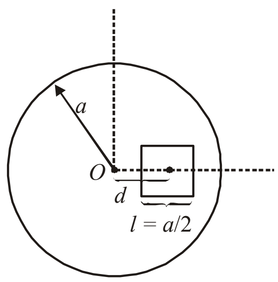
6

System of Particles and Rotational Motion

TOPIC 1 Centre of Mass, Centre of Gravity & Principle of Moments

1. The centre of mass of a solid hemisphere of radius 8 cm is x cm from the centre of the flat surface. Then value of x is ______. **[NA Sep. 06, 2020 (II)]**

2. A square shaped hole of side $l = \frac{a}{2}$ is carved out at a distance $d = \frac{a}{2}$ from the centre 'O' of a uniform circular disk of radius a. If the distance of the centre of mass of the remaining portion from O is $-\frac{a}{X}$, value of X (to the nearest integer) is ___________. **[NA Sep. 02, 2020 (II)]**

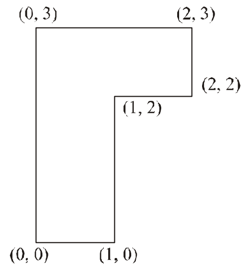


3. A rod of length L has non-uniform linear mass density given by $\rho(x) = a + b\left(\frac{x}{L}\right)^2$, where a and b are constants and $0 \le x \le L$. The value of x for the centre of mass of the rod is at: **[9 Jan. 2020 II]**

(a) $\frac{3}{2}\left(\frac{a+b}{2a+b}\right)L$ (b) $\frac{3}{4}\left(\frac{2a+b}{3a+b}\right)L$

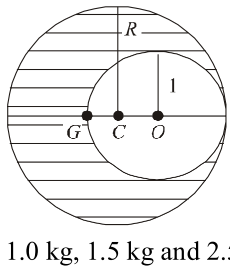
(c) $\frac{4}{3}\left(\frac{a+b}{2a+3b}\right)L$ (d) $\frac{3}{2}\left(\frac{2a+b}{3a+b}\right)L$

4. The coordinates of centre of mass of a uniform flag shaped lamina (thin flat plale) of mass 4 kg. (The coordinates of the same are shown in figure) are: **[8 Jan. 2020 I]**



(a) (1.25 m, 1.50 m) (b) (0.75 m, 1.75 m)
(c) (0.75 m, 0.75 m) (d) (1 m, 1.75 m)

5. As shown in fig. when a spherical cavity (centred at O) of radius 1 is cut out of a uniform sphere of radius R (centred at C), the centre of mass of remaining (shaded) part of sphere is at G, i.e on the surface of the cavity. R can be determined by the equation: **[8 Jan. 2020 II]**



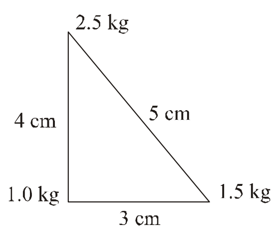
(a) $(R^2 + R + 1)(2 - R) = 1$

(b) $(R^2 - R - 1)(2 - R) = 1$

(c) $(R^2 - R + 1)(2 - R) = 1$

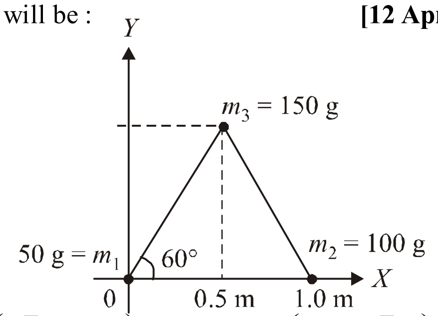
(d) $(R^2 + R - 1)(2 - R) = 1$

6. Three point particles of masses 1.0 kg, 1.5 kg and 2.5 kg are placed at three corners of a right angle triangle of sides 4.0 cm, 3.0 cm and 5.0 cm as shown in the figure. The center of mass of the system is at a point: **[7 Jan. 2020 I]**



(a) 0.6 cm right and 2.0 cm above 1 kg mass
(b) 1.5 cm right and 1.2 cm above 1 kg mass
(c) 2.0 cm right and 0.9 cm above 1 kg mass
(d) 0.9 cm right and 2.0 cm above 1 kg mass

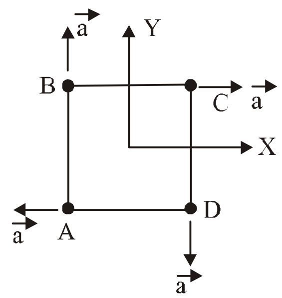
7. Three particles of masses 50 g, 100 g and 150 g are placed at the vertices of an equilateral triangle of side 1 m (as shown in the figure). The (x, y) coordinates of the centre of mass will be : **[12 Apr. 2019 II]**



(a) $\left(\frac{\sqrt{3}}{4}m, \frac{5}{12}m\right)$ (b) $\left(\frac{7}{12}m, \frac{\sqrt{3}}{8}m\right)$

(c) $\left(\frac{7}{12}m, \frac{\sqrt{3}}{4}m\right)$ (d) $\left(\frac{\sqrt{3}}{8}m, \frac{7}{12}m\right)$

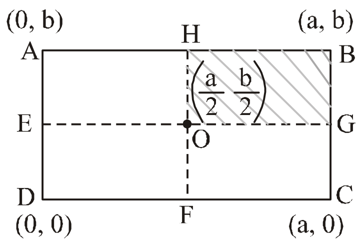
8. Four particles A, B, C and D with masses $m_A = m$, $m_B = 2m$, $m_C = 3m$ and $m_D = 4m$ are at the corners of a square. They have accelerations of equal magnitude with directions as shown. The acceleration of the centre of mass of the particles is : **[8 April 2019 I]**



(a) $\frac{a}{5}(\hat{i} - \hat{j})$ (b) a

(c) Zero (d) $\frac{a}{5}(\hat{i} + \hat{j})$

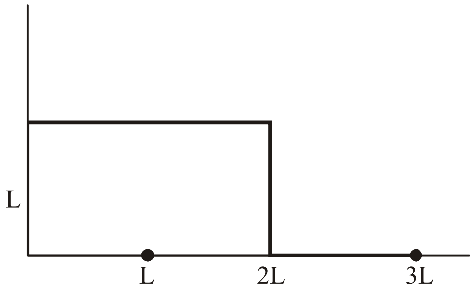
9. A uniform rectangular thin sheet ABCD of mass M has length a and breadth b, as shown in the figure. If the shaded portion HBGO is cut-off, the coordinates of the centre of mass of the remaining portion will be : **[8 Apr. 2019 II]**



(a) $\left(\frac{3a}{4}, \frac{3b}{4}\right)$ (b) $\left(\frac{5a}{3}, \frac{5b}{3}\right)$

(c) $\left(\frac{2a}{3}, \frac{2b}{3}\right)$ (d) $\left(\frac{5a}{12}, \frac{5b}{12}\right)$

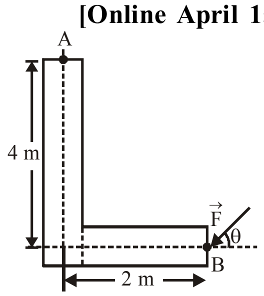
10. The position vector of the centre of mass r_{cm} of an asymmetric uniform bar of negligible area of cross-section as shown in figure is: **[12 Jan. 2019 I]**



(a) $\vec{r}\text{cm} = \frac{13}{8}L\hat{x} + \frac{5}{8}L\hat{y}$ (b) $\vec{r}\text{cm} = \frac{5}{8}L\hat{x} + \frac{13}{8}L\hat{y}$

(c) $\vec{r}\text{cm} = \frac{3}{8}L\hat{x} + \frac{11}{8}L\hat{y}$ (d) $\vec{r}\text{cm} = \frac{11}{8}L\hat{x} + \frac{3}{8}L\hat{y}$

11. A force of 40 N acts on a point B at the end of an L-shaped object, as shown in the figure. The angle θ that will produce maximum moment of the force about point A is given by: **[Online April 15, 2018]**



(a) $\tan\theta = \frac{1}{4}$

(b) $\tan\theta = 2$

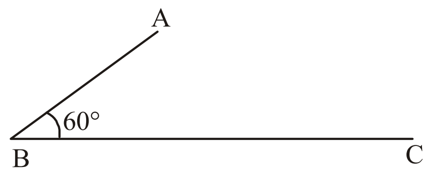
(c) $\tan\theta = \frac{1}{2}$

(d) $\tan\theta = 4$

12. In a physical balance working on the principle of moments, when 5 mg weight is placed on the left pan, the beam becomes horizontal. Both the empty pans of the balance are of equal mass. Which of the following statements is correct? **[Online April 8, 2017]**

(a) Left arm is longer than the right arm
(b) Both the arms are of same length
(c) Left arm is shorter than the right arm
(d) Every object that is weighed using this balance appears lighter than its actual weight.

13. In the figure shown ABC is a uniform wire. If centre of mass of wire lies vertically below point A, then $\frac{BC}{AB}$ is close to : **[Online April 10, 2016]**



(a) 1.85 (b) 1.5
(c) 1.37 (d) 3

14. Distance of the centre of mass of a solid uniform cone from its vertex is z_0. If the radius of its base is R and its height is h then z_0 is equal to : **[2015]**

(a) $\frac{5h}{8}$ (b) $\frac{3h^2}{8R}$ (c) $\frac{h^2}{4R}$ (d) $\frac{3h}{4}$

15. A uniform thin rod AB of length L has linear mass density $\mu(x) = a + \frac{bx}{L}$, where x is measured from A. If the CM of the rod lies at a distance of $\left(\frac{7}{12}\right)L$ from A, then a and b are related as : **[Online April 11, 2015]**

(a) $a = 2b$ (b) $2a = b$
(c) $a = b$ (d) $3a = 2b$

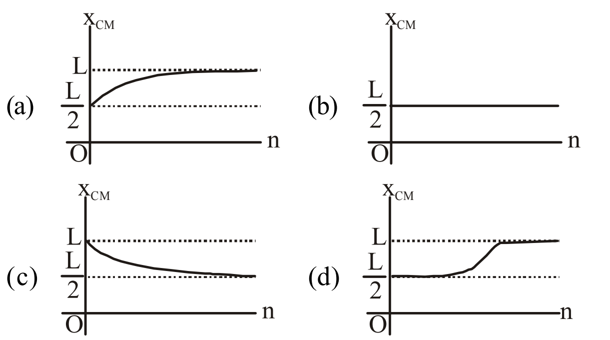
16. A thin bar of length L has a mass per unit length λ, that increases linearly with distance from one end. If its total mass is M and its mass per unit length at the lighter end is λ_O, then the distance of the centre of mass from the lighter end is: **[Online April 11, 2014]**

(a) $\frac{L}{2} - \frac{\lambda_o L^2}{4M}$ (b) $\frac{L}{3} + \frac{\lambda_o L^2}{8M}$
(c) $\frac{L}{3} + \frac{\lambda_o L^2}{4M}$ (d) $\frac{2L}{3} - \frac{\lambda_o L^2}{6M}$

17. A boy of mass 20 kg is standing on a 80 kg free to move long cart. There is negligible friction between cart and ground. Initially, the boy is standing 25 m from a wall. If he walks 10 m on the cart towards the wall, then the final distance of the boy from the wall will be **[Online April 23, 2013]**

(a) 15m (b) 12.5m (c) 15.5m (d) 17m

18. A thin rod of length 'L' is lying along the x-axis with its ends at $x = 0$ and $x = L$. Its linear density (mass/length) varies with x as $k\left(\frac{x}{L}\right)^n$, where n can be zero or any positive number. If the position x_{CM} of the centre of mass of the rod is plotted against 'n', which of the following graphs best approximates the dependence of x_{CM} on n? **[2008]**



19. A circular disc of radius R is removed from a bigger circular disc of radius $2R$ such that the circumferences of the discs coincide. The centre of mass of the new disc is α/R form the centre of the bigger disc. The value of α is **[2007]**

(a) 1/4 (b) 1/3 (c) 1/2 (d) 1/6

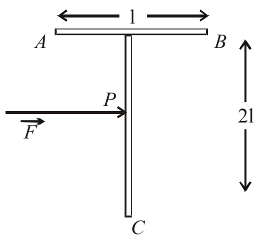
20. Consider a two particle system with particles having masses m_1 and m_2. If the first particle is pushed towards the centre of mass through a distance d, by what distance should the second particle is moved, so as to keep the centre of mass at the same position? **[2006]**

(a) $\frac{m_2}{m_1}d$ (b) $\frac{m_1}{m_1 + m_2}d$
(c) $\frac{m_1}{m_2}d$ (d) d

21. A body A of mass M while falling vertically downwards under gravity breaks into two parts; a body B of mass $\frac{1}{3}$ M and a body C of mass $\frac{2}{3}$ M. The centre of mass of bodies B and C taken together shifts compared to that of body A towards **[2005]**

(a) does not shift
(b) depends on height of breaking
(c) body B
(d) body C

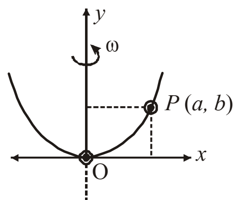
22. A 'T' shaped object with dimensions shown in the figure, is lying on a smooth floor. A force '$\vec{F}$' is applied at the point P parallel to AB, such that the object has only the translational motion without rotation. Find the location of P with respect to C. **[2005]**



(a) $\frac{3}{2}\ell$ (b) $\frac{2}{3}\ell$ (c) ℓ (d) $\frac{4}{3}\ell$

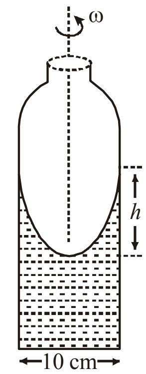
TOPIC 2 Angular Displacement, Velocity and Aceleration

23. A bead of mass m stays at point $P(a, b)$ on a wire bent in the shape of a parabola $y = 4Cx^2$ and rotating with angular speed ω (see figure). The value of ω is (neglect friction) : **[Sep. 02, 2020 (I)]**



(a) $2\sqrt{2gC}$ (b) $2\sqrt{gC}$
(c) $\sqrt{\frac{2gC}{ab}}$ (d) $\sqrt{\frac{2g}{C}}$

24. A cylindrical vessel containing a liquid is rotated about its axis so that the liquid rises at its sides as shown in the figure. The radius of vessel is 5 cm and the angular speed of rotation is ω rad s^{-1}. The difference in the height, h (in cm) of liquid at the centre of vessel and at the side will be : **[Sep. 02, 2020 (I)]**



(a) $\frac{2\omega^2}{25g}$ (b) $\frac{5\omega^2}{2g}$

(c) $\frac{25\omega^2}{2g}$ (d) $\frac{2\omega^2}{5g}$

25. A spring mass system (mass m, spring constant k and natural length l) rests in equilibrium on a horizontal disc. The free end of the spring is fixed at the centre of the disc. If the disc together with spring mass system, rotates about it's axis with an angular velocity ω, ($k >> m\omega^2$) the relative change in the length of the spring is best given by the option: **[9 Jan. 2020 II]**

(a) $\sqrt{\frac{2}{3}}\left(\frac{m\omega^2}{k}\right)$ (b) $\frac{2m\omega^2}{k}$

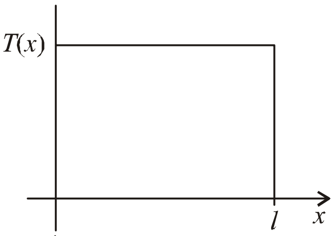
(c) $\frac{m\omega^2}{k}$ (d) $\frac{m\omega^2}{3k}$

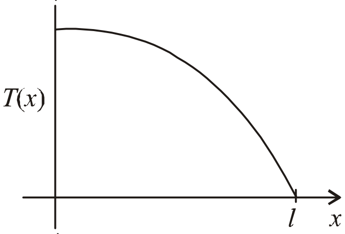
26. A particle of mass m is fixed to one end of a light spring having force constant k and unstretched length l. The other end is fixed. The system is given an angular speed ω about the fixed end of the spring such that it rotates in a circle in gravity free space. Then the stretch in the spring is: **[8 Jan. 2020 I]**

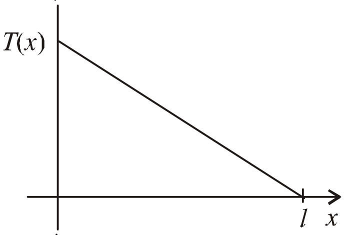
(a) $\frac{ml\omega^2}{k-\omega m}$ (b) $\frac{ml\omega^2}{k-m\omega^2}$

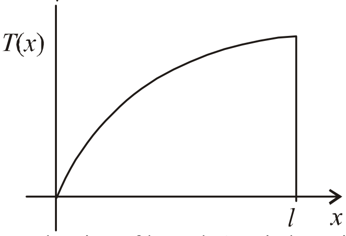
(c) $\frac{ml\omega^2}{k+m\omega^2}$ (d) $\frac{ml\omega^2}{k+m\omega}$

27. A uniform rod of length l is being rotated in a horizontal plane with a constant angular speed about an axis passing through one of its ends. If the tension generated in the rod due to rotation is T(x) at a distance x from the axis, then which of the following graphs depicts it most closely? **[12 Apr. 2019 II]**

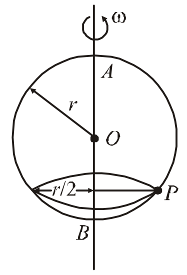
(a) 

(b) 

(c) 

(d) 

28. A smooth wire of length $2\pi r$ is bent into a circle and kept in a vertical plane. A bead can slide smoothly on the wire. When the circle is rotating with angular speed ω about the vertical diameter AB, as shown in figure, the bead is at rest with respect to the circular ring at position P as shown. Then the value of ω^2 is equal to : **[12 Apr. 2019 II]**



(a) $\frac{\sqrt{3g}}{2r}$ (b) $2g/(r\sqrt{3})$

(c) $(g\sqrt{3})/\mathrm{r}$ (d) $2g/r$

29. A long cylindrical vessel is half filled with a liquid. When the vessel is rotated about its own vertical axis, the liquid rises up near the wall. If the radius of vessel is 5 cm and its rotational speed is 2 rotations per second, then the difference in the heights between the centre and the sides, in cm, will be : **[12 Jan. 2019 II]**

(a) 2.0 (b) 0.1

(c) 0.4 (d) 1.2

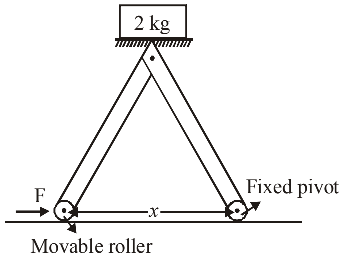
30. A particle is moving with a uniform speed in a circular orbit of radius R in a central force inversely proportional to the n^{th} power of R. If the period of rotation of the particle is T, then: **[2018]**

(a) $T \propto R^{3/2}$ for any n. (b) $T \propto R^{n/2+1}$

(c) $T \propto R^{(n+1)/2}$ (d) $T \propto R^{n/2}$

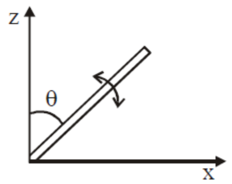
31. The machine as shown has 2 rods of length 1 m connected by a pivot at the top. The end of one rod is connected to the floor by a stationary pivot and the end of the other rod has a roller that rolls along the floor in a slot.

As the roller goes back and forth, a 2 kg weight moves up and down. If the roller is moving towards right at a constant speed, the weight moves up with a : **[Online April 9, 2017]**



(a) constant speed
(b) decreasing speed
(c) increasing speed
(d) speed which is $\frac{3}{4}$th of that of the roller when the weight is 0.4 m above the ground

32. A slender uniform rod of mass M and length ℓ is pivoted at one end so that it can rotate in a vertical plane (see figure). There is negligible friction at the pivot. The free end is held vertically above the pivot and then released. The angular acceleration of the rod when it makes an angle θ with the vertical is **[2017]**



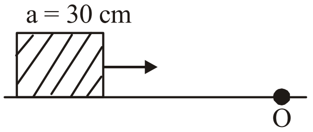
(a) $\frac{3g}{2\ell}\cos\theta$ (b) $\frac{2g}{3\ell}\cos\theta$

(c) $\frac{3g}{2\ell}\sin\theta$ (d) $\frac{2g}{2\ell}\sin\theta$

33. Concrete mixture is made by mixing cement, stone and sand in a rotating cylindrical drum. If the drum rotates too fast, the ingredients remain stuck to the wall of the drum and proper mixing of ingredients does not take place. The maximum rotational speed of the drum in revolutions per minute (rpm) to ensure proper mixing is close to :
(Take the radius of the drum to be 1.25 m and its axle to be horizontal): **[Online April 10, 2016]**

(a) 27.0 (b) 0.4 (c) 1.3 (d) 8.0

34. A cubical block of side 30 cm is moving with velocity 2 ms^{-1} on a smooth horizontal surface. The surface has a bump at a point O as shown in figure. The angular velocity (in rad/s) of the block immediately after it hits the bump, is : **[Online April 9, 2016]**



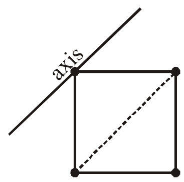
(a) 13.3 (b) 5.0 (c) 9.4 (d) 6.7

35. Two point masses of mass $m_1 = fM$ and $m_2 = (1-f)M$ ($f < 1$) are in outer space (far from gravitational influence of other objects) at a distance R from each other. They move in circular orbits about their centre of mass with angular velocities ω_1 for m_1 and ω_2 for m_2. In that case **[Online May 19, 2012]**

(a) $(1-f)\,\omega_1 = f\omega$
(b) $\omega_1 = \omega_2$ and independent of f
(c) $f\omega_1 = (1-f)\omega_2$
(d) $\omega_1 = \omega_2$ and depend on f

TOPIC 3 Torque, Couple and Angular Momentum

36. Four point masses, each of mass m, are fixed at the corners of a square of side l. The square is rotating with angular frequency ω, about an axis passing through one of the corners of the square and parallel to its diagonal, as shown in the figure. The angular momentum of the square about this axis is : **[Sep. 06, 2020 (I)]**



(a) $ml^2\omega$ (b) $4\,ml^2\omega$

(c) $3\,ml^2\omega$ (d) $2\,ml^2\omega$

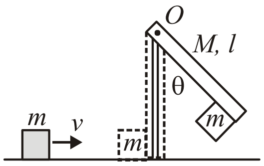
37. A thin rod of mass 0.9 kg and length 1 m is suspended, at rest, from one end so that it can freely oscillate in the vertical plane. A particle of move 0.1 kg moving in a straight line with velocity 80 m/s hits the rod at its bottom most point and sticks to it (see figure). The angular speed (in rad/s) of the rod immediately after the collision will be ______________. **[NA Sep. 05, 2020 (II)]**

38. A person of 80 kg mass is standing on the rim of a circular platform of mass 200 kg rotating about its axis at 5 revolutions per minute (rpm). The person now starts moving towards the centre of the platform. What will be the rotational speed (in rpm) of the platform when the person reaches its centre __________.

