}$ or $x = n\pi, n \in Z$

B. $\cos 3x + \cos x - \cos 2x = 0$

or $\quad 2\cos\frac{3x+x}{2}\cos\frac{3x-x}{2} - \cos 2x = 0$

or $\quad 2\cos 2x \cos x - \cos 2x = 0$

or $\quad \cos 2x\,(2\cos x - 1) = 0$

If $\cos 2x = 0$, $2x = (2n+1)\frac{\pi}{2} \Rightarrow x = (2n+1)\frac{\pi}{4}$

If $2\cos x - 1 = 0$, $\cos x = \frac{1}{2} \Rightarrow \cos x = \cos\frac{\pi}{3}$

$\Rightarrow \quad x = 2n\pi \pm \frac{\pi}{3}$

C. $\sin 2x + \cos x = 0$

$\Rightarrow \quad 2\cos x \cos x \cos x = 0$

$\Rightarrow \quad \cos x\,(2\sin x + 1) = 0$

$\Rightarrow \quad \cos x = 0$

or $\quad 2\sin x + 1 = 0$

$\Rightarrow \quad \cos x = 0$

or $\quad \sin x = -\frac{1}{2}$

$\Rightarrow \quad \cos x = 0$

or $\quad \sin x = \sin\left(\pi + \frac{\pi}{6}\right)$

$\Rightarrow \quad \cos x = 0$

or $\quad \sin x = \sin\frac{7\pi}{6}$

$\Rightarrow \quad x = (2n+1)\frac{\pi}{2}$

or $\quad x = n\pi + (-1)^n\frac{7\pi}{6}, n \in Z$

Hence, general solution is

$x = (2n+1)\frac{\pi}{2}$ or $x = n\pi + (-1)^n\frac{7\pi}{6}$

where $n \in Z$

D. $\sec^2 2x = 1 - \tan 2x$

$\Rightarrow \quad 1 + \tan^2 2x = 1 - \tan 2x = 0$

$\Rightarrow \quad \tan^2 2x + \tan 2x = 0$

$\Rightarrow \quad \tan 2x(\tan 2x + 1) = 0$

If $\tan 2x = 0 \Rightarrow 2x = n\pi$ or $x = \frac{n\pi}{2}$

If $\tan 2x + 1 = 0 \Rightarrow \tan 2x = -1$

$= \tan\left(\pi - \frac{\pi}{4}\right) = \tan\frac{3\pi}{4}$

$\Rightarrow \quad 2x = n\pi + \frac{3\pi}{4}$ or $x = \frac{n\pi}{2} + \frac{3\pi}{8}$

E. We have, $(\sin 5x + \sin x) + \sin 3x = 0$

$\Rightarrow \quad 2\sin\frac{5x+x}{2}\cos\frac{5x-x}{2} + \sin 3x = 0$

or $\quad 2\sin 3x\cos 2x + \sin 3x = 0$

or $\quad \sin 3x\,(2\cos 2x + 1) = 0$

If $\sin 3x = 0 \Rightarrow 3x = n\pi$ or $x = \frac{n\pi}{3}$

If $2\cos 2x + 1 = 0$, $\cos 2x = \frac{-1}{2}$

$= \cos\left(\pi - \frac{\pi}{3}\right) = \cos\frac{2\pi}{3}$

$\therefore \; 2x = 2n\pi \pm \frac{2\pi}{3}$ or $x = n\pi \pm \frac{\pi}{3}$

88. (d) A. LHS $= \frac{\cos 9x - \cos 5x}{\sin 17x - \sin 3x}$

$$= \frac{-2\sin\frac{9x+5x}{2}\sin\frac{9x-5x}{2}}{2\cos\frac{17x+3x}{2}\sin\frac{17x-3x}{2}}$$

$$= \frac{-\sin 7x.\sin 2x}{\cos 10x\,.\,\sin 7x} = -\frac{\sin 2x}{\cos 10x} = \text{RHS}$$

B. L.H.S. $= \frac{\sin 5x + \sin 3x}{\cos 5x + \cos 3x}$

$$= \frac{2\sin\frac{5x+3x}{2}\cos\frac{5x-3x}{2}}{2\cos\frac{5x+3x}{2}\cos\frac{5x-3x}{2}}$$

$$= \frac{\sin 4x}{\cos 4x} = \tan 4x = \text{R.H.S.}$$

C. L.H.S. $= \frac{\sin x + \sin 3x}{\cos x + \cos 3x} = \frac{\sin 3x + \sin x}{\cos 3x + \cos x}$

$$= \frac{2\sin\frac{3x+x}{2}\cos\frac{3x-x}{2}}{2\cos\frac{3x+x}{2}\cos\frac{3x-x}{2}} = \frac{\sin 2x\cos x}{\cos 2x\cos x}$$

$$= \frac{\sin 2x}{\cos 2x} = \tan 2x$$

D. L.H.S. $= \frac{\sin x - \sin y}{\cos x + \cos y} = \frac{2\cos\frac{x+y}{2}\sin\frac{x-y}{2}}{2\cos\frac{x+y}{2}\cos\frac{x-y}{2}}$

$$= \frac{\sin\frac{x-y}{2}}{\cos\frac{x-y}{2}} = \tan\frac{x-y}{2} = \text{R.H.S.}$$

E. L.H.S. $= \frac{\sin x - \sin 3x}{\sin^2 x - \cos^2 x} = \frac{-(\sin 3x - \sin x)}{-(\cos^2 x - \sin^2 x)}$

$$= \frac{\sin 3x - \sin x}{\cos^2 x - \sin^2 x} = \frac{2\cos\frac{3x+x}{2}\sin\frac{3x-x}{2}}{\cos 2x}$$

$= \dfrac{2\cos 2x \times \sin x}{\cos 2x}$ [$\because \cos 2x = \cos^2 x - \sin^2 x$]

$= 2\sin x$

89. (a) Since x lies in the second quadrant

$\sin x = 3/5$ given

$\cos x = -\sqrt{1-\sin^2 x}$ ($\because x$ lies in II quadrant)

$= -\sqrt{1-\dfrac{9}{25}} = -\dfrac{4}{5}$

$\sec x = -\dfrac{5}{4}$, $\tan x = -\dfrac{3}{4}$

$\operatorname{cosec} x = \dfrac{5}{3}$, $\cot x = -\dfrac{4}{3}$

INTEGER TYPE QUESTIONS

90. (c) As, we know that

$\operatorname{cosec}(-\theta) = -\operatorname{cosec}\theta$

$\therefore$ $\operatorname{cosec}(-1410°) = -\operatorname{cosec}(360 \times 4 - 30)°$

$= -(-\operatorname{cosec} 30°)$

$= \operatorname{cosec} 30°$ [$\because \operatorname{cosec}(2n\pi - \theta) = -\operatorname{cosec}\theta$]

