LOGARITHMS

I. Logarithm : If *a* is a positive real number, other than 1 and $a^m = x$, then we write : $m = \log_a x$ and we say that the value of log x to the base a is m.

Example :

(i) $10^3 = 1000 \Rightarrow \log_{10} 1000 = 3$

(ii) $3^4 = 81 \Rightarrow \log_3 81 = 4$

(i) $2^{-3} = \frac{1}{8} \Rightarrow \log_2 \frac{1}{8} = -3$

(i) $(.1)^2 = 0.1 \Rightarrow \log_{(.1)} 0.1 = 2$

❖ EXAMPLES ❖

Ex.1 : Evalute :

(i) $\log_3 27$ (ii) $\log_7\left(\frac{1}{343}\right)$ (iii) $\log_{100}{(0.01)}$

Sol. : (i) Let $\log_3 27 = n$

Then, $3^n = 27 = 3^3$

or $n = 3$ $\therefore \log_3 27 = 3$

(ii) Let $\log_7\left(\frac{1}{343}\right) = n$

Then, $7^n = \frac{1}{343} = \frac{1}{7^3} = 7^{-3}$ or $n = -3$

$\therefore \log_7\left(\frac{1}{343}\right) = -3$

(iii) Let $\log_{100} (0.01) = n$

Then, $(100)^n = 0.01 = \frac{1}{100} = (100)^{-1}$

or $n = -1$

$\therefore \log_{100} (0.01) = -1$

Ex.2: Evalute :

(i) $\log_7 1 = 0$ (ii) $\log_{34} 34$ (iii) $36^{\log_6 4}$

Sol.: (i) We know that $\log_a 1 = 0$, so $\log_7 1 = 0$

(ii) We know that $\log_a a = 1$, so $\log_{34} 34 = 0$

(iii) We know that $a^{\log_a x} = x$.

Now,

$$36^{\log_6 4} = (6^2)^{\log_6 4} = 6^{2(\log_6 4)} = 6^{\log_6 (4^2)} = 6^{\log_6 16} = 16$$

Ex.3 : If $\log_{\sqrt{8}} x = 3\frac{1}{3}$, find the value of x.

Sol. : $\log_{\sqrt{8}} x = \frac{10}{3} \Leftrightarrow x = (\sqrt{8})^{10/3}$

$$x = (2^{3/2})^{10/3} = 2^{\left(\frac{3}{2}\times\frac{10}{3}\right)} = 2^5 = 32$$

Ex.4 : Evaluate

(i) $\log_5 3 \times \log_{27} 25$ (ii) $\log_9 27 - \log_{27} 9$

Sol. : (i) $\log_5 3 \times \log_{27} 25 = \frac{\log 3}{\log 5} \times \frac{\log 25}{\log 27}$

$= \frac{\log 3}{\log 5} \times \frac{\log(5^2)}{\log(3^3)} = \frac{\log 3}{\log 5} \times \frac{2\log 5}{3\log 3} = \frac{2}{3}$

(ii) Let $\log_9 27 = n$

Then, $9^n = 27$

$\Leftrightarrow 3^{2n} = 3^3$

$\Leftrightarrow 2n = 3$

$\Leftrightarrow n = \frac{3}{2}$

Again, let $\log_{27} 9 = m$

Then, $27^m = 9 \Leftrightarrow 3^{3m} = 3^2 \Leftrightarrow 3m = 2 \Leftrightarrow m = \frac{2}{3}$ $\therefore \log_9 27 - \log_{27} 9 = (n - m) =$

$\left(\frac{3}{2} - \frac{2}{3}\right) = \frac{5}{6}$

Ex.5 : Simplify : $\left(\log\frac{75}{16} - 2\log\frac{5}{9} + \log\frac{32}{243}\right)$

Sol. : $\log\frac{75}{16} - 2\log\frac{5}{9} + \log\frac{32}{243} = \log\frac{75}{16} - \log\left(\frac{5}{9}\right)^2 + \log\frac{32}{243}$

$= \log\frac{75}{16} - \log\frac{25}{81} + \log\frac{32}{243}$

$= \log\left(\frac{75}{16} \times \frac{32}{243} \times \frac{81}{25}\right) = \log 2$

Ex.6 : Find the value of x which satisfies the relation

$\log_{10} 3 + \log_{10} (4x + 1) = \log_{10} (x + 1) + 1$

Sol. : $\log_{10} 3 + \log_{10} (4x + 1) = \log_{10} (x + 1) + 1$

$\Leftrightarrow \log_{10} 3 + \log_{10} (4x + 1) = \log_{10} (x + 1) + \log_{10} 10$