[NA Sep. 03, 2020 (I)]

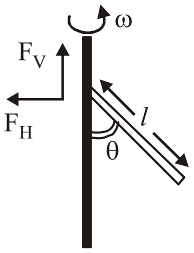
39. A block of mass $m = 1$ kg slides with velocity $v = 6$ m/s on a frictionless horizontal surface and collides with a uniform vertical rod and sticks to it as shown. The rod is pivoted about O and swings as a result of the collision making angle θ before momentarily coming to rest. If the rod has mass $M = 2$ kg, and length $l = 1$ m, the value of θ is approximately: (take $g = 10$ m/s^2)

[Sep. 03, 2020 (I)]



(a) 63° (b) 55°
(c) 69° (d) 49°

40.



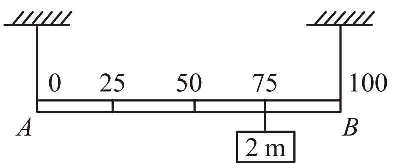
A uniform rod of length 'l' is pivoted at one of its ends on a vertical shaft of negligible radius. When the shaft rotates at angular speed ω the rod makes an angle θ with it (see figure). To find θ equate the rate of change of angular momentum (direction going into the paper) $\frac{ml^2}{12}\omega^2 \sin\theta\cos\theta$ about the centre of mass (CM) to the torque provided by the horizontal and vertical forces F_H and F_V about the CM. The value of θ is then such that :

[Sep. 03, 2020 (II)]

(a) $\cos\theta = \frac{2g}{3l\omega^2}$ (b) $\cos\theta = \frac{g}{2l\omega^2}$

(c) $\cos\theta = \frac{g}{l\omega^2}$ (d) $\cos\theta = \frac{3g}{2l\omega^2}$

41.

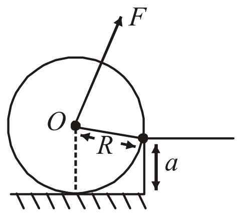


Shown in the figure is rigid and uniform one meter long rod AB held in horizontal position by two strings tied to its ends and attached to the ceiling. The rod is of mass 'm' and has another weight of mass 2 m hung at a distance of 75 cm from A. The tension in the string at A is :

[Sep. 02, 2020 (I)]

(a) 0.5 mg (b) 2 mg
(c) 0.75 mg (d) 1 mg

42. A uniform cylinder of mass M and radius R is to be pulled over a step of height a ($a < R$) by applying a force F at its centre 'O' perpendicular to the plane through the axes of the cylinder on the edge of the step (see figure). The minimum value of F required is : **[Sep. 02, 2020 (I)]**



(a) $Mg\sqrt{1-\left(\frac{R-a}{R}\right)^2}$ (b) $Mg\sqrt{\left(\frac{R}{R-a}\right)^2-1}$

(c) $Mg\frac{a}{R}$ (d) $Mg\sqrt{1-\frac{a^2}{R^2}}$

43. Consider a uniform rod of mass $M = 4$m and length l pivoted about its centre. A mass m moving with velocity v making angle $\theta = \frac{\pi}{4}$ to the rod's long axis collides with one end of the rod and sticks to it. The angular speed of the rod-mass system just after the collision is:

[8 Jan. 2020 I]

(a) $\frac{3}{7\sqrt{2}}\frac{v}{l}$ (b) $\frac{3}{7}\frac{v}{l}$

(c) $\frac{3\sqrt{2}}{7}\frac{v}{l}$ (d) $\frac{4}{7}\frac{v}{l}$

44. A particle of mass m is moving along a trajectory given by

$x = x_0 + a\cos\omega_1 t$

$y = y_0 + b\cos\omega_2 t$

The torque, acting on the particle about the origin, at $t = 0$ is: **[10 Apr. 2019 I]**

(a) $m(-x_0 b + y_0 a)\omega_1^2\hat{k}$ (b) $+my_0 a\omega_1^2\hat{k}$

(c) zero (d) $-m(x_0 b\omega_2^2 - y_0 a\omega_1^2)\hat{k}$

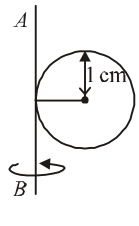
45. The time dependence of the position of a particle of mass $m = 2$ is given by $\vec{r}(t) = 2t\hat{i} - 3t^2\hat{j}$. Its angular momentum, with respect to the origin, at time t = 2 is :

[10 Apr. 2019 II]

(a) $48(\hat{i}+\hat{j})$ (b) $36\hat{k}$

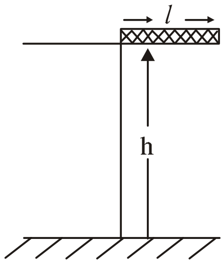
(c) $-34(\hat{k}-\hat{i})$ (d) $-48\hat{k}$

46. A metal coin of mass 5 g and radius 1 cm is fixed to a thin stick AB of negligible mass as shown in the figure The system is initially at rest. The constant torque, that will make the system rotate about AB at 25 rotations per second in 5s, is close to : **[10 Apr. 2019 II]**



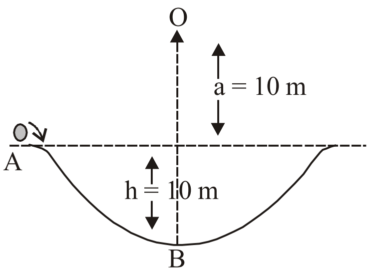
(a) 4.0×10^{-6} Nm (b) 1.6×10^{-5} Nm
(c) 7.9×10^{-6} Nm (d) 2.0×10^{-5} Nm

47. A rectangular solid box of length 0.3 m is held horizontally, with one of its sides on the edge of a platform of height 5m. When released, it slips off the table in a very short time $\tau = 0.01$ s, remaining essentially horizontal. The angle by which it would rotate when it hits the ground will be (in radians) close to : **[8 Apr. 2019 II]**



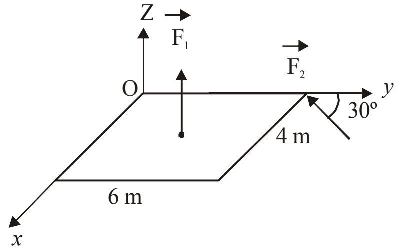
(a) 0.5 (b) 0.3 (c) 0.02 (d) 0.28

48. A particle of mass 20 g is released with an initial velocity 5 m/s along the curve from the point A, as shown in the figure. The point A is at height h from point B. The particle slides along the frictionless surface. When the particle reaches point B, its angular momentum about O will be : (Take g = 10 m/s^2) **[12 Jan. 2019 II]**



(a) 2 kg-m^2/s (b) 8 kg-m^2/s
(c) 6 kg-m^2/s (d) 3 kg-m^2/s

49. A slab is subjected to two forces $\vec{F_1}$ and $\vec{F_2}$ of same magnitude F as shown in the figure. Force $\vec{F_2}$ is in XY-plane while force F_1 acts along z-axis at the point $(2\vec{i}+3\vec{j})$. The moment of these forces about point O will be : **[11 Jan. 2019 I]**



(a) $\left(3\hat{i}-2\hat{j}+3\hat{k}\right)$F (b) $\left(3\hat{i}-2\hat{j}-3\hat{k}\right)$F
(c) $\left(3\hat{i}+2\hat{j}-3\hat{k}\right)$F (d) $\left(3\hat{i}+2\hat{j}+3\hat{k}\right)$F

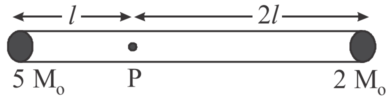
50. The magnitude of torque on a particle of mass 1 kg is 2.5 Nm about the origin. If the force acting on it is 1 N, and the distance of the particle from the origin is 5m, the angle between the force and the position vector is (in radians): **[11 Jan. 2019 II]**

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{8}$ (d) $\frac{\pi}{4}$

51. To mop-clean a floor, a cleaning machine presses a circular mop of radius R vertically down with a total force F and rotates it with a constant angular speed about its axis. If the force F is distributed uniformly over the mop and if coefficient of friction between the mop and the floor is μ, the torque, applied by the machine on the mop is: **[10 Jan. 2019 I]**

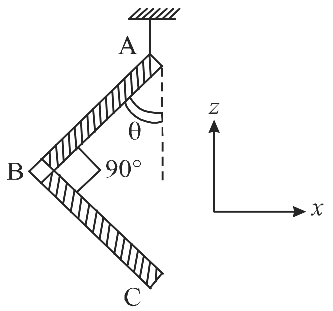
(a) μ FR/3 (b) μ FR/6
(c) μ FR/2 (d) $\frac{2}{3}\mu$ μFR

52. A rigid massless rod of length $3l$ has two masses attached at each end as shown in the figure. The rod is pivoted at point P on the horizontal axis (see figure). When released from initial horizontal position, its instantaneous angular acceleration will be: **[10 Jan. 2019 II]**



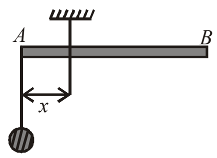
(a) $\frac{g}{13l}$ (b) $\frac{g}{3l}$
(c) $\frac{g}{2l}$ (d) $\frac{7g}{3l}$

53. An L-shaped object, made of thin rods of uniform mass density, is suspended with a string as shown in figure. If AB = BC, and the angle made by AB with downward vertical is θ, then: **[9 Jan. 2019 I]**



(a) $\tan\theta = \frac{1}{2\sqrt{3}}$ (b) $\tan\theta = \frac{1}{2}$
(c) $\tan\theta = \frac{2}{\sqrt{3}}$ (d) $\tan\theta = \frac{1}{3}$

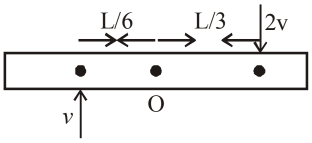
54.



A uniform rod AB is suspended from a point X, at a variable distance from x from A, as shown. To make the rod horizontal, a mass m is suspended from its end A. A set of (m, x) values is recorded. The appropriate variable that give a straight line, when plotted, are: **[Online April 15, 2018]**

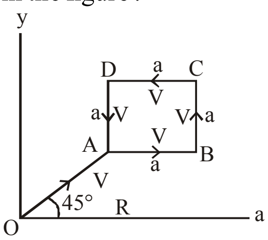
(a) $m, \frac{1}{x}$ (b) $m, \frac{1}{x^2}$ (c) m, x (d) m, x^2

55. A thin uniform bar of length L and mass 8m lies on a smooth horizontal table. Two point masses m and 2m moving in the same horizontal plane from opposite sides of the bar with speeds 2v and v respectively. The masses stick to the bar after collision at a distance $\frac{L}{3}$ and $\frac{L}{6}$ respectively from the centre of the bar. If the bar starts rotating about its center of mass as a result of collision, the angular speed of the bar will be: **[Online April 15, 2018]**



(a) $\frac{v}{6L}$ (b) $\frac{6v}{5L}$ (c) $\frac{3v}{5L}$ (d) $\frac{v}{5L}$

56. A particle of mass m is moving along the side of a square of side 'a', with a uniform speed v in the x-y plane as shown in the figure : **[2016]**



Which of the following statements is false for the angular momentum $\vec{L}$ about the origin?

(a) $\vec{L} = mv\left[\frac{R}{\sqrt{2}} + a\right]\hat{k}$ when the particle is moving from B to C.

(b) $\vec{L} = \frac{mv}{\sqrt{2}} R\hat{k}$ when the particle is moving from D to A.

(c) $\vec{L} = -\frac{mv}{\sqrt{2}} R\hat{k}$ when the particle is moving from A to B.

(d) $\vec{L} = mv\left[\frac{R}{\sqrt{2}} - a\right]\hat{k}$ when the particle is moving from C to D.

57. A particle of mass 2 kg is on a smooth horizontal table and moves in a circular path of radius 0.6 m. The height of the table from the ground is 0.8 m. If the angular speed of the particle is 12 rad s^{-1}, the magnitude of its angular momentum about a point on the ground right under the centre of the circle is : **[Online April 11, 2015]**

(a) 14.4 kg m^2s^{-1} (b) 8.64 kg m^2s^{-1}
(c) 20.16 kg m^2s^{-1} (d) 11.52 kg m^2s^{-1}

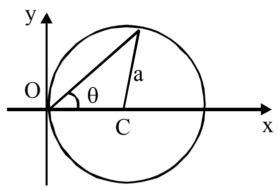
58. A bob of mass m attached to an inextensible string of length l is suspended from a vertical support. The bob rotates in a horizontal circle with an angular speed ω rad/s about the vertical. About the point of suspension: **[2014]**

(a) angular momentum is conserved.
(b) angular momentum changes in magnitude but not in direction.
(c) angular momentum changes in direction but not in magnitude.
(d) angular momentum changes both in direction and magnitude.

59. A ball of mass 160 g is thrown up at an angle of 60° to the horizontal at a speed of 10 ms^{-1}. The angular momentum of the ball at the highest point of the trajectory with respect to the point from which the ball is thrown is nearly ($g = 10\ ms^{-2}$) **[Online April 19, 2014]**

(a) 1.73 kg m^2/s (b) 3.0 kg m^2/s
(c) 3.46 kg m^2/s (d) 6.0 kg m^2/s

60. A particle is moving in a circular path of radius a, with a constant velocity v as shown in the figure. The centre of circle is marked by 'C'. The angular momentum from the origin O can be written as: **[Online April 12, 2014]**



(a) va (1 + cos 2θ) (b) va (1 + cos θ)
(c) va cos 2θ (d) va

61. A particle of mass 2 kg is moving such that at time t, its position, in meter, is given by $\vec{r}(t) = 5\hat{i} - 2t^2\hat{j}$. The angular momentum of the particle at $t = 2s$ about the origin in kg m^{-2} s^{-1} is : **[Online April 23, 2013]**

(a) $-80\hat{k}$ (b) $(10\hat{i} - 16\hat{j})$
(c) $-40\hat{k}$ (d) $40\hat{k}$

62. A bullet of mass 10 g and speed 500 m/s is fired into a door and gets embedded exactly at the centre of the door. The door is 1.0 m wide and weighs 12 kg. It is hinged at one end and rotates about a vertical axis practically without friction. The angular speed of the door just after the bullet embeds into it will be : **[Online April 9, 2013]**

(a) 6.25 rad/sec (b) 0.625 rad/sec
(c) 3.35 rad/sec (d) 0.335 rad/sec

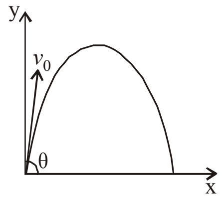
63. A stone of mass m, tied to the end of a string, is whirled around in a circle on a horizontal frictionless table. The length of the string is reduced gradually keeping the angular momentum of the stone about the centre of the circle constant. Then, the tension in the string is given by $T = Ar^n$, where A is a constant, r is the instantaneous radius of the circle. The value of n is equal to **[Online May 26, 2012]**

(a) – 1 (b) – 2 (c) – 4 (d) – 3

64. A thin horizontal circular disc is rotating about a vertical axis passing through its centre. An insect is at rest at a

point near the rim of the disc. The insect now moves along a diameter of the disc to reach its other end. During the journey of the insect, the angular speed of the disc. **[2011]**
(a) continuously decreases
(b) continuously increases
(c) first increases and then decreases
(d) remains unchanged

65. A small particle of mass m is projected at an angle θ with the x-axis with an initial velocity v_0 in the x-y plane as shown in the figure. At a time $t < \frac{v_0 \sin\theta}{g}$, the angular momentum of the particle is **[2010]**



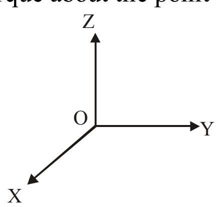
(a) $-mg\, v_0 t^2 \cos\theta\, \hat{j}$ (b) $mg\, v_0 t \cos\theta \hat{k}$
(c) $-\frac{1}{2} mg v_0 t^2 \cos\theta\, \hat{k}$ (d) $\frac{1}{2} mg v_0 t^2 \cos\theta\, \hat{i}$

where $\hat{i}, \hat{j}$ and $\hat{k}$ are unit vectors along *x, y* and *z*-axis respectively.

66. Angular momentum of the particle rotating with a central force is constant due to **[2007]**
(a) constant torque
(b) constant force
(c) constant linear momentum
(d) zero torque

67. A thin circular ring of mass *m* and radius *R* is rotating about its axis with a constant angular velocity ω. Two objects each of mass *M* are attached gently to the opposite ends of a diameter of the ring. The ring now rotates with an angular velocity ω' = **[2006]**
(a) $\frac{\omega(m+2M)}{m}$ (b) $\frac{\omega(m-2M)}{(m+2M)}$
(c) $\frac{\omega m}{(m+M)}$ (d) $\frac{\omega m}{(m+2M)}$

68. A force of $-F\hat{k}$ acts on *O*, the origin of the coordinate system. The torque about the point (1, –1) is **[2006]**

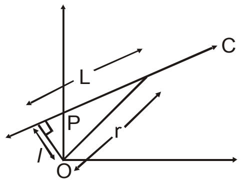


(a) $F(\hat{i}-\hat{j})$ (b) $-F(\hat{i}+\hat{j})$
(c) $F(\hat{i}+\hat{j})$ (d) $-F(\hat{i}-\hat{j})$

69. A solid sphere is rotating in free space. If the radius of the sphere is increased keeping mass same, which one of the following will not be affected ? **[2004]**
(a) Angular velocity
(b) Angular momentum
(c) Moment of inertia
(d) Rotational kinetic energy

70. Let $\vec{F}$ be the force acting on a particle having position vector $\vec{r}$, and $\vec{T}$ be the torque of this force about the origin. Then **[2003]**
(a) $\vec{r}.\vec{T} = 0$ and $\vec{F}.\vec{T} \neq 0$
(b) $\vec{r}.\vec{T} \neq 0$ and $\vec{F}.\vec{T} = 0$
(c) $\vec{r}.\vec{T} \neq 0$ and $\vec{F}.\vec{T} \neq 0$
(d) $\vec{r}.\vec{T} = 0$ and $\vec{F}.\vec{T} = 0$

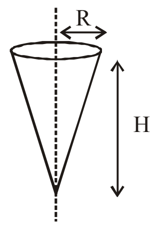
71. A particle of mass *m* moves along line *PC* with velocity *v* as shown. What is the angular momentum of the particle about *P*? **[2002]**



(a) *mvL* (b) *mvl* (c) *mvr* (d) zero.

TOPIC 4 Moment of Inertia and Rotational K.E.

72. Shown in the figure is a hollow icecream cone (it is open at the top). If its mass is *M*, radius of its top, *R* and height, *H*, then its moment of inertia about its axis is : **[Sep. 06, 2020 (I)]**



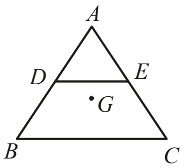
(a) $\frac{MR^2}{2}$ (b) $\frac{M(R^2+H^2)}{4}$
(c) $\frac{MH^2}{3}$ (d) $\frac{MR^2}{3}$

73. The linear mass density of a thin rod AB of length L varies from A to B as $\lambda(x) = \lambda_0\left(1+\frac{x}{L}\right)$, where *x* is the distance from A. If M is the mass of the rod then its moment of inertia about an axis passing through A and perpendicular to the rod is : **[Sep. 06, 2020 (II)]**
(a) $\frac{5}{12}ML^2$ (b) $\frac{7}{18}ML^2$
(c) $\frac{2}{5}ML^2$ (d) $\frac{3}{7}ML^2$

74. A wheel is rotating freely with an angular speed ω on a shaft. The moment of inertia of the wheel is I and the moment of inertia of the shaft is negligible. Another wheel of moment of inertia 3I initially at rest is suddenly coupled to the same shaft. The resultant fractional loss in the kinetic energy of the system is : **[Sep. 05, 2020 (I)]**

(a) $\frac{5}{6}$ (b) $\frac{1}{4}$
(c) 0 (d) $\frac{3}{4}$

75. *ABC* is a plane lamina of the shape of an equilateral triangle. *D*, *E* are mid points of *AB*, *AC* and *G* is the centroid of the lamina. Moment of inertia of the lamina about an axis passing through *G* and perpendicular to the plane *ABC* is I_0. If part *ADE* is removed, the moment of inertia of the remaining part about the same axis is $\frac{NI_0}{16}$ where *N* is an integer. Value of *N* is _______. **[NA Sep. 04, 2020 (I)]**

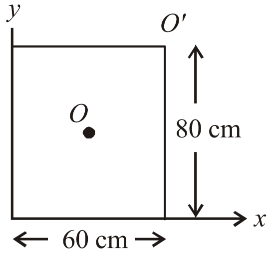


76. A circular disc of mass *M* and radius *R* is rotating about its axis with angular speed ω_1. If another stationary disc having radius $\frac{R}{2}$ and same mass *M* is dropped co-axially on to the rotating disc. Gradually both discs attain constant angular speed ω_2. The energy lost in the process is *p*% of the initial energy. Value of *p* is ___________. **[NA Sep. 04, 2020 (I)]**

77. Consider two uniform discs of the same thickness and different radii $R_1 = R$ and $R_2 = \alpha R$ made of the same material. If the ratio of their moments of inertia I_1 and I_2, respectively, about their axes is $I_1 : I_2 = 1 : 16$ then the value of α is : **[Sep. 04, 2020 (II)]**

(a) $2\sqrt{2}$ (b) $\sqrt{2}$
(c) 2 (d) 4

78.



