Circles

Exercise 12.1

Q. 1. Name the following parts from the adjacent figure where 'O' is the centre of the circle.

i. $\overline{AO}$

ii. $\overline{AB}$

iii. BC

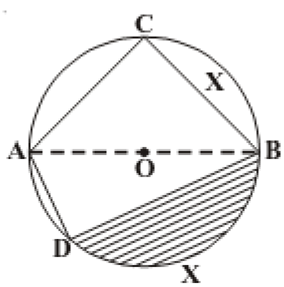
iv. $\overline{AC}$

v. DCB

vi. ACB

vii. $\overline{AD}$

viii. shaded region



Answer : i. Radius

ii. Diametre

iii. Minor arc

iv. Chord

v. Major arc

vi. Semi-circle

vii. Chord

viii. Minor segment

Q. 2. State true or false.

i. A circle divides the plane on which it lies into three parts. ( )
ii. The area enclosed by a chord and the minor arc is minor segment. ( )
iii. The area enclosed by a chord and the major arc is major segment. ( )
iv. A diameter divides the circle into two unequal parts. ( )
v. A sector is the area enclosed by two radii and a chord ( )
vi. The longest of all chords of a circle is called a diameter. ( )
vii. The mid point of any diameter of a circle is the centre. ( )

Answer : i. True

ii. True

iii. True

iv. False

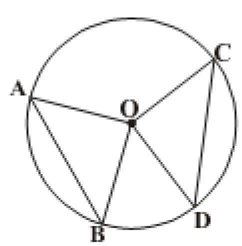
v. False

vi. True

vii. True

Exercise 12.2

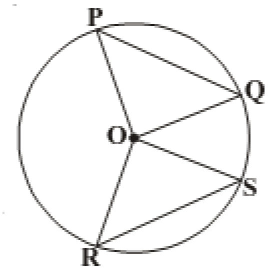
Q. 1. In the figure, if AB = CD and ∠AOB = 90° find ∠COD



Answer : We know “Angles subtended by equal chords at the center of a circle are equal”.

∴ ∠COD = ∠AOB = 90^0

Q. 2. In the figure, PQ = RS and ∠ORS = 48°. Find ∠OPQ and ∠ROS.



Answer : In ΔORS,

∠ORS = ∠OSR (radius of circle)

∴ ∠OSR = ∠ORS = 48^0

∴ ∠OSR + ∠ORS + ∠ROS = 180°

⇒ 48° + 48° + ∠ROS = 180°

⇒ 96° + ∠ROS = 180°

⇒ ∠ROS = 180°-96°

⇒ ∠ROS = 180°-96°

⇒ ∠ROS = 84°

We know "Angles subtended by equal chords at the center of a circle are equal".

∴ ∠POQ = ∠ROS = 84^0

In ΔPOQ,

∴∠POQ + ∠OPQ + ∠OQP = 180°

84^0 + x + x = 180°

84^0 + 2x = 180°

2x = 180°-84°

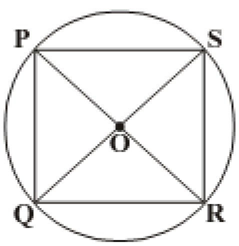
$2x = 96^{\circ}$

$x = \frac{96}{2}$

$x = 48^{\circ}$

$\therefore \angle OPQ = 48^{\circ}$

Q. 3. In the figure PR and QS are two diameters. Is PQ = RS?



Answer : Considering a circle, where PR and QS being a diameter.

Let center of circle be O.

In triangles POQ and SOR

PO = OR (Radius of circle)

OS = OQ (Radius of circle)

Angle POQ = Angle SOR (Vertically opposite angles are always equal)

Hence triangle POQ is congruent to triangle SOR (By Side Angle Side Axiom)

PQ = RS (By C.P.C.T.C)

Hence, proved.

