CHAPTER 2

ELECTROSTATIC POTENTIAL AND CAPACITANCE

Syllabus

- *Electric potential, potential difference, electric potential due to a point charge, a dipole and system of charges; equi-potential surfaces, electrical potential energy of a system of two point charges and of electric dipole in an electrostatic field.*
- *Conductors and insulators, free charges and bound charges inside a conductor. Dielectrics and electric polarisation, capacitors and capacitance, combination of capacitors in series and in parallel, capacitance of a parallel plate capacitor with and without dielectric medium between the plates, energy stored in a capacitor.*

Chapter Analysis

List of Topic	2016	2017	2018
Electric Potential	2 Q. (1 mark) 1 Q. (3 marks)	1 Q. (1 mark) 1 Q. (3 marks)	1 Q. (3 marks)
Capacitance	1 Q. (5 marks)	1 Q. (2 marks) 1 Q. (3 marks)	

TOPIC-1
Electric Potential

Revision Notes

Electric potential

- Electric potential is the amount of work done by an external force in moving a unit positive charge from one point to another in electrostatic field without producing an acceleration.
- It is written as $V = \dfrac{W}{q}$

  where, W = work done in moving charge q through the field, q = charge being moved through the field.
- The SI units of electric potential are $\dfrac{\text{J}}{\text{C}}$, Volt, $\dfrac{\text{Nm}}{\text{C}}$

Potential difference

- Electric potential difference is defined as the amount of work done to carrying a unit charge from one point to another in an electric field.

  Electric potential difference = $\dfrac{\text{Work}}{\text{Charge}} = \dfrac{\Delta PE}{\text{Charge}} = \dfrac{W}{q}$

Between two points A and B, $W_{AB} = -V_{AB} \times q$
where, $V_{AB} = V_B - V_A$ is potential difference between A and B.

- In a region of space having an electric field, the work done by electric field dW, when positive point charge q, is displaced by a distance ds, then,

$$dW = q\vec{E}.\vec{ds}$$

$$\Delta V = V_{AB} = V_B - V_A = -\frac{W_{AB}}{q} = -\frac{\int_A^B q\vec{E}.\vec{ds}}{q} = -\int_A^B \vec{E}.\vec{ds}$$

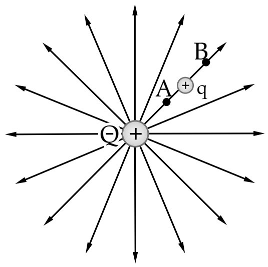
Electric potential due to point charge

- The electric potential by point charge q, at a distance r from the charge, can be written as

$$V_E = \frac{1}{4\pi\varepsilon_0}.\frac{q}{r}$$

where, ε_0 is permittivity of vacuum
- Electric potential is a scalar quantity.
- Dimension of Electric potential is $[M L^2 T^{-3} A^{-1}]$
- For a single point charge q the potential difference between A and B is given by,

$$\Delta V = V_B - V_A = -\int_A^B \vec{E}.\vec{ds} = -\int_A^B Eds\cos 0° = -E\int_A^B ds$$



where, E is the field due to a point charge, $ds = dr$, so that,

$$V_B - V_A = -\int_{r_A}^{r_B} \frac{q}{4\pi\varepsilon_0}\cdot\frac{dr}{r^2} = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r}\right]_{r_A}^{r_B} = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r_B} - \frac{1}{r_A}\right]$$

- If $r_B = \infty$, then $V_B = 0$ so,

$$V_A = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r_A} = \frac{kq}{r_A}$$

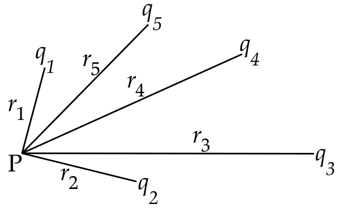
Dipole and system of charges

- Electric dipole is two charged objects having equal but opposite electric charges which are separated by a distance.
- The net potential due to a dipole at any point on its equatorial line is always zero. So work done in moving a charge on equatorial line is always zero.
- Electric potential due to dipole at a point at distance r and making an angle θ with the dipole moment p is given by, $V = \frac{1}{4\pi\varepsilon_0}\cdot\frac{p\cos\theta}{r^2} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{\vec{p}.\hat{r}}{r^2} (r >> a)$
- Potential at a point due to system of charges is the sum of potentials due to individual charges.
- In a system of charges $q_1, q_2, q_3, \ldots q_n$ having positive vectors $r_1, r_2, r_3, \ldots r_n$ relative to point P, the potential at point P due to total charge configuration is algebraic sum of potentials due to individual charges, so,

$$V = V_1 + V_2 + V_3 + \ldots + V_n$$

$$= \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{r_3} + \ldots + \frac{q_n}{r_n}\right)$$

$$V = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{q_i}{r_i}$$



- It is known that in a uniformly charged spherical shell, electric potential outside the shell is given as :

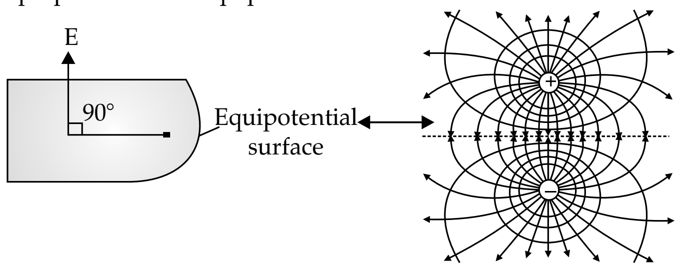
$$V = \frac{1}{4\pi\varepsilon_0}.\frac{q}{r} \qquad (r \geq R)$$

where, q is total charge on shell and R is shell radius.

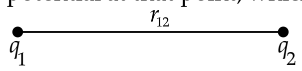
Equipotential surfaces

Equipotential Surface is a surface in space on which all points have same potential. It requires no work to move the charge on such surface, hence the surface will have no E component, so E will be at right angle to the surface.

- Work done in moving a charge over equipotential surface is zero.
- Electric field is always perpendicular to equipotential surface.



- Spacing among equipotential surfaces allows to locate regions of strong and weak fields.
- Equipotential surfaces **never intersect** each other. If they intersect then the intersecting point of two equipotential surfaces results in two values of electric potential at that point, which is impossible.

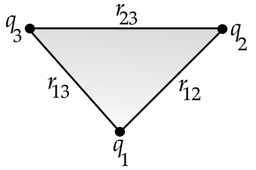


- Potential energy of a system of two charges

  $U = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_{12}}$

- Potential energy of a system of three charges

  $U = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}}\right)$

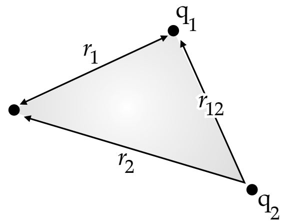


- Potential energy due to single charge in an external field

  Potential energy of q at r in a external field

  $U = qv(\vec{r})$

  Here, $v(\vec{r})$ is the external potential at point r



- Potential energy due to two charges in an external field

  $U = q_1 V(\vec{r_1}) + q_2 V(\vec{r_2}) + \frac{1}{4 \quad {}_0} \cdot \frac{q_1 q_2}{r_{12}}$

- Potential energy of a dipole in an external field :

  When a dipole of charge $q_1 = +q$ and $q_2 = -q$ having separation '$2a$' is placed in an external field ($\vec{E}$).

  $U(\theta) = -pE\cos\theta$

  Here, $p = 2aq$ and θ is the angle between electric field and dipole

Key Formulae

- Electric Potential $V = \frac{W}{q}$, measured in volt; 1 volt = 1 Joule / coulomb.
- Electric potential difference or "voltage" $(\Delta V) = V_f - V_i = \frac{\Delta U}{q} = \frac{W}{q}$
- Electric potential due to a point charge q at a distance r away : $V = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r}$
- Finding V from E : $V_f - V_i = -\int_i^f \vec{E}.d\vec{S}$

- Potential energy of two point charges in absence of external electric field : $U = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1q_2}{r_{12}}\right]$
- Potential energy of two point charges in presence of external electric field : $q_1V(r_1) + q_2V(r_2) + \frac{q_1q_2}{4\pi\varepsilon_0 r_{12}}$

Note : All symbols are in their usual meanings.

Objective Answer Type Questions **(1 mark each)**

Q. 1. The electrostatic potential on the surface of a charged conducting sphere is 100 V.
Two statements are made in this regard :
S_1 : At any point inside the sphere, electric field intensity is zero.
S_2 : At any point inside the sphere, the electrostatic potential is 100 V.
Which of the following is a correct statement?
(a) S_1 is true, but S_2 is false.
(b) Both S_1 and S_2 are false.
(c) S_1 is true, S_2 is also true and S_1 is the cause of S_2.
(d) S_1 is true, S_2 is also true but the statements are independent. [NCERT Exemplar]

Ans. Correct option : (c)

Explanation : The relation between electric field intensity E and potential (V) is,

$$E = -\frac{dV}{dr}$$

where, Electric field intensity, $E = 0$ inside the sphere

So that, $\frac{dV}{dr} = 0$

This means that V = constant. So, if $E = 0$ inside charged sphere, the potential is constant or $V = 100$ everywhere inside the sphere and it verifies the shielding effect also. So, it verifies the option (c).

Q. 2. Equipotential surface at a great distance from a collection of charges whose total sum is not zero are approximately
(a) spheres. (b) planes.
(c) paraboloids. (d) ellipsoids.
[NCERT Exemplar]

Ans. Correct option : (a)

Explanation : For equipotential surface, these surfaces are perpendicular to the field lines. So there must be electric field, which cannot be without charge.

So the algebraic sum of all charges must not be zero. Equipotential surface at a great distance means that space of charge is negligible as compared to distance. So the collection of charges is considered as a point charge.

Electric potential due to point charge is,

$$V = k_e\frac{q}{r}$$

which explains that electric potentials due to point charge is same for all equidistant points. The locus of these equidistant points, which are at same potential, forms spherical surface.

Q. 3. A positively charged particle is released from rest in an uniform electric field. The electric potential energy of the charge
(a) remains constant because the electric field is uniform.
(b) increases because the charge moves along the electric field.
(c) decreases because the charge moves along the electric field.
(d) decreases because the charge moves opposite to the electric field. [NCERT Exemplar]

Ans. Correct option : (c)

Explanation : As we know that, an equipotential surface is always perpendicular to the direction of electric field. Positive charge experiences the force in the direction of electric field. When a positive charge is released from rest in uniform electric field, its velocity increases in the direction of electric field. So K.E. increases, and the P.E. decreases due to law of conservation of energy.

Q. 4. Figure shows some equipotential lines distributed in space. A charged object is moved from point A to point B.

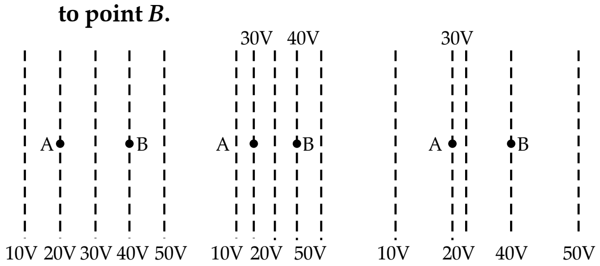


Fig I **Fig II** **Fig III**

(a) The work done in Figure (I) is the greatest.
(b) The work done in Figure (II) is least.

(c) The work done is the same in Figure (I), Figure (II) and Figure (III).

(d) The work done in Figure (III) is greater than Figure (II), but equal to that in Figure (I).

[NCERT Exemplar]

Ans. Correct option : (c)

Explanation : The work done by the electrostatic force is given by $W_{12} = q(V_2 - V_1)$

As the potential difference between A and B in all three figures are equal, 20 V, so work done by any charge in moving from A to B surface will be equal.

Very Short Answer Type Questions (1 mark each)

Q. 1. A point charge Q is placed at point 'O' as shown in figure. Is the potential at point A, *i.e.*, V_A, greater, smaller or equal to potential, V_B at point B, when Q is (i) positive, and (ii) negative charge?

U [Foreign I, II, III 2017]

O• *A*• *B*•

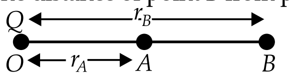
Ans. (i) $V_A > V_B$ ½

(ii) $V_A < V_B$ ½

[CBSE Marking Scheme 2017]

Detailed Answer :

Let r_A is the distance of point A from point charge Q and r_B is the distance of point B from point charge Q.



Potential at point A :

$$V(r_A) = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{r_A}$$

Potential at point B :

$$V(r_B) = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{r_B}$$

Since $r_A < r_B$, so when :

charge Q is positive; $V_A > V_B$ ½

charge Q is negative; $V_A < V_B$ ½

Q. 2. Does the charge given to a metallic sphere depend on whether it is hollow or solid?

U [Delhi I, II, III 2017]

Ans. No ½

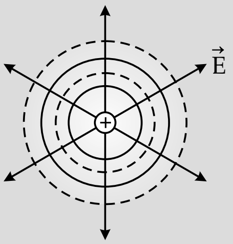
Because the charge resides only on the surface of the conductor ½

[CBSE Marking Scheme 2017]

Q. 3. What is the geometrical shape of equipotential surfaces due to an isolated charge ?

R [Delhi I, II, III 2013]

Ans. For an isolated charge, the equipotential surfaces are concentric spherical shells and the distance between the shells increases with the decrease in electric field. ½



½

[CBSE Marking Scheme 2013]

Q. 4. Why are electric field lines perpendicular at a point on an equipotential surface of a conductor ?

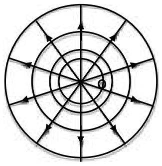
U [O.D. I, II, III 2015]

Ans. If it were not so, the presence of a component of the field along the surface would violate its equipotential nature. **1**

[CBSE Marking Scheme 2015]

Detailed Answer :

In an equipotential surface, the potential for a point charge is given by $V = \frac{kQ}{r}$. As electric field lines point radially away from the charge, they are perpendicular to equipotential surface. If the electric field lines are not perpendicular to the surface of conductor, there exists a non-zero component of electric field along the surface of conductor where charges could not be at rest. ½



½

Q. 5. For any charge configuration, equipotential surface through a point is normal to the electric field. Justify. U [Delhi Set I, II, III 2014]

OR

Why must electrostatic field at the surface of a charged conductor be normal to the surface at every point ? Give reason. [Foreign 2014]

Ans. If the electric field was not normal to the surface, then it would have a component along the surface which would cause work to be done in moving a charge on an equipotential surface. **1**

[CBSE Marking Scheme 2014]

Detailed Answer :

Equipotential lines are curved lines with similar altitude that pertains to electric potential or voltage. These lines are always perpendicular to electric field which create equipotential surfaces in three dimensions. The movement of charge along equipotential surface requires no work as movement is perpendicular to electric field. ½

As work done in moving test charge along equipotential surface is zero, there appear fixed potential values across every point on equipotential surface, so

$$W = Fs\cos\theta = 0$$

where,

F = Electric force

s = Magnitude of displacement of charge

In case of non-zero displacement, it exists only when

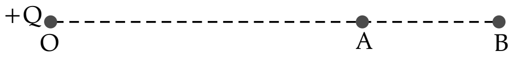
$$\cos\theta = 0$$

$$\Rightarrow \qquad \theta = 90°$$

Hence, force acting on point charge is perpendicular to equipotential surface. ½

Q. 6. A point charge $+Q$ is placed at point O as shown in the figure. Is the potential difference $V_A - V_B$ positive, negative or zero ?

U [Delhi Set I, II, III 2016]



Ans. Positive. **[CBSE Marking Scheme 2016] 1**

Detailed Answer :

Let $OA = r_A$, $OB = r_B$

$$V_A - V_B = \frac{1}{4\pi\varepsilon_0} \cdot Q\left(\frac{1}{r_A} - \frac{1}{r_B}\right)$$

Since $\frac{1}{r_A} - \frac{1}{r_B}$ is positive quantity. Hence, $V_A - V_B$ is positive. **1**

Answering Tip

- If $r_A < r_B$

  Then $\frac{1}{r_A} > \frac{1}{r_B}$

  So, $\frac{1}{r_A} - \frac{1}{r_B} = +\text{ve}$

AI Q. 7. A charge 'q' is moved from a point A above a dipole of dipole moment 'p' to a point B below the dipole in equatorial plane without acceleration. Find the work done in the process.

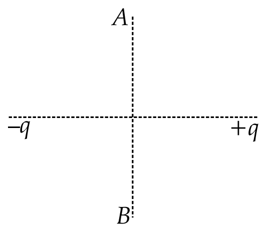
U [O.D. I, II, III 2016]

Ans. No work is done.

$W = qV_{AB} = q \times 0 = 0$ **1**

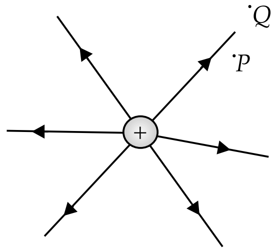
[CBSE Marking Scheme 2016]

Detailed Answer :



It is seen that potential due to dipole at any point on equatorial line in equatorial plane is zero. From the figure, points A & B lies in equatorial plane of dipole, where work done in moving charge 'q' from point A to B without acceleration will be zero. **1**

Q. 8. Figure shows the field lines due to a positive charge. Is the work done by the field in moving a small positive charge from Q to P, positive or negative? Give reason. **A [Foreign 2014]**



Ans. Negative; ½

This happens as the charge is displaced against the force exerted by the field. ½

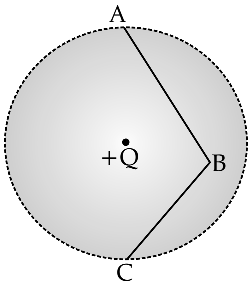
[CBSE Marking Scheme 2014]

Q. 9. In the given figure, charge $+Q$ is placed at the centre of a dotted circle. Work done in taking another charge $+q$ from A to B is W_1 and from B to C is W_2. Which one of the following is correct ?