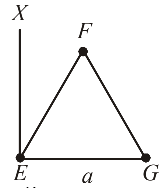
For a uniform rectangular sheet shown in the figure, the ratio of moments of inertia about the axes perpendicular to the sheet and passing through *O* (the centre of mass) and *O'* (corner point) is : **[Sep. 04, 2020 (II)]**

(a) 2/3 (b) 1/4
(c) 1/8 (d) 1/2

79. Moment of inertia of a cylinder of mass *M*, length *L* and radius *R* about an axis passing through its centre and perpendicular to the axis of the cylinder is $I = M\left(\frac{R^2}{4} + \frac{L^2}{12}\right)$. If such a cylinder is to be made for a given mass of a material, the ratio *L/R* for it to have minimum possible *I* is : **[Sep. 03, 2020 (I)]**

(a) $\frac{2}{3}$ (b) $\frac{3}{2}$
(c) $\sqrt{\frac{3}{2}}$ (d) $\sqrt{\frac{2}{3}}$

80. An massless equilateral triangle *EFG* of side '*a*' (As shown in figure) has three particles of mass *m* situated at its vertices. The moment of inertia of the system about the line *EX* perpendicular to *EG* in the plane of *EFG* is $\frac{N}{20}ma^2$ where *N* is an integer. The value of *N* is _________. **[Sep. 03, 2020 (II)]**

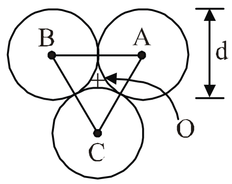


81. Two uniform circular discs are rotating independently in the same direction around their common axis passing through their centres. The moment of inertia and angular velocity of the first disc are 0.1 kg-m^2 and 10 rad s^{-1} respectively while those for the second one are 0.2 kg-m^2 and 5 rad s^{-1} respectively. At some instant they get stuck together and start rotating as a single system about their common axis with some angular speed. The kinetic energy of the combined system is : **[Sep. 02, 2020 (II)]**

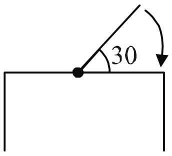
(a) $\frac{10}{3}$ J (b) $\frac{20}{3}$ J
(c) $\frac{5}{3}$ J (d) $\frac{2}{3}$ J

82. Three solid spheres each of mass *m* and diameter *d* are stuck together such that the lines connecting the centres form an equilateral triangle of side of length *d*. The ratio $\frac{I_0}{I_A}$ of moment of inertia I_0 of the system about an axis passing the centroid and about center of any of the spheres I_A and perpendicular to the plane of the triangle is: **[9 Jan. 2020 I]**

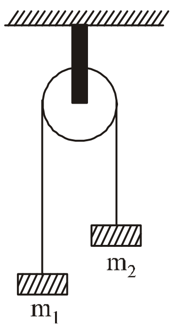
(a) $\frac{13}{23}$
(b) $\frac{15}{13}$
(c) $\frac{23}{13}$
(d) $\frac{13}{15}$



83. One end of a straight uniform 1 m long bar is pivoted on horizontal table. It is released from rest when it makes an angle 30° from the horizontal (see figure). Its angular speed when it hits the table is given as $\sqrt{n}s^{-1}$, where n is an integer. The value of n is ________ . **[9 Jan. 2020 I]**



84. A uniformly thick wheel with moment of inertia I and radius R is free to rotate about its centre of mass (see fig). A massless string is wrapped over its rim and two blocks of masses m_1 and m_2 ($m_1 > m_2$) are attached to the ends of the string. The system is released from rest. The angular speed of the wheel when m_1 descents by a distance h is: **[9 Jan. 2020 II]**



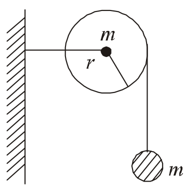
(a) $\left[\dfrac{2(m_1-m_2)gh}{(m_1+m_2)R^2+1}\right]^{1/2}$

(b) $\left[\dfrac{2(m_1+m_2)gh}{(m_1+m_2)R^2+1}\right]^{1/2}$

(c) $\left[\dfrac{(m_1-m_2)}{(m_1+m_2)R^2+1}\right]^{1/2} gh$

(d) $\left[\dfrac{m_1+m_2}{(m_1+m_2)R^2+1}\right]^{1/2} gh$

85. As shown in the figure, a bob of mass m is tied by a massless string whose other end portion is wound on a fly wheel (disc) of radius r and mass m. When released from rest the bob starts falling vertically. When it has covered a distance of h, the angular speed of the wheel will be: **[7 Jan. 2020 I]**



(a) $\dfrac{1}{r}\sqrt{\dfrac{4gh}{3}}$ (b) $r\sqrt{\dfrac{3}{2gh}}$

(c) $\dfrac{1}{r}\sqrt{\dfrac{2gh}{3}}$ (d) $r\sqrt{\dfrac{3}{4gh}}$

86. The radius of gyration of a uniform rod of length l, about an axis passing through a point $\dfrac{l}{4}$ away from the centre of the rod, and perpendicular to it, is: **[7 Jan. 2020 I]**

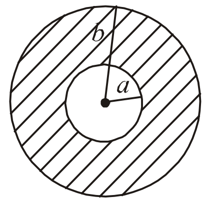
(a) $\dfrac{1}{4}l$ (b) $\dfrac{1}{8}l$ (c) $\sqrt{\dfrac{7}{48}}l$ (d) $\sqrt{\dfrac{3}{8}}l$

87. Mass per unit area of a circular disc of radius a depends on the distance r from its centre as $\sigma(r) = A + Br$. The moment of inertia of the disc about the axis, perpendicular to the plane and passing through its centre is: **[7 Jan. 2020 II]**

(a) $2\pi a^4\left(\dfrac{A}{4}+\dfrac{aB}{5}\right)$ (b) $2\pi a^4\left(\dfrac{aA}{4}+\dfrac{B}{5}\right)$

(c) $\pi a^4\left(\dfrac{A}{4}+\dfrac{aB}{5}\right)$ (d) $2\pi a^4\left(\dfrac{A}{4}+\dfrac{B}{5}\right)$

88. A circular disc of radius b has a hole of radius a at its centre (see figure). If the mass per unit area of the disc varies as $\left(\dfrac{\sigma_0}{r}\right)$, then the radius of gyration of the disc about its axis passing through the centre is : **[12 Apr. 2019 I]**



(a) $\sqrt{\dfrac{a^2+b^2+ab}{2}}$ (b) $\dfrac{a+b}{2}$

(c) $\sqrt{\dfrac{a^2+b^2+ab}{3}}$ (d) $\dfrac{a+b}{3}$

89. Two coaxial discs, having moments of inertia I_1 and $\dfrac{I_1}{2}$, are rotating with respective angular velocities ω_1 and $\dfrac{\omega_1}{2}$, about their common axis. They are brought in contact with each other and thereafter they rotate with a common angular velocity. If E_f and E_i are the final and initial total energies, then $(E_f - E_i)$ is : **[10 Apr. 2019 I]**

(a) $-\dfrac{I_1\omega_1^2}{12}$ (b) $\dfrac{I_1\omega_1^2}{6}$ (c) $\dfrac{3}{8}I_1\omega_1^2$ (d) $-\dfrac{I_1\omega_1^2}{24}$

90. A thin disc of mass M and radius R has mass per unit area $\sigma(r) = kr^2$ where r is the distance from its centre. Its moment of inertia about an axis going through its centre of mass and perpendicular to its plane is : **[10 Apr. 2019 I]**

(a) $\dfrac{MR^2}{3}$ (b) $\dfrac{2MR^2}{3}$

(c) $\dfrac{MR^2}{6}$ (d) $\dfrac{MR^2}{2}$

91. A solid sphere of mass M and radius R is divided into two unequal parts. The first part has a mass of $\dfrac{7M}{8}$ and is converted into a uniform disc of radius 2R. The second part is converted into a uniform solid sphere. Let I_1 be the moment of inertia of the new sphere about its axis. The ratio I_1/I_2 is given by : **[10 Apr. 2019 II]**

(a) 185 (b) 140 (c) 285 (d) 65

92. A stationary horizontal disc is free to rotate about its axis. When a torque is applied on it, its kinetic energy as a function of θ, where θ is the angle by which it has rotated, is given as $k\theta^2$. If its moment of inertia is I then the angular acceleration of the disc is: **[9 April 2019 I]**

(a) $\frac{k}{4I}\theta$ (b) $\frac{k}{I}\theta$ (c) $\frac{k}{2I}\theta$ (d) $\frac{2k}{I}\theta$

93. Moment of inertia of a body about a given axis is 1.5 kg m^2. Initially the body is at rest. In order to produce a rotational kinetic energy of 1200 J, the angular acceleration of 20 rad/s^2 must be applied about the axis for a duration of: **[9 Apr. 2019 II]**

(a) 2.5s (b) 2s (c) 5s (d) 3s

94. A thin smooth rod of length L and mass M is rotating freely with angular speed ω_0 about an axis perpendicular to the rod and passing through its center. Two beads of mass m and negligible size are at the center of the rod initially. The beads are free to slide along the rod. The angular speed of the system, when the beads reach the opposite ends of the rod, will be: **[9 Apr. 2019 II]**

(a) $\frac{M\omega_0}{M+m}$ (b) $\frac{M\omega_0}{M+3m}$

(c) $\frac{M\omega_0}{M+6m}$ (d) $\frac{M\omega_0}{M+2m}$

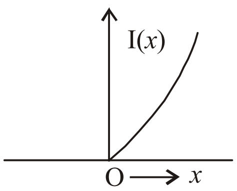
95. A thin circular plate of mass M and radius R has its density varying as $\rho(r) = \rho_0 r$ with ρ_0 as constant and r is the distance from its center. The moment of Inertia of the circular plate about an axis perpendicular to the plate and passing through its edge is $I = a\,MR^2$. The value of the coefficient a is: **[8 April 2019 I]**

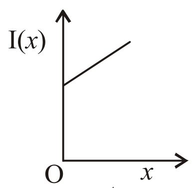
(a) $1/2$ (b) $3/5$ (c) $8/5$ (d) $3/2$

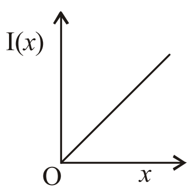
96. Let the moment of inertia of a hollow cylinder of length 30 cm (inner radius 10 cm and outer radius 20 cm), about its axis be 1. The radius of a thin cylinder of the same mass such that its moment of inertia about its axis is also I, is: **[12 Jan. 2019 I]**

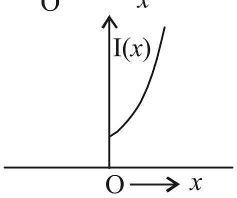
(a) 12 cm (b) 16 cm
(c) 14 cm (d) 18 cm

97. The moment of inertia of a solid sphere, about an axis parallel to its diameter and at a distance of x from it, is 'I(x)'. Which one of the graphs represents the variation of I(x) with x correctly? **[12 Jan. 2019 II]**

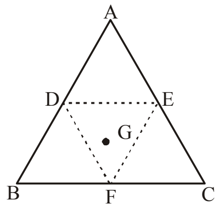
(a)


(b)


(c)


(d)


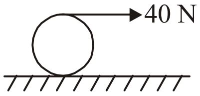
98. An equilateral triangle ABC is cut from a thin solid sheet of wood. (See figure) D, E and F are the mid-points of its sides as shown and G is the centre of the triangle. The moment of inertia of the triangle about an axis passing through G and perpendicular to the plane of the triangle is I_0. If the smaller triangle DEF is removed from ABC, the moment of inertia of the remaining figure about the same axis is I. Then : **[11 Jan. 2019 I]**



(a) $I = \frac{15}{16}I_0$ (b) $I = \frac{3}{4}I_0$

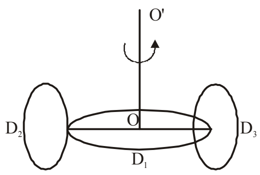
(c) $I = \frac{9}{16}I_0$ (d) $I = \frac{I_0}{4}$

99. a string is wound around a hollow cylinder of mass 5 kg and radius 0.5 m. If the string is now pulled with a horizontal force of 40 N, and the cylinder is rolling without slipping on a horizontal surface (see figure), then the angular acceleration of the cylinder will be (Neglect the mass and thickness of the string) **[11 Jan. 2019 II]**



(a) 20 rad/s^2 (b) 16 rad/s^2
(c) 12 rad/s^2 (d) 10 rad/s^2

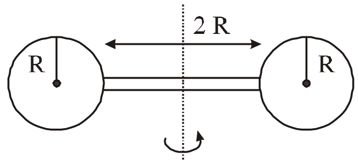
100. A circular disc D_1 of mass M and radius R has two identical discs D_2 and D_3 of the same mass M and radius R attached rigidly at its opposite ends (see figure). The moment of inertia of the system about the axis OO', passing through the centre of D_1, as shown in the figure, will be : **[11 Jan. 2019 II]**



(a) MR^2 (b) $3MR^2$

(c) $\frac{4}{5}MR^2$ (d) $\frac{2}{3}MR^2$

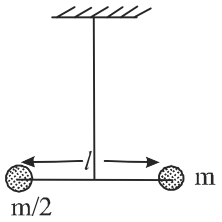
101. Two identical spherical balls of mass M and radius R each are stuck on two ends of a rod of length 2R and mass M (see figure). The moment of inertia of the system about the axis passing perpendicularly through the centre of the rod is: **[10 Jan. 2019 II]**



(a) $\frac{137}{15}MR^2$ (b) $\frac{17}{15}MR^2$

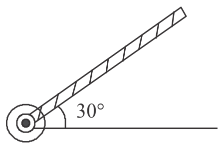
(c) $\frac{209}{15}MR^2$ (d) $\frac{152}{15}MR^2$

102. Two masses m and $\frac{m}{2}$ are connected at the two ends of a massless rigid rod of length l. The rod is suspended by a thin wire of torsional constant k at the centre of mass of the rod-mass system (see figure). Because of torsional constant k, the restoring toruque is $\tau = k\theta$ for angular displacement θ. If the rod is rotated by θ_0 and released, the tension in it when it passes through its mean position will be: **[9 Jan. 2019 I]**



(a) $\frac{3k\theta_0^2}{l}$ (b) $\frac{2k\theta_0^2}{l}$ (c) $\frac{k\theta_0^2}{l}$ (d) $\frac{k\theta_0^2}{2l}$

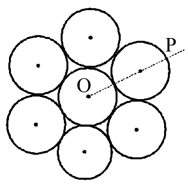
103. A rod of length 50 cm is pivoted at one end. It is raised such that if makes an angle of 30° from the horizontal as shown and released from rest. Its angular speed when it passes through the horizontal (in rads^{-1}) will be (g = 10 ms^{-2}) **[9 Jan. 2019 II]**



(a) $\sqrt{\frac{30}{7}}$ (b) $\sqrt{30}$

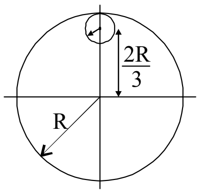
(c) $\sqrt{\frac{20}{3}}$ (d) $\sqrt{\frac{30}{2}}$

104. Seven identical circular planar disks, each of mass M and radius R are welded symmetrically as shown. The moment of inertia of the arrangement about the axis normal to the plane and passing through the point P is: **[2018]**



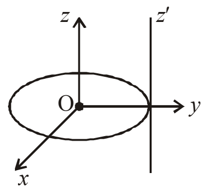
(a) $\frac{19}{2}MR^2$ (b) $\frac{55}{2}MR^2$ (c) $\frac{73}{2}MR^2$ (d) $\frac{181}{2}MR^2$

105. From a uniform circular disc of radius R and mass 9 M, a small disc of radius $\frac{R}{3}$ is removed as shown in the figure. The moment of inertia of the remaining disc about an axis perpendicular to the plane of the disc and passing, through centre of disc is : **[2018]**



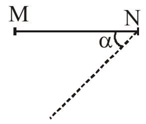
(a) 4 MR^2 (b) $\frac{40}{9}$MR^2 (c) 10 MR^2 (d) $\frac{37}{9}$MR^2

106. A thin circular disk is in the xy plane as shown in the figure. The ratio of its moment of inertia about z and z′ axes will be **[Online April 16, 2018]**



(a) 1 : 2 (b) 1 : 4 (c) 1 : 3 (d) 1 : 5

107. A thin rod MN, free to rotate in the vertical plane about the fixed end N, is held horizontal. When the end M is released the speed of this end, when the rod makes an angle α with the horizontal, will be proportional to: (see figure) **[Online April 15, 2018]**



(a) $\sqrt{\cos\alpha}$ (b) $\cos\alpha$

(c) $\sin\alpha$ (d) $\sqrt{\sin\alpha}$

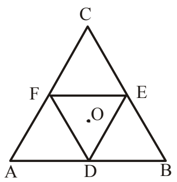
108. The moment of inertia of a uniform cylinder of length ℓ and radius R about its perpendicular bisector is I. What is the ratio ℓ/R such that the moment of inertia is minimum ? **[2017]**

(a) 1 (b) $\frac{3}{\sqrt{2}}$ (c) $\sqrt{\frac{3}{2}}$ (d) $\frac{\sqrt{3}}{2}$

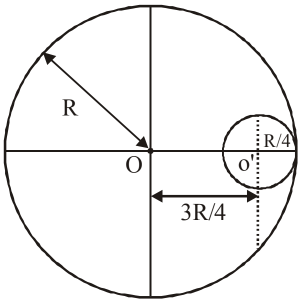
109. Moment of inertia of an equilateral triangular lamina ABC, about the axis passing through its centre O and perpendicular to its plane is I_o as shown in the figure. A cavity DEF is cut out from the lamina, where D, E, F are the mid points of the sides. Moment of inertia of the remaining part of lamina about the same axis is : **[Online April 8, 2017]**

(a) $\frac{7}{8}I_o$ (b) $\frac{15}{16}I_o$

(c) $\frac{3I_o}{4}$ (d) $\frac{31I_o}{32}$



110. A circular hole of radius $\frac{R}{4}$ is made in a thin uniform disc having mass M and radius R, as shown in figure. The moment of inertia of the remaining portion of the disc about an axis passing through the point O and perpendicular to the plane of the disc is : **[Online April 9, 2017]**



(a) $\frac{219MR^2}{256}$

(b) $\frac{237MR^2}{512}$

(c) $\frac{19MR^2}{512}$

(d) $\frac{197MR^2}{256}$

111. From a solid sphere of mass M and radius R a cube of maximum possible volume is cut. Moment of inertia of cube about an axis passing through its center and perpendicular to one of its faces is : **[2015]**

(a) $\frac{4MR^2}{9\sqrt{3}\pi}$ (b) $\frac{4MR^2}{3\sqrt{3}\pi}$

(c) $\frac{MR^2}{32\sqrt{2}\pi}$ (d) $\frac{MR^2}{16\sqrt{2}\pi}$

112. Consider a thin uniform square sheet made of a rigid material. If its side is '*a*' mass *m* and moment of inertia I about one of its diagonals, then :**[Online April 10, 2015]**

(a) $I > \frac{ma^2}{12}$ (b) $\frac{ma^2}{24} < I < \frac{ma^2}{12}$

(c) $I = \frac{ma^2}{24}$ (d) $I = \frac{ma^2}{12}$

113. A ring of mass M and radius R is rotating about its axis with angular velocity ω. Two identical bodies each of mass m are now gently attached at the two ends of a diameter of the ring. Because of this, the kinetic energy loss will be: **[Online April 25, 2013]**

(a) $\frac{m(M+2m)}{M}\omega^2 R^2$ (b) $\frac{Mm}{(M+m)}\omega^2 R^2$

(c) $\frac{Mm}{(M+2m)}\omega^2 R^2$ (d) $\frac{(M+m)M}{(M+2m)}\omega^2 R^2$

114. This question has Statement 1and Statement 2. Of the four choices given after the Statements, choose the one that best describes the two Statements.

Statement 1: When moment of inertia *I* of a body rotating about an axis with angular speed ω increases, its angular momentum *L* is unchanged but the kinetic energy *K* increases if there is no torque applied on it.

Statement 2: $L = I\omega$, kinetic energy of rotation $= \frac{1}{2}I\omega^2$ **[Online May 12, 2012]**

(a) Statement 1 is true, Statement 2 is true, Statement 2 is not the correct explanation of Statement 1.

(b) Statement 1 is false, Statement 2 is true.

