22. ELECTROMAGNETIC INDUCTION AND ELECTROMAGNETIC WAVES

1. INTRODUCTION

The phenomenon of electromagnetic induction has acquired prime importance in today's world in the field of Electrical and Electronics Engineering. We have studied that a current produces a magnetic field. The phenomenon of electromagnetic induction is thereverse effect wherein a magnetic field produces a current. Applications of this phenomenon are found in modern electric power generation and transmission systems and various electronic devices. This phenomenon enables us to convert the kinetic energy of a coil rotating and/or translating in a magnetic field into electrical energy. So, by applying this phenomenon, energy stored in various forms like, nuclear, thermal, wind etc. can be converted into electrical energy. The operating principle of electric motors, generators and transformers is based on this phenomenon. Other applications include musical instruments, induction stove used in our kitchen, and induction furnace used in foundries.

2. MAGNETIC FIELD LINES AND MAGNETIC FLUX

Let us first discuss the concept of magnetic field lines and magnetic flux. We can represent any magnetic field by magnetic field lines. Unlike the electric lines of force it is wrong to call them magnetic lines of force, because they do not point in the direction of the force on a charge. The force on a moving charged particle is always perpendicular to the magnetic field (or magnetic field lines) at the particle's position.

The idea of magnetic field lines is the same as it is for electric field lines. The magnetic field at any point is tangential to the field line at that point. Where the field lines are close, the magnitude of field is large, where the field lines are far apart, the field magnitude is small. Also, because the direction $\vec{B}$ at each point is unique, field lines never intersect. Unlike the electric field lines, magnetic lines form closed loops.

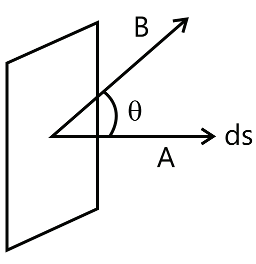


Figure 22.1: Magnetic flux through elementary area ds

SI unit of magnetic field is Tesla (T). 1 T=10^4 Gauss.

Magnetic flux (ϕ) through an area ds in magnetic field B is defined as $\phi = \vec{B} \cdot \vec{ds}$ ... (i)

Physically it represents total lines of induction passing through a given

Area, equation (i) can be written as

$\phi = B\,ds\cos\theta$ ... (ii)

Where θ is angle between B and area vector ds. (see Figure 22.1) According to equation

(ii) flux can change not only due to magnetic field and area but also due to orientation of area w.r.t. B.

Dimensional formula of flux is $\left[ML^2 T^{-2} A^{-1}\right]$

Note down the following points regarding the magnetic flux:

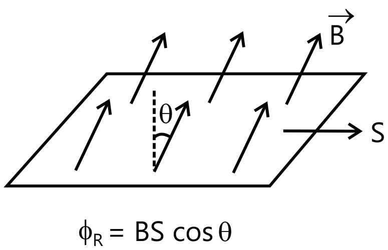
(a) Magnetic flux is a scalar quantity (dot product of two vector quantities is scalar quantity)

(b) The SI unit of magnetic flux is tesla-meter^2 (1T-m^2). This unit is called weber (1 Wb)

1Wb=1Tm^2=Nm/A

Thus, unit of magnetic field is also weber/m^2(1Wb/m^2), or 1 T=1Wb/m^2

(c) In the special case in which $\vec{B}$ is uniform over a plane surface with total area S, than $\phi_B = BA\cos\theta$ (see Figure 22.2)



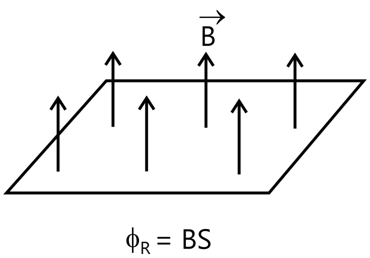


Figure 22.2: Determination of flux for relative orientation of B and S

If $\vec{B}$ is perpendicular to the surface, then cos θ =1 and ϕ_B=BS

Illustration 1: At certain location in the northern hemisphere, the earth's magnetic field has a magnitude of 42 μT and points downward at 57^0 to vertical. The flux through a horizontal surface of area 2.5m^2 will be (cos 57^0 =0.545) **(JEE MAIN)**

