Lecture - 13

Four Cases of complete mismatch on a line ($|\Gamma| = 1$):-

Case-(1):-

$$Z_L = jR_0 = \text{Inductive load}$$

$$Z_0 = R_0 = \text{lossless line}$$

$$|\Gamma| = 1$$

$V_f \rightarrow$ Sin

$V_r \rightarrow$ cos

$$\Gamma = \frac{jR_0 - R_0}{jR_0 + R_0} = \frac{j-1}{j+1}$$

$$= \frac{\sqrt{2}\,\angle 135}{\sqrt{2}\,\angle 45^\circ}$$

$$= 1\,\angle 90^\circ = j$$

Case-(II) :-

$Z_L = -jR_0$ = capacitive - Load

$Z_0 = R_0$ = lossless line

$$\Gamma = \frac{-jR_0 - R_0}{-jR_0 + R_0} = \frac{-j-1}{-j+1}$$

$$= \frac{\sqrt{2}\angle 45^\circ}{\sqrt{2}\angle 135} = -j$$

Note:-
Real Power dissipation in any resistive load is always zero.

Case-(III) :-

$Z_L = 0 \Rightarrow$ S.C load

$Z_0 = R_0$ = lossless line

$$\Gamma = \frac{0 - Z_0}{0 + Z_0} = -1 = 1\angle 180^\circ$$

$V_f \to \sin$

$V_r = -\sin$

$V_f + V_r = 0$

Voltage at S.C is always zero

Case-(IV) :-

$Z_L = \infty \Rightarrow$ O.C load

$Z_0 = R_0$ = lossless line

$$\Gamma = \frac{1 - Z_0/Z_L}{1 + Z_0/Z_L} = 1\angle 0$$

$V_f + V_r = V_{max}$

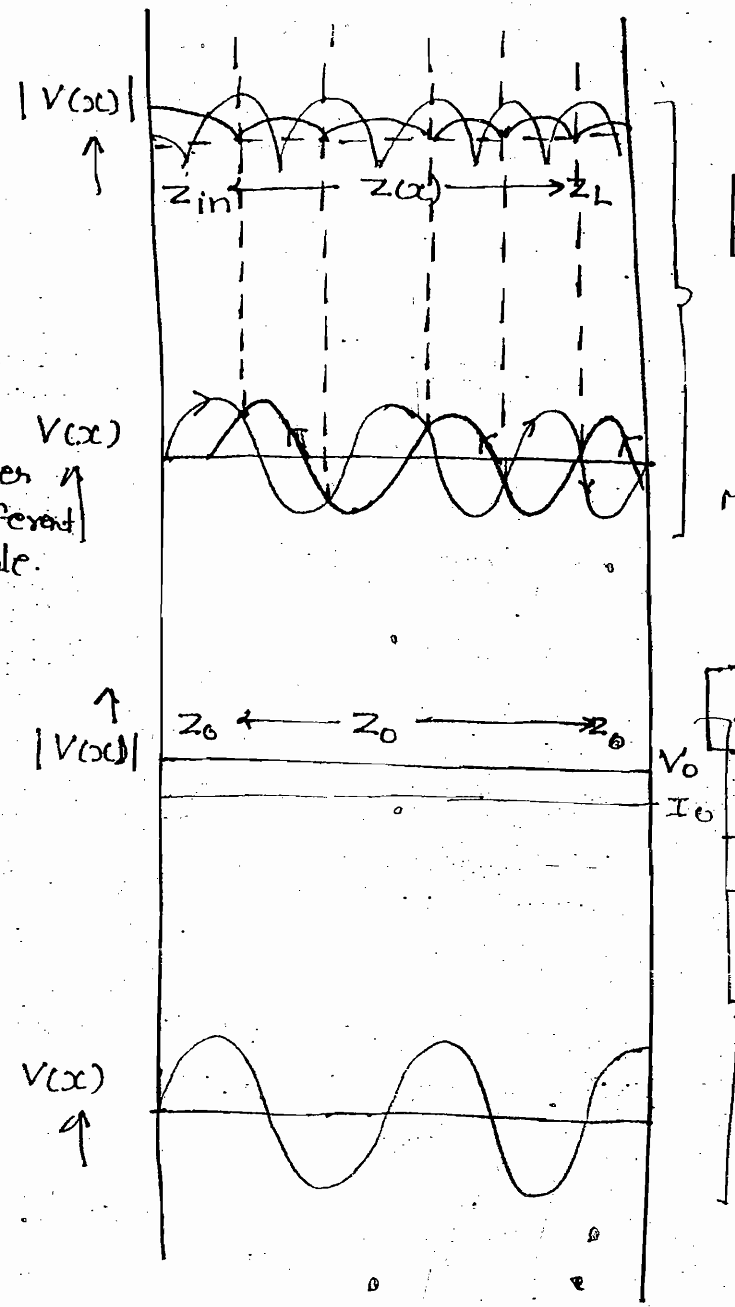
Voltage at O.C is always maximum

$|\Gamma| = 1$

$V_f \to \sin$

$V_r = \sin$

Standing Wave and SWR :-



These two waves have different amplitude.

$$\beta x - (-\beta x + \theta) = 2n\pi$$

$n = 0, 1, 2 \ldots$ |V(x)|

$$\boxed{2\beta x_{max} = 2n\pi + \theta}$$

Voltage Maxima
→ current minima
→ Impedance maxima

Mismatched line

$$\boxed{2\beta x_{min} = (2n+1)\pi + \theta}$$

Voltage minimas
→ current maximas
→ Impedance minima.

Matched line.