(c) Statement 1 is true, Statement 2 is true, Statement 2 is correct explanation of the Statement 1.

(d) Statement 1 is true, Statement 2 is false.

115. A solid sphere having mass m and radius r rolls down an inclined plane. Then its kinetic energy is **[Online May 7, 2012]**

(a) $\frac{5}{7}$ rotational and $\frac{2}{7}$ translational

(b) $\frac{2}{7}$ rotational and $\frac{5}{7}$ translational

(c) $\frac{2}{5}$ rotational and $\frac{3}{5}$ translational

(d) $\frac{1}{2}$ rotational and $\frac{1}{2}$ translational

116. A circular hole of diameter R is cut from a disc of mass *M* and radius *R*; the circumference of the cut passes through the centre of the disc. The moment of inertia of the remaining portion of the disc about an axis perpendicular to the disc and passing through its centre is **[Online May 7, 2012]**

(a) $\left(\frac{15}{32}\right)MR^2$ (b) $\left(\frac{1}{8}\right)MR^2$

(c) $\left(\frac{3}{8}\right)MR^2$ (d) $\left(\frac{13}{32}\right)MR^2$

117. A mass *m* hangs with the help of a string wrapped around a pulley on a frictionless bearing. The pulley has mass *m* and radius *R*. Assuming pulley to be a perfect uniform circular disc, the acceleration of the mass *m*, if the string does not slip on the pulley, is: **[2011]**

(a) g (b) $\frac{2}{3}g$ (c) $\frac{g}{3}$ (d) $\frac{3}{2}g$

118. A pulley of radius 2 m is rotated about its axis by a force $F = (20t - 5t^2)$ newton (where *t* is measured in seconds) applied tangentially. If the moment of inertia of the pulley about its axis of rotation is 10 kg-m^2 the number of rotations made by the pulley before its direction of motion is reversed, is: **[2011]**

(a) more than 3 but less than 6

(b) more than 6 but less than 9

(c) more than 9

(d) less than 3

119. A thin uniform rod of length *l* and mass *m* is swinging freely about a horizontal axis passing through its end. Its maximum angular speed is ω. Its centre of mass rises to a maximum height of **[2009]**

(a) $\frac{1}{6}\frac{l\omega}{g}$ (b) $\frac{1}{2}\frac{l^2\omega^2}{g}$

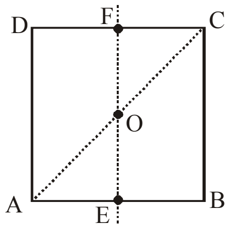
(c) $\frac{1}{6}\frac{l^2\omega^2}{g}$ (d) $\frac{1}{3}\frac{l^2\omega^2}{g}$

120. Consider a uniform square plate of side '*a*' and mass '*M*'. The moment of inertia of this plate about an axis perpendicular to its plane and passing through one of its corners is **[2008]**

(a) $\frac{5}{6}Ma^2$ (b) $\frac{1}{12}Ma^2$

(c) $\frac{7}{12}Ma^2$ (d) $\frac{2}{3}Ma^2$

121. For the given uniform square lamina $ABCD$, whose centre is O, **[2007]**



(a) $I_{AC} = \sqrt{2}\, I_{EF}$ (b) $\sqrt{2} I_{AC} = I_{EF}$

(c) $I_{AD} = 3I_{EF}$ (d) $I_{AC} = I_{EF}$

122. Four point masses, each of value m, are placed at the corners of a square $ABCD$ of side ℓ. The moment of inertia of this system about an axis passing through A and parallel to BD is **[2006]**

(a) $2m\ell^2$ (b) $\sqrt{3}m\ell^2$

(c) $3m\ell^2$ (d) $m\ell^2$

123. The moment of inertia of a uniform semicircular disc of mass M and radius r about a line perpendicular to the plane of the disc through the centre is **[2005]**

(a) $\frac{2}{5}Mr^2$ (b) $\frac{1}{4}Mr$

(c) $\frac{1}{2}Mr^2$ (d) Mr^2

124. One solid sphere A and another hollow sphere B are of same mass and same outer radii. Their moment of inertia about their diameters are respectively I_A and I_B Such that **[2004]**

(a) $I_A < I_B$ (b) $I_A > I_B$

(c) $I_A = I_B$ (d) $\frac{I_A}{I_B} = \frac{d_A}{d_B}$

where d_A and d_B are their densities.

125. A circular disc X of radius R is made from an iron plate of thickness t, and another disc Y of radius $4R$ is made from an iron plate of thickness $\frac{t}{4}$. Then the relation between the moment of inertia I_X and I_I is **[2003]**

(a) $I_Y = 32 I_X$ (b) $I_Y = 16 I_X$

(c) $I_Y = I_X$ (d) $I_Y = 64 I_X$

126. A particle performing uniform circular motion has angular frequency is doubled & its kinetic energy halved, then the new angular momentum is **[2003]**

(a) $\frac{L}{4}$ (b) $2L$

(c) $4L$ (d) $\frac{L}{2}$

127. Moment of inertia of a circular wire of mass M and radius R about its diameter is **[2002]**

(a) $MR^2/2$ (b) MR^2 (c) $2MR^2$ (d) $MR^2/4$

128. Initial angular velocity of a circular disc of mass M is ω_1. Then two small spheres of mass m are attached gently to diametrically opposite points on the edge of the disc. What is the final angular velocity of the disc? **[2002]**

(a) $\left(\frac{M+m}{M}\right)\omega_1$ (b) $\left(\frac{M+m}{m}\right)\omega_1$

(c) $\left(\frac{M}{M+4m}\right)\omega_1$ (d) $\left(\frac{M}{M+2m}\right)\omega_1$

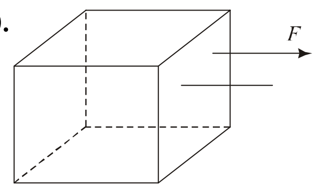
TOPIC 5 Rolling Motion

129. A uniform sphere of mass 500 g rolls without slipping on a plane horizontal surface with its centre moving at a speed of 5.00 cm/s. Its kinetic energy is: **[8 Jan. 2020 II]**

(a) 8.75×10^{-4} J (b) 8.75×10^{-3} J

(c) 6.25×10^{-4} J (d) 1.13×10^{-3} J

130.



Consider a uniform cubical box of side a on a rough floor that is to be moved by applying minimum possible force F at a point b above its centre of mass (see figure). If the coefficient of friction is $\mu = 0.4$, the maximum possible value of $100 \times \frac{b}{a}$ for box not to topple before moving is ________. **[NA 7 Jan. 2020 II]**

131. A solid sphere and solid cylinder of identical radii approach an incline with the same linear velocity (see figure). Both roll without slipping all throughout. The two climb maximum heights h_{sph} and h_{cyl} on the incline. The ratio $\frac{h_{sph}}{h_{cyl}}$ is given by : **[8 Apr. 2019 II]**

(a) $\frac{2}{\sqrt{5}}$ (b) 1 (c) $\frac{14}{15}$ (d) $\frac{4}{5}$

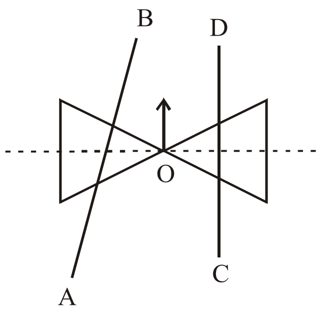
132. The following bodies are made to roll up (without slipping) the same inclined plane from a horizontal plane: (i) a ring of radius R, (ii) a solid cylinder of radius $\frac{R}{2}$ and (iii) a solid sphere of radius $\frac{R}{4}$. If, in each case, the speed of the center of mass at the bottom of the incline is same, the ratio of the maximum heights they climb is: **[9 April 2019 I]**

(a) 4 : 3 : 2 (b) 10 : 15 : 7
(c) 14:15:20 (d) 2:3:4

133. A homogeneous solid cylindrical roller of radius R and mass M is pulled on a cricket pitch by a horizontal force. Assuming rolling without slipping, angular acceleration of the cylinder is: **[10 Jan. 2019 I]**

(a) $\frac{3F}{2mR}$ (b) $\frac{F}{3mR}$
(c) $\frac{F}{2mR}$ (d) $\frac{2F}{3mR}$

134. A roller is made by joining together two cones at their vertices O. It is kept on two rails AB and CD, which are placed asymmetrically (see figure), with its axis perpendicular to CD and its centre O at the centre of line joining AB and Cd (see figure). It is given a light push so that it starts rolling with its centre O moving parallel to CD in the direction shown. As it moves, the roller will tend to: **[2016]**

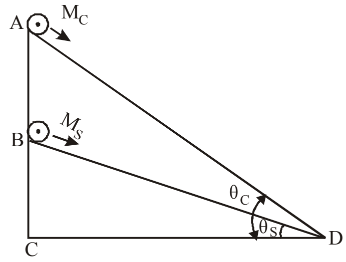


(a) go straight.
(b) turn left and right alternately.
(c) turn left.
(d) turn right.

135. A uniform solid cylindrical roller of mass 'm' is being pulled on a horizontal surface with force F parallel to the surface and applied at its centre. If the acceleration of the cylinder is 'a' and it is rolling without slipping then the value of 'F' is: **[Online April 10, 2015]**

(a) ma (b) $\frac{5}{3}$ma
(c) $\frac{3}{2}$ma (d) 2 ma

136. A cylinder of mass M_c and sphere of mass M_s are placed at points A and B of two inclines, respectively (See Figure). If they roll on the incline without slipping such that their accelerations are the same, then the ratio $\frac{\sin\theta_c}{\sin\theta_s}$ is: **[Online April 9, 2014]**

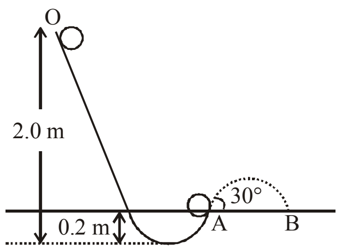


(a) $\sqrt{\frac{8}{7}}$ (b) $\sqrt{\frac{15}{14}}$
(c) $\frac{8}{7}$ (d) $\frac{15}{14}$

137. A loop of radius r and mass m rotating with an angular velocity ω_0 is placed on a rough horizontal surface. The initial velocity of the centre of the hoop is zero. What will be the velocity of the centre of the hoop when it ceases to slip ? **[2013]**

(a) $\frac{r\omega_0}{4}$ (b) $\frac{r\omega_0}{3}$
(c) $\frac{r\omega_0}{2}$ (d) $r\omega_0$

138. A tennis ball (treated as hollow spherical shell) starting from O rolls down a hill. At point A the ball becomes air borne leaving at an angle of 30° with the horizontal. The ball strikes the ground at B. What is the value of the distance AB ?
(Moment of inertia of a spherical shell of mass m and radius R about its diameter $= \frac{2}{3}mR^2$) **[Online April 22, 2013]**

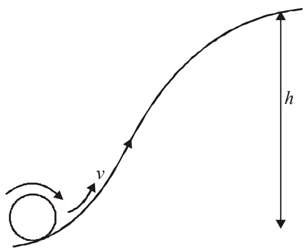


(a) 1.87 m (b) 2.08 m
(c) 1.57 m (d) 1.77 m

139. A thick-walled hollow sphere has outside radius R_0. It rolls down an incline without slipping and its speed at the bottom is v_0. Now the incline is waxed, so that it is practically frictionless and the sphere is observed to slide down (without any rolling). Its speed at the bottom is observed to be $5v_0/4$. The radius of gyration of the hollow sphere about an axis through its centre is **[Online May 26, 2012]**

(a) $3R_0/2$ (b) $3R_0/4$
(c) $9R_0/16$ (d) $3R_0$

140. A solid sphere is rolling on a surface as shown in figure, with a translational velocity v ms^{-1}. If it is to climb the inclined surface continuing to roll without slipping, then minimum velocity for this to happen is **[Online May 12, 2012]**



(a) $\sqrt{2gh}$ (b) $\sqrt{\frac{7}{5}gh}$

(c) $\sqrt{\frac{7}{2}gh}$ (d) $\sqrt{\frac{10}{7}gh}$

141. A round uniform body of radius R, mass M and moment of inertia I rolls down (without slipping) an inclined plane making an angle θ with the horizontal. Then its acceleration is **[2007]**

(a) $\dfrac{g\sin\theta}{1-MR^2/I}$ (b) $\dfrac{g\sin\theta}{1+I/MR^2}$

(c) $\dfrac{g\sin\theta}{1+MR^2/I}$ (d) $\dfrac{g\sin\theta}{1-I/MR^2}$

Hints & Solutions

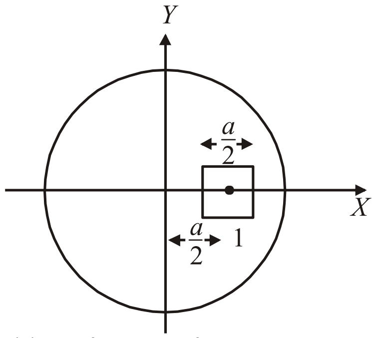
1. **(3)** Centre of mass of solid hemisphere of radius R lies at a distance $\frac{3R}{8}$ above the centre of flat side of hemisphere.

$\therefore h_{cm} = \frac{3R}{8} = \frac{3 \times 8}{8} = 3$ cm

2. **(23.00)** Let σ be the mass density of circular disc.
Original mass of the disc, $m_0 = \pi a^2 \sigma$

Removed mass, $m = \frac{a^2}{4}\sigma$

Remaining, mass, $m' = \left(\pi a^2 - \frac{a^2}{4}\right)\sigma = a^2\left(\frac{4\pi - 1}{4}\right)\sigma$



New position of centre of mass

$$X_{CM} = \frac{m_0 x_0 - mx}{m_0 - m} = \frac{\pi a^2 \times 0 - \frac{a^2}{4} \times \frac{a}{2}}{\pi a^2 - \frac{a^2}{4}}$$

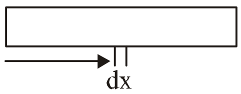
$$= \frac{-a^3/8}{\left(\pi - \frac{1}{4}\right)a^2} = \frac{-a}{2(4\pi - 1)} = \frac{-a}{8\pi - 2} = -\frac{a}{23}$$

$\therefore x = 23$

3. **(b)** Given,

Linear mass density, $\rho(x) = a + b\left(\frac{x}{L}\right)^2$

$$X_{CM} = \frac{\int x dm}{\int dm}$$



$$\int dm = \int_0^L \rho(x) dx$$

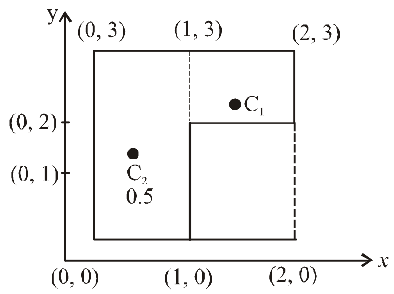
$$= \int_0^L \left[a + b\left(\frac{x}{L}\right)^2\right] dx = aL + \frac{bL}{3}$$

$$\int_0^L x dm = \int_0^L \left(ax + \frac{bx^3}{L^2}\right) dx = \left(\frac{aL^2}{2} + \frac{bL^2}{4}\right)$$

$$\therefore \quad X_{CM} = \frac{\left(\frac{aL^2}{2} + \frac{bL^2}{4}\right)}{aL + \frac{bL}{3}}$$

$$\Rightarrow X_{CM} = \frac{3L}{4}\left(\frac{2a + b}{3a + b}\right)$$

4. **(b)**



For given Lamina

$\quad\quad x \;\; y$

$m_1 = 1, C_1 = (1.5, 2.5)$

$m_2 = 3, C_2 = (0.5, 1.5)$

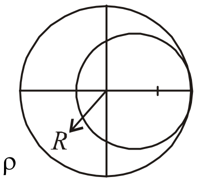
$$X_{cm} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} = \frac{1.5 + 1.5}{4} = 0.75$$

$$Y_{cm} = \frac{m_1 y_1 + m_2 y_2}{m_1 + m_2} = \frac{2.5 + 4.5}{4} = 1.75$$

$\therefore$ Coordinate of centre of mass of flag shaped lamina (0.75, 1.75)

5. **(a)** Mass of sphere = volume of sphere x density of sphere

$= \frac{4}{3}\pi R^3 \rho$



Mass of cavity $M_{cavity} = \frac{4}{3}\pi(1)^3\rho$

Mass of remaining

$$M_{(Remaining)} = \frac{4}{3}\pi R^3 \rho - \frac{4}{3}\pi(1)^3 \rho$$

Centre of mass of remaining part,

$$X_{COM} = \frac{M_1 r_1 + M_2 r_2}{M_1 + M_2}$$

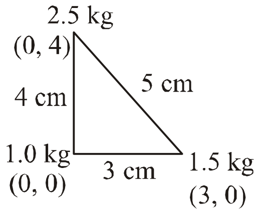
$$\Rightarrow -(2 - R) = \frac{\left[\frac{4}{3}\pi R^3 \rho\right]0 + \left[\frac{4}{3}\pi(1)^3(-\rho)\right][R - 1]}{\frac{4}{3}\pi R^3 \rho + \frac{4}{3}\pi(1)^3(-\rho)}$$

$$\Rightarrow \frac{(R-1)}{(R^3-1)} = 2-R$$

$$\Rightarrow \frac{(R-1)}{(R-1)(R^2+R+1)} = 2-R$$

$$\Rightarrow (R^2+R+1)(2-R) = 1$$

6. **(d)**



$$X_{cm} = \frac{m_1x_1+m_2x_2+m_3x_3}{m_1+m_2+m_3}$$

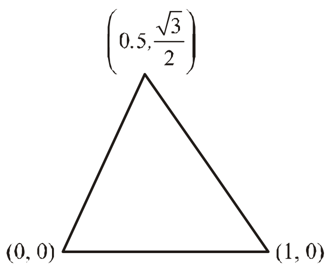
$$X_{cm} = \frac{1\times 0+1.5\times 3+2.5\times 0}{1+1.5+2.5} = \frac{1.5\times 3}{5} = 0.9cm$$

$$Y_{cm} = \frac{m_1y_1+m_2y_2+m_3y_3}{m_1+m_2+m_3}$$

$$Y_{cm} = \frac{1\times 0+1.5\times 0+2.5\times 4}{1+1.5+2.5} = \frac{2.5\times 4}{5} = 2cm$$

Hence, centre of mass of system is at point (0.9, 2)

7. **(c)** $x_{cm} = \frac{50\times 0+100\times 1+150\times 0.5}{50+100+150} = \frac{7}{12}$ m



$$y_{cm} = \frac{50\times 0+100\times 0+150\times\frac{\sqrt{3}}{2}}{50+100+150} = \frac{\sqrt{3}}{4}\text{ m}$$

8. **(a)** Acceleration of centre of mass (a_{cm}) is given by

$$\therefore\ \vec{a}_{cm} = \frac{m_1\vec{a}_1+m_2\vec{a}_2+........}{m_1+m_2+........}$$

$$= \frac{(2m)a\hat{j}+3m\times a\hat{i}+ma(-\hat{i})+4m\times a(-\hat{j})}{2m+3m+4m+m}$$

$$= \frac{2a\hat{i}-2a\hat{j}}{10} = \frac{a}{5}(\hat{i}-\hat{j})$$

9. **(d)** With respect to point θ, the CM of the cut-off portion $\left(\frac{a}{4}, \frac{b}{4}\right)$.

Using, $x_{CM} = \frac{MX-mx}{M-m}$

$$= \frac{M\times 0-\frac{M}{4}\times\frac{a}{4}}{M-\frac{M}{4}} = -\frac{a}{12}$$

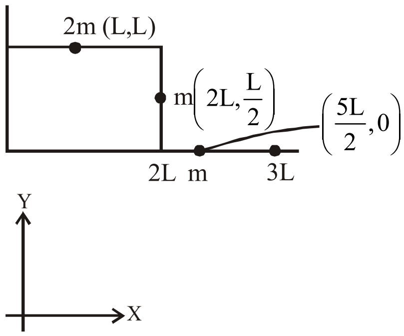
and $y_{CM} = -\frac{b}{12}$

So CM coordinates one

$$x_0 = \frac{a}{2}-\frac{a}{12} = \frac{5a}{12}$$

and $y_0 = \frac{b}{2}-\frac{b}{12} = \frac{5b}{12}$

10. **(a)**



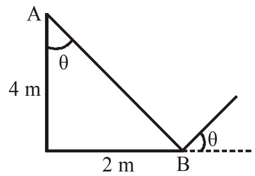
x-coordinate of centre of mass is

$$X_{cm} = \frac{2mL+2mL+\frac{5mL}{2}}{4m} = \frac{13}{8}L$$

y-coordinate of centre of mass is

$$Y_{cm} = \frac{2m\times L+m\times\left(\frac{L}{2}\right)+m\times 0}{4m} = \frac{5L}{8}$$

11. **(c)** To produce maximum moment of force line of action of force must be perpendicular to line AB.

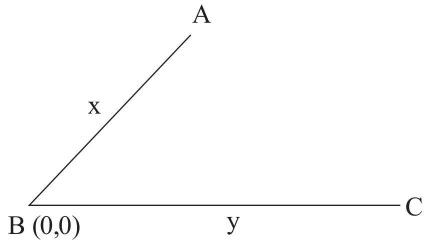


$$\therefore\ \tan\theta = \frac{2}{4} = \frac{1}{2}$$

12. **(c)** According to principle of moments when a system is stable or balance, the anti-clockwise moment is equal to clockwise moment.
i.e., load × load arm = effort × effort arm
When 5 mg weight is placed, load arm shifts to left side, hence left arm becomes shorter than right arm.