$\Leftrightarrow \log_{10} \{3 (4x + 1) = \log_{10} [10 (x + 1)]$

$\Leftrightarrow 3 (4x + 1) = 10 (x + 1)$

$\Leftrightarrow 12x + 3 = 10x + 10$

$\Leftrightarrow 2x + 7 \Leftrightarrow x = \frac{7}{2}$

Ex.7 : Simplify :

$$\left[\frac{1}{\log_{xy}(xyz)}+\frac{1}{\log_{yz}(xyz)}+\frac{1}{\log_{zx}(xyz)}\right]$$

Sol. : Given expression

$= \log_{xyz}(xy) + \log_{xyz}(yz) + \log_{xyz}(zx)$

$$\left[\because \log_a x = \frac{1}{\log_x a}\right]$$

$= \log_{xyz}(xy \times yz \times zx) = \log_{xyz}(xyz)^2$

$= 2\log_{xyz}(xyz) = 2 \times 1 = 2$

Ex.8 : If $\log_{10} 2 = 0.30103$, find the value of $\log_{10} 50$.

Sol. : $\log_{10} 50 = \log_{10}\left(\frac{100}{2}\right) = \log_{10}100 - \log_{10} 2$

$= 2 - 0.30103 = 1.69897$

Ex.9 : If $\log 2 = 0.3010$, and $\log 3 = 0.4771$, find the values of:

(i) $\log 25$ (ii) $\log 4.5$

Sol. : (i) $\log 25 = \log\left(\frac{100}{4}\right) = \log 100 - \log 4$

$= 2 - 2\log 2$

$= (2 - 2 \times 0.3010)$

$= 1.398$

(ii) $\log 4.5 = \log\left(\frac{9}{2}\right)$

$= \log 9 - \log 2$

$= 2\log 3 - \log 2$

$= (2 \times 0.4771 - 0.3010)$

$= 0.6532$

Ex.10: If $\log 2 = 0.30103$, find the number of digits in 2^{56}

Sol. : $\log(2^{56}) = 56\log 2 = (56 \times 0.30103) = 16.85768$

Its characteristic is 16. Hence, the number of digits in 2^{56} is 17

EXERCISE

Q.1 The value of $\log_2 16$ is

(A) $\frac{1}{8}$ (B) 4 (C) 8 (D) 16

Q.2 The value of $\log_{343} 7$ is

(A) $\frac{1}{3}$ (B) –3 (C) $-\frac{1}{3}$ (D) 3

Q.3 The value of $\log_5\left(\frac{1}{125}\right)$ is:

(A) 3 (B) –3 (C) $\frac{1}{3}$ (D) $-\frac{1}{3}$

Q.4 The value of $\log_{\sqrt{2}} 32$ is:

(A) $\frac{5}{2}$ (B) 5 (C) 10 (D) $\frac{1}{10}$

Q.5 The value of $\log_{10} (.0001)$ is

(A) $\frac{1}{4}$ (B) $-\frac{1}{4}$ (C) – 4 (D) 4

Q.6 The value of $\log_{(.01)} (1000)$ is :

(A) $\frac{1}{3}$ (B) $-\frac{1}{3}$ (C) $\frac{3}{2}$ (D) $-\frac{3}{2}$

Q.7 The logarithm of 0.0625 to the base 2 is:

(A) –4 (B) –2
(C) 0.25 (D) 0.5

Q.8 If $\log_3 x = -2$, then x is equal to:

(A) –9 (B) –6 (C) –8 (D) $\frac{1}{9}$

Q.9 If $\log_8 x = \frac{2}{3}$, then the value of x is:

(A) $\frac{3}{4}$ (B) $\frac{4}{3}$ (C) 3 (D) 4

Q.10 If $\log_x\left(\frac{9}{16}\right) = -\frac{1}{2}$, then x is equal to:

(A) $-\frac{3}{4}$ (B) $\frac{3}{4}$ (C) 3 (D) $\frac{256}{81}$

Q.11 If $\log_x 4 = 4.0$ then the value of x is:

(A) 1 (B) 4 (C) 16 (D) 32

Q.12 If $\log_{10000} x = -\frac{1}{4}$, then x is equal to:

(A) $\frac{1}{10}$ (B) $\frac{1}{100}$

(C) $\frac{1}{1000}$ (D) $\frac{1}{10000}$

Q.13 If $\log_x 4 = \frac{1}{4}$, then x is equal to:

(A) 16 (B) 64 (C) 128 (D) 256

Q.14 If $\log_x (0.1) = -\frac{1}{3}$, then the value of x is:

(A) 10 (B) 100

(C) 1000 (D) $\frac{1}{1000}$

Q.15 If $\log_{32} x = 0.8$, then x is equal to:

(A) 25.6 (B) 16 (C) 10 (D) 12.8

Q.16 If $\log_x y = 100$ and $\log_2 x = 10$, then the value of y is:

(A) 2^{10} (B) 2^{100}

(C) 2^{1000} (D) 2^{10000}

Q.17 The value of $\log_{(-1/3)} 81$ is equal to:

(A) –27 (B) –4

(C) 4 (D) 27

Q.18 The value of $\log_{2\sqrt{3}}(1728)$ is:

(A) 3 (B) 5

(C) 6 (D) 9

Q.19 $\frac{\log\sqrt{8}}{\log 8}$ is equal to:

(A) $\frac{1}{\sqrt{8}}$ (B) $\frac{1}{4}$ (C) $\frac{1}{2}$ (D) $\frac{1}{8}$

Q.20 Which of the following statements is not correct?

(A) $\log_{10} 10 = 1$

(B) $\log (2 + 3) = \log (2 \times 3)$

(C) $\log_{10} 1 = 0$

(D) $\log (1 + 2 + 3) = \log 1 + \log 2 + \log 3$

Q.21 The value of $\log_2 (\log_5 625)$ is :

(A) 2 (B) 5 (C) 10 (D) 15

Q.22 If $\log_2 [\log_3 (\log_2 x)] = 1$, then x is equal to:

(A) 0 (B) 12 (C) 128 (D) 512

Q.23 The value of $\log_2 \log_2 \log_3 \log_3 27^3$ is:

(A) 0 (B) 1 (C) 2 (D) 3

Q.24 If $a^x = b^y$, then:

(A) $\log\frac{a}{b} = \frac{x}{y}$ (B) $\frac{\log a}{\log b} = \frac{x}{y}$

(C) $\frac{\log a}{\log b} = \frac{y}{x}$ (D) none of these

Q.25 log 360 is equal to:
(A) 2 log 2 + 3 log 3
(B) 3 log 2 + 2 log 3
(C) 3 log 2 + 2 log 3 – log 5
(D) 3 log 2 + 2 log 3 + log 5

Q.26 The value of $\left(\frac{1}{3}\log_{10}125 - 2\log_{10}4 + \log_{10}32\right)$ is :

(A) 0 (B) $\frac{4}{5}$ (C) 1 (D) 2

Q.27 $2\log_{10}5 + \log_{10}8 - \frac{1}{2}\log_{10}4 = ?$
(A) 2 (B) 4
(C) $2 + 2\log_{10}2$ (D) $4 - 4\log_{10}2$

Q.28 If $\log_a(ab) = x$, then $\log_b(ab)$ is:

(A) $\frac{1}{x}$ (B) $\frac{x}{x+1}$ (C) $\frac{x}{1-x}$ (D) $\frac{x}{x-1}$

Q.29 If log 2 = x, log 3 = y and log 7 = z, then the value of log $\left(4.\sqrt[3]{63}\right)$ is:

(A) $2x+\frac{2}{3}y-\frac{1}{3}z$ (B) $2x+\frac{2}{3}y+\frac{1}{3}z$

(C) $2x-\frac{2}{3}y+\frac{1}{3}z$ (D) $-2x+\frac{2}{3}y+\frac{1}{3}z$

Q.30 If $\log_4 x + \log_2 x = 6$, then x is equal to:
(A) 2 (B) 4 (C) 8 (D) 16

Q.31 If $\log_8 x + \log_8\frac{1}{6} = \frac{1}{3}$, then the value of x is:
(A) 12 (B) 16 (C) 18 (D) 24

Q.32 If $\log_{10}125 + \log_{10}8 = x$, then x is equal to:

(A) $\frac{1}{3}$ (B) .064 (C) –3 (D) 3

Q.33 The value of $(\log_9 27 + \log_8 32)$ is:

(A) $\frac{7}{2}$ (B) $\frac{19}{6}$ (C) 4 (D) 7

Q.34 $(\log_5 3) \times (\log_3 625)$ equals:
(A) 1 (B) 2 (C) 3 (D) 4

Q.35 $(\log_5 5)(\log_4 9)(\log_3 2)$ is equal to:

(A) 1 (B) $\frac{3}{2}$ (C) 2 (D) 5

Q.36 If $\log_{12} 27 = a$, then $\log_6 16$ is:

(A) $\frac{3-a}{4(3+a)}$ (B) $\frac{3+a}{4(3-a)}$

(C) $\frac{4(3+a)}{(3-a)}$ (D) $\frac{4(3-a)}{(3+a)}$

Q.37 If $\log_{10} 5 + \log_{10} (5x + 1) = \log_{10} (x + 5) + 1$, then x is equal to:
(A) 1 (B) 3 (C) 5 (D) 10

Q.38 If $\log_5 (x^2 + x) - \log_5 (x + 1) = 2$, then the value of x is:
(A) 5 (B) 10 (C) 25 (D) 32

Q.39 The value of $\left(\frac{1}{\log_3 60}+\frac{1}{\log_4 60}+\frac{1}{\log_5 60}\right)$ is:
(A) 0 (B) 1 (C) 5 (D) 60

Q.40 The value of $(\log_3 4) (\log_4 5) (\log_5 6) (\log_6 7) (\log_7 8) (\log_8 9)$ is:
(A) 2 (B) 7 (C) 8 (D) 33

Q.41 The value of $16^{\log_4 5}$ is:

(A) $\frac{5}{64}$ (B) 5 (C) 16 (D) 25

Q.42 If $\log x + \log y = \log (x + y)$, then
(A) $x = y$ (B) $xy = 1$

(C) $y = \frac{x-1}{x}$ (D) $y = \frac{x}{x-1}$

Q.43 If $\log\frac{a}{b}+\log\frac{b}{a}=\log(a+b)$, then:
(A) $a + b = 1$ (B) $a - b = 1$
(C) $a = b$ (D) $a^2 - b^2 = 1$

Q.44 $\left[\log\left(\frac{a^2}{bc}\right)+\log\left(\frac{b^2}{ac}\right)+\log\left(\frac{c^2}{ab}\right)\right]$ is equal to:
(A) 0 (B) 1 (C) 2 (D) abc

Q.45 $(\log_b a \times \log_c b \times \log_a c)$ is equal to:
(A) 0 (B) 1
(C) abc (D) $a + b + c$

Q.46 $\left[\frac{1}{(\log_a bc)+1}+\frac{1}{(\log_b ca)+1}+\frac{1}{(\log_c ab)+1}\right]$
is equal to:

(A) 1 (B) $\frac{3}{2}$ (C) 2 (D) 3

Q.47 The value of

$$\left[\frac{1}{\log_{(p/q)} x}+\frac{1}{\log_{(q/r)} x}+\frac{1}{\log_{(r/p)} x}\right]$$ is

(A) 0 (B) 1 (C) 2 (D) 3

Q.48 If $\log_{10} 7 = a$, then $\log_{10}\left(\frac{1}{70}\right)$ is equal to:

(A) $-(1 + a)$ (B) $(1 + a)^{-1}$

(C) $\frac{a}{10}$ (D) $\frac{1}{10a}$

Q.49 If $a = b^x$, $b = c^y$ and $c = a^z$, then the value of xyz is equal to:

(A) –1 (B) 0 (C) 1 (D) abc

Q.50 If log 27 = 1.431, then the value of log 9 is:

(A) 0.934 (B) 0.945

(C) 0.954 (D) 0.958

ANSWER KEY

Q.No	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Ans.	B	A	B	C	C	D	A	D	D	D	D	A	D	C	B	C	B	C	C	B
Q.No	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
Ans.	A	D	A	C	D	C	A	D	B	D	A	D	B	D	A	D	B	C	B	A
Q.No	41	42	43	44	45	46	47	48	49	50										
Ans.	D	D	A	A	B	A	A	A	C	C										

HINTS & SOLUTION

Sol.1 Let $\log_2 16 = n$.
Then, $2^n = 16 = 2^4$
$\Rightarrow n = 4$
$\therefore \log_2 16 = 4$

Sol.2 Let $\log_{343} 7 = n$.
Then, $(343)^n = 7$
$\Leftrightarrow (7^3)^n = 7$
$\Leftrightarrow 3n = 1 \Leftrightarrow n = \frac{1}{3}$
$\therefore \log_{343} 7 = \frac{1}{3}$

