# 8. Solids

## Questions Pg-193

### 1. Question

A square of side 5 centimetres, and four isosceles triangles of base 5 centimetres and height 8 centimetres, are to be put together to make a square pyramid. How many square centimeters of paper is needed?

**Answer**

Given: side of a square = 5cm

There are 4 isosceles triangle

Base (b) = 5cm

Height (h) = 8cm

Area of a square base (A) = side × side

= 5 × 5

Area of a square base (A) = 25 $cm^2$

Area of one isosceles triangle = $\frac{1}{2} \times b \times h$

$= \frac{1}{2} \times 5 \times 8$

Area of one isosceles triangle = $20cm^2$

Area of 4 isosceles triangle = 4 × 20 = $80cm^2$

Total surface area = 25 + 80 = $105cm^2$

### 2. Question

A toy is in the shape of a square pyramid of base edge 16 centimetres and slant height 10 centimetres. What is the total cost of painting 500 such toys, a 80 rupees per square metre?

**Answer**

Given: side of a square = 16cm

Slant Height (l) = 10 cm

We know that,

Surface area of a pyramid = $\frac{1}{2} \times p \times l + A$

Where, p = perimeter of the base

l = slant height

A = area of the base

Area of a square base (A) = side × side

= 16 × 16

Area of a square base (A) = 256 $cm^2$

Perimeter of a square base (p) = 4 × side

= 4 × 16

Perimeter of a square base (p) = 64 cm

Since, Surface area of a pyramid = $\frac{1}{2} \times p \times l + A$

⇒ Surface area of a pyramid = $\frac{1}{2} \times 64 \times 10 + 256 = 576 \text{ cm}^2$

Total surface area of pyramid shaped toys = 500 × 576

= 288000$cm^2$

= 28.8$m^2$

Cost of 1$m^2$ = 80Rs.

Cost of 28.8$m^2$ = 28.8 × 80 = 2304 Rs.

**3. Question**

The lateral faces of a square pyramid are equilateral triangles and the length of a base edge is 30 centimetres. What is its surface area?

**Answer**

Given lateral faces of square pyramid is equilateral triangle

side = 30cm

Since it is a equilateral triangle

lateral Base (b) = 30 cm

Area of a square base (A) = side × side

= 30 × 30

Area of a square base (A) = 900 $cm^2$

Area of an equilateral triangle $= \frac{\sqrt{3}}{4} (\text{side})^2$

$= \frac{\sqrt{3}}{4} (30)^2$

Area of an equilateral triangle = 225√3 $cm^2$

Area of an 4 equilateral triangle = 4× 225√3 = 900√3 $cm^2$

Total surface area = 900 + 900√3 = 900(1 + √3)$cm^2$

**4. Question**

The perimeter of the base of square pyramid is 40 centimetres and the total length of all its edges is 92 centimetres. Calculate its surface area.

**Answer**

Let b be the base edge

And l be the lateral edge

Then we can write:

4b + 4l = 92 cm .....(1)

But 4b = 40 cm.

So we get $b = \frac{40}{4} = 10 \text{ cm}$

Substituting this value of b in (1) we get:

40 + 4l = 92

$\Rightarrow$ 4l = 52

$\Rightarrow$ l = 13cm

Altitude of one isosceles triangle = $\sqrt{(13^2 - 5^2)}$

= $\sqrt{(169 - 25)}$

= $\sqrt{144}$ = 12cm

Now, we can calculate the total surface area

Base area = 10 × 10 = 100cm$^2$

Area of one isosceles triangle = $\frac{1}{2} \times \text{base} \times \text{height}$

$= \frac{1}{2} \times 10 \times 12$

Area of one isosceles triangle = 60cm$^2$

Area of 4 isosceles triangle = 4 × 60 = 240cm$^2$

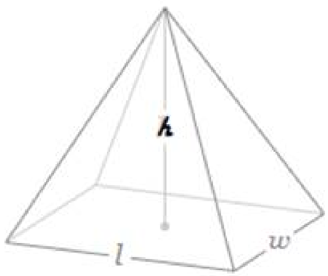
Total surface area = 100 + 240 = 340cm$^2$

**5. Question**

Can we make a square pyramid with the lateral surface area equal to the base area?

**Answer**

Consider pyramid as shown in figure given along.




Lateral surface are of pyramid is given by formula

$$\text{LSA} = l\sqrt{\left(\frac{w}{2}\right)^2 + h^2} + w\sqrt{\left(\frac{l}{2}\right)^2 + h^2}$$

As the base is given to be square, So l=w=x,

$$\text{LSA} = 2x\sqrt{\left(\frac{x}{2}\right)^2 + h^2}$$

And base area = $x^2$

So for base area to be equal to LSA,

$$x^2 = 2x\sqrt{\left(\frac{x}{2}\right)^2 + h^2}$$

$$\frac{x}{2} = \sqrt{\left(\frac{x}{2}\right)^2 + h^2}$$

$$\left(\frac{x}{2}\right)^2 = \left(\frac{x}{2}\right)^2 + h^2$$

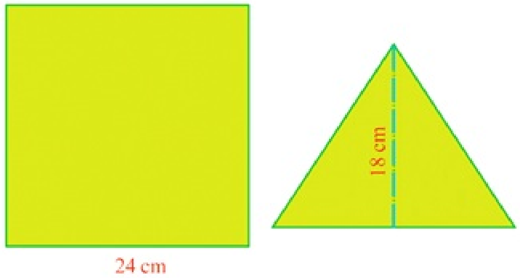
Or, h=0

Thus, it is not possible to have a pyramid with lateral surface area equal to base area.

## Questions Pg-195

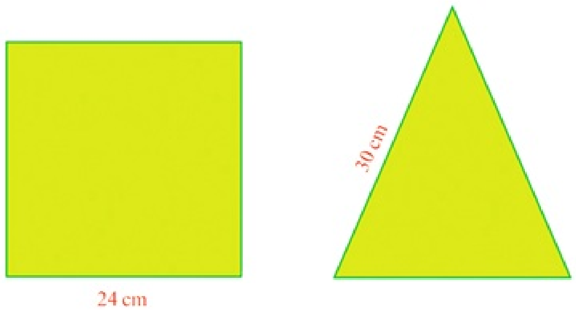
### 1. Question

Using a square and four triangles with dimensions as specified in the picture, a pyramid is made.