Sol: The magnetic flux through any surface is $\vec{\phi} = \vec{B}\cdot\vec{A}$

Using the formula of flux $\phi = BA\cos\theta$

we get the flux through the area as $\phi = BA\cos 57^0 = 42\times10^{-6}\times2.5\times0.545 = 57\times10^{-6}$ Wb.

3. ELECTROMAGNETIC INDUCTION

If a magnet is brought to a coil which is connected with a galvanometer, an electric current is produced in the circuit (See Figure 22.3). The direction of the current so induced in the circuit, is reversed when the magnet recedes away from the coil. The current so produced lasts long, as there is relative motion between the magnet and the coil.

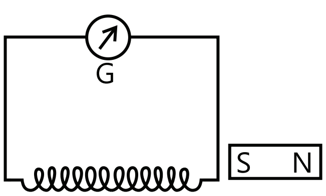


Figure 22.3: Induced current in coil due to relative movement of magnet

It is shown that whenever the magnetic flux linked with a closed circuit changes, an induced e.m.f. is produced in the circuit and lasts as long as the flux changes. Such currents are produced due to induced electromotive force and the phenomenon is called electromagnetic induction. The magnitude and direction of induced electromagnetic force is given by the following Faraday's and Lenz's laws respectively.

3.1 Faraday's First Law

Whenever the magnetic flux linked with a closed circuit changes, an induced electromotive force is produced which produces an induced current in the circuit which lasts as long as the change lasts.

3.2 Faraday's Second Law

The induced e.m.f. is equal to negative of rate of change of flux through the circuit. $e = \frac{-d\phi}{dt}$

The negative sign shows that the induced e.m.f. opposes the changes in the magnetic flux.

If the coil has N number of turns, then $e = -\frac{Nd\phi}{dt}$.

4. LENZ'S LAW

The direction of induced electromotive force is such that it opposes the cause that produces the electromagnetic induction.

If the magnetic flux changes from ϕ_1 to ϕ_2 in time t, the average induced e.m.f. is given by $e(avg) = -\frac{N(\phi_2 - \phi_1)}{t}$

When the magnetic flux ϕ through a closed circuit of known resistance R changes, the quantity of induced charge q can be found as below:

$$\text{As } e = -N\left(\frac{\Delta\phi}{\Delta t}\right), \quad i = \frac{e}{R} = \frac{N}{R}\left(\frac{\Delta\phi}{\Delta t}\right); \; q = i\Delta t = \frac{N}{R}\left(\frac{\Delta\phi}{\Delta t}\right) \quad \Delta t = \frac{N\Delta\phi}{R} = \frac{\text{Total change of flux}}{\text{Resistance}}$$

Furthermore, the direction of induced e.m.f. is that of the induced current. Lenz's law follows from the law of conservation of energy.

4.1 Fleming's Right Hand Rule

It states that if the thumb and the first two fingers of the right hand are stretched mutually perpendicular to each other and if the forefinger gives the direction of the magnetic field and the thumb gives the direction of motion of the conductor, then the central finger gives the direction of the induced current.

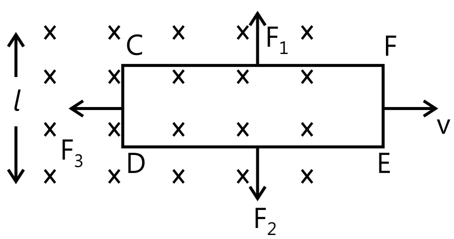


Figure 22.4: Loop moving in magnetic field

The current in the above mentioned loop is in anticlockwise direction. If the loop CDEF (See Figure 22.4) is moved towards right with velocity v, the induced current I will be flowing in clock wise direction and this current will produce forces F_1 and F_2 on arms CF and DE respectively, which being equal and opposite will cancel. Force F_3 on arm CD= BI l where CD=l

$\therefore F_3 = Bl\left(\frac{Blv}{R}\right) = \frac{B^2l^2v}{R}$ where R is the resistance of closed loop.