$= 2.$

91. (b) Given expression

$= \cos\dfrac{10\pi}{13} + \cos\dfrac{8\pi}{13} + \cos\dfrac{3\pi}{13} + \cos\dfrac{5\pi}{13}$

$= \left(\cos\dfrac{10\pi}{13} + \cos\dfrac{3\pi}{13}\right) + \left(\cos\dfrac{8\pi}{13} + \cos\dfrac{5\pi}{13}\right)$

$= 2\cos\left(\dfrac{13\pi}{2\times 13}\right)\cdot\cos\left(\dfrac{7\pi}{2\times 13}\right)$

$+ 2\cos\left(\dfrac{13\pi}{2\times 13}\right)\cos\left(\dfrac{3\pi}{2\times 13}\right)$

$= 2\cos\dfrac{\pi}{2}\left(\cos\dfrac{7\pi}{26} + \cos\dfrac{3\pi}{26}\right)$ $\left[\because \cos\dfrac{\pi}{2} = 0\right]$

$= 0.$

92. (a) $\sin\theta + \cos\theta = 1$

Squaring on both sides, we get

$\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = 1$

$\therefore$ $\sin\theta\cos\theta = 0.$

93. (a) $\cos A = \dfrac{3}{5}$, $\cos B = \dfrac{4}{5}$

$\sin A = -\sqrt{1-\dfrac{9}{25}} = \dfrac{-4}{5}$

$\sin B = -\sqrt{1-\dfrac{16}{25}} = \dfrac{-3}{5}$

($\because \angle A$ and $\angle B$ in the 4^{th} quad.)

$\therefore$ $2\sin A + 4\sin B = 2\left(\dfrac{-4}{5}\right) + 4\left(\dfrac{-3}{5}\right) = -4 = -a$

94. (a) $\sin 765° = \sin(360 \times 2 + 45)° = \sin 45° = \dfrac{1}{\sqrt{2}}$

95. (b) $\operatorname{cosec}(-1410°) = -\operatorname{cosec}(360 \times 4 - 30)°$

$= -(-\operatorname{cosec} 30°) = \operatorname{cosec} 30° = 2$

96. (c) $\tan\dfrac{19\pi}{3} = \tan\left(6\pi + \dfrac{\pi}{3}\right)$

$= \tan\left[2\pi\times 3 + \dfrac{\pi}{3}\right] = \tan\dfrac{\pi}{3} = \sqrt{3}$

97. (b) $\sin\left(\dfrac{-11\pi}{3}\right) = -\sin\left(\dfrac{11\pi}{3}\right) = -\sin\left(4\pi - \dfrac{\pi}{3}\right)$

$= -\sin\left(2\pi\times 2 - \dfrac{\pi}{3}\right) = -\left(-\sin\dfrac{\pi}{3}\right) = \sin\dfrac{\pi}{3} = \dfrac{\sqrt{3}}{2}$

98. (c) $\left(1+\cos\dfrac{\pi}{6}\right)\left(1+\cos\dfrac{\pi}{3}\right)\left(1+\cos\dfrac{2\pi}{3}\right)$

$\left(1+\cos\dfrac{7\pi}{6}\right) = \left(1+\dfrac{\sqrt{3}}{2}\right)\left(1+\dfrac{1}{2}\right)\left(1-\dfrac{1}{2}\right)\left(1-\dfrac{\sqrt{3}}{2}\right)$

$= \left(1-\dfrac{3}{4}\right)\left(1-\dfrac{1}{4}\right) = \dfrac{1}{4}\times\dfrac{3}{4} = \dfrac{3}{16}$

99. (c) $\tan\theta = \dfrac{1}{\sqrt{7}}$ $\Rightarrow$ $\cot\theta = \sqrt{7}$

Given expression $= \dfrac{1+\cot^2\theta - 1 - \tan^2\theta}{1+\cot^2\theta + 1 + \tan^2\theta}$

$= \dfrac{\cot^2\theta - \tan^2\theta}{2+\cot^2\theta + \tan^2\theta} = \dfrac{(\sqrt{7})^2 - \left(\dfrac{1}{\sqrt{7}}\right)^2}{2+(\sqrt{7})^2 + \left(\dfrac{1}{\sqrt{7}}\right)^2}$

$= \dfrac{48}{64} = \dfrac{3}{4} = \dfrac{m}{m+1} \Rightarrow m = 3$

100. (b) $\cos^2 x = 1 - \sin^2 x = 1 - \dfrac{24}{25} = \dfrac{1}{25}$

$\Rightarrow \cos x = \dfrac{-1}{5}$

($\because$ $\sin x$ and $\cos x$ are negative in III quad)

$\therefore$ $\cot x = \dfrac{\cos x}{\sin x} = \dfrac{1}{2\sqrt{6}}$

101. (d) $\sin^2\theta = 1 - \cos^2\theta = \left(1-\dfrac{9}{25}\right) = \dfrac{16}{25}$

$\Rightarrow \sin\theta = \dfrac{-4}{5}$

$\therefore$ $\tan\theta = \left(\dfrac{-4}{5}\times\dfrac{5}{-3}\right) = \dfrac{4}{3}$

$\cot\theta = \dfrac{3}{4}$

$\Rightarrow \operatorname{cosec}\theta = \dfrac{-5}{4}$ and $\sec\theta = \dfrac{-5}{3}$

$\therefore$ $\left(\dfrac{\operatorname{cosec}\theta + \cot\theta}{\sec\theta - \tan\theta}\right) = \dfrac{-2}{4}\times\dfrac{3}{-9} = \dfrac{1}{6}$

102. (b) $3\sin\dfrac{\pi}{6}\sec\dfrac{\pi}{3} - 4\sin\dfrac{5\pi}{6}\cot\dfrac{\pi}{4}$

$= 3\times\dfrac{1}{2}\times 2 - 4\sin\left(\pi - \dfrac{\pi}{6}\right)\times 1$

$= 3 - 4\sin\dfrac{\pi}{6} = 3 - 4\times\dfrac{1}{2} = 1$

103. (a) $2\sin^2\frac{\pi}{6}+\text{cosec}^2\frac{7\pi}{6}\cos^2\frac{\pi}{3}$

$=2\times\left(\frac{1}{2}\right)^2+\text{cosec}^2\left(\pi+\frac{\pi}{6}\right)\cos^2\frac{\pi}{3}$

$=\frac{2}{4}+\text{cosec}^2\frac{\pi}{6}\cos^2\frac{\pi}{3}$

$=\frac{1}{2}+(2)^2\left(\frac{1}{2}\right)^2=\frac{3}{2}=\frac{m}{m-1}\quad\therefore m=3$

104. (d) $\cot^2\frac{\pi}{6}+\text{cosec}\frac{5\pi}{6}+3\tan^2\frac{\pi}{6}$

$=(\sqrt{3})^2+\text{cosec}\left(\pi-\frac{\pi}{6}\right)+3\left(\frac{1}{\sqrt{3}}\right)^2$