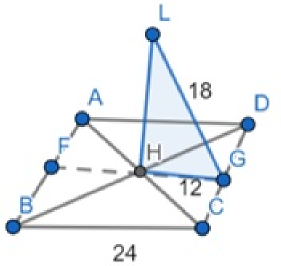

What is the height of this pyramid?

What if the square and triangles are like this?




**Answer**

From the first figure we get.




Slant Height LG = 18cm

Base BC = 24cm

Height of pyramid $LH = \sqrt{LG^2 - \left(\frac{BC}{2}\right)^2}$

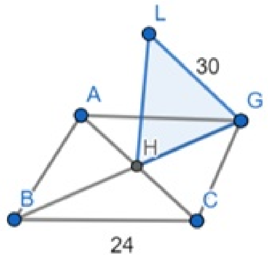
$$\text{Height AC} = \sqrt{18^2 - \left(\frac{24}{2}\right)^2}$$

= √ (324 – $12^2$)

= √ (324 – 144)

Height AC = √180 cm = 6√5 cm

From the second diagram we get,




LH = Height of pyramid

HG = Half of a diagonal

BC = base edge = 24cm

LG = lateral edge = 30cm

BG = Full diagonal = $\sqrt{(24^2 + 24^2)} = 24\sqrt{2}$ cm

$$HG = \frac{24\sqrt{2}}{2} = 12\sqrt{2}\ \text{cm}$$

By Pythagoras theorem

HL = $\sqrt{(LG^2 - HG^2)}$

= $\sqrt{(30^2 - (12\sqrt{2})^2)}$

= $\sqrt{(900 - 288)}$

= $\sqrt{612}$ cm

**2. Question**

A square pyramid of base edge 10 centimetres and height 12 centimetres is to be made of paper. What should be the dimension of the triangles?

**Answer**

Base edge (b) = 10cm

Height (h) = 12cm

$$\text{Slant height} = \sqrt{h^2 + \left(\frac{b}{2}\right)^2}$$

$$= \sqrt{12^2 + \left(\frac{10}{2}\right)^2}$$

= $\sqrt{(144 + 5^2)}$

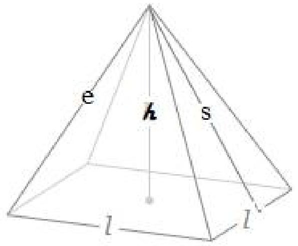
= $\sqrt{(144 + 25)}$

Slant Height = $\sqrt{169}$ = 13cm

**3. Question**

Prove that in any square pyramid, the squares of the height, slant height and lateral edge are in arithmetic sequence.

**Answer**




In the diagram,

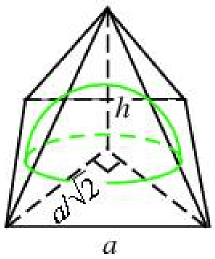
e = lateral edge

s = slant height

h = height of pyramid

l = side of square base

With the help of diagram given below we get that, diagonal of square base is of length l√2




From the figures and Pythagoras theorem,

$s^2 = h^2 + \left(\frac{l}{2}\right)^2$

$s^2 = h^2 + \frac{l^2}{4}$ (1)

Also, $e^2 = h^2 + \left(\frac{l\sqrt{2}}{2}\right)^2$

$e^2 = h^2 + \frac{l^2}{2}$ (2)

By comparing (1) and (2) we get that,

$h^2$, $s^2$ and $e^2$ are in AP with,

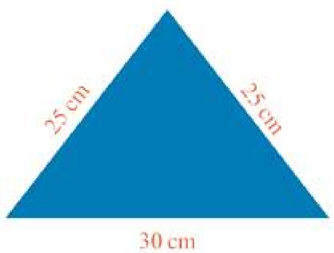
First term, a= $h^2$

Common difference, d= $\frac{l^2}{4}$

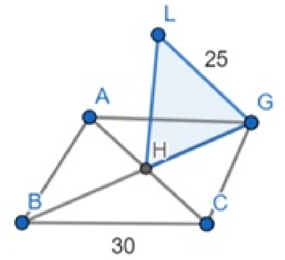
Hence, proved.

**4. Question**

A square pyramid is to be made with the triangle shown here as a lateral face. What would be its height? What if the base edge is 40 centimetres instead of 30 centimetres?



**Answer**




LH = Height of pyramid

HG = Half of a diagonal

BC = base edge = 30cm

LG = lateral edge = 25cm

BG = Full diagonal = √($30^2$+ $30^2$) = 30√2 cm

$$HG = \frac{30\sqrt{2}}{2} = 15\sqrt{2}\text{ cm}$$

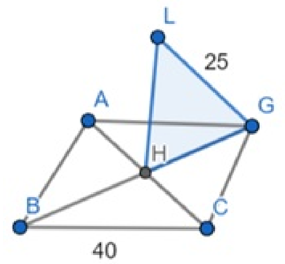
By Pythagoras theorem

HL = √($LG^2$ + $HG^2$)

= √($25^2$ + $(15\sqrt{2})^2$)

= √(625 + 450)

= √1075 cm




LH = Height of pyramid

HG = Half of a diagonal

BC = base edge = 40cm

LG = lateral edge = 25cm

BG = Full diagonal = √($40^2$+ $40^2$) = 40√2 cm

$$HG = \frac{40\sqrt{2}}{2} = 20\sqrt{2}\text{ cm}$$

By Pythagoras theorem

HL = √($LG^2$ + $HG^2$)

= √($25^2$ + $(20\sqrt{2})^2$)

= √(625 + 800)

= √1425 cm

## Questions Pg-197

**1. Question**

What is the volume of a square pyramid of base edge 10 centimetres and slant height 15 centimetres?