13. **(c)** Centre of mass $x_{cm} = \frac{x}{2}\frac{(\rho x)\left(\frac{x}{2}\right)\frac{1}{2}+\rho y^{y/2}}{\rho(x+y)}$

$$\Rightarrow \frac{1}{2}+\frac{y}{x} = \frac{y^2}{x^2}$$

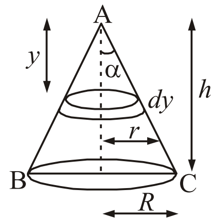


$$\therefore \frac{Bc}{AB}\frac{y}{x}=\frac{1+\sqrt{3}}{2}=1.37$$

14. **(d)** Let density of cone = ρ.

Centre of mass, $ycm=\frac{\int ydm}{\int dm}$

$$=\frac{\int_0^h y\pi r^2 dy\rho}{\frac{1}{3}\pi R^2 h\rho}=\frac{\int_0^h r^2 ydy}{\frac{1}{3}R^2 h} \quad ...(i)$$



For a cone, we know that

$$\frac{r}{R}=\frac{y}{n} \quad \therefore r=\frac{y}{n}R$$

$$ycm=\frac{\int_0^h 3y^3 dy}{h^3}=\frac{3\left[\frac{y^4}{4}\right]_0^h}{h^3}=\frac{3}{4}h$$

15. **(b)** Centre of mass of the rod is given by:

$$x_{cm}=\frac{\int_0^L (ax+\frac{bx^2}{L})dx}{\int_0^L (a+\frac{bx}{L})dx}$$

$$=\frac{\frac{aL^2}{2}+\frac{bL^2}{3}}{aL+\frac{bL}{2}}=\frac{L\left(\frac{a}{2}+\frac{b}{3}\right)}{a+\frac{b}{2}}$$

Now $\frac{7L}{12}=\frac{\frac{a}{2}+\frac{b}{3}}{a+\frac{b}{2}}$

On solving we get, $b = 2a$

16. **(c)** **17.** **(d)**

18. **(a)** The linear mass density $\lambda=k\left(\frac{x}{L}\right)^n$

Here $\frac{x}{L}\le 1$

With increase in the value of n, the centre of mass shift towards the end $x = L$ This is satisfied by only option (a).

$$x_{CM}=\frac{\int_0^L xdm}{\int_0^L dm}=\frac{\int_0^L x(\lambda dx)}{\int_0^L \lambda\, dx}=\frac{\int_0^L k\left(\frac{x}{L}\right)^n .xdx}{\int_0^L k\left(\frac{x}{L}\right)^n dx}$$

$$=\frac{k\left[\frac{x^{n+2}}{(n+2)L^n}\right]_0^L}{\left[\frac{k\,x^{n+1}}{(n+1)L^n}\right]_0^L}=\frac{L(n+1)}{n+2}$$

For $n=0, x_{CM}=\frac{L}{2}$; $n=1$,

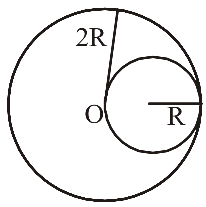
$x_{CM}=\frac{2L}{3}$; $n=2$, $x_{CM}=\frac{3L}{4}$;....

For $n\to\infty x_{cm}=L$

Moment of inertia of a square plate about an axis through its centre and perpendicular to its plane is.

19. **(b)** Let σ be the mass per unit area of the disc. Then the mass of the complete disc

$=\sigma(\pi(2R)^2)$



The mass of the removed disc $=\sigma(\pi R^2)=\pi\sigma R^2$

Let us consider the above situation to be a complete disc of radius $2R$ on which a disc of radius R of negative mass is superimposed. Let O be the origin. Then the above figure can be redrawn keeping in mind the concept of centre of mass as :

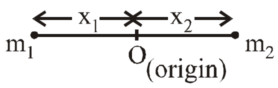
$$\underset{O}{4\pi\sigma R^2} \xleftrightarrow{\quad R \quad} -\pi\sigma R^2$$

$$x_{c.m}=\frac{\left(6\pi(2R)^2\right)\times 0+\left(-6\left(\pi R^2\right)\right)R}{4\pi\sigma R^2-\pi\sigma R^2}$$

$$\therefore \quad x_{c.m}=\frac{-\pi\sigma R^2\times R}{3\pi\sigma R^2}$$

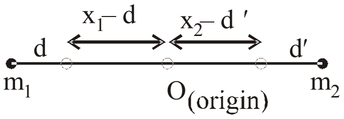
$$\therefore \quad x_{c.m}=-\frac{R}{3}=\alpha R\Rightarrow\alpha=\frac{1}{3}$$

20. **(c)** Initially,



$$0 = \frac{m_1(-x_1) + m_2 x_2}{m_1 + m_2} \Rightarrow m_1 x_1 = m_2 x_2 \text{ ...(1)}$$

Let the particles is displaced through distanced away from centre of mass



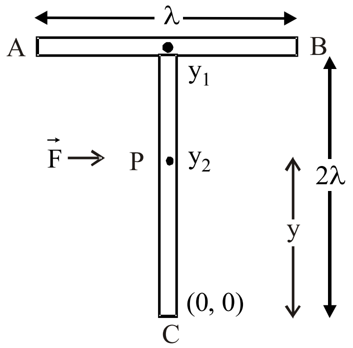
$$\therefore 0 = \frac{m_1(d - x_1) + m_2(x_2 - d')}{m_1 + m_2}$$

$$\Rightarrow 0 = m_1 d - m_1 x_1 + m_2 x_2 - m_2 d'$$

$$\Rightarrow d' = \frac{m_1}{m_2} d \qquad \text{[From (1).]}$$

21. **(a)** The centre of mass of bodies B and C taken together does not shift as no external force acts. The centre of mass of the system continues its original path. It is only the internal forces which comes into play while breaking.

22. **(d)**



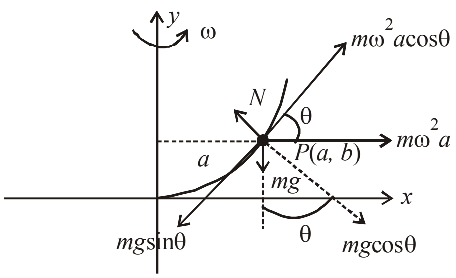
To have translational motion without rotation, the force $\vec{F}$ has to be applied at centre of mass. i.e. the point 'P' has to be at the centre of mass

Taking point C at the origin position, positions of y, and y_2 are $r_1 = 2l$, $r_2 = l$ and $ml = m$ and $m_2 = 2m$

$$y = \frac{m_1 y_1 + m_2 y_2}{m_1 + m_2} = \frac{m \times 2\ell + 2m \times \ell}{3m} = \frac{4\ell}{3}$$

23. **(a)** $y = 4Cx^2 \Rightarrow \dfrac{dy}{dx} = \tan\theta = 8Cx$

At P, $\tan\theta = 8Ca$



For steady circular motion

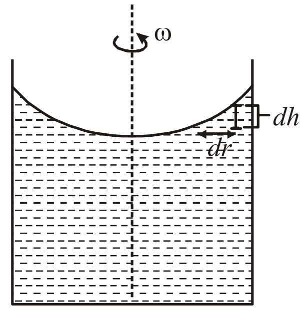
$$m\omega^2 a\cos\theta = mg\sin\theta$$

$$\Rightarrow \omega = \sqrt{\frac{g\tan\theta}{a}}$$

$$\therefore \omega = \sqrt{\frac{g \times 8aC}{a}} = 2\sqrt{2gC}$$

24. **(c)** Here, $\rho dr\omega^2 r = \rho g dh$

$$\Rightarrow \omega^2 \int_0^R r\,dr = g\int_0^h dh$$



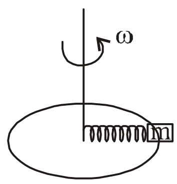
$$\Rightarrow \frac{\omega^2 R^2}{2} = gh \qquad \text{(Given } R = 5 \text{ cm)}$$

$$\therefore h = \frac{\omega^2 R^2}{2g} = \frac{25\omega^2}{2g}$$

25. **(c)** Free body diagram in the frame of disc

$$\xleftarrow{kx} \boxed{m} \longrightarrow m\omega^2(\lambda_0 + x)$$

$$\therefore \quad m\omega^2(\ell_0 + x) = kx$$



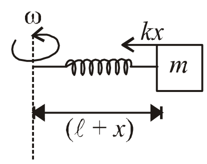
$$\Rightarrow \quad x = \frac{m\ell_0\omega^2}{k - m\omega^2}$$

For $k >> m\omega^2$

$$\Rightarrow \quad \frac{x}{\ell_0} = \frac{m\omega^2}{k}$$

26. **(b)** At elongated position (x),

$$F_{\text{radial}} = \frac{mv^2}{r} = mr\omega^2$$



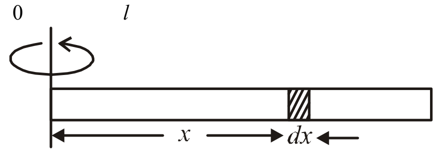
$$\therefore kx = m(\ell + x)\omega^2$$

$(\because r = \ell + x$ here)

$$kx = m\ell\omega^2 + mx\omega^2$$

$$\therefore x = \frac{m\ell\omega^2}{k - m\omega^2}$$

27. **(d)** $\displaystyle\int_0^T (-dT) = \int_l^x (dm)\omega^2 x$

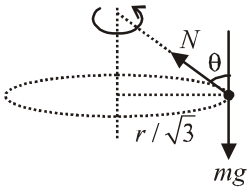


$$-T = \int_l^x \left(\frac{m}{l}dx\right)\omega^2 x$$

$$\text{or} \quad T = \frac{m\omega^2}{l}(l^2 - x^2)$$

It is a parabola between T and x.

28. (b) $N \sin\theta = m\omega^2 (r/2)$ ...(i)



$$\sin\theta = \frac{r/2}{r} = \frac{1}{2} \Rightarrow \theta = 30^\circ$$

and $N\cos\theta = mg$ ...(ii)

or $\tan\theta = \dfrac{\omega^2 r}{2g}$

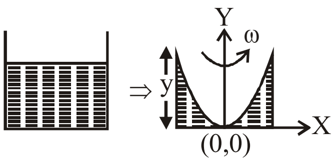
or $\tan 30^\circ = \dfrac{\omega^2 r}{2g}$

or $\dfrac{1}{\sqrt{3}} = \dfrac{\omega^2 r}{2g}$

$\therefore \quad \omega^2 = \dfrac{2g}{r\sqrt{3}}$.

29. (a) Using $v^2 = u^2 + 2gy$ $[\therefore u = 0 \text{ at } (0,0)]$

$v^2 = 2gy$ $[\therefore v = \omega x]$



$$\Rightarrow y = \frac{\omega^2 x^2}{2g} = \frac{(2\times 2\pi)^2 \times (0.05)^2}{20} \simeq 2\text{cm}$$

30. (c) $m\omega^2 R = \text{Force} \propto \dfrac{1}{R^n}$ $\left(\text{Force} = \dfrac{mv^2}{R}\right)$

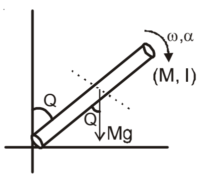
$$\Rightarrow \quad \omega^2 \propto \frac{1}{R^{n+1}} \Rightarrow \quad \omega \propto \frac{1}{R^{\frac{n+1}{2}}}$$

Time period $T = \dfrac{2\pi}{\omega}$

Time period, $T \propto R^{\frac{n+1}{2}}$

31. (b) decreasing speed

32. (c) Torque at angle θ



$\tau = Mg\sin\theta . \dfrac{l}{2}$

Also $\tau = I\alpha$

$\therefore \quad I\alpha = Mg\sin\theta\dfrac{l}{2}$

$\dfrac{Ml^2}{3}.\alpha = Mg\sin\theta\dfrac{l}{2}$ $\left[\because I_{rod} = \dfrac{Ml^2}{3}\right]$

$\Rightarrow \quad \dfrac{l\alpha}{3} = g\dfrac{\sin\theta}{2} \quad \therefore \quad \alpha = \dfrac{3g\sin\theta}{2l}$

33. (a) For just complete rotation

$v = \sqrt{Rg}$ at top point

The rotational speed of the drum

$$\Rightarrow \omega = \frac{v}{R} = \sqrt{\frac{g}{R}} = \sqrt{\frac{10}{1.25}}$$

The maximum rotational speed of the drum in revolutions per minute

$$\omega(\text{rpm}) = \frac{60}{2\pi}\sqrt{\frac{10}{1.25}} = 27.$$

34. (b) Angular momentum, $mvr = I\omega$

Moment of Inertia (I) of cubical block is given by

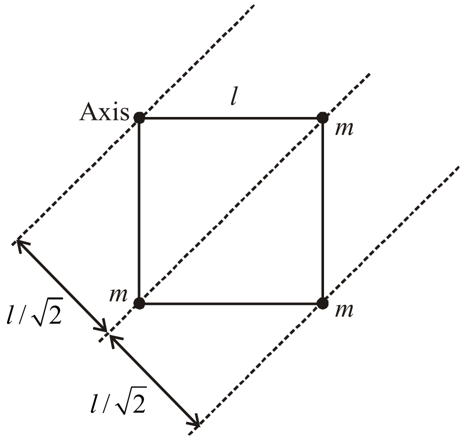
$$I = m\left(\frac{R^2}{6} + \left(\frac{R}{\sqrt{2}}\right)^2\right) \qquad \therefore \ \omega = \frac{m.2\frac{R}{2}}{m\left[\frac{R^2}{6} + \left(\frac{R}{\sqrt{2}}\right)^2\right]}$$

$$\Rightarrow \omega = \frac{12}{8R} = \frac{3}{2\times 0.3} = \frac{10}{2} = 5 \text{ rad/s}.$$

35. (b) Angular velocity is the angular displacement per unit time i.e., $\omega = \dfrac{\Delta\theta}{\Delta t}$

Here $\omega_1 = \omega_2$ and independent of f.

36. (c) Angular momentum, $L = I\omega$

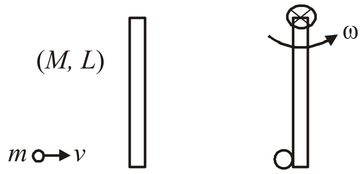


$$I = m(0)^2 + m\left(\frac{l}{\sqrt{2}}\right)^2 \times 2 + m(\sqrt{2}l)^2$$

$$= \frac{2ml^2}{2} + 2ml^2 = 3ml^2$$

Angular momentum $L = I\omega = 3ml^2\omega$

37. (20)



Before collision After collision

Using principal of conservation of angular momentum we have

$\vec{L}_i = \vec{L}_f \Rightarrow mvL = I\omega$

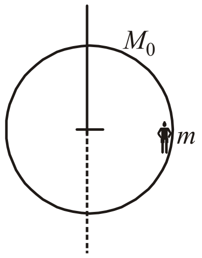
$\Rightarrow mvL = \left(\frac{ML^2}{3} + mL^2\right)\omega$

$\Rightarrow 0.1 \times 80 \times 1 = \left(\frac{0.9 \times 1^2}{3} + 0.1 \times 1^2\right)\omega$

$\Rightarrow 8 = \left(\frac{3}{10} + \frac{1}{10}\right)\omega \Rightarrow 8 = \frac{4}{10}\omega$

$\Rightarrow \omega = 20$ rad/sec.

38. **(9.00)**
Here $M_0 = 200$ kg, $m = 80$ kg
Using conservation of angular momentum, $L_i = L_f$



$I_1\omega_1 = I_2\omega_2$

$I_1 = (I_M + I_m) = \left(\frac{M_0R^2}{2} + mR^2\right)$

$I_2 = \frac{1}{2}M_0R^2$ and $\omega_1 = 5$ rpm

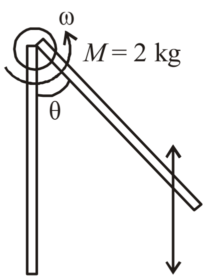
$\therefore \omega_2 = \left(\frac{M_0R^2}{2} + mR^2\right) \times \frac{5}{\frac{M_0R^2}{2}}$

$= \frac{5R^2}{R^2} \times \frac{(80+100)}{100} = 9$ rpm.

39. **(a)** Using conservation of angular momentum

$mvl = \left(ml^2 + \frac{2ml^2}{3}\right)\omega \Rightarrow mvl = \frac{5}{3}ml^2\omega \Rightarrow \omega = \frac{3v}{5l}$

or, $\omega = \frac{3 \times 6}{5 \times 1} = \frac{18}{5}$ rad/s



Now using energy conservation, after collision

$\frac{1}{2}I\omega^2 = 2mg\frac{l}{2}(1 - \cos\theta) + mgl(1 - \cos\theta)$

$\Rightarrow \frac{1}{2}\left(\frac{5}{3}ml^2\right)\frac{9v^2}{25l^2} = 2mgl(1 - \cos\theta)$

$\Rightarrow \frac{3}{5 \times 2}mv^2 = 2mgl(1 - \cos\theta)$

$\frac{3}{10} \times \frac{36}{2 \times 10} = 1 - \cos\theta \Rightarrow 1 - \frac{27}{50} = \cos\theta$

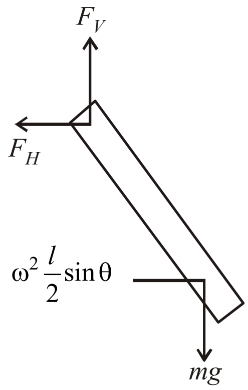
or, $\cos\theta = \frac{23}{50}$ $\quad\therefore \theta \simeq 63°$.

40. **(d)** Vertical force = mg

Horizontal force = Centripetal force = $m\omega^2\frac{l}{2}\sin\theta$

Torque due to vertical force $= mg\frac{l}{2}\sin\theta$

Torque due to horizontal force $= m\omega^2\frac{l}{2}\sin\theta\frac{l}{2}\cos\theta$



Net Torque = Angular momentum

$mg\frac{l}{2}\sin\theta - m\omega^2\frac{l}{2}\sin\theta\frac{l}{2}\cos\theta = \frac{ml^2}{12}\omega^2\sin\theta\cos\theta$

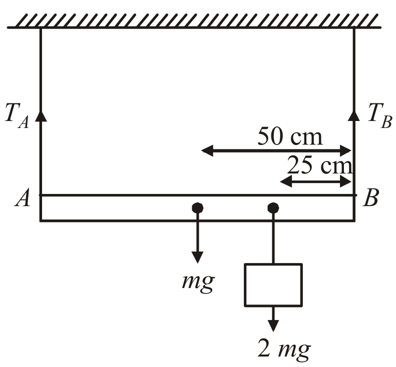
$\Rightarrow \cos\theta = \frac{3}{2}\frac{g}{\omega^2 l}$

41. **(d)** Net torque, τ_{net} about B is zero at equilibrium

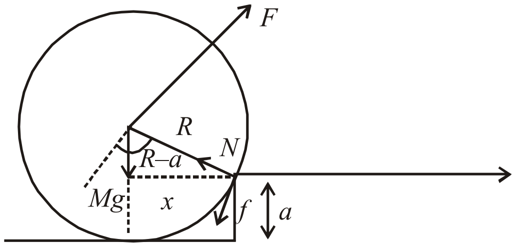
$\therefore T_A \times 100 - mg \times 50 - 2mg \times 25 = 0$

$\Rightarrow T_A \times 100 = 100mg$

$\Rightarrow T_A = 1mg$ (Tension in the string at A)



42. **(a)**

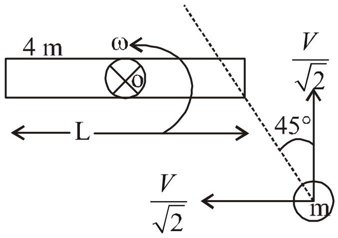


For step up, $F \times R \geq Mg \times x$

$x = \sqrt{R^2 - (R-a)^2}$ from figure

$$F_{\min} = \frac{Mg}{R} \times \sqrt{R^2 - (R-a)^2} = Mg\sqrt{1 - \left(\frac{R-a}{R}\right)^2}$$

43. **(c)**



About point O angular momentum

$L_{initial} = L_{final}$

$$\Rightarrow \frac{mV}{\sqrt{2}} \times \frac{1}{2} = \left[\frac{4mL^2}{12} + \frac{mL^2}{4}\right] \times \omega$$

$$\therefore \omega = \frac{6V}{7\sqrt{2}L} = \frac{3\sqrt{2}V}{7L}$$

44. **(b)** Given that, $x = x_0 + a \cos \omega_1 t$

$y = y_0 + b \sin \omega_2 t$

$$\frac{dx}{dt} = V_x$$

$$\Rightarrow v_x = -a\omega_1 \sin(\omega_1 t), \text{ and } \frac{dy}{dt} = v_y = b\omega_2 \cos(\omega_2 t)$$

$$\frac{dv_x}{dt} = a_x = -a\omega_1^2 \cos(\omega_1 t), \quad \frac{dv_y}{dt} = a_y = -b\omega_2^2 \sin(\omega_2 t)$$

At $t = 0$, $x = x_0 + a$, $y = y_0$

$a_x = -a\omega_1^2$, $a_y = 0$

Now, $\vec{\tau} = \vec{r} \times \vec{F} = m(\vec{r} \times \vec{a})$

$$= [(x_0 + a)\hat{i} + y_0\hat{j}] \times m(-a\,\omega_1^2\,\hat{i}) = +my_0\,a\,\omega_1^2\hat{k}$$

