WORK, ENERGY & POWER

1. WORK

1.1 Introduction to Work:

In Physics, work stands for 'mechanical work'.

Work is said to be done by a force when the body is displaced actually through some distance in the direction of the applied force.

However, when there is no displacement in the direction of the applied force, no work is said to be done, i.e., work done is zero, when displacement of the body in the direction of the force is zero.

Suppose a constant force $\vec{F}$ acting on a body produces a displacement $\vec{s}$ in the body along the positive x-direction, as shown in the figure

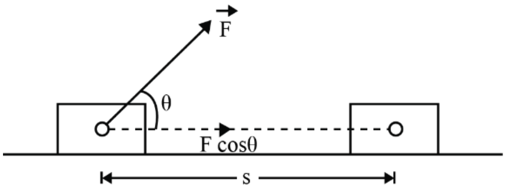


Fig. 5.1

If θ is the angle which $\vec{F}$ makes with the positive direction of the displacement, then the component of $\vec{F}$ in the direction of displacement is (F cosθ). As work done by the force is the product of component of force in the direction of the displacement and the magnitude of the displacement,

$$\boxed{W = (F\cos\theta)\, s} \qquad ...(1)$$

If displacement is exactly in the direction of force applied, θ = 0°. Then from (1),

W = (F cos 0°) s = F s

Equation (1) can be rewritten as $\boxed{W = \vec{F}.\vec{s}}$ ...(2)

Thus, work done by a force is the dot product of force and displacement.

In terms of rectangular component, $\vec{F}$ and $\vec{s}$ may be written as

$\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$ and $\vec{s} = x\hat{i} + y\hat{j} + z\hat{k}$

From (2), $W = \vec{F}.\vec{s}$

$$W = \left(F_x\hat{i} + F_y\hat{j} + F_z\hat{k}\right).\left(x\hat{i} + y\hat{j} + z\hat{k}\right)$$

$$\boxed{W = xF_x + yF_y + zF_z}$$

Obviously, work is a scalar quantity, i.e., it has magnitude only and no direction. However, work done by a force can be positive or negative or zero.

NOTE:

Work done is positive, negative or zero depending upon the angle between force and displacement

1.2 Dimensions and Units of Work

As work = force × distance

$W = (M^1L^1T^{-2}) \times L$

$$\boxed{W = \left[M^1\,L^2\,T^{-2}\right]}$$

This is the dimensional formula of work.

The **units** of work are of two types:

1. Absolute units 2. Gravitational units

(a) Absolute unit

1. **Joule.** It is the absolute unit of work in SI.

   Work done is said to be one joule, when a force of one newton actually moves a body through a distance of one metre in the direction of applied force.

   From $W = Fs\cos\theta$

   1 joule = 1 newton × 1 metre × cos 0° = 1 N–m

2. **Erg.** It is the absolute unit of work in cgs system.

   Work done is said to be one erg, when a force of one dyne actually moves a body through a distance of one cm in the direction of applied force.

   From $W = Fs\cos\theta$

   $1\,\text{erg} = 1\,\text{dyne} \times 1\,\text{cm} \times \cos 0° = 10^{-5}\,\text{N} \times 10^{-2}\,\text{m} \times 1$

   $1\,\text{erg} = 10^{-7}\,\text{J}$

(b) Gravitational units

These are also called the practical units of work.

1. **Kilogram-metre** (kg–m). It is the gravitational unit of work in SI.

   Work done is said to be one kg–m, when a force of 1 kg f move a body through a distance of 1 m in the direction of the applied force.

From $W = F\cos\theta$

1 kg–m = 1 kg f × 1 m × cos 0° = 9.8 N × 1 m = 9.8 joule, i.e.,

$\boxed{1\,kg - m = 9.8\,J}$

2. **Gram-centimetre** (g-cm). It is the gravitational unit of work in cgs system.
   Work done is said to be one g-cm, when a force of 1 g f moves a body through a distance of 1 cm. in the direction of the applied force.
   From $W = Fs\cos\theta$
   1 g-cm = 1 g f × 1 cm × cos 0°
   1 g-cm = 980 dyne × 1 cm × 1
   $\boxed{1g - cm = 980\,erg}$

1.3. Nature of Work Done

Although work done is a scalar quantity, its value may be positive, negative or even zero, as described below:

(a) Positive work

As $W = \vec{F}.\vec{s} = Fs\cos\theta$

∴ when θ is acute (< 90°), cos θ is positive. Hence, work done is positive.

For example :

When a body falls freely under the action of gravity, θ = 0°, cos θ = cos 0° = + 1. Therefore, work done by gravity on a body falling freely is positive.

(b) Negative work

As $W = \vec{F}.\vec{s} = Fs\cos\theta$

∴ When θ is obtuse (> 90°), cos θ is negative. Hence, work done is negative.

For example :

When a body is thrown up, its motion is opposed by gravity. The angle θ between gravitational force and the displacement is 180°. As cos θ = cos 180° = –1, therefore, work done by gravity on a body moving upwards is negative.

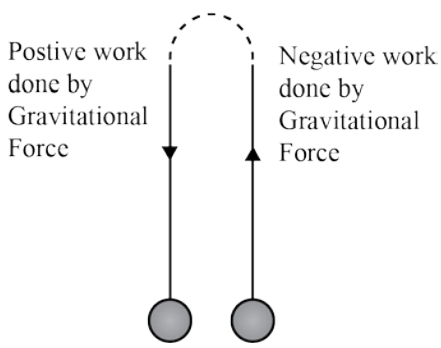


Fig. 5.2

(c) Zero work

When force applied $\vec{F}$ or the displacement $\vec{s}$ or both are zero, work done W = F s cos θ is zero. Again, when angle θ between $\vec{F}$ and $\vec{s}$ is 90°, cos θ = cos 90° = 0. Therefore work done is zero.

For example :

When we push hard against a wall, the force we exert on the wall does no work, because $\vec{s} = 0$. However, in this process, our muscles are contracting and relaxing alternately and internal energy is being used up. That is why we do get tired.

And

In case of a particle moving in a circle with constant speed the centripetal force acting on the particle is always perpendicular to its velocity so angle θ between $\vec{F}$ and $\vec{s}$ is 90°, [cos θ = cos 90° = 0]. Therefore, work done is zero.

1.4. Work done by a Variable Force

If the force is variable, then the work done is

$$W = \int_{x_A}^{x_B} F(dx)$$

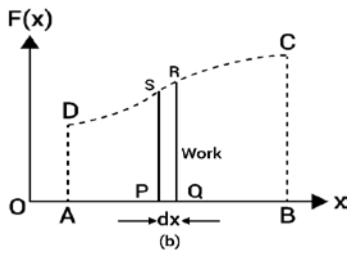


Fig. 5.3

$$W = \int_{x_A}^{x_B} \text{area of the strip PQRS}$$

= total area under the curve between F and x-axis for $x \in [x_A, x_B]$

$\boxed{W = \text{Area ABCDA}}$

Hence, work done by a variable force is numerically equal to the area under the force-displacement curve and the displacement axis.

2. KINETIC ENERGY

2.1. Introduction to Kinetic Energy:

The kinetic energy of a body is the energy possessed by the body by virtue of its motion.

For example:

(i) A bullet fired from a gun can pierce through a target on account of kinetic energy of the bullet.

(ii) Windmills work on the kinetic energy of air. For example, sailing ships use the kinetic energy of wind.

(iii) Water mills work on the kinetic energy of water. For example, fast flowing stream has been used to grind corn.

(iv) A nail is driven into a wooden block on account of kinetic energy of the hammer striking the nail.

Formula for Kinetic Energy

$$\boxed{\text{K.E. of body} = \frac{1}{2} m v^2}$$

2.2. Relation Between Kinetic Energy and Linear Momentum

Let m = mass of a body, v = velocity of the body.

$\therefore$ Linear momentum of the body, p = mv

and K.E. of the body $= \frac{1}{2}mv^2 = \frac{1}{2m}\left(m^2v^2\right)$

$$\therefore \quad \boxed{\text{K.E.} = \frac{p^2}{2m}}$$

This is an important relation. It shows that a body cannot have K.E. without having linear momentum. The reverse is also true.

Further, if p = constant, $\text{K.E} \propto \frac{1}{m}$

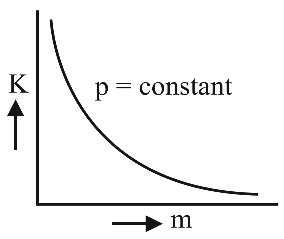
This is shown in figure (a)

If K.E. = constant, $p^2 \propto m$ or

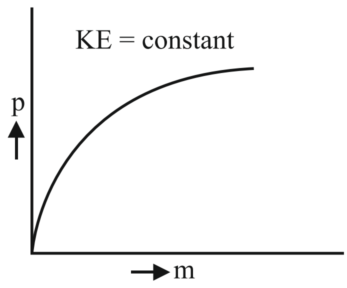
This is shown in figure (b).

If m = constant, $p^2 \propto \text{K.E}$ or

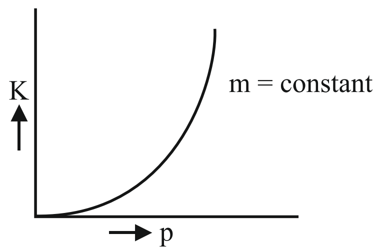
This is shown in figure (c)



(a)



(b)



(c)

3. WORK ENERGY THEOREM

According to this principle, work done by net force in displacing a body is equal to change in kinetic energy of the body.

Thus, when a force does some work on a body, the kinetic energy of the body increases by the same amount. Conversely, when an opposing (retarding) force is applied on a body, its kinetic energy decreases. The decrease in kinetic energy of the body is equal to the work done by the body against the retarding force. Thus, according to work energy principle, work and kinetic energy are equivalent quantities.

Proof : To prove the work-energy theorem, we confine ourselves to motion in one dimension.

Suppose m = mass of a body, u = initial velocity of the body, F = force applied on the body along it's direction of motion, a = acceleration produced in the body, v = final velocity of the body after t second.

Small amount of work done by the applied force on the body, dW = F (ds) when ds is the small distance moved by the body in the direction of the force applied.

$$\text{Now,} \quad F = ma = m\left(\frac{dv}{dt}\right)$$

$$dW = F(ds) = m\left(\frac{dv}{dt}\right)ds = m\left(\frac{dv}{dt}\right)$$

$$dW = mv\,dv \qquad \left(\because \frac{ds}{dt} = v\right)$$

Total work done by the applied force on the body in increasing its velocity from u to v is

$$W = \int_u^v mv\,dv = m\int_u^v v\,dv = m\left[\frac{v^2}{2}\right]_u^v$$

$$W = \frac{1}{2}m\left(v^2 - u^2\right) = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$$

But $\frac{1}{2}mv^2 = K_f$ = final K.E. of the body and

$\frac{1}{2}mu^2 = K_i$ = initial K.E. of the body

$W = K_f - K_i$ = change in K.E. of body

i.e., work done on the body = increase in K.E. of body

4. POTENTIAL ENERGY

4.1. Conservative and Non-Conservative Force

Conservative force

A force is said to be conservative if work done by or against the force in moving a body depends only on the initial and final positions of the body, and not on the nature of path followed between the initial and the final positions.

This means, work done by or against a conservative force in moving a body over any path between fixed initial and final positions will be the same.

For example, gravitational force is a conservative force.

Properties of Conservative forces:

1. Work done by or against a conservative force, in moving a body from one position to the other depends only on the initial position and final position of the body.
2. Work done by or against a conservative force does not depend upon the nature of the path followed by the body in going from initial position to the final position.
3. Work done by or against a conservative force in moving a body through any round trip (i.e., closed path, where final position coincides with the initial position of the body) is always zero.

Non-conservative Forces

A force is said to be non-conservative, if work done by or against the force in moving a body from one position to another, depends on the path followed between these two positions.

For example, frictional force is non-conservative forces.

Potential Energy and the Associated Conservative Force:

We know how to find potential energy associated with a conservative force. Now we learn how to obtain the conservative force if potential energy function is known. Consider work done dW by a conservative force in moving a particle through an infinitely small path length $d\vec{s}$ as shown in the figures.

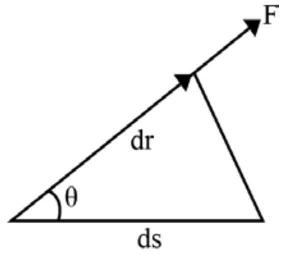


Fig. 5.4

$$dU = U_f - U_i = -dW = -\vec{F}.d\vec{s} = -Fds\cos\theta$$

From the above equation, the magnitude F of the conservative force can be expressed.

$$F = -\frac{dU}{ds\cos\theta} = -\frac{dU}{dr}$$

If we assume an infinitely small displacement in the direction of the force, magnitude of the force is given by the following equation.

$$F = -\frac{dU}{dr}$$

Here minus sign suggests that the force acts in the direction of decreasing potential energy.

Also, $F_x = -\frac{\partial U}{\partial x}, F_y = -\frac{\partial U}{\partial y}, F_Z = -\frac{\partial U}{\partial z}$

4.2 Introduction to Potential Energy

The potential energy of a body is defined as the energy possessed by the body by virtue of its position or configuration in some conservative field.

Thus, potential energy is the energy that can be associated with the configuration (or arrangement) of a system of objects that exert forces on one another. Obviously, if configuration of the system changes, then its potential energy changes.

Two important types of potential energy are :

1. Gravitational potential energy
2. Elastic potential energy.

Also $W_C = -\Delta U$

W_C : Work done by conservative force

4.3. Gravitational Potential Energy

Gravitational potential energy of a body is the energy possessed by the body by virtue of its position above the surface of the earth.

To calculate gravitational potential energy, suppose

m = mass of a body

g = acceleration due to gravity on the surface of earth.

h = height through which the body is raised, as shown in the figure.

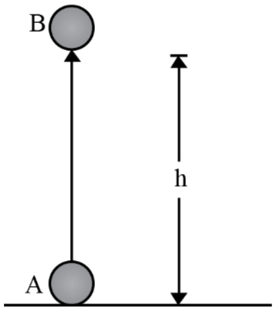


Fig. 5.5

If we assume that height h is not too large and the value of g is practically constant over this height,

$$W_g = mg \times \cos 180°$$

$$W_g = -mgh$$

$$\Delta U = -W_g$$

$$\Delta U = mgh$$

$$U_B - U_A = mgh$$

Considering $U_A = 0, U_B = U = mgh$

4.4. Spring Potential Energy

Potential energy of a spring is the energy associated with the state of compression or expansion of an elastic spring.

To calculate it, consider an elastic spring OA of negligible mass. The end O of the spring is fixed to a rigid support and a body of mass m is attached to the free end A. Let the spring be oriented along x–axis and the body of mass m lies on a perfectly frictionless horizontal table.

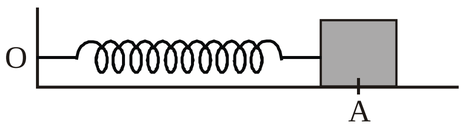


Fig.5.6

The position of the body A, when spring is unstretched is chosen as the origin.

When the spring is compressed or elongated, it tends to regain to its original length, on account of elasticity. The force trying to bring the spring back to its original configuration is called restoring force or spring force.

For a small stretch or compression, spring obeys Hooke's law.

Restoring Force $\propto$ stretch or compression

$$\vec{F} \propto -\vec{x}, \vec{F} = -k\vec{x}$$

where k is a constant of the spring and is called spring constant.

It is established that for a spring, $k \propto \frac{1}{\ell}$,

ℓ : Natural length of spring

i.e., smaller the length of the spring, greater will be the force constant and vice-versa.

The negative sign in equation indicates that the restoring force is directed always towards the equilibrium position.

Let the body be displaced further through an infinitesimally

small distance dx, against the restoring force.

Small amount of work done in increasing the length of the spring by dx is

dW = – F dx = kx dx

Total work done in giving displacement x to the body can be obtained by integrating from x = 0 to x = x, i.e.,

$$W = \int_{x=0}^{x=x} k\,x\,dx = k\left[\frac{x^2}{2}\right]_{x=0}^{x=x} = k\left[\frac{x^2}{2} - 0\right] = \frac{1}{2}kx^2$$

This work done is stored in the spring at the point B in the form of P.E

$$\therefore \text{ P.E. at B} = W = \frac{1}{2}kx^2$$

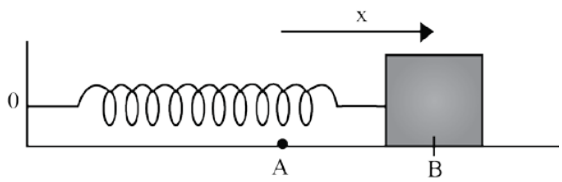


Fig. 5.7

The variation of potential energy with distance x is shown in figure

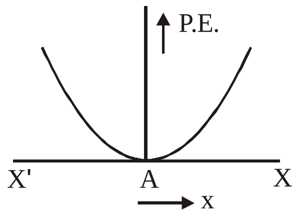


Fig. 5.8

5. MECHANICAL ENERGY AND ITS CONSERVATION

The mechanical energy (E) of a body is the sum of kinetic energy (K) and potential energy (U) of the body

i.e., $\boxed{E = K + U}$

Obviously, mechanical energy of a body is a scalar quantity measured in joule.

We can show that the total mechanical energy of a system is conserved if the force, doing work on the system are conservative.

This is called the principle of conservation of total mechanical energy.