**Answer**

Given: base edge (b) = 10cm

Slant height (l) = 15cm

$$\text{height} = \sqrt{l^2 - \left(\frac{b}{2}\right)^2}$$

$$= \sqrt{15^2 - \left(\frac{10}{2}\right)^2}$$

= $\sqrt{(225 - 5^2)}$

= $\sqrt{(225 - 25)}$

Height = $\sqrt{200} = 10\sqrt{2}$ cm

Base area = $(\text{base edge})^2$

= $(10)^2$

Base area = $100cm^2$

We know that,

$$\text{Volume of a pyramid} = \frac{1}{3} \times \text{base area} \times \text{height}$$

$$= \frac{1}{3} \times 100 \times 10\sqrt{3}$$

$$= \frac{1}{3} \times 1000\sqrt{3}\ \text{cm}^3$$

**2. Question**

Two square pyramids have the same volume. The base edge of one is half that of the other. How many times the height of the second pyramid is the height of the first?

**Answer**

Let the volume of first pyramid be $V_1$

Let the base edge of first pyramid be $a_1$

Let the height of first pyramid be $h_1$

Let the volume of second pyramid be $V_2$

Let the area of first pyramid be $a_2$

Let the height of first pyramid be $h_2$

We know that,

Volume of a pyramid = $\frac{1}{3} \times \text{base area} \times \text{height}$

Volume of a first pyramid $V_1 = \frac{1}{3} a_1^2 h_1$

Volume of a second pyramid $V_2 = \frac{1}{3} a_2^2 h_2$

Given: $V_1 = V_2$

$\Rightarrow \frac{1}{3}a_1^2h_1 = \frac{1}{3}a_2^2h_2$ ....(1)

Given: $a_2 = \frac{a_1}{2}$

Substitute in (1)

$\Rightarrow \frac{1}{3}a_1^2h_1 = \frac{1}{3}\left(\frac{a_1}{2}\right)^2 h_2$

$\Rightarrow h_2 = 4h_1$

**3. Question**

The base edges of two square pyramids are in the ratio 1 : 2 and their heights in the ratio 1 : 3. The volume of the first is 180 cubic centimeters. What is the volume of the second?

**Answer**

Let the volume of first pyramid be $V_1$

Let the base edge of first pyramid be $a_1$

Let the height of first pyramid be $h_1$

Let the volume of second pyramid be $V_2$

Let the area of first pyramid be $a_2$

Let the height of first pyramid be $h_2$

We know that,

Volume of a pyramid = $\frac{1}{3} \times \text{base area} \times \text{height}$

Volume of a first pyramid $V_1 = \frac{1}{3}a_1^2h_1 = 180\text{cm}^3$ .....(1)

Volume of a second pyramid $V_2 = \frac{1}{3}a_2^2h_2$ ...(2)

Given: $\frac{a_1}{a_2} = \frac{1}{2}$

$\Rightarrow a_2 = 2a_1$ ...(a)

Given: $\frac{h_1}{h_2} = \frac{1}{3}$

$\Rightarrow h_2 = 3h_1$ ...(b)

Consider, Volume of a second pyramid $V_2 = \frac{1}{3}a_2^2h_2$

$\Rightarrow V_2 = \frac{1}{3}(2a_1)^2(3h_1)$ (from a and b)

$\Rightarrow V_2 = 12 \times \frac{1}{3}(a_1)^2(h_1)$

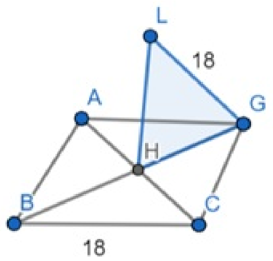
$\Rightarrow V_2 = 12 \times V_1$

$\Rightarrow V_2 = 12 \times 180 = 2160\text{cm}^3$

**4. Question**

All edges of a square pyramid are 18 centimetres. What is its volume?

**Answer**




LH = Height of pyramid

HG = Half of a diagonal

BC = base edge = 18cm

LG = lateral edge = 18cm

BG = Full diagonal = $\sqrt{(18^2 + 18^2)}$ = $18\sqrt{2}$ cm

$$HG = \frac{18\sqrt{2}}{2} = 9\sqrt{2}\text{ cm}$$

By Pythagoras theorem

HL = $\sqrt{(LG^2 - HG^2)}$

= $\sqrt{(18^2 - (9\sqrt{2})^2)}$

= $\sqrt{(324 - 162)}$

= $\sqrt{162}$ cm

We know that,

Volume of a pyramid = $\frac{1}{3} \times \text{base area} \times \text{height}$

Volume of a pyramid = $\frac{1}{3}(18)^2\sqrt{162}$

$$= \frac{1}{3}(18)^2 \times 9 \times 9\sqrt{2} = 972\sqrt{2}\text{cm}^3$$

**5. Question**

The slant height of a square pyramid is 25 centimetres and its surface area is 896 square centimetres. What is its volume?

**Answer**

Let the base edge be (2a) cm

Then area of one isosceles triangle = $\frac{1}{2} \times \text{base} \times \text{altitude}$

$= \frac{1}{2} \times (2a) \times 25 = 25a\text{ cm}^2$.

Area of four isosceles triangles = $4 \times 25a = 100a\text{ cm}^2$.

Base area = $(2a)^2 = 4a^2\text{ cm}^2$.