$$V(x) = V_0 e^{j\beta x} + V_0 |\Gamma| e^{j\theta} e^{-j\beta x}$$
$$= V_0 e^{j\beta x} + V_0 |\Gamma| e^{j(-\beta x + \theta)}$$

The magnitude plot of a single harmonic matched line as a constant value straight line graph. The magnitude plot of two harmonics mismatched line as shown below :-

$|V(x)| = V_{max} = V_0 + V_0 |\Gamma|$ when they are inphase

$$\beta x - (-\beta x + \theta) = 2n\pi$$

$n = 0, 1, 2$ |V(x)| ↑

$$\boxed{2\beta x_{max} = 2n\pi + \theta}$$

Voltage Maxima Positions.

$|V(x)| = V_{min} = V_0 - V_0|\Gamma|$ when they are out of phase

$$\boxed{2\beta x_{min} = (2n+1)\pi + \theta}$$

$\longrightarrow$ Voltage Minima

$\rightarrow$ The magnitude plot of two harmonics travelling in opposite direction having periodic interference results in addition and cancelation leading to standing wave formation

Note 1 :—

The current also has identical standing wave pattern with the voltage maxima coincide with current minima

Note 2 :—

As the impedance is different at different point of line voltage and current distribution is also extremely spread on the line such that

$$Z_{max} = \frac{V_{max}}{I_{min}}$$

$$Z_{min} = \frac{V_{min}}{I_{max}}$$

Note 3 :-

$$\left.\begin{array}{l} \beta x - \frac{2\pi}{\lambda}x - \text{Periodic} - 2\pi \\ x \rightarrow \text{Periodic} - \lambda \end{array}\right\} \text{Harmonic}$$

$$\left.\begin{array}{l} 2\beta x - \frac{2\pi}{\lambda/2}x \rightarrow \text{Periodic} - 2\pi \\ x \longrightarrow \text{Periodic} - \lambda/2 \end{array}\right\} \text{Standing wave}$$

Maxima to Maxima distance on standing wave is $\lambda/2$.