$=3+\text{cosec}\frac{\pi}{6}+1=3+2+1=6$

105. (b) LHS $=\cos\left(\frac{3\pi}{2}+x\right)\cos(2\pi+x)$

$\left[\cot\left(\frac{3\pi}{2}-x\right)+\cot(2\pi+x)\right]$

Now, $\cos\left(\frac{3\pi}{2}+x\right)=\sin x,\cos(2\pi+x)=\cos x$ and

$\cot\left(\frac{3\pi}{2}-x\right)=\tan x,\cot(2\pi+x)=\cot x$

L.H.S. $=\sin x.\cos x[\tan x+\cot x]$

$=\sin x.\cos x\left[\frac{\sin x}{\cos x}+\frac{\cos x}{\sin x}\right]$

$=\sin x.\cos x\left[\frac{\sin^2 x+\cos^2 x}{\cos x\sin x}\right]$

$=(\sin x.\cos x)\frac{1}{\cos x\sin x}=1\ [\because \sin^2 x+\cos^2 x=1]$

106. (b) L.H.S. $\cot x\cot 2x-\cot 2x\cot 3x-\cot 3x\cot x$

We have $3x=x+2x$

$\cot 3x=\cot(x+2x)=\frac{\cot x\cot 2x-1}{\cot x+\cot 2x}$

By cross multiplication

$\cot 3x(\cot x+\cot 2x)=\cot x\cot 2x-1$

$\cot x\cot 3x+\cot 2x\cot 3x=\cos x\cot 2x-1$

$\therefore\ \cos x\cot 2x-\cot 2x\cot 3x-\cot 3x\cot x=1$

ASSERTION - REASON TYPE QUESTIONS

107. (d) If the radii of the two circles are r_1 and r_2 and l is the length of arc in either case, then

$l=r_1$ (circular measure of 30°) $=r_1\left(\frac{30\pi}{180}\right)$

and also $l=r_2$ (circular measure of 70°) $=r_2\left(\frac{70\pi}{180}\right)$.

So, we must have $\frac{r_1\pi}{6}=\frac{7r_2\pi}{18}\Rightarrow\frac{r_1}{r_2}=\frac{7}{3}$.

108. (d) $\because\ \tan\left(\frac{\pi}{2}\sin\theta\right)=\cot\left(\frac{\pi}{2}\cos\theta\right)$

$=\tan\left(\frac{\pi}{2}-\frac{\pi}{2}\cos\theta\right)$

$\therefore\ \frac{\pi}{2}\sin\theta=n\pi+\frac{\pi}{2}-\frac{\pi}{2}\cos\theta$

$\Rightarrow\ \sin\theta+\cos\theta=2n+1,\ n\in I$

$\because\ -\sqrt{2}\le\sin\theta+\cos\theta\le\sqrt{2}$

$\therefore\ n=0,-1$

Then, $\sin\theta+\cos\theta=1,-1$.

109. (a) Given equation is

$\tan\theta+\tan\left(\theta+\frac{\pi}{3}\right)+\tan\left(\theta+\frac{2\pi}{3}\right)=3$

$\Rightarrow\tan\theta+\frac{\tan\theta+\sqrt{3}}{1-\sqrt{3}\tan\theta}+\frac{\tan\theta-\sqrt{3}}{1+\sqrt{3}\tan\theta}=3$

$\Rightarrow\tan\theta+\frac{\left(\tan\theta+\sqrt{3}\right)\left(1+\sqrt{3}\tan\theta\right)+\left(\tan\theta-\sqrt{3}\right)\times\left(1-\sqrt{3}\tan\theta\right)}{\left(1-\sqrt{3}\tan\theta\right)\left(1+\sqrt{3}\tan\theta\right)}=3$

$\Rightarrow\tan\theta+\frac{8\tan\theta}{1-3\tan^2\theta}=3$

$\Rightarrow\frac{\tan\theta-3\tan^3\theta+8\tan\theta}{1-3\tan^2\theta}=3$

$\Rightarrow\frac{3\left(3\tan\theta-\tan^3\theta\right)}{1-3\tan^2\theta}=3$

$\Rightarrow 3\tan 3\theta=3\Rightarrow\tan 3\theta=1$

$\Rightarrow\tan 3\theta=\tan\frac{\pi}{4}\Rightarrow 3\theta=n\pi+\frac{\pi}{4},\ n\in I$

$\Rightarrow\theta=\frac{n\pi}{3}+\frac{\pi}{12},\ n\in I.$

110. (d) Reason is true.

$\therefore$ Degree measure of (–2) radian $=\frac{180}{\pi}\times-2$

$=\frac{180}{22}\times-2\times7\qquad\left[\because\pi=\frac{22}{7}\right]$

$=\left(-\frac{1260}{11}\right)^{o}=-\left(114\frac{6}{11}\right)^{o}=-114^{\circ}\left(\frac{6}{11}\times60\right)'$

$=-114^{\circ}32'\left(\frac{8}{11}\times60\right)''=-114^{\circ}32'43.6''$

$=-114^{\circ}32'44''$ (approx.)

So, Assertion is false.

111. (a) I. $\frac{\cos(\pi+x)\cos(-x)}{\sin(\pi-x)\cos\left(\frac{\pi}{2}+x\right)}=\frac{(-\cos x)(\cos x)}{(\sin x)(-\sin x)}$

$\left[\begin{array}{l}\because\cos(\pi+\theta)=-\cos\theta\\ \cos(-\theta)=\cos\theta\\ \sin(\pi-\theta)=\sin\theta\\ \sin(-\theta)=-\sin\theta\end{array}\right]$

$= \dfrac{\cos^2 x}{\sin^2 x} = \cot^2 x$

So, both the Assertion and Reason are true and Reason is the correct explanation of Assertion.

112. (a) We have,

$\tan 2x = -\cot\left(x+\dfrac{\pi}{3}\right) = \tan\left(\dfrac{\pi}{2}+x+\dfrac{\pi}{3}\right)$

$\Rightarrow \quad \tan 2x = \tan\left(x+\dfrac{5\pi}{6}\right)$

Therefore, $2x = n\pi + \left(x+\dfrac{5\pi}{6}\right)$, where $n \in Z$

$(\because \tan x = \tan y \Rightarrow x = n\pi + y,\ n \in Z)$

$\Rightarrow \quad x = n\pi + \dfrac{5\pi}{6}$, where $n \in Z$.

113. (b) Both are correct statements. Reason is not the correct explanation for the Assertion.

114. (b) Both Assertion and Reason is correct but Reason is not correct explanation.

115. (b) Both Assertion and Reason is correct. Reason is not the correct explanation for Assertion.

Reason : $40°20' = 40\frac{1}{3}$ degree

$= \dfrac{\pi}{180} \times \dfrac{|2|}{3}$ radian $= \dfrac{|2|\pi}{540}$ radian.

116. (d) Assertion is incorrect. The second hand rotates through 360° in a minute.