Total surface area = $(100a + 4a^2)\text{cm}^2$

Given total surface area = 896

$\therefore 100a + 4a^2 = 896$

$\Rightarrow a^2 + 25a - 224 = 0$

Using, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

We get,

$$\Rightarrow a = \frac{-25 \pm \sqrt{25^2 - 4(1)(-224)}}{2}$$

$$\Rightarrow a = \frac{-25 \pm \sqrt{25^2 - 4(1)(-224)}}{2}$$

$$\Rightarrow a = \frac{-25 \pm 39}{2}$$

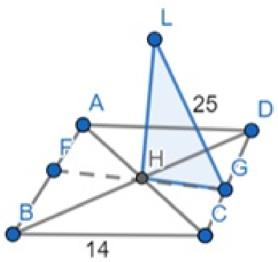
$$\Rightarrow a = \frac{-25 + 39}{2} \text{ or } \frac{-25 - 39}{2}$$

$$\Rightarrow a = \frac{14}{2} \text{ or } \frac{-64}{2}$$

⇒ a = 7 or – 32

Since, base edge cannot be negative

∴ a = 7 cm




LG = Slant height = 25cm

HG = Half of base = 7cm

Apply Pythagoras theorem

LH = $\sqrt{(25^2 - 7^2)}$

= $\sqrt{(625 - 49)}$

= $\sqrt{576}$ = 24cm

Volume of a pyramid = $\frac{1}{3} \times \text{base area} \times \text{height}$

Volume of a pyramid = $\frac{1}{3}(14)^2 24 = 1568\text{cm}^3$

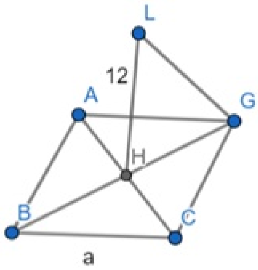
**6. Question**

All edges of a square pyramid are of the same length and its height is 12 centimetres. What is its volume?

**Answer**

Let all the edges of the square prism be 'a'.

The lateral edge BC is also 'a'.




HL = height of pyramid

HG = half of a diagonal of the base

LG = Lateral edge = a Cm

Full diagonal BG = $\sqrt{(a^2 + a^2)} = a\sqrt{2}$ cm

Half diagonal $HG = \frac{a\sqrt{2}}{2}$.

Applying Pythagoras theorem, we get:

LH = $\sqrt{(LG^2 - HG^2)}$

$= \sqrt{(a^2 - \left(\frac{a\sqrt{2}}{2}\right)^2}$

$= \sqrt{(a^2 - \frac{a^2}{2}}$

$= \frac{a}{\sqrt{2}}$

Given: LH = 12cm

Thus $\frac{a}{\sqrt{2}} = 12$

⇒ a = $12\sqrt{2}$ cm

Volume of a pyramid $= \frac{1}{3} \times \text{base area} \times \text{height}$

Volume of a pyramid $= \frac{1}{3}\left(12\sqrt{2}\right)^2 12 = 1152\text{cm}^3$

**7. Question**

What is the surface area of a square pyramid of base perimeter 64 centimetres and volume 1280 cubic centimetres?

**Answer**

Base perimeter is given as 64 cm.

So base edge $= \frac{64}{4} = 16$ cm

Volume is given as 1280 $cm^3$.

We know that,

$$\text{Volume of a pyramid} = \frac{1}{3} \times \text{base area} \times \text{height}$$

$$1280 = \frac{1}{3}(16)^2 h$$

$$1280 = \frac{1}{3} \times 256 \times h$$

⇒ h = 15 cm

Let height of pyramid (h) = 15 cm

half of base edge = 8 cm

slant height = l

Applying Pythagoras theorem, we get:

l = √($15^2$ + $8^2$)

= √(225 + 64)

= √289

= 17 cm

Then area of one isosceles triangle = $\frac{1}{2} \times \text{base} \times \text{altitude}$

$$= \frac{1}{2} \times 16 \times 17 = 136\text{cm}^2$$

Area of four isosceles triangles = 4×136 = 544 $cm^2$

Base area = $16^2$ = 256 $cm^2$.

Total surface area = (544 + 256) = 800 $cm^2$

## Questions Pg-200

**1. Question**

What are the radius of the base and slant height of a cone made by rolling up a sector of central angle 60° cut out from a circle of radius 10 centimetres?

**Answer**

Given that radius of the circle is 10 cm.

This will be same as the radius of the sector, $r_s$ = 10

This rs will be the slant height l of the cone.

Slant height l = 10 cm

Central angle θ = 60°

the length of arc will be $\frac{\pi r_s \theta}{180}$

Thus, the length of arc will be $60 \times \frac{\pi \times 10}{180} = \frac{10\pi}{3}$.

But this is same as the circumference of the base of the cone

So if $r_b$ is the radius of the base of the cone

$$2\pi r_b = \pi \times \frac{10}{3}$$

$$r_b = \pi \times \frac{5}{3} = 1.67\text{cm}$$

**2. Question**

What is the central angle of the sector to be used to make a cone of base radius 10 centimetres and slant height 25 centimetres?

**Answer**

Given that slant height of the cone should be 25 cm.

So radius of the sector $r_s$ = 25 cm

To find: Central angle θ

Radius of the base $r_b$ = 10 cm

So circumference of the base = $2\pi r_b = 2\pi\times 10 = 20\pi$ cm ..(1)

This circumference is equal to the length of the arc

The length of arc will be $\frac{\pi r_s \theta}{180}$

Thus, the length of arc will be $\theta \times \frac{\pi\times 25}{180} = \frac{5\pi\theta}{36}$. ...(2)

Equating the results in (1) and (2):

$$20\pi = \theta \times \frac{5\pi}{36}$$

$\Rightarrow \theta = 144°$

**3. Question**

What is the ratio of the base – radius and slant height of a cone made by rolling up a semicircle?

**Answer**

Here, The sector is a semicircle

Let $r_s$ be the radius of the sector.

Then slant height of the cone will be $r_s$.

Central angle θ of a semi circle = 180°

The arc length of a semicircle is 'half the circumference of the full circle'

The 'circumference of the full circle' is $2\pi r_s$.

So half of it is πrs.

This is same as the circumference of the base of the cone

So if $r_b$ is the radius of the base of the cone,

$2\pi r_b = \pi r_s$

$$\Rightarrow \frac{r_b}{r_s} = \frac{1}{2}$$

But $r_s$ is the slant height.

$$\frac{\text{base radius}}{\text{slant height}} = \frac{1}{2}$$

## Questions Pg-201

**1. Question**

What is the area of the curved surface of a cone of base radius 12 centimetres and slant height 25 centimetres?