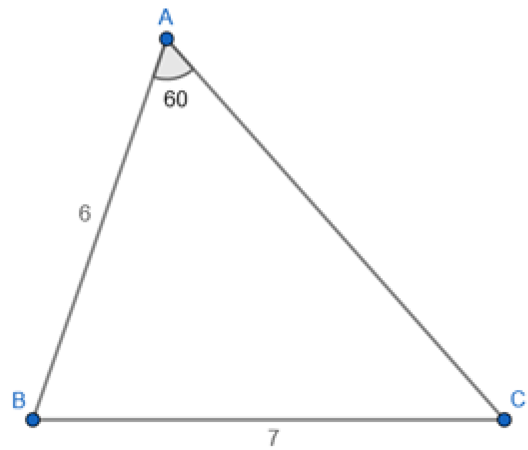
Exercise 12.3

Q. 1. Draw the following triangles and construct circumcircles for them.

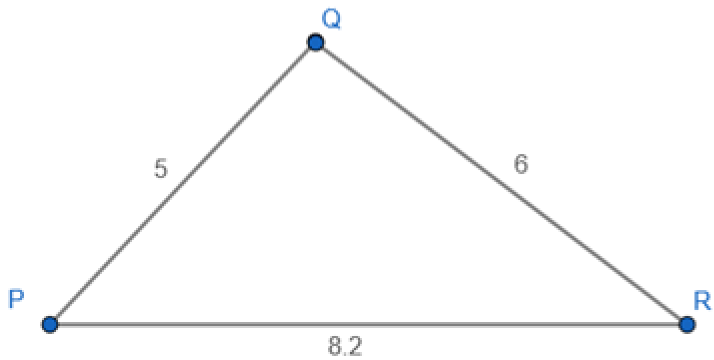
(i) In $\triangle$ ABC, AB = 6cm, BC = 7cm and $\angle A = 60°$
(iii) In $\triangle$ XYZ, XY = 4.8cm, $\angle X = 60°$ and $\angle Y = 70°$
(iii) In $\triangle$ XYZ, XY = 4.8cm, $\angle X = 60°$ and $\angle Y = 70°$

Answer :

(i)



(ii)



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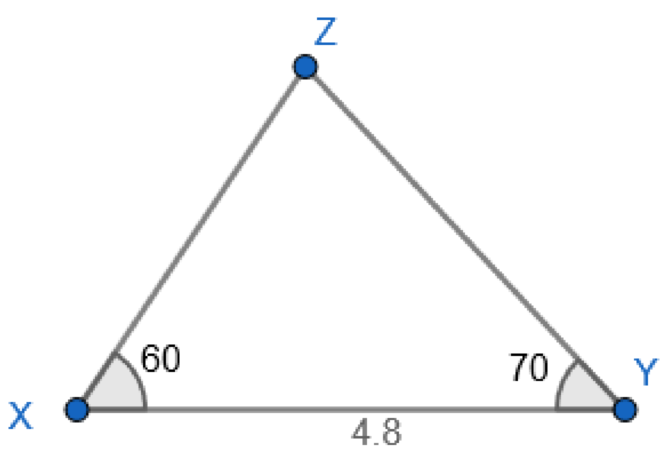
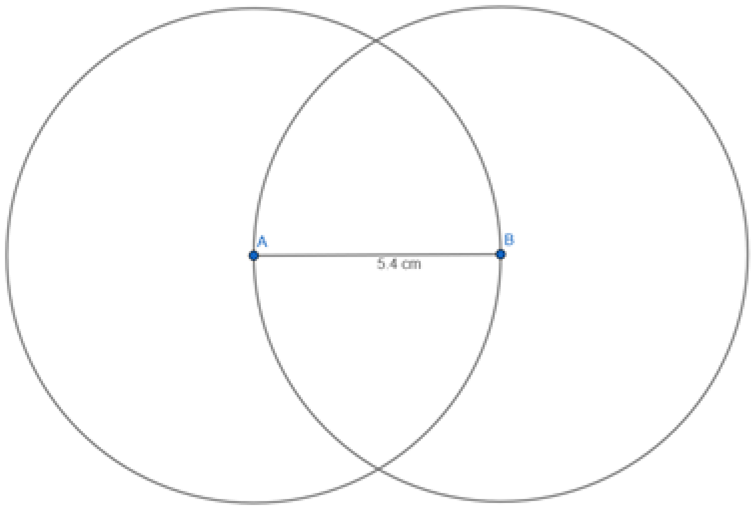