Standing Wave Ratio (SWR):-

It can be VSWR or CSWR

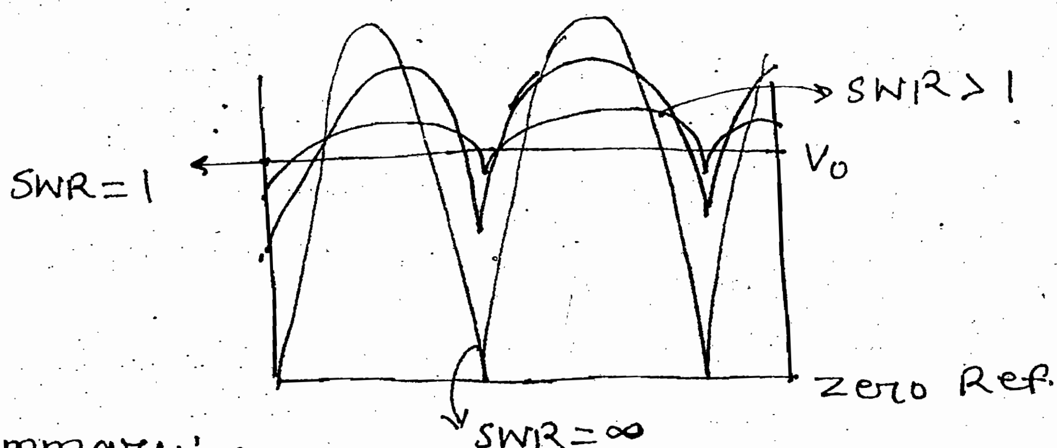
$$= \frac{V_{max}}{V_{min}} \text{ or } \frac{I_{max}}{I_{min}}$$

$$\boxed{SWR = \frac{V_0 + V_0|\Gamma|}{V_0 - V_0|\Gamma|} = \frac{1+|\Gamma|}{1-|\Gamma|}}$$

$$0 \leq |\Gamma| \leq 1$$

$$\boxed{1 \leq SWR < \infty}$$

Matched line — Complete Mismatch



Summary:-

$Z_0 \downarrow \uparrow$
$V_0 \uparrow \downarrow$ } SWR
$I_0 \uparrow \downarrow$

SWR is a measure of mismatch on the line

$$Z_{max} = \frac{V_{max}}{I_{min}}$$

$$= \frac{V_0 + V_0|\Gamma|}{I_0 - I_0|\Gamma|}$$

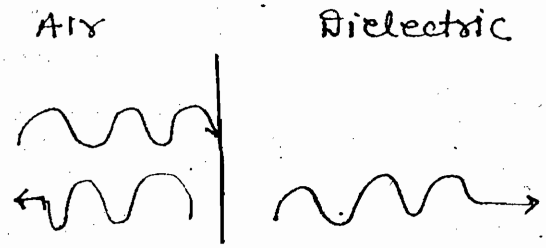
$$Z_{max} = Z_0 \cdot SWR$$

$$Z_{min} = Z_0/SWR$$

Workbook :-

8.

$$ESWR = 5 = \frac{1+|\Gamma|}{1-|\Gamma|}$$



$$|\Gamma| = \frac{5-1}{5+1} = \frac{2}{3}$$

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{2}{3}\angle\theta = \pm\frac{2}{3}$$

θ → 0 or 180°

η_1 → 120π (real)

η_2 → $\frac{120\pi}{\sqrt{\epsilon_R}}$ (real)

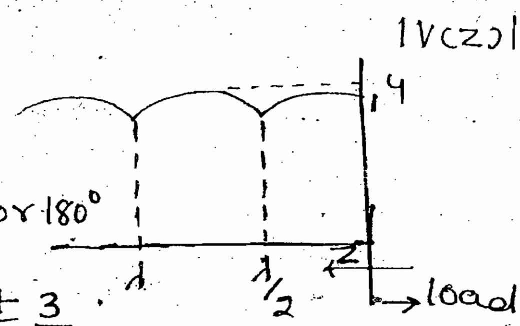
As $\eta_2 < \eta_1$

$$\Gamma = -\frac{2}{3} = \frac{\eta_2 - 120\pi}{\eta_2 + 120\pi} \Rightarrow \eta_2 = 24\pi$$

9.