**Answer**

Given: base radius $r_b$ = 12cm

Slant height l = 25cm

We know that, Curved surface area of cone = $\pi r_b l$

Where $r_b$ = base radius and l = slant height

Curved surface area of cone = π× 25× 12 = 300π $cm^2$

**2. Question**

What is the surface area of a cone of base diameter 30 centimetres and height 40 centimetres?

**Answer**

height of the cone = 40 cm

Base diameter = 30cm

Base radius $r_b = \frac{30}{2} = 15\text{ cm}$

l = slant height

Applying Pythagoras theorem, we get:

$l = \sqrt{(h^2 + r_b^2)}$

$= \sqrt{(40^2 + 15^2)}$

= √(1600 + 225) = √1825 cm

We know that, Curved surface area of cone = $\pi r_b l$

Where $r_b$ = base radius and l = slant height

Curved surface area of cone = π× 15× √18255 = 640.8π $cm^2$

Surface area of base = $\pi r_b^2 = \pi \times 15^2 = 225\pi\ cm^2$.

Total surface area = (640.8 + 225)π = 865.8π $cm^2$

**3. Question**

A conical fire work is of base diameter 10 centimetres and height 12 centimetres. 10000 such fireworks are to be wrapped in colour paper. The price of the colour paper is 2 rupees per square metre. What is the total cost?

**Answer**

height of the cone (h) = 12 cm

Diameter of a cone = 10cm

Base radius $r_b = \frac{10}{2} = 5\text{ cm}$

l = slant height

Applying Pythagoras theorem, we get:

$l = \sqrt{(h^2 + r_b^2)}$

$= \sqrt{(12^2 + 5^2)}$

= √(144 + 25) = √169 cm = 13cm

We know that, Curved surface area of cone = $\pi r_b l$

Where $r_b$ = base radius and l = slant height

Curved surface area of cone = π× 5× 13 = 65π $cm^2$

Surface area of base = $\pi r_b^2 = \pi \times 5^2 = 25\pi$ $cm^2$.

Total surface area = $(65 + 25)\pi = 90\pi$ $cm^2 = 90\times 3.14 = 282.6 cm^2$

Surface area of 10000 fire works = $282.6 \times 10000 = 2826000$ $cm^2$.

$$2826000 \text{ cm}^2 = \frac{282600}{10000} \text{ m}^2 = 282.6 \text{ m}^2.$$

So cost of colour paper = 282.6 × 2 = Rs 565.20

**4. Question**

Prove that for a cone made by rolling up a semicircle, the area of the curved surface is twice the base area.

**Answer**

Let the radius of the sector be $r_s$

Central angle θ is 180° (∵ the sector is a semicircle)

Let the radius of the base be $r_b$

So circumference of the base = $2\pi r_b$ ...(1)

This circumference is equal to the length of the arc

Since, the arc length of a semicircle is 'half the circumference of the

full circle'

The 'circumference of the full circle' is 2πrs.

So half of it is πrs. ...(2)

Equating the results in (1) and (2):

$2\pi r_b = \pi r_s$

$\Rightarrow 2r_b = r_s$. ..(3)

Find the area of the sector

The area of the sector is area of the semicircle which is $\frac{1}{2} \times \pi r_s^2$

Let us substitute for $r_s$ using the result in (3). We get:

$$\text{Area of the sector} = \frac{1}{2} \times \pi r_s^2$$

$$= \frac{1}{2}\pi(2r_b)^2$$

$$= \frac{1}{2}4\pi r_b^2$$

$= 2\pi r_b^2$

Area of the sector is same as the area of curved surface.

Area of the curved surface of the cone = $2\pi r_b^2$ ...(4)

To find the base area.

radius of the base of the cone = $r_b$.

So area of the base of the cone = $\pi r_b^2$. ...(5)

Comparing the results in (4) and (5), we get:

Area of the curved surface of the cone = Twice the base area

## Questions Pg-202

### 1. Question

The base radius and height of a cylindrical block of wood are 15 centimetres and 40 centimetres. What is the volume of the largest cone that can be carved out of this?

**Answer**

Base radius = 15cm

Height of cylindrical block = 40cm

Volume of the cone = $\frac{1}{3} \times \pi r_b^2 h$

$= \frac{1}{3} \times \pi 15^2 \times 40$

= 5 × 15 × 40 π

= 3000π $cm^3$

### 2. Question

The base radius and height of a solid metal cylinder are 12 centimetres and 20 centimetres. By melting it and recasting, how many cones of base radius 4 centimetres and height 5 centimetres can be made?

**Answer**

For the cylinder:

Radius of base = 12 cm

Height = 20 cm

Volume of the cone = $\frac{1}{3} \times \pi r_b^2 h$

$= \frac{1}{3} \times \pi 12^2 \times 20$

= 4 × 12 × 20 π

= 2880π $cm^3$

For the cone:

Radius of base = $r_b$ = 4 cm

Height = h = 5 cm

So volume of one cone $= \frac{1}{3} \times \pi r_b^2 \times h$

$= \frac{1}{3} \times \pi(4)^2 \times 5$

$= \frac{80\pi}{3}$ $cm^3$.

So number of cones that can be obtained $= \frac{\text{Volume ofcylinder}}{\text{Volume of one cone}}$

$= \frac{2880\pi}{\frac{80\pi}{3}}$

= 108

**3. Question**

A sector of central angle 216° is cut out from a circle of radius 25 centimetres and is rolled up into a cone. What are the base radius and height of the cone? What is its volume?

**Answer**

Given that radius of the circle is 25 cm.