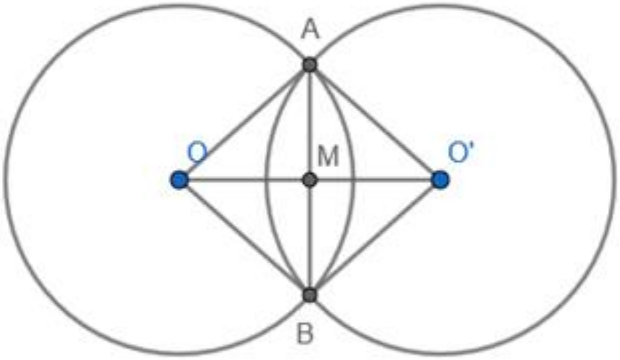
Q. 2. Draw two circles passing through A, B where AB = 5.4cm

Answer :



Q. 3. If two circles intersect at two points, then prove that their centres lie on the perpendicular bisector of the common chord.

Answer :



Let two circles O and O' intersect at two points A and B so that AB is the common chord of two circles.

OO' is the line segment joining the centers.

Let OO' intersect AB at M

Now Draw line segments OA, OB , O'A and O'B

In ΔOAO' and OBO' , we have
OA = OB (radii of same circle)
O'A = O'B (radii of same circle)
O'O = OO' (common side)
⇒ ΔOAO' ≅ ΔOBO' (SSS congruency)

⇒ ∠AOO' = ∠BOO'
⇒ ∠AOM = ∠BOM ......(i)

Now in ΔAOM and ΔBOM we have

OA = OB (radii of same circle)
∠AOM = ∠BOM (from (i))
OM = OM (common side)
⇒ ΔAOM ≅ ΔBOM (SAS congruency)
⇒ AM = BM and ∠AMO = ∠BMO

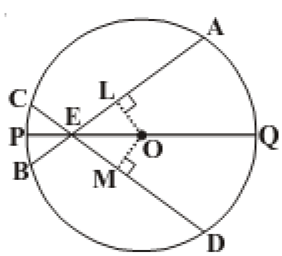
But

∠AMO + ∠BMO = 180°
⇒ 2∠AMO = 180°
⇒ ∠AMO = 90°

Thus, AM = BM and ∠AMO = ∠BMO = 90°

Hence OO' is the perpendicular bisector of AB.

Q. 4. If two intersecting chords of a circle make equal angles with diameter passing through their point of intersection, prove that the chords are equal.



Answer : Given that AB and CD are two chords of a circle, with center O intersecting at a point E.

PQ is a diameter passing through E, such that ∠ AEQ = ∠ DEQ

Draw OL ⊥ AB and OM ⊥ CD.

In right angled ΔOLE

∠LOE + 90° + ∠ LEO = 180° (Angle sum property of a triangle)
∴∠LOE = 90° – ∠LEO

= 90° – ∠AEQ = 90° – ∠DEQ

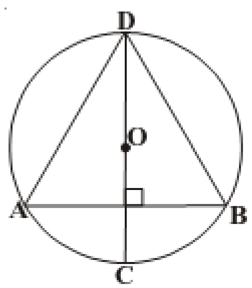
= 90° – ∠MEO = ∠MOE

In triangles OLE and OME,
∠LEO = ∠MEO
∠LOE = ∠MOE (Proved)