117. (c) Assertion is correct and Reason is incorrect.

CRITICAL THINKING TYPE QUESTIONS

118. (b) $\tan 20° + 2\tan 50° - \tan 70°$

$= \dfrac{\sin 20°}{\cos 20°} - \dfrac{\sin 70°}{\cos 70°} + 2\tan 50°$

$= \dfrac{\sin 20°\cos 70° - \cos 20°\sin 70°}{\cos 20°\cos 70°} + 2\tan 50°$

$= \dfrac{\sin(20° - 70°)}{\frac{1}{2}\left[\cos(70°+20°) + \cos(70°-20°)\right]} + 2\tan 50°$

$= \dfrac{2\sin(-50°)}{\cos 90° + \cos 50°} + 2\tan 50°$

$= \dfrac{-2\sin 50°}{0 + \cos 50°} + 2\tan 50°$

$= -2\tan 50° + 2\tan 50° = 0.$

119. (d) $\sin(2\alpha) = \sin(\alpha+\beta+\alpha-\beta)$

$= \sin(\alpha+\beta)\cos(\alpha-\beta) + \cos(\alpha+\beta)\sin(\alpha-\beta)$

$= \dfrac{5}{13}.\dfrac{4}{5} + \dfrac{12}{13}.\dfrac{3}{5} = \dfrac{56}{65}$

120. (d) $\cos\theta = \dfrac{1}{\sqrt{2}} = \cos\left(\dfrac{\pi}{4}\right)$

$\theta = 2n\pi \pm \dfrac{\pi}{4}$; $n \in I$

Put $n = 1$, $\theta = \dfrac{9\pi}{4}, \dfrac{7\pi}{4}$

$\tan\theta = -1 = \tan\left(\dfrac{-\pi}{4}\right) \Rightarrow \theta = n\pi - \pi/4,\ n \in I$

Put $n = 1$, $\theta = \dfrac{3\pi}{4}$

Put $n = 2$, $\theta = \dfrac{7\pi}{4}$

The common value which satisfies both these equation is $\left(\dfrac{7\pi}{4}\right)$. Hence the general value is $2n\pi + \dfrac{7\pi}{4}$

121. (d) The given expression

$= \dfrac{\sqrt{3}}{\sin 20°} - \dfrac{1}{\cos 20°} = \dfrac{\sqrt{3}\cos 20° - \sin 20°}{\sin 20°\cos 20°}$

$= \dfrac{2\left(\frac{\sqrt{3}}{2}\cos 20° - \frac{1}{2}\sin 20°\right)}{\sin 20°\cos 20°}$

$= \dfrac{2(\sin 60°\cos 20° - \cos 60°\sin 20°)}{\sin 20°\cos 20°}$

$= \dfrac{2\sin(60° - 20°)}{\sin 20°\cos 20°} = \dfrac{2\sin 40°}{\sin 20°\cos 20°}$

$= \dfrac{4\sin 40°}{2\sin 20°\cos 20°} = \dfrac{4\sin 40°}{\sin 40°} = 4$

122. (d) We have $\sin^2\theta - \sin\theta - 2 = 0$

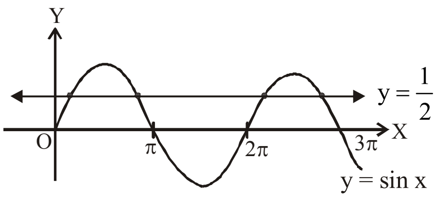
$\Rightarrow (\sin\theta + 1)(\sin\theta - 2) = 0$

As $\sin\theta \neq 2$

$\therefore \sin\theta = -1 = \sin\dfrac{3\pi}{2}$

$\therefore \theta = \dfrac{3\pi}{2} = \dfrac{6\pi}{4} \in \left(\dfrac{5\pi}{4}, \dfrac{7\pi}{4}\right)$

123. (a)



$2\sin^2 x + 5\sin x - 3 = 0$

$\Rightarrow (\sin x + 3)(2\sin x - 1) = 0$

$\Rightarrow \sin x = \dfrac{1}{2}$ and $\sin x \neq -3$

$\therefore$ In $[0, 3\pi]$, x has 4 values.

124. (b) $\text{LHS} = 2\cos\dfrac{\pi}{13}\cos\dfrac{9\pi}{13} + \cos\dfrac{3\pi}{13} + \cos\dfrac{5\pi}{13}$

$= \cos\left(\dfrac{9\pi}{13}+\dfrac{\pi}{13}\right) + \cos\left(\dfrac{9\pi}{13}-\dfrac{\pi}{13}\right) + \cos\dfrac{3\pi}{13} + \cos\dfrac{5\pi}{13}$

$= \cos\dfrac{10\pi}{13} + \cos\dfrac{8\pi}{13} + \cos\dfrac{3\pi}{13} + \cos\dfrac{5\pi}{13}$

$= \cos\left(\pi - \dfrac{3\pi}{13}\right) + \cos\left(\pi - \dfrac{5\pi}{13}\right) + \cos\dfrac{3\pi}{13} + \cos\dfrac{5\pi}{13}$

$= -\cos\dfrac{3\pi}{13} - \cos\dfrac{5\pi}{13} + \cos\dfrac{3\pi}{13} + \cos\dfrac{5\pi}{13}$

$= 0 = \text{RHS} \quad [\because \cos(\pi - \theta) = -\cos\theta]$

125. (a) Given value

$= (\sin 47° + \sin 61°) - (\sin 11° + \sin 25°)$
$= 2 \sin 54° \cos 7° - 2 \sin 18° \cos 7°$
$= 2 \cos 7° (\sin 54° - \sin 18°)$
$= 2 \cos 7° \, 2 \cos 36° \sin 18°$
$= 2\cos 7° \dfrac{2\sin 18° \cos 18°}{\cos 18°} \times \cos 36°$
$= \cos 7° \dfrac{2\sin 36° \cos 36°}{\cos 18°}$
$= \cos 7° \dfrac{\sin 72°}{\cos 18°} = \cos 7°$ $[\because \sin 72° = \cos 18°]$

126. (c) Since $\sin\theta + \cos\theta = \sqrt{2}\left[\frac{1}{\sqrt{2}}\sin\theta + \frac{1}{\sqrt{2}}\cos\theta\right]$

$= \sqrt{2}\left[\sin\theta\cos\frac{\pi}{4} + \cos\theta\sin\frac{\pi}{4}\right] = \sqrt{2}\sin\left(\theta + \frac{\pi}{4}\right)$

which lies between $-\sqrt{2}$ and $\sqrt{2}$

$[\because \sin\left(\theta + \frac{\pi}{4}\right)$ lies between -1 and $1]$

127. (d) $\tan 2\theta \tan\theta = 1 \Rightarrow \dfrac{2\tan\theta}{1-\tan^2\theta} . \tan\theta = 1$