This will be same as the radius of the sector $r_s = 25$ cm

This $r_s$ will be the slant height of the cone

Slant height = 25 cm

Central angle θ = 216°

The length of arc will be $\frac{\pi r_s \theta}{180}$

Thus, the length of arc will be $216 \times \frac{\pi \times 25}{180} = 30\pi \text{cm}.$

But this is same as the circumference of the base of the cone

So if $r_b$ is the radius of the base of the cone

⇒ $2\pi r_b = 30\pi$

⇒ $r_b = 30$cm

Also,

height of the cone = h

base radius = $r_b$ = 15 cm

slant height = l = 25 cm

Applying Pythagoras theorem, we get:

$h = \sqrt{(l^2 - r_b^2)}$

$= \sqrt{(25^2 - 15^2)}$

$= \sqrt{(625 - 225)} = \sqrt{400} = 20$ cm

$\text{So volume of one cone} = \frac{1}{3} \times \pi r_b^2 \times h$

$= \frac{1}{3} \times \pi(15)^2 \times 20$

$= 1500\pi$ cm$^3$

**4. Question**

The base radii of two cones are in the ratio 3 : 5 and their heights are in the ratio 2 : 3. What is the ratio of their volumes?

**Answer**

Given that $\frac{r_{b_1}}{r_{b_2}} = \frac{3}{5}$

Also given that $\frac{h_1}{h_2} = \frac{2}{3}$

We know that, Volume of the cone = $\frac{1}{3} \times \pi r_b^2 h$

$$\Rightarrow \frac{V_1}{V_2} = \frac{\frac{1}{3} \times \pi r_{b_1}^2 h_1}{\frac{1}{3} \times \pi r_{b_2}^2 h_2}$$

$$\Rightarrow \frac{V_1}{V_2} = \frac{r_{b_1}^2 h_1}{r_{b_2}^2 h_2}$$

$$\Rightarrow \frac{V_1}{V_2} = \left(\frac{r_{b1}}{r_{b2}}\right)^2 \times \frac{h_1}{h_2}$$

$$\Rightarrow \frac{V_1}{V_2} = \left(\frac{3}{5}\right)^2 \times \frac{2}{3} = \frac{6}{25}$$

**5. Question**

The cones have the same volume and their base radii are in the ratio 4 : 5. What is the ratio of their heights?

**Answer**

Given that $\frac{r_{b_1}}{r_{b_2}} = \frac{4}{5}$.

Also given that $V_1 = V_2$

We know that, Volume of the cone = $\frac{1}{3} \times \pi r_b^2 h$

$$\Rightarrow \frac{1}{3} \times \pi r_{b_1}^2 h_1 = \frac{1}{3} \times \pi r_{b_2}^2 h_2$$

$$\Rightarrow r_{b_1}^2 h_1 = r_{b_2}^2 h_2$$

$$\Rightarrow \frac{r_{b_1}^2}{r_{b_2}^2} = \frac{h_2}{h_1}$$

$$\Rightarrow \left(\frac{4}{5}\right)^2 = \frac{h_2}{h_1}$$

$$\Rightarrow \frac{h_2}{h_1} = \frac{16}{25}$$

$$\Rightarrow \frac{h_1}{h_2} = \frac{25}{16}$$

## Questions Pg-205

**1. Question**

The surface area of a solid sphere is 120 square centimetres. If it is cut into two halves, what would be the surface area of each hemisphere?

**Answer**

Let r cm be the radius of the total sphere.

Then it's surface area would be $4\pi r^2$ $cm^2$

This surface area is given as 120 $cm^2$.

we can equate the two:

$4\pi r^2 = 120$

$\Rightarrow \pi r^2 = 30$

$\Rightarrow r^2 = \frac{30}{\pi}$ .....(1)

Curved surface area of a hemisphere = $\frac{1}{2} \times 4\pi r^2 = 2\pi r^2$

Base area = $\pi r^2$

So total surface area of a hemisphere = $2\pi r^2 + \pi r^2 = 3\pi r^2$

Substitute value from (1)

So total surface area of each hemisphere = $3\pi \times \frac{30}{\pi} = 90 \text{ cm}^2$

**2. Question**

The volume of two spheres are in the ratio 27 : 64. What is the ratio of their radii? And the ratio of their surface areas?

**Answer**

Let the volume and radius of the first sphere be $V_1$ and $r_1$

Respectively

Let the volume and radius of the second sphere be $V_2$ and $r_2$

Respective

Then volume of first sphere = $V_1 = \frac{4}{3} \times [\pi(r_1)^3]$

volume of second sphere = $V_2 = \frac{4}{3} \times [\pi(r_2)^3]$

Given that $\frac{V_1}{V_2} = \frac{27}{64}$

$$\frac{\frac{4}{3} \times [\pi(r_1)^3]}{\frac{4}{3} \times [\pi(r_2)^3]} = \frac{27}{64}$$

$$\frac{(r_1)^3}{r_2^3} = \frac{27}{64}$$

$$\left(\frac{r_1}{r_2}\right)^3 = \frac{27}{64}$$

$$\frac{r_1}{r_2} = \frac{3}{4}$$

Let $S_1$ and $S_2$ be the surface areas. Then we get:

$$\frac{S_1}{S_2} = \frac{4 \times [\pi(r_1)^2]}{4 \times [\pi(r_2)^2]}$$

$$\frac{S_1}{S_2} = \frac{(r_1)^2}{(r_2)^2}$$

$$\frac{S_1}{S_2} = \left(\frac{r_1}{r_2}\right)^2$$

$$\frac{S_1}{S_2} = \left(\frac{r_1}{r_2}\right)^2 = \left(\frac{3}{4}\right)^2$$

$$\frac{S_1}{S_2} = \frac{9}{16}$$

**3. Question**

The base radius and length of a metal cylinder are 4 centimetres and 10 centimetres. If it is melted and recast into spheres of radius 2 centimetres, how many spheres can be made?

**Answer**

Total volume available for melting = Volume of the cylinder

$= \pi r^2 h$

$= \pi \times 4^2 \times 10$

$= 160\pi \text{ cm}^3$

Volume of one sphere $= \frac{4}{3}\pi r^3 = \frac{4}{3} \times \pi \times 2^3 = \frac{32}{3} \times \pi \text{ cm}^3$.

$$\text{Number of spheres} = \frac{[\text{Total volume}]}{[\text{Volume of one sphere}]}$$

$$\text{Number of spheres} = \frac{160\pi}{\frac{32}{3} \times \pi}$$

Number of spheres = 15

**4. Question**

a metal sphere of radius 12 centimetres is melted and recast into 27 small spheres. What is the radius of each sphere?