45. **(d)** We have given $\vec{r} = 2t\hat{i} - 3t^2\hat{j}$

$$\vec{r}(\text{at } t = 2) = 4\hat{i} - 12\hat{j}$$

Velocity, $\vec{v} = \frac{d\vec{r}}{dt} = 2\hat{i} - 6t\hat{j}$

$$\vec{v}(\text{at } t = 2) = 2\hat{i} - 12\hat{j}$$

$$\vec{L} = mvr \sin\theta\hat{n} = m(\vec{r} \times \vec{v})$$

$$= 2(4\hat{i} - 12\hat{j}) \times (2\hat{i} - 12\hat{j}) = -48\hat{k}$$

46. **(d)** Angular acceleration,

$$\alpha = \frac{\omega - \omega_0}{t} = \frac{25 \times 2\pi - 0}{5} = 10\,\pi\,\text{rad/s}^2$$

$\tau = I\alpha$

$$\Rightarrow \tau = \left(\frac{5}{4}mR^2\right)\alpha \approx \left(\frac{5}{4}\right)(5 \times 10^{-3})(10^{-4})10\pi$$

$= 2.0 \times 10^{-5}$ Nm

47. **(a)**

48. **(c)** According to work-energy theorem

$$mgh = \frac{1}{2}mv_B^2 - \frac{1}{2}mv_A^2$$

$$2gh = v_B^2 - v_A^2$$

$$2 \times 10 \times 10 = v_B^2 - 5^2$$

$\Rightarrow v_B = 15$ m/s

Angular momentum about O,

$L_O = mvr$

$= 20 \times 10^{-3} \times 20$

$L_O = 6$ kg.m²/s

49. **(a)** Given, $\vec{F}_1 = \frac{F}{2}(-\hat{i}) + \frac{F\sqrt{3}}{2}(-\hat{j})$

$\vec{r}_1 = 0\hat{i} + 6\hat{j}$

Torque due to F_1 force

$$\vec{\tau}_{F_1} = \vec{r}_1 \times \vec{F}_1 = 6\hat{j} \times \left(\frac{F}{2}(-\hat{i}) + \frac{F\sqrt{3}}{2}(-\hat{j})\right) = 3F(\hat{k})$$

Torque due to F_2 force

$$\vec{\tau}_{F_2} = (2\hat{i} + 3\hat{j}) \times F\hat{k} = 3F\hat{i} + 2F(-\hat{j})$$

$$\vec{\tau}_{net} = \vec{\tau}_{F_1} + \vec{\tau}_{F_2} = 3F\hat{i} + 2F(-\hat{j}) + 3F(\hat{k})$$

$$= (3\hat{i} - 2\hat{j} + 3\hat{k})F$$

50. **(a)** Torque about the origin $= \vec{\tau} = \vec{r} \times \vec{F}$

$= r\,F \sin\theta \Rightarrow 2.5 = 1 \times 5 \sin\theta$

$$\sin\theta = 0.5 = \frac{1}{2}$$

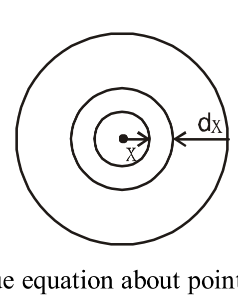
$$\Rightarrow \theta = \frac{\pi}{6}$$

51. **(d)** Consider a strip of radius x and thickness dx,

Torque due to friction on this strip

Net torque = $\sum$ Torque on ring

$$\int d\tau = \int_0^R \frac{\mu F.2\pi x dx}{\pi R^2}$$



$$\Rightarrow \tau = \frac{2\mu F}{R^2} \cdot \frac{R^3}{3}$$

$$\tau = \frac{2\mu FR}{3}$$

52. **(a)** Applying torque equation about point P.

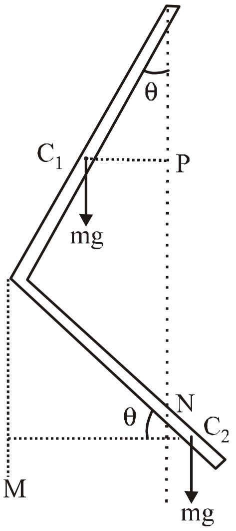
$\tau = I\,\alpha = [2M_0(2\ell)^2 + 5M_0\ell^2]\alpha$

$\Rightarrow 5M_0 g\ell - 4M_0 g\ell = [2M_0(2\ell)^2 + 5M_0\ell^2]\alpha$

$\Rightarrow M_0 g\ell = (13M_0 g\ell^2)\alpha$

$\therefore \alpha = \dfrac{g}{13\ell}$

53. (d) Given that, the rod is of uniform mass density and AB = BC



Let mass of one rod is m.

Balancing torque about hinge point.

mg (C_1P) = mg (C_2N)

$$mg\left(\frac{L}{2}\sin\theta\right) = mg\left(\frac{L}{2}\cos\theta - L\sin\theta\right)$$

$$\Rightarrow \frac{3}{2}mgL\sin\theta = \frac{mgL}{2}\cos\theta$$

$$\Rightarrow \frac{\sin\theta}{\cos\theta} = \frac{1}{3} \text{ or, } \tan\theta = \frac{1}{3}$$

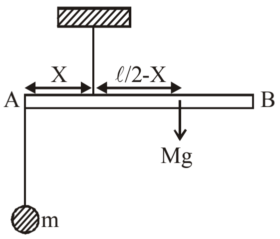
54. (a) Balancing torque w.r.t. point of suspension

$$mg\,x = Mg\left(\frac{\ell}{2} - x\right)$$

$$\Rightarrow mx = M\frac{\ell}{2} - Mx$$

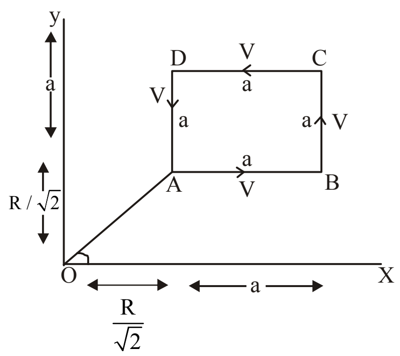
$$m = \left(M\frac{\ell}{2}\right)\frac{1}{x} - M$$

$y = \alpha\dfrac{1}{x} - C$ Straight line equation.



55. (a)

56. (a) We know that $|L| = mvr_\perp$



In none of the cases, the perpendicular distance $r_\perp$ is $\left(\dfrac{R}{\sqrt{2}} + a\right)$

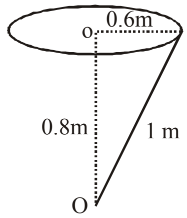
57. (a) Angular momentum,

$L_0 = mvr \sin 90°$

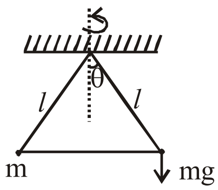
$= 2 \times 0.6 \times 12 \times 1 \times 1$

[As $V = r\omega$, Sin 90° = 1]

So, $L_0 = 14.4$ kgm^2/s



58. (c) Torque working on the bob of mass m is, $\tau = mg \times \ell \sin\theta$. (Direction parallel to plane of rotation of particle)



As τ is perpendicular to $\vec{L}$, direction of L changes but magnitude remains same.

59. (c) Given : m = 0.160 kg

θ = 60°

v = 10 m/s

Angular momentum $L = \vec{r} \times m\vec{v}$

= H mv cos θ

$$= \frac{v^2\sin^2\theta}{2g}\cos\theta \qquad \left[H = \frac{v^2\sin^2\theta}{2g}\right]$$

$$= \frac{10^2 \times \sin^2 60° \times \cos 60°}{2\times 10}$$

$= 3.46$ kg m^2/s

60. (a)

61. (a) Angular momentum L = m (v × r)

$$= 2\,\text{kg}\left(\frac{dr}{dt}\times r\right) = 2\,\text{kg}(4t\,j \times 5i - 2t^2\hat{j})$$

$= 2$ kg $(-20\,t\,\hat{k}) = 2$ kg $\times -20 \times 2$ m^{-2} s^{-1} $\hat{k}$

$= -80\,\hat{k}$

62. (b) **63. (d)**

64. (c) Angular momentum, $L = I\omega \Rightarrow L = mr^2\omega$

As insect moves along a diameter, the effective mass and hence moment of inertia (I) first decreases then increases so from principle of conservation of angular momentum, angular speed ω first increases then decreases.

65. (c) $\vec{L} = m(\vec{r} \times \vec{v})$

$$\vec{L} = m\left[v_0\cos\theta t\,\hat{i} + (v_0\sin\theta t - \frac{1}{2}gt^2)\hat{j}\right]$$

$$\times\left[v_0 \cos\theta \hat{i} + (v_0 \sin\theta - gt)\hat{j}\right]$$

$$= mv_0 \cos\theta t\left[-\frac{1}{2}gt\right]\hat{k}$$

$$= -\frac{1}{2}mgv_0 t^2 \cos\theta\hat{k}$$

66. (d) We know Torque $\vec{\tau_c} = \frac{d\vec{L_c}}{dt}$

where $\vec{L_c}$ = Angular momentum about the center of mass of the body. Central forces act along the center of mass. Therefore torque about center of mass is zero.

$\therefore \tau = \frac{dL}{dt} = 0 \qquad \Rightarrow \vec{L_c} = \text{constt.}$

67. (d) Applying conservation of angular momentum $I'\omega' = I\omega$

$(mR^2 + 2MR^2)\omega' = mR^2\omega$

$\Rightarrow (m + 2m)R^2\omega' = mR^2\omega$

$$\Rightarrow \omega' = \omega\left[\frac{m}{m+2M}\right]$$

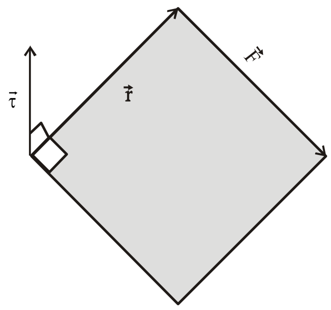
68. (c) Torque $\vec{\tau} = \vec{r} \times \vec{F} = (\hat{i} - \hat{j}) \times (-F\hat{k})$

$= F[-\hat{i} \times \hat{k} + \hat{j} \times \hat{k}] = F(\hat{j} + \hat{i}) = F(\hat{i} + \hat{j})$

$\left[\text{Since } \hat{k} \times \hat{i} = \hat{j} \text{ and } \hat{j} \times \hat{k} = \hat{i}\right]$

69. (b) Angular momentum will remain the same since no external torque act in free space.

70. (d) We know that $\vec{\tau} = \vec{r} \times \vec{F}$



Vector $\vec{\tau}$ is perpendicular to both $\vec{r}$ and $\vec{F}$. We also know that the dot product of two vectors which have an angle of 90° between them is zero.

$\therefore \vec{r} \cdot \vec{T} = 0$ and $\vec{F} \cdot \vec{T} = 0$

71. (d) Angular momentum (L)

= (linear momentum) × (perpendicular distance of the line of action of momentum from the axis of rotation)

As the particle moves with velocity V along line PC, the line of motion passes through P.

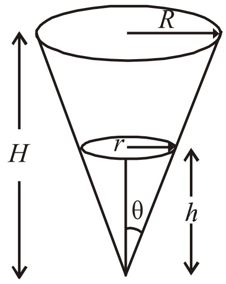
$\therefore L = mv \times r$

$= mv \times 0$

$= 0$

72. (d) Hollow ice-cream cone can be assume as several parts of discs having different radius, so

$I = \int dI = \int dm(r^2)$ ...(i)



From diagram,

$\frac{r}{h} = \tan\theta = \frac{R}{H}$ or $r = \frac{R}{H}h$ ...(ii)

Mass of element, $dm = \rho(\pi r^2)dh$ ...(iii)

From eq. (i), (ii) and (iii),

Area of element, $dA = 2\pi r dl = 2\pi r \frac{dh}{\cos\theta}$

Mass of element, $dm = \frac{2Mh \tan dh}{R\sqrt{R^2 + H^2}\cos\theta}$

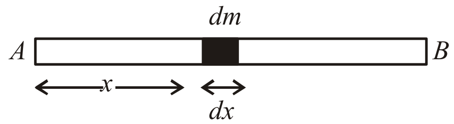
(here, $r = h\tan\theta$)

$$I = \int dI = \int_0^H dm(r^2) = \int_0^H \rho(\pi r^2)dh\left(\frac{R}{H}\cdot h\right)^2$$

$$= \int_0^H \rho\left(\pi\left(\frac{R}{H}\cdot h\right)^4\right)dh$$

Solving we get, $I = \frac{MR^2}{2}$

73. (b)



Mass of the small element of the rod

$dm = \lambda \cdot dx$

Moment of inertia of small element,

$$dI = dm \cdot x^2 = \lambda_0\left(1 + \frac{x}{L}\right)\cdot x^2 \; dx$$

Moment of inertia of the complete rod can be obtained by integration

$$I = \lambda_0 \int_0^L \left(x^2 + \frac{x^3}{L}\right)dx$$

$$= \lambda_0\left|\frac{x^3}{3} + \frac{x^4}{4L}\right|_0^L = \lambda_0\left[\frac{L^3}{3} + \frac{L^3}{4}\right]$$

$$\Rightarrow I = \frac{7\lambda_0 L^3}{12} \qquad \text{...(i)}$$

Mass of the thin rod,

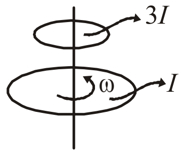
$$M = \int_0^L \lambda\, dx = \int_0^L \lambda_0 \left(1 + \frac{x}{L}\right) dx = \frac{3\lambda_0 L}{2}$$

$$\therefore \lambda_0 = \frac{2M}{3L}$$

$$\therefore I = \frac{7}{12}\left(\frac{2M}{3L}\right) L^3 \Rightarrow I = \frac{7}{18} ML^2$$

74. **(d)** By angular momentum conservation, $L_c = L_f$

$$\omega I + 3I \times 0 = 4I\omega' \Rightarrow \omega' = \frac{\omega}{4}$$



$$(KE)_i = \frac{1}{2} I\omega^2$$

$$(KE)_f = \frac{1}{2}(3I + I)\omega'^2$$

$$= \frac{1}{2} \times (4I) \times \left(\frac{\omega}{4}\right)^2 = \frac{I\omega^2}{8}$$

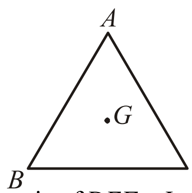
$$\Delta KE = \frac{1}{2} I\omega^2 - \frac{1}{8} I\omega^2 = \frac{3}{8} I\omega^2$$

$\therefore$ Fractional loss in K.E. $= \dfrac{\Delta KE}{KE_{l_i}} = \dfrac{\frac{3}{8} I\omega^2}{\frac{1}{2} I\omega^2} = \dfrac{3}{4}.$

75. **(11)** Let mass of triangular lamina $= m$, and length of side $= l$, then moment of inertia of lamina about an axis passing through centroid G perpendicular to the plane.

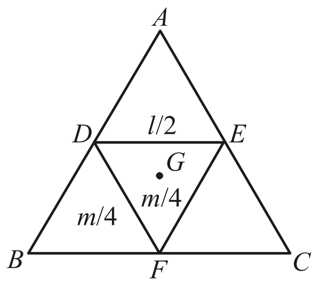
$$I_0 \propto ml^2$$

$$I_0 = kml^2$$



Let moment of inertia of $DEF = I_1$ about G

So, $I_1 \propto \left(\dfrac{m}{4}\right)\left(\dfrac{l}{2}\right)^2 \propto \dfrac{ml^2}{16}$ or $I_1 = \dfrac{I_0}{16}$



Let $I_{ADE} = I_{BDF} = I_{EFC} = I_2$

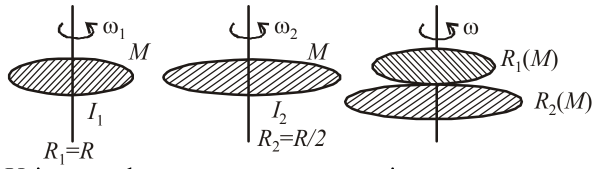
$$\therefore 3I_2 + I_1 = I_0 \Rightarrow 3I_2 + \frac{I_0}{16} = I_0 \Rightarrow I_2 = \frac{5I_0}{16}$$

Hence, moment of inertia of $DECB$ i.e., after removal part ADE

$$= 2I_2 + I_1 = 2\left(\frac{5I_0}{16}\right) + \left(\frac{I_0}{16}\right) = \frac{11I_0}{16} = \frac{NI_0}{16}$$

Therefore value of $N = 11$.

76. **(20)** As we know moment of inertia disc, $I_{\text{disc}} = \dfrac{1}{2} MR^2$



Using angular momentum conservation

$$I_1\omega_1 + I_2\omega_2 = (I_1 + I_2) \times \omega_f$$

$$\frac{MR^2}{2} \times \omega + 0 = \left(\frac{MR^2}{2} + \frac{MR^2}{8}\right)\omega_f \Rightarrow \omega_f = \frac{4}{5}\omega$$

Initial K.E., $K_i = \dfrac{1}{2} I\omega^2 = \dfrac{1}{2}\left(\dfrac{MR^2}{2}\right)\omega^2 = \dfrac{MR^2\omega^2}{4}$

Final K.E., $K_f = \dfrac{1}{2}\left(\dfrac{MR^2}{2} + \dfrac{MR^2}{8}\right)\dfrac{16}{25}\omega^2 = \dfrac{MR^2\omega^2}{5}$

Percentage loss in kinetic energy % loss

$$= \frac{\frac{MR^2\omega^2}{4} - \frac{MR^2\omega^2}{5}}{\frac{MR^2\omega^2}{4}} \times 100 = 20\% = P\%$$

Hence, value of $P = 20$.

77. **(c)** Let p be the density of the discs and t is the thickness of discs.

Moment of inertia of disc is given by

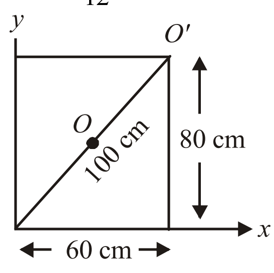
$$I = \frac{MR^2}{2} = \frac{[\rho(\pi R^2)t]R^2}{2}$$

$I \propto R^4$ (As ρ and t are same)

$$\frac{I_2}{I_1} = \left(\frac{R_2}{R_1}\right)^4 \Rightarrow \frac{16}{1} = \alpha^4 \Rightarrow \alpha = 2$$

78. **(b)** Moment of inertia of rectangular sheet about an axis passing through O,

$$I_O = \frac{M}{12}(a^2 + b^2) = \frac{M}{12}[(80)^2 + (60)^2]$$

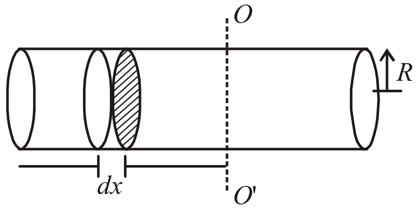


From the parallel axis theorem, moment of inertia about O',

$$I_{O'} = I_O + M(50)^2$$

$$\frac{I_O}{I_{O'}} = \frac{\frac{M}{12}(80^2+60^2)}{\frac{M}{12}(80^2+60^2)+M(50)^2} = \frac{1}{4}$$

79. **(c)** Let there be a cylinder of mass m length L and radius R. Now, take elementary disc of radius R and thickness dx at a distance of x from axis OO' then moment of inertia about OO' of this element.



$$dI = \frac{dmR^2}{4} + dmx^2$$

$$\Rightarrow I = \int dI = \int \frac{dmR^2}{4} + \int_{n=L/2}^{n=-L/2} \frac{M}{L} dx \times x^2$$

Given : $I = \frac{MR^2}{4} + \frac{ML^2}{12}$

$$\Rightarrow I = \frac{M}{4} \times \frac{V}{\pi L} + \frac{ML^2}{12} \Rightarrow I = \frac{MV}{4\pi L} + \frac{ML^2}{12}$$

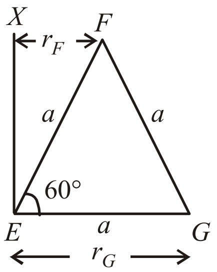
$$\frac{dI}{dL} = -\frac{mV}{4\pi L^2} + \frac{M \times 2L}{12} = 0$$

$$\Rightarrow V = \frac{2}{3}\pi L^3 \Rightarrow \pi R^2 L = \frac{2}{3}\pi L^3$$

$$\therefore \frac{L}{R} = \sqrt{\frac{3}{2}}$$

80. **(25)**

Moment of inertia of the system about axis XE.



$$I = I_E + I_F + I_G$$

$$\Rightarrow I = m(r_E)^2 + m(r_F)^2 + m(r_G)^2$$

$$\Rightarrow I = m \times 0^2 + m\left(\frac{a}{2}\right)^2 + ma^2 = \frac{5}{4}ma^2 = \frac{25}{20}ma^2$$

$\therefore N = 25.$

81. **(b)** Initial angular momentum $= I_1\omega_1 + I_2\omega_2$

Let ω be angular speed of the combined system.