$\Rightarrow 2\tan^2\theta = 1 - \tan^2\theta \Rightarrow 3\tan^2\theta = 1$

$\Rightarrow \tan\theta = \pm\frac{1}{\sqrt{3}} = \tan\left(\pm\frac{\pi}{6}\right)$

$\Rightarrow \theta = n\pi \pm \frac{\pi}{6} \; (n \in Z) = (6n \pm 1)\frac{\pi}{6}$

or $\tan 2\theta = \cot\theta = \tan\left(\frac{\pi}{2} - \theta\right)$

$\Rightarrow 2\theta = n\pi + \frac{\pi}{2} - \theta \Rightarrow 3\theta = n\pi + \frac{\pi}{2}$

$\Rightarrow \theta = \frac{n\pi}{3} + \frac{\pi}{6} = (2n+1)\frac{\pi}{6}$

128. (b) The given equation can be written as

$\frac{1}{2}(\sin 8x + \sin 2x) = \frac{1}{2}(\sin 8x + \sin 4x)$

or $\sin 2x - \sin 4x \Rightarrow -2\sin x \cos 3x = 0$

Hence $\sin x = 0$ or $\cos 3x = 0$.

That is, $x = n\pi \; (n \in I)$, or $3x = k\pi + \frac{\pi}{2} \; (k \in I)$.

Therefore, since $x \in [0, \pi]$, the given equation is satisfied if $x = 0, \pi, \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}$

129. (c) $\tan(\cot x) = \cot(\tan x) = \tan\left(\frac{\pi}{2} - \tan x\right)$

$\Rightarrow \cot x = n\pi + \frac{\pi}{2} - \tan x$

$[\because \tan\theta = \tan\alpha \Rightarrow \theta = n\pi + \alpha]$

$\Rightarrow \cot x + \tan x = n\pi + \frac{\pi}{2}$

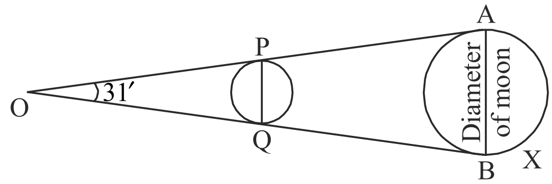
$\Rightarrow \frac{\cos x}{\sin x} + \frac{\sin x}{\cos x} = (2n+1)\frac{\pi}{2}$

$\Rightarrow \frac{1}{\sin x \cos x} = (2n+1)\frac{\pi}{2}$

$\Rightarrow \frac{1}{\sin 2x} = \frac{(2n+1)\pi}{4}$

$\therefore \sin 2x = \frac{4}{(2n+1)\pi}$

130. (c) The coin will just hide the full moon if the lines joining the observer's eye O to the ends A and B of moon's diameter touch the coin at the ends P and Q of its diameter.



Here, $\angle POQ = \angle AOB = 31'$

$= \left(\frac{31}{60}\right)^o = \frac{31}{60} \times \frac{\pi}{180}$ radian.

Since, this angle is very small, the diameter PQ of the coin can be regarded as an arc of a circle whose centre is O and radius equal to the distance of the coin from O.

$\therefore \quad \frac{31\pi}{60 \times 180} = \frac{1}{r} \quad \left(\because \quad \theta = \frac{\ell}{r}\right)$

$\Rightarrow \quad r = \frac{60 \times 180}{31\pi}$

$\Rightarrow \quad r = \frac{60 \times 180 \times 7}{31 \times 22} = 110.9$ cm.

131. (a) Radius of the wheel = 35 cm

$\therefore$ Circumference of the wheel = $2\pi \times 35$ cm

$= 2 \times \frac{22}{7} \times 35$ cm = 220 cm.

Hence, the linear distance travelled by a point of the rim in one revolution = 220 cm.

Number of revolutions made by the wheel in 3 minutes

$= 20 \times 3 \times 60 = 3600$

$\therefore$ The linear distance travelled by a point of the rim in 3 minutes = $220 \times 3600 = 792000$ cm

$= \frac{792000}{100000}$ km = 7.92 km.

132. (b) In 60 minutes, the minute hand of a watch completes one revolution. Therefore, in 40 minutes, the minute hand turns through $\frac{2}{3}$ of a revolution. Therefore, $\theta = \frac{2}{3} \times 360°$ or $\frac{4\pi}{3}$ radian. Hence, the required distance travelled is given by

$l = r\theta = 1.5 \times \frac{4\pi}{3} = 2\pi = 2 \times 3.14 = 6.28$ cm.

133. (c) Let r_1 and r_2 be the radii of the two circles. Given that

$\theta_1 = 65° = \frac{\pi}{180} \times 65 = \frac{13\pi}{36}$ radian

and $\theta_2 = 110° = \frac{\pi}{180} \times 110 = \frac{22\pi}{36}$ radian

Let l be the length of each of the arc.
Then, $l = r_1\theta_1 = r_2\theta_2$, which gives

$\frac{13\pi}{36} \times r_1 = \frac{22\pi}{36} \times r_2$, i.e. $\frac{r_1}{r_2} = \frac{22}{13}$

Hence, $r_1 : r_2 = 22 : 13$.

134. (d) $\tan A + \cot A = 4$ …(i)

Squaring (i) both sides, we get
$\tan^2 A + \cot^2 A + 2 = 16$
$\Rightarrow \tan^2 A + \cot^2 A = 14$ …(ii)
Squaring (ii) both sides, we get
$(\tan^2 A + \cot^2 A)^2 = 196$
$\Rightarrow \tan^4 A + \cot^4 A = 196 - 2$
$\Rightarrow \tan^4 A + \cot^4 A = 194.$

135. (b) Given, $\frac{\sin A}{\sin B} = m$

$\Rightarrow \sin A = m \sin B$ ... (i)

and $\frac{\cos A}{\cos B} = n$

$\Rightarrow \cos A = n \cos B$ ... (ii)

Squaring (i) and (ii) and then adding, we get
$1 = m^2 \sin^2 B + n^2 \cos^2 B$

$\Rightarrow \frac{1}{\cos^2 B} = m^2 \frac{\sin^2 B}{\cos^2 B} + n^2$ [Dividing by $\cos^2 B$]

$\Rightarrow \sec^2 B = m^2 \tan^2 B + n^2$
$\Rightarrow 1 + \tan^2 B = m^2 \tan^2 B + n^2$
$\Rightarrow 1 - n^2 = (m^2 - 1)\tan^2 B$

$\Rightarrow \tan^2 B = \frac{1-n^2}{m^2-1}$

$\Rightarrow \tan B = \pm\sqrt{\frac{1-n^2}{m^2-1}}.$

136. (d) $\tan(A - B) = 1 \Rightarrow A - B = 45°$ or $225°$

$\sec(A + B) = \frac{2}{\sqrt{3}} \Rightarrow A + B = 30°$ or $330°$

$A + B = 330° = 2\pi - \frac{\pi}{6} = \frac{11\pi}{6}$ …(i)

and $A - B = 225° = \frac{5\pi}{4}$ …(ii)