(i) $W_1 > W_2$

(ii) $W_1 = W_2$

(iii) $W_1 < W_2$



U [CBSE SQP 2017-18]

Ans. (ii)

As $V_A - V_B = V_B - V_C$ magnitude of work done is same.

[CBSE Marking Scheme 2017] 1

Detailed Answer :

As we know $E = -\frac{dV}{dr}$ and inside a ring there is zero electric field which means constant potential

$\therefore \qquad V_A - V_B = V_B - V_C$

$\therefore$ Magnitude of work done is same. **1**

Answering Tip

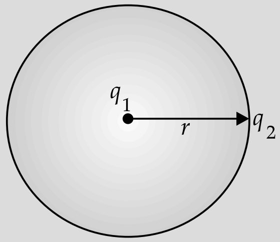
- Points A and B are at same potential lies on a equipotential surface.

AI Q. 10. What is the amount of work done in moving a point charge around a circular arc of radius r at the center where another point charge is located ? **A&E [O.D. Comptt. I, II, III 2014]**

Ans. Let q_1 be the charge inside the circle and q_2 be the charge moving around the circular arc, then work done

$$W = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r^2}.$$

The work done in moving the charge over the circular arc is zero, because it is moving over an equipotential surface.



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[CBSE Marking Scheme 2014]

Q. 11. An electron is accelerated through a potential difference V. Write the expression for its final speed, if it was initially at rest.

A&E [Delhi/O.D. Comptt. I, II, III 2018]

Ans. $v = \sqrt{\frac{2eV}{m}}$ **1**

[CBSE Marking Scheme, 2018]

Detailed Answer :

Kinetic energy of electron = Electric energy gain by the electron

$$\Rightarrow \quad \frac{1}{2}m(v^2 - u^2) = eV$$

$$\Rightarrow \quad \frac{1}{2}mv^2 = eV \qquad [\text{as } u = 0]$$

$$\therefore \quad v = \sqrt{\frac{2eV}{m}}$$ **1**

Short Answer Type Questions-I

(2 marks each)

Q. 1. Write two properties of equipotential surfaces. Depict equipotential surfaces due to an isolated point charge. Why do the equipotential surfaces get closer as the distance between the equipotential surfaces and the source charge decreases ?

R [O.D. Comptt. I, II, III 2012]

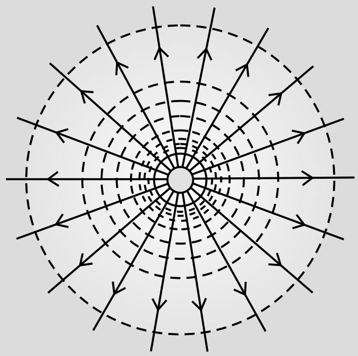
Ans. Characteristics of equipotential surfaces :

(i) These surfaces are always perpendicular to field lines. ½

(ii) No work is done in moving the test charge from one point of equipotential surface to the other. ½

On reducing the distance of source charge, the electric field becomes strong and equipotential surfaces come closer. ½

For an isolated point charge, the equipotential surfaces are shown below.



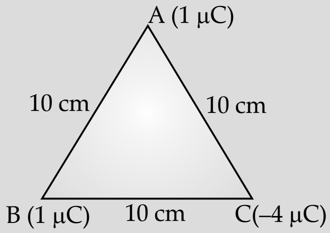
½

[CBSE Marking Scheme, 2012]

Q. 2. Calculate the amount of work done to dissociate a system of three charges 1 μC, 1 μC and – 4 μC placed on the vertices of an equilateral triangle of side 10 cm. **A [O.D. Comptt. I 2013]**

Ans. Given :

$$q_A = 1 \times 10^{-6}\,\text{C} = q_B$$
$$q_C = -4 \times 10^{-6}\,\text{C}$$
$$r = 10 \text{ cm} = 0.1 \text{ m}$$



$$W = W_{AB} + W_{BC} + W_{AC}$$

$$W = \frac{1}{4\pi\varepsilon_0 r}[q_A q_B + q_B q_C + q_C q_A]$$

$$W = 9 \times 10^9 \times \frac{1}{0.1}[1\times1 - 4\times1 - 4\times1]\times10^{-12}\,\text{J}$$

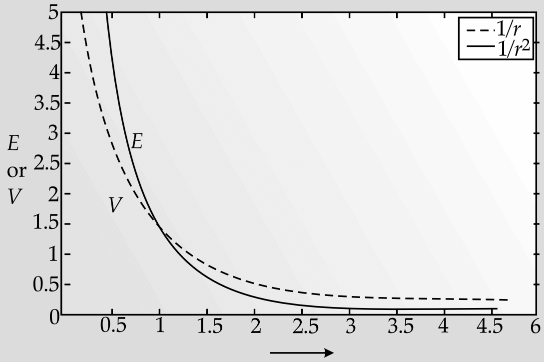
$$W = \frac{-9\times10^9\times7\times10^{-12}}{0.1}\,\text{J}$$

$$W = -0.63\,\text{J}$$ **2**

[CBSE Marking Scheme 2013]

AI Q. 3. Draw a plot showing the variation of (i) electric field (*E*) and (ii) electric potential (*V*) with distance *r* due to a point charge *Q*. **U [Delhi I, II, III 2012]**

Ans.



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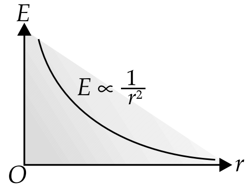
[CBSE Marking Scheme 2012]

Detailed Answer :

Due to point charge :

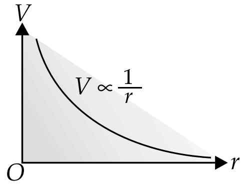
Electric field, $E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{r^2}$

$\because \quad E \propto \frac{1}{r^2}$ ½

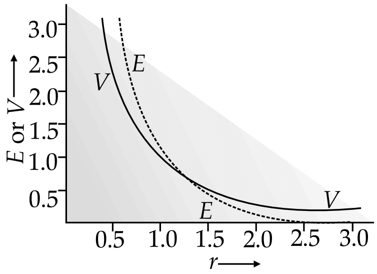


Electric potential, $V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{r}$

$\because \quad V \propto \frac{1}{r}$ ½

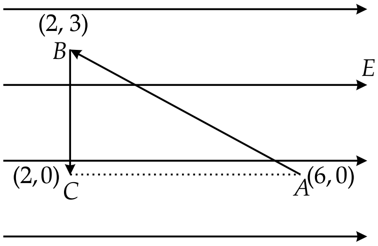


Now on plotting electric field and electric potential with distance *r*



1

Q. 4. ***A*** **test charge** ***q*** **is moved without acceleration from** ***A*** **to** ***C*** **along the path from** ***A*** **to** ***B*** **and then from** ***B*** **to** ***C*** **in electric field** ***E*** **as shown in the figure.**



(i) Calculate the potential difference between ***A*** **and** ***C*****.**

(ii) At which point (of the two) is the electric potential more and why ? A **[O.D. I, II, III 2012]**

Ans. Since, $E = -dV/dr$

$E = (V_C - V_A)/4$ ½

Therefore, $V_A - V_C = -4E$ ½

At point *C*, potential is more ½

Electric field is in the direction in which the potential decreases. ½

[CBSE Marking Scheme 2012]

Detailed Answer :

(i) From the figure, as work done is independent of path, so moving a charge *q* from point *A* to *B* and further from point *B* to *C*, we will take a direct path from point *A* to *C*. So potential difference between points *A* and *C* is given as :

$$V_C - V_A = -E.dl$$

$$= -\int_A^C Edl\cos\theta$$ ½

As direction of test charge '*q*' is opposite to direction of electric field *E* along *AC*, so angle between $\vec{E}$ and $\vec{dl}$ will be 180°, hence we can write this as :

$$V_C - V_A = -E.dl$$

$$= -\int_A^C Edl\cos\theta$$ ½

$$V_C - V_A = (6-2)E = 4E$$

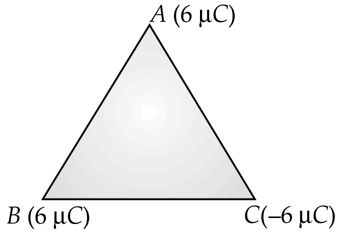
(ii) As direction of electric field is high to low potential, so $V_C > V_A$

Also, $V_C - V_A = 4E$ with *E* as positive ½

Potential increases when charge is moved against the direction of electric field, so potential will be more at point *C*. ½

Q. 5. Find the amount of work done in arranging the three point charges, on the vertices of an equilateral triangle ***ABC*****, of side 10 cm, as shown in the figure.**

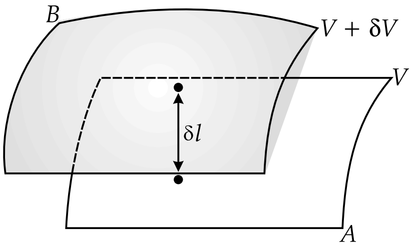
A **[Comptt. O.D. I, II, III 2013; CBSE SQP 2010]**



Ans. Try yourself, Similar to Q. 2 of Short Answer Type Questions I **2**

Q. 6. Two closely spaced equipotential surfaces ***A*** **and** ***B*** **with potentials** ***V*** **and** ***V*** **+ δ*****V*****, (where δ*****V*** **is the change in** ***V*****), are kept δ*****l*** **distance apart as shown in the figure. Deduce the relation between the electric field and the potential gradient between them. Write the two important conclusions concerning the relation between the electric field and electric potentials.**

U **[Delhi Comptt. I, II, III 2014]**



Ans. Work done in moving a unit positive charge along distance δ*l*

$$|E_l|\ \delta l = V_A - V_B$$ ½

$$= V - (V + \delta V) = -\delta V$$

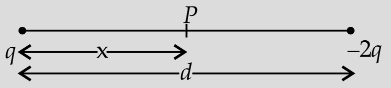
$$\Rightarrow \qquad |E_l| = -\frac{\delta V}{\delta l}$$ ½

(i) Electric field is in the direction in which the potential decreases the steepest. ½

(ii) The magnitude of electric field is given by the change in the magnitude of potential per unit displacement, normal to the equipotential surface at the point. **[CBSE Marking Scheme 2014]** ½

Q. 7. Two point charges q and $-2q$ are kept d distance apart. Find the location of the point relative to charge q at which potential due to this system of charges is zero. A **[Comptt. O.D. I, II, III 2014]**

Ans.



½

Let P be the required point at a distance x from charge q

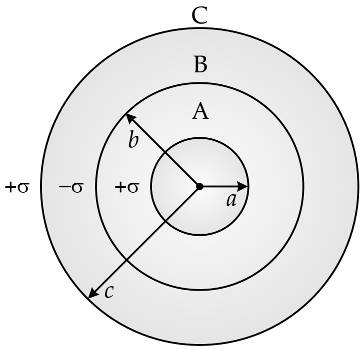
$$\therefore \quad \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{x} + \frac{1}{4\pi\varepsilon_0}\cdot\frac{(-2q)}{(d-x)} = 0$$ ½

$$\frac{1}{x} = \frac{2}{d-x}$$ ½

$$\Rightarrow \qquad x = \frac{d}{3}$$

$\therefore$ The required point is at a distance $\frac{d}{3}$ from the charge q. **[CBSE Marking Scheme 2014]** ½

Q. 8. Three concentric metallic shells A, B and C of radii a, b and c ($a < b < c$) have surface charge densities $+\sigma$, $-\sigma$ and $+\sigma$ respectively as shown in the figure.



If shells A and C are at the same potential, then obtain the relation between the radii a, b and c. U **[Foreign Set-III, 2014]**

Ans. $q_A = \sigma.\, 4\pi a^2$, $q_B = -\sigma.\, 4\pi b^2$, $q_c = \sigma.\, 4\pi c^2$,

$$V_A = k\left[\frac{q_A}{a}+\frac{q_B}{b}+\frac{q_C}{c}\right]$$

$$V_C = k\left[\frac{q_A+q_B+q_C}{c}\right]$$

$\because$, $\quad V_A = V_C$, we have

$$k\left[\frac{q_A}{a}+\frac{q_B}{b}+\frac{q_C}{c}\right] = k\left[\frac{q_A+q_B+q_C}{c}\right]$$ 1

$$\therefore \quad \frac{q_A}{a}+\frac{q_B}{b} = \frac{q_A+q_B}{c}$$

Putting the values of q_A and q_B,

We get $\quad a + b = c$ 1

Q. 9. N spherical droplets, each of radius r, have been charged to have a potential V each. If all these droplets were to coalesce to form a single large drop, what would be the potential of this large drop ?

(It is given that the capacitance of a sphere of radius x equals $4\pi\varepsilon_0 kr$.) A **[CBSE SQP 2013]**

Ans. Total (initial) charge on all the droplets

$$= N \times (4\pi\varepsilon_0 krV)$$ ½

Also $$N \times \frac{4}{3}\pi r^3 = \frac{4}{3}\pi R^3$$ ½

$$\therefore \quad R = N^{1/3}\, r$$

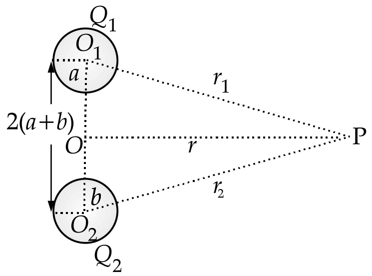
If V' is the potential of the large drop, we have

$$4\pi\varepsilon_0 kR \times V' = N \times 4\pi\varepsilon_0 kr \times V$$ ½

$$\therefore \quad V' = \frac{Nr}{R}.V = N^{2/3}V$$ ½

AI **Q. 10. Find the $P.E.$ associated with a charge q if it were present at the point P with respect to the 'set-up' of two charged spheres, arranged as shown. Here O is the mid-point of the line O_1O_2.**

A **[CBSE SQP 2013]**



Ans. $$r_1 = O_1P = \sqrt{r^2+(2a+b)^2}$$ ½

$$r_2 = O_2P = \sqrt{r^2+(a+2b)^2}$$ ½

$$\therefore \quad V = \frac{1}{4\pi\varepsilon_0}\left[\frac{Q_1}{r_1}+\frac{Q_2}{r_2}\right]$$ ½

$\therefore$ P.E. of charge, q, at $P = qV$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{Q_1}{[r^2+(2a+b)^2]^{1/2}}+\frac{Q_2}{[r^2+(a+2b)^2]^{1/2}}\right]$$ ½

Q. 11. Two point charges q_1 and q_2 are located at $\vec{r_1}$ and $\vec{r_2}$ respectively in an external electric field E. Obtain the expression for the total work done in assembling this configuration.

A **[Comptt. Delhi I, II, III 2014]**

Ans. Work done in bringing the charge q_1 from infinity to the position $\vec{r_1}$

$$W_1 = q_1\, V(\vec{r_1})$$ ½

Work done in bringing the charge q_2 to the position $\vec{r_2}$

$$W_2 = q_2 V(\vec{r_2}) + \frac{q_1 q_2}{4\pi\varepsilon_0 r_{12}}$$ ½ + ½

Hence, total work done in assembling the two charges

$$W = W_1 + W_2$$

$$W = q_1 V(\vec{r_1}) + q_2 V(\vec{r_2}) + \frac{q_1 q_2}{4\pi\varepsilon_0 r_{12}}$$ ½

[CBSE Marking Scheme 2014]

Short Answer Type Questions-II (3 marks each)

Q. 1. (i) Write two properties by which electric potential is related to the electric field.

(ii) Two point charges q_1 and q_2, separated by a distance of r_{12} are kept in an external electric field. Derive an expression for the potential energy of the system of two charges in the field.

A **[Delhi Compt. I, II, III 2017]**

Ans. (i) Properties 1

(ii) Derivation of expression of potential energy 2

(i) (a) Electric field is in the direction in which potential decreases at the maximum rate. ½

(b) Magnitude of electric field is given by change in the magnitude of potential per unit displacement normal to a charged conducting surface. ½

(ii) Work done in bringing the charge q_1 to a point against external electric field

$$W_1 = q_1 V \vec{r_1}$$ ½

Work done in bringing the charge q_2 against the external electric field and electric field produced due to charge q_1

$$W_2 = q_2 V(\vec{r_2}) + \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r_{12}}$$ ½

Therefore, Total work done = Electrostatic potential energy :

$$U = q_1 V(\vec{r_1}) + q_2 V(\vec{r_2}) + \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r_{12}}$$ 1

[CBSE Marking Scheme, 2017]

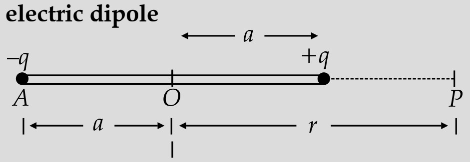
Q. 2. (i) Derive the expression for the electric potential due to an electric dipole at a point on its axial line.