VSWR = 4

$$|\Gamma| = \frac{4-1}{4+1} = \frac{3}{5}$$



$$\Gamma = \frac{Z_L - 50}{Z_L + 50} = \frac{3}{5}\angle\theta = \pm\frac{3}{5}$$

θ → 0° or 180°

Γ → real, $Z_L - 50$ → real, $Z_L + 50$ → real

Vmin is exactly at the load, Zmin is at the load

$Z_L < Z_0$

$\Rightarrow$ Γ is negative

$$\Gamma = -\frac{3}{5} = \frac{Z_L - 50}{Z_L + 50} \Rightarrow Z_L = 12.5\,\Omega$$

→ 1st voltage maxima occurs at $\lambda/4$

$n=0$

$$2 \cdot \frac{2\pi}{\lambda} \cdot \frac{\lambda}{4} = 2n\pi + \theta \Rightarrow \boxed{\theta = \pi}$$

Note:-

→ If Z_L, Z_0 are real

$$Z_L > Z_0$$

Γ_{is} is real & positive

Γ's phase is 0

$$2\beta x_{max} = 2n\pi + \theta \Rightarrow x_{max} = 0$$

First voltage maxima exactly occurs

eg:- OC line where $\Gamma = +1$

→ If Z_L, Z_0 are real

$$Z_L < Z_0$$

Γ is real and negative

First voltage minima exactly occurs

eg:- SC line where $\Gamma = -1$

→ $$VSWR = \frac{1 + \left|\frac{Z_L - Z_0}{Z_L + Z_0}\right|}{1 - \left|\frac{Z_L - Z_0}{Z_L + Z_0}\right|}$$

If Z_L, Z_0 are real & $Z_L > Z_0$

$$VSWR = \frac{1 + \frac{Z_L - Z_0}{Z_L + Z_0}}{1 - \frac{Z_L - Z_0}{Z_L + Z_0}}$$

$$\boxed{VSWR = \frac{Z_L}{Z_0}}$$

If Z_L, Z_0 are real

$$Z_L < Z_0$$

$$\boxed{VSWR = ?}$$

20 S 24. $\Gamma(\lambda/8) = ?$

$|\Gamma|$ Γ at load $= 0.6 \angle 60^\circ$

Γ $\Gamma(x) = \dfrac{Z(x) - Z_0}{Z(x) + Z_0} = \dfrac{Z_L - Z_0}{Z_L + Z_0} e^{-}$

$$\Gamma(\lambda/8) = \Gamma(0)\, e^{-j2\beta \cdot \frac{\lambda}{8}}$$

$$= 0.6\, e^{j60}\, e^{-j2 \cdot \frac{2\pi}{\lambda} \cdot \frac{\lambda}{8}}$$

1st

21. ← 17 $= 0.6\, e^{j60}\, e^{-j90}$

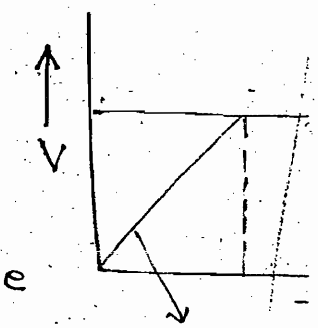
7cm

$n=0$ $= 0.6 \angle -30^\circ$

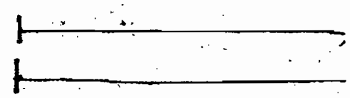
2 25. 30 V D.C Battery

D.C Transients

During the transient state even DC has time variations and all AC fundamentals are applicable