Solving (i) and (ii), we get

$2B = \frac{11\pi}{6} - \frac{5\pi}{4} \Rightarrow 2B = \frac{7\pi}{12} \Rightarrow B = \frac{7\pi}{24}.$

137. (a) $4 \sin\alpha \sin\left(\alpha + \frac{\pi}{3}\right)\sin\left(\alpha + \frac{2\pi}{3}\right)$

$= 2\sin\alpha\left\{2\sin\left(\alpha + \frac{2\pi}{3}\right)\sin\left(\alpha + \frac{\pi}{3}\right)\right\}$

$= 2 \sin\alpha [2 \sin(\alpha + 120°) \sin(\alpha + 60°)]$
$= 2 \sin\alpha [\cos(\alpha + 120° - \alpha - 60°) - \cos(\alpha + 120° + \alpha + 60°)]$
$= 2 \sin\alpha [\cos 60° - \cos(180° + 2\alpha)]$

$= 2 \sin\alpha \cdot \frac{1}{2} - 2\sin\alpha(-\cos 2\alpha)$

$= \sin\alpha + 2\cos 2\alpha \sin\alpha$
$= \sin\alpha + \sin(2\alpha + \alpha) - \sin(2\alpha - \alpha)$
$= \sin\alpha + \sin 3\alpha - \sin\alpha = \sin 3\alpha.$

138. (c) $[\sin x + \cos x]^{1 + 2\sin x \cos x} = 2$

$\Rightarrow (\sin x + \cos x)^{(\sin x + \cos x)^2} = 2$

$\Rightarrow (\sin x + \cos x)^{(\sin x + \cos x)^2} = (\sqrt{2})^{(\sqrt{2})^2}$ … (i)

Comparing (i) both sides, we get
$\sin x + \cos x = \sqrt{2}$

$\Rightarrow \sin\left(x + \frac{\pi}{4}\right) = 1 = \sin\frac{\pi}{2}$

$\Rightarrow x = n\pi + (-1)^n \frac{\pi}{2} - \frac{\pi}{4}$

So, $x = \frac{\pi}{4}$, when $n = 0$.

139. (c) Given that
$\tan\theta + \sec\theta = p$ …(i)
and we know that
$\Rightarrow \sec^2\theta - \tan^2\theta = (\sec\theta - \tan\theta)p$
(multiplying both the sides by $(\sec\theta - \tan\theta)$)
$\Rightarrow (\sec\theta - \tan\theta)\, p = 1$

$\Rightarrow \sec\theta - \tan\theta = \frac{1}{p}$ …(ii)

On solving equations (i) and (ii), we get

$2\sec\theta = \frac{p^2+1}{p} \Rightarrow \sec\theta = \frac{p^2+1}{2p}$

140. (c) We have,

$\sec\theta + \tan\theta = \sqrt{3}$ … (i)

$\Rightarrow \sec\theta - \tan\theta = \frac{1}{\sqrt{3}}$ … (ii)

$[\because \sec^2\theta - \tan^2\theta = 1]$

By solving (i) and (ii), we get

$\tan\theta = \frac{1}{2}\left(\sqrt{3} - \frac{1}{\sqrt{3}}\right) = \frac{1}{\sqrt{3}}$

$\therefore \tan\theta = \tan\left(\frac{\pi}{6}\right)$

$\Rightarrow \theta = n\pi + \frac{\pi}{6}$

$\therefore$ Solutions for $0 \le \theta \le 2\pi$ are $\frac{\pi}{6}$ and $\frac{7\pi}{6}$.
Hence, there are two solutions.

141. (c) Given equation is $\cos x - \sin x = \frac{1}{\sqrt{2}}$

Dividing equation by $\sqrt{2}$,

$\frac{1}{\sqrt{2}}\cos x - \frac{1}{\sqrt{2}}\sin x = \frac{1}{2}$

$\Rightarrow \cos\left(\frac{\pi}{4} + x\right) = \cos\frac{\pi}{3}$

$\Rightarrow \frac{\pi}{4} + x = 2n\pi \pm \frac{\pi}{3}$

$x = 2n\pi + \frac{\pi}{3} - \frac{\pi}{4} = 2n\pi + \frac{\pi}{12}$

or $x = 2n\pi - \frac{\pi}{3} - \frac{\pi}{4} = 2n\pi - \frac{7\pi}{12}$.

142. (a) $4\sin^2\theta + 2(\sqrt{3}+1)\cos\theta = 4 + \sqrt{3}$

$\Rightarrow 4 - 4\cos^2\theta + 2(\sqrt{3}+1)\cos\theta = 4 + \sqrt{3}$

$\Rightarrow 4\cos^2\theta - 2(\sqrt{3}+1)\cos\theta + \sqrt{3} = 0$

$\Rightarrow \cos\theta = \frac{2(\sqrt{3}+1) \pm \sqrt{4(\sqrt{3}+1)^2 - 16\sqrt{3}}}{8}$

$\Rightarrow \cos\theta = \frac{\sqrt{3}}{2}$ or $\frac{1}{2}$

$\Rightarrow \theta = 2n\pi \pm \frac{\pi}{6}$ or $2n\pi \pm \frac{\pi}{3}$.

143. (a) $2\sin^2 x + 5\sin x - 3 = 0$

$\Rightarrow \sin x = \frac{-5 \pm \sqrt{25+24}}{4} = \frac{-5 \pm 7}{4} = -3, \frac{1}{2}$

But $\sin x \neq -3$

$\therefore \sin x = \frac{1}{2}$

$\therefore$ Number of solution in $[0, 3\pi]$ will be equal to 4.

144. (b) $\sin\theta + \cos\theta = 1$

Dividing by $\sqrt{1^2 + 1^2} = \sqrt{2}$,

$\frac{1}{\sqrt{2}}\sin\theta + \frac{1}{\sqrt{2}}\cos\theta = \frac{1}{\sqrt{2}}$

$\Rightarrow \sin\left(\theta + \frac{\pi}{4}\right) = \frac{1}{\sqrt{2}} = \sin\frac{\pi}{4}$

$\Rightarrow \theta + \frac{\pi}{4} = n\pi + (-1)^n\frac{\pi}{4}$

$\Rightarrow \theta = n\pi + (-1)^n\frac{\pi}{4} - \frac{\pi}{4}$.

145. (a) $\sin 6\theta + \sin 4\theta + \sin 2\theta = 0$

$\Rightarrow 2\sin 4\theta\cos 2\theta + \sin 4\theta = 0$

$\Rightarrow \sin 4\theta(2\cos 2\theta + 1) = 0$

$\Rightarrow 2\cos 2\theta = -1 \Rightarrow \cos 2\theta = -\frac{1}{2} = \cos\left(\frac{2\pi}{3}\right)$

$\Rightarrow 2\theta = 2n\pi \pm \frac{2\pi}{3} \Rightarrow \theta = n\pi \pm \frac{\pi}{3}$

and $\sin 4\theta = 0 \Rightarrow 4\theta = n\pi \Rightarrow \theta = \frac{n\pi}{4}$

$\theta = \frac{n\pi}{4}$ or $n\pi \pm \frac{\pi}{3}$.