(ii) Depict the equipotential surface due to electric dipole. R **[Delhi II 2017]**

Ans. (i) Derivation of expression for electric potential due to electric dipole on axial line 2

(ii) Depiction of equipotential surfaces due to an electric dipole 1

(i)



Potential due to charge $-q$ at A :

$$V_A = \frac{1}{4\pi\varepsilon_0} \cdot \frac{-q}{(r+a)}$$ ½

Potential due to charge $+q$ at B :

$$V_B = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{(r \quad a)}$$ ½

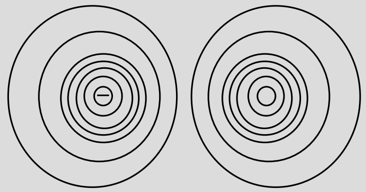
Potential at point P, $V = V_A + V_B$

Net potential at point P :

$$V = \frac{q}{4\pi\varepsilon_0}\left[\frac{-1}{(r+a)} + \frac{1}{(r-a)}\right]$$ ½

$$V = \frac{q}{4\pi\varepsilon_0} \cdot \frac{q \times 2a}{(r^2 - a^2)}$$ ½

(ii)



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[CBSE Marking Scheme 2017]

AI **Q. 3. Define an equipotential surface. Draw equipotential surfaces :**

(i) in the case of a single point charge and

(ii) in a constant electric field in z-direction.

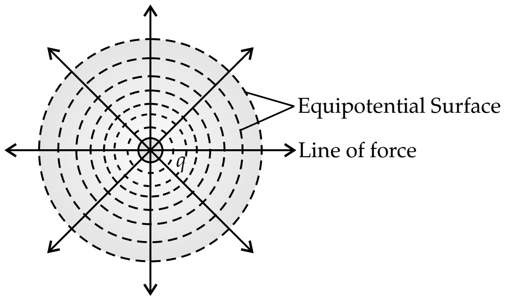
Why the equipotential surfaces about a single charge are not equidistant ?

(iii) Can electric field exist tangential to an equipotential surface ? Give reason.

R **[O.D. I, II, III 2016]**

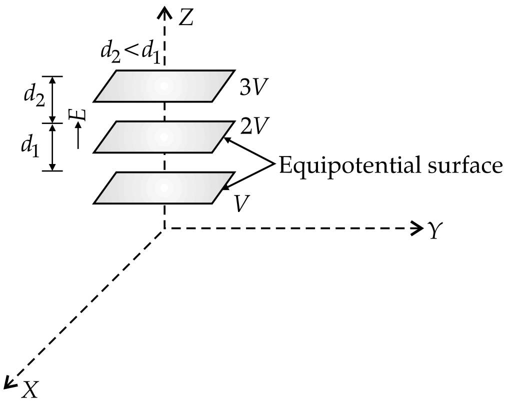
Ans. Equipotential surface is a surface which has equal potential at every point on it.

(i) Equipotential surfaces due to single point charge are concentric spheres having charge at the centre.



1

(ii) In constant electric field along z-direction, the perpendicular distance between equipotential surfaces remains same.



For single charge, equipotential surface will be series of concentric spherical shells with charge at centre,

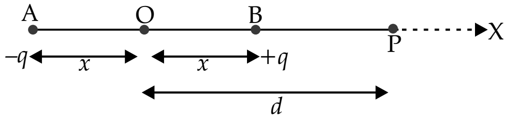
$$dr \propto \frac{1}{E}$$

the separation dr between equipotential surfaces will go on increasing with decrease in electric field. **1**

(iii) No, because if the surface is not equipotential then it would mean that there is tangential component of electric field along surface.
This component will result in motion of electrons, but since we have static fields, this is not possible. **1**

Q. 4. Obtain the expression for the potential due to an electric dipole of dipole moment p at a point 'd' on the axial line. A **[O.D. Comptt. I, II, III 2013]**

Ans. Consider an electric dipole of charges $+q$ and $-q$ separated by $2x$ distance being placed in free space. Let P be the point at which the electric field is to be determined due to the electric dipole.



Let the potential at point P due to positive charge be V_+ and the potential at point P due to negative charge V_-

$$V_+ = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{(d-x)}$$

$$V_- = \frac{1}{4\pi\varepsilon_0} \cdot \frac{-q}{(d+x)}$$

$\therefore$ The total potential at point P is given by

$$V = V_+ + V_-$$

$$V = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{d-x} - \frac{q}{d+x}\right)$$

$$V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{2xq}{(d^2-x^2)}$$

$\therefore$ $$2qx = p$$

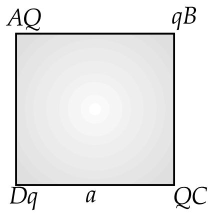
$$V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{p}{(d^2-x^2)}$$

Now, if $x <<< d$

$$x^2 \cong 0$$

$$V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{p}{d^2}$$ **3**

AI **Q. 5. Four point charges Q, q, Q and q are placed at the corners of a square of side 'a' as shown in the figure.**



Find the
(i) resultant electric force on a charge Q, and
(ii) potential energy of this system.
[Delhi/O.D. CBSE 2018]

Ans. (i) Finding the resultant force on a charge Q **2**
(ii) Potential Energy of the system **1**

(i) Let us find the force on the charge Q at the point C
Force due to the other charge Q

$$F_1 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q^2}{(a\sqrt{2})^2}$$

$$\vec{F_1} = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q^2}{2a^2}\right) \text{(along } AC) \quad \text{½}$$

Force due to the charge q (at B),

$$\vec{F_2} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{qQ}{a^2} \text{ along } BC$$

Force due to the charge q (at D),

$$\vec{F_3} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{qQ}{a^2} \text{ along } DC \quad \text{½}$$

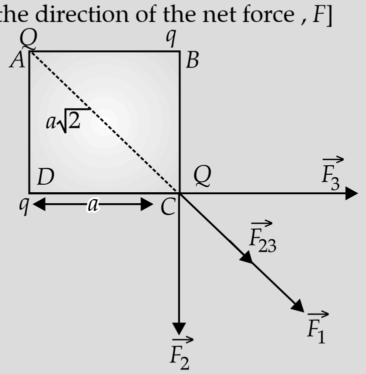
Resultant of these two equal forces $\vec{F_2}$ & $\vec{F_3}$

$$\vec{F_{23}} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{qQ(\sqrt{2})}{a^2} \text{ (along } AC) \quad \text{½}$$

$\therefore$ Net force on charge Q (at point C)

$$\vec{F} = \vec{F_1} + \vec{F_{23}} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{a^2}\left[\frac{Q}{2} + \sqrt{2}q\right] \quad \text{½}$$

This force is directed along AC.
(For the charge Q, at the point A, the force will have the same magnitude but will be directed along CA)
[**Note :** Don't deduct marks if the student does not write the direction of the net force , F]



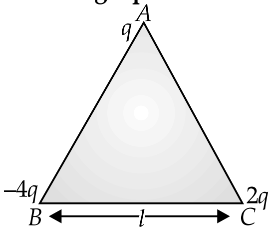
(ii) Potential energy of the system

$$= \frac{1}{4\pi\varepsilon_0}\left[4\frac{qQ}{a} + \frac{q^2}{a\sqrt{2}} + \frac{Q^2}{a\sqrt{2}}\right]$$

$$= \frac{1}{4\pi\varepsilon_0 a}\left[4qQ + \frac{q^2}{\sqrt{2}} + \frac{Q^2}{\sqrt{2}}\right]$$ **1**

[CBSE Marking Scheme 2018]

Q. 6. (i) Three point charges q, – $4q$ and $2q$ are placed at the vertices of an equilateral triangle ABC of side 'l' as shown in the figure. Obtain the expression for the magnitude of the resultant electric force acting on the charge q.



(ii) Find out the amount of the work done to separate the charges at infinite distance. **3**

A [Delhi/OD CBSE 2018]

Ans. (i) Finding the magnitude of the resultant force on charge q **2**

(ii) Finding the work done **1**

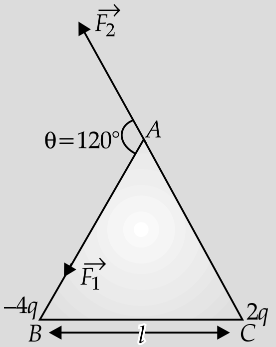
(i) Force on charge q due to the charge – $4q$ ½

$$\vec{F}_1 = \frac{1}{4\pi\varepsilon_0}\left(\frac{4q^2}{l^2}\right), \text{ along } AB$$

Force on the charge q, due to the charge $2q$

$$\vec{F}_2 = \frac{1}{4\pi\varepsilon_0}\left(\frac{2q^2}{l^2}\right), \text{ along } CA$$

The forces F_1 and F_2 are inclined to each other at an angle of 120°



Hence, resultant electric force on charge q

$$F = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$$ ½

$$= \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos 120°}$$

$$= \sqrt{F_1^2 + F_2^2 - F_1F_2}$$ ½

$$= \left(\frac{1}{4\pi\varepsilon_0}\cdot\frac{q^2}{l^2}\right)\sqrt{16+4-8}$$

$$= \frac{1}{4\pi\varepsilon_0}\left(\frac{2\sqrt{3}q^2}{l^2}\right)$$ ½

(ii) Net *P.E.* of the system

$$= \frac{1}{4\pi\varepsilon_0}\cdot\frac{q^2}{l}[-4+2-8]$$

$$= \frac{(-10)}{4\pi\varepsilon_0}\cdot\frac{q^2}{l}$$ ½

$$\therefore \quad \text{Work done} = \frac{10\,q^2}{4p\varepsilon_0 l} = \frac{5q^2}{2\pi\varepsilon_0 l}$$ ½

[CBSE Marking Scheme 2018]

Detailed Answer :

(ii) The amount of work done to separate the charges at infinite distance is equal to the (-ve) potential energy of the given system. Now we know that the potential energy of three (03) charges at the corners of an equilateral triangle *ABC* of side l is given by

$$U_{PE} = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1q_2}{r_{12}} + \frac{q_1q_3}{r_{13}} + \frac{q_2q_3}{r_{23}}\right]$$

given $q_1 = q$, $q_2 = -4q$, $q_3 = 2q$, $r_{12} = r_{13} = r_{23} = l$

$$\Rightarrow \quad U_{PE} = \frac{1}{4\pi\varepsilon_0}\left[\frac{q(-4q)}{l} + \frac{q.2q}{l} + \frac{2q(-4q)}{l}\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\cdot\frac{q^2}{l}[-4+2-8]$$

$$U_{PE} = -\frac{10q^2}{4\pi\varepsilon_0 l}$$

Therefore work done, $W = -U_{PE} = -\left(\dfrac{-10q^2}{4\pi\varepsilon_0 l}\right)$

$$\Rightarrow \quad \boxed{W = +\frac{5q^2}{2\pi\varepsilon_0 l}}$$ **1**

Q. 7. A particle, having a charge +5 μC, is initially at rest at the point x = 30 cm on the x-axis. The particle begins to move due to the presence of a charge Q that is kept fixed at the origin. Find the kinetic energy of the particle at the instant it has moved 15 cm from its initial position if (a) Q = +15 μC and (b) Q = – 15 μC

U [SQP 2018-19]

Ans. From energy conservation,

$$U_i + K_i = U_f + K_f$$

$$kQq/r_i + 0 = kQq/r_f + K_f$$ ½

$$K_f = kQq(1/r_i - 1/r_f)$$ ½

When Q is +15 μC, q will move 15 cm away from it. Hence r_f = 45 cm

$K_f = 9 \times 10^9 \times 15 \times 10^{-6} \times 5 \times 10^{-6}\,[1/(30 \times 10^{-2}) - 1/(45\times10^{-2})]$ ½

= 0·75 J ½

When Q is – 15 μC, q will move 15 cm towards it. Hence r_f = 15 cm

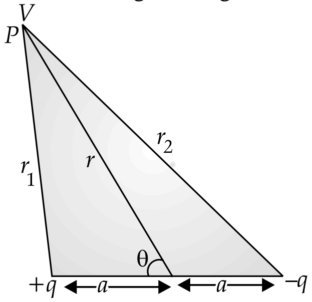
$K_f = 9 \times 10^9 \times (-15 \times 10^{-6}) \times 5 \times 10^{-6}\,[(1/30 \times 10^{-2}) - 1/(15 \times 10^{-2})]$ ½

= 2·25 J ½

[CBSE Marking Scheme 2018-19]

Long Answer Type Questions (5 marks each)

Q. 1. Derive an expression for potential due to a dipole for distances large compared to the size of the dipole. How is the potential due to dipole different from that due to single charge ?



A [SQP 2017]

Ans. (i) Derivation 4
(ii) Difference 1

(i) Consider origin at the centre of dipole. As per superposition principle, potential due to dipole will be the sum of potentials due to charges q and $-q$

$$V = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{r_1} - \frac{q}{r_2}\right]$$ ½

Where,
r_1 and r_2 = distances of point P from q and $-q$.
From the arrangements,

$$r_1^2 = r^2 + a^2 - 2ar\cos\theta$$ ½
$$r_2^2 = r^2 + a^2 + 2ar\cos\theta$$ ½

If r is greater than a, and taking terms upto first order in a/r

$$r_1^2 = r^2\left[1 - \frac{2a\cos\theta}{r} + \frac{a^2}{r^2}\right]$$
$$= r^2\left[1 - \frac{2a\cos\theta}{r}\right]$$ ½

Also, $$r_2^2 \cong r^2\left[1 + \frac{2a\cos\theta}{r}\right]$$ ½

With the help of Binomial theorem, keeping terms upto first order in a/r;

$$\frac{1}{r_1} \cong \frac{1}{r}\left[1 - \frac{2a\cos\theta}{r}\right]^{-\frac{1}{2}}$$
$$\cong \frac{1}{r}\left[1 + \frac{a}{r}\cos\theta\right]$$

$$\frac{1}{r_2} \cong \frac{1}{r}\left[1 + \frac{2a\cos\theta}{r}\right]^{-\frac{1}{2}}$$
$$\cong \frac{1}{r}\left[1 - \frac{a}{r}\cos\theta\right]$$

As $$p = qa$$ 1

$$V = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q(2a)\cos\theta}{r^2}$$

$$V = \frac{p}{4\pi\varepsilon_0}\cdot\frac{\cos\theta}{r^2}$$

Now, $$p\cos\theta = \vec{p}.\hat{r}$$

where, $\hat{r}$ is unit vector along position vector. Hence electric potential of dipole for distances large compared to size of dipole is given as : ½

$$V = \frac{1}{4\pi\varepsilon_0}\cdot\frac{\vec{p}.\hat{r}}{r^2} \text{ for } r >> a$$

(ii) For potential at any point on axis, $\theta = [0, \pi]$

$$V = \pm\frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r}$$

It is observed that :

potential is positive when $\theta = 0$ ½

potential is negative when $\theta = \pi$

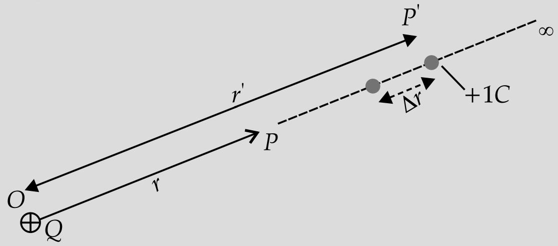
Hence, electric potential falls at large distance, as $\frac{1}{r^2}$ and not as $\frac{1}{r}$ ½

[CBSE Marking Scheme 2017]

AI **Q. 2. (i) Obtain the expression for the potential to show due to a point charge.**
(ii) Potential, due to an electric dipole (length 2*a*) varies as the 'inverse square' of the distance of the 'field point' from the centre of the dipole for $r > a$.

U **[Comptt. Delhi. I, II, III 2016]**

Ans. (i)



½

Consider a point charge 'Q' kept at point O. Let P be the field point at distance r.
At some intermediate point P' the electrostatic force on the unit positive charge is : ½

$$= \frac{Q \times 1}{4\pi\varepsilon_0 r'^2}$$

Work done against this force from r' to $r' + \Delta r'$ is

$$\Delta W = \frac{Q}{4\pi\varepsilon_0 r'^2}\Delta r'.$$ ½

Total work done 'W' by the external Force from ∞ to r

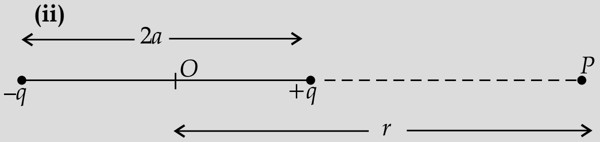
$$W = -\int_\infty^r \frac{Q}{4\pi\varepsilon_0 r'^2}\Delta r'$$ ½

$$W = \frac{Q}{4\pi\varepsilon_0 r}$$

Hence potential at this point

$$V = W = \frac{Q}{4\pi\varepsilon_0 r}$$ ½

(ii)



Potential at point P due to charge $(-q)$ ½

$$V_1 = \frac{-1}{4\pi\varepsilon_0}\cdot\frac{q}{(r+a)}$$

Potential due to charge $+q$

$$V_2 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{(r-a)}$$ ½

Hence total potential at point P

$$V = V_1 + V_2 = \frac{q}{4\pi\varepsilon_0}\left[\frac{-1}{(r+a)} + \frac{1}{(r-a)}\right]$$

$$= \frac{q\times 2a}{4\pi\varepsilon_0(r^2-a^2)}$$ ½

$$V = \frac{1}{4\pi\varepsilon_0}\cdot\frac{p}{(r^2-a^2)}$$

where, $\vec{p} = q\times 2a$ = dipole moment for $r >> a$ ½

$$\Rightarrow \quad V = \frac{1}{4\pi\varepsilon_0}\cdot\frac{p}{r^2}$$

$$\Rightarrow \quad V \propto \frac{1}{r^2}.$$ ½

[CBSE Marking Scheme 2016]

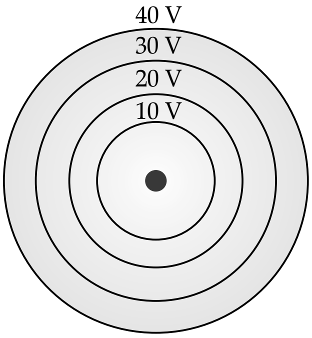
AI Q. 3. (i) Two isolated metal spheres A and B have radii R and $2R$ respectively, and same charge q. Find which of the two spheres have greater :

(a) capacitance and

(b) energy density just outside the surface of the spheres.