Final angular momentum $= I_1\omega + I_2\omega$

According to conservation of angular momentum

$$(I_1 + I_2)\omega = I_1\omega_1 + I_2\omega_2$$

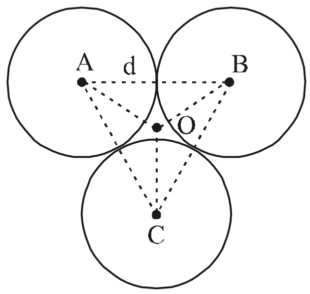
$$\Rightarrow \omega = \frac{I_1\omega_1 + I_2\omega_2}{I_1 + I_2} = \frac{0.1 \times 10 + 0.2 \times 5}{0.1 + 0.2} = \frac{20}{3}$$

Final rotational kinetic energy

$$K_f = \frac{1}{2}I_1\omega^2 + \frac{1}{2}I_2\omega^2 = \frac{1}{2}(0.1+0.2)\times\left(\frac{20}{3}\right)^2$$

$$\Rightarrow K_f = \frac{20}{3} \text{ J}$$

82. **(a)** Moment of inertia,



$$I_1 = \frac{2}{5}m\left(\frac{d}{2}\right)^2 + m(AO)^2$$

and $AO = \frac{d}{\sqrt{3}}$

Moment of inertia about 'O'

$$I_0 = 3I_1 = 3\left[\frac{2}{5}m\left(\frac{d}{2}\right)^2 + m\left(\frac{d}{\sqrt{3}}\right)^2\right]$$

$$\Rightarrow I_0 = \frac{13}{10}Md^2$$

And $I_A = 2\left[\frac{2}{5}M\left(\frac{d}{2}\right)^2 + Md^2\right] + \frac{2}{5}M\left(\frac{d}{2}\right)^2$

$$\Rightarrow I_A = \frac{23}{10}Md^2$$

$$\therefore \frac{I_O}{I_A} = \frac{\frac{13}{10}Md^2}{\frac{23}{10}Md^2} = \frac{13}{23}$$

83. **(15)** Here, length of bar, l = 1 m

angle, $\theta = 30°$

$$\Delta PE = \Delta KE \text{ or } mgh = \frac{1}{2}I\omega^2$$

$$\Rightarrow (mg)\frac{l}{2}\sin 30° = \frac{1}{2}\left(\frac{ml^2}{3}\right)\omega^2$$

$$\Rightarrow mg\frac{l}{2}\times\frac{1}{2} = \frac{1}{2}\left(\frac{ml^2}{3}\right)\omega^2$$

$$\Rightarrow \omega = \sqrt{15}\ \text{rad/s}$$

84. (a) Using principal of conservation of energy

$$(m_1 - m_2)gh = \frac{1}{2}(m_1 + m_2)v^2 + \frac{1}{2}I\omega^2$$

$$\Rightarrow \quad (m_1 - m_2)gh = \frac{1}{2}(m_1 + m_2)(\omega R)^2 + \frac{1}{2}I\omega^2$$

$$(\because v = \omega R)$$

$$\Rightarrow \quad (m_1 - m_2)gh = \frac{\omega^2}{2}\left[(m_1 + m_2)R^2 + I\right]$$

$$\Rightarrow \quad \omega = \sqrt{\frac{2(m_1 - m_2)gh}{(m_1 + m_2)R^2 + I}}$$

85. (a) When the bob covered a distance 'h'

Using $mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$

$$= \frac{1}{2}m(\omega r)^2 + \frac{1}{2}\times\frac{mr^2}{2}\times\omega^2 \quad (\because v = \omega r \textit{ no slipping})$$

$$\Rightarrow \quad mgh = \frac{3}{4}m\omega^2 r^2$$

$$\Rightarrow \omega = \sqrt{\frac{4gh}{3r^2}} = \frac{1}{r}\sqrt{\frac{4gh}{3}}$$

86. (c) Moment inertia of the rod passing through a point $\frac{\ell}{4}$ away from the centre of the rod

$I = Ig + m\ell^2$

$$\Rightarrow I = \frac{MI^2}{12} + M\times\left(\frac{I^2}{16}\right) = \frac{7MI^2}{48}$$

Using $I = MK^2 = \frac{7MI^2}{48}$ (K = radius of gyration)

$$\Rightarrow K = \sqrt{\frac{7}{48}}I$$

87. (a) Given,
mass per unit area of circular disc, $\sigma = A + Br$
Area of the ring = $2\pi r dr$
Mass of the ring, $dm = \sigma 2\pi r dr$
Moment of inertia,

$$I = \int dmr^2 = \int \sigma\, 2\pi r dr.r^2$$

$$\Rightarrow \quad I = 2\pi\int_0^a (A + Br)r^3 dr = 2\pi\left[\frac{Aa^4}{4} + \frac{Ba^5}{5}\right]$$

$$\Rightarrow \quad I = 2\pi a^4\left[\frac{A}{4} + \frac{Ba}{5}\right]$$

88. (c) $I = \int_a^b (dm)r^2$

$$= \int_a^b \left(\frac{\sigma_0}{r}\times 2\pi r\, dr\right)r^2 = \frac{2\pi\sigma_0}{3}\,|\, r^3\,|_a^b$$

$$= \frac{2\pi\sigma_0}{3}(b^3 - a^3)$$

Mass of the disc,

$$m = \int_a^b \frac{\sigma_0}{r}\times 2\pi r\, dr = 2\pi\sigma_0(b - a)$$

Radius of gyration,

$$k = \sqrt{\frac{I}{m}}$$

$$= \sqrt{\frac{(2\pi\sigma_0/3)(b^3 - a^3)}{2\pi\sigma_0(b - a)}} = \sqrt{\frac{a^2 + b^2 + ab}{3}}$$

89. (d) As no external torque is acting so angular momentum should be conserved
$(I_1 + I_2)\,\omega = I_1\omega_1 + I_2\omega_2$ [ω_c = common angular velocity of the system, when discs are in contact]

$$\omega_c = \frac{I_1\omega_1 + \frac{I_1\omega_1}{4}}{I_1 + \frac{I_1}{2}}\left(\frac{5}{4}\times\frac{2}{3}\right)\omega_1$$

$$\omega_c = \frac{5\omega_1}{6}$$

$$E_f - E_i = \frac{1}{2}(I_1 + I_2)\omega_c^2 - \frac{1}{2}I_1\omega_1^2 - \frac{1}{2}I_2\omega_2^2$$

Put $I_2 = I_1/2$ and $\omega_c = \frac{5\omega_1}{6}$ $5\omega_1/6$

We get :

$$E_f - E_i = -\frac{I_1\omega_1^2}{24}$$

90. (b) As from the question density $(\sigma) = kr^2$

Mass of disc $M = \int_0^R (kr^2)2\pi r dr = 2\pi k\frac{R^4}{4} = \frac{\pi k R^4}{2}$

$$\Rightarrow k = \frac{2M}{\pi R^4} \qquad \text{....(i)}$$

$\therefore$ Moment of inertia about the axis of the disc.

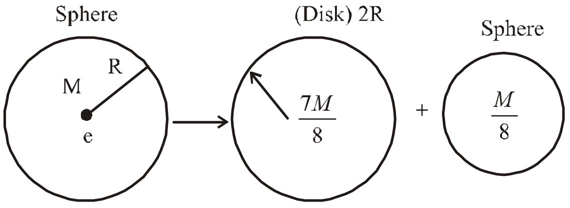
$$I = \int dI = \int (dm)r^2 = \int \sigma dA r^2$$

$$= \int (kr^2)(2\pi r dr)r^2$$

$$= \int_0^R 2\pi k\, r^5 dr = \frac{\pi k R^6}{3} = \frac{\pi\times\left(\frac{2M}{\pi R^4}\right)\times R^6}{3} = \frac{2}{3}MR^2$$

[putting value of k from eqn ....(i)]

91. (b)



$V=\frac{4}{3}\pi R^3$ $\quad\left(\frac{7V}{8}\right)$

$$I_1=\left(\frac{7M}{8}\right)(2R)^2\frac{1}{2}=\left(\frac{7}{16}\times 4\right)MR^2=\frac{14}{8}\text{mR}^2$$

$$I_2=\frac{2}{5}\left(\frac{M}{8}\right)r^2$$

$$\Rightarrow I_2=\frac{2}{5}\left(\frac{M}{8}\right)\left(\frac{R^2}{4}\right)=\frac{MR^2}{80}$$

$$\left[\begin{array}{l}\frac{4}{3}\pi r^3\rho=\frac{1}{8}\frac{4}{3}\pi R^3\times\rho\\ \Rightarrow r=R/2\end{array}\right]$$

$$\frac{I_1}{I_2}=\frac{14\times 80}{8}=140$$

92. (d) $\frac{1}{2}I\omega^2=kQ^2$

or $\omega=\left(\sqrt{\frac{2k}{I}}\right)Q$

or $\alpha=\frac{d\omega}{\pi}=\sqrt{\frac{2K}{I}}\left(\frac{dQ}{dt}\right)=\left(\sqrt{\frac{2k}{I}}\right)\omega$

$=\left(\sqrt{\frac{2k}{I}}\right)\left(\sqrt{\frac{2k}{I}}\right)\theta=\frac{2k\theta}{I}$

93. (b) $\omega=\alpha t=20t$

Given, $\frac{1}{2}I\omega^2=1200$

or $\frac{1}{2}\times 1.5\times(20t)^2=1200$

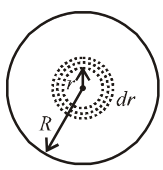
or $t=2$ s

94. (c) $I_i\omega_i=I_f\omega_f$

or $\left(\frac{ML^2}{12}\right)\omega_0=\left(\frac{ML^2}{12}+2m\left(\frac{L}{2}\right)^2\right)\omega_f$

$\therefore \omega_f=\left(\frac{M\omega_0}{M+6m}\right)$

95. (c) Taking a circular ring of radius r and thickness dr as a mass element, so total mass,



$$M=\int_0^R\rho_0 r\times 2\pi r dr=\frac{2\pi\rho_0 R^3}{3}$$

$$I_C=\int_0^R\rho_0 r\times 2\pi r dr\times r^2=\frac{2\pi\rho_0 R^5}{5}$$

Using parallel axis theorem

$$\therefore\ I=I_C+MR^2=2\pi\rho_0 R^5\left(\frac{1}{3}+\frac{1}{5}\right)=\frac{16\pi\rho_0 R^5}{15}$$

$$=\frac{8}{5}\left[\frac{2}{3}\pi\rho_0 R^3\right]R^2=\frac{8}{5}MR^2$$

96. (b)

97. (d) According to parallel axes theorem

$$I=\frac{2}{5}mR^2+mx^2$$

Hence graph (d) correctly depicts I vs x.

98. (a) Let mass of the larger triangle = M

Side of larger triangle = ℓ

Moment of inertia of larger triangle = ma^2

Mass of smaller triangle = $\frac{M}{4}$

Length of smaller triangle = $\frac{\ell}{2}$

Moment of inertia of removed triangle

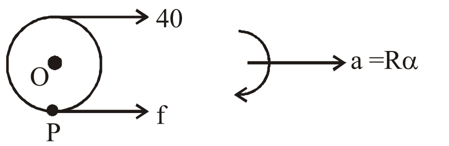
$=\frac{M}{4}\left(\frac{a}{2}\right)^2$

$$\therefore \frac{I_{removed}}{I_{original}}=\frac{\frac{M}{4}}{M}\cdot\frac{\left(\frac{a}{2}\right)^2}{(a)^2}$$

$I_{removed}=\frac{I_0}{16}$

So, $I=I_0-\frac{I_0}{16}=\frac{15I_0}{16}$

99. (b)



From newton's second law

$40+f=m(R\alpha)$ .....(i)

Taking torque about 0 we get

$40\times R-f\times R=I\alpha$

$40\times R-f\times R=mR^2\alpha$

$40-f=mR\alpha$ ...(ii)

Solving equation (i) and (ii)

$$\alpha=\frac{40}{mR}=16\,\text{rad/s}^2$$

100. (b) Moment of inertia of disc D_1 about OO′ = $I_1=\frac{MR^2}{2}$

M.O.I of D_2 about OO′

$$=I_2=\frac{1}{2}\left(\frac{MR^2}{2}\right)+MR^2=\frac{MR^2}{4}+MR^2$$

M.O.I of D_3 about OO′

$$=I_3=\frac{1}{2}\left(\frac{MR^2}{2}\right)+MR^2=\frac{MR^2}{4}+MR^2$$

so, resultant M.O.I about OO′ is $I = I_1 + I_2 + I_3$

$$\Rightarrow I = \frac{MR^2}{2} + 2\left(\frac{MR^2}{4} + MR^2\right)$$

$$= \frac{MR^2}{2} + \frac{MR^2}{2} + 2MR^2 = 3\ MR^2$$

101. (a) For Ball

using parallel axes theorem, for ball moment of inertaia,

$$I_{ball} = \frac{2}{5}MR^2 + M(2R)^2 = \frac{22}{5}MR^2$$

For two balls $I_{balls} = 2\times\frac{22}{5}\ MR^2$=and,

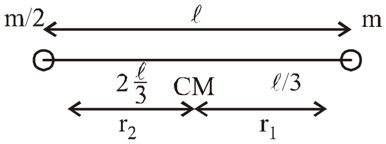
$$Irod = \frac{M(2R)^2}{12} = \frac{MR^2}{3}$$

$$I_{system} = I_{balls} + I_{rod}$$

$$= \frac{44}{5}MR^2 + \frac{MR^2}{3} = \frac{137}{15}MR^2$$

102. (c) As we know, $\omega = \sqrt{\frac{k}{I}}$

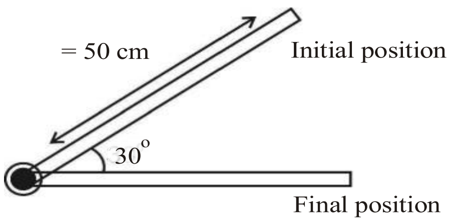
$$\omega = \sqrt{\frac{3k}{m\ell^2}} \quad \left[\because\ I_{rod} = \frac{1}{3}m\ell^2\right]$$



Tension when it passes through the mean position,

$$= m\omega^2\theta_0^2\frac{\ell}{3} = m\frac{3k}{m\ell^2}\theta_0^2\frac{\ell}{3} = \frac{k\theta_0^2}{\ell}$$

103. (d)



By the low of conservation of energy,
P.E. of rod = Rotational K.E.

$$mg\frac{\ell}{2}\text{Sin}\,\theta = \frac{1}{2}I\omega^2$$

$$\Rightarrow mg\frac{\ell}{2}\text{Sin}\,30° = \frac{1}{2}\frac{m\ell^2}{3}\omega^2 \quad \Rightarrow mg\frac{\ell}{2}\times\frac{1}{2} = \frac{1}{2}\frac{m\ell^2}{3}\omega^2$$

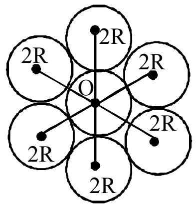
For complete length of rod,

$$\omega = \sqrt{3g/2(2\ell)} = \sqrt{\frac{30}{2}}\ \text{rod}\,s^{-1}$$

104. (d) Using parallel axes theorem, moment of inertia about 'O'

$$I_o = I_{cm} + md^2$$

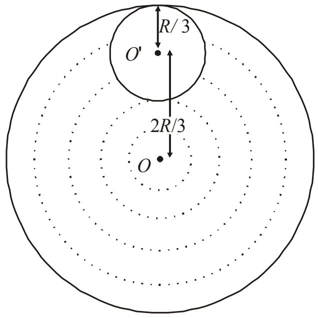
$$= \frac{7MR^2}{2} + 6(M\times(2R)^2) = \frac{55MR^2}{2}$$



Again, moment of inertia about point P, $I_p = I_o + md^2$

$$= \frac{55MR^2}{2} + 7M(3R)^2 = \frac{181}{2}MR^2$$

105. (a) Let σ be the mass per unit area.



The total mass of the disc
$= \sigma\times\pi R^2 = 9M$
The mass of the circular disc cut

$$= \sigma\times\pi\left(\frac{R}{3}\right)^2 = \sigma\times\frac{\pi R^2}{9} = M$$

Let us consider the above system as a complete disc of mass $9M$ and a negative mass M super imposed on it.

Moment of inertia (I_1) of the complete disc = $\frac{1}{2}9MR^2$ about an axis passing through O and perpendicular to the plane of the disc.

M.I. of the cut out portion about an axis passing through O' and perpendicular to the plane of disc

$$= \frac{1}{2}\times M\times\left(\frac{R}{3}\right)^2$$

$\therefore$ *M.I.* (I_2) of the cut out portion about an axis passing through O and perpendicular to the plane of disc

$$= \left[\frac{1}{2}\times M\times\left(\frac{R}{3}\right)^2 + M\times\left(\frac{2R}{3}\right)^2\right]$$

[Using perpendicular axis theorem]

$\therefore$ The total *M.I.* of the system about an axis passing through O and perpendicular to the plane of the disc is

$I = I_1 + I_2$

$$= \frac{1}{2}9MR^2 - \left[\frac{1}{2}\times M\times\left(\frac{R}{3}\right)^2 + M\times\left(\frac{2R}{3}\right)^2\right]$$

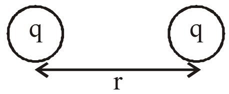
$$= \frac{9MR^2}{2} - \frac{9MR^2}{18} = \frac{(9-1)MR^2}{2} = 4MR^2$$

106. (c) As we know, moment of inertia of a disc about an axis passing through C.G. and perpendicular to its plane,

$$I_z = \frac{mR^2}{2}$$

Moment of inertia of a disc about a tangential axis perpendicular to its own plane,

$$I_z' = \frac{3}{2}mR^2$$



$$\therefore \quad I_z/I_z' = \frac{mR^2}{2} \Big/ \frac{3mR^2}{2} = 1/3$$

107. (a) When the rod makes an angle α

Displacement of centre of mass $= \frac{l}{2}\cos\alpha$

$$mg\frac{l}{2}\cos\alpha = \frac{1}{2}I\omega^2$$

$$mg\frac{l}{2}\cos\alpha = \frac{ml^2}{6}\omega^2$$ ($\because$ M.I. of thin uniform rod about an axis passing through its centre of mass and perpendicular to the rod $I = \frac{ml^2}{12}$)

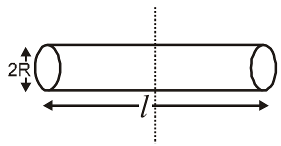
$$\Rightarrow \quad \omega = \sqrt{\frac{3g\cos\alpha}{l}}$$

Speed of end $= \omega \times l = \sqrt{3g\cos\alpha l}$

i.e., Speed of end, $\omega \propto \sqrt{\cos\alpha}$

108. (c) As we know, moment of inertia of a solid cylinder about an axis which is perpendicular bisector

$$I = \frac{mR^2}{4} + \frac{ml^2}{12}$$



$$I = \frac{m}{4}\left[R^2 + \frac{l^2}{3}\right]$$

$$= \frac{m}{4}\left[\frac{V}{\pi l} + \frac{l^2}{3}\right] \quad \Rightarrow \quad \frac{dl}{dl} = \frac{m}{4}\left[\frac{-V}{\pi l^2} + \frac{2l}{3}\right] = 0$$

$$\frac{V}{\pi l^2} = \frac{2l}{3} \quad \Rightarrow V = \frac{2\pi l^3}{3}$$

$$\pi R^2 l = \frac{2\pi l^3}{3} \Rightarrow \frac{l^2}{R^2} = \frac{3}{2} \text{ or, } \frac{l}{R} = \sqrt{\frac{3}{2}}$$

109. (b) According to theorem of perpendicular axes, moment of inertia of triangle (ABC)

$I_0 = kml^2$ ..... (i)

BC = 1

Moment of inertia of a cavity DEF

$$I_{DEF} = K\frac{m}{4}\left(\frac{l}{2}\right)^2$$

$$= \frac{k}{16}ml^2$$

From equation (i),

$$I_{DEF} = \frac{I_0}{16}$$

Moment of inertia of remaining part

$$I_{remain} = I_0 - \frac{I_0}{16} = \frac{15I_0}{16}$$

110. (b) Moment of Inertia of complete disc about 'O' point

$$I_{total} = \frac{MR^2}{2}$$

Radius of removed disc = R/4

$\therefore$ Mass of removed disc = M/16

[As $M \propto R^2$]

M.I of removed disc about its own axis (O')

$$= \frac{1}{2}\frac{M}{16}\left(\frac{R}{4}\right)^2 = \frac{MR^2}{512}$$

M.I of removed disc about O

$I_{removed\ disc} = I_{cm} + mx^2$

$$= \frac{MR^2}{512} + \frac{M}{16}\left(\frac{3R}{4}\right)^2 = \frac{19\ MR^2}{512}$$

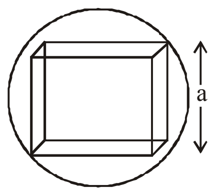
M.I of remaining disc

$$I_{remaining} = \frac{MR^2}{2} - \frac{19}{512}MR^2 = \frac{237}{512}MR^2$$

111. (a) Here $a = \frac{2}{\sqrt{3}}R$

$$\text{Now, } \frac{M}{M'} = \frac{\frac{4}{3}\pi R^3}{a^3}$$

$$= \frac{\frac{4}{3}\pi R^3}{\left(\frac{2}{\sqrt{3}}R\right)^3} = \frac{\sqrt{3}}{2}\pi.$$



$$M' = \frac{2M}{\sqrt{3}\pi}$$

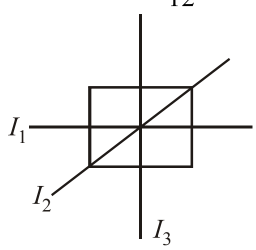
Moment of inertia of the cube about the given axis,

$$I = \frac{M'a^2}{6}$$

$$= \frac{\frac{2M}{\sqrt{3}\pi} \times \left(\frac{2}{\sqrt{3}}R\right)^2}{6} = \frac{4MR^2}{9\sqrt{3}\pi}$$

112. (d) For a thin uniform square sheet

$$I_1 = I_2 = I_3 = \frac{ma^2}{12}$$



113. (c) Kinetic energy $_{\text{(rotational)}}$ $K_R = \frac{1}{2}I\omega^2$

Kinetic energy $_{\text{(translational)}}$ $K_T = \frac{1}{2}Mv^2$ $(v = R\omega)$

M.I.$_{\text{(initial)}}$ $I_{ring} = MR^2$; $\omega_{initial} = \omega$

M.I.$_{\text{(new)}}$ $I'_{\text{(system)}} = MR^2 + 2mR^2$

$\omega'_{\text{(system)}} = \frac{M\omega}{M+2m}$

Solving we get loss in K.E.