Power to pull the loop $= F_3 v = \frac{B^2l^2v^2}{R}$

This work is completely converted to heat due to current flowing in the heat produced in the loop $= I^2R = \frac{B^2l^2v^2}{R}$.

Problem Solving Tactic

Never try to use Fleming right hand rule while actually solving a problem. Instead always try to imagine situation and apply Lenz's law, which is very fundamental and easy to understand.

Illustration 2: Space is divided by the line AD into two regions. Region I is field free and the region II has a uniform magnetic field B directed into the plane of paper.

ACD is semicircular conducting loop of radius r with center at O, the plane of the loop being in the plane of the paper. The loop is now made to rotate with a constant angular velocity ω about an axis passing through O and perpendicular to the plane of the paper.

The effective resistance of the loop is R.

(a) Obtained an expression for the magnitude of the induced current in the loop.

(b) Show the direction of the current when the loop is entering into the region II.

(c) Plot a graph between the induced e.m.f. and the time of rotation for two periods of rotation. **(JEE MAIN)**

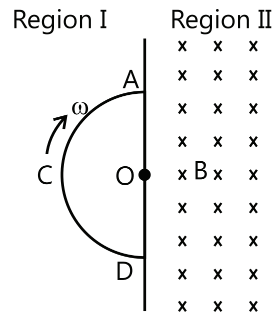


Figure 22.5

Sol: The current induced in the loop is such that it opposes the change in the magnetic flux linked with the loop.

(a) When the loop is in region I, the magnetic flux linked with the loop is zero. When the loop enters in magnetic field in region II.

The magnetic flux linked with it, is given by $\phi = BA$

$\therefore$ e.m.f. induced is $E = -\frac{d\phi}{dt} = -\frac{d(BA)}{dt} = -B\frac{dA}{dt}$ (Numerically)

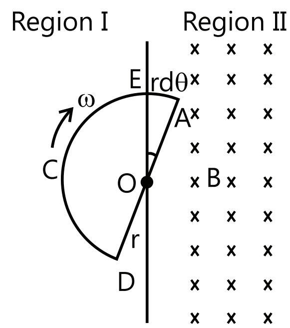


Figure 22.6

Let $d\theta$ be the angle by which the loop is rotated in time dt, then from Figure 22.6 dA= Area of the triangle OEA=(1/2)r•r $d\theta$

$\therefore E = B\frac{1}{2}.\frac{r \times r\,d\theta}{dt} = \frac{1}{2}Br^2\frac{d\theta}{dt} = \frac{1}{2}Br^2\omega$ Using Ohm's law, induct current $I = \frac{e.m.f}{R} = \frac{1}{2}\frac{Br^2\omega}{R}$

Note: dA can also be calculated in the following way; The area corresponding 2π (angle) is πr^2.

$\therefore$ Area corresponding to unit angle $d\theta = \frac{\pi r^2}{2\pi}$

Area corresponding to angle $d\theta = \frac{\pi r^2}{2\pi} \times d\theta$; $\quad \therefore\ dA = \frac{\pi r^2}{2\pi} \times d\theta = \frac{1}{2}r^2 d\theta$

(b) According to Lenz's law, the direction of current induced is to oppose the change in magnetic flux. So that magnetic field induced must be upward. In this way, the direction of current must be anticlockwise.

(c) The graph is shown in Figure 22.7. When the loop enters the magnetic field, the magnetic flux linked increases and e.m.f. e=(1/2) $Br^2\omega$ is produced in one direction. When the loop comes out of the field, the flux decreases and e.m.f. is