(ii) (a) Show that the equipotential surfaces are closed together in the regions of strong field and far apart in the regions of weak field. Draw equipotential surfaces for an electric dipole.

(b) Concentric equipotential surfaces due to a charged body placed at the centre are shown. Identify the polarity of the charge and draw the electric field lines due to it.



U **[CBSE SQP 2015-16]**

Ans. (i) (a) $C_A = 4\pi\varepsilon_0 R,\ C_B = 4\pi\varepsilon_0(2R)$ ½

$\therefore \quad C_B > C_A$ ½

(b) $U = \frac{1}{2}\varepsilon_0 E^2$ ½

$$E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{A\varepsilon_0}$$

$$\therefore \quad U \propto \frac{1}{A^2}$$

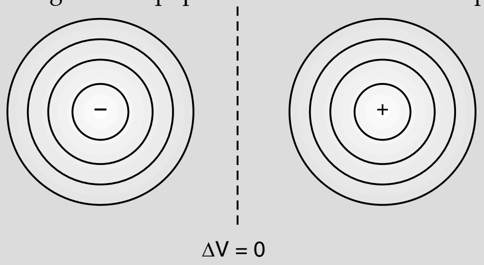
$\therefore \quad U_A > U_B$ ½

(ii) (a) $E = -\frac{dV}{dr}$ ½

For same change in dV, $E \propto \frac{1}{dr}$ ½

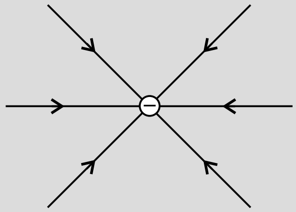
where, 'dr' represents the distance between equipotential surfaces.

Diagram of equipotential surface due to a dipole **1**



(b) Polarity of charge : – negative ½

The direction of electric field is radially inward.

 ½

[CBSE Marking Scheme 2015]

Q. 4. Two point charges q and $-q$ are located at points $(0, 0, -a)$ and $(0, 0, a)$ respectively.

(i) Find the electrostatic potential at $(0, 0, z)$ and $(x, y, 0)$.

(ii) How much work is done in moving a small test charge from the point (5,0,0) to (– 7, 0, 0) along the x-axis ?

(iii) How would your answer change if the path of the test charge between the same points is not along the x-axis but along any other random path ?

(iv) If the above point charges are now placed in the same positions in the uniform external electric field $\vec{E}$, what would be the potential energy of the charge system in its orientation of unstable equilibrium ?

Justify your answer in each case.

A **[Comptt. Delhi/OD I, II, III 2018]**

Ans. (i) Finding the electrostatic potential 2

(ii) Finding the work done 1

(iii) Effect of change of path 1

(iv) Potential energy of the system (with justification in each case) 1

(i) We have, for a point charge,

$$V = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r}$$ ½

(a) At point (0, 0, z) :
Potential due to the charge ($+q$)

$$V_+ = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{(z+a)}$$ ½

Potential due to the charge ($-q$)

$$V_- = \frac{1}{4\pi\varepsilon_0}\cdot\frac{(-q)}{(z-a)}$$

Total Potential at (0, 0, z)

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{1}{z+a} - \frac{1}{z-a}\right]$$ ½

$$= \frac{-2qa}{4\pi\varepsilon_0(z^2-a^2)}$$

(b) At point (x, y, 0)
Potential due to the charge $+ q$

$$V_+ = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{\sqrt{x^2+y^2+a^2}}$$

Potential due to the charge ($- q$)

$$V_- = \frac{1}{4\pi\varepsilon_0}\cdot\frac{-q}{\sqrt{x^2+y^2+a^2}}$$

Total potential at (x, y, 0)

$$= \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{\sqrt{x^2+y^2+a^2}} - \frac{1}{\sqrt{x^2+y^2+a^2}}\right) = 0$$ ½

Note : Give full credit of part (b) if a student writes that the point (x, y, 0) is equidistant from charges $+q$ and $-q$, Hence total potential due to them at the given point will be zero.

(ii) Work done = $q[V_1 - V_2]$ ½

$V_1 = 0$ and $V_2 = 0$

$\therefore$ Work done = 0

Where V_1 and V_2 are the total potential due to dipole at point (5, 0, 0) and (– 7, 0, 0) ½

(iii) There would be no change

This is because the electrostatic field is a conservative field. **1**

(**Alternatively :** The work done, in moving a test charge between two given points is independent of the path taken) **1**

(iv) The two given charges make an electric dipole of dipole moment $\vec{p} = q.\overrightarrow{2a}$

P.E. in position of unstable equilibrium (where $\vec{p}$ and $\vec{E}$ are antiparallel to each other) ½

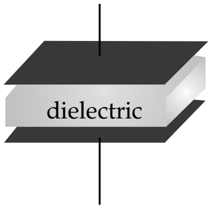
$= + pE = 2aqE$ ½

[CBSE Marking Scheme, 2018]

TOPIC-2
Capacitance

Revision Notes

Conductors and insulators



- Conductors are the materials through which charge can move freely. **Examples :** metals, semi-metals as carbon, graphite, antimony and arsenic.
- Insulators are materials in which the electrical current will not flow easily. Such materials cannot be grounded and do not easily transfer electrons. **Examples :** plastics and glass.

Dielectrics

- These are the material in which induced dipole moment is linearly proportional to applied electric field.
- Electrical displacement or electrical flux density $D = \varepsilon_r\varepsilon_0 E$.
  where, ε_r= relative permittivity, ε_0 = permittivity of free space and E is electric field.
- If a dielectric is kept in between the plates of capacitor, capacitance increases by factor 'κ' (kappa) known as dielectric constant, so $C = \kappa\varepsilon_0\frac{A}{d}$
  where, A = area
  κ = dielectric constant of material also called relative permittivity $\kappa = \varepsilon_r = \frac{\varepsilon}{\varepsilon_0}$

Material	Dielectric Constant (κ)	Dielectric strength (10^6 V/m)
Air	1.00059	3
Paper	3.7	16
Pyrex Glass	5.6	14
Water	80	-

- In dielectric, polarisation and production of induced charge takes place when dielectric is kept in an external electric field.

Electric polarization

- Electric polarization P is the difference between electric fields D (induced) and E (imposed) in dielectric due to bound and free charges written as $P = \frac{D-E}{4\pi}$
- In term of electric susceptibility : $P = \chi_e E$
- In MKS : $P = \varepsilon_0 \chi_e E$,
- The dielectric constant κ is always greater than 1 as $\chi_e > 0$

Capacitor

- A capacitor is a device which is used to store charge.
- Amount of charge 'Q' stored by the capacitor depends on voltage applied and size of capacitor.
- Capacitor consists of two similar conducting plates placed in front of each other where one plate is connected to positive terminal while other plate is connected to negative terminal.
- Electric charge stored between plates of capacitor is directly proportional to potential difference between its plates, *i.e.*,

$$Q = CV$$

where, C = Capacitance of capacitor, V= potential difference between the plates
- In capacitor, energy is stored in the form of electrical energy, in the space between the plates.

Capacitance

- Capacitance of a capacitor is ratio of magnitude of charge stored on the plate to potential difference between the plates, written as $C = \frac{Q}{\Delta V}$

where, C = capacitance in farads (F), Q = charge in Coulombs (C), ΔV = electric potential difference in Volts (V),
- SI unit of capacitance is farad (F)
- $1\,F = \frac{1\,C}{1\,V} = 9 \times 10^{11}$ stat farad,

Where, stat-farad is electrostatic unit of capacitance in C.G.S. system
- Capacitance of a conductor depends on size, shape, medium and other conductors in surrounding.
- Parallel plate capacitor with dielectric among its plates has capacitance which is given as :

$$C = \kappa\varepsilon_0 \frac{A}{d},$$

where, $\varepsilon_0 = 8.85 \times 10^{-12}$ F/m
- Capacitor having capacitance of 1 Farad is too large for electronics applications, so components with lesser values of capacitance such as μ (micro), n (nano) and p (pico) are applied such as :

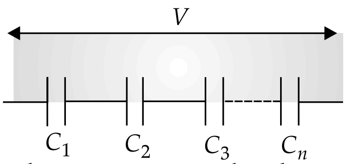
PREFIX	MULTIPLIER	
μ	10^{-6} (millionth)	$1\,\mu F = 10^{-6}\,F$
n	10^{-9} (thousand-millionth)	$1\,nF = 1^{-9}\,F$
p	10^{-12} (million-millionth)	$1\,pF = 10^{-12}\,F$

Combination of capacitors in series and parallel

Capacitors in series

- (i) If a number of capacitors of capacitances $C_1, C_2, C_3, \ldots\ldots\ldots C_n$ are connected in series, then their equivalent capacitance is given by :

$$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots\ldots\ldots\ldots \frac{1}{C_n}$$



- In series combination, the charge on each capacitor is same, but the potential difference on each capacitor depends on their respective capacitance, *i.e.*,

$$q_1 = q_2 = q_3 \ldots\ldots\ldots\ldots q_n = q$$

- If $V_1, V_2, V_3, \ldots\ldots\ldots, V_n$ be the potential differences across the capacitors and V be the emf of the charging battery, then

$$V = V_1 + V_2 + V_3 + \ldots\ldots\ldots + V_n$$

- As charge on each capacitor is same, therefore

$$q = V_1C_1 = V_2C_2 = V_3C_3 \text{ ....}$$

the potential difference is inversely proportional to the capacitance, *i.e.*,

$$V \propto \frac{1}{C}$$

- In series, potential difference across largest capacitance is minimum.
- The equivalent capacitance in series combination is less than the smallest capacitance in combination.

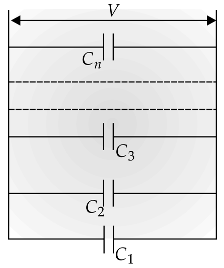
Capacitors in parallel

- (i) If a number of capacitors of capacitances C_1, C_2, C_3...............C_n are connected in parallel, then their equivalent capacitance is given by,

$$C_p = C_1 + C_2 + C_3 + \text{. .... } + C_n$$

- In parallel combination, the potential difference across each capacitor is same and equal to the emf of the charging battery, *i.e.*,

$$V_1 = V_2 = V_3 = \text{..............} = V_n = V$$



while the charge on different capacitors may be different.

- If q_1, q_2, q_3, ..., q_n be the charges on the different capacitors, then

$$q_1 + q_2 + q_3 + \text{..............} + q_n = VC_p$$

- As potential drop across each capacitor is same, so

$$\Rightarrow \qquad V = \frac{q_1}{C_1} = \frac{q_2}{C_2} = \frac{q_3}{C_3} = \text{.......} = \frac{q_n}{C_n}$$

- The charges on capacitors are directly proportional to capacitances, *i.e.*, $q \propto C$
- Parallel combination is useful when large capacitance with large charge gets accumulated on combination.
- Force of attraction between parallel plate capacitor will be $F = \frac{1}{2}\left[\frac{QV}{d}\right] = \frac{1}{2}QE$ where Q is charge on capacitor.

Capacitance of parallel plate capacitor with and without dielectric medium between the plates

- Parallel plate capacitor is a capacitor with two identical plane parallel plates separated by a small distance where space between them is filled by dielectric medium
- The electric field between two large parallel plates is given as :

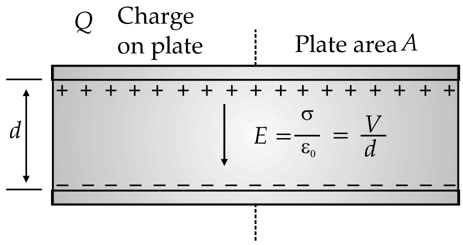
$$E = \frac{\sigma}{\varepsilon},$$

Where, σ = charge density and ε = permittivity

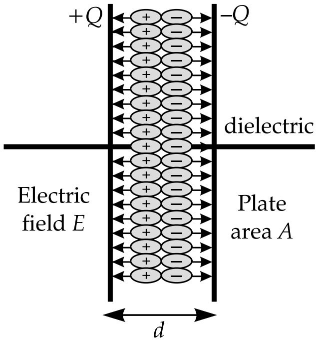
Surface charge density,

$$\sigma = \frac{Q}{A},$$

where, Q = charge on plate and A = plate area



- Capacitance of parallel-plate capacitor with area A separated by a distance d is written as $C = \varepsilon_r\varepsilon_0 \frac{A}{d}$



- If a dielectric slab is placed in between the plates of a capacitor, then its capacitance will increase by certain amount.
- Capacitance of parallel plate capacitor depends on plate area A, distance d between the plates, medium between the plates (κ) and not on charge on the plates or potential difference between the plates.
- If we have number of dielectric slabs of same area as the plates of the capacitor and thicknesses $t_1, t_2, t_3, ...$ and dielectric constant $\kappa_1, \kappa_2, \kappa_3. ....$ between the plates, then the capacitance of the capacitor is given by

$$C = \frac{\varepsilon_0 A}{\frac{t_1}{\kappa_1} + \frac{t_2}{\kappa_2} + \frac{t_3}{\kappa_3} + ...},$$

Where, $d = t_1 + t_2 + t_3 + ...$

- If slab of conductor of thickness t is introduced between the plates, then

$$C = \frac{\varepsilon_0 A}{\frac{t}{\kappa} + \frac{(d-t)}{1}} = \frac{\varepsilon_0 A}{\frac{t}{\infty} + \frac{(d-t)}{1}}$$

$$C = \frac{\varepsilon_0 A}{d-t} \qquad (\because \kappa = \infty \text{ for a conductor})$$

- When the medium between the plates consists of slabs of same thickness but areas $A_1, A_2, A_3, ...$ and dielectric constants $\kappa_1, \kappa_2, \kappa_3 ...$, then capacitance is given by

$$C = \frac{\varepsilon_0(\kappa_1 A_1 + \kappa_2 A_2 + \kappa_3 A_3 ....)}{d}$$

$$\therefore \qquad \kappa = \frac{C_m}{C_0} = \frac{\text{capacitance in medium}}{\text{capacitance in vacuum}}$$

- When space between the plates is partly filled with medium of thickness t and dielectric constant κ, then capacitance will be :

$$C = \frac{\varepsilon_0 A}{d - t + \frac{t}{\kappa}} = \frac{\varepsilon_0 A}{d - t\left(1 - \frac{1}{\kappa}\right)}$$

When there is no medium between the plates, then $\kappa = 1$, so

$$C_{\text{vacuum}} = \frac{\varepsilon_0 A}{d}$$

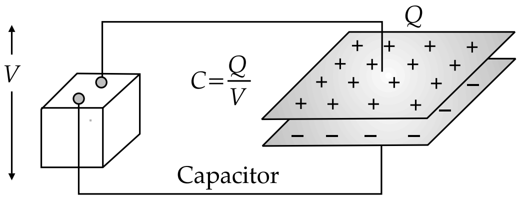
- Capacitance of spherical conductor of radius R in a medium of dielectric constant κ is given by

$$C = 4\pi\varepsilon_0 \kappa R$$

Energy stored in capacitor

- In capacitor, energy gets stored when a work is done on moving a positive charge from negative conductor to positive conductor against the repulsive forces.

$$U = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}QV = \frac{1}{2}CV^2$$



- **Polar atom :** Atom in which positive and negative charges possess asymmetric charge distribution about its centre.
- **Polarisation :** The stretching of atoms of a dielectric slab under an applied electric field.
- **Dielectric strength :** The maximum value of electric field that can be applied to dielectric without its electric breakdown.
- **Dielectric :** It is an electrically insulated or non-conducting material considered for its electric susceptibility.
- **Permittivity :** It is a property of a dielectric medium that shows the forces which electric charges placed in medium exerts on each other.

OR

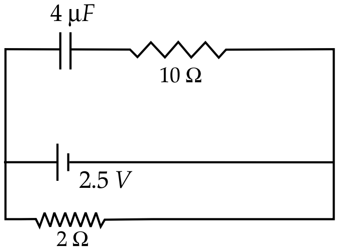
It is the measure of resistance that is encountered when forming an electric field in a particular medium. More specifically, permittivity describes the amount of charge needed to generate one unit of electric flux in a particular medium.

