Squares and Square Roots

- If a natural number m can be expressed as n^2, where n is also a natural number, then m is a square number. The square numbers are also called **perfect squares.**

  Examples: 1, 4, 9, 16, 25, 36 … are perfect squares.

- **Prime factorization method of finding the square roots of numbers**

  The square root of 67600 can be found by prime factorization method as follows:
  The number 67600 can be prime factorized as:
  $\underline{2 \times 2} \times \underline{2 \times 2} \times \underline{5 \times 5} \times \underline{13 \times 13}$
  The numbers, 2, 2, 5, 13, occur in pairs. Therefore,
  $\sqrt{67600} = 2 \times 2 \times 5 \times 13 = 260$

  Example: Find the smallest number by which 252 can be multiplied to make it a perfect square.

  Solution: We have, $252 = \underline{2 \times 2} \times \underline{3 \times 3} \times 7$
  The number 7 does not occur in pair. Therefore, if we multiply 252 by 7, then it will become a perfect square.
  Therefore, $252 \times 7 = \underline{2 \times 2} \times \underline{3 \times 3} \times \underline{7 \times 7}$
  That is, 1764 is a perfect square and $\sqrt{1764} = 42$

- The square of a number with units digit 5, say $(a5)^2$, can be written as follows.

  $(a5)^2 = a(a + 1)\times100\times100 + 25$
  For example,
  $625^2 = 390625 = (62 \times 63)\times100\times100 + 25$
  $= 390600 + 25$

- Perfect squares exhibit some special properties.

  - The square of even numbers are even and square of odd numbers are odd.
  - The unit place of a perfect square can never be 2, 3, 7 and 8.
  - By observing the last digit of a number, we can find the last digit of the square of the number.

    - If a number has 1 or 9 at its units place, then its square ends in 1.

    - If a number ends with 4 or 6, then its square end with 6.

    - If a number ends with 2 or 8, then its square ends with 4.

    - If a number ends with 5, then its square ends with 5.

    - If a number ends with 0, then its square also ends with 0.

    - If a number ends with 3 or 7, then its square ends with 9.

  - If a square number ends with 0, then the number of zeroes at the end is even.
  - If a number ends with n number of zeroes, then its square ends with $2n$ zeroes.

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  - If a square number ends with 0, then the number of zeroes at the end is even.
  - If a number ends with n number of zeroes, then its square ends with $2n$ zeroes.

- If we add two consecutive triangular numbers, then we obtain a square number.

  For example, $10 + 15 = 25 = 5^2$
  $21 + 28 = 49 = 7^2$

- The sum of first n odd natural numbers is n^2.

For example, the sum of first 7 odd natural numbers is 49 i.e., $1 + 3 + 5 + 7 + 9 + 11 + 13 = 49 = 7^2$

Also, we can say that if a natural number cannot be expressed as a sum of successive odd natural numbers starting from 1, then it is not a perfect square.

- Square of any odd number can be expressed as the sum of two consecutive positive integers. For example, $9^2 = 81 = 40 + 41$

- We can express the product of two consecutive even or odd natural numbers as follows.

  $(a + 1) \times (a - 1) = a^2 - 1$
  For example, $47 \times 49 = (48 - 1) \times (48 + 1) = (48)^2 - 1$
  $52 \times 54 = (53 - 1) \times (53 + 1) = (53)^2 - 1$

- There are $2n$ non-perfect square numbers between the squares of the numbers, n and $(n+1)$.

  For example, $5^2 = 25$ and $6^2 = 36$
  There are 10 non-perfect square numbers between them, i.e., 26, 27, 28, 29, 30, 31, 32, 33, 34, 35.

- The square of some numbers can be found through various patterns of numbers.
- Sum of first n odd numbers = n^2

  Example: 81 can be written as the sum of first 9 odd numbers.
  We know that, $81 = 9^2$
  Therefore, we can express 9^2, i.e., 81 as the sum of first 9 odd numbers, as the sum of first n odd numbers is n^2.
  Therefore, $81 = 1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17$

- We can find the squares of numbers having more than one digit by making use of the identity,

  $(a + b)^2 = a \times (a + b) + b \times (a + b)$
  For example,
  $(45)^2 = (40 + 5)^2$
  $= (40) \times (40 + 5) + 5 \times (40 + 5)$
  $= 40^2 + 40 \times 5 + 5 \times 40 + 5^2$
  $= 1600 + 200 + 200 + 25$
  $= 2025$

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$= 390600 + 25$

- Square root is the inverse operation of square. The square root of a number is denoted by the symbol $\sqrt{\ }$.
  For example, $6^2 = 36$
  $\Rightarrow$ 36 is the square of 6.
  $\Rightarrow$ The square root of 36 is 6.
  $\Rightarrow \sqrt{36} = 6$.

- **Prime factorization method of finding the square roots of numbers**

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  Solution: We have, $252 = \underline{2 \times 2} \times \underline{3 \times 3} \times 7$
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- **Finding square root of perfect squares by division method**
  The steps of finding the square root of 1369 by division method are as follows:

  Step1: Firstly, place bars over every pair of digits starting from the digit at ones place. We obtain $\overline{13}.\overline{69}$.

  Step2: Find the largest number whose square is less than or equal to the number under the extreme left bar.
  Take this number as the divisor and the number under the extreme left bar as the dividend. Divide and obtain the remainder.

	3
3	$\overline{13}.\overline{69}$
	-9
9	4

  Step3: Bring down the number under the next bar to the right of the remainder.
  Therefore, the new dividend is 469.

Double the divisor and enter it with the blank on its right.

	3.
3	$\overline{13}.\overline{69}$
	-9
9	4.69

Step 4: Guess the largest possible digit to fill the blank, which becomes the new digit in the quotient, such that when the new digit is multiplied to the new quotient, the product is less than or equal to the dividend.
In this case, $97 \times 7 = 469$
Therefore, the quotient is 7.
Also, the remainder becomes 0 and no bar is left.
Therefore, $\sqrt{1369} = 3.7$

- **Estimating square roots of numbers**

The value of $\sqrt{180}$ to the nearest whole number can be estimated as follows:
It is known that, $13^2 = 169$ and $14^2 = 196$.
$\therefore 169 < 180 < 196$
$\Rightarrow 13^2 < \sqrt{180} < 14^2$
169 is closer to 180 than 196.
Therefore, $\sqrt{180}$ is approximately equal to 13.