For simplicity, we assume the motion to be one dimensional

only. Suppose a body undergoes a small displacement Δx under the action of a conservative force $F(x)$. According to work energy theorem,

change in K.E. = work done

$$\Delta K = F(x).\Delta x$$

As the force is conservative, the potential energy function $U(x)$ is defined as

$$-\Delta U = F(x).\Delta x \text{ or } \Delta U = -F(x).\Delta x$$

Adding, we get $\Delta K = F(x).\Delta x$

$$\Delta K = -\Delta U, \Delta(K + U) = 0$$

which means $\boxed{(K + U) = E = \text{constant}}$

5.1 Illustration of the Law of Conservation of Mechanical Energy

To illustrate the law further, let us calculate kinetic energy K.E., potential energy P.E. and total energy T.E. of a body falling freely under gravity.

Let m be the mass of the body held at A, at a height h above the ground, as shown in the figure.

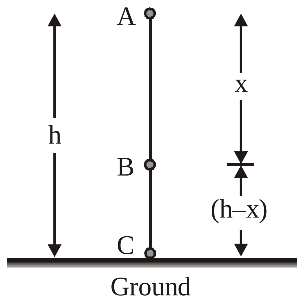


Fig.5.9

As the body is at rest at A, therefore,

At A : K. E. of the body = 0

P.E. of the body = mgh where g is acceleration due to gravity at A.

T.E. of the body $= K.E + P.E = 0 + mgh$

$$E_A = mgh \quad ....(1)$$

Let the body be allowed to fall freely under gravity, when it strikes the ground at C with a velocity v.

From $v^2 - u^2 = 2as$

$$v^2 - 0 = 2gh$$

$$v^2 = 2gh \; ...(2)$$

$\therefore$ At C : K.E. of the body $= \frac{1}{2}mv^2 = \frac{1}{2}m(2\,gh) = mgh$

P.E. of the body = mgh = mg (0) = 0

Total energy of the body = K.E. + P.E.

$$E_C = mgh + 0 = mgh \; ...(3)$$

In the free fall, let the body cross any point B with a velocity v_1, where AB = x

From $v^2 - u^2 = 2$ as

$$v_1^2 - 0 = 2(g)x \qquad ....(4)$$

At B : K.E. of the body $= \frac{1}{2}mv^2 = \frac{1}{2}m(2gx) = mgx$

Height of the body at B above the ground

$= CB = (h - x)$

P.E. of the body at B = mg (h – x)

Total energy of the body at B = K.E. + P.E.

$$E_B = mgx + mg(h - x) = mgx + mgh - mgx$$

$$E_B = mgh \; ...(5)$$

From (1), (3), (5) we find that

$\boxed{E_A = E_B = E_C = mgh}$ which proves conservation of mechanical energy

6. POTENTIAL ENERGY AND NATURE OF EQUILIBRIUM

As we know $F = -\frac{dU}{dr}$ So, Force = negative of slope of u versus r graph.

The state of stable and unstable equilibrium is associated with a point location, where the potential energy function assumes a minimum and maximum value respectively, and the neutral equilibrium is associated with region of space, where the potential energy function assumes a constant value.

For the sake of simplicity, consider a one-dimensional potential energy function U of a central force F. Here r is the radial coordinate of a particle. The central force F experienced by the particle equals to the negative of the slope of the potential energy function. Variation in the force with r is also shown in the figure.

At locations $r = r_1, r = r_2$, and in the region $r \geq r_3$, where potential energy function assumes a minimum, a maximum, and a constant value respectively, the force becomes zero and the particle is in the state of equilibrium

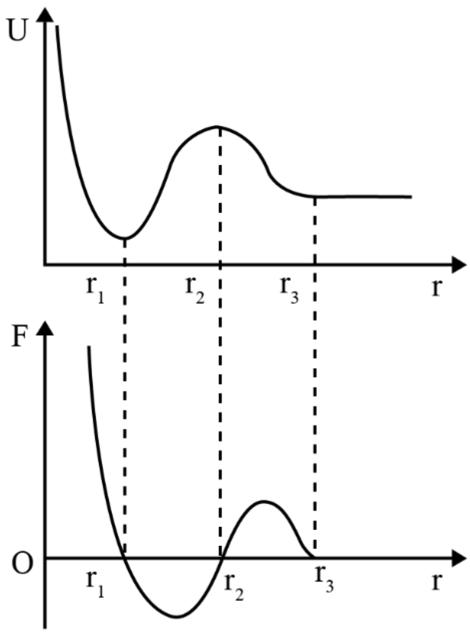


Fig.5.10

NOTE:

Force is negative of the slope of the potential energy function

6.1 Stable Equilibrium

At $r = r_1$ the potential energy function is a minima and the force on either side acts towards the point $r = r_1$. If the particle is displaced on either side and released, the force tries to restore it at $r = r_1$. At this location the particle is in the state of stable equilibrium. The dip in the potential energy curve at the location of stable equilibrium is known as potential well. A particle when disturbed from the state of stable equilibrium starts oscillations about the location of stable equilibrium. At the locations of stable equilibrium, we have

$$F(r) = -\frac{\partial U}{\partial r} = 0; \text{ and } \frac{\partial F}{\partial r} < 0; \text{ and } \frac{\partial^2 U}{\partial r^2} > 0$$

6.2 Unstable Equilibrium

At $r = r_2$ the potential energy function is a maxima, the force acts away from the point $r = r_2$. If the particle is displaced slightly on either side, it will not return to the location $r = r_2$. At this location, the particle is in the state of unstable equilibrium. At the locations of unstable equilibrium, we have

$$F(r) = -\frac{\partial U}{\partial r} = 0; \text{ therefore } \frac{\partial F}{\partial r} > 0; \text{ and } \frac{\partial^2 U}{\partial r^2} < 0$$

6.3 Neutral Equilibrium

In the region $r = r_3$, the potential energy function is constant and the force is zero everywhere. In this region, the particle is in the state of neutral equilibrium. At the locations of neutral equilibrium, we have

$$F(r) = -\frac{\partial U}{\partial r} = 0 \text{ therefore } \frac{\partial F}{\partial r} = 0 \text{ and } \frac{\partial^2 U}{\partial r^2} = 0$$

7. POWER

Power of a person or machine is defined as the time rate at which work is done by it.

i.e., Power = Rate of doing work = $\frac{\text{work done}}{\text{time taken}}$

Thus, power of a body measures how fast it can do the work.

Units of power

The absolute unit of power in SI is **watt**, which is denoted by W.

From P = W/t

1 watt = $\frac{1\,\text{joule}}{1\,\text{sec}}$, i.e., $\boxed{1\,W = 1\,Js^{-1}}$

Power of a body is said to be one watt, if it can do one joule of work in one second.

$\boxed{1\text{ h.p.} = 746\text{ W}}$

NOTE:

Power is also described in terms of rate at which energy is consumed.

$$P = \frac{dW}{dt}$$

Now, dW = $\vec{F}.\overrightarrow{ds}$, where $\vec{F}$ is the force applied and $\overrightarrow{ds}$ is the small displacement.

$$P = \frac{\vec{F}.d\vec{s}}{dt}$$

But $\frac{d\vec{s}}{dt} = \vec{v}$, the instantaneous velocity.

$$\boxed{P = \vec{F}.\vec{v}}$$

Dimensions of power can be deduced as:

$$\boxed{\begin{aligned} &P = \frac{W}{t} \\ &\Rightarrow [P] = \frac{M^1L^2T^{-2}}{T^1} = \left[M^1L^2T^{-3}\right] \end{aligned}}$$

Solved Examples

Example - 1

The sign of work done by a force on a body is important to understand. State carefully if the following quantities are positive or negative:

(a) work done by a man in lifting a bucket out of a well by means of a rope tied to the bucket.

(b) work done by gravitational force in the above case,

(c) work done by friction on a body sliding down an inclined plane,

(d) work done by an applied force on a body moving on a rough horizontal plane with uniform velocity,

(e) work done by the resistive force of air on a vibrating pendulum in bringing it to rest.

Sol. (a) Positive: In the given case, force and displacement are in the same direction. Hence, the sign of work done is positive. In this case, the work is done on the bucket.

(b) Negative: in the given case, the direction of force (vertically downward) and displacement (vertically upward) are opposite to each other. Hence, the sign of work done is negative.

(c) Negative: since the direction of frictional force is opposite to the direction of motion, the work done by frictional force is negative in this case.

(d) Positive: Here the body is moving on a rough horizontal plane. Frictional force opposes the motion of the body. Therefore, in order to maintain a uniform velocity, a uniform force must be applied to the body. Since the applied force acts in the direction of motion of the body, the work done is positive.

(e) Negative: the resistive force of air acts in the direction opposite to the direction of motion of the pendulum. Hence, the work done is negative in this case.

Example – 2

A body of mass 2 kg initially at rest moves under the action of an applied horizontal force of 7 N on a table with coefficient of kinetic friction = 0.1. Compute the

(a) work done by the applied force in 10 s,

(b) work done by friction in 10 s,

(c) work done by the net force on the body in 10 s,

(d) change in kinetic energy of the body in 10 s, and interpret your results.

Sol. Mass of the body, m = 2 kg

Applied force, F = 7 N

Coefficient of kinetic friction, $\mu = 0.1$

Initial velocity, u = 0

Time, t = 10 s

Frictional force is given as:

$f = \mu mg =$

$0.1 \times 2 \times 9.8 = 1.96$ N

Total force = 7-1.96 = 5.04

Total acceleration of the body:

$a = 2.52\ ms^{-2}$

The distance travelled by the body is given by the equation of motion :

$s = ut + (1/2)at^2$

$= 0 + (1/2) \times 2.52 \times (10)^2 = 126$ m

(a) Work done by the applied force,

$W_a = F \times s = 7 \times 126 = 882$ J

(b) Work done by the frictional force.

$W_f = F \times s = -1.96 \times 126 = -247 J$

(c) Net force = 7 + (-1.96) = 5.04 N

Work done by the net force,

$W_{net} = 5.04 \times 126 = 635 J$

(d) From the first equation of motion, final velocity can be calculated as :

$v = u + at = 0 + 2.52 \times 10 = 25.2$ m/s

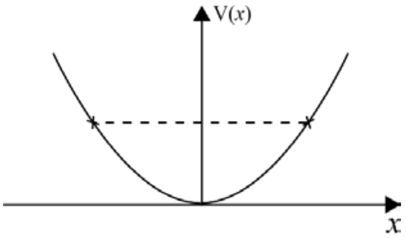
Change in kinetic energy

$= (1/2)mv^2 - (1/2)\ mu^2$

$= (1/2) \times 2(v^2 - u^2) = (25.2)^2 - 0^2 = 635$ J

Example – 3

The potential energy function for a particle executing linear simple harmonic motion is given by $U(x) = \frac{kx^2}{2}$ where k is the force constant of the oscillator. For k = 0.5 N m^{-1}, the graph of V (x) versus x is shown in fig. Show that a particle of total energy 1 J moving under this potential must 'turn back' when it reaches x = $\pm$ 2m.



Sol.

Total energy of the particle, E = 1J

Force constant, k = 0.5 N m^{-1}.

Kinetic energy of the particle, $K = \frac{1}{2}mv^2$.

According to the conservation law :

E = U + K

$$1 = \frac{1}{2}kx^2 + \frac{1}{2}mv^2$$

At the moment of 'turn back', velocity (and hence K) becomes zero,

$$\therefore 1 = \frac{1}{2}kx^2$$

$$\frac{1}{2} \times 0.5x^2 = 1$$

$x^2 = 4$

$x = \pm 2$

Hence, the particle turns back when it reaches x = $\pm$ 2 m.

Example – 4

Underline the correct alternative:

(a) When a conservative force does positive work on a body, the potential energy of the body increases/ decreases/remains unaltered.

(b) Work done by a body against friction always results in a loss of its kinetic/potential energy.

Sol.

(a) Decreases: A conservative force does a positive work on a body when it displaces the body in the direction of force. As a result, the body advances toward the centre of force. It decreases the separation between the two, thereby decreasing the potential energy of the body.

(b) Kinetic energy: The work done against the direction of friction reduces the velocity of a body. Hence, there is a loss of kinetic energy of the body.

Example – 5

State if each of the following statements is true or false. Give reasons for your answer.

(a) Total energy of a system is always conserved, no matter what internal and external forces on the body are present.

(b) Work done in the motion of a body over a closed loop is zero for every force in nature.

Sol.

(a) False The external forces on the body may change the total energy of the body.

(b) False The work done in the motion of a body over a closed loop is zero for a conservation force only.

Example – 6

A body is initially at rest. It undergoes one-dimensional motion with constant acceleration. The power delivered to it at time t is proportional to

(i) $t^{1/2}$ (ii) t (iii) $t^{3/2}$ (iv) t^2

Sol.

From

v = u + at

v = 0 + at = at

As power, $P = F \times v$

$\therefore P = (ma) \times at = ma^2 t$

As m and a are constants, therefore, $P \propto t$

Hence, right choice is (ii) t

Example – 7

A body is moving unidirectionally under the influence of a source of constant power. Its displacement in time t is proportional to

(i) $t^{1/2}$ (ii) t (iii) $t^{3/2}$ (iv) t^2

Sol.

As power, P = force velocity

$P = [MLT^{-2}][LT^{-1}] = [ML^2T^{-3}]$ = constant

As, $P = [ML^2T^{-3}]$

$\therefore L^2T^{-3}$ = constant

or, L^2/T^3 = constant

$\therefore L^2 \propto T^3$ or, $L \propto T^{3/2}$

Hence, right choice is (iii) $t^{3/2}$

Example – 8

A body constrained to move along the z-axis of a coordinate system is subject to a constant force

F given by $\vec{F} = -\hat{i} + 2\hat{j} + 3\hat{k}$ N

Where i, j, k are unit vectors along the x-, y- and z-axis of the system respectively. What is the work done by this force in moving the body a distance of 4 m along the z-axis?

Sol.

Force exerted on the body,

$\vec{F} = -\hat{i} + 2\hat{j} + 3\hat{k}$ N

Displacement, $\vec{S} = 4\hat{k}$ m

Work done, $W = \vec{F}.\vec{s}$

$= (-\hat{i} + 2\hat{j} + 3\hat{k}).(4\hat{k})$

$= 0 + 0 + 3 \times 4$

$= 12J$

Hence, 12 J of work is done by the force on the body.

Example – 9

A pump on the ground floor of a building can pump up water to fill a tank of volume 30 m^3 in 15 min. If the tank is 40 m above the ground, and the efficiency of the pump is 30%, how much electric power is consumed by the pump?

Sol.

Volume of the tank, V = 30 m^3

Time of operation, t = 15 min = 15×60 = 900 s

Height of the tank, h = 40m

Efficiency of the pump, η = 30%

Density of water, $\rho = 10^3$ kg/m^3

Mass of water, $m = V\rho = 30 \times 10^3$ kg

Output power can be obtained as :

P_o = Work done/Time = mgh/t

$= 30 \times 10^3 \times 9.8 \times 40/900 = 13.067 \times 10^3$ W

For input power P_i, efficiency, is given by the relation:

$= \frac{P_0}{P_i} = 30\%$

$P_i = 13.067 \times 100 \times 10^3 /30$

$= 0.436 \times 10^5$ W = 43.6 kW

Example – 10

A body of mass 0.5 kg travels in a straight line with velocity $v = ax^{3/2}$ where $a = 5\ m^{1/2}\ s^{-1}$. What is the work done by the net force during its displacement from x = 0 to x = 2m?

Sol.

Mass of the body, m = 0.5 kg

Velocity of the body is governed by the equation,

$v = ax^{3/2}$ and $a = 5\ m^{1/2}\ s^{-1}$

Initial velocity, u at (x = 0) = 0

Final velocity v at (x = 2m) = $10\sqrt{2}$ m/s

Work done, W = Change in kinetic energy

$= (1/2)\ m\ (v^2 - u^2)$

$= (1/2) \times 0.5\ [(10\sqrt{2})^2 - 0^2]$

$= (1/2) \times 0.5 \times 10 \times 10 \times 2$

= 50 J

Example – 11

A man rowing a boat upstream is at rest with respect to shore.

(a) Is he doing any work ?

(b) If he stops rowing and moves down with the stream, is any work being done on him ?

Sol. (a) No work is being done by the net force because displacement of boat relative to the shore is zero.

(b) When he stops rowing, force of water flow will produce displacement with respect to the shore. Therefore, work is done by force of flowing water. KE of the person will increase.

Example – 12

A stone is dropped from the top of a high tower. Will the mechanical energy of the stone be conserved or not if the force of friction due to air is not neglected?

Sol. Mechanical energy is conserved only when forces involved are conservative. As force of friction due to air is non-conservative, therefore, mechanical energy of the stone is not conserved.

Example – 13

A man can jump higher on moon than on earth. With same effort, can a runner improve his timing for 100 m race on moon as compared to that on earth ?

Sol. The man can jump higher on moon than on earth, because the accleration due to gravity on moon is less than acceleration due to gravity on earth. However, acceleration due to gravity has no effect on horizontal motion. Therefore, a runner cannot improve his timing on the moon for 100 metre race.

Example – 14

Work done by external forces is always equal to the gain in kinetic energy. Is it always true ?

Sol. Yes, This is the universal work-energy theorem.

Example – 15

Assume that the Earth revolves around the Sun in a perfectly circular orbit. Does the Sun do any work on the Earth ?

Sol. While the force is along the radius, the displacement is a along the tangent. Since radius and tangent are perpendicular, therefore, E and S are also perpenedicular. Consequently, work done is zero.

Example – 16

A block of mass 5 kg is being raised vertically upwards by the help of a string attached to it. It rises with an acceleration of 2 m/s^2. The block rises by 2.5 m. Match the correct choices :

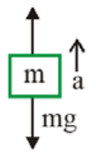
Column-I	**Column-II**
(A) Work done by gravity	(P) 122.55
(B) Work done by tension	(Q) 147.55
(C) Net work done on the block	(R) –122.55
	(S) 25 J

Sol. Let us first calculate the tension.