Key Formulae

- Capacitance, $C = \frac{Q}{V}$, measured in Farad; 1 F = 1 coulomb/volt
- Parallel plate capacitor :
$$C = \kappa\varepsilon_0 \frac{A}{d}$$
- Cylindrical capacitor :
$$C = 2\pi\kappa\varepsilon_0 \frac{L}{\ln(b / a)}$$
where, L= length [m], b= radius of the outer conductor [m], a = radius of the inner conductor [m]
- Spherical capacitor :
$$C = 4\pi\kappa\, \varepsilon_0 \left(\frac{ab}{b-a}\right)$$
where, b = radius of the outer conductor [m], a = radius of the inner conductor [m]
- Maximum charge on a capacitor :
$$Q = VC$$
- For capacitors connected in series, the charge Q is equal for each capacitor as well as for the total equivalent. If the **dielectric constant** κ is changed, the capacitance is multiplied by κ, the voltage is divided by κ and Q is unchanged. In vacuum $\kappa = 1$ and when dielectrics are used, replace ε_0 with $\kappa\, \varepsilon_0$.
- Electrical energy stored in a capacitor : [Joules (J)]
$$U_E = \frac{QV}{2} = \frac{CV^2}{2} = \frac{Q^2}{2C}$$
- Surface charge density or Charge per unit area : [C/m^2]
$$\sigma = \frac{q}{\text{A}}$$
- **Energy density :**
  - Electric energy density is also called Electrostatic pressure.
  - Electric force between plates of capacitor
$$F = \frac{1}{2}\varepsilon_0 E^2{\cdot}A$$
  - Energy stored in terms of Energy density
$$\frac{E}{A \times d} = \frac{1}{2}\varepsilon_0 E^2$$
$$U = \frac{1}{2}\varepsilon_0 E^2$$
where, U = energy per unit volume [J/m^3], ε_0= permittivity of free space, = 8.85×10^{-12} C^2/Nm^2, E = energy [J]
  - **Capacitors in series :**
$$\frac{1}{C_{eff}} = \frac{1}{C_1} + \frac{1}{C_2} \cdots$$
  - **Capacitors in parallel :**
$$C_{eff} = C_1 + C_2 \ldots$$

Objective Type Questions **(1 mark each)**

Q. 1. A capacitor of 4 μF is connected as shown in the circuit Figure. The internal resistance of the battery is 0.5 Ω. The amount of charge on the capacitor plates will be :



(a) 0 μC **(b) 4 μC**
(c) 16 μC **(d) 8 μC**

[NCERT Exemplar]

Ans. Correct option : (d)

Explanation : As capacitor offer infinite resistance for DC circuit. So current from cell will not flow across branch of 4 μF and 10 Ω. So current will flow across 2 ohm branch.

So current flows through across 2 Ω resistance from left to right is,

$$I=\frac{V}{(R+r)}$$

$$=\frac{2.5\text{ V}}{(2+0.5)}$$

$$= 1\text{ A}$$

So Potential Difference (PD) across 2 Ω resistance $V = RI = 2\times1 = 2$ Volt

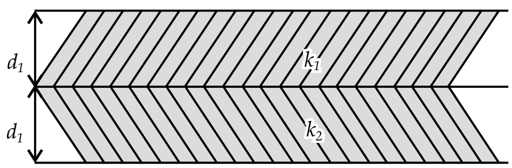
As battery, capacitor and 2 branches are in parallel. So PD will remain same across all three branches.

As current does not flow through capacitor branch so no potential drop will be across 10 Ω.

So PD across 4 F capacitor = 2 Volt

$Q = CV = 2\,\mu\text{F}\times 2\text{ V} = 8\,\mu\text{C}$

Q. 2. A parallel plate capacitor is made of two dielectric blocks in series. One of the blocks has thickness d_1 and dielectric constant κ_1 and the other has thickness d_2 and dielectric constant κ_2 as shown in Figure. This arrangement can be thought as a dielectric slab of thickness d (= d_1+d_2) and effective dielectric constant κ. The κ is :



(a) $\dfrac{k_1d_1+k_2d_2}{d_1+d_2}$ **(b)** $\dfrac{k_1d_1+k_2d_2}{k_1+k_2}$

(c) $\dfrac{k_1k_2(d_1+d_2)}{(k_1d_1+k_2d_2)}$ **(d)** $\dfrac{2k_1k_2}{k_1+k_2}$

[NCERT Exemplar]

Ans. Correct option : (c)

Explanation : Capacitance of a parallel plate capacitor filled with dielectric of constant k_1 and thickness d_1 is,

$$C_1=\frac{k_1\varepsilon_0A}{d_1}$$

Similarly, for other capacitance of a parallel plate capacitor filled with dielectric of constant k_2 and thickness d_2 is,

$$C_2=\frac{k_2\varepsilon_0A}{d_2}$$

Both capacitors are in series so equivalent capacitance C is related as :

$$\frac{1}{C}=\frac{1}{C_1}+\frac{1}{C_2}=\frac{d_1}{k_1\varepsilon_0A}+\frac{d_2}{\text{k}_2\varepsilon_0A}$$

$$=\frac{1}{\varepsilon_0A}\left[\frac{k_2d_1+k_1d_2}{k_1k_2}\right]$$

So, $$C=\frac{k_1k_2\varepsilon\ A}{(k_1d_2+k_2d_1)} \quad \ldots\text{(i)}$$

$$C'=\frac{k\varepsilon_0A}{d} \quad \ldots\text{(ii)}$$

where, $d = (d_1+d_2)$

So, multiply the numerator and denominator of eqn. (i) with (d_1+d_2),

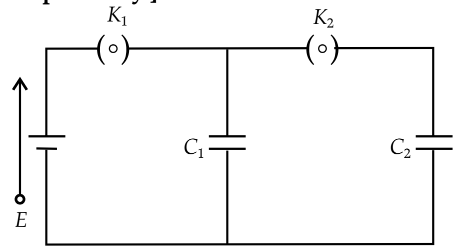
$$C=\frac{k_1k_2\varepsilon_0A}{(k_1d_2+k_2d_1)}\cdot\frac{(d_1+d_2)}{(d_1+d_2)}=\frac{k_1k_2}{(k_1d_2+k_2d_1)}\cdot\frac{\varepsilon_0A}{(d_1+d_2)} \quad \ldots\text{(iii)}$$

Comparing eqns. (ii) and (iii), the dielectric constant of new capacitor is :

$$k=\frac{k_1k_2(d_1+d_2)}{(k_1d_2+k_2d_1)}$$

Q. 3. In the circuit shown in Figure, initially key K_1 is closed and key :

K_2 is open. Then K_1 is opened and K_2 is closed (order is important). [Take Q_1' and Q_2' as charges on C_1 and C_2 and V_1 and V_2 as voltage respectively.]



Then

(a) charge on C_1 gets redistributed such that $V_1 = V_2$

(b) charge on C_1 gets redistributed such that $Q_1' = Q_2'$

(c) charge on C_1 gets redistributed such that $C_1V_1 + C_2V_2 = C_1 E$

(d) charge on C_1 gets redistributed such that $Q_1' + Q_2' = Q$ [NCERT Exemp. Q. 2.11, Page 12]

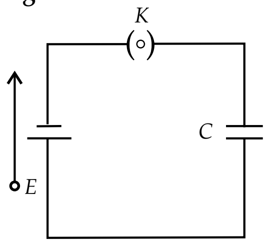
Ans. Correct options : (a) and (d)

Explanation : When key k_1 is closed and key k_2 is open, the capacitor C_1 is charged by cell and when k_1 is opened and k_2 is closed, the charge stored by capacitor C_1 gets redistributed between C_1 and C_2.

So, the charge on C_1 gets redistributed such that $Q_1' + Q_2' = Q$

As C_1 and C_2 both are in parallel combination, so their potential will be equal, *i.e.*, $V_1 = V_2$. It verifies the answer (a).

Q. 4. A parallel plate capacitor is connected to a battery as shown in Figure. Consider two situations :



A : Key K is kept closed and plates of capacitors are moved apart using insulating handle.

B : Key K is opened and plates of capacitors are moved apart using insulating handle.

Choose the correct option(s).

(a) In A : Q remains same but C changes.

(b) In B : V remains same but C changes.

(c) In A : V remains same and hence Q changes.

(d) In B : Q remains same and hence V changes.

[NCERT Exemp. Q. 2.13, Page 13]

Ans. Correct options : (c) and (d)

Explanation :

(i) In case A : when the space between the plates of capacitor increases, the capacitance decreases by relation,

$$C=\frac{K\varepsilon_0 A}{d}$$

But battery remains same, *i.e.*, potential difference across plate remains 'V' same. So by $Q = CV$ relation, Q also decreases verifies answer (c) and discards answer (a).

(ii) In case B : K is open, and capacitance decreases by moving apart plates of capacitor, so by relation $Q = CV$, here K is open, so charge Q remains same in turn V will increase on decreasing C. Hence answer (d) is verified.

Very Short Answer Type Question (1 mark)

AI **Q. 1. Does the charge given to metallic sphere depend on whether it is hollow or solid ? Give reasons for your answer.** A **[Delhi I, II, III 2017]**

Ans. No, ½

Because the charge resides only on the surface of the conductor. ½

[CBSE Marking Scheme 2017]

Detailed Answer :

No, the charge given to a metallic sphere does not depend on whether the metallic sphere is hollow or solid. ½

The amount of charge held by a capacitor is given by

$$Q = CV$$

where, Q = charge, C = capacitance, V = Voltage

The reason that capacitance C and charge Q is not affected by whether or not the sphere is hollow or solid since in perfect conductor, like charges are free to take equilibrium positions in response to mutual electrostatic repulsion where charges move to outer surface of sphere and get uniformly distributed over surface of sphere. ½

Short Answer Type Questions-I (2 marks each)

AI **Q. 1. The battery remains connected to a parallel plate capacitor and a dielectric slab is inserted between the plates. What will be effect on its (i) potential difference (ii) capacity (iii) electric field and (iv) energy stored ?** A **[CBSE SQP I 2017]**

Ans. When a battery remains connected,

(i) potential difference V remains constant ½

(ii) capacity C increases ½

(iii) electric field will remain same ½

(iv) energy stored $\frac{1}{2}CV^2$ increases as C increases ½

[CBSE Marking Scheme 2017]

Detailed Answer :

When a battery remains connected to a parallel plate capacitor and if a dielectric slab is inserted between the plates of capacitor, then

(i) there will be no change in the potential difference as the capacitor remained connected with the battery. ½

(ii) capacity or capacitance will increase since with the introduction of dielectric slab, capacitance of capacitor will result $C = \frac{\kappa\varepsilon_0 A}{d}$ where $\kappa > 1$ resulting an increase in C. ½

(iii) Electric field will remain same as there will be no change in potential difference and distance between the plates. ½

(iv) Energy stored will be increased since from the expression $U = \frac{1}{2}CV^2$, potential difference V remains same while C increases which finally increases the energy of capacitor. ½

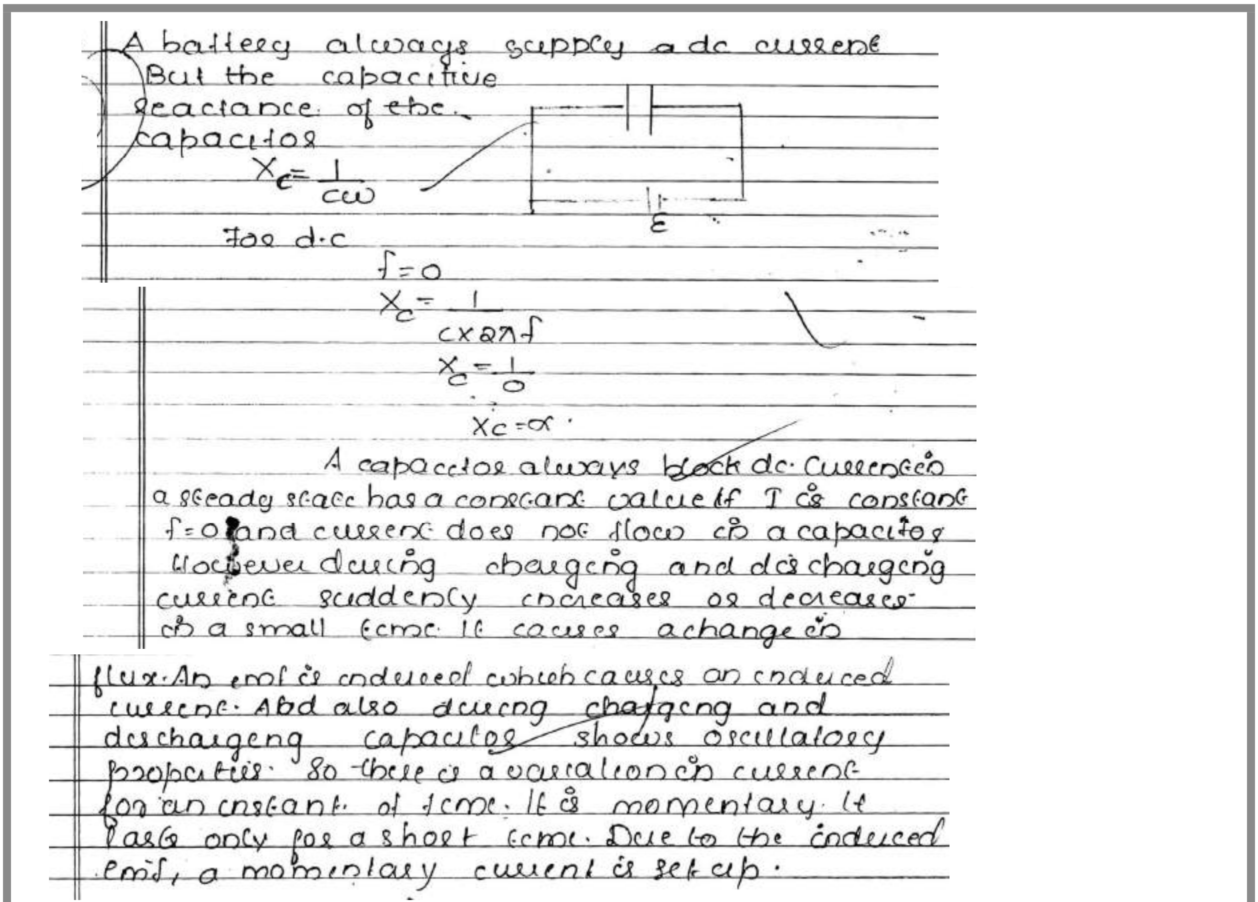
Q. 2. Why does current in a steady state not flow in a capacitor connected across a battery ? However momentary current does flow during charging or discharging of capacitor. Explain. **A [O.D. III 2017]**

Ans. (i) In steady state no current flows because we have two sources (battery and fully charged capacitor) of equal potential connected in opposition. **1**

(ii) During charging/discharging, there is a momentary flow of current as the potential of the two sources are not equal to each other. **1**

[CBSE Marking Scheme 2017]

OR



A battery always supply a dc current
But the capacitive
reactance of the
capacitor
$X_C = \frac{1}{C\omega}$

For d.c
$f = 0$
$X_C = \frac{1}{C \times 2\pi f}$
$X_C = \frac{1}{0}$
$X_C = \infty$

A capacitor always block dc. Current in a steady state has a constant value if I is constant f=0 and current does not flow in a capacitor However during charging and discharging current suddenly increases or decreases in a small time it causes a change in flux. An emf is induced which causes an induced current. And also during charging and discharging capacitor shows oscillatory properties. So there is a variation in current for an instant of time. It is momentary. It lasts only for a short time. Due to the induced emf, a momentary current is set up.

[Topper's Answer, 2017]

Q. 3. A parallel plate capacitor of capacitance C is charged to a potential V. It is then connected to another uncharged capacitor having the same capacitance. Find out the ratio of the energy stored in the combined system to that of stored initially in the single capacitor. **A [O.D. I, II, III 2014]**

Ans. Energy stored in a capacitor

$$U = \frac{1}{2}CV^2 \text{ or } \frac{1}{2}\frac{q^2}{C} \qquad ½$$

Capacitance of the (parallel) combination

$$= C + C = 2C \qquad ½$$

Here, the total charge, Q, remains the same :

$$\therefore \quad \text{Initial energy} = \frac{1}{2}\cdot\frac{q^2}{C}$$

and

$$\text{Final energy} = \frac{1}{2}\frac{q^2}{2C} \qquad ½$$

$$\therefore \quad \text{Ratio of energies} = \frac{\text{final energy}}{\text{initial energy}}$$

$$= \frac{\frac{1}{4}\frac{q^2}{C}}{\frac{1}{2}\frac{q^2}{C}} \qquad ½$$

$$= \frac{2}{4} = \frac{1}{2}$$

[CBSE Marking Scheme 2014]

Q. 4. A slab of material of dielectric constant κ has the same area as that of the plates of a parallel plate capacitor but has the thickness *d*/2, where *d* is the separation between the plates. Find out the expression for its capacitance when the slab is inserted between the plates of the capacitor.

U **[O.D. I, II, III 2013]**

Ans. Initially when there is vacuum between the two plates, the capacitance of the two parallel plates is,

$$C_0 = \frac{\varepsilon_0 A}{d} \qquad \tfrac{1}{2}$$

where, A is the area of parallel plates.

Suppose that the capacitor is connected to a battery, an electric field E_0 is produced.

Now if we insert the dielectric slab of thickness $t = d/2$, the electric field reduces to E.

Now the gap between the plates is divided in two parts, for distance t there is electric field E and for the remaining distance $(d - t)$, the electric field is E_0. ½

If V be the potential difference between the plates of the capacitor, then

$$V = Et + E_0(d-t)$$

$$V = \frac{Ed}{2} + \frac{E_0 d}{2} \qquad \left(\because t = \frac{d}{2}\right)$$

$$= \frac{d}{2}(E + E_0) \qquad \left(\because \frac{E_0}{E} = \kappa\right)$$

$$\Rightarrow \quad V = \frac{d}{2}\left(\frac{E_0}{\kappa} + E_0\right)$$

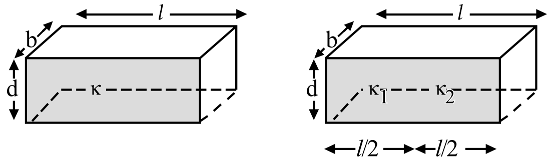
$$= \frac{dE_0}{2\kappa}(\kappa + 1)$$

Now, $$E_0 = \frac{\sigma}{\varepsilon_0} = \frac{q}{\varepsilon_0 A}$$

$$\Rightarrow \quad V = \frac{d}{2\kappa} \cdot \frac{q}{\varepsilon_0 A}(\kappa + 1)$$

We know, $$C = \frac{q}{V} = \frac{2\kappa\varepsilon_0 A}{d(\kappa+1)} \qquad 1$$

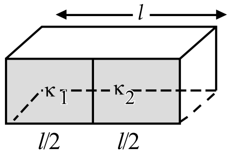
Q. 5. Two identical capacitors of plate dimension *l* × *b* and plate separation *d* have dielectric slabs filled in between the space of the plates as shown in the figures.