Sol.3 Let $\log_5\left(\frac{1}{125}\right) = n$ then,
$5^n = \frac{1}{125}$
$\Leftrightarrow 5^n = 5^{-3} \Leftrightarrow n = -3$
$\therefore \log_5\left(\frac{1}{125}\right) = -3$

Sol.4 Let $\log_{\sqrt{2}} 32 = n$. Then,
$(\sqrt{2})^n = 32$
$\Leftrightarrow (2)^{n/2} = 2^5$
$\Leftrightarrow \frac{n}{2} = 5$
$\Leftrightarrow n = 10$
$\therefore \log_{\sqrt{2}} 32 = 10$

Sol.5 Let $\log_{10} (0.0001) = n$
Then, $10^n = .0001$
$\Leftrightarrow 10^n = \frac{1}{10000} = \frac{1}{10^4}$
$\Leftrightarrow 10^n = 10^{-4}$
$\Leftrightarrow n = -4$
$\therefore \log_{10} (.0001) = -4$

Sol.6 Let $\log_{(.01)} = (1000) = n$
Then $(.01)^n = (1000)$
$\Leftrightarrow \left(\frac{1}{100}\right)^n = 10^3$
$\Leftrightarrow -2n = 3 \Leftrightarrow n = -\frac{3}{2}$

Sol.7 Let $\log_2 0.0625 = n$

Then, $2^n = 0.0625 = \frac{625}{10000}$

$\Leftrightarrow\ 2^n = \frac{1}{16}$

$\Leftrightarrow\ 2^n = 2^{-4}$

$\Leftrightarrow n = -4$

$\therefore \log_2 0.0625 = -4$

Sol.8 $\log_3 x = -2$

$\Leftrightarrow\ x = 3^{-2} = \frac{1}{3^2} = \frac{1}{9}$

Sol.9 $\log_8 x = \frac{2}{3}$

$\Leftrightarrow x = 8^{2/3} = (2^3)^{2/3}$

$x = 2^{\left(3 \times \frac{2}{3}\right)}$

$x = 2^2 = 4$

Sol.10 $\log_x\left(\frac{9}{16}\right) = -\frac{1}{2}$

$\Leftrightarrow\ x^{-1/2} = \frac{9}{16}$

$\Leftrightarrow\ \frac{1}{\sqrt{x}} = \frac{9}{16}$

$\Leftrightarrow \sqrt{x} = \frac{16}{9}$

$\Leftrightarrow\ x = \left(\frac{16}{9}\right)^2 = \frac{256}{81}$

Sol.11 $\log_x 4 = 0.4$

$\Leftrightarrow\ \log_x 4 = \frac{4}{10} = \frac{2}{5}$

$\Leftrightarrow x^{2/5} = 4$

$\Leftrightarrow x = 4^{5/2} = (2^2)^{5/2}$

$\Leftrightarrow\ x = 2^{\left(2 \times \frac{5}{2}\right)} = 2^5$

$\Leftrightarrow x = 32$

Sol.12 $\log_{10000} x = -\frac{1}{4}$

$\Leftrightarrow x = (10000)^{-1/4}$

$x = (10^4)^{-1/4}$

$x = 10^{-1} = \frac{1}{10}$

Sol.13 $\log_x 4 = \frac{1}{4}$

$\Leftrightarrow x^{1/4} = 4$

$\Leftrightarrow x = 4^4 = 256$

Sol.14 $\log_x(0.1) = -\frac{1}{3}$

$\Leftrightarrow x^{-1/3} = 0.1$

$\Leftrightarrow \frac{1}{x^{1/3}} = 0.1$

$\Leftrightarrow x^{1/3} = \frac{1}{0.1} = 10$

$\Leftrightarrow x = (10)^3 = 1000.$

Sol.15 $\log_{32} x = 0.8$
$\Leftrightarrow x = (32)^{0.8}$
$x = (2^5)^{4/5} = 2^4 = 16$

Sol.16 $\log_2 x = 10 \Rightarrow x = 2^{10}$
$\therefore \log_x y = 100 \Rightarrow y = x^{100} = (2^{10})^{100} \Rightarrow y = 2^{1000}$

Sol.17 Let $\log_{(-1/3)}81 = x$. Then,

$\left(-\frac{1}{3}\right)^x = 81 = 3^4 = (-3)^4 = \left(-\frac{1}{3}\right)^{-4}$