$V_f + V_r =$ Adding



$\Gamma = \dfrac{10}{30} = \dfrac{Z_L - 50}{Z_L + 50}$

$\Rightarrow Z_L = 100\,\Omega$

12. ε

$V_f =$

$\leftarrow V_r$

$I_L = \dfrac{30}{Z_L}\,\Omega$
(Steady state)

$n=0$

2. 2 $= \dfrac{30}{100}$

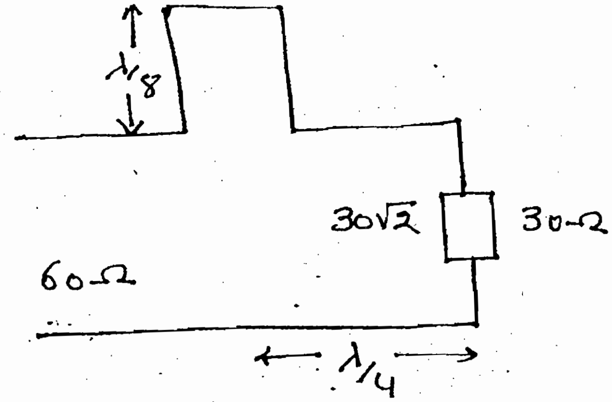
$= 0.3$ Amp.

26.

$$VSWR = \frac{1+|\Gamma|}{1-|\Gamma|}$$

$$= \frac{\sqrt{17}+1}{\sqrt{17}-1}$$

$$= 1.63$$



$$Z_{L_1} = Z_{in2} + Z_{in3}$$

$$= 60 + j30\,\Omega$$

$$Z_{in2} = j30\,\Omega$$

$$Z_{in3} = \frac{(30\sqrt{2})^2}{30} = 60\,\Omega$$

$$\Gamma = \frac{Z_{L_1} - jZ_0}{Z_{L_1} + jZ_0} = \frac{60 + j30 - 60}{60 + j80} = \frac{j}{j+4}$$

$$|\Gamma| = \frac{1}{\sqrt{17}}$$

27.

$$V_f = 25 \sin(bx - 75^\circ)$$

$$\Gamma = 0.6 \angle -30^\circ$$

$$V_r = ?$$

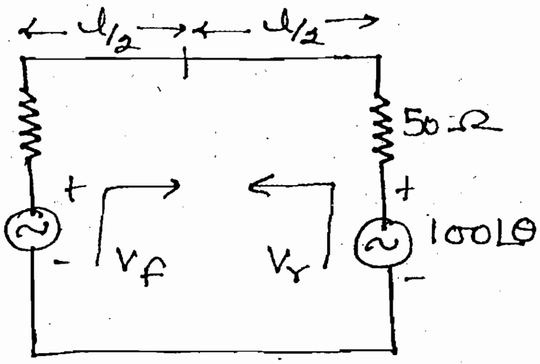
$$\frac{|V_r|}{|V_f|} = 0.6 \qquad |V_r| = 15$$

$$V_r \text{ phase} = V_f \text{ phase} + \Gamma_{phase}$$

$$= -75^\circ - 30 = -105^\circ$$

$$V_r = 15 \sin(-bx - 105^\circ)$$

28. V_f & V_r are inphase and add up at centre



I_f & I_r cancel at the centre

$$|I_f| = |I_r|$$

$$\Rightarrow I_{min} = 0$$

$$Z_{max} = \frac{V_{max}}{I_{min}} = \infty$$

29. $$VSWR = \frac{Z_L}{Z_0} = \frac{75}{50} = 1.5$$

Smith Chart (Circle Diagrams) :-

→ It is used to calculate Γ and VSWR for a known Z_L & Z_0

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$= \frac{Z_L/Z_0 - 1}{Z_L/Z_0 + 1}$$

→ It is used normalised or relative ratio of Z_L/Z_0 for its calculation

→ It is a rectangular graph or polar plot of Γ_r Vs Γ_i having two families ~~of~~ on circles

→ constant R circles

→ constant X circles.