Obtain the relation between the dielectric constants κ, κ_1 and κ_2. A **[O.D. Comptt. I, II, III 2013]**

Ans. The capacitor can be considered as split into two capacitors connected in parallel.

Here $$C_1 = \frac{\kappa_1 \varepsilon_0 A/2}{d},\; C_2 = \frac{\kappa_2 \varepsilon_0 A/2}{d} \qquad \tfrac{1}{2}$$



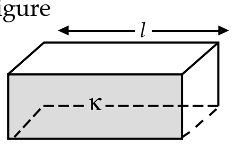
In parallel combination,

$$C_{eq} = C_1 + C_2$$

$$C_{eq} = \frac{\kappa_1 \varepsilon_0 A}{2d} + \frac{\kappa_2 \varepsilon_0 A}{2d} \qquad \tfrac{1}{2}$$

$$C_{eq} = \frac{\varepsilon_0 A}{2d}(\kappa_1 + \kappa_2) \qquad \text{...(i) } \tfrac{1}{2}$$

From this figure



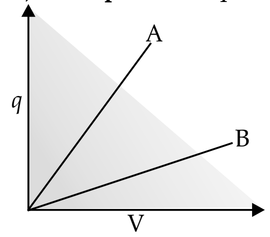
$$C_{eq} = \frac{\kappa \varepsilon_0 A}{2d} \qquad \text{...(ii) } \tfrac{1}{2}$$

From (i) and (ii)

$$\frac{\kappa \varepsilon_0 A}{2d} = \frac{\varepsilon_0 A}{2d}(\kappa_1 + \kappa_2)$$

$$\kappa = \kappa_1 + \kappa_2$$

Q. 6. The given graph shows variation of charge *q* versus potential difference *V* for two capacitors C_1 and C_2. Both the capacitors have same plate separation but plate area of C_2 is greater than that of C_1. Which line (*A* or *B*) corresponds to C_1 and why ?



[Comptt. O.D. I, II, III 2014]

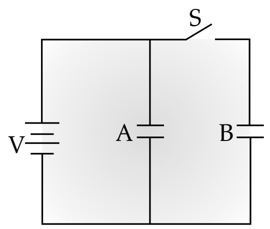
Ans. For C_1

Line *B* 1

Since slope (*q*/*V*) of Line *B* is lesser than that of Line *A*. **[CBSE Marking Scheme 2014] 1**

Short Answer Type Questions-II (3 marks each)

Q. 1. Two identical parallel plate capacitors *A* and *B* are connected to a battery of *V* volts with the switch *S* closed. The switch is now opened and the free space between the plates of the capacitors is filled with a dielectric of dielectric constant κ. Find the ratio of the total electrostatic energy stored in both capacitors before and after the introduction of the dielectric.



U **[O.D. I, II, III 2017]**

Ans. Formula for energy stored ½

Energy stored before 1

Energy stored after 1

Ratio ½

$$\text{Energy stored} = \frac{1}{2}CV^2 = \left(\frac{1}{2}\frac{Q^2}{C}\right) \quad ½$$

Net capacitance with switch S closed

$$= C + C = 2C \quad ½$$

$$\text{Energy stored} = \frac{1}{2}\times 2C \times V^2 = CV^2 \quad ½$$

After the switch S is opened, capacitance of each capacitor = $\kappa \times C$

$\therefore$ Energy stored in capacitor A

$$A = \frac{1}{2}\kappa CV^2$$

For capacitor B,

$$\text{Energy stored} = \frac{1}{2}\frac{Q^2}{\kappa C} = \frac{1}{2}\frac{C^2V^2}{\kappa C}$$

$$= \frac{1}{2}\frac{CV^2}{\kappa} \quad ½$$

$$\therefore \text{ Total Energy stored} = \frac{1}{2}\kappa CV^2 + \frac{1}{2}\frac{CV^2}{\kappa}$$

$$= \frac{1}{2}CV^2\left(\kappa + \frac{1}{\kappa}\right) \quad ½$$

$$= \frac{1}{2}CV^2\left(\frac{\kappa^2+1}{\kappa}\right)$$

$$\therefore \quad \text{Required ratio} = \frac{2CV^2\kappa}{CV^2(\kappa^2+1)}$$

$$= \frac{2\kappa}{(\kappa^2+1)} \quad ½$$

[CBSE Marking Scheme 2017]

Q. 2. A capacitor of unknown capacitance is connected across a battery of V volt. A charge of 360 μC is stored in it. When the potential across the capacitor is reduced by 120 V, the charge stored in the capacitor becomes 120 μC. Calculate V and the unknown capacitance. What would have been the charge on the capacitor if the voltage were increased by 120 V? U **[Delhi (Comptt.) I 2017]**

Ans. Calculation of V and unknown capacitance 2

Calculation of charge when voltage is increased by 120 V 1

Capacitance $\quad C = \dfrac{Q_1}{V_1}$ ½

Also, $\quad C = \dfrac{Q_2}{V_2}$ and $C = \dfrac{Q_3}{V_3}$ ½

$$\frac{360\mu C}{V} = \frac{120\mu C}{(V-120)} \quad ½$$

So, $\quad 3V - 360V = V$

$$V = 180\ V \quad ½$$

$$C = \frac{360\ \mu V}{180\ V} = 2\ \mu F \quad ½$$

$$2\mu F = \frac{Q_3}{300} \quad ½$$

$$Q_3 = 600\ \mu\ C \quad ½$$

[CBSE Marking Scheme 2017]

Detailed Answer :

(i) Let the initial voltage = V volts

Charge stored, $Q_1 = 360\ \mu C$

$$Q_1 = CV_1 = CV \quad \text{...(i) ½}$$

Now the changed potential

$$V_2 = (V - 120) \text{ volts}$$

Charge stored, $\quad Q_2 = 120\ \mu C$

$$Q_2 = CV_2 \quad \text{...(ii) ½}$$

Now divide equation (i) by (ii), we have:

$$\frac{Q_1}{Q_2} = \frac{CV_1}{CV_2}$$

$$\frac{360\ \mu C}{120\ \mu C} = \frac{V}{V-120} \quad ½$$

On solving, we get V = 180 volts

Now value of unknown capacitance C can be calculated as

$$C = \frac{Q_1}{V} = \frac{360\times10^{-6}}{180}\text{ F}$$

$$= 2 \times 10^{-6} F = 2\ \mu F \quad ½$$

(ii) When the voltage applied increases to 120 V, then

$$V_3 = 180 + 120 = 300\ V \quad ½$$

Finally the charge stored in the capacitor will be :

$$Q_3 = CV_3$$

$$= 2 \times 10^{-6} \times 300\ C$$

$$= 600\ \mu C \quad ½$$

Q. 3. A capacitor of unknown capacitance is connected across a battery of V volt. A charge of 240 μC is stored in it. When the potential across the capacitor is reduced by 80 V, the charge stored in the capacitor becomes 80 μC. Calculate V and the unknown capacitance. What would have been the charge in the capacitor if the voltage were increased by 80 V ? U **[Delhi (Compt) II 2017]**

Ans. Calculation of V and unknown capacitance 2

Calculation of charge when voltage is increased by 80 V 1

Try yourself, Similar to Q.2, Short Answer Type II

[CBSE Marking Scheme 2017]

Q. 4. A capacitor of unknown capacitance is connected across a battery of V volt. A charge of 120 μC is stored in it. When the potential across the capacitor is reduced by 40 V, the charge stored in the capacitor becomes 40 μC. Calculate V and the unknown capacitance. What would have been charge in the capacitor if the voltage is increased by 40 V ? U **[Delhi (Compt) III 2017]**

Ans. Calculation of V and unknown capacitance 2
Calculation of charge when voltage is increased by 40 V 1
Try yourself, Similar to Q.2, Short Answer Type II
[CBSE Marking Scheme 2017]

Q. 5. A parallel plate capacitor of capacitance C is charged to a potential V by a battery. Without disconnecting the battery, the distance between the plates is tripled and a dielectric medium of κ = 10 is introduced between the plates of the capacitor. Explain giving reasons, how will the following be affected :
(i) capacitance of the capacitor
(ii) charge on the capacitor, and
(iii) energy density of the capacitor.
U **[O.D. (Compt) I, II, III 2017]**

Ans. Effect on capacitance 1
Effect on charge 1
Effect on energy 1

(i) $$C = \frac{\varepsilon_0 A}{d} \quad ½$$

$$C' = \frac{\kappa\varepsilon_0 A}{d'} = 10\frac{\varepsilon_0 A}{3d} = \frac{10}{3}C \quad ½$$

(ii) V remains same since battery is not disconnected ½

$$\therefore \quad Q' = C'V = \frac{10}{3}CV = \frac{10}{3}Q \quad ½$$

(iii) Energy density, $U_d = \frac{1}{2}\varepsilon_0 E^2$

$$E = \frac{(V)}{d}$$

$$U'_d = \frac{1}{2}\kappa\varepsilon_0 E'^2 \quad ½$$

$$= \frac{10}{2}\varepsilon_0\left(\frac{V}{d'}\right)^2$$

$$= \frac{10}{9}\left(\frac{1}{2}\varepsilon_0 E^2\right)$$

$$= \frac{10}{9}U_d \quad ½$$

[CBSE Marking Scheme 2017]

Q. 6. (i) How many electrons must be added to one plate and removed from other so as to store 25.0 J of energy in a 5.0 nF parallel plate capacitor ?
(ii) How would you modify this capacitor so that it can store 50.0 J of energy without changing the charge on its plates ? U **[SQP 2017-18]**

Ans. (i) $C = 5 \times 10^{-9}$ F, $U = 25$ J

$$U = \frac{Q^2}{2C}$$

$$Q^2 = 2UC \quad ½$$
$$= 2 \times 25 \times 5 \times 10^{-9}$$
$$Q = 5{\cdot}0 \times 10^{-4}\ C \quad ½$$
$$Q = ne \quad ½$$
$$n = \frac{Q}{e} = \frac{5{\cdot}0\times10^{-4}C}{1{\cdot}60\times10^{-19}}$$
$$= 3{\cdot}125 \times 10^{15} \text{ electrons} \quad ½$$

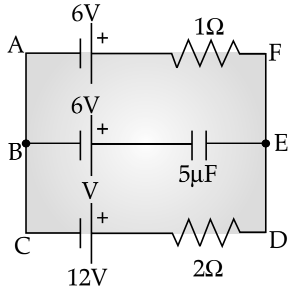
(ii) Without changing charge on the plates, we can make C half.

$$C = \frac{\varepsilon_0 A}{d}$$

Double the plate separation or inserting dielectric of dielectric constant of a value such that C will become half. 1

[CBSE Marking Scheme 2017]

Q. 7. In the given circuit, with steady current, calculate the potential difference across the capacitor and the charge stored in it. U **[Foreign I 2017]**



Ans. Value of current 1
Value of voltage 1
Value of charge 1
In the loop $ACDFA$

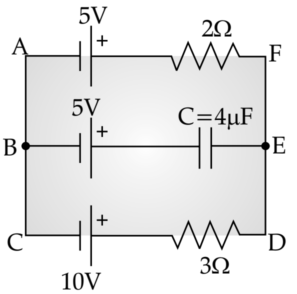
$$I = \frac{12-6}{(1+2)} = 2A \quad 1$$

$$V_{AF} = V_{BE} \quad ½$$
$$= 6 + 2 = 6 + V_C$$
$$V_C = 2\ V \quad ½$$

Charge $Q = CV_C = 5\ \mu F \times 2\ V$
$= 10\ \mu C$ 1

[CBSE Marking Scheme 2017]

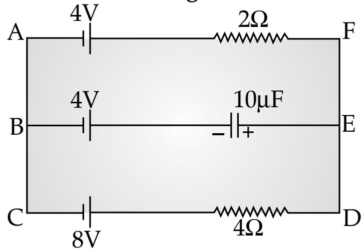
Q. 8. In the given circuit, with steady current, calculate the potential drop across the capacitor in terms of V.



U**[Foreign II 2017]**

Ans. Calculation of current 1½
Calculation of potential across capacitor 1½
Try yourself, Similar to Q. 7 Short Answer Type II
[CBSE Marking Scheme 2017]

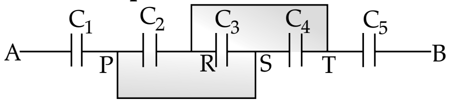
AI **Q. 9. In the given circuit, with steady current, calculate the potential difference across the capacitor and the charge stored in it.**



U **[Foreign III 2017]**

Ans. Value of current 1
Value of voltage 1
Value of charge 1
Try yourself, Similar to Q. 7 Short Answer Type II
[CBSE Marking Scheme 2017]

AI **Q. 10. (i) Find equivalent capacitance between A and B in the combination given below. Each capacitor is of 2 μF capacitance.**



(ii) If a DC source of 7 V is connected across AB, how much charge is drawn from the source and what is the energy stored in the network ? U **[Delhi I 2017]**

Ans. (i) Calculation of equivalent capacitance 1
(ii) Calculation of charge and energy stored 1+1

(i) Capacitors C_2, C_3, and C_4 are in parallel

$\therefore$ $C_{234} = C_2 + C_3 + C_4$

$\therefore$ $C_{234} = 6\ \mu F$ ½

Capacitors C_1, C_{234}, and C_5 are in series

$$\therefore \quad \frac{1}{C_{eq}} = \frac{1}{C_1}+\frac{1}{C_{234}}+\frac{1}{C_5}$$

$$= \frac{1}{2}+\frac{1}{6}+\frac{1}{2} = \frac{7}{6}\mu F$$

$$C_{eq} = \frac{6}{7}\mu F$$ ½

(ii) Charge drawn from the source

$$Q = C_{eq}V$$ ½

$$= \frac{6}{7}\times 7\ \mu C = 6\ \mu C$$ ½

$$\text{Energy stored, } U = \frac{Q^2}{2C_{eq}}$$

$$= \frac{6\times 6\times 10^{-12}\times 7}{2\times 6\times 10^{-6}} J$$ ½

$$= 21\ \mu J$$ ½

[CBSE Marking Scheme 2017]

Q. 11. A 12 pF capacitor is connected to a 50 V battery. How much electrostatic energy is stored in the capacitor ? If another capacitor of 6 pF is connected in series with it with the same battery connected across the combination, find the charge stored and potential difference across each capacitor.
U **[Delhi II 2017]**

Ans. Calculation of electrostatic energy in 12 pF capacitor 1
Total charge stored in combination 1
Potential difference across each capacitor ½ + ½

Energy stored, in the capacitor of capacitance 12 pF,

$$U = \frac{1}{2}CV^2$$

$$= \frac{1}{2}\times 12\times 10^{-12}\times 50\times 50 J$$

$$= 1.5\times 10^{-8} J$$ ½

C = Equivalent capacitance of 12 pF and 6 pF, in series, is given by ½

$$\frac{1}{C} = \frac{1}{12}+\frac{1}{6} = \frac{1+2}{12}$$

$\therefore$ $C = 4\ pF$

Charge stored across each capacitor ½

$$q = CV$$
$$= 4\times 10^{-12}\times 50\ C$$
$$= 2\times 10^{-10}\ C$$

Charge on each capacitor 12 pF as well as 6 pF

$\therefore$ Potential difference across capacitor C_1 ½

$$\therefore \quad V_1 = \frac{2\times 10^{-10}}{12\times 10^{-12}}$$

$$= \frac{50}{3} V$$ ½

$\therefore$ Potential difference across capacitor C_2

$$\therefore \quad V_2 = \frac{2\times 10^{-10}}{6\times 10^{-12}}$$

$$= \frac{100}{3} V$$ ½

[CBSE Marking Scheme 2017]

Q. 12. Two identical capacitors of 12 pF each are connected in series across a battery of 50 V. How much electrostatic energy is stored in the combination ? If these were connected in parallel across the same battery, how much energy will be stored in the combination now ?
Also find the charge drawn from the battery in each case. U **[Delhi III 2017]**

Ans. Equivalent capacitance in series ½
Energy in series combination ½
Charge in series combination ½
Equivalent capacitance in parallel combination ½
Energy in parallel combination ½
Charge in parallel combination ½

In series combination :

$$\frac{1}{C_s} = \left(\frac{1}{12}+\frac{1}{12}\right)(pF)^{-1} \quad ½$$

$\therefore \quad C_s = 6 \times 10^{-12}$ F

$$U_s = \frac{1}{2}CV^2$$

$$U_s = \frac{1}{2} \times 6 \times 10^{-12} \times 50 \times 50\, J$$

$U_s = 75 \times 10^{-10}\, J$ ½

$q_s = C_s V$

$= 6 \times 10^{-12} \times 50\, C$

$= 300 \times 10^{-12}\, C$

$= 3 \times 10^{-10}\, C$ ½

In parallel combination :