From force diagram :

$T - mg = 5\,a$ $\qquad$ $T = 5\,(9.8 + 2) = 59$ N.

As the T and displacement $\bar{S}$ are in same direction (upwards), work done by the tension T is:



$W = T\,s = 59\,(2.5) = 147.5$ J

Work done by the gravity = $-$ mgs = $-$ 5 (9.8) (2.5)

= $-$ 122.5 J

Net work done on block = work done by T + work done by mg

= 147.5 + ($-$122.5) = 25 J

The answer is A $\rightarrow$ R, B $\rightarrow$ Q, C $\rightarrow$ S.

Example – 17

A pump is required to lift 1000 kg of water per minute from a well 20 m deep and eject it at a rate of 20 m/s.

(a) How much work is done in lifting water ?

(b) How much work is done in giving it a KE ?

(c) What HP (horse power) engine is required for the purpose of lifting water ?

Sol. (a) Work done in lifting water = gain in PE (potential energy = $1000 \times g \times 20 = 1.96 \times 10^5$

(b) Work = $1000 \times g \times 20 = 1.96 \times 10^5$ J per minute

Work done (per minute) in giving it KE = 1/2 mv^2

= 1/2 (1000) (20)^2 = 2×10^5 J per minute

(c) Power of the engine = work done per second

$= 1/60\ (1.96 + 2)\ 10^5\ \text{J} = 6.6 \times 10^3\ \text{W (watts)}$

Since 1 HP = 746 W,

Hence, power required = 8.85 HP

Example – 18

An object of mass 5 kg falls from rest through a vertical distance of 20 m and attains a velocity of 10 m/s. How much work is done by the resistance of the air on the object ? ($g = 10\ \text{m/s}^2$)

Sol. Applying Work-Energy theorem,

work done by all the force = change in kinetic energy

or $W_{mg} + W_{air} = \frac{1}{2}mv^2$

$$\therefore \quad W_{air} = \frac{1}{2}mv^2 - W_{mg}$$

$$= \frac{1}{2}mv^2 - mgh$$

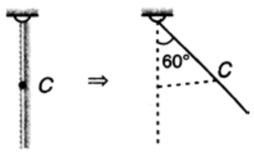
$$= \frac{1}{2} \times 5 \times (10)^2 - (5) \times (10) \times (20)$$

= – 750 J

Example – 19

A rod of length 1.0 m and mass 0.5 kg fixed at one end is initially hanging, vertical. The other end is now raised until it makes an angle 60° with the vertical. How much work is required ?

Sol. For increase in gravitational potential energy of a rod we see the centre of the rod.



W = change in potential energy

$$= mg\frac{\ell}{2}(1 - \cos\theta)$$

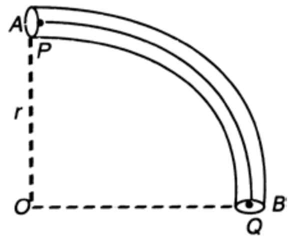
Substituting the values, we have

$$W = (0.5)(9.8)\left(\frac{1.0}{2}\right)(1 - \cos 60°)$$

= 1.225 J

Example – 20

A smooth narrow tube in the form of an arc AB of a circle of centre O and radius r is fixed so that A is vertically above O and OB is horizontal. Particles P of mass m and Q of mass 2 m with a light inextensible string of length $\left(\frac{\pi r}{2}\right)$ connecting them are placed inside the tube with P at A and Q at B and released from rest. Assuming the string remains taut during motion, find the speed of particles when P reaches B.



Sol. All surface are smooth. Therefore, mechanical energy of the system will remain conserved.

$\therefore$ Decrease in PE of both the block

= increase in KE of both the blocks

$$\therefore \quad (mgr) + (2mg)\left(\frac{\pi r}{2}\right) = \frac{1}{2}(m + 2m)v^2$$

$$\text{Or} \quad v = \sqrt{\frac{2}{3}(1 + \pi)gr}$$

Example – 21

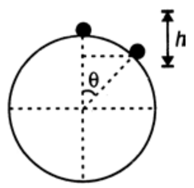
A small mass m starts from rest and slides down the smooth spherical surface of radius R. Assume zero potential energy at the top. Find :

(a) the change in potential energy

(b) the kinetic energy

(c) the speed of the mass as a function of the angle θ made by the radius through the mass with the vertical.

Sol. In the figure $h = R(1 - \cos\theta)$



(a) As the mass comes down, potential energy will decrease. Hence,

$\Delta U = -mgh = -mgR(1 - \cos\theta)$

(b) Magnitude of decrease in potential energy = increase in kinetic energy

$\therefore$ Kinetic energy = mgh

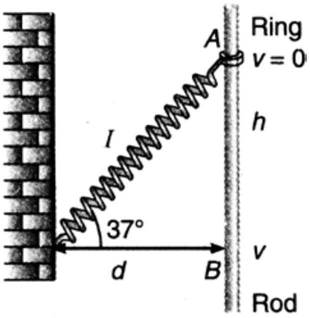
= mgR (1 – cos θ)

(c) $\frac{1}{2}mv^2 = mgR(1-\cos\theta)$

$\therefore \quad v = \sqrt{2gR(1-\cos\theta)}$

Example – 22

One end of a light spring of natural length d and spring constant k is fixed on a rigid wall and the other is attached to a smooth ring of mass m which can slide without friction on a vertical rod fixed at a distance d from the wall. Initially the spring makes an angle of 37° with the horizontal as shown in figure. When the system is released from rest, find the speed of the ring when the spring becomes horizontal [sin 37° = 3/5]



Sol. If *l* is the stretched length of the spring, then from figure

$\frac{d}{\ell} = \cos 37° = \frac{4}{5}$, i.e., $\ell = \frac{5}{4}d$

so the stretch $y = \ell - d = \frac{5}{4}d - d = \frac{d}{4}$

and $h = \ell \sin 37° = \frac{5}{4}d \times \frac{3}{5} = \frac{3}{4}d$

Now taking point B as reference level and applying law of conservation of mechanical energy between A and B,

$E_A = E_B$

Or $mgh + \frac{1}{2}ky^2 = \frac{1}{2}mv^2$ [as for B, h = 0 and y = 0]

Or $\frac{3}{4}mgd + \frac{1}{2}k\left(\frac{d}{4}\right)^2 = \frac{1}{2}mv^2$

[as for A, $h = \frac{3}{4}$ d and y = $\frac{1}{4}$ d]

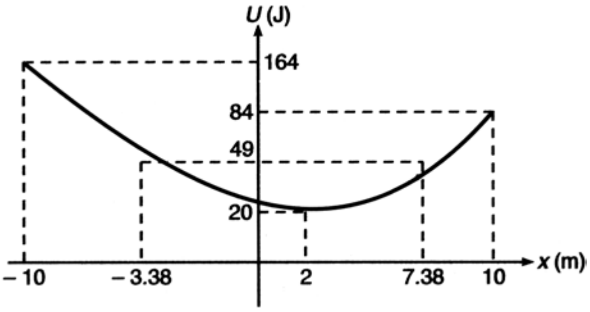
Or $v = d\sqrt{\frac{3g}{2d} + \frac{k}{16m}}$

Example – 23

A single conservative force F (x) acts on a 1.0 kg particle that moves along the x–axis. The potential energy U (x) is given by :

U (x) = 20 + $(x – 2)^2$

where x is in meters. At x = 5.0 m the particle has a kinetic energy of 20 J.



(a) What is the mechanical energy of the system ?

(b) The maximum kinetic energy of the particle, and

(c) The value of x at which it occurs.

(d) Determine the equation for F (x) as a function of x.

(e) For what (finite) value of x does F (x) = 0 ?

Sol.

(a) Potential energy at x = 5.0 m is

U = 20 + $(5 – 2)^2$ = 29 J

$\therefore$ Mechanical energy

E = K + U = 20 + 29 = 49 J

(b) and (c) :

Maximum kinetic energy is at x = 2m, where the potential energy is minimum and this maximum kinetic energy is,

$K_{max} = E - U_{min} = 49 - 20 = 29$ J

(d) $F = -\frac{dU}{dx} = 2(x-2) = (2x-4)$

(e) F (x) = 0, at x = 2.0 m

where potential energy is minimum (the position of stable equilibrium).

Example – 24

A running man has half the kinetic energy of that of a boy of half of his mass. The man speeds up by 1 m/s so as to have same kinetic energy as that of the boy, the original speed of the man is :

(a) $\left(\sqrt{2}-1\right)m/s$ (b) $\sqrt{2}\,m/s$

(c) $\dfrac{1}{\left(\sqrt{2}-1\right)}m/s$ (d) $\dfrac{1}{\sqrt{2}}m/s$

Sol. $K_{man} = \frac{1}{2}K_{boy}$

$$\frac{1}{2}mv^2 = \frac{1}{2}\left(\frac{1}{2}\left(\frac{m}{2}\right)u^2\right)$$

$$v = \frac{u}{2}$$

$$K_{man} = K_{boy}$$

$$\frac{1}{2}m(v+1)^2 = \left(\frac{1}{2}\left(\frac{m}{2}\right)u^2\right)$$

$$v+1 = \frac{u}{\sqrt{2}}$$

$$v+1 = \frac{2v}{\sqrt{2}} \Rightarrow \left(2-\sqrt{2}\right)v = \sqrt{2}$$

$$v = \frac{\sqrt{2}}{2-\sqrt{2}} \Rightarrow v = \frac{1}{\sqrt{2}-1}ms^{-1}$$

EXERCISE – 1: BASIC OBJECTIVE QUESTIONS

Work

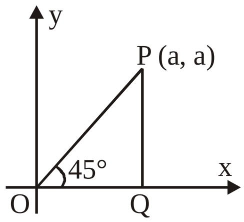
1. A force of $\left(10\hat{i}-3\hat{j}+6\hat{k}\right)$ N acts on a body of 5 kg and displaces it from A $\left(6\hat{i}+5\hat{j}-3\hat{k}\right)m$ to B $\left(10\hat{i}-2\hat{j}+7\hat{k}\right)m$. The work done is

(a) zero (b) 121 J
(c) 100 J (d) 221 J

2. A body is under the action of two equal and opposite forces, each of 3 N. The body is displaced by 2m. The work done is:

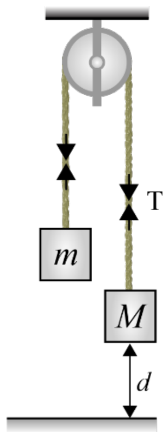
(a) + 6 J (b) – 6 J
(c) 0 (d) none of the above

3. A particle is moved from (0, 0) to (a, a) under a force $\vec{F}=(3\hat{i}+4\hat{j})$ from two paths. Path 1 is OP and path 2 is OQP. Let W_1 and W_2 be the work done by this force in these two paths. Then:



(a) $W_1 = W_2$ (b) $W_1 = 2W_2$
(c) $W_2 = 2W_1$ (d) $W_2 = 4W_1$

4. The net work done by the tension in the figure when the bigger block of mass M touches the ground is:

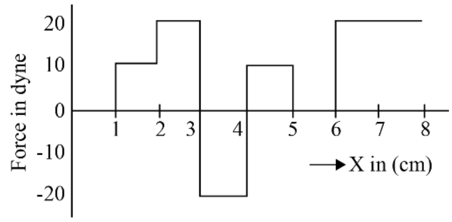


(a) + Mgd (b) – (M + m)gd
(c) – mgd (d) zero

5. A ball of mass 5 kg experiences a force F = $(2x^2 + x)$ N. Work done in displacing the ball by 2 m from origin is:

(a) $\frac{22}{3}$J (b) $\frac{44}{3}$J
(c) $\frac{32}{3}$J (d) $\frac{16}{3}$J

6. The relationship between force and position is shown in figure (in one dimensional case). The work done by the force in displacing a body from x = 1 cm to x = 5 cm is:



(a) 20 erg (b) 60 erg
(c) 70 erg (d) 700 erg

7. Under the action of a force, a 2 kg body moves such that its position x as a function of time t is given by x = $\frac{t^3}{3}$, where x is in metre and t in second. The work done by the force in the first two seconds is:

(a) 1600 J (b) 160 J
(c) 16 J (d) 1.6 J

8. A particle moves along the x-axis from x = 0 to x = 5 m under the influence of a force given by F = $7 - 2x + 3x^2$. Work done in the process is:

(a) 70 (b) 270
(c) 35 (d) 135

9. A particle moves under a force F = Cx from x = 0 to x = x_1. The work done is:

(a) Cx_1^2 (b) $\frac{Cx_1^2}{2}$
(c) 0 (d) Cx_1^3

10. A particle of mass 0.5 kg is displaced from position $\vec{r}_1$ (2, 3, 1) to $\vec{r}_2$ (4, 3, 2) by applying a force of magnitude 30 N which is acting along $\left(\hat{i}+\hat{j}+\hat{k}\right)$. The work done by the force is

(a) $10\sqrt{3}\,J$ (b) $30\sqrt{3}\,J$
(c) 30 J (d) none of these

11. A box is dragged across a floor by a rope which makes an angle of 45° with the horizontal. The tension in the rope is 100 N while the box is dragged 10 m. The work done is:

(a) 607.1 J (b) 707.1 J

(c) 1414.2 J (d) 900 J

12. A horizonal force F pulls a 10 kg carton across the floor at a constant speed. If the coefficient of sliding friction between carton and floor is 0.50, the work done by F in moving the carton through 5 m is: [take $g = 10\, m s^{-2}$]

(a) 196 J (b) 210.5 J

(c) 245 J (d) 254 J

13. The work done by a force $\vec{F} = \left(-6x^3\,\hat{i}\right)$ N is displacing a particle from x = 4 m to x = –2 m is

(a) – 240 J

(b) 360 J

(c) 420 J

(d) will depend upon the path

14. A body of mass 500 g is taken up an inclined plane of length 10 m and height 5 m, and then released to slide down to the bottom. The coefficient of friction between the body and the plane is 0.1. What is the amount of work done by friction in the round trip?

(a) 5 J (b) 15 J

(c) $5\sqrt{3}\,J$ (d) $\frac{5}{\sqrt{3}}\,J$

15. A mass M is lowered with the help of a string by a distance x at a constant acceleration $\frac{g}{2}$. The magnitude of work done by the string will be:

(a) Mgx (b) $\frac{1}{2}Mgx^2$

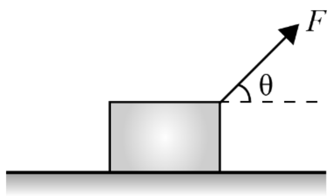
(c) $\frac{1}{2}$ Mgx (d) Mgx^2

16. The work done by pseudo forces is

(a) positive (b) negative

(c) zero (d) all of these

17. A block of mass m is pulled along a horizontal surface by applying a force at an angle q with the horizontal. If the block travels with a uniform velocity and has a displacement d and the coefficient of friction is m, then the work done by the applied force is



(a) $\frac{\mu mgd}{\cos\theta + \mu\sin\theta}$ (b) $\frac{\mu mgd\cos\theta}{\cos\theta + \mu\sin\theta}$

(c) $\frac{\mu mgd\sin\theta}{\cos\theta + \mu\sin\theta}$ (d) $\frac{\mu mgd\cos\theta}{\cos\theta - \mu\sin\theta}$

18. A uniform chain of length L and mass M is lying on a smooth table and one third of its length is hanging vertically down over the edge of the table. If g is acceleration due to gravity, work required to pull the hanging part on to the table is

(a) MgL (b) $\frac{MgL}{3}$

(c) $\frac{MgL}{9}$ (d) $\frac{MgL}{18}$

19. A car covers a distance of 10 km along an inclined plane under the action of a horizontal force of 5 N. The work done on car is 25 kJ. The inclination of the plane to horizontal is:

(a) 0° (b) 30°

(c) 60° (d) 90°

Kinetic Energy

20. The P.E. and KE of a helicopter flying horizontally at a height 400 m are in the ratio 5 : 2. The velocity of helicopter is

(a) 28 m/s (b) 14 m/s

(c) 56 m/s (d) 30 m/s

21. A 120 g mass has a velocity $\vec{v} = \left(2\hat{i} + 5\hat{j}\right) ms^{-1}$ at a certain instant. K.E. of the body at that instant is

(a) 3.0 J (b) 1.74 J

(c) 4.48 J (d) 5.84 J

22. A body is moving under the action of a force. Suddenly, force is increased to such an extent that its kinetic energy is increased by 100%. The momentum increases by:

(a) 100% (b) 60%

(c) 40% (d) 20%

23. A man has a box of weight 10 kg. The energy of the box, when the man runs with a constant velocity of 2 m/sec along with the box behind the bus, is:

(a) 10 joule (b) 30 joule

(c) 20 joule (d) 2 joule

24. What is the shape of the graph between the speed and kinetic energy of a body?

(a) straight line (b) hyperbola

(c) parabola (d) exponential

25. If the linear momentum is increased by 50%, then kinetic energy will be increased by:

(a) 50% (b) 100%

(c) 125% (d) 25%

26. A running man has half the KE that a boy of half his mass has. The man speeds up by 1m/s and then has the same KE as that of boy. The original speeds of man and boy in m/s are:

(a) $(\sqrt{2}+1),(\sqrt{2}-1)$ (b) $(\sqrt{2}+1),2(\sqrt{2}+1)$

(c) $\sqrt{2},\sqrt{2}$ (d) $(\sqrt{2}+1),2(\sqrt{2}-1)$

27. An object moving horizontally with kinetic energy of 800 J experiences a constant opposing force of 100 N while moving from a to b (where ab = 2m). The energy of particle at b is:

(a) 700 J (b) 400 J

(c) 600 J (d) 300 J

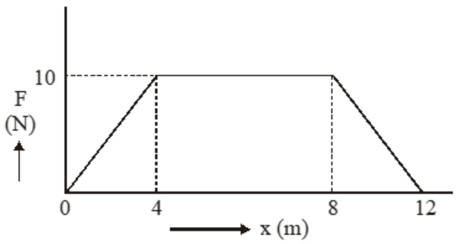
28. A particle moves on a rough horizontal ground with some initial velocity v_0. If $\frac{3}{4}$th of its K.E. is lost in friction in time t_0, the coefficient of friction between the particle and the ground is

(a) $\frac{v_0}{2gt_0}$ (b) $\frac{v_0}{4gt_0}$

(c) $\frac{3v_0}{4gt_0}$ (d) $\frac{v_0}{gt_0}$

Work Energy Theorem

29. A particle of mass 0.1 kg is subjected to a force which varies with distance as shown in figure. If it starts its journey from rest at x = 0, its velocity at x = 12 m is



(a) 0 m/s (b) 40 m/s

(c) $20\sqrt{2}\ m/s$ (d) 20 m/s

30. What average force is necessary to stop a bullet of mass 20 gm and speed 250 m/sec as it penetrates wood to a distance of 12 cm:

(a) 3.4×10^3 newton

(b) 5.2×10^3 newton

(c) 4.0×10^3 newton

(d) 3.6×10^3 newton

31. How much work must be done by a force on 50 kg body in order to accelerate it from rest to 20 m/s in 10 s?

(a) 10^3 J (b) 10^4 J

(c) 2×10^3 J (d) 4×10^4 J

32. The displacement of a body of mass 2 kg varies with time t as $s = t^2 + 2t$, where s is in meters and t is in seconds. The work done by all the forces acting on the body during the time interval t = 2s to t = 4s is

(a) 36 J (b) 64 J

(c) 100 J (d) 120 J

33. An object of mass m is allowed to fall from rest along a rough inclined plane. The speed of the object on reaching the bottom of the plane is proportional to

(a) m^0 (b) m

(c) m^2 (d) m^{-1}

34. A spring of spring constant 1000 N/m is compressed through 5 cm and is used to push a metal ball of mass 0.1 kg. The velocity with which the metal ball moves is

(a) 5 m/s (b) 7.5 m/s

(c) 10 m/s (d) 2.5 m/s

35. A block of mass 4 kg falls from a height of 3 m on a spring of force constant 1500 N/m. Calculate maximum compression of spring (g = 9.8 N/kg)

(a) 1.35 m (b) 0.42 m

(c) 0.735 m (d) 0.676 m

36. A truck weighing 1000 kg changes its speed from 36 km/h to 72 km/h in 2 minutes. Thus, the work done by the engine on the truck is:

(a) 2.5×10^5 J (b) 3.5×10^5 J

(c) 1.5×10^5 J (d) 5.5×10^5 J

37. The work done in time t on a body of mass m which is accelerated from rest to a speed v in time t_1 as a function of time t is given by:

(a) $\frac{1}{2}m\frac{v}{t_1}t^2$ (b) $m\frac{v}{t_1}t^2$

(c) $\frac{1}{2}\left(\frac{mv}{t_1}\right)^2 t^2$ (d) $\frac{1}{2}m\frac{v^2}{t_1^2}t^2$

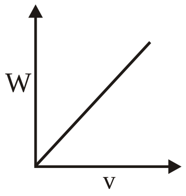
38. What average force is necessary to stop a bullet of mass 20 gm and speed 250 m/sec as it penetrates wood to a distance of 12 cm:

(a) 3.4×10^3 newton (b) 5.2×10^3 newton

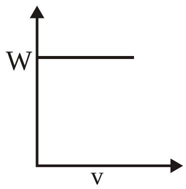
(c) 4.0×10^3 newton (d) 3.6×10^3 newton

39. A particle at rest on a frictionless table is acted upon by a horizontal force which is constant in magnitude and direction. A graph is plotted of the work done on the particle W, against the speed of the particle v. If there are no frictional forces acting on the particle, the graph will look like:

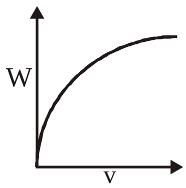
(a)



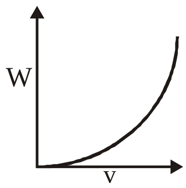
(b)



(c)



(d)



40. A body of mass 2 kg is moved from a point A to a point B by an external agent in a conservative force field. If the velocity of the body at the points A and B are 5 m/s and 3 m/s respectively and the work done by the external agent is –10 J, then the change in potential energy between points A and B is

(a) 6 J (b) 36 J

(c) 16 J (d) none of these

41. A block of mass 0.5 kg has an initial velocity of 10 m/s. down an inclined plane of angle 30°, the coefficient of friction between the block and the inclined surface is 0.2. The velocity of the block after it travels a distance of 10 m is:

(a) 17 m/s (b) 13 m/s

(c) 24 m/s (d) 8 m/s

42. A block is moved from rest through a distance of 4 m along a straight-line path. The mass of the block is 5 kg and the force acting on it is 20 N. If kinetic energy acquired by the block be 40 J, at what angle to the path is the force acting?

(a) 30º (b) 60º

(c) 45º (d) 0º

43. A particle is moving in a conservative force field from point A to point B. U_A and U_B are the potential energies of the particle at point A and B and W_C is the work done by conservative forces in the process of taking the particle from A and B:

(a) $W_C = U_B - U_A$ (b) $W_C = U_A - U_B$

(c) $U_A > U_B$ (d) $U_B > U_A$

44. Work done by the conservative forces on a system is equal to

(a) the change in kinetic energy of the system

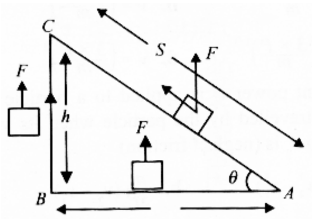
(b) negative of the change in potential energy of the system

(c) the change in total mechanical energy of the system

(d) none of these

Potential Energy

45. If we shift a body in equilibrium from A to C in a gravitational field via path AC or ABC



(a) the work done by the force $\vec{F}$ for both paths will be same

(b) $W_{AC} > W_{ABC}$

(c) $W_{AC} < W_{ABC}$

(d) None of the above

46. A meter stick of mass 400 g is pivoted at one end and displaced through an angle 60°. The increase in its potential energy is:

(a) 1 J (b) 10 J

(c) 100 J (d) 1000 J

47. A man weighing 60 kg lifts a body of mass 15 kg to the top of a building 10 m high in 3 minutes. His efficiency is

(a) 20% (b) 10%

(c) 30% (d) 40%

48. A spring of spring constant 8 N/cm has an extension of 5 cm. The minimum work done in joule in increasing the extension from 5 cm to 15 cm is

(a) 16 J (b) 8 J

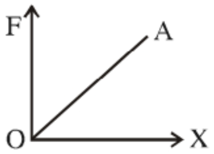
(c) 4 J (d) 32 J

49. The potential energy of a certain spring when stretched through a distance 'S' is 10 joules. The amount of work (in joule) that must be done on this spring to stretch it through an additional distance 'S' will be:

(a) 30 (b) 40

(c) 10 (d) 20

50. The force required to stretch a spring varies with the distance as shown in the figure. If the experiment is performed with the above spring of half the length, the line OA will:



(a) shift towards F–axis

(b) shift towards X–axis

(c) remain as it is

(d) become double in length

51. Two springs have their force constants k_1 and k_2. Both are stretched till their elastic energies are equal. Then, ratio of stretching forces F_1/F_2 is equal to:

(a) $k_1 : k_2$ (b) $k_2 : k_1$

(c) $\sqrt{k_1} : \sqrt{k_2}$ (d) $k_2^2 : k_1^2$

52. On changing the length of a spring by 0.1 m there is a change of 5 J in its potential energy. The force constant of the spring is:

(a) 80 Nm^{-1} (b) 10.0 Nm^{-1}

(c) 90 Nm^{-1} (d) 1000 Nm^{-1}

53. A rod of mass m and length ℓ is lying on a horizontal table. Work done in making it stand on one end will be:

(a) $mg\ell$ (b) $\frac{mg\ell}{2}$

(c) $\frac{mg\ell}{4}$ (d) $2mg\ell$

Conservation of Mechanical Energy

54. A toy gun uses a spring of very large value of force constant k. When charged before triggering in the upward direction, the spring is compressed by a small distance x. If mass of shot is m, on being triggered it will go up to a height of:

(a) $\frac{kx^2}{mg}$ (b) $\frac{x^2}{kmg}$

(c) $\frac{kx^2}{2mg}$ (d) $\frac{(kx)^2}{mg}$

55. A body is attached to the lower end of a vertical spiral spring and it is gradually lowered to its equilibrium position. This stretches the spring by a length d. If the same body attached to the same spring is allowed to fall suddenly, what would be the maximum stretching in this case?

(a) d (b) 2d

(c) 3d (d) $\frac{1}{2}$d

56. A sphere of mass 2 kg is moving on a frictionless horizontal table with velocity n. It strikes with a spring (force constant = 1 N/m) and compresses it by 4 m. The velocity (n) of the sphere is:

(a) 4 m/s (b) $2\sqrt{2}$ m/s

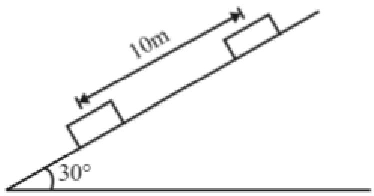
(c) 2 m/s (d) $\sqrt{2}$ m/s

57. An elastic string of unstretched length L and force constant k is stretched by a small length x. It is further stretched by another small length y. The work done in the second stretching is

(a) $\frac{1}{2}ky^2$ (b) $\frac{1}{2}k\left(x^2+y^2\right)$

(c) $\frac{1}{2}ky\left(2x+y\right)$ (d) $\frac{1}{2}k\left(x+y\right)^2$

58. A block of mass 0.5 kg has an initial velocity of 10 m/s down an inclined plane 30°, the coefficient of friction between the block and the inclined surface is 0.2. The velocity of the block after it travels a distance of 10 m figure is



(a) 17 m/s (b) 13 m/s

(c) 24 m/s (d) 8 m/s

59. A coconut of mass 1.0 kg falls to earth from a height of 10 m. The kinetic energy of the coconut, when it is 4 m above ground is:

(a) 0.588 joule (b) 58.8 joule
(c) 5.88 joule (d) 588 joule

60. Calculate the K.E. and P.E. of the ball halfway up, when a ball of mass 0.1 kg is thrown vertically upwards with an initial speed of 20 ms^{-1}.

(a) 10 J, 20 J (b) 10 J, 10 J
(c) 15 J, 8 J (d) 8 J, 16 J

61. If a body of mass 3 kg is dropped from top of a tower of height 250 m, then its kinetic energy after 3 sec. will be

(a) 1126 J (b) 1048 J
(c) 735 J (d) 1296.5 J

62. A body of mass 2 kg moves down the quadrant of a circle of radius 4 m. The velocity on reaching the lowest point is 8 m/s. What is work done against friction?

(a) 14.4 J (b) 28.8 J
(c) 64 J (d) Zero

63. The KE of a 500-gram stone is 100 J. Against a force of 50 N, how long will it travel?

(a) 0.2 s (b) 0.1 s
(c) 0.3 s (d) 0.4 s

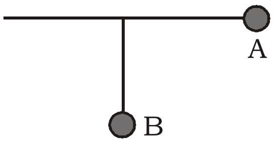
64. If water falls from a dam into a turbine wheel 19.6 m below, then velocity of water at turbine, is (Take g = 9.8 m/s^2)

(a) 9.8 m/s (b) 19.6 m/s
(c) 39.2 m/s (d) 98.0 m/s

65. Three particles A, B and C are projected from the top of a tower with the same speed. A is thrown straight upwards B straight down and C horizontally. They hit the ground with speeds v_A, v_B and v_C, then which of the following is correct:

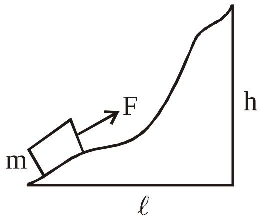
(a) $v_A = v_B > v_C$ (b) $v_A = v_B = v_C$
(c) $v_A > v_B = v_C$ (d) $v_B > v_C > v_A$

66. A pendulum of length 2 m left at A. When it reaches B, it loses 10% of its total energy due to air resistance. The velocity at B is:



(a) 6 m/s (b) 1 m/s
(c) 2 m/s (d) 8 m/s

67. A body of mass m was slowly pulled up the hill by a force F which at each point was directed along the tangent to the trajectory. All surfaces are smooth. Find the work performed by this force:



(a) mg ℓ (b) – mg ℓ
(c) mgh (d) zero

68. A particle is released from the top of two inclined rough surfaces of height 'h' each. The angle of inclination of the two planes are 30° and 60° respectively. All other factors (e.g. coefficient of friction, mass of block etc.) are same in both the cases. Let K_1 and K_2 be kinetic energies of the particle at the bottom of the plane in two cases. Then

(a) $K_1 = K_2$ (b) $K_1 > K_2$
(c) $K_1 < K_2$ (d) data insufficient

69. A particle is released from a height H. At certain height its kinetic energy is two times its potential energy. Height and speed of particle at that instant are

(a) $\frac{H}{3}, \sqrt{\frac{2gH}{3}}$ (b) $\frac{H}{3}, 2\sqrt{\frac{gH}{3}}$
(c) $\frac{2H}{3}, \sqrt{\frac{2gH}{3}}$ (d) $\frac{H}{3}, \sqrt{2gH}$

70. A body is falling with velocity 1 m/s at a height 3 m from the ground. The speed at height 2 m from the ground will be:

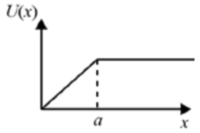
(a) 4.54 m/s (b) 1 m/s
(c) 6 m/s (d) 5.32 m/s

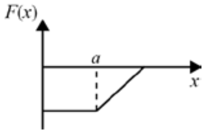
71. If v be the instantaneous velocity of the body dropped from the top of a tower, when it is located at height h, then which of the following remains constant?

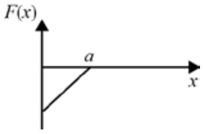
(a) $gh + v^2$ (b) $gh + \frac{v^2}{2}$
(c) $gh - \frac{v^2}{2}$ (d) $gh - v^2$

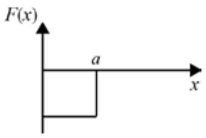
Potential Energy Graphs

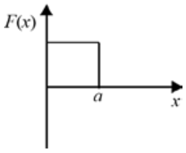
72. The potential energy of a particle is represented in the figure. The force acting on the system will be represented by



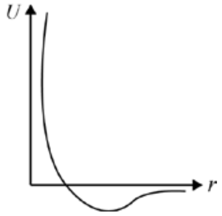
(a) 

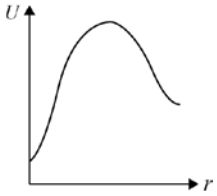
(b) 

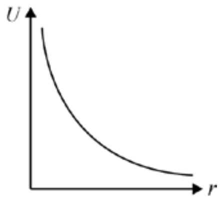
(c) 

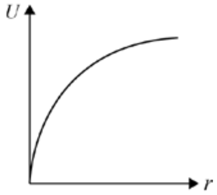
(d) 

73. The diagrams represent the potential energy (U) of a function of the inter-atomic distance r. Which diagram corresponds to stable molecules found in nature.

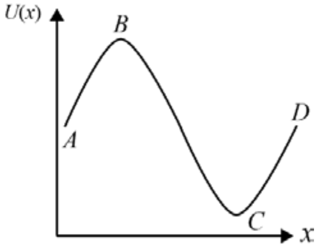
(a) 

(b) 

(c) 

(d) 

74. The potential energy of a particle varies with distance x as shown in the graph.



The force acting on the particle is zero at

(a) C (b) B

(c) B and C (d) A and D

Power

75. A pump of 200 W power is lifting 2 kg water from an average depth of 10 m per second. Velocity of water delivered by the pump is (g = 9.8 m/s^2)

(a) 3 m/s (b) 2 m/s

(c) 4 m/s (d) 1 m/s

76. A machine gun fires 360 bullets per minute, with a velocity of 600 m/s. If the power of the gun is 5.4 kW, mass of each bullet is (assume 100% efficiency)

(a) 5 kg (b) 0.5 kg

(c) 5 g (d) 0.5 g

77. A train of mass 100 ton is moving up an incline of 1 in 100 at a constant speed of 36 km ph. If the friction per ton is 100 N, then power of the engine is

(a) 198 kW (b) 96 kW

(c) 298 kW (d) 398 kW

78. The power of a water pump is 2 kW. If g = 10 m/s^2, the amount of water it can raise in one minute to a height of 10 m is

(a) 2000 litre (b) 1000 litre

(c) 100 litre (d) 1200 litre

79. A man is riding on a cycle with velocity 7.2 km/hr up a hill having a slope 1 in 20. Total mass of the man and cycle is 100 kg. The power of man is:

(a) 98 W (b) 49 W

(c) 196 W (d) 147 W

80. Power applied to a particle varies with time as

$P = (3t^2 - 2t + 1)$ W, where t is in second. Find the change in its kinetic energy between time t = 2s and t = 4s.