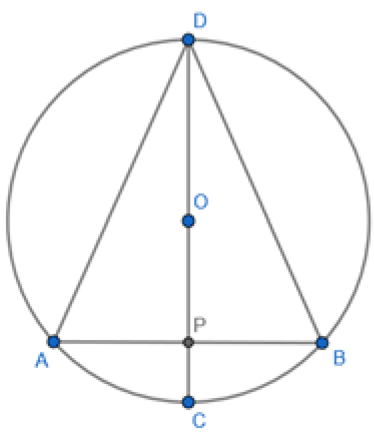
OE = OE (Common side)
∴ ΔOLE ≅ ΔOME
⇒ OL = OM (CPCT)

Thus, AB = CD

Q. 5. In the adjacent figure, AB is a chord of circle with centre O. CD is the diameter perpendicular to AB. Show that AD = BD.



Answer :



Given, AB is a chord of circle with centre O. CD is the diameter

Perpendicular to AB.

We know that line drawn from the center of a circle to the chord

Perpendicular to it bisects the chord.

∴AP = BP

In ΔADP and ΔBDP,

AP = BP

APD = BPD = 90°

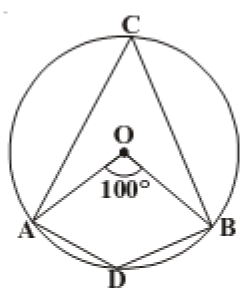
PD = PD (Common)

∴ $\Delta ADP \cong \Delta BDP$

∴ AD = BD (CPCT)

Exercise 12.4

Q. 1. In the figure, ‘O’ is the centre of the circle. ∠AOB = 100° find ∠ADB.



Answer : Given, ∠AOB = 100°

Join AB, which is a chord.

∴ $\angle ACB = \frac{\angle AOB}{2}$ (Angle subtended at the center is twice the angle subtended at circumference by the same chord)

⇒∠ ACB = 50°

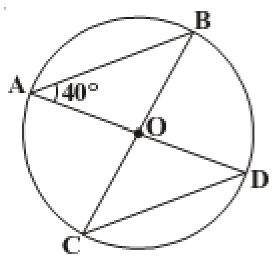
Now, ACBD is a quadrilateral and angles on the opposite sides in the quadrilateral are supplementary.

∴∠ ADB = 180°-∠ ACB

⇒∠ ADB = 180°-50°

$\Rightarrow \angle ADB = 130^\circ$

Q. 2. In the figure, ∠BAD = 40° then find ∠BCD.



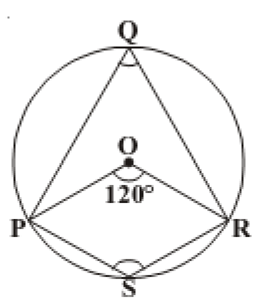
Answer : Given, ∠BAD = 40°

Join BD, which is a chord.

We know “Two or more angles subtended by the chord at the circumference are same”.

∴ ∠BCD = ∠BAD = 40°

Q. 3. In the figure, O is the centre of the circle and ∠POR = 120°. Find ∠PQR and∠PSR



Answer : Given, ∠POR = 120°

We know that “Angle subtended at the center is twice the angle subtended at circumference by the same chord”.

$\therefore \angle PQR = \frac{\angle POR}{2}$

$\Rightarrow \angle PQR = \frac{120^\circ}{2}$

$\Rightarrow \angle PQR = 60^\circ$

Now, PQRS is a quadrilateral and angles on the opposite sides in the quadrilateral are supplementary.

$\therefore \angle PSR = 180^\circ - \angle PQR$

$\Rightarrow \angle PSR = 180^\circ - 60^\circ$

$\Rightarrow \angle PSR = 120^\circ$

Q. 4. If a parallelogram is cyclic, then it is a rectangle. Justify.

Answer : Let ABCD be a cyclic parallelogram.

A rectangle is a parallelogram with one angle 90°. So, we have to prove angle 90°.

