CHAPTER 05

Work, Power and Energy

Work Done

- **By a constant force**

  $W = \mathbf{F} \cdot \mathbf{s} = \mathbf{F} \cdot (\mathbf{r}_f - \mathbf{r}_i) = Fs\cos\theta$

  = Force $\times$ displacement in the direction of force.

  = Displacement $\times$ component of force in the direction of displacement.

  $= +\mathbf{Fs}$ if $\theta = 0°$

  $= -\mathbf{Fs}$ if $\theta = 180°$

  $= 0$ if $\theta = 90°$, where θ is the angle between $\mathbf{F}$ and $\mathbf{s}$.

- **By a variable force** $W = \int_{x_i}^{x_f} F \cdot x$, where $F = f(x)$

- **By area under F - x graph** If force is a function of x, we can find work done by area under F - x graph with projection along x-axis. In this method, magnitude of work done can be obtained by area under F-x graph, but sign of work done should be decided by you.

  If force and displacement both are positive or negative, work done will be positive. If one is positive and other is negative, then work done will be negative.

- Work done by a force may be positive, negative or zero, depending upon the angle (θ) between the force vector $\mathbf{F}$ and displacement vector $\mathbf{s}$. Work done by a force is zero when $\theta = 90°$, it is positive when $0° \leq \theta < 90°$ and negative when $90° < \theta \leq 180°$.

  For example, when a person lifts a body, the work done by the lifting force is positive (as $\theta = 0°$) but work done by the force of gravity is negative (as $\theta = 180°$).

- Work depends on frame of reference. For example, if a person is pushing a box inside a moving train, then work done as seen from the frame of reference of train is $\mathbf{F} \cdot \mathbf{s}$ while as seen from the ground it is $\mathbf{F} \cdot (\mathbf{s} + \mathbf{s}_0)$. Here $\mathbf{s}_0$, is the displacement of train relative to ground.

Note $1 \text{J} = 10^7$ erg

Conservative and Non-Conservative Forces

- In case of conservative forces, work done is path independent.
- Potential energy is defined only for conservative forces.
- If only conservative forces are acting on a system, its mechanical energy should remain constant.
- In case of non-conservative forces, work done is dependent upon the path.
- Examples of conservative forces are gravitational force between two point masses, electrostatic force between two charges and the spring force.
- Examples of non-conservative forces are force of friction, viscous force etc.

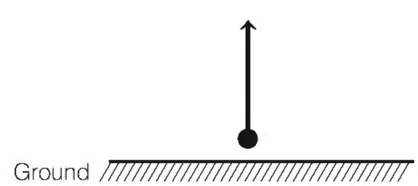
Potential Energy

- Potential energy is defined only for conservative forces.
- In a conservative force field, difference in potential energy between two points is the negative of work done by conservative forces in displacing the body (or system) from some initial position to final position. Hence,

$$\Delta U = -W$$

or $$U_B - U_A = -W_{A \to B}$$

- Change in potential energy is equal to the negative of work done by the conservative force ($\Delta U = -\Delta W$). If work done by the conservative force is negative, change in potential energy will be positive or potential energy of the system will increase and *vice-versa*.



This can be explained by a simple example. Suppose a ball is taken from the ground to some height, work done by gravity is negative, i.e. change in potential energy should increase or potential energy of the ball will increase.

$$\Delta W_{gravity} = -\text{ve}$$

$\therefore$ $$\Delta U = +\text{ve} \qquad (\because \Delta U = -\Delta W)$$

or $$U_f - U_i = +\text{ve}$$

- Absolute potential energy at a point can be defined with respect to a reference point, where potential energy is assumed to be zero.

Reference point corresponding to gravitational potential energy and electrostatic potential energy is assumed at infinity.

Reference point corresponding to spring potential energy is taken at a point at natural length of spring.

Now, negative of work done in displacing the body from reference point (say O) to the point under consideration (say P) is called absolute potential energy at P. Thus,

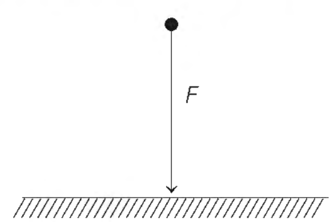
$$U_P = -W_{O \to P}$$

- For increasing or decreasing in gravitational potential energy of a particle (for small heights) we can write,

$$\Delta U = \pm mgh$$

Here, h is the difference in heights of particle. In case of a rigid body, h of centre of mass of the rigid body is seen.

- $F = -\dfrac{dU}{dr}$, i.e. conservative forces always act in a direction, where potential energy of the system is decreased. This can also be shown as in figure.



If a ball is dropped from a certain height. The force on it (its weight) acts in a direction in which its potential energy decreases.

Relation between Conservative Force (F) and its Potential Energy (*U*)

Conversion of *U* into F

- $\mathbf{F} = -\left[\dfrac{\partial U}{\partial x}\hat{\mathbf{i}} + \dfrac{\partial U}{\partial y}\hat{\mathbf{j}} + \dfrac{\partial U}{\partial z}\hat{\mathbf{k}}\right]$
- $F = -\dfrac{dU}{dr}$ or $-\dfrac{dU}{dx}$

or $-$ (slope of U - r or U - x graph)

Conversion of F into *U*

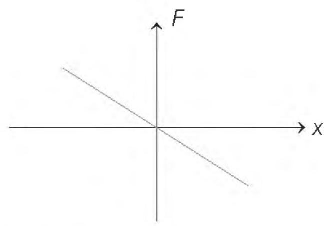
- $dU = -\mathbf{F} \cdot d\mathbf{r}$

Here, $\mathbf{F}$ will be given in the question and $d\mathbf{r} = (dx\hat{\mathbf{i}} + dy\hat{\mathbf{j}} + dz\hat{\mathbf{k}})$ is a standard vector.

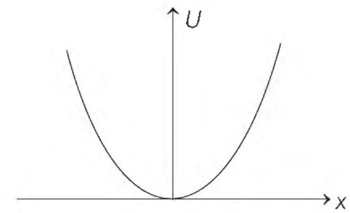
- $dU = -Fdr$ or $-Fdx$

Spring Force

- $F = -kx$ or $F \propto -x$



- $U_x = \dfrac{1}{2}kx^2$



Work Done and Energy

- Work done by conservative forces is equal to minus of change in potential energy,

$$W_c = -\Delta U = -(U_f - U_i) = U_i - U_f$$

- Work done by all the forces is equal to change in kinetic energy,

$$W_{\text{all}} = \Delta K = K_f - K_i \qquad \text{(Work-energy theorem)}$$

- Work done by the forces other than the conservative forces (non-conservative + external forces) is equal to change in mechanical energy

$$W_{nc} + W_{\text{ext}} = \Delta E = E_f - E_i = (K_f + U_f) - (K_i + U_i)$$

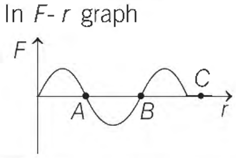
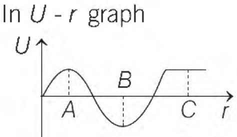
- If there are no non-conservative forces, then

$$W_{\text{ext}} = \Delta E = E_f - E_i$$

Further, in this case, if no information is given regarding the change in kinetic energy, then we can take it zero. In that case,

$$W_{\text{ext}} = \Delta U = U_f - U_i$$

Types of Equilibrium

Physical situation	**Stable equilibrium**	**Unstable equilibrium**	**Neutral equilibrium**
Net force	Zero	Zero	Zero
Potential energy	Minimum	Maximum	Constant
When displaced from mean (equilibrium) position.	A restoring nature of force will act on the body, which brings the body back towards mean position.	A force will act which moves the body away from mean position.	Force is again zero.
In *F*- *r* graph 	At point *A*	At point *B*	At point *C*
In *U* - *r* graph 	At point *B*	At point *A*	At point *C*

Power

- **Average power**

$$P_{av} = \frac{\text{total work done}}{\text{total time taken}} = \frac{W_{total}}{t}$$

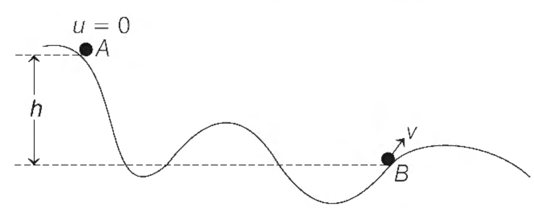
- **Instantaneous power**

$$P_i \text{ or } P = \text{rate of doing work done}$$
$$= \frac{dW}{dt} = \mathbf{F} \cdot \mathbf{v} = Fv\cos\theta$$

- If θ (between $\mathbf{F}$ and $\mathbf{v}$) is 90°, then power of the force is zero. If θ is acute, then power is positive and if θ is obtuse, then power is negative.

Some Important Points Related to Work and Energy

- Suppose a particle is released from point A with $u = 0$.



If friction is absent everywhere, then velocity at B will be

$v = \sqrt{2gh}$ (Irrespective of the track it follows from A to B)

Here, $h = h_A - h_B$

- In circular motion, centripetal force acts towards the centre. This force is perpendicular to small displacement $d\mathbf{s}$ and velocity $\mathbf{v}$. Hence, work done by it is zero and power of this force is also zero.
- If friction is absent, then

$E_i = E_f$ (where, E = mechanical energy)

If friction exists, then

$$E_i - E_f = \text{WDAF}$$

Here, WDAF = work done against friction
$= \mu N d$

where, μ = coefficient of friction
N = normal reaction
and d = distance travelled on rough surface.

Note *In most of the cases,*
$N = mg$ on horizontal ground and
$N = mg\cos\theta$ on inclined ground

- Work done by friction $= E_f - E_i = -$ (WDAF)