$\therefore x = -4$ i.e., $\log_{(-1/3)} 81 = -4$

Sol.18 Let $\log_{2\sqrt{3}}(1728) = x$

Then, $(2\sqrt{3})^x = 1728 = (12)^3 = [(2\sqrt{3})^2]^3$

$(2\sqrt{3})^x = (2\sqrt{3})^6$

$\therefore x = 6$, i.e. $\log_{2\sqrt{3}}(1728) = 6$

Sol.19 $\frac{\log\sqrt{8}}{\log 8} = \frac{\log(8)^{1/2}}{\log 8} = \frac{\frac{1}{2}\log 8}{\log 8} = \frac{1}{2}$

Sol.20 (a) Since $\log_a a = 1$,
So $\log_{10} 10 = 1$
(b) $\log(2 + 3) = 5$ and
$\log(2 \times 3) = \log 6 = \log 2 + \log 3$
$\therefore \log(2 + 3) \neq \log(2 \times 3)$
(c) Since $\log_a 1 = 0$, so $\log_{10} 1 = 0$
(d) $\log(1 + 2 + 3) = \log 6 = \log(1 \times 2 \times 3)$
$= \log 1 + \log 2 + \log 3$
So, (b) is incorrect

Sol.21 Let $\log_5 625 = x$. Then $5^x = 625 = 5^4$ or $x = 4$
Let $\log_2(\log_5 625) = y$. Then, $\log_2 4 = y$
or $2^y = 4$
or $y = 12$
$\therefore \log_2(\log_5 625) = 2$

Sol.22 $\log_2[\log_3(\log_2 x)] = 1 = \log_2 2$
$\Leftrightarrow \log_3(\log_2 x) = 2$
$\Leftrightarrow \log_2 x = 3^2 = 9$
$\Leftrightarrow x = 2^9 = 512$

Sol.23 $\log_2 \log_2 \log_3 (\log_3 27^3)$

$= \log_2 \log_2 \log_3 [\log_3 (3^3)^3] = \log_2 \log_2 \log_3 [\log_3 (3)^9]$

$= \log_2 \log_2 \log_3 (9 \log_3 3) = \log_2 \log_2 \log_3 9$

$\because \log_3 3 = 1]$

$= \log_2 \log_2 [\log_3 (3)^2] = \log_2 \log_2 (2 \log_3 3)$

$= \log_2 \log_2 2 = \log_2 1 = 0$

Sol.24 $a^x = b^y \Rightarrow \log a^x = \log b^y$

$\Rightarrow x \log a = y \log b \Rightarrow \frac{\log a}{\log b} = \frac{y}{x}$

Sol.25 $360 = (2 \times 2 \times 2) \times (3 \times 3) \times 5$

So, $\log 360$

$= \log (2^3 \times 3^2 \times 5) = \log 2^3 + \log 3^2 + \log 5$

$= 3 \log 2 + 2 \log 3 + \log 5$

Sol.26 $\frac{1}{3}\log_{10} 125 - 2\log_{10} 4 + \log_{10} 32$

$= \log_{10}(125)^{1/3} - \log_{10}(4)^2 + \log_{10} 32$

$= \log_{10} 5 - \log_{10} 16 + \log_{10} 32$

$= \log_{10}\left(\frac{5\times 32}{16}\right) = \log_{10} 10 = 1$

Sol.27 $2 \log_{10} 5 + \log_{10} 8 - \frac{1}{2}\log_{10} 4$

$= \log_{10}(5^2) + \log_{10} 8 - \log_{10}(4^{1/2})$

$\log_{10} 25 + \log_{10} 8 - \log_{10} 2 = \log_{10}\left(\frac{25\times 8}{2}\right)$

$= \log_{10} 100 = 2$

Sol.28 $\log_a (ab) = x$

$\Leftrightarrow \frac{\log ab}{\log a} = x \quad \Leftrightarrow \frac{\log a + \log b}{\log a} = x$

$\Leftrightarrow 1 + \frac{\log b}{\log a} = x \quad \Leftrightarrow \frac{\log b}{\log a} = x - 1$

$\Leftrightarrow \frac{\log a}{\log b} = \frac{1}{x-1} \quad \Leftrightarrow 1 + \frac{\log a}{\log b} = 1 + \frac{1}{x-1}$

$\Leftrightarrow \frac{\log b + \log a}{\log b} = \frac{x}{x-1}$

$\Leftrightarrow \frac{\log(ab)}{\log b} = \frac{x}{x-1} \quad \Leftrightarrow \log_b (ab) = \frac{x}{x-1}$

Sol.29 $\log (4.\sqrt[3]{63}) = \log 4 + \log(\sqrt[3]{63}) = \log 4 + \log(63)^{1/3}$