Since ABCD is a parallelogram,

$\angle A = \angle C$

In cyclic parallelogram ABCD,

$\angle A + \angle C = 180^\circ$

$\angle A + \angle A = 180^\circ$

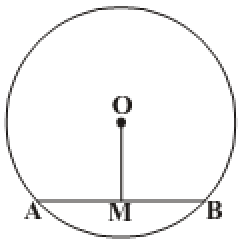
$2\angle A = 180^\circ$

$\angle A = \frac{180^\circ}{2}$

$\angle A = 90^\circ$

Hence, proved.

Q. 5. In the figure, 'O' is the centre of the circle. OM = 3cm and AB = 8cm. Find the radius of the circle



Answer : Joining OA we see that OA is radius of the circle.

Let radius (OA) be r cm.

Given, AB = 8cm

OM = 3cm

We know that “perpendicular from center divides the chord into equal parts”.

∴ AM = $\frac{AB}{2}$

⇒ AM = 4 cm

In right angled triangle OAM,

$OA^2 = OM^2 + AM^2$ (Pythagoras Thm)

⇒ $OA^2 = 3^2 + 4^2$

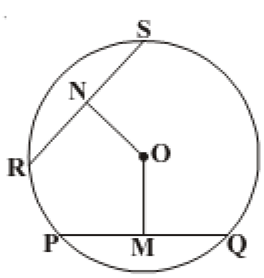
⇒ $OA^2 = 9 + 16$

⇒ $OA^2 = 25$

⇒ OA = $\sqrt{25}$

⇒ OA = 5 cm

Q. 6. In the figure, ‘O’ is the centre of the circle and OM, ON are the perpendiculars from the centre to the chords PQ and RS. If OM = ON and PQ = 6cm. Find RS.

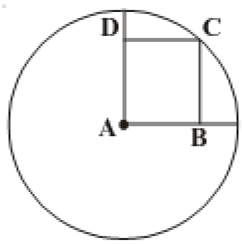


Answer : Given, OM = ON

We know that “If two chords are equidistant from the center then the two chords are equal in length”.

∴ PQ = RS = 6 cm

Q. 7. A is the centre of the circle and ABCD is a square. If BD = 4cm then find the radius of the circle.



Answer : Given, BD = 4cm

We know that “a square has equal diagonals” and BD is a diagonal.

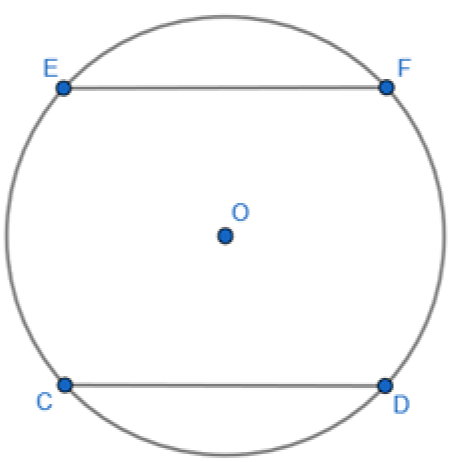
AC, which is the radius of the circle, is also a diagonal of the square.

∴ AC = BD = 4 cm

Thus, Radius of the circle = 4 cm

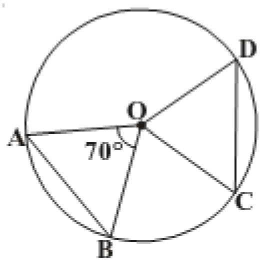
Q. 8. Draw a circle with any radius and then draw two chords equidistant from the centre.

Answer :



Circle with center O and two chords CD and EF equidistant from center.

Q. 9. In the given figure 'O' is the centre of the circle and AB, CD are equal chords. If ∠AOB = 70°. Find the angles of the △OCD.



Answer : We know "Angles subtended by equal chords at the center of a circle are equal".