$C_p = (12 + 12)\, p$F ½

$\therefore \quad C_p = 24 \times 10^{-12}$ F

$$U_p = \frac{1}{2}\times 24\times 10^{-12}\times 2500\, J$$

$U_p = 3 \times 10^{-8}$ J

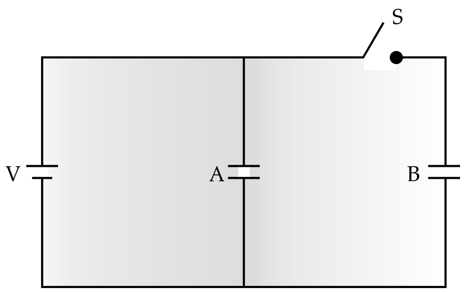
$q_p = C_p V$ ½

$= 24 \times 10^{-12} \times 50\, C$

$= 1{\cdot}2 \times 10^{-9}\, C$ ½

[CBSE Marking Scheme 2017]

13. Two identical parallel plate capacitors A and B are connected to a battery of V volts with the switch S closed. The switch is now opened and the free space between the plates of the capacitors is filled with a dielectric of dielectric constant κ. Find the ratio of the total electrostatic energy stored in both capacitors before and after the introduction of the dielectric. A **[O.D. Set I, II, III 2017]**



Ans. Initially:

$C_A^0 = \frac{\varepsilon_0 A}{d} = C \quad V_0$

$C_B^0 = \frac{\varepsilon_0 A}{d} = C \quad V_0$

Later

$C_A = K C_A^0 = KC$

$C_B = KC$

Total electrostatic energy stored before dielectric

$= \frac{1}{2}C_A V_A^2 + \frac{1}{2}C_B V_B^2$

$\frac{1}{2}\times CV^2 + \frac{1}{2}CV^2$

$\times 2 \times \frac{1}{2}CV^2 = CV^2$

After across A

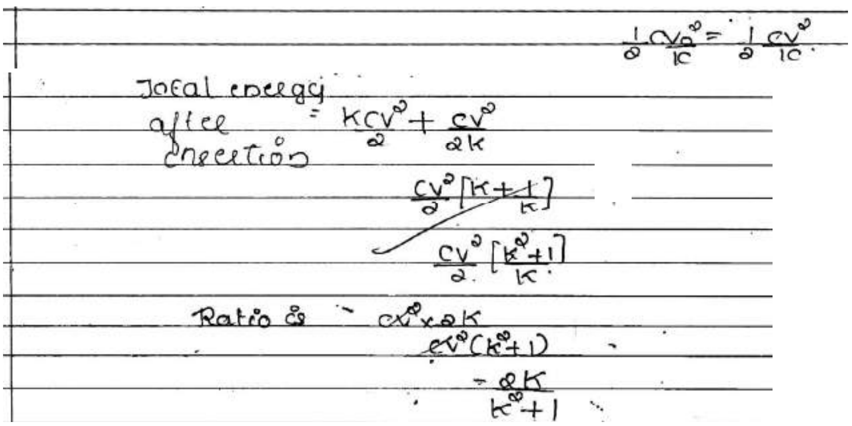
$\frac{1}{2}\times C_A \cdot V^2$

$\frac{1}{2}\times KC\times V^2$

$= K\times\frac{1}{2}CV^2$

across B $\frac{1}{2}C_B \times V_{new}^2$

$\frac{1}{2}C_B\times \frac{V_0^2}{K^2} = \frac{1}{2}\times KC\times\frac{V_0^2}{K^2}$


$\frac{1}{2}\frac{CV_0^2}{K} = \frac{1}{2}\frac{CV^2}{K}$

Total energy after insertion $= \frac{KCV^2}{2} + \frac{CV^2}{2K}$

$\frac{CV^2}{2}\left[K + \frac{1}{K}\right]$

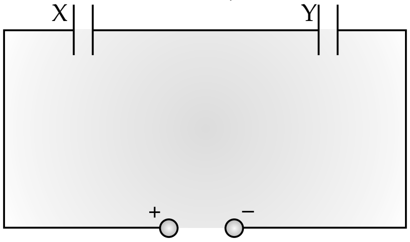
$\frac{CV^2}{2}\left[\frac{K^2+1}{K}\right]$

Ratio is $= \frac{CV^2 \times 2K}{CV^2(K^2+1)}$

$= \frac{2K}{K^2+1}$

[Topper's Answers 2017]

AI **Q. 14. Two parallel plate capacitors X and Y have the same area of plates and same separation between them. X has air between the plates while Y contains a dielectric medium of $\varepsilon_r = 4$.**



(i) Calculate capacitance of each capacitor if equivalent capacitance of the combination is 4 μF.

(ii) Calculate the potential difference between the plates of X and Y.

(iii) Estimate the ratio of electrostatic energy stored in X and Y.

U **[Delhi I, II, III 2016, Set D, 2009, Set F 2008]**

Ans. (i) Let $C_X = C$

$C_Y = 4C$

(as it has a dielectric medium of $\varepsilon_r = 4$)

For series combination of two capacitors

$$\frac{1}{C} = \frac{1}{C_X} + \frac{1}{C_Y}$$

$$\Rightarrow \quad \frac{1}{4\mu F} = \frac{1}{C} + \frac{1}{4C}$$

$$\frac{1}{4\mu F} = \frac{5}{4C}$$

$$\Rightarrow \quad C = 5\ \mu F$$

Hence $C_X = 5\ \mu F$ ½

$C_Y = 20\ \mu F$ ½

(ii) Total charge $Q = CV$

$= 4\ \mu F \times 15\ V = 60\ \mu C$

$$V_X = \frac{Q}{C_X} = \frac{60\ \mu C}{5\ \mu F} = 12\ V$$ ½

$$V_Y = \frac{Q}{C_Y} = \frac{60\ \mu C}{20\ \mu F} = 3\ V$$ ½

(iii)
$$\frac{E_x}{E_y} = \frac{\frac{Q^2}{2C_X}}{\frac{Q^2}{2C_Y}} = \frac{C_Y}{C_X} = \frac{20}{5} = 4:1$$ **1**

(Also accept any other correct alternative method)

[CBSE Marking Scheme 2016]

Q. 15. A parallel plate capacitor, of capacitance 20 μF, is connected to a 100 V, supply. After sometime, the battery is disconnected, and the space, between the plates of the capacitor is filled with a dielectric of dielectric constant 5. Calculate the energy stored in the capacitor.

(i) Before

(ii) After the dielectric has been put in between its plates. A **[O.D. Comptt. I, II, III, 2016]**

Ans. Charge stored, $Q = CV = 20 \times 10^{-6} \times 100$ C ½

$= 2000\ \mu C$

New value of capacitance

$= 5 \times 20\ \mu F$

$= 100\ \mu F$

Energy stored in a capacitor

$$= \frac{1}{2}\frac{Q^2}{C}\left(= \frac{1}{2}CV^2 = \frac{1}{2}QV\right)$$ ½

(i) Energy stored before dielectric

$$= \frac{1}{2} \times \frac{[2000 \times 10^{-6}] \times (2000 \times 10^{-6})}{20 \times 10^{-6}}$$

$= 0.1\ J$ **1**

(ii) Energy stored after the dielectric is introduced

($\because$ there is no change in the value of Q)

$$= \frac{1}{2} \times \frac{2000 \times 10^{-6} \times 2000 \times 10^{-6}}{100 \times 10^{-6}}$$ ½

$= 0.02\ J$ ½

[CBSE Marking Scheme, 2016]

Q. 16. (i) A parallel plate capacitor (C_1) having charge Q is connected, to an identical uncharged capacitor C_2 in series. What would be the charge accumulated on the capacitor C_2 ?

(ii) Three identical capacitors each of capacitance 3 μF are connected in turn in series and in parallel combination to the common source of V volt. Find out the ratio of the energies stored in two configurations.

A [O.D. Set I, II, III 2016]

Ans. (i) Both capacitors would come to common potential.

$V_{common} = \frac{\text{Total charge}}{\text{Net capacitance}} = \frac{Q}{C_1C_2}(C_1+C_2)$

$\Rightarrow$ Charge on second capacitor $(Q') = \frac{Q(C_1+C_2)}{C_1 C_2}$

(ii) Capacitance in series:

$\frac{1}{C_{net}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \Rightarrow \frac{1}{C_s} = \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = 1 \Rightarrow C_s = 1$

Parallel combination

$C_{net} = C_1 + C_2 + C_3 \Rightarrow C_p = 3+3+3 = 9 \Rightarrow C_p = 9\ \mu F$

Ratio of energies $= \frac{1}{2}C_sV^2 : \frac{1}{2}C_pV^2 = 1:9$

$\therefore$ Ratio of energies in series and parallel $= 1:9$

[Topper's Answer 2016]

Q. 17. Two capacitors of unknown capacitances C_1 and C_2 are connected first in series and then in parallel across a battery of 100 V. If the energy stored in the two combinations is 0·045 J and 0.25 J respectively, determine the value of C_1 and C_2. Also calculate the charge on each capacitor in parallel combination.

A [Delhi I, II, III 2015]

Ans. Energy stored in a capacitor

$$U = \frac{1}{2}CV^2 \quad ½$$

In series combination

$$0.045 = \frac{1}{2}\frac{C_1C_2}{C_1+C_2}(100)^2$$

$$\Rightarrow \frac{C_1C_2}{C_1+C_2} = 0.09 \times 10^{-4} \quad ...(i)\ ½$$

In parallel combination

$$0.25 = \frac{1}{2}(C_1+C_2)(100)^2$$

$$\Rightarrow C_1 + C_2 = 0.5 \times 10^{-4} \quad ...(ii)\ ½$$

On simplifying (i) and (ii)

$$C_1C_2 = 0.045 \times 10^{-8}$$
$$(C_1 - C_2)^2 = (C_1 + C_2)^2 - 4C_1C_2$$
$$= (0.5 \times 10^{-4})^2 - 4 \times 0.045 \times 10^{-8}$$
$$= 0.25 \times 10^{-8} - 0.180 \times 10^{-8}$$
$$(C_1 - C_2)^2 = 0.07 \times 10^{-8}$$
$$(C_1 - C_2) = 2.6 \times 10^{-5} = 0.25 \times 10^{-4} \quad ...(iii)$$

From (ii) and (iii) we have ½

$C_1 = 0.38 \times 10^{-4}$ F and $C_2 = 0.12 \times 10^{-4}$ F

Charges on capacitors C_1 and C_2 in parallel combination

$Q_1 = C_1V = (0.38 \times 10^{-4} \times 100) = 0.38 \times 10^{-2}\ C$ ½

$Q_2 = C_2V = (0.12 \times 10^{-4} \times 100) = 0.12 \times 10^{-2}\ C$ ½

Alternatively,

$$U = \frac{1}{2}CV^2$$

and
$$0.045 = \frac{1}{2}\left(\frac{C_1C_2}{C_1+C_2}\right)(100)^2$$

$$0.25 = \frac{1}{2}(C_1 + C_2)(100)^2$$

If the student is unable to calculate C_1 and C_2, award him/her full 2 marks.

Also if the student just writes

$Q_1 = C_1V = C_1(100)$ and $Q_2 = C_2V = C_2(100)$

award him/her one mark for this part of the question.

[CBSE Marking Scheme, 2015]

Q. 18. Eight identical spherical drops, each carrying a charge 1 nC are at a potential of 900 V each. All these drops combine together to form a single large drop. Calculate the potential of this large drop.

(Assume no wastage of any kind and take the capacitance of a sphere of radius r as proportional to r). **U [CBSE SQP 2014]**

Ans. Let the radius of each drop be r. The capacitance C of each drop is kr, where k is a constant.

Also $q = CV$, $V = 900$ volt ½

$\therefore$ Charge on each drop $= q = (kr \times 900)\ C$ ½

$\therefore$ Total charge on all the eight drops = $Q = 8q$

$= 7200kr$ ½

Let R be the radius of the large drop. Then

$$\frac{4\pi}{3} R^3 = 8 \times \frac{4\pi}{3} r^3$$

$\therefore$ $R = (8)^{1/3} r = 2r$ ½

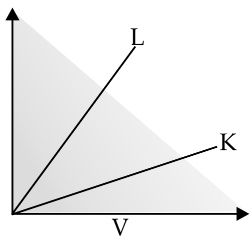
$\therefore$ Capacitance C' of the large drop = $kR = 2kr$ ½

$\therefore$ Potential of the large drop = $\frac{Q}{C'} = \frac{7200kr}{2kr}$ volt

$= 3600\ V$ ½

[CBSE Marking Scheme 2014]

Q. 19. The following graph shows the variation of charge *Q* with voltage *V* for two capacitors *K* and *L* in which capacitor is more electrostatic energy stored ?



C **[Foreign 2008]**

Ans. The slope of *Q-V* graph of capacitor gives capacitance, therefore, capacitor *L* have greater capacitance. Since, two capacitors are given same voltage *V*, therefore energy stored in a capacitor will be given by formula.

$$U = \frac{1}{2}CV^2 \text{ and for same } V$$

$$U \propto C$$

$\because$ $C_L > C_K$

$\Rightarrow$ $U_L > U_K$

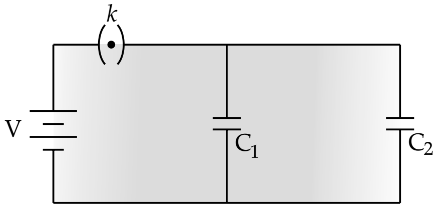
$\Rightarrow$ Capacitor *L* with store more electrostatic energy. **3**

OR

Try yourself, Similar to Q. 6 Short Answer type I

[CBSE Marking Scheme, 2008] 3

Q. 20. Two parallel plate capacitors of capacitances C_1 and C_2 such that $C_1 = 2C_2$ are connected across a battery of *V* volts as shown in the figure. Initially the key (κ, is kept closed to fully charge the capacitors. The key is now thrown open and a dielectric slab of dielectric constant 'κ' is inserted in the two capacitors to completely fill the gap between the plates. Find the ratio of (i) the net capacitance and (ii) the energies stored in the combination, before and after the introduction of the dielectric slab.

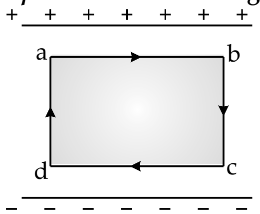


A **[Comptt. Delhi I, II, III 2014]**

Ans. Try yourself, Similar to Q. 13 Short Answer Type II

Q. 21. (i) Obtain the expression for the energy stored per unit volume in a charged parallel plate capacitor.

(ii) The electric field inside a parallel plate capacitor is E. Find the amount of work done in moving a charge *q* over a closed rectangular loop *a b c d a*.



A **[Delhi I, II, III 2014]**

Ans. (i) Work done by the source of potential, in storing an additional charge (dq), is

$$dW = V.dq$$

But $V = \frac{q}{C}$

$\Rightarrow$ $dW = \frac{q}{C}\,dq$ ½

Total work done in storing the charge q,

$$\int dW = \int_0^q \frac{q}{C} dq$$

$$W = \frac{1}{C}\left(\frac{q^2}{2}\right)_0^q = \frac{q^2}{2C}$$ ½

This work is stored as electrostatic energy in the capacitor.

$\therefore$ $U = \frac{1}{2} CV^2$ $(\because\ q = CV)$

Energy stored per unit volume = $\frac{\frac{1}{2}CV^2}{Ad}$

$$U = \frac{\frac{1}{2}\left(\frac{\varepsilon_0 A}{d}\right)(Ed)^2}{Ad}$$

$$U = \frac{1}{2}\varepsilon_0 E^2$$ **1**

(ii) Work done in moving the charge q from a to b, and from c to d is zero because electric field is perpendicular to the displacement. ½

Work done from b to c = – Work done from d to a ½

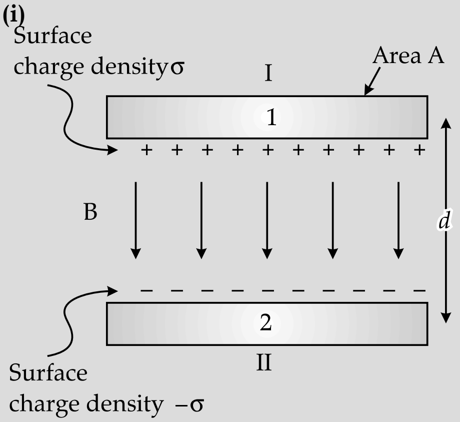
$\therefore$ Total work done in moving a charge q over the closed loop = 0

[CBSE Marking Scheme 2014]

AI Q. 22. (i) Derive the expression for the capacitance of a parallel plate capacitor having plate area A and plate separation d.

(ii) Two charged spherical conductors of radii R_1 and R_2 when connected by a conducting wire acquire charges q_1 and q_2 respectively. Find the ratio of their surface charge densities in terms of their radii. A **[Delhi I, II, III 2014] [NCERT Exemplar]**

Ans. (i)



Electric field between the plates of capacitor

$$E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{A\varepsilon_0} \quad \frac{1}{2}$$

$$\therefore \quad V = Ed = \frac{Qd}{A\varepsilon_0} \quad \frac{1}{2}$$

Capacitance, $$C = \frac{Q}{V} = \frac{\varepsilon_0 A}{d} \quad \mathbf{1}$$

(ii) When the two charged spherical conductors are connected by a conducting wire, they acquire the same potential. ½

i.e., $$\frac{Kq_1}{R_1} = \frac{Kq_2}{R_2} \Rightarrow \frac{q_1}{q_2} = \frac{R_1}{R_2}$$

Hence, ratio of surface charge densities,

$$\frac{\sigma_1}{\sigma_2} = \frac{q_1/4\pi R_1^2}{q_2/4\pi R_2^2}$$

$$\frac{\sigma_1}{\sigma_2} = \frac{q_1 R_2^2}{q_2 R_1^2}$$

$$\frac{\sigma_1}{\sigma_2} = \frac{R_1}{R_2} \times \frac{R_2^2}{R_1^2} = \frac{R_2}{R_1} \quad \frac{1}{2}$$

[CBSE Marking Scheme, 2014]

Q. 23. In a parallel plate capacitor with air between the plates, each plate has an area of 6×10^{-3} m^2 and the separation between the plates is 3 mm.

(i) Calculate the capacitance of the capacitor.

(ii) If this capacitor is connected to 100 V supply, what would be the charge on each plate ?

(iii) How would charge on the plates be affected, if a 3 mm thick mica sheet of κ = 6 is inserted between the plates while the voltage supply remains connected ? A **[Foreign 2014]**

Ans. (i) $$C = \frac{\varepsilon_0 A}{d} \quad \frac{1}{2}$$

$$C = \frac{8.85\times10^{-12}\times6\times10^{-3}}{3\times10^{-3}} \text{ F}$$

$$C = 17.7 \times 10^{-12} \text{ F} = (17.7\ pF) \quad \frac{1}{2}$$

(ii) $$Q = CV \quad \frac{1}{2}$$

$$Q = 17.7 \times 10^{-12} \times 100\ C$$

$$Q = 17.7 \times 10^{-10}\ C = 1.77\ nC \quad \frac{1}{2}$$

(iii) $$Q' = \kappa Q \quad \frac{1}{2}$$

$$= 6 \times 17.7 \times 10^{-10}\ C$$

$$= 106.2 \times 10^{-10}\ C = 10.62 \times 10^{-9}$$

$$= 10.62\ nC \quad \frac{1}{2}$$

[CBSE Marking Scheme 2014]

AI Q. 24. A capacitor of unknown capacitance is connected across a battery of V volts. The charge stored in it is 360 μC. When potential across the capacitor is reduced by 120 V, the charge stored in it becomes 120 μC.

Calculate :

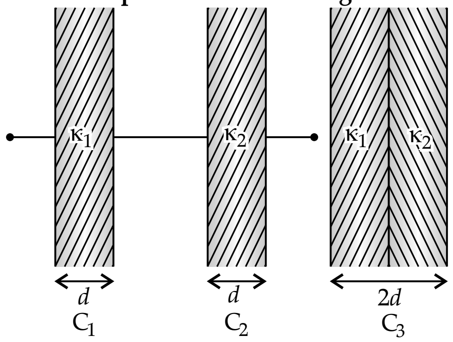
(i) The potential V and the unknown capacitance C.

(ii) What will be the charge stored in the capacitor, if the voltage applied had increased by 120 V ?

A **[Delhi I, II, III 2013]**

Ans. Try yourself, Similar to Q. 2, Short Answer Type II

Q. 25. The capacitors C_1 and C_2 having plates of area A each, are connected in series, as shown. Compare the capacitance of this combination with the capacitor C_3, again having plates of area A each, but 'made up' as shown in the figure.



U **[CBSE SQP 2013]**

Ans. We have $$C_1 = \frac{A\varepsilon_0\kappa_1}{d} \quad \frac{1}{2}$$

and $$C_2 = \frac{A\varepsilon_0\kappa_2}{d}$$

$$\therefore \quad C_{eq} = \frac{C_1C_2}{C_1+C_2} = \frac{A\varepsilon_0}{d}\left(\frac{\kappa_1\kappa_2}{\kappa_1+\kappa_2}\right) \quad \frac{1}{2}$$

Now, capacitor C_3 can be considered as made up of two capacitors C_1 and C_2, each of plate area A and separation d, connected in series. ½

We have $$C_1' = \frac{A\varepsilon_0\kappa_1}{d}$$

and $$C_2' = \frac{A\varepsilon_0 K_2}{d} \quad \frac{1}{2}$$

$$\Rightarrow \quad C_3 = \frac{C_1'C_2'}{C_1'+C_2'} = \frac{A\varepsilon_0}{d}\left(\frac{\kappa_1\kappa_2}{\kappa_1+\kappa_2}\right)$$

$$\therefore \quad \frac{C_3}{C_{eq}} = 1 \quad ½$$

Hence, the net capacitance of the combination is equal to that of C_3. ½

Q. 26. (i) Deduce the expression for the electrostatic energy stored in a capacitor of capacitance 'C' and having charge 'Q'.

(ii) How will the (a) energy stored and (b) the electric field inside the capacitor be affected when it is completely filled with a dielectric material of dielectric constant 'κ' ? U **[O.D. I, II, III 2012]**

Ans. (i) Expression of electrostatic energy stored in a capacitor **2**

(ii) Effect on energy stored and electric field in capacitor due to dielectric material. **1**

Potential difference between the plates of capacitor ½

$$V = q/C$$

Work done in adding an additional charge dq on the capacitor

$$dW = V \times dq$$

$$= \left(\frac{q}{C}\right) \times dq \quad ½$$

$\therefore$ Total energy stored in the capacitor,

$$U = \int dW = \int_0^Q \frac{q}{C}\, dq = \frac{1}{2}\frac{Q^2}{C} \quad 1$$

When battery is disconnected :

(a) Energy stored will be decreased or energy stored $= \frac{1}{\kappa}$ times the initial energy. ½

(b) Electric field would decrease. ½

$$E' = E/\kappa$$

[CBSE Marking Scheme 2012]

Long Answer Type Questions **(5 marks each)**

Q. 1. (i) Define the capacitance of a capacitor. Obtain the expression for the capacitance of a parallel plate capacitor in vacuum in terms of plate area A and separation d between the plates.

(ii) A slab of material of dielectric constant κ has the same area as the plates of a parallel plate capacitor but has a thickness $\frac{2d}{4}$. Find the ratio of the capacitance with dielectric inside it to its capacitance without the dielectric.

U **[Foreign I, II, III 2017]**

Ans. (i) Definition of capacitance **1**

Obtaining capacitance **2**

(ii) Ratio of capacitances **2**

(i) Capacitance equals the magnitude of the charge on each plate needed to raise the potential difference between the plates by unity. **1**

OR

(i) Try yourself, similar Q.22 (i), Short Answer Type Question-II

(ii) Capacitance without dielectric,

$$C = \frac{A\varepsilon_0}{d}$$

Capacitance when filled with dielectric having thickness $\frac{3d}{4}$

$$C' = \frac{A\varepsilon_0}{\left(d - t + \frac{t}{\kappa}\right)} \quad ½$$

$$= \frac{A\varepsilon_0}{\left(d - \frac{3d}{4} + \frac{3d}{4\kappa}\right)} \quad \left[\text{As } t = \frac{3d}{4}\right]$$

$$= \frac{4\varepsilon_0 \kappa A}{d(\kappa+3)}$$

$$\text{Ratio} \quad \frac{C'}{C} = \frac{A\varepsilon_0 4\kappa}{d(\kappa+3)} \times \frac{d}{A\varepsilon_0} \quad 1$$

$$= \frac{4\kappa}{(\kappa+3)} \quad ½$$

[CBSE Marking Scheme 2017]

AI **Q. 2. (i) Define the S.I. unit of capacitance.**

(ii) Obtain expression for the capacitance of a parallel plate capacitor.

(iii) Derive the expression for the affective capacitance of a series combination of n-capacitors.

R **[Delhi Comptt. I, II, III, 2016]**

Ans. (i) When a charge of one coulomb, produces a potential difference of one volt between the plates of a capacitor, the capacitance is one farad. **1**

[Alternatively,

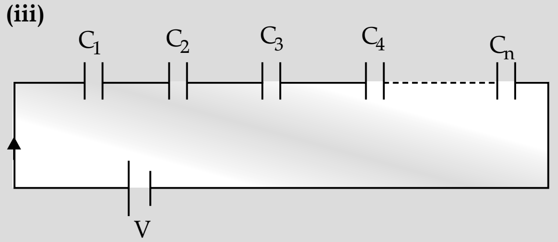
$$Q = CV \Rightarrow C = \frac{Q}{V}$$

When Q = 1 coulomb, V = 1 volt

$$1 \text{ farad} = \frac{1 \text{ coulomb}}{1 \text{ volt}}$$

(ii) Try yourself, Similar to Q. 22 (i), Short Answer Type Question-II **2**

(iii)



½

In series combination, charge on each capacitor is same.

Let it be Q,

$$V_1 = \frac{Q}{C_1} \qquad ½$$

$$V_2 = \frac{Q}{C_2}$$

$$V_n = \frac{Q}{C_n} \qquad ½$$

Total potential

$$V = V_1 + V_2 + V_3 + ... + V_n$$

$$V = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3} + ... + \frac{Q}{C_n}$$

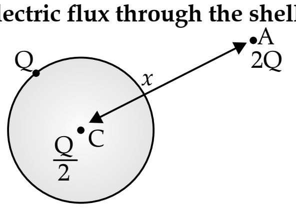
$$\frac{V}{Q} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + ... + \frac{1}{C_n} \qquad ½$$

$$\Rightarrow \quad \frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + ... + \frac{1}{C_n}.$$

[CBSE Marking Scheme 2016]

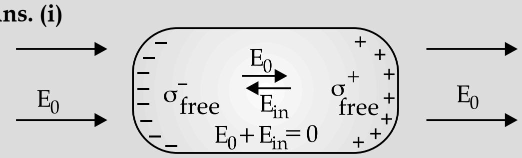
Q. 3. (i) Explain, using suitable diagrams, the difference in the behaviour of (a) conductor and (b) dielectric in the presence of an external electric field. Define the terms polarization of a dielectric and write its relation with susceptibility.

(ii) A thin metallic spherical shell of radius R carries a charge Q on its surface. A point charge $\frac{Q}{2}$ is placed at its centre C and another charge $+2Q$ is placed outside the shell at a distance x from the centre as shown in the figure. Find (a) the force on the charge at the centre of shell and at the point A, (b) the electric flux through the shell.

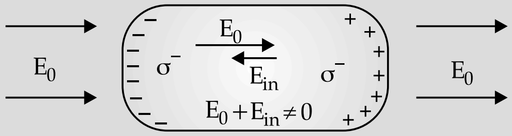


U **[Delhi I, II, III 2015]**

Ans. (i)



Conductor ½



Dielectric ½

(a) In the presence of electric field, the free charge carriers, in a conductor, move the charge distribution in the conductor re-adjusting itself so that the net electric field within the conductor becomes zero. ½

(b) In a dielectric, the external electric field induces a net dipole moment, by stretching/reorienting the molecules. The electric field, due to this induced dipole moment, opposes, but does not exactly cancel, the external electric field. ½

Polarisation : Induced dipole moment, per unit volume, is called the polarization. For linear isotropic dielectrics having a susceptibility χ_C, we have

$$P = \chi_C E \qquad \mathbf{1}$$

(ii) (a) Net Force on the charge $\frac{Q}{2}$, placed at the centre of the shell, is zero. **1**

Force on charge $2Q$ kept at point A

$$F = E \times 2Q$$

$$= \frac{1\left(\frac{3Q}{2}\right)2Q}{4\pi\varepsilon_0 x^2}$$

$$= \frac{6Q^2}{8\pi\varepsilon_0 x^2} = \frac{3Q^2}{4\pi\varepsilon_0 x^2}$$

OR

$$\frac{(k)3Q^2}{x^2}$$

where, $k = \frac{1}{4\pi\varepsilon_0}$

(b) Electric flux through the shell

$$\phi = \frac{Q}{2\varepsilon_0} \qquad \mathbf{1}$$

$\because$ Charge enclose is $\frac{Q}{2}$

[CBSE Marking Scheme 2015]

AI **Q. 4. (i) Compare the individual dipole moment and the specimen dipole moment for H_2O molecule and O_2 molecule when placed in**

(a) Absence of external electric field.

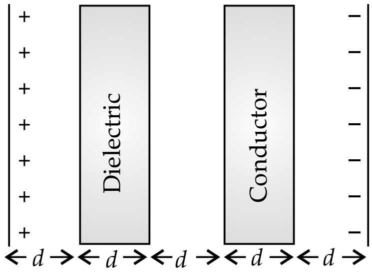
(b) Presence of external electric field.

Justify your answer.

(ii) Given two parallel conducting plates of area A and charge densities $+\sigma$ and $-\sigma$. A dielectric slab of constant κ and a conducting slab of thickness d each are inserted in between them as shown.

(a) Find the potential difference between the plates.

(b) Plot E versus x graph, taking $x = 0$ at positive plate and $x = 5d$ at negative plate.



U **[CBSE SQP 2015-16]**

Ans. (i)

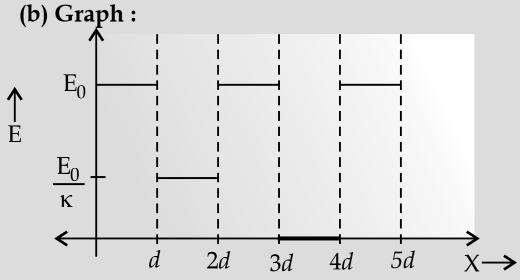
	Non-polar (O_2)	**Polar (H_2O)**
In absence of electric field		
Individual	No dipole moment exists.	Dipole moment exists.
Specimen	No dipole moment exists.	Dipoles are randomly oriented. Net P=0
In presence of electric field		
Individual	Dipole moment exists (molecules become polarised.)	Torque acts on the molecules to align them parallel to *E*.
Specimen	Dipole moment exists.	Net dipole moment exists parallel to *E*.

3

(ii) (a) $V = E_0 d + \frac{E_0}{\kappa} d + E_0 d + 0 + E_0 d$ ½

$V = 3 E_0 d + \frac{E_0}{\kappa} d$ ½

(b) Graph :



1

[CBSE Marking Scheme 2015]

Q. 5. (i) If two similar large plates, each of area *A* having surface charge densities +σ and –σ are separated by a distance *d* in air, find the expressions for

(a) field at points between the two plates and on outer side of the plates.

Specify the direction of the field in each case.

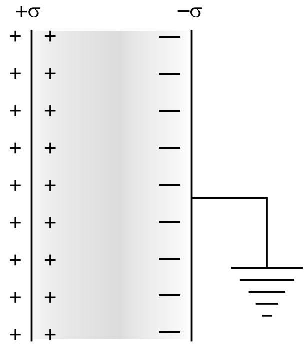
(b) the potential difference between the plates.

(c) the capacitance of the capacitor so formed.

(ii) Two metallic spheres of radii *R* and 2*R* are charged so that both of these have same surface charge density σ. If they are connected to each other with a conducting wire, in which direction will the charge flow and why ? U **[O.D. I, II, III 2016]**

Ans.

(i)



(a) The electric field at point between the plates

$$E = \frac{\sigma}{\varepsilon_0}$$

and directed from positive plate to negative plate. **1**

(b) Potential difference $V = E \times$ separation between two plates

$$\therefore \quad V = \frac{\sigma}{\varepsilon_0} \times d$$

$$V = \frac{\sigma}{\varepsilon_0} d$$ **1**

(c) Charge on capacitor = magnitude of charge on any of two plates

$$q = \sigma A$$

and potential difference,

$$V = \frac{\sigma d}{\varepsilon_0}$$

$$\therefore \text{ Capacitance, } C = \frac{q}{V} = \frac{\sigma A}{\left(\frac{\sigma d}{\varepsilon_0}\right)}$$

$$C = \frac{\varepsilon_0 A}{d}$$ **1**

(ii) When two charged conducting metallic spheres are connected with a conducting wire, then charge flows between the two, till their potentials become equal or redistribution of charges takes place *i.e.*, charge is divided on both the spheres. **2**

AI **Q. 6. A capacitor of capacitance C_1 is charged to a potential V_1 while another capacitor of capacitance C_2 is charged to a potential difference V_2. The capacitors are now disconnected from their respective charging batteries and connected in parallel to each other.**

(i) Find the total energy stored in the two capacitors before they are connected.

(ii) Find the total energy stored in the parallel combination of the two capacitors.

(iii) Explain the reason for the difference of energy in parallel combination in comparison to the total energy before they are connected.

R **[Comptt. Delhi/O.D. I, II, III 2018]**

Ans. (i) Finding the total energy before the capacitors are connected **1**

(ii) Finding the total energy in the parallel combination **3**

(iii) Reason for difference **1**

(i) We have

Energy stored in a capacitor $= \frac{1}{2}CV^2$ ½

$\therefore$ Energy stored in the charged capacitors

$$E_1 = \frac{1}{2}C_1V_1^2$$

And $$E_2 = \frac{1}{2}C_2V_2^2$$

$\therefore$ Total energy stored $= \frac{1}{2}C_1V_1^2 + \frac{1}{2}C_2V_2^2$ ½

(b) Let V be the potential difference across the parallel combination.

Equivalent capacitance = $(C_1 + C_2)$ ½

Since charge is a conserved quantity, we have

$$(C_1 + C_2)V = C_1V_1 + C_2V_2$$ ½

$$\Rightarrow \quad V = \left[\frac{C_1V_1 + C_2V_2}{(C_1 + C_2)}\right]$$ **1**

$\therefore$ Total energy stored in the parallel combination

$$= \frac{1}{2}(C_1 + C_2)V^2$$ ½

$$= \frac{1}{2}\frac{(C_1V_1 + C_2V_2)^2}{(C_1 + C_2)}$$ ½

(c) The total energy of the parallel combination is different (less) from the total energy before the capacitors are connected. This is because some energy gets used up due to the movement of charges. **1**

[CBSE Marking Scheme, 2018]

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