146. (c) $\sqrt{2}\sec\theta + \tan\theta = 1$

$\Rightarrow \frac{\sqrt{2}}{\cos\theta} + \frac{\sin\theta}{\cos\theta} = 1$

$\Rightarrow \sin\theta - \cos\theta = -\sqrt{2}$

Dividing by $\sqrt{2}$ on both sides, we get

$\frac{1}{\sqrt{2}}\sin\theta - \frac{1}{\sqrt{2}}\cos\theta = -1$

$\Rightarrow \frac{1}{\sqrt{2}}\cos\theta - \frac{1}{\sqrt{2}}\sin\theta = 1$

$\Rightarrow \cos\left(\theta + \frac{\pi}{4}\right) = \cos(0)$

$\Rightarrow \theta + \frac{\pi}{4} = 2n\pi \pm 0 \Rightarrow \theta = 2n\pi - \frac{\pi}{4}$.

147. (c) $12\cot^2\theta - 31\operatorname{cosec}\theta + 32 = 0$

$\Rightarrow 12(\operatorname{cosec}^2\theta - 1) - 31\operatorname{cosec}\theta + 32 = 0$

$\Rightarrow 12\operatorname{cosec}^2\theta - 31\operatorname{cosec}\theta + 20 = 0$

$\Rightarrow 12\operatorname{cosec}^2\theta - 16\operatorname{cosec}\theta - 15\operatorname{cosec}\theta + 20 = 0$

$\Rightarrow (4\operatorname{cosec}\theta - 5)(3\operatorname{cosec}\theta - 4) = 0$

$\Rightarrow \operatorname{cosec}\theta = \frac{5}{4}, \frac{4}{3}$

$\therefore \sin\theta = \frac{4}{5}, \frac{3}{4}$.

148. (b) We have $\sec^2\theta = \frac{4}{3}$

$\Rightarrow \cos^2\theta = \frac{3}{4}$

$\Rightarrow \cos^2\theta = \cos^2\left(\frac{\pi}{6}\right)$

$\Rightarrow \theta = n\pi \pm \frac{\pi}{6}$ .... $\left[\begin{array}{l}\text{If } \cos^2\theta = \cos^2\alpha \\ \Rightarrow \theta = n\pi \pm \alpha\end{array}\right]$

149. (a) We have $\tan 5\theta = \cot 2\theta$

$\Rightarrow \tan 5\theta = \tan\left(\frac{\pi}{2} - 2\theta\right) \ldots \left[\because \tan\left(\frac{\pi}{2} - \theta\right) = \cot\theta\right]$

$\Rightarrow 5\theta = n\pi + \frac{\pi}{2} - 2\theta \Rightarrow 7\theta = n\pi + \frac{\pi}{2}$

$\Rightarrow \theta = \frac{n\pi}{7} + \frac{\pi}{14}$.

150. (c) Since, none of the x, y and $(x + y)$ is multiple of π, we find that $\sin x$, $\sin y$ and $\sin(x + y)$ are non-zero. Now,

$\cot(x + y) = \frac{\cos(x+y)}{\sin(x+y)} = \frac{\cos x\cos y - \sin x\sin y}{\sin x\cos y + \cos x\sin y}$

On dividing numerator and denominator by $\sin x \sin y$, we have

$\cot(x + y) = \frac{\cot x\cot y - 1}{\cot y + \cot x}$

On replacing y by (–y) in above identity, we get

$\cot(x - y) = \frac{\cot x \cdot \cot y + 1}{\cot y - \cot x}$.

151. (a) Given, $3\tan(\theta - 15) = \tan(\theta + 15)$

$\frac{\tan A}{\tan B} = \frac{3}{1}$,

where $A = \theta + 15°$, $B = \theta - 15°$

On applying componendo and dividendo, we get

$\Rightarrow \dfrac{\tan A + \tan B}{\tan A - \tan B} = \dfrac{3+1}{3-1} \Rightarrow \dfrac{\frac{\sin A}{\cos A} + \frac{\sin B}{\cos B}}{\frac{\sin A}{\cos A} - \frac{\sin B}{\cos B}} = 2$

$\Rightarrow \dfrac{\sin(A+B)}{\sin(A-B)} = 2$

$\Rightarrow \sin 2\theta = 2 \sin 30°$

$\Rightarrow \sin 2\theta = 2 \cdot \dfrac{1}{2} = 1 = \sin \dfrac{\pi}{2}$

$\Rightarrow 2\theta = n\pi + (-1)^n \dfrac{\pi}{2}$

$\Rightarrow \theta = \dfrac{n\pi}{2} + (-1)^n \dfrac{\pi}{4}.$

152. (c) Let $\theta = \alpha + \beta$. Then, $\tan \alpha = K \tan \beta$

or $\dfrac{\tan \alpha}{\tan \beta} = \dfrac{K}{1}$

Applying componendo and dividendo, we have

$\dfrac{\tan \alpha + \tan \beta}{\tan \alpha - \tan \beta} = \dfrac{K+1}{K-1}$

or $\dfrac{\sin \alpha \cos \beta + \cos \alpha \sin \beta}{\sin \alpha \cos \beta - \cos \alpha \sin \beta} = \dfrac{K+1}{K-1}$

i.e., $\dfrac{\sin(\alpha+\beta)}{\sin(\alpha-\beta)} = \dfrac{K+1}{K-1}$

Given that, $\alpha - \beta = \phi$ and $\alpha + \beta = \theta$. Therefore,

$\dfrac{\sin \theta}{\sin \phi} = \dfrac{K+1}{K-1}$ or $\sin \theta = \dfrac{K+1}{K-1} \sin \phi.$

153. (a) We have, $m \sin \theta = n \sin(\theta + 2\alpha)$

$\Rightarrow \dfrac{\sin(\theta+2\alpha)}{\sin \theta} = \dfrac{m}{n}$

Using componendo and dividendo, we get

$\dfrac{\sin(\theta+2\alpha) + \sin \theta}{\sin(\theta+2\alpha) - \sin \theta} = \dfrac{m+n}{m-n}$

$\Rightarrow \dfrac{2 \sin\left(\frac{\theta+2\alpha+\theta}{2}\right) \cdot \cos\left(\frac{\theta+2\alpha-\theta}{2}\right)}{2 \cos\left(\frac{\theta+2\alpha+\theta}{2}\right) \cdot \sin\left(\frac{\theta+2\alpha-\theta}{2}\right)} = \dfrac{m+n}{m-n}$