∴ ∠DOC = ∠AOB = 70°

In ΔOCD,

∴∠OCD + ∠ODC + ∠DOC = 180°

$x + x + 70^{\circ} = 180^{\circ}$

$70^{0} + 2x = 180^{\circ}$

$2x = 180^{\circ}-70^{\circ}$

$2x = 110^{\circ}$

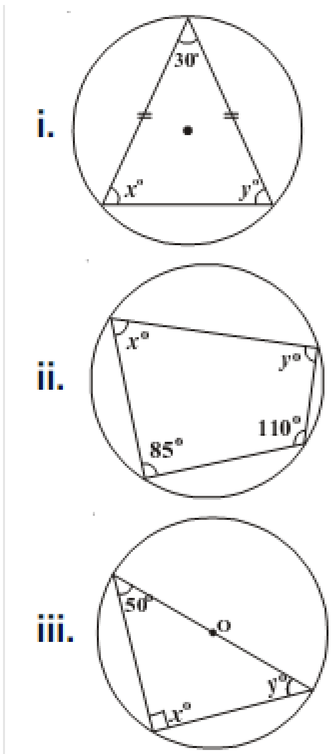
$x = \frac{110}{2}$

$x = 55^{\circ}$

∴ ∠OCD = ∠ODC = 55°

Exercise 12.5

Q. 1. Find the values of x and y in the figures given below.



Answer : (i) We know “Sum of all angles of a triangle is 180°”.

∴ $x + y + 30^\circ = 180^\circ$

Since it is an isosceles triangle, x = y.

∴ $x + x + 30^\circ = 180^\circ$

⇒ $2x + 30^\circ = 180^\circ$

⇒ $2x = 180^\circ - 30^\circ$

⇒ $2x = 150^\circ$

⇒ $x = \frac{150}{2}$

⇒ $x = 75^\circ$

∴ $x = y = 75^\circ$

(ii) We know that “Angles on the opposite sides in the cyclic quadrilateral are supplementary”.

$\therefore x + 110^\circ = 180^\circ$

$\Rightarrow x = 180^\circ - 110^\circ$

$\Rightarrow x = 70^\circ$

$\therefore y + 85^\circ = 180^\circ$

$\Rightarrow y = 180^\circ - 85^\circ$

$\Rightarrow y = 95^\circ$

(iii) Given, $x = 90^\circ$

$\therefore x + y + 50^\circ = 180^\circ$

$\Rightarrow 90^\circ + y + 50^\circ = 180^\circ$

$\Rightarrow y + 140^\circ = 180^\circ$

$\Rightarrow y = 180^\circ - 140^\circ$

$\Rightarrow y = 40^\circ$

Q. 2. Given that the vertices A, B, C of a quadrilateral ABCD lie on a circle. Also $\angle A + \angle C = 180^\circ$, then prove that the vertex D also lie on the same circle.

Answer : In a quadrilateral, if the sum of opposite angles is 180°, then it is a cyclic quadrilateral.

In quad. ABCD, A + C = 180°,

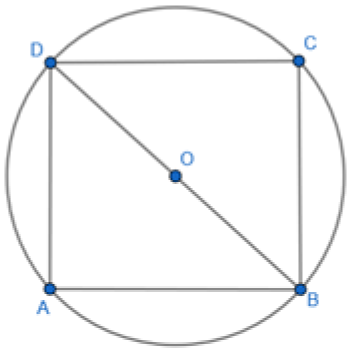
Therefore, ABCD is a cyclic quad.

In a cyclic quad., the vertices lie on the same circle. Thus, D also lies on the same circle.

Hence, proved.

Q. 3. Prove that a cyclic rhombus is a square.

Answer : To prove rhombus inscribed in a circle is a square, we need to prove that either any one of its interior angles is equal to 90° or its diagonals are equal.



∠ABD = ∠DBC = b

∠ADB = ∠BDC = a

In the figure, diagonal BD is angular bisector of angle B and angle D.