(a) 32 J (b) 46 J

(c) 61 J (d) 102 J

EXERCISE – 2: PREVIOUS YEARS JEE MAINS QUESTIONS

1. A car of weight W is on an inclined road that rises by 100 m over a distance of 1 km and applies a constant frictional force $\frac{W}{20}$ on the car. While moving uphill on the road at a speed of $10\,\text{ms}^{-1}$, the car needs power P. If it needs power $\frac{P}{2}$ while moving downhill at speed v then value of v is: **(2016)**

(a) $20\,\text{ms}^{-1}$ (b) $15\,\text{ms}^{-1}$

(c) $10\,\text{ms}^{-1}$ (d) $5\,\text{ms}^{-1}$

2. A particle of mass M is moving in a circle of fixed radius R in such a way that its centripetal acceleration at time t is given by n^2Rt^2 where n is a constant. The power delivered to the particle by the force acting on it, is: (all quantities are measured in S.I unit) **(2016)**

(a) Mn^2R^2t (b) MnR^2t

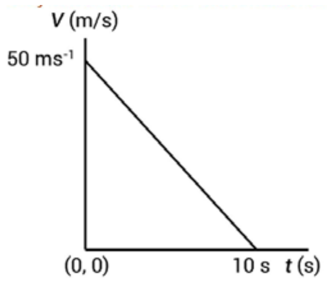
(c) MnR^2t^2 (d) $\frac{1}{2}Mn^2R^2t^2$

3. Concrete mixture is made by mixing cement, stone and sand in a rotating cylindrical drum. If the drum rotates too fast, the ingredients remain stuck to the wall of the drum and proper mixing of ingredients does not take place. The maximum rotational speed of the drum in revolutions per minute (r p m) to ensure proper mixing is close to: (Take the radius of the drum to be 1.25 m and its axis to be horizontal): (given $\sqrt{2} = 1.414$ and $g = 10\,\text{m/s}^2$) **(2016)**

(a) 0.4 (b) 1.3

(c) 8.0 (d) 27.0

4. Velocity-time graph for a body of mass 10 kg is shown in figure. Work-done on the body in first two second of the motion is: **(2016)**



(a) 12000 J (b) -12000 J

(c) -4500 J (d) -9300 J

5. A time dependent force F=6 t acts on a particle of mass 1 kg. If the particle starts from rest, the work done by the force during the first 1 sec. will be: **(2017)**

(a) 4.5 J (b) 22 J

(c) 9 J (d) 18 J

6. A particle is moving in a circular path of radius a under the action of an attractive potential $U = -\frac{k}{2r^2}$. its total energy is: **(2018)**

(a) $\frac{k}{2a^2}$ (b) Zero

(c) $-\frac{3}{2}\frac{k}{a^2}$ (d) $-\frac{k}{4a^2}$

7. A body of mass m starts moving from rest along x-axis so that its velocity varies as $v = a\sqrt{s}$ where a is a constant and s is the distance covered by the body. The total work done by all the forces acting on the body in the first t seconds after the start of the motion is: **(2018)**

(a) $\frac{1}{8}ma^4t^2$ (b) $8ma^4t^2$

(c) $4ma^4t^2$ (d) $\frac{1}{4}ma^4t^2$

8. Two particles of the same mass m are moving in circular orbits because of force, given by $F(r) = \frac{-16}{r} - r^3$. The first particle is at a distance r=1, and the second, at r =4. The best estimate for the ratio of kinetic energies of the first and the second particle is closest to: **(2018)**

(a) 6×10^{-2} (b) 3×10^{-3}

(c) 10^{-1} (d) 6×10^{2}

9. A force acts on a 2kg object so that its position (in m) is given as a function of time (in s) as $x = 3t^2 + 5$. What is the work done by this force in first 5 second? **(2019)**

(a) 850 J (b) 950 J

(c) 875 J (d) 900 J

10. A particle which is experiencing a force, given by $\vec{F} = 3\vec{i} - 12\vec{j}$, undergoes a displacement of $\vec{d} = 4\vec{i}$. If the particle had a kinetic energy of 3J at the beginning of the displacement, what is its kinetic energy at the end of the displacement? **(2019)**

(a) 9 J (b) 12 J

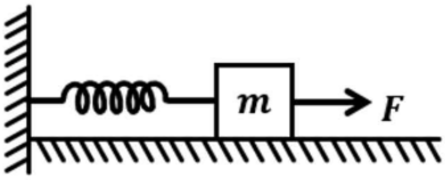
(c) 10 J (d) 15 J

11. A uniform cable of mass ' M ' and length ' L ' is placed on a horizontal surface such that its $\left(\frac{1}{n}\right)^{th}$ part is hanging below the edge of the surface. To lift the hanging part of the cable up to the surface, the work done should be: **(2019)**

(a) $\frac{MgL}{2n^2}$ (b) $\frac{MgL}{n^2}$

(c) $\frac{2MgL}{n^2}$ (d) $nMgL$

12. A block of mass m, lying on a smooth horizontal surface, is attached to a spring (of negligible mass) of spring constant k. The other end of the spring is fixed, as shown in the figure. The block is initially at rest in its equilibrium position. If now the block is pulled with a constant force F, the maximum speed of the block is: **(2019)**

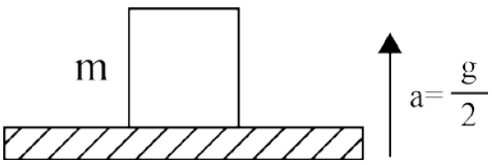


(a) $\frac{2F}{\sqrt{mk}}$ (b) $\frac{2F}{\pi\sqrt{mk}}$

(c) $\frac{\pi F}{\sqrt{mk}}$ (d) $\frac{F}{\sqrt{mk}}$

13. A block of mass m is kept on a platform which starts from rest with constant acceleration g/2 upwards, as shown in figure. The work done by normal reaction on block in time t is:

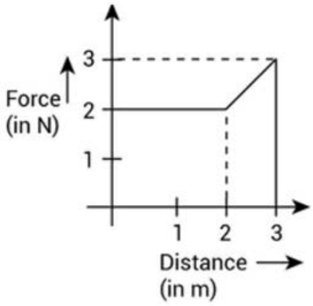
(2019)



(a) $-\frac{mg^2t^2}{8}$ (b) $\frac{mg^2t^2}{8}$

(c) 0 (d) $\frac{3mg^2t^2}{8}$

14. A particle moves in one dimension from rest under the influence of a force that varies with the distance travelled by the particle as shown in the figure. The kinetic energy of the particle (in J) after it has travelled 3 m is: **(2019)**



(a) 2.5 J (b) 4 J

(c) 5 (d) 6.5 J

15. A 60 HP electric motor lifts an elevator with a maximum total load capacity of 2000 kg. If the frictional force on the elevator is 4000 N, the speed of the elevator at full load is close to (Given $1HP = 746\,W, g = 10\,m/s^2$) **(2019)**

(a) $1.5\,m/s$ (b) $2.0\,m/s$

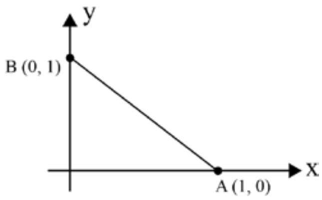
(c) $1.7\,m/s$ (d) $1.9\,m/s$

16. An elevator in a building can carry a maximum of 10 persons, with the average mass of each person being 68 kg. The mass of the elevator itself is 920 kg and it moves with a constant speed of 3 m/s. The frictional force opposing the motion is 6000 N. If the elevator is moving up with its full capacity, the power delivered by the motor to the elevator $\left(g = 10\,m/s^2\right)$ must be at least **(2020)**

(a) 66000 W (b) 63360 W

(c) 48000 W (d) 56300 W

17. Particle moves from point A to point B along the line shown in figure under the action of force $\vec{F} = -x\hat{i} + y\hat{j}$ .Determine the work done on the particle by $\vec{F}$ in moving the particle from point A to point B (all quantities are in S I units) **(2020)**

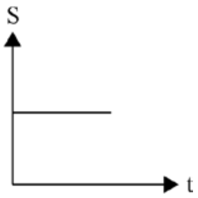


(a) 1 J (b) 1/2 J

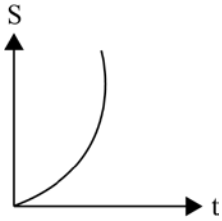
(c) 2 J (d) 3/2 J

18. A particle is moving unidirectionally on a horizontal plane under the action of a constant power supplying energy source. The displacement (s) - time (t) graph that describes the motion of the particle is (graphs are drawn schematically and are not to scale): **(2020)**

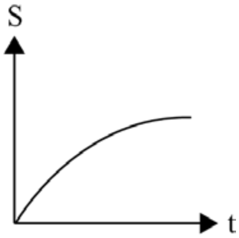
(a)



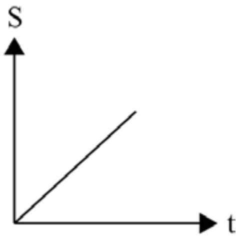
(b)



(c)



(d)



19. A person pushes a box on a rough horizontal platform surface. He applies a force of 200 N over a distance of 15 m. Thereafter, he gets progressively tired and his applied force reduces linearly with distance to 100 N. The total distance through which the box has been moved is 30 m. What is the work done by the person during the total movement of the box? **(2020)**

(a) 5690 J (b) 5250 J

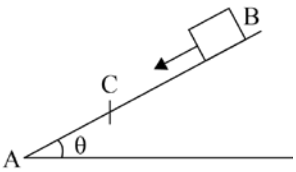
(c) 2780 J (d) 3280 J

20. If the potential energy between two molecules is given by $U = -\frac{A}{r^6} + \frac{B}{r^{12}}$, then at equilibrium, separation between molecules, and the potential energy are: **(2020)**

(a) $\left(\frac{2B}{A}\right)^{1/6}, -\frac{A^2}{4B}$ (b) $\left(\frac{2B}{A}\right)^{1/6}, -\frac{A^2}{2B}$

(c) $\left(\frac{B}{A}\right)^{1/6}, 0$ (d) $\left(\frac{B}{2A}\right)^{1/6}, -\frac{A^2}{2B}$

21. A small block starts slipping down from a point B on an inclined plane AB, which is making an angle θ with the horizontal section B C is smooth and the remaining section C A is rough with a coefficient of friction μ. It is found that the block comes to rest as it reaches the bottom (point A) of the inclined plane. If BC = 2 AC, the coefficient of friction is given by $\mu = k \tan\theta$. The value of k is **(2020)**



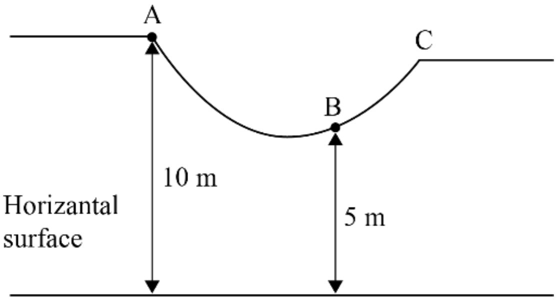
22. A body of mass 2 kg is driven by an engine delivering a constant power of $1 J/s$. The body starts from rest and moves in a straight line. After 9 seconds, the body has moved a distance (in m) **(2020)**

23. A small bob tied at one end of a thin string of length 1 m is describing a vertical circle so that the maximum and minimum tension in the string is in the ratio 5: 1. The velocity of the bob at the highest position is ...m/s. (Take $g = 10\,m/s^2$) **(2021)**

24. The potential energy (U) of a diatomic molecule is a function dependent on r (interatomic distance) as $U = \frac{\alpha}{r^{10}} - \frac{\beta}{r^5} - 3$ Where, α and β are positive constants. The equilibrium distance between two atoms will be $\left(\frac{2\alpha}{\beta}\right)^{a/b}$, where a/b is in lowest form and a=_______________, **(2021)**

25. Two solids A and B of mass 1 kg and 2 kg respectively are moving with equal linear momentum. The ratio of then kinetic energies $(KE)_A : (KE)_B$ will be $\frac{A}{1}$ so the value of A will be **(2021)**

26. As shown in the figure, a particle of mass 10 kg is placed at a point A. When the particle is slightly displaced to its right, it starts moving and reaches the point B. The speed of the particle at B is x m/s.



$\left(\text{Take } g = 10\,\text{m/s}^2\right)$

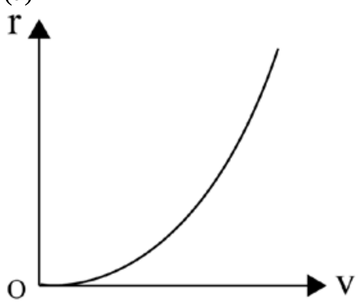
The value of 'x' to the nearest integer is.........

(2021)

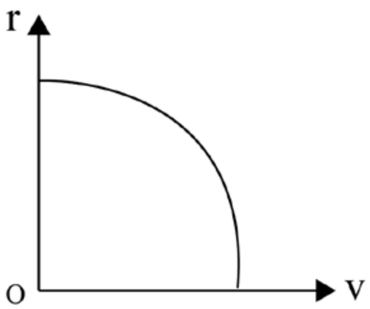
27. A ball of mass 4 kg, moving with a velocity of $10\,\text{ms}^{-1}$, collides with a spring of length 8 m and force constant $100\,\text{Nm}^{-1}$. The length of the compressed spring is x in m. The value of x, to the nearest integer, is **(2021)**

28. A particle of mass m moves in a circular orbit under the central potential field, $U(r) = -\frac{C}{r}$, where C is a positive constant. The correct radius - velocity graph of the particle's motion is: **(2021)**

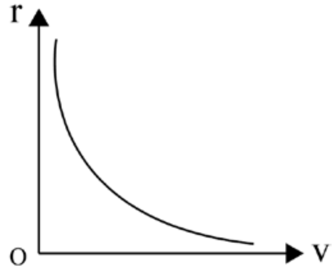
(a)



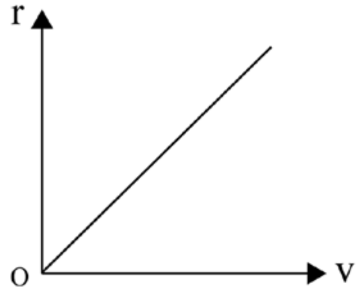
(b)



(c)



(d)



29. A boy is rolling a 0.5 kg ball on the frictionless floor with the speed of 20 m/s. The ball gets deflected by an obstacle on the way. After deflection it moves with 5 % of its initial kinetic energy. What is the speed of the ball now? **(2021)**

(a) $19.0\,\text{m/s}$ (b) $4.47\,\text{m/s}$

(c) $14.41\,\text{m/s}$ (d) $1.00\,\text{m/s}$

30. The constant power delivering machine has towed a box, which was initially at rest, along a horizontal straight line. The distance moved by the box in time 't' is proportional to: **(2021)**

(a) $t^{1/2}$ (b) $t^{2/3}$

(c) t (d) $t^{3/2}$

EXERCISE – 3: ADVANCED OBJECTIVE QUESTIONS

Objective Questions I [Only one correct option]

1. Force acting on a particle is $\left(2\hat{i}+3\hat{j}\right)$N. Work done by this force is zero, when a particle is moved on the line $3y + kx = 5$. Here, value of k is:

(a) 2 (b) 4

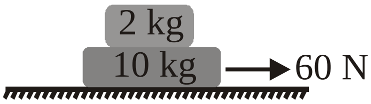
(c) 6 (d) 8

2. A small block of mass m is kept on a rough inclined surface of inclination θ fixed in an elevator. The elevator goes up with a uniform velocity v and the block does not slide on the wedge. The work done by the force of friction on the block in a time t will be:

(a) Zero (b) mgvt $\cos^2\theta$

(c) mgvt $\sin^2\theta$ (d) $\frac{1}{2}$ mgvt sin2θ

3. A plank of mass 10 kg and a block of mass 2 kg are placed on a horizontal plane as shown in the figure.



There is no friction between plane and plank. The coefficient of friction between block and plank is 0.5, A force of 60 N is applied on plank horizontally In first 2 s the work done by the friction on the block is:

(a) – 100 J (b) 100 J

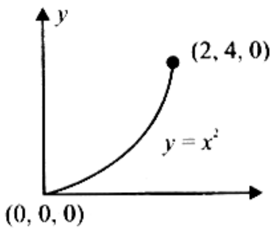
(c) zero (d) 200 J

4. A force of $\vec{F}=2x\hat{i}+2\hat{j}+3z^2\hat{k}$ N is acting on a particle. Find the work done by this force in displacing the body from (1, 2, 3) m to (3, 6, 1) m.

(a) –10 J (b) 100 J

(c) 10 J (d) 1 J

5. A force $\vec{F}=\left(3xy-5z\right)\hat{j}+4z\hat{k}$ is applied on a particle. The work done by the force when the particle moves from point (0, 0, 0) to point (2, 4, 0) as shown in the figure is



(a) $\frac{280}{5}$ (b) $\frac{140}{5}$

(c) $\frac{232}{5}$ (d) $\frac{192}{5}$

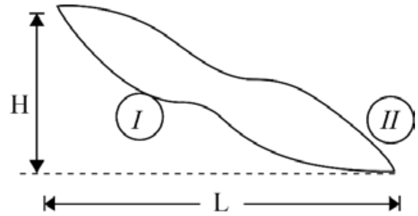
6. If a person is pushing a box inside a moving train with a force $\vec{F}$, the work done in the frame of the earth will be:

(a) $\vec{F}.\vec{s}_0$ (b) $\vec{F}.\vec{s}$

(c) $\vec{F}.\left(\vec{s}+\vec{s}_0\right)$ (d) Zero

(where $\vec{s}$ is the displacement of the box in the train and $\vec{s}_0$ is the displacement of the train relative to the ground.)