**Answer**

Total volume available for melting = Volume of the sphere

$= \frac{4}{3}\pi r^3$

$= \frac{4}{3}\pi 12^3$

$= 2304\pi \text{ cm}^3$

Let 'r' be the radius of one small sphere.

Then volume of one small sphere = $= \frac{4}{3}\pi r^3$

$$\text{Number of spheres} = \frac{[\text{Total volume}]}{[\text{Volume of one sphere}]}$$

$$\text{Number of spheres} = \frac{2304\pi}{\frac{4}{3} \times \pi r^3}$$

$$\text{Number of spheres} = \frac{1728}{r^3}$$

But number of spheres is given as 27.

So we can write:

$$27 = \frac{1728}{r^3}$$

$$r^3 = \frac{1728}{27} = 64$$

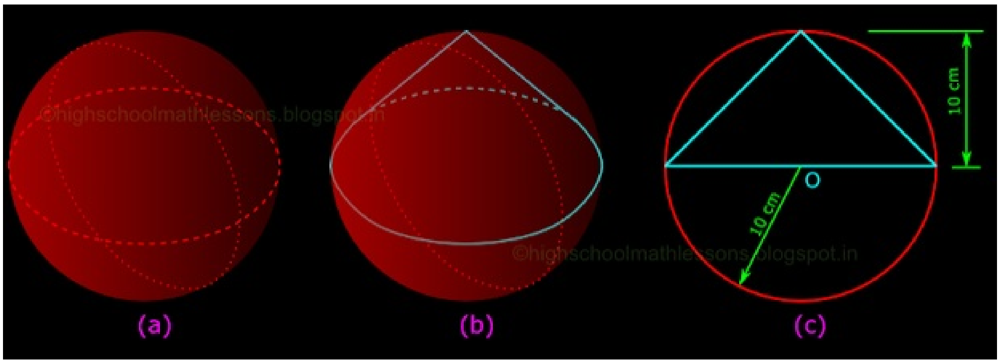
R = 4cm

**5. Question**

Form a solid sphere of radius 10 centimetres, a cone of height 16 centimetres is carved out. What fraction of the volume of the sphere is the volume of the cone?

**Answer**

1. Consider the red sphere in fig (a) below.




• Two ellipses are drawn inside it: A dotted ellipse and a dashed ellipse

• The dashed ellipse represents a circle whose centre is same as the centre of the sphere

♦ Also this circle is horizontal

• So this circle divides the sphere into an upper hemisphere and a lower hemisphere

• This circle is taken as the base of the cone in fig.b.

• We can see that, the cone fits perfectly in the upper hemisphere.

• This is shown more clearly in fig.c

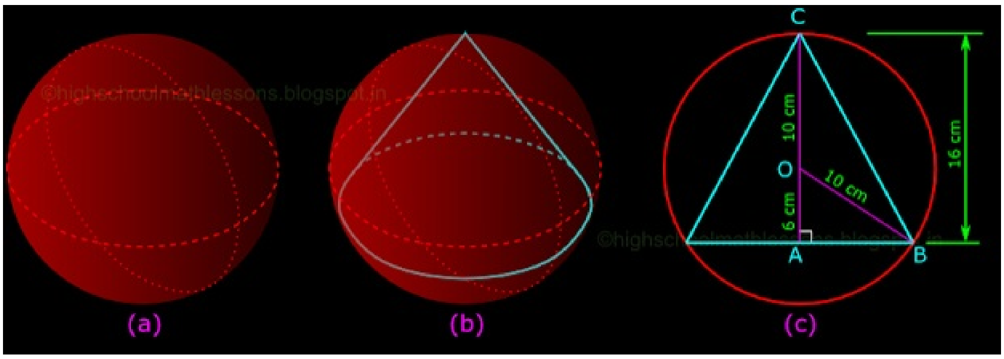
2. From fig.c we can see that, the height of the cone will be the height of the hemisphere, which is 10 cm

• But cone given in the question has a height of 16 cm.

• So the given cone does not fit inside the upper hemisphere alone.

♦ It will occupy some portion of the lower hemisphere also

• This is shown in fig (b) below. In that fig. we can see that the, base of the new cone is below the dashed ellipse




3. In fig (c), the measurements are given

• One half of the cone is represented by the right triangle ABC

• The distance of the apex C from the centre O will be the radius of the sphere, which is 10 cm

• So the remaining distance OA will be (16 – 10) = 6 cm

• Distance OB will also be the radius 10 cm

• Applying Pythagoras theorem to the right triangle OAB, we get:

$$AB^2 = OB^2 - OA^2$$

$\Longrightarrow AB^2 = 10^2 - 6^2$

$\Longrightarrow AB^2 = 100 - 36$

$\Longrightarrow AB^2 = 64$

$\Longrightarrow AB = 8$ cm

4. Thus we have:

• Height of the cone, h = 16 cm

• Radius of the cone, $r_c = 8$ cm

• So Volume, $V_c = \frac{1}{3}\pi(r_c)^2h = \frac{1}{3} \times \pi \times 8^2 \times 16$

• Volume of sphere, $V_s = \frac{4}{3}\pi r^3 = \frac{4}{3} \times \pi \times 10^3$