$= \frac{Mm}{(M+2m)}\omega^2R^2$

114. (b) As $L = I\omega$ so L increases with increase in ω. Kinetic energy$_{\text{(rotational)}}$ depends on an angular velocity 'ω' and moment of inertia of the body I.

i.e., $K.E._{\text{(rotational)}} = \frac{1}{2}I\omega^2$

115. (b) $K.E_{\text{rotational}} = \frac{1}{2}I\omega^2$

$= \frac{1}{2}\frac{2}{5}\omega r^2 d^2 \left(\because I_{\text{Solid sphere}} = \frac{2}{5}mr^2\right)$

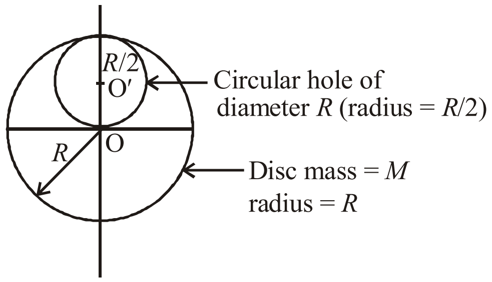
$K.E_{\text{translational}} = \frac{1}{2}mv^2$

$\therefore \frac{K.E_{\text{rotational}}}{K.E_{\text{translational}}} = \frac{2}{5}$

Hence option (b) is correct

116. (d) M.I. of complete disc about its centre O.

$I_{\text{Total}} = \frac{1}{2}MR^2$ ...(i)



Mass of circular hole (removed)

$= \frac{M}{4} \left(\text{As } M = \pi R^2 t \therefore M \propto R^2\right)$

M.I. of removed hole about its own axis

$= \frac{1}{2}\left(\frac{M}{4}\right)\left(\frac{R}{2}\right)^2 = \frac{1}{32}MR^2$

M.I. of removed hole about O'

$I_{\text{removed hole}} = I_{\text{cm}} + mx^2$

$= \frac{MR^2}{32} + \frac{M}{4}\left(\frac{R}{2}\right)^2$

$= \frac{MR^2}{32} + \frac{MR^2}{16} = \frac{3MR^2}{32}$

M.I. of complete disc can also be written as

$I_{\text{Total}} = I_{\text{removed hole}} + I_{\text{remaining disc}}$

$I_{\text{Total}} = \frac{3MR^2}{32} + I_{\text{remaining disc}}$ ...(ii)

From eq. (i) and (ii),

$\frac{1}{2}MR^2 = \frac{3MR^2}{32} + I_{\text{remaining disc}}$

$\Rightarrow I_{\text{remaining disc}}$

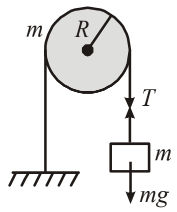
$= \frac{MR^2}{2} - \frac{3MR^2}{32} = \left(\frac{13}{32}\right)MR^2$

117. (b) For translational motion,

$mg - T = ma$ .....(1)

For rotational motion,

$T.R = I\alpha$



$\Rightarrow T.R = \frac{1}{2}mR^2\alpha$

Also, acceleration, $a = R\alpha$

$\therefore T = \frac{1}{2}mR\alpha = \frac{1}{2}ma$

Substituting the value of T is equation (1) we get mg -

$\frac{1}{2}ma = ma \Rightarrow a = \frac{2}{3}g$

118. (a) Given,

Force, $F = (20t - 5t^2)$

Radius, $r = 2m$

Torque, $T = rf = I\alpha$

$\Rightarrow 2(20t - 5t^2) = 10\alpha$

$\therefore \alpha = 4t - t^2$

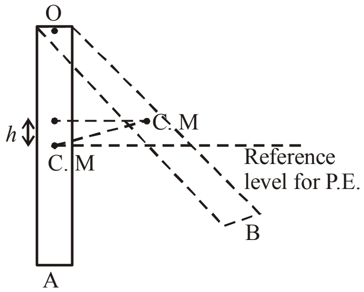
$\Rightarrow \frac{d\omega}{dt} = 4t - t^2 \Rightarrow \int_0^{\omega} d\omega = \int_0^t \left(4t - t^2\right)dt$

$\Rightarrow \omega = 2t^2 - \frac{t^3}{3}$ (as $\omega = 0$ at $t = 0$, 6s)

$\int_0^{\theta} d\theta = \int_0^6 \left(2t^2 - \frac{t^3}{3}\right)dt$

$\Rightarrow \theta = 36 \text{ rad} \Rightarrow 2\pi n = 36 \Rightarrow n = \frac{36}{2\pi} < 6$

119. (c)



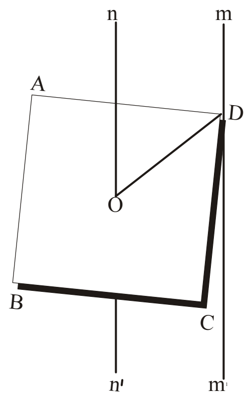
The moment of inertia of the rod about O is $\frac{1}{3}m\ell^2$. The kinetic energy of the rod at position A = $\frac{1}{2}I\omega^2$ where I is the moment of inertia of the rod about O. When the rod is in position B, its angular velocity is zero. In this case, the energy of the rod is mgh where h is the maximum height to which the centre of mass (C.M) rises

Gain in potential energy = Loss in kinetic energy

$$\therefore\ mgh = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{1}{3}ml^2\right)\omega^2$$

$$\Rightarrow\ h = \frac{\ell^2\omega^2}{6g}$$

120. (d) $I_{nn'} = \frac{1}{12}M(a^2 + a^2) = \frac{Ma^2}{6}$



Also, $DO = \frac{DB}{2} = \frac{\sqrt{2}a}{2} = \frac{a}{\sqrt{2}}$

By parallel axes the orem, moment of inertia of plate about an axis through one of its corners.

$$I_{mm'} = I_{nn'} + M\left(\frac{a}{\sqrt{2}}\right)^2 = \frac{Ma^2}{6} + \frac{Ma^2}{2}$$

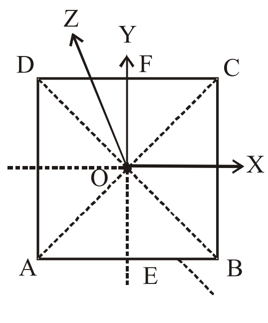
$$= \frac{Ma^2 + 3Ma^2}{6} = \frac{2}{3}Ma^2$$

121. (d) By the theorem of perpendicular axes,

$I = I_{EF} + I_{GH}$

Here, I is the moment of inertia of square lamina about an axis through O and perpendicular to its plane.

$\therefore\ I_{EF} = I_{GH}$ (By Symmetry of Figure)



$$\therefore\ I_{EF} = \frac{I}{2} \qquad ...(i)$$

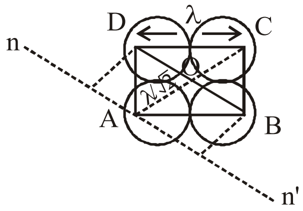
Again, by the same theorem $I = I_{AC} + I_{BD} = 2I_{AC}$

($\because\ I_{AC} = I_{BD}$ by symmetry of the figure)

$$\therefore\ I_{AC} = \frac{I}{2} \qquad ...(ii)$$

From (i) and (ii), we get, $I_{EF} = I_{AC}$.

122. (c)



$I_{nn'}$ = $M.I$ due to the point mass at B +
$M.I$ due to the point mass at D +
$M.I$ due to the point mass at C.

$$I_{nn'} = m\left(\frac{\ell}{\sqrt{2}}\right)^2 + m\left(\frac{\ell}{\sqrt{2}}\right)^2 + m\left(\sqrt{2}\ell\right)^2$$

$$\Rightarrow I_{nn'} = 2 \times m\left(\frac{\ell}{\sqrt{2}}\right)^2 + m(\sqrt{2}\ell)^2$$

$$= m\ell^2 + 2m\ell^2 = 3m\ell^2$$

123. (c) The disc may be assumed as combination of two semi circular parts. Therefore, circular disc will have twice the mass of semicircular disc.

Moment of inertia of disc = $\frac{1}{2}(2m)r^2 = Mr^2$

Let I be the moment of inertia of the uniform semicircular disc

$$\Rightarrow\ 2I = 2Mr^2 \Rightarrow I = \frac{Mr^2}{2}$$

124. (a) The moment of inertia of solid sphere A about its diameter $I_A = \frac{2}{5}MR^2$.

The moment of inertia of a hollow sphere B about its diameter $I_B = \frac{2}{3}MR^2$.

$\therefore I_A < I_B$

125. (d) We know that density $(d) = \frac{mass(M)}{volume(V)}$

$\therefore\ M = d \times V = d \times (\pi R^2 \times t)$.

The moment of inertia of a disc is given by $I = \frac{1}{2}MR^2$

$$\therefore\ I_x = \frac{1}{2}MxR_x^2 = \frac{1}{2}(d \times \pi R^2 \times t)R^2$$

$= \frac{\pi d}{2} t \times R^4$

$I_y = \frac{1}{2} M_y R_y^2 = \frac{1}{2}\left[\pi\left(4R^2\right)\left(\frac{1}{4}\right) d\right] \times (4R)^2$

$\therefore \frac{I_X}{I_Y} = \frac{t_X R_X^4}{t_Y R_Y^4} = \frac{t \times R^4}{\frac{t}{4} \times (4R)^4} = \frac{1}{64}$

126. (a) Rotational kinetic energy $= \frac{1}{2} I\omega^2$,

Angular momentum, $L = I\omega \Rightarrow I = \frac{L}{\omega}$

$\therefore K.E. = \frac{1}{2}\frac{L}{\omega} \times \omega^2 = \frac{1}{2} L\omega$

$L' = \frac{2K.E}{\omega}$

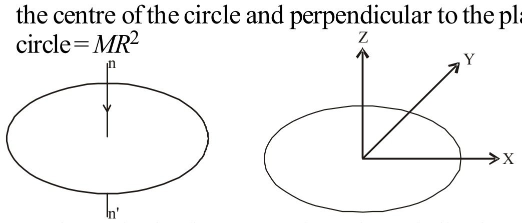
When ω is doubled and K.E is haled.

New angular momentum,

$L' = \frac{\frac{2K.E}{2}}{2\omega}$

$\Rightarrow \therefore L' = \frac{L}{4}$

127. (a) M. I of a circular wire about an axis nn' passing through the centre of the circle and perpendicular to the plane of the circle $= MR^2$



As shown in the figure, X-axis and Y-axis lie along diameter of the ring . Using perpendicular axis theorem

$I_X + I_Y = I_Z$

Here, I_X and I_Y are the moment of inertia about the diameter.

$\Rightarrow 2 I_X = MR^2$ $[\because I_X = I_Y$ (by symmetry) and $I_Z = MR^2]$

$\therefore I_X = \frac{1}{2} MR^2$

128. (c) Moment of inertia of circular disc

$I_1 = \frac{1}{2} MR^2$

When two small sphere are attached on the edge of the disc, the moment of inertia becomes

$I_2 = \frac{1}{2} MR^2 + 2mR^2$

When two small spheres of mass m are attached gently, the external torque, about the axis of rotation, is zero and therefore the angular momentum about the axis of rotation is constant.

$\therefore I_1\omega_1 = I_2\omega_2 \Rightarrow \omega_2 = \frac{I_1}{I_2}\omega_1$

$\therefore \omega_2 = \frac{\frac{1}{2} MR^2}{\frac{1}{2} MR^2 + 2mR^2} \times \omega_1 = \frac{M}{M + 4m}\omega_1$

129. (a) $K.E$ of the sphere = translational $K.E$ + rotational $K.E$

$= \frac{1}{2} mv^2 + \frac{1}{2} I\omega^2$

Where, I = moment of inertia,

ω = Angular, velocity of rotation

m = mass of the sphere

v = linear velocity of centre of mass of sphere

$\because$ Moment of inertia of sphere $I = \frac{2}{5} mR^2$

$\therefore K.E = \frac{1}{2} mv^2 + \frac{1}{2} \times \frac{2}{5} mR^2 \times \omega^2$

$\Rightarrow K.E = \frac{1}{2} mv^2 + \frac{1}{2} \times \frac{2}{5} mR^2 \times \left(\frac{v}{R}\right)^2 \left(\because \omega = \frac{v}{R}\right)$

$\Rightarrow KE = \frac{1}{2}\left(\frac{2}{5} mR^2 + mR^2\right)\left(\frac{v}{R}\right)^2$

$\Rightarrow KE = \frac{1}{2} mR^2 \times \frac{7}{5} \times \frac{v^2}{R^2} = \frac{7}{10} \times \frac{1}{2} \times \frac{25}{10^4}$

$\Rightarrow KE = \frac{35}{4} \times 10^{-4}$ joule

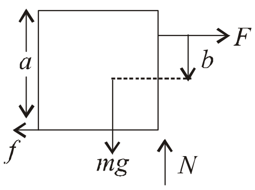
$\Rightarrow$ KE $= 8.75 \times 10^{-4}$ joule

130. (50) For the box to be slide

$F = \mu mg = 0.4$ mg

For no toppling

$F\left(\frac{a}{2} + b\right) \le mg\frac{a}{2}$



$\Rightarrow 0.4\, mg\left(\frac{a}{2} + b\right) \le mg\frac{a}{2}$

$\Rightarrow 0.2\, a + 0.4\, b \le 0.5\, a$

$\Rightarrow \frac{b}{a} \le \frac{3}{4}$

i.e. $b \le 0.75\, a$ but this is not possible.

As the maximum value of b can be equal to $0.5a$.

$\Rightarrow \frac{100b}{a} = 50$

131. (c) For sphere,

$\frac{1}{2} mv^2 + I\omega^2 = \frac{1}{2} mgh$

or $\frac{1}{2} mv^2 + \frac{1}{2}\left(\frac{2}{5} mR^2\right)\frac{v^2}{R^2} = mgh$ or $h = \frac{7v^2}{10g}$

For cylinder

$\frac{1}{2} mv^2 + \frac{1}{2}\left(\frac{mR^2}{2}\right) = mgh'$

or $h' = \dfrac{3v^2}{4g}$

$\therefore \quad \dfrac{h}{h'} = \dfrac{7v^2/10g}{3v^2/4g} = \dfrac{14}{15}$

132.(Bonus) $mgh = \dfrac{1}{2}mv_{cm}^2 + \dfrac{1}{2}I_{cm}\omega^2$

$= \dfrac{1}{2}mv_{cm}^2 + \dfrac{1}{2}I_{cm}\left(\dfrac{v_{cm}}{R}\right)^2$

$= \dfrac{1}{2}\left(m + \dfrac{I_{cm}}{R^2}\right)v_{cm}^2$

For ring : mgh $= \dfrac{1}{2}\left(m + \dfrac{mR^2}{R^2}\right)v_{cm}^2$

$\therefore h = \dfrac{v_{cm}^2}{g}.$

For solid cylinder, $mgh = \dfrac{1}{2}\left(m + \dfrac{mR^2}{2R^2}\right)v_{cm}^2$

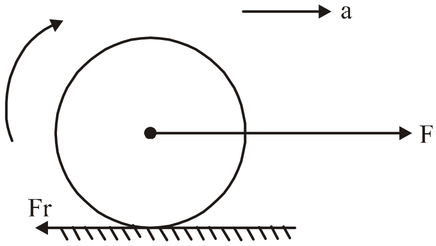
$\therefore h = \dfrac{3v_{cm}^2}{4g}$

For sphere, $mgh = \dfrac{1}{2}\left(m + \dfrac{2}{5}\dfrac{mR^2}{R^2}\right)v_{cm}^2$

$\therefore h = \dfrac{7v_{cm}^2}{10g}$

Ratio of heights $1 : \dfrac{3}{4} : \dfrac{7}{10} \Rightarrow 20 : 15 : 14$

133. (d)



$F - fr = ma$ ...(i)

$f_r R = I\alpha = \dfrac{mR^2}{2}\alpha$ ...(ii)

for pure rolling

$a = \alpha R$ ...(iii)

from (1) (2) and (3)

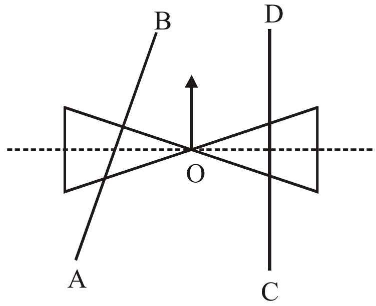
$F - \dfrac{mR\alpha}{2} = m\alpha R$

$F = \dfrac{3}{2}mR\alpha$

$\alpha = \dfrac{2F}{3mR}$

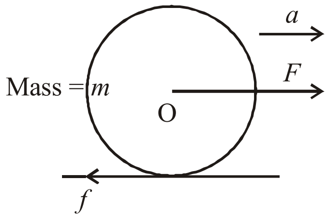
134. (c) As shown in the diagram, the normal reaction of AB on roller will shift towards O.

This will lead to tending of the system of cones to turn left.



135. (c) From figure,

$ma = F - f$ ....(i)



And, torque $\tau = I\alpha$

$\dfrac{mR^2}{2}\alpha = fR$

$\dfrac{mR^2}{2}\dfrac{a}{R} = fR \quad \left[\because \alpha = \dfrac{a}{R}\right]$

$\dfrac{ma}{2} = f$ ...(ii)

Put this value in equation (i),

$ma = F - \dfrac{ma}{2}$ or $F = \dfrac{3ma}{2}$

136.(d) As we know,

Acceleration, $a = \dfrac{mg\sin\theta}{m + \dfrac{I}{r^2}}$

For cylinder, $a_c = \dfrac{M_c.g.\sin\theta_c}{M_c + \dfrac{1}{2}\dfrac{M_cR^2}{R^2}} = \dfrac{M_c.g.\sin\theta_c}{M_c + \dfrac{M_cR^2}{2R^2}}$

or, $a_c = \dfrac{2}{3}g\sin\theta_c$

For sphere,

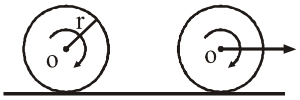
$a_s = \dfrac{M_s g\sin\theta_s}{M_s + \dfrac{I_s}{r^2}} = \dfrac{M_s g\sin\theta_s}{M_s + \dfrac{2}{5}\dfrac{MR^2}{R^2}}$

or, $a_s = \frac{5}{7} g \sin\theta_s$

given, $a_c = a_s$

i.e., $\frac{2}{3} g \sin\theta_c = \frac{5}{7} g \sin\theta_s$

$\therefore \quad \frac{\sin\theta_c}{\sin\theta_s} = \frac{\frac{5}{7}g}{\frac{2}{3}g} = \frac{15}{14}$

137. (c) 

From conservation of angular momentum about any fix point on the surface,

$mr^2\omega_0 = 2mr^2\omega$

$\Rightarrow \quad \omega = \omega_0/2 \Rightarrow v = \frac{\omega_0 r}{2} \quad [\because v = r\omega]$

138. (b) Velocity of the tennis ball on the surface of the earth or ground

$v = \sqrt{\frac{2gh}{1+\frac{k^2}{R^2}}}$ (where k = radius of gyration of spherical shell = $\sqrt{\frac{2}{3}}R$)

Horizontal range AB = $\frac{v^2 \sin 2\theta}{g}$

$$= \frac{\left(\sqrt{\frac{2gh}{1+k^2/R^2}}\right)^2 \sin(2\times 30°)}{g} = 2.08\,\text{m}$$

139. (b) When body rolls dawn on inclined plane with velocity V_0 at bottom then body has both rotational and translational kinetic energy.

Therefore, by law of conservation of energy,

$P.E. = K.E_{trans} + K.E_{rotational}$

$= \frac{1}{2}mV_0^2 + \frac{1}{2}I\omega^2$

$= \frac{1}{2}mV_0^2 + \frac{1}{2}mk^2\frac{V_0^2}{R_0^2}$ ...(i)

$\left[\because I = mk^2, \omega = \frac{V}{R_0}\right]$

When body is sliding down then body has only translatory motion.

$\therefore P.E. = K.E_{trans}$

$= \frac{1}{2}m\left(\frac{5}{4}v_0\right)^2$ ...(ii)

Dividing (i) by (ii) we get

$$\frac{P.E.}{P.E.} = \frac{\frac{1}{2}mv_0^2\left[1+\frac{K^2}{R_0^2}\right]}{\frac{1}{2}\times\frac{25}{16}\times mV_0^2} = \frac{25}{16} = 1+\frac{K^2}{R_0^2} \Rightarrow \frac{K^2}{R_0^2} = \frac{9}{16}$$

or, $K = \frac{3}{4}R_0$.

140. (d) Minimum velocity for a body rolling without slipping

$$v = \sqrt{\frac{2gh}{1+\frac{K^2}{R^2}}}$$

For solid sphere, $\frac{K^2}{R^2} = \frac{2}{5}$

$$\therefore \quad v = \sqrt{\frac{2gh}{1+\frac{K^2}{R^2}}} = \sqrt{\frac{10}{7}gh}$$

141. (b) Acceleration of the body rolling down an inclined plane is given by.

$$a = \frac{g\sin\theta}{1+\frac{I}{MR^2}}$$