7. A body is lifted over route I and then route II such that force is always tangent to the path. Coefficient of friction is same for both the paths. Work done



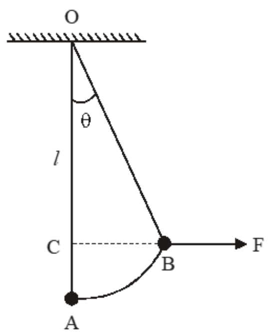
(a) on both the routes is same

(b) on route I is more

(c) on route II is more

(d) on both the routes is zero

8. An object of mass m is tied to a string of length *l* and a variable horizontal force is applied on it, which starts at zero and gradually increases (it is pulled extremely slowly so that equilibrium exists at all times) until the string makes an angle θ with the vertical. Work done by the force F is:



(a) mg*l* (1 – sin θ) (b) mg*l*

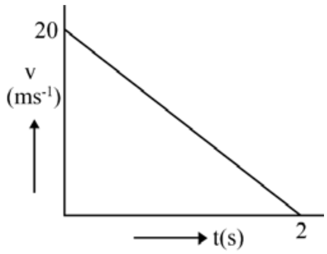
(c) mg*l* (1 – cos θ) (d) mg*l* (1 – tan θ)

9. A particle moves along the x–axis from x= x_1 to x = x_2 under the influence of a force given by F = 2x. Work done in the process is

(a) zero (b) $x_2^2 - x_1^2$

(c) $2x_2\,(x_2 - x_1)$ (d) $2x_1\,(x_1 - x_2)$

10. Velocity time graph of a particle of mass 2 kg moving in a straight line is as shown in figure. Work done by all forces on the particle is:



(a) 400 J (b) –400 J

(c) –200 J (d) 200 J

11. Work done by a conservative force on a system is equal to

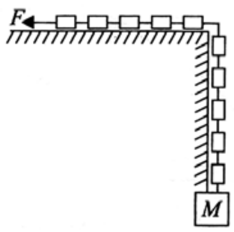
(a) the change in kinetic energy of the system

(b) the change in potential energy of the system

(c) the change in total mechanical energy of the system

(d) none of the above

12. A chain (uniform) of mass m and length ℓ has a small block of mass M attached to one of its ends and hangs from the surface of a table, with one-third its length resting on it. It is pulled by a constant horizontal force to lift the block slowly to the table surface. The work done by the force is



(a) $(M + m)g\left(\frac{2\ell}{3}\right)$ (b) $\left(M+\frac{m}{3}\right)\left(\frac{g\ell}{3}\right)$

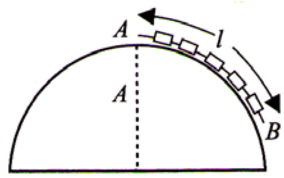
(c) $(M+m)\left(\frac{g\ell}{3}\right)$ (d) $\left(M+\frac{m}{3}\right)\left(\frac{2g\ell}{3}\right)$

13. A particle located in a one-dimensional potential field has its potential energy function as U(x) $=\frac{a}{x^4}-\frac{b}{x^2}$ where a and b are positive constants. The position of equilibrium x-corresponds to

(a) $\frac{b}{2a}$ (b) $\sqrt{\frac{2a}{b}}$

(c) $\sqrt{\frac{2b}{a}}$ (d) $\frac{a}{2b}$

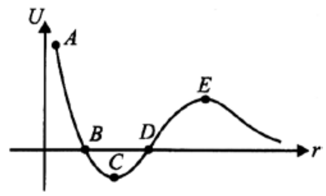
14. A uniform chain AB of mass m and length ℓ is placed with one end A at the highest point of a hemisphere of radius R. Referring to the top of the hemisphere as the datum level, the potential energy of the chain is (given that $\ell < \frac{\pi R}{2}$)



(a) $\frac{mR^2 g}{\ell}\left(\frac{\ell}{R}-\sin\frac{\ell}{R}\right)$ (b) $\frac{mR^2 g}{2\ell}\left(\frac{\ell}{R}-\sin\frac{\ell}{R}\right)$

(c) $\frac{mR^2 g}{2\ell}\left(\sin\frac{\ell}{R}-\frac{\ell}{R}\right)$ (d) $-\frac{mR^2 g}{\ell}\left(\frac{\ell}{R}-\sin\frac{\ell}{R}\right)$

15. The given plot shows the variation of U, the potential energy of interaction between two particles, with the distance separating them, r



1. B and D are equilibrium points.
2. C is a point of stable equilibrium.
3. The force of interaction between the two particles is attractive between points C and B, and repulsive between points D and E on the curve.
4. The force of interaction between the particles is repulsive between points C and A.

Which of the above statements are correct?

(a) 1 and 3 (b) 1 and 4

(c) 2 and 4 (d) 2 and 3

16. The potential energy function associated with the force $\vec{F} = 4xy\,\hat{i} + 2x^2\,\hat{j}$ is

(a) U = $-2x^2y$ (b) U = $-4x^2y$ + constant

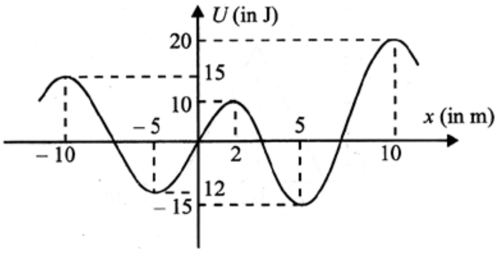
(c) U = $2x^2y$ + constant (d) not defined

17. The potential energy for a force field is given by U (x, y) = cos (x + y). The force acting on a particle at position given by coordinates $\left(0, \frac{\pi}{4}\right)$ is

(a) $-\frac{1}{\sqrt{2}}\left(\hat{i}+\hat{j}\right)$ (b) $\frac{1}{\sqrt{2}}\left(\hat{i}+\hat{j}\right)$

(c) $\left(\frac{1}{2}\hat{i}+\frac{\sqrt{3}}{2}\hat{j}\right)$ (d) $\left(\frac{1}{2}\hat{i}-\frac{\sqrt{3}}{2}\hat{j}\right)$

18. In the given figure the variation of potential energy of a particle of mass m = 2 kg is represented w.r.t. its x-coordinate. The particle moves under the effect of the conservative force along the x-axis. which of the following statements is incorrect about the particle?



(a) If it is released at the origin, it will move in negative x-axis

(b) If it is released at x = 2 + Δ where Δ → 0, then its maximum speed will be 5 m/s and it will perform oscillatory motion

(c) If initially x = –10 and then it will cross x = 10

(d) x = –5 and x = +5 are unstable equilibrium positions of the particle

19. The potential energy for a body of mass m that is acted on by a very massive body is given by $U = -mgx + \frac{kx^3}{3}$. The corresponding force is

(a) – mg + kx^2 (b) mg – kx^2

(c) mg – kx (d) – mg + kx

20. A ball is released from the top of a tower. The ratio of work done by force of gravity in first, second and third second of the motion of ball is

(a) 1 : 2 : 3 (b) 1 : 4 : 16

(c) 1 : 3 : 5 (d) 1 : 9 : 25

21. A man throws the bricks to a height of 12 m where they reach with a speed of 12 m/s. If he throws the bricks such that they just reach that height, what percentage of energy will be saved (g = 9.8 m/s^2)

(a) 29% (b) 46%

(c) 37.5% (d) 50%

22. A man raises 1 kg wt. to a height of 100 cm and holds it there for 30 minutes. How much work has he performed?

(a) 1 × 9.8 J (b) 1 × 9.8 × 30 × 60 J

(c) 1 × 9.8 × 30 J (d) 1 × 9.8 × 30 erg.

23. A ball is dropped from a height of 20 cm. Ball rebounds to a height of 10 cm. What is the loss of energy?

(a) 25% (b) 75%

(c) 50% (d) 100%

24. A machine, which is 75% efficient, uses 12 J of energy in lifting up a 1 kg mass through a certain distance. The mass is then allowed to fall through that distance. the velocity at the end of its fall is (in ms^{-1})

(a) $\sqrt{24}$ (b) $\sqrt{32}$

(c) $\sqrt{18}$ (d) $\sqrt{9}$

25. A rope ladder with a length l carrying a man with a mass m at its end is attached to the basket of balloon with a mass M. The entire system is in equilibrium in the air. As the man climbs up the ladder into the balloon, the balloon descends by a height h. Then the potential energy of the man

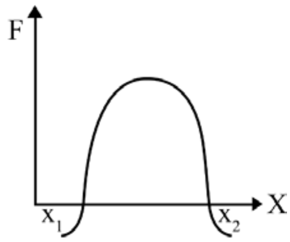
(a) increases by mg (l – h)

(b) increases by mg$l$

(c) increase by mgh

(d) increases by mg (2l – h)

26. The force acting on a body moving along x–axis varies with the position of the particle as shown in the figure. The body is in stable equilibrium at:



(a) x = x_1 (b) x = x_2

(c) both x_1 and x_2 (d) neither x_1 nor x_2

27. If the speed of a vehicle increases by 2 m/s, its K.E. is doubled. The original speed of the vehicle was

(a) $\left(\sqrt{2}+1\right) m/s$ (b) $\sqrt{2}\left(\sqrt{2}+1\right) m/s$

(c) $2\left(\sqrt{2}+1\right) m/s$ (d) $\sqrt{2}\ m/s.$

28. A particle moves in a straight line with retardation proportional to its displacement. Its loss of KE for any displacement x is proportional to

(a) x (b) x^2

(c) x^0 (d) e^x

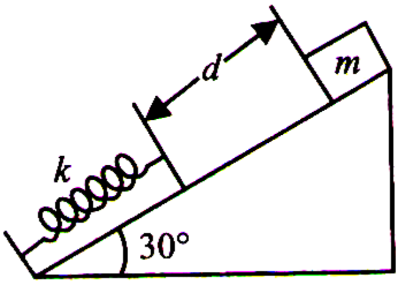
29. An engine pumps water continuously through a hole. Speed with which water passes through the hole nozzle is v and k is the mass per unit length of the water jet as it leaves the nozzle. Find the rate at which kinetic energy is being imparted to the water.

(a) $\frac{1}{2}kv^2$ (b) $\frac{1}{2}kv^3$

(c) $\frac{v^2}{2k}$ (d) $\frac{v^3}{2k}$

30. A block of 4 kg mass starts at rest and slides a distance d down a frictionless incline (angle 30°) where it runs into a spring of negligible mass. The block slides an additional 25 cm before it is brought to rest momentarily by compressing the spring. The force constant of spring is 400 N/m. the value of d is then

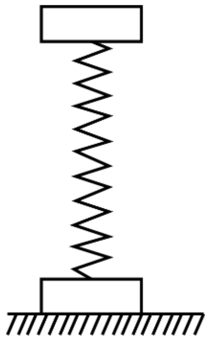
(take g = 10 ms^{-2})



(a) 25 cm (b) 37.5 cm

(c) 62.5 cm (d) none of the above

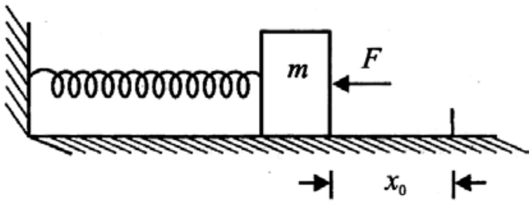
31. Two discs, each having mass m, are attached rigidly to the ends of a vertical spring. One of the discs rests on a horizontal surface and the other produces a compression x_0 on the spring when it is in equilibrium. How much further must the spring be compressed so that when the force causing compression is removed, the extension of the spring will be able to lift the lower disc off the table



(a) x_0 (b) $2x_0$

(c) $3x_0$ (d) $1.5x_0$

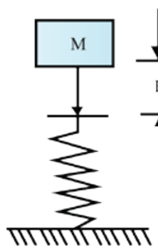
32. A spring of stiffness k is kept compressed by applying horizontal force on m by a length x_0 $\left(=\frac{mg}{k}\right)$. If the force F is withdrawn suddenly, the block oscillates and finally stops. In consequence, frictional loss is equal to 50% of the initial potential energy stored in the spring. The coefficient of friction the between block and the ground is



(a) $\frac{1}{2}$ (b) $\frac{2}{3}$

(c) $\frac{1}{\sqrt{2}}$ (d) $\frac{1}{\sqrt{3}}$

33. A mass m is allowed to fall on a pedestal fixed on the top of a vertical spring. If the height of the mass was H from the pedestal and the compression of the spring is d then the spring's force factor is given by

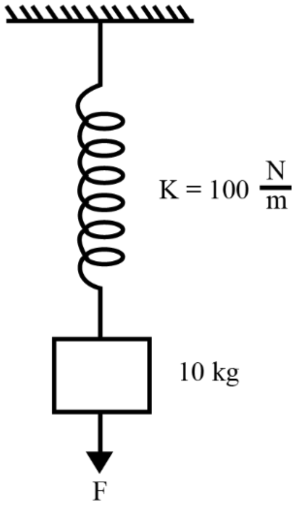


(a) $Mg\frac{(H+d)}{d^2}$ (b) $2Mg\frac{(H-d)}{d^2}$

(c) $\frac{Mg}{2}\frac{H}{d^2}$ (d) $2Mg\frac{(H+d)}{d^2}$

34. A vertical spring of force constant 100 N/m is attached with a hanging mass of 10 kg. Now an external force is applied on the mass so that the spring is stretched by additional 2 m. The work done by the force F is

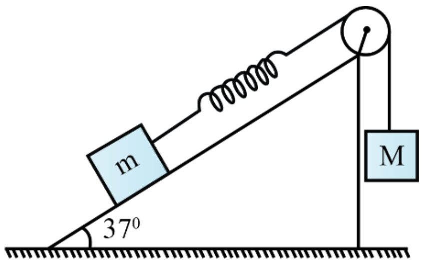
($g = 10\ m/s^2$)



(a) 200 J (b) 400 J

(c) 450 J (d) 600 J

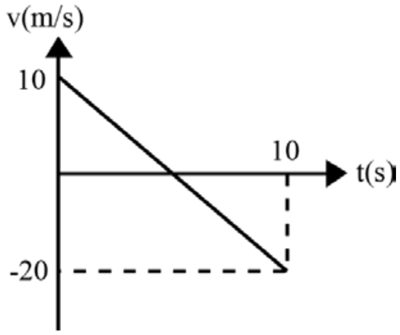
35. A block of mass m is attached with a massless spring of force constant k. The block is placed over a rough inclined surface for which the coefficient of friction is $\mu = 3/4$. The minimum value of M required to move the block up the plane is (Neglect mass of string and pulley and friction in pulley)



(a) $\frac{3}{5}m$ (b) $\frac{4}{5}m$

(c) $\frac{6}{5}m$ (d) $\frac{3}{2}m$

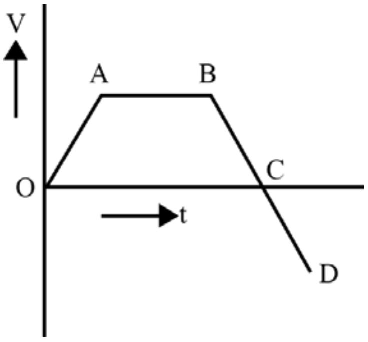
36. Velocity-time graph of a particle moving in a straight line is as shown in figure. Mass of the particle is 2 kg. Work done by all the forces acting on the particle in time interval between t = 0 to t = 10 s is



(a) 300 J (b) –300 J

(c) 400 J (d) –400 J

37. A plot of velocity versus time is shown in figure. A single force acts on the body. The correct statement is:



(a) in moving from C to D, work done by the force on the body is positive.

(b) in moving from B to C, work done by the force on the body is positive.

(c) in moving from A to B, the body does work on the system.

(d) in moving from O to A, work is done by the body and is negative.