$\Rightarrow \dfrac{2\sin(\theta+\alpha) \cdot \cos \alpha}{2\cos(\theta+\alpha) \cdot \sin \alpha} = \dfrac{m+n}{m-n}$

$\Rightarrow \tan(\theta + \alpha) \cdot \cot \alpha = \dfrac{m+n}{m-n}$

154. (c) $5 \tan \theta = 4 \Rightarrow \tan \theta = \dfrac{4}{5}$

$\therefore \sin \theta = \dfrac{4}{\sqrt{41}}$ and $\cos \theta = \dfrac{5}{\sqrt{41}}$

$\dfrac{5 \sin \theta - 3 \cos \theta}{5 \sin \theta + 2 \cos \theta} = \dfrac{5 \times \frac{4}{\sqrt{41}} - 3 \times \frac{5}{\sqrt{41}}}{5 \times \frac{4}{\sqrt{41}} + 2 \times \frac{5}{\sqrt{41}}}$

$= \dfrac{20-15}{20+10} = \dfrac{5}{30} = \dfrac{1}{6}.$

155. (c) $\dfrac{1 + \sin A - \cos A}{1 + \sin A + \cos A}$

$= \dfrac{2\sin^2 \frac{A}{2} + 2 \sin \frac{A}{2} \cos \frac{A}{2}}{2\cos^2 \frac{A}{2} + 2 \sin \frac{A}{2} \cos \frac{A}{2}}$ $\left(\begin{array}{l}\text{as} \\ \sin A = 2\sin\frac{A}{2}\cos\frac{A}{2} \\ \cos A = 2\cos^2\frac{A}{2} - 1 \\ \cos A = 1 - 2\sin^2\frac{A}{2}\end{array}\right)$

$= \dfrac{2 \sin \frac{A}{2}\left(\sin \frac{A}{2} + \cos \frac{A}{2}\right)}{2 \cos \frac{A}{2}\left(\cos \frac{A}{2} + \sin \frac{A}{2}\right)} = \tan \dfrac{A}{2}.$

Trick: Put A = 60°

Then, $\dfrac{1 + \left(\frac{\sqrt{3}}{2}\right) - \left(\frac{1}{2}\right)}{1 + \left(\frac{\sqrt{3}}{2}\right) + \left(\frac{1}{2}\right)} = \dfrac{1+\sqrt{3}}{3+\sqrt{3}} = \dfrac{1}{\sqrt{3}},$

which is given by option (c), i.e. $\tan \dfrac{60°}{2} = \dfrac{1}{\sqrt{3}}.$

156. (d) $\dfrac{1}{4}\left\{\sqrt{3} \cos 23° - \sin 23°\right\}$

$= \dfrac{1}{2}\{\cos 30° \cos 23° - \sin 30° \sin 23°\}$

$= \dfrac{1}{2}\cos\{30° + 23°\} = \dfrac{1}{2} \cos 53°.$

157. (c) Given equation $\cos x + \cos y + \cos \alpha = 0$ and $\sin x + \sin y + \sin \alpha = 0$.

The given equation may be written as

$\cos x + \cos y = -\cos \alpha$ and $\sin x + \sin y = -\sin \alpha$.

Therefore,

$2 \cos\left(\dfrac{x+y}{2}\right) \cos\left(\dfrac{x-y}{2}\right) = -\cos \alpha$ ... (i)

$2 \sin\left(\dfrac{x+y}{2}\right) \cos\left(\dfrac{x-y}{2}\right) = -\sin \alpha$ ... (ii)

Divide (i) by (ii), we get

$\dfrac{2 \cos\left(\frac{x+y}{2}\right) \cos\left(\frac{x-y}{2}\right)}{2 \sin\left(\frac{x+y}{2}\right) \cos\left(\frac{x-y}{2}\right)} = \dfrac{\cos \alpha}{\sin \alpha}$

$\Rightarrow \cot\left(\dfrac{x+y}{2}\right) = \cot \alpha.$

158. (a) $\sin 12° \sin 24° \sin 48° \sin 84°$

$= \dfrac{1}{4}(2 \sin 12° \sin 48°)(2 \sin 24° \sin 84°)$

$= \dfrac{1}{2}(\cos 36° - \cos 60°)(\cos 60° - \cos 108°)$

$= \dfrac{1}{4}\left(\cos 36° - \dfrac{1}{2}\right)\left(\dfrac{1}{2} + \sin 18°\right)$

$= \dfrac{1}{4}\left\{\dfrac{1}{4}\left(\sqrt{5}+1\right) - \dfrac{1}{2}\right\}\left\{\dfrac{1}{2} + \dfrac{1}{4}\left(\sqrt{5}-1\right)\right\} = \dfrac{1}{16}$

and $\cos 20° \cos 40° \cos 60° \cos 80°$

$= \frac{1}{2}[\cos(60° - 20°) \cos 20° \cos(60° + 20°)]$

$= \frac{1}{2}\left[\frac{1}{4}\cos 3(20°)\right] = \frac{1}{8}\cos 60° = \frac{1}{2} \times \frac{1}{8} = \frac{1}{16}.$

159. (b) We have, $\frac{2 \sin \alpha}{1 + \cos \alpha + \sin \alpha} = y$

Then, $\frac{4 \sin \frac{\alpha}{2} \cos \frac{\alpha}{2}}{2\cos^2 \frac{\alpha}{2} + 2 \sin \frac{\alpha}{2} \cos \frac{\alpha}{2}} = y$

$\Rightarrow \frac{2 \sin \frac{\alpha}{2}}{\cos \frac{\alpha}{2} + \sin \frac{\alpha}{2}} \times \frac{\left(\sin \frac{\alpha}{2} + \cos \frac{\alpha}{2}\right)}{\left(\sin \frac{\alpha}{2} + \cos \frac{\alpha}{2}\right)} = y$

$\Rightarrow \frac{1 - \cos \alpha + \sin \alpha}{1 + \sin \alpha} = y.$

Trick: Put value of $\theta = 30°$ and check.

160. (b) Given, $\sin 2\theta + \sin 2\phi = \frac{1}{2}$ … (i)

and $\cos 2\theta + \cos 2\phi = \frac{3}{2}$ … (ii)

Square and adding,

$\therefore \ (\sin^2 2\theta + \cos^2 2\theta) + (\sin^2 2\phi + \cos^2 2\phi)$

$+ 2[\sin 2\theta \sin 2\phi + \cos 2\theta \cos 2\phi] = \frac{1}{4} + \frac{9}{4}$

$\Rightarrow \cos 2\theta \cos 2\phi + \sin 2\theta \sin 2\phi = \frac{1}{4}$

$\Rightarrow \cos(2\theta - 2\phi) = \frac{1}{4} \Rightarrow \cos 2(\theta - \phi) = \frac{1}{4}$

$\Rightarrow 2\cos^2(\theta - \phi) - 1 = \frac{1}{4} \Rightarrow \cos^2(\theta - \phi) = \frac{5}{8}$