In triangle ABD and BCD,

AD = BC (sides of rhombus are equal)

AB = CD (sides of rhombus are equal)

BD = BD (common side)

△ABD ≅ △BCD. (SSS congruency)

In the figure,

2a + 2b = 180° (as, opposite angles of a cyclic quadrilateral are always supplementary)

2(a + b) = 180°

a + b = 90°

In △ABD,

Angle A = 180°-(a + b)

= 180° - 90°

= 90°

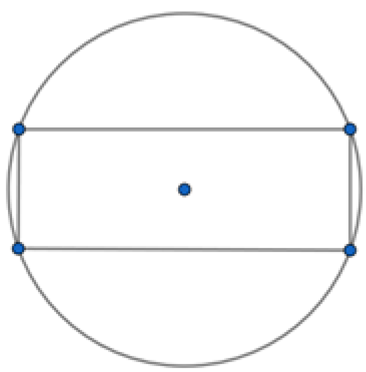
Therefore, proved that one of its interior angle is 90°

Hence, rhombus inscribed in a circle is a square.

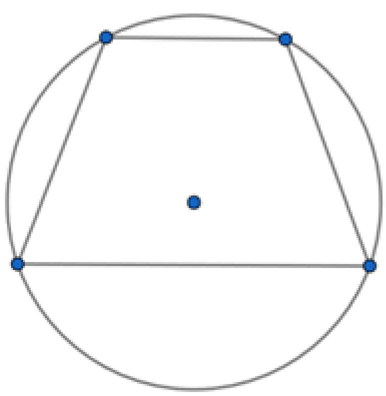
Q. 4. For each of the following, draw a circle and inscribe the figure given. If a polygon of the given type can't be inscribed, write not possible.

(a) Rectangle
(b) Trapezium
(c) Obtuse triangle
(d) Non-rectangular parallelogram
(e) Accute issosceles triangle
(f) A quadrilateral PQRS with $\overline{PR}$ as diameter.

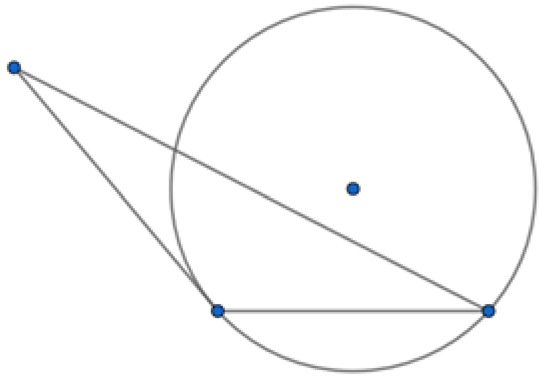
Answer : (a) Rectangle = Possible



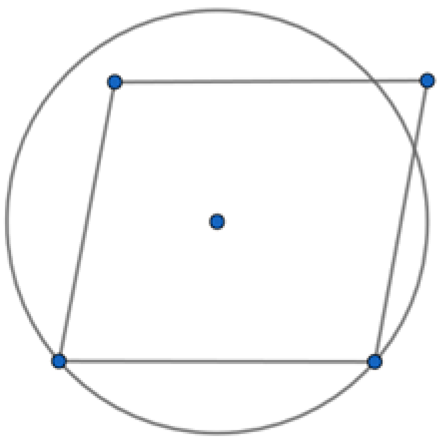
(b) Trapezium = Possible



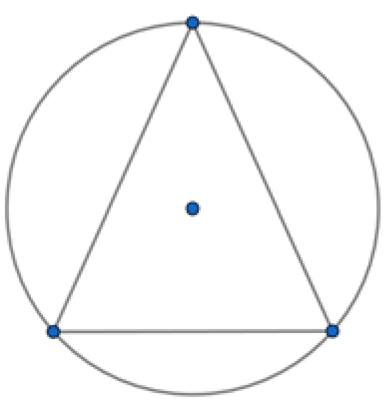
(c) Obtuse triangle = Not Possible



(d) Non-rectangular parallelogram = Not Possible



(e) Acute isosceles triangle = Possible



(f) A quadrilateral PQRS with $\overline{PR}$ as diameter = Possible