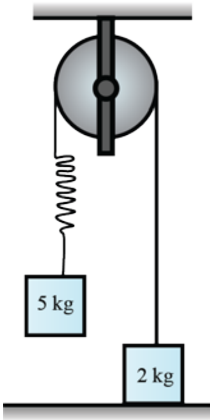
38. A mass-spring system oscillates such that the mass moves on a rough surface having coefficient of friction μ. It is compressed by a distance a from its normal length and, on being released, it moves to a distance b from its equilibrium position. The decrease in amplitude for one half-cycle (–a to b) is:

(a) $\frac{\mu mg}{k}$ (b) $\frac{2\mu mg}{k}$

(c) $\frac{\mu g}{k}$ (d) $\frac{k}{\mu mg}$

39. System shown in figure is released from rest. Pulley and spring is massless, and friction is absent everywhere. The speed of 5 kg block when 2 kg block leaves the contact with ground is:

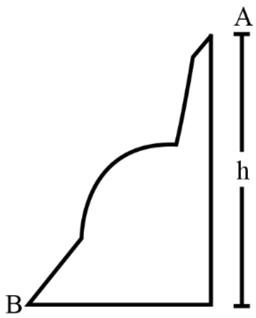
(Take force constant of spring k = 40 N/m and g = 10 m/s^2)



(a) $\sqrt{2}$ m/s (b) $2\sqrt{2}\,m/s$

(c) 2 m/s (d) $4\sqrt{2}\,m/s$

40. In the given curved road, if particle is released from A then



(a) kinetic energy at B must be mgh

(b) kinetic energy at B may be zero

(c) kinetic energy at B must be less than mgh

(d) kinetic energy at B must not be equal to zero

41. A uniform flexible chain of mass m and length 2l hangs in equilibrium over a smooth horizontal pin of negligible diameter. One end of the chain is given a small vertical displacement so that the chain slips over the pin. The speed of chain when it leaves pin is:

(a) $\sqrt{2g\ell}$ (b) $\sqrt{g\ell}$

(c) $\sqrt{4g\ell}$ (d) $\sqrt{3g\ell}$

42. A meter stick of mass 400 g is pivoted at one end and displaced through an angle 60°. The increase in its potential energy is:

(a) 1 J (b) 10 J

(c) 100 J (d) 1000 J

43. A uniform chain has a mass M and length L. It is placed on a frictionless table with length l_0 hanging over the edge. The chain begins to slide down. Then, the speed v with which the end slides down from the edge is given by:

(a) $v=\sqrt{\frac{g}{L}(L+l_0)}$ (b) $v=\sqrt{\frac{g}{L}(L-l_0)}$

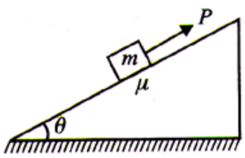
(c) $v=\sqrt{\frac{g}{L}(L^2-l_0^2)}$ (d) $v=\sqrt{2g(L-l_0)}$

44. A block of mass m is moving with a constant acceleration ‘a’ on a rough plane. If the coefficient of friction between the block and the ground is μ, the power delivered by the external agent after a time t from the beginning is equal to

(a) ma^2t (b) μmgat

(c) μm(a + μg)gt (d) m(a + μg)at

45. A block of mass m is being pulled up the rough incline by an agent delivering constant power P. The coefficient of friction between the block and the incline is μ. The maximum speed of the block during the course of ascent is



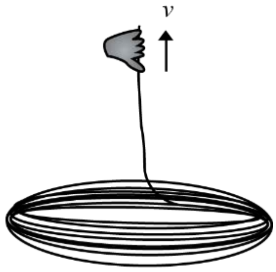
(a) $v=\dfrac{P}{mg\sin\theta+\mu mg\cos\theta}$

(b) $v=\dfrac{P}{mg\sin\theta-\mu mg\cos\theta}$

(c) $v=\dfrac{2P}{mg\sin\theta-\mu mg\cos\theta}$

(d) $v=\dfrac{3P}{mg\sin\theta-\mu mg\cos\theta}$

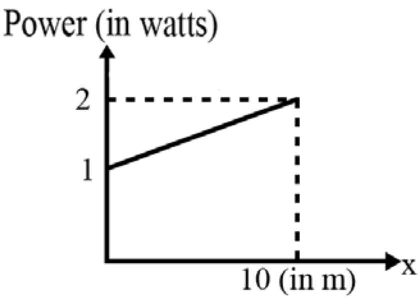
46. A uniform rope of linear mass density λ and length ℓ is coiled on a smooth horizontal surface. One end is pulled up with constant velocity v. Then the average power applied by the external agent in pulling the entire rope just off the ground is



(a) $\frac{1}{2}\lambda\ell v^2+\frac{\lambda\ell^2 g}{2}$ (b) $\lambda\ell gv$

(c) $\frac{1}{2}\lambda v^3+\frac{\lambda\ell vg}{2}$ (d) $\lambda\ell gv+\frac{1}{2}\lambda v^3$

47. A particle A of mass 10/7 kg is moving in the positive direction of x. Its initial position is x = 0 and initial velocity at x = 0 is 1m/s. Velocity at x = 10m is



(a) 4 m/s (b) 2 m/s

(c) $3\sqrt{2}$ m/s (d) $\frac{100}{3}$ m/s

48. Power supplied to a particle of mass 2 kg varies with time as $P = 3t^2/2$ watt, here t is in second. Velocity of particle at t = 0 is u = 0. The velocity of particle at time t = 2 s will be

(a) 1 m/s (b) 4 m/s

(c) 2 m/s (d) $2\sqrt{2}\ m/s$

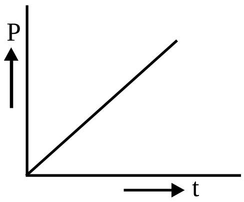
49. A constant power P is applied to a particle of mass m. The distance travelled by the particle when its velocity increases from v_1 to v_2 is (neglect friction)

(a) $\frac{3P}{m}\left(v_2^2-v_1^2\right)$ (b) $\frac{m}{3P}\left(v_2-v_1\right)$

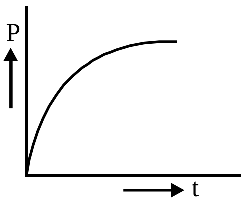
(c) $\frac{m}{3P}\left(v_2^3-v_1^3\right)$ (d) $\frac{m}{3P}\left(v_2^2-v_1^2\right)$

50. A motor drives a body along a straight line with a constant force. The power P developed by the motor must vary with time t as shown in figure.

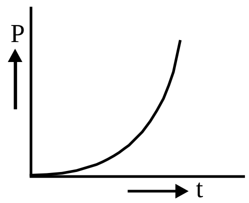
(a)



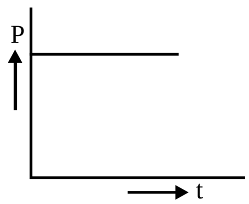
(b)



(c)



(d)



51. A dam is situated at a height of 550 m above sea level and supplies water to a powerhouse which is at a height of 50m above sea level. 2000 kg of water passes through the turbines per second. What would be the maximum electrical power output of the power house if the whole system were 80% efficient?

(a) 8 MW (b) 10 MW

(c) 12.5 MV (d) 16 MV

52. A 500 kg car, moving with a velocity of 36 km h^{-1} on a straight road unidirectionally, doubles its velocity in one minute. The power delivered by the engine for doubling the velocity is

(a) 750 W (b) 1050 W

(c) 1150 W (d) 1250 W

53. The power of a water jet flowing through an orifice of radius r with velocity v is

(a) zero (b) $500\ \pi r^2 v^2$

(c) $500\ \pi r^2 v^3$ (d) $\pi r^4 v$

54. A force F acting on a body depends on its displacement S as $F \propto S^{-1/3}$. The power delivered by F will depend on displacement as:

(a) $S^{2/3}$ (b) $S^{-5/3}$

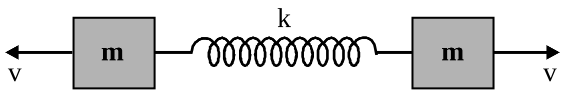
(c) $S^{1/2}$ (d) S^0

55. A pendulum of mass 1 kg and length ℓ = 1 m is released from rest at angle $\theta = 60°$. The power delivered by all the forces acting on the bob at angle $\theta = 30°$ will be: ($g = 10\ m/s^2$)

(a) 13.5 W (b) 20.4 W

(c) 24.6 W (d) zero

56. A bob of mass m accelerates uniformly from rest to v_1 in time t_1. As a function of t, the instantaneous power delivered to the body is



(a) $\frac{mv_1t}{t_2}$ (b) $\frac{mv_1t}{t_1}$

(c) $\frac{mv_1t^2}{t_1}$ (d) $\frac{mv_1^2t}{t_1^2}$

57. The potential energy of a particle of mass 1 kg is, $U = 10 + (x - 2)^2$. Here, U is in joules and x in metres on the positive x-axis. Particle travels up to x = +6 m. Choose the correct statement:

(a) On negative x-axis particle travels up to x = – 2m

(b) The maximum kinetic energy of the particle is 16 J

(c) Both (a) and (b) are correct

(d) Both (a) and (b) are wrong

58. The potential Energy as a function of the force between two atoms in a diatomic molecule is given by $U(x) = \frac{a}{x^{12}} - \frac{b}{x^6}$, where a and b are positive constants and x is the distance between the atoms. The position of stable equilibrium for the system of the two atoms is given as:

(a) $x = \frac{a}{b}$ (b) $x = \sqrt{\frac{a}{b}}$

(c) $x = \frac{\sqrt{3a}}{b}$ (d) $x = \sqrt[6]{\left(\frac{2a}{b}\right)}$

Objective Questions II

[One or more than one correct option]

59. Work done by force of friction

(a) can be zero (b) can be positive

(c) can be negative (d) any of these

60. When work done by force of gravity is negative,

(a) PE increases (b) KE decreases

(c) PE remains constant (d) PE decreases

61. Which of the following may or may not be conserved?

(a) Energy (b) Potential energy

(c) Mechanical energy (d) Kinetic energy

62. Internal forces can change

(a) kinetic energy (b) mechanical energy

(c) energy (d) momentum

63. In which of the following cases, no work is done by the force:

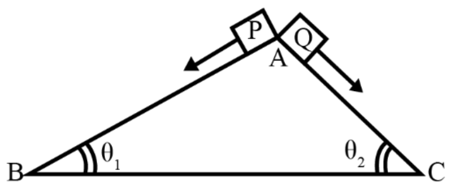
(a) A man carrying a bucket of water, walking on a level road with a uniform velocity

(b) A drop of rain falling vertically with a constant velocity

(c) A man whirling a stone tied to a string in a circle with a constant speed.

(d) A man walking up on a staircase

64. Two inclined frictionless tracks of different inclinations meet at A from where two blocks P and Q of different masses are allowed to slide down from rest at the same time, on each track, as shown in the figure. Then [$given: \theta_2 > \theta_1$]



(a) both blocks will reach the bottom at the same time

(b) block Q will reach the bottom earlier than block P

(c) both blocks will reach the bottom with the same speed

(d) block Q will reach the bottom with a higher speed than block P.

65. A body of mass m is moving in a straight line at a constant speed v. Its kinetic energy is K and the magnitude of its momentum is p. Which of the following relations are correct?

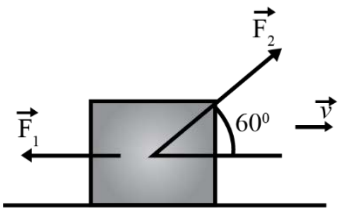
(a) p = $\sqrt{2mK}$ (b) p = $\sqrt{\frac{2K}{m}}$

(c) 2K= pv (d) v = $\sqrt{\frac{2K}{p}}$

Numerical Value Type Questions

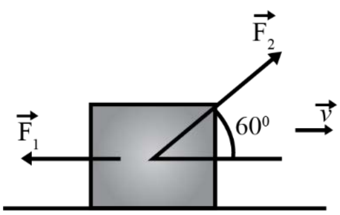
66. A projectile is thrown with initial velocity U at an angle θ to the horizontal. Its velocity when it is at the highest point is $\sqrt{\frac{2}{5}}$ times the velocity when it is at height half of the maximum height. The angle of projection θ with horizontal is π/*. What is the value *?

67. Two constant forces $\vec{F}_1$ and $\vec{F}_2$ are acting on a bock as shown. The magnitude of the force $\vec{F}_1$ is 2 N and that of $\vec{F}_2$ is 4 N. The velocity of the block at a certain instant is 3.0 m/sec.



Find the power(in watt) due to each force and the net power.

68. Two constant forces $\vec{F}_1$ and $\vec{F}_2$ are acting on a bock as shown. The magnitude of the force $\vec{F}_1$ is 2 N and that of $\vec{F}_2$ is 4 N. The velocity of the block at a certain instant is 3.0 m/sec.



If the magnitude of the force $\vec{F}_2$ is increased to 6 N, what is the net power (in watt) at this instant?

69. A body of 0.2 kg is suspended through a spring, so that the spring is stretched by 1.0 cm at equilibrium. A particle of mass 0.12 kg is slowly dropped on the body after the impact. Find the maximum extension of the spring (in cm). $[g = 10ms^{-2}]$

Assertion & Reason

For the following questions choose the correct answer from the codes (A), (B), (C) and (D) defined as follows.

(A) If both Assertion and Reason are true and the Reason is correct explanation of the Assertion.

(B) If both Assertion and Reason are true but Reason is not correct explanation of the Assertion.

(C) If Assertion is true but Reason is false.

(D) If Assertion is false but Reason is true.

70. **Assertion:** Stopping distance $= \frac{\text{Kinetic energy}}{\text{Stopping force}}$

Reason: Work done in stopping a body is equal to KE of the body.

(a) A (b) B

(c) C (d) D

71. **Assertion:** Two springs of force constants k_1 and k_2 are stretched by the same force. If $k_1 > k_2$, then work done in stretching the first spring (W_1) is less than work done in stretching the second spring (W_2).

Reason: $F = k_1x_1 = k_2x_2$

$$\therefore \frac{x_1}{x_2} = \frac{k_2}{k_1}$$

$$\frac{W_1}{W_2} = \frac{\frac{1}{2}k_1x_1^2}{\frac{1}{2}k_2x_2^2} = \frac{k_1}{k_2}\left(\frac{k_2}{k_1}\right)^2 = \frac{k_2}{k_1}$$

As $k_1 > k_2$, $W_1 < W_2$

(a) A (b) B

(c) C (d) D

72. **Assertion:** A weightlifter does not work in holding the weight up.

Reason: Work done is zero because distance moved is zero.

(a) A (b) B

(c) C (d) D

73. **Assertion:** Mass and energy are not conserved separately but are conserved as a single entity 'mass-energy'.

Reason: This is because one can be obtained at the cost of the other as per Einstein equation.

$E = mc^2$

(a) A (b) B

(c) C (d) D

74. **Assertion:** Energy released when a mass of one microgram disappears in a process is 9×10^7 J.

Reason: It follows from $E = \frac{1}{2}mv^2$.

(a) A (b) B

(c) C (d) D

75. **Assertion:** In a circular motion, work done by centripetal force is not zero always.

Reason: If speed of the particle increases or decreases in circular motion, net force acting on the particle does not remain towards centre.

(a) A (b) B

(c) C (d) D

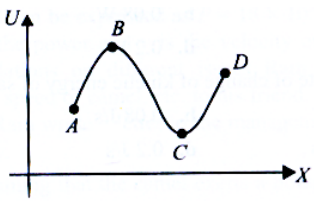
76. **Assertion:** Under the action of a conservative force of constant magnitude, work done is path independent.

Reason: Work done by force of gravity is path independent only near the surface of Earth.

(a) A (b) B

(c) C (d) D

77. **Assertion:** The potential energy of a particle varies with distance x as shown in the graph.



The force acting on the particle is zero at point B and C.

Reason: The slope of the U-x curve is zero at point B and C.

(a) A (b) B

(c) C (d) D

Match the following

Each question has two columns. Four options are given representing matching of elements from Column-I and Column-II. Only one of these four options corresponds to a correct matching.

For each question, choose the option corresponding to the correct matching.

78. A man pushes a block of 30 kg along a level floor at a constant speed with a force directed at 45° below the horizontal. If the coefficient of friction is 0.20, then match the following: $[g = 10ms^{-2}]$

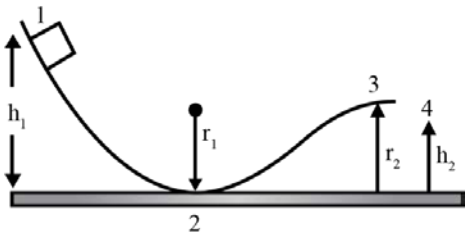
Column I	**Column II**
(a) Work done by all forces exerted by the surface on the block in 20 m	(p) zero
(b) Work done by the force of gravity	(q) 1500 J
(c) Work done by the man on the block in pushing it through 10 m	(r) 750 J
(d) Net force on the block	(s) -1500 J

Paragraph Type Questions

Using the following passage, solve Q. 79 to 80

Passage

A block of mass m is released from a height h_1 along a smooth track as shown in the figure.