5. Taking ratios, we get:

$$\frac{V_c}{V_s} = \frac{\frac{1}{3} \times \pi \times 8^2 \times 16}{\frac{4}{3} \times \pi \times 10^3}$$

$$\frac{V_c}{V_s} = \frac{8^2 \times 16}{4 \times 10^3}$$

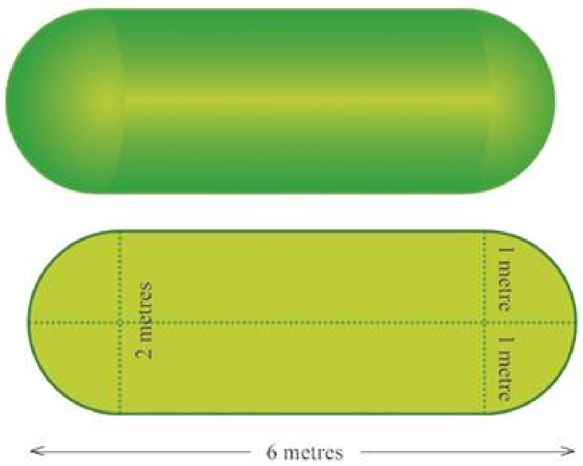
$$\frac{V_c}{V_s} = \frac{32}{125}$$

$$V_c = \frac{32}{125}V_s$$

• So 'volume of the cone' is $\frac{32}{125}$ of the 'volume of the sphere'

**6. Question**

The picture shows the dimensions of a petrol tank.




How many litres of petrol can it hold?

**Answer**

The tank has two hemispherical parts and one cylindrical part

The yellow dashed line indicates the axis of the tank

From the fig., it is clear that radius of the hemisphere is 1 m.

So it's volume = $V_h = \frac{2}{3}\pi r^3 = \frac{2}{3}\times\pi\times 1^3 = \frac{2}{3}\times\pi\ m^3$

Thus volume of two hemispheres = $2\times\frac{2}{3}\times\pi = \frac{4}{3}\times\pi\ m^3$

Height of a hemisphere will be equal to it's radius.

So length of the cylindrical part = [6 – (2×1)] = 4 m

Volume of cylinder = $V_c = \pi r^2 h = \pi\times 1^2\times 4 = 4\pi\ m^3$

Thus total volume = $\frac{4}{3}\times\pi + 4\pi = \frac{16}{3}\times\pi\ m^3$.

We know that 1 liter is the volume of a cube of edge 10 cm

Now, $\left(\frac{16}{3}\times\pi\right)\ m^3 = \left[\left(\frac{16}{3}\times\pi\right)\times 1000000\right]\ cm^3$ = 16746666.67 cm3.

(∵ 1 m = 100 cm)

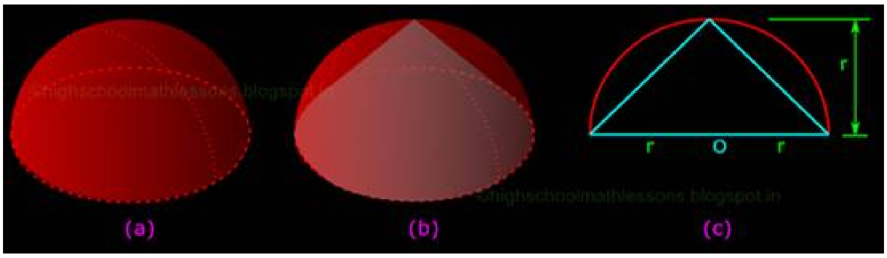
Thus the no. of liters = 16746666.67⁄1000 = 16746.67 liters

**7. Question**

A solid sphere is cut into two hemispheres. From one, a square pyramid and from the other a cone. each of maximum possible size are carved out. What is the ratio of their volumes?

**Answer**

1. Consider the red hemisphere in fig (a) below.




• A dotted ellipse and a dashed curve are drawn inside it

• The dashed ellipse represents the base of the hemisphere

• For maximum possible volume, this base is taken as the base of the cone in fig. (b)

• We can see that, the cone fits perfectly in the hemisphere.

• This is shown more clearly in (c)

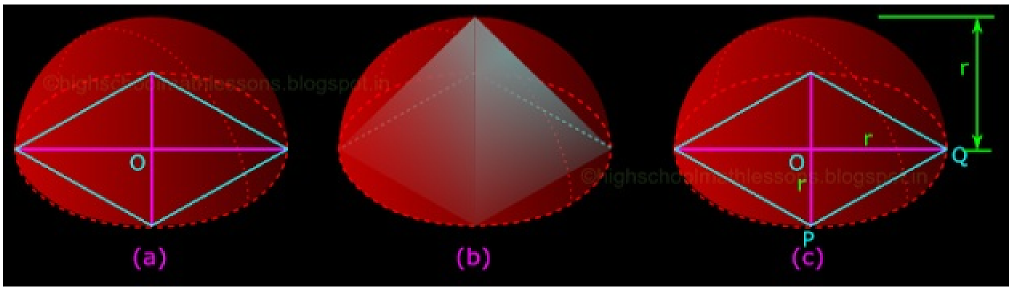
2. From fig.c, we have:

• Height of the cone, $h_c$ = r

• Radius of the cone, $r_c$ = r

• So Volume, $V_c = \frac{1}{3}\pi(r_c)^2 h = \frac{1}{3}\times\pi\times r^2\times r = \frac{1}{3}\times\pi\times r^3$

3. Consider the red hemisphere in fig (a) below. It is the same hemisphere of radius r, that we saw for the cone above




• A square is drawn in the base of the hemisphere

• This square is the base of the pyramid

4. For maximum possible volume, the diagonal of the square must be equal to the diameter of the circle

• So in fig. c, we can write:

OP = OQ = half of diameter = radius = r

5. OPQ is a right triangle. We can apply Pythagoras theorem

• Then base edge = PQ = $\sqrt{(OP^2 + OQ^2)} = \sqrt{(r^2 + r^2)} = \sqrt{2r^2} = \sqrt{2}$ r

6. So volume of the pyramid, $V_p = \frac{1}{3} \times \text{ base area } \times \text{ height}$

$$= \frac{1}{3} \times \sqrt{[2]}r \times \sqrt{[2]}r \times r = \frac{2}{3} \times r^3$$

7. Now we can take the ratio:

$$\frac{V_p}{V_c} = \frac{\left(\frac{2}{3} \times r^3\right)}{\frac{1}{3} \times \pi \times r^3} = \frac{2}{\pi}$$

• Thus we get:

$V_p : V_c = 2 : \pi$