$= \log(2^2) + \log(7\times 3^2)^{1/3}$

$= 2\log 2 + \frac{1}{3}\log 7 + \frac{2}{3}\log 3 = 2x + \frac{1}{3}z + \frac{2}{3}y$

Sol.30 $\log_4 x + \log_2 x = 6$

$\Leftrightarrow \frac{\log x}{\log 4} + \frac{\log x}{\log 2} = 6$

$\Leftrightarrow \frac{\log x}{2\log 2} + \frac{\log x}{\log 2} = 6$

$\Leftrightarrow$ 3 log x = 12 log 2

$\Leftrightarrow$ log x = 4 log 2

$\Leftrightarrow$ log x = log (2^4) = log 16

$\Leftrightarrow$ x = 16

Sol.31 $\log_8 x + \log_8\left(\frac{1}{6}\right) = \frac{1}{3}$

$\Leftrightarrow \frac{\log x}{\log 8} + \frac{\log \frac{1}{6}}{\log 8} = \frac{1}{3}$

$\Leftrightarrow \log x + \log \frac{1}{6} = \frac{1}{3}\log 8$

$\Leftrightarrow \log x + \log \frac{1}{6} = \log(8^{1/3}) = \log 2$

$\Leftrightarrow \log x = \log 2 - \log \frac{1}{6} = \left(2 \times \frac{6}{1}\right) = \log 12$

$\therefore$ x = 12

Sol.32 $\log_{10} 125 + \log_{10} 8 = x \Rightarrow \log_{10}(125 \times 8) = x$

$\Rightarrow x = \log_{10}(1000) = \log_{10}(10)^3 = 3\log_{10} 10 = 3$

Sol.33 Let $\log_9 27 = x$ Then,

$9x = 27 \Leftrightarrow (3^2)^x = 3^3 \Leftrightarrow 2x = 3 \Leftrightarrow x = \frac{3}{2}$

Let $\log_8 32 = y$. Then,

$8^y = 32 \Leftrightarrow (2^3)^y = 2^5 \Leftrightarrow 3y = 5 \Leftrightarrow y = \frac{5}{3}$

$\therefore \log_9 27 + \log_8 32 = \left(\frac{3}{2} + \frac{5}{3}\right) = \frac{19}{6}$

Sol.34 Given expression = $\left(\frac{\log 3}{\log 5} \times \frac{\log 625}{\log 3}\right)$

$= \frac{\log 625}{\log 5} = \frac{\log(5^4)}{\log 5} = \frac{4\log 5}{\log 5} = 4$

Sol.35 Given expression = $\frac{\log 9}{\log 4} \times \frac{\log 2}{\log 3}$ [$\therefore \log_5 5 = 1$]

$= \frac{\log 3^2}{\log 2^2} \times \frac{\log 2}{\log 3} \times \frac{2\log 3}{2\log 2} \times \frac{\log 2}{\log 3} = 1$

Sol.36 $\log_{12} 27 = a \Rightarrow \dfrac{\log 27}{\log 12} = a$

$\Rightarrow \dfrac{\log 3^3}{\log(3\times 2^2)} = a \quad \Rightarrow \dfrac{3\log 3}{\log 3 + 2\log 2} = a \quad \Rightarrow \dfrac{\log 3 + 2\log 2}{3\log 3} = \dfrac{1}{a} \quad \Rightarrow \dfrac{\log 3}{3\log 3} + \dfrac{2\log 2}{3\log 3} = \dfrac{1}{a}$

$\Rightarrow \dfrac{2}{3}\dfrac{\log 2}{\log 3} = \dfrac{1}{a} - \dfrac{1}{3} = \left(\dfrac{3-a}{3a}\right)$

$\Rightarrow \dfrac{\log 2}{\log 3} = \left(\dfrac{3-a}{2a}\right) \quad \Rightarrow \log 3 = \left(\dfrac{2a}{3-a}\right)\log 2$

$\Rightarrow \log_6 16 = \dfrac{\log 16}{\log 6} = \dfrac{\log 2^4}{\log(2\times 3)}$

$= \dfrac{4\log 2}{\log 2 + \log 3} = \dfrac{4\log 2}{\log 2\left[1 + \left(\dfrac{2a}{3-a}\right)\right]}$