where $R = \text{Real}\left[Z_L/Z_0\right]$

$X = \text{Imag}\left[Z_L/Z_0\right]$

$$\Gamma_r + j\Gamma_i = \frac{R+jX-1}{R+jX+1} = \frac{(R-1)+jX}{(R+1)+jX}$$

$$\left(\Gamma_r - \frac{R}{R+1}\right)^2 + \Gamma_i^2 = \left(\frac{1}{R+1}\right)^2$$ constant R circles Equation.

$$(\Gamma_r - 1)^2 + \left(\Gamma_i - \frac{1}{X}\right)^2 = \left(\frac{1}{X}\right)^2$$ constant X circles Equation.

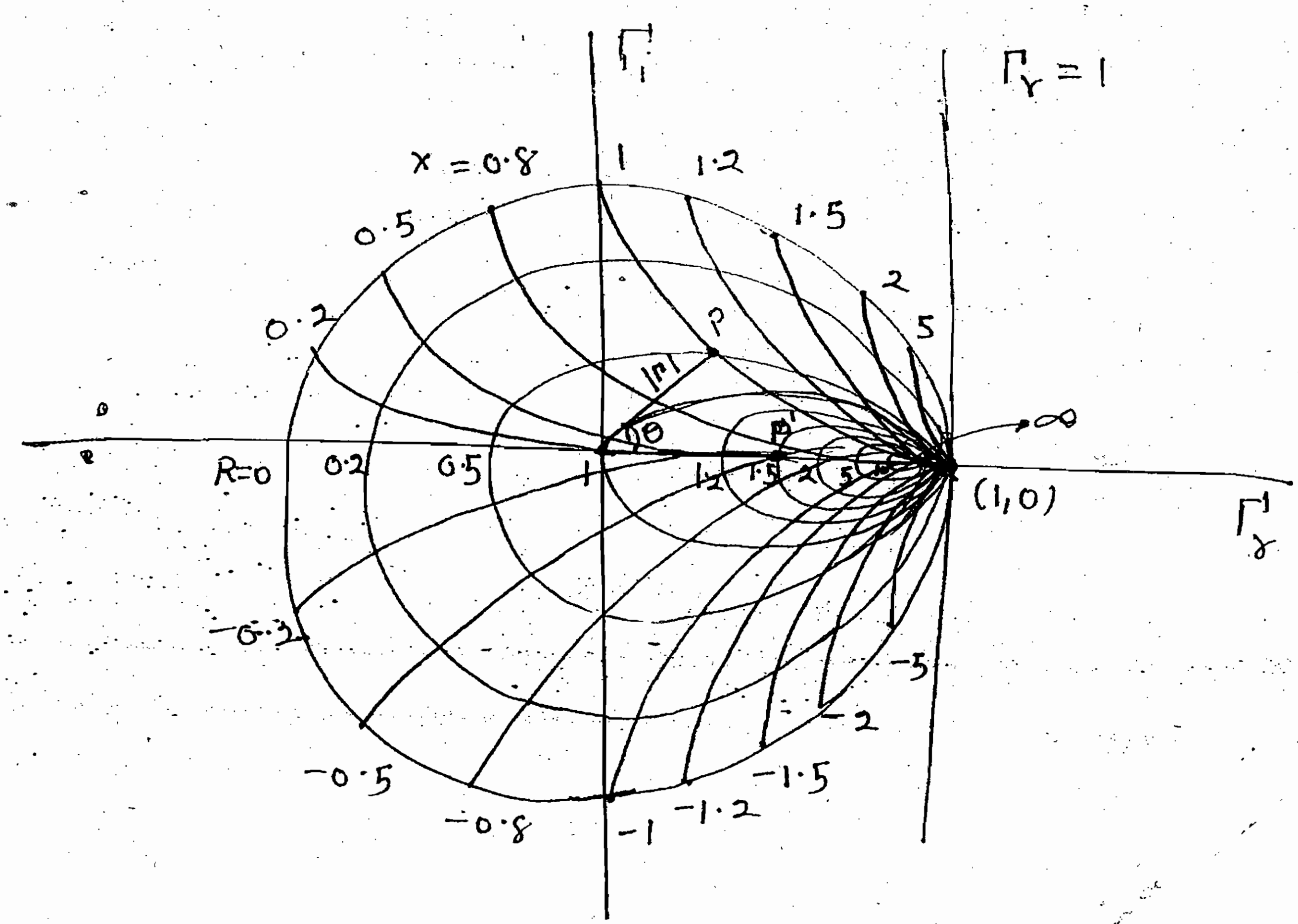
Properties of constant R circles:-

(a) Centres – $\left(\frac{R}{R+1}, 0\right)$

(b) Radius – $\left(\frac{1}{R+1}\right)$

(c) They all pass through $(1,0)$

(d) Range $(0, \infty)$



1. The R=0 circle is the biggest possible R circle and it is pheriphary or boundary of smith chart.

2. All circles R=0 to 1 extend into the four quadrants the circles with R>1 confined in the 1st and IV quadrants

3. All the R-circles are concurrent with their centres on a straight line and have a common tangent $\Gamma_r = 1$ axis

Properties of constant X-circles:-

a) Centres - $(1, \frac{1}{x})$ (b) Radius - $(1/x)$

c) They all pass through (1,0)

d) Range $(-\infty, \infty)$

1. x=0 circle is the biggest possible X circle and it is Γ_r axis

2. All circles with X=0 to 1 extend into the first two quadrant but circles with X>1 confined in the 1st quadrant

3. All the X-circles are concurrent and have a common tangent with Γ_r axis

Note:-

The centres of R circles lie on the common tangent of X-circle and vice-versa. Hence R and X constitute a pair of orthogonal families of circles ~~Be~~

2. A horizontal movement on the X=0 circle ~~has~~ as R value increasing and a clockwise movement on the R=0 circle as X increasing inductively (+ry)

Calculation of Γ :-

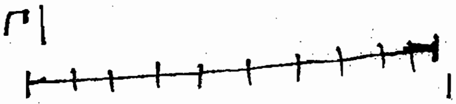
For a known Z_L and Z_0 calculate

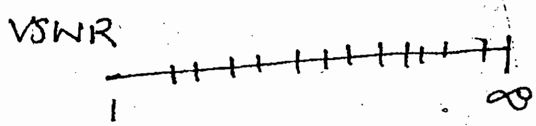
$\frac{Z_L}{Z_0} = R + jX$ and locate the corresponding point of intersection of R and X circle

→ Join the point P to origin the OP length $= |\Gamma|$

→ OP segments inclination with Γ_r axis is Γ's phase

→ End





Calculation of VSWR :-

→ For a known $|\Gamma|$ identify a point P' on the +ve $|\Gamma|$ real axis such that OP' length $= |\Gamma|$. The R circle value at P' is VSWR

$$R + jX \Big|_{X=0} = \frac{Z_L}{Z_0} = VSWR$$

Γ_r axis

Identify following points on the Smith's Chart :-

(1) $Z_L = jR_0$, $Z_0 = R_0$

$$\frac{Z_L}{Z_0} = \underset{R=0}{0} + j\underset{X=1}{1}$$

$$\Gamma = j$$

(11) $Z_L = -jR_0$

$Z_0 = R_0$

bottom

(III) $Z_L = 0$

$Z_0 = R_0$

$$\frac{Z_L}{Z_0} = \underbrace{0}_{R} + j\,\underbrace{0}_{X}$$

$\Gamma = -1$ Extremely left

(IV) $Z_L = \infty$

$Z_0 = R_0$

Extremely Right

(V) $Z_L = R_0$

$Z_0 = R_0$

$\longrightarrow$ centre $\Gamma = 0$