79. Determine the force exerted on the block by the track at point 2, where radius of curvature is r_1.

(a) $mg + \frac{m(2gh_1)}{r_1}$ (b) $2mg + \frac{m(2gh_1)}{r_1}$

(c) $\frac{mg}{2} + \frac{m(2gh_1)}{r_1}$ (d) $mg + \frac{m(gh_1)}{r_1}$

80. Determine the minimum safe value of radius of curvature at point 3, so that the block does not fly off the track.

(a) $(h_1 - h_2)$ (b) $2(h_1 - h_2)$

(c) $\frac{(h_1 - h_2)}{3}$ (d) $\frac{2(h_1 - h_2)}{3}$

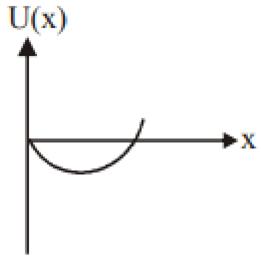
EXERCISE – 4: PREVIOUS YEARS JEE ADVANCED QUESTIONS

Objective Questions I [Only one correct option]

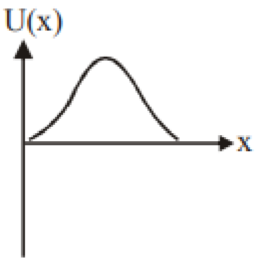
1. A particle, which is constrained to move along x-axis, is subjected to a force in the same direction which varies with the distance x of the particle from the origin as $F(x) = -kx + ax^3$. Here, k and a are positive constant. For $x \geq 0$, the functional form of the potential energy $U(x)$ of the particle is:

(2002)

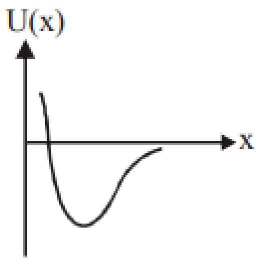
(a)



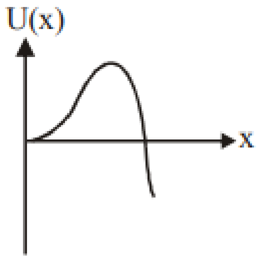
(b)



(c)



(d)



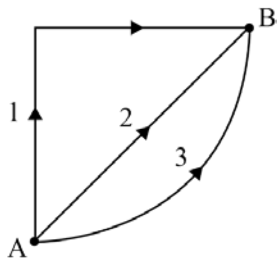
2. An ideal spring with spring-constant k is hung from the ceiling and a block of mass M is attached to its lower end. The mass is released with the spring initially unstretched. Then the maximum extension in the spring is:

(2002)

(a) $\frac{4Mg}{k}$ (b) $\frac{2Mg}{k}$

(c) $\frac{Mg}{k}$ (d) $\frac{Mg}{2k}$

3. If W_1, W_2 and W_3 represent the work done in moving a particle from A to B along three different paths 1, 2 and 3 respectively (as shown) in the gravitational field of a point mass m. Find the correct relation between W_1, W_2 and W_3 : **(2003)**



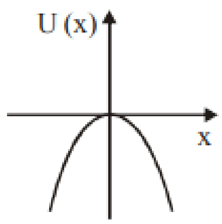
(a) $W_1 > W_2 > W_3$ (b) $W_1 = W_2 = W_3$

(c) $W_1 < W_2 < W_3$ (d) $W_2 > W_1 > W_3$

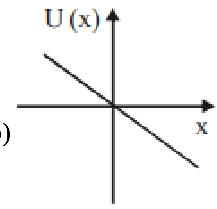
4. A particle is placed at the origin and a force $F = kx$ is acting on it (where k is a positive constant). If $U(0) = 0$, the graph of $U(x)$ versus x will be (where U is the potential energy function):

(2004)

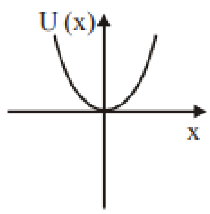
(a)



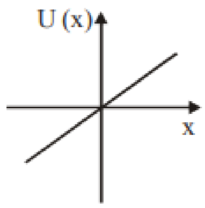
(b)



(c)

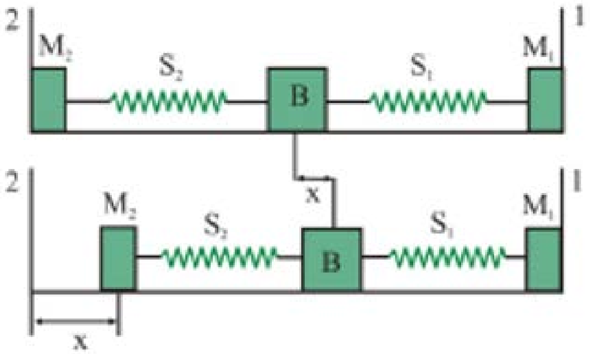


(d)



5. A block (B) is attached to two unstretched springs S_1 and S_2 with spring constants k and 4k, respectively. The other ends are attached to two supports M_1 and M_2 not attached to the walls. The springs and supports have negligible mass. There is no friction anywhere. The block B is displaced towards wall 1 by a small distance x and released. The block returns and moves a maximum distance y towards wall 2. Displacements x and y are measured with respect to the equilibrium position of the block B. The ratio $\frac{y}{x}$ is

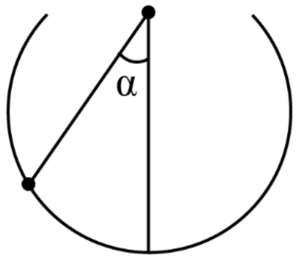
(2008)



(a) 4 (b) 2

(c) $\frac{1}{2}$ (d) $\frac{1}{4}$

6. An insect crawls up a hemispherical surface very slowly (see figure) The coefficient of friction between the insect and the surface is $\frac{1}{3}$. If the line joining the centre of the hemispherical surface to the insect makes an angle α with the vertical, the maximum possible value of α is given by:



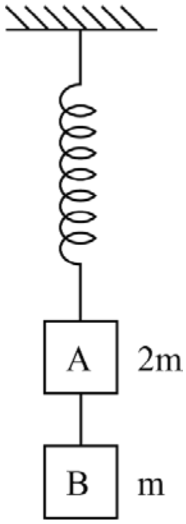
(2009)

(a) $\cot\alpha = 3$ (b) $\tan\alpha = 3$

(c) $\sec\alpha = 3$ (d) $\text{cosec}\ \alpha = 3$

7. Two blocks A and B of masses 2m and m, respectively are connected by a massless and inextensible string. The whole system is suspended by a massless spring as shown in the fig. The magnitudes of acceleration of A and B, immediately after the string is cut, are respectively

(2009)

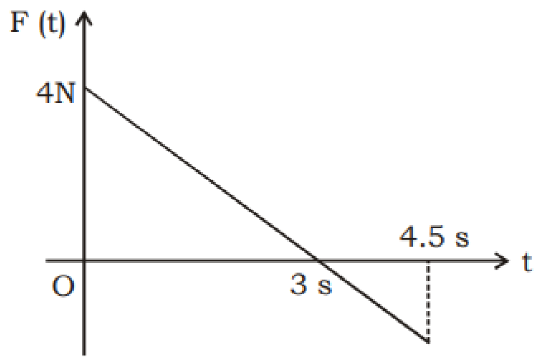


(a) $g, \frac{g}{2}$ (b) $\frac{g}{2}, g$

(c) g, g (d) $\frac{g}{2}, \frac{g}{2}$

8. A block of mass 2kg is free to move along the x–axis. It is at rest and from $t = 0$ onwards it is subjected to a time-dependent force F(t) in the x direction. The force F(t) varies with t as shown in the figure. The kinetic energy of the block after 4.5 seconds is

(2010)

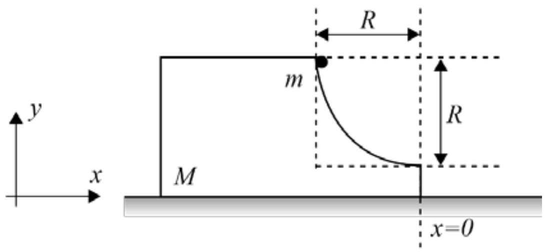


(a) 4.50 J (b) 7.50 J

(c) 5.06 J (d) 14.06 J

Objective Questions II
[One or more than one correct option]

9. A block of mass M has a circular cut with a frictionless surface as shown. The block rests on the horizontal frictionless surface of a fixed table. Initially the right edge of the block is at $x = 0$, in a co-ordinate system fixed to the table. A point mass m is released from rest at the topmost point of the path as shown and it slides down. When the mass loses contact with the block, its position is x and the velocity is *v*. At that instant, which of the following options is/are correct? **(2017)**



(a) The velocity of the point mass m is: $v = \sqrt{\dfrac{2gR}{1+\dfrac{m}{M}}}$

(b) The velocity of the block M is: $V = -\dfrac{m}{M}\sqrt{2gR}$

(c) The position of the point mass is $x = -\sqrt{2}\dfrac{mR}{M+m}$

(d) The x component of displacement of the centre of mass of the block M is: $-\dfrac{mR}{M+m}$

10. A particle of mass m is initially at rest is at the origin. It is subjected to a force and starts moving along the x-axis. Its kinetic energy K changes with time as $\dfrac{dK}{dt} = \gamma t$, where γ is a positive constant of appropriate dimensions. Which of the following statements is (are) true? **(2018)**

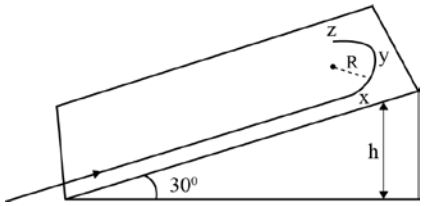
(a) The force applied on the particle is constant

(b) The speed of the particle is proportional to time

(c) The distance of the particle from the origin increases linearly with time

(d) The force is conservative

11. A student skates up a ramp that makes an angle 30º with the horizontal. He/she starts (as shown in the figure) at the bottom of the ramp with speed v_0 and wants to turn around over a semi-circular path xyz of radius R during which he/she reaches a maximum height h (at point y) from the ground as shown in the figure. Assume that the energy loss is negligible, and the force required for this turn at the highest point is provided by his/her weight only. Then (g is the acceleration due to gravity) **(2020)**



(a) $v_0^2 - 2gh = \dfrac{1}{2}gR$

(b) $v_0^2 - 2gh = \dfrac{\sqrt{3}}{2}gR$

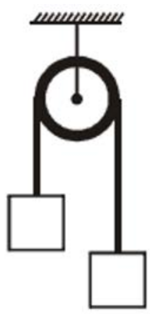
(c) the centripetal force required at points x and z is zero

(d) the centripetal force required is maximum at points x and z

Numerical Value Type Questions

12. A bullet is fired at a target. Its velocity is decreased by 50% after penetrating 21 cm into the target. Find the additional thickness (in cm) that the bullet will penetrate before coming to rest. **(2008)**

13. A light inextensible string that goes over a smooth fixed pulley as shown in the figure connects two blocks of masses 0.36 and 0.72 kg. Taking $g = 10 ms^{-2}$, find the work done (in joules) by the string on the block of mass 0.36 kg during the first second after the system is released from rest. **(2009)**

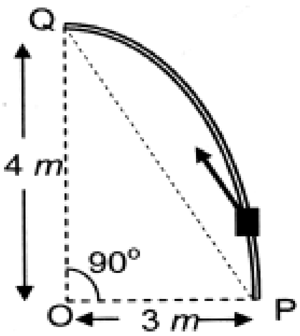


14. A bob of mass m, suspended by a string of length l_1, is given a minimum velocity required to complete a full circle in the vertical plane. At the highest point, it collides elastically with another bob of mass m suspended by a string of length l_2, which is initially at rest. Both the strings are massless and inextensible. If the second bob, after collision acquires the minimum speed required to complete a full circle in the vertical plane, the ratio $\frac{l_1}{l_2}$ is **(2013)**

15. A particle of mass 0.2 kg is moving in one dimension under a force that delivers a constant power 0.5 W to the particle. If the initial speed (in ms^{-1}) of the particle is zero, the speed (in ms^{-1}) after 5s is

(2013)

16. Consider an elliptically shaped rail PQ in the vertical plane with OP =3m and OQ = 4m. A block of mass 1 kg is pulled along the rail from P to Q with a force of 18 N, which is always parallel to line PQ (see the figure given) Assuming no frictional losses, the kinetic energy of the block when it reaches Q is (n×10) Joules. The value of n is (take acceleration due to gravity = 10 ms^{-2}) **(2014)**



Assertion & Reason

For the following questions choose the correct answer from the codes (A), (B), (C) and (D) defined as follows.

(A) Statement I is true, Statement II is also true; Statement II is the correct explanation of Statement I.

(B) Statement I is true, Statement II is also true; Statement II is not the correct explanation of Statement I.

(C) Statement I is true, Statement II is false.

(D) Statement I is false, Statement II is true

17. **Statement–I:** A block of mass m starts moving on a rough horizontal surface with a velocity v. It stops due to friction between the block and the surface after moving through a certain distance. The surface is now tilted to an angle of 30° with the horizontal and the same block is made to go up on the surface with the same initial velocity v. The decrease in the mechanical energy in the second situation is smaller than that in the first situation.

Statement-II : The coefficient of friction between the block and the surface decreases with the increase in the angle of inclination. **(2015)**

(a) A (b) B

(c) C (d) D

Match the following

18. A particle of unit mass is moving along the x-axis under the influence of a force and its total energy is conserved. Four possible forms of the potential energy of the particle are given in column I (a and U_0 constant). Match the potential energies in column I to the corresponding statement(s) in column II.

(2015)

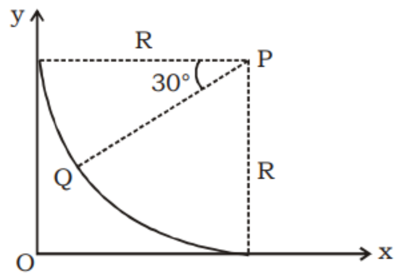
Column - I	Column - II
(A) $U_1(x)=\frac{U_0}{2}\left[1-\left(\frac{x}{a}\right)^2\right]^2$	(P) the force acting on the particle is zero at $x=a$
(B) $U_2(x)=\frac{U_0}{2}\left(\frac{x}{a}\right)^2$	(Q) The force acting on the particle is zero at $x=0$
(C) $U_3(x)=\frac{U_0}{2}\left(\frac{x}{a}\right)^2 e^{\left[-\left(\frac{x}{a}\right)^2\right]}$	(R) The force acting on the particle is zero at $x=-a$
(D) $U_4(x)=\frac{U_0}{4}\left[\frac{x}{a}-\frac{1}{3}\left(\frac{x}{a}\right)^3\right]$	(S) The particle experiences an attractive force towards $x=0$ in the region $\lvert x\rvert < a$.
	(T) The particle with total energy $\frac{U_0}{4}$ can the oscillate about the point $x=-a$

Paragraph Type Questions

Using the following passage, solve Q.19 to Q.20

Passage

A small block of mass 1 kg is released from rest at the top of a rough track. The track is a circular arc of radius 40m. The block slides along the track without toppling and a frictional force act on it in the direction opposite to the instantaneous velocity. The work done in overcoming the friction up to the point Q, as shown in the figure, is 150 J. (Take the acceleration due to gravity, $g=10\ ms^{-2}$).



19. The speed of the block when it reaches the point Q

(2013)

(a) $5\ ms^{-1}$ (b) $10\ ms^{-1}$

(c) $10\sqrt{3}\ ms^{-1}$ (d) $20\ ms^{-1}$

20. The magnitude of the normal reaction that acts on the block at the point Q is **(2013)**

(a) $7.5\ N$ (b) $8.6\ N$

(c) $11.5\ N$ (d) $22.5\ N$

Answer Key

CHAPTER -5 | WORK, ENERGY AND POWER

EXERCISE - 1 :
BASIC OBJECTIVE QUESTIONS

1. (b)	**2.** (c)	**3.** (a)	**4.** (d)
5. (a)	**6.** (a)	**7.** (c)	**8.** (d)
9. (b)	**10.** (b)	**11.** (b)	**12.** (d)
13. (b)	**14.** (c)	**15.** (c)	**16.** (d)
17. (b)	**18.** (d)	**19.** (c)	**20.** (c)
21. (b)	**22.** (c)	**23.** (c)	**24.** (c)
25. (c)	**26.** (b)	**27.** (c)	**28.** (a)
29. (b)	**30.** (b)	**31.** (b)	**32.** (b)
33. (a)	**34.** (a)	**35.** (b)	**36.** (c)
37. (d)	**38.** (b)	**39.** (d)	**40.** (a)
41. (b)	**42.** (b)	**43.** (b)	**44.** (b)
45. (a)	**46.** (a)	**47.** (a)	**48.** (b)
49. (a)	**50.** (a)	**51.** (c)	**52.** (d)
53. (b)	**54.** (c)	**55.** (b)	**56.** (b)
57. (c)	**58.** (b)	**59.** (b)	**60.** (b)
61. (d)	**62.** (a)	**63.** (a)	**64.** (b)
65. (b)	**66.** (a)	**67.** (c)	**68.** (c)
69. (b)	**70.** (a)	**71.** (b)	**72.** (c)
73. (a)	**74.** (c)	**75.** (b)	**76.** (c)
77. (a)	**78.** (d)	**79.** (a)	**80.** (b)

EXERCISE - 2 :
PREVIOUS YEARS JEE MAIN QUESTIONS

1. (b)	**2.** (a)	**3.** (d)	**4.** (c)
5. (a)	**6.** (b)	**7.** (a)	**8.** (d)
9. (d)	**10.** (d)	**11.** (b)	**12.** (d)
13. (d)	**14.** (d)	**15.** (d)	**16.** (a)
17. (a)	**18.** (b)	**19.** (b)	**20.** (a)
21. (3.00)	**22.** (18.00)	**23.** (5.00)	**24.** (1.00)
25. (2.00)	**26.** (10.00)	**27.** (6.00)	**28.** (c)
29. (b)	**30.** (d)		

EXERCISE - 3:
ADVANCED OBJECTIVE QUESTION

1. (a)	**2.** (c)	**3.** (b)	**4.** (a)
5. (d)	**6.** (c)	**7.** (a)	**8.** (c)
9. (b)	**10.** (b)	**11.** (b)	**12.** (d)
13. (b)	**14.** (d)	**15.** (c)	**16.** (b)
17. (b)	**18.** (d)	**19.** (b)	**20.** (c)
21. (c)	**22.** (a)	**23.** (c)	**24.** (c)
25. (a)	**26.** (b)	**27.** (c)	**28.** (b)
29. (b)	**30.** (b)	**31.** (b)	**32.** (c)
33. (d)	**34.** (a)	**35.** (a)	**36.** (a)
37. (a)	**38.** (b)	**39.** (b)	**40.** (b)
41. (b)	**42.** (a)	**43.** (c)	**44.** (d)
45. (a)	**46.** (c)	**47.** (a)	**48.** (c)
49. (c)	**50.** (a)	**51.** (a)	**52.** (d)
53. (c)	**54.** (d)	**55.** (a)	**56.** (d)
57. (c)	**58.** (d)	**59.** (a,b,c,d)	**60.** (a,b)
61. (b,c,d)	**62.** (a,b)	**63.** (a,b,c)	**64.** (b,c)
65. (a,c)	**66.** (3)	**67.** (0)	**68.** (3)
69. (1.2)	**70.** (a)	**71.** (a)	**72.** (a)
73. (a)	**74.** (c)	**75.** (d)	**76.** (d)
77. (a)	**78.** ($a \to s, b \to p, c \to r, d \to p$)		
79. (a)	**80.** (b)		

EXERCISE - 4:
PREVIOUS YEARS JEE ADVANCED QUESTIONS

1. (d)	**2.** (b)	**3.** (b)	**4.** (a)
5. (c)	**6.** (a)	**7.** (b)	**8.** (c)
9. (a,d)	**10.** (a,b,d)	**11.** (a,d)	**12.** (7)
13. (8)	**14.** (5)	**15.** (5)	**16.** (5)
17. (c)			
18. [$A \to P, Q, R, T; B \to Q, S; C \to P, Q, R, S; D \to P, R, T$]			
19. (b)	**20.** (a)		