$= \dfrac{4}{\left(\dfrac{3+a}{3-a}\right)} = \dfrac{4(3-a)}{(3+a)}$

Sol.37 $\log_{10} 5 + \log_{10}(5x+1) = \log_{10}(x+5) + 1$

$\Rightarrow \log_{10} 5 + \log_{10}(5x+1) = \log_{10}(x+5) + \log_{10} 10$

$\Rightarrow \log_{10}[5(5x+1)] = \log_{10}[10(x+5)]$

$\Rightarrow 5(5x+1) = 10(x+5)$

$\Rightarrow 5x + 1 = 2x + 10$

$\Rightarrow 3x = 9$

$\Rightarrow x = 3$

Sol.38 $\log_5(x^2+x) - \log_5(x+1) = 2 \Rightarrow \log_5\left(\dfrac{x^2+x}{x+1}\right) = 2$

$\Rightarrow \log_5\left[\dfrac{x(x+1)}{x+1}\right] = 2$

$\Rightarrow \log_5 x = 2 \Rightarrow x = 5^2 = 25$

Sol.39 Given expression

$= \log_{60} 3 + \log_{60} 4 + \log_{60} 5$

$= \log_{60}(3\times 4\times 5) = \log_{60} 60 = 1$

Sol.40 Given expression =

$\left(\dfrac{\log 4}{\log 3}\times\dfrac{\log 5}{\log 4}\times\dfrac{\log 6}{\log 5}\times\dfrac{\log 7}{\log 6}\times\dfrac{\log 8}{\log 7}\times\dfrac{\log 9}{\log 8}\right)$

$= \dfrac{\log 9}{\log 3} = \dfrac{\log 3^2}{\log 3} = \dfrac{2\log 3}{\log 3} = 2$

Sol.41 We know that : $a^{\log_a x} = x$.

$\therefore 16^{\log_4 5} = (4^2)^{\log_4 5} = 4^{2\log_4 5} = 4^{\log_4 (5^2)} = 4^{\log_4 25} = 25$

Sol.42 $\log x + \log y = \log(x + y) \Rightarrow \log(x + y) = \log(xy)$

$\Rightarrow x + y = xy \Rightarrow y(x - 1) = x \Rightarrow y = \frac{x}{x-1}$

Sol.43 $\log\frac{a}{b} + \log\frac{b}{a} = \log(a + b)$

$\Rightarrow \log(a+b) = \log\left(\frac{a}{b} \times \frac{b}{a}\right) = \log 1$

So, $a + b = 1$

Sol.44 Given expression $= \log\left(\frac{a^2}{bc} \times \frac{b^2}{ac} \times \frac{c^2}{ab}\right) = \log 1 = 0$

Sol.45 Given expression $= \left(\frac{\log a}{\log b} \times \frac{\log b}{\log c} \times \frac{\log c}{\log a}\right) = 1$

Sol.46 Given expression

$= \frac{1}{\log_a bc + \log_a a} + \frac{1}{\log_b ca + \log_b b} + \frac{1}{\log_c ab + \log_c c}$

$= \frac{1}{\log_a (abc)} + \frac{1}{\log_b (abc)} + \frac{1}{\log_c (abc)}$

$= \log_{abc} a + \log_{abc} b + \log_{abc} c$

$= \log_{abc}(abc) = 1$

Sol.47 Given expression

$= \log_x\left(\frac{p}{q}\right) + \log_x\left(\frac{q}{r}\right) + \log_x\left(\frac{r}{p}\right)$

$= \log_x\left(\frac{p}{q} \times \frac{q}{r} \times \frac{r}{p}\right) = \log_x 1 = 0$

Sol.48 $\log_{10}\left(\frac{1}{70}\right) = \log_{10} 1 - \log_{10} 70 = -\log_{10}(7 \times 10)$

$= -(\log_{10} 7 + \log_{10} 10) = -(a+1)$

Sol.49 $a = b^x$, $b = c^y$, $c = a^z$

$\Rightarrow x = \log_b a$, $y = \log_c b$, $z = \log_a c$

$\Rightarrow xyz = (\log_b a) \times (\log_c b) \times (\log_a c)$

$\Rightarrow xyz = \left(\frac{\log a}{\log b} \times \frac{\log b}{\log c} \times \frac{\log c}{\log a}\right) = 1$

Sol.50 $\log 27 = 1.431 \Rightarrow \log(3^3) = 1.431 \Rightarrow 3 \log 3 = 1.431$

$\Rightarrow \log 3 = 0.477$

$\therefore \log 9 = \log(3^2) = 2 \log 3 = (2 \times 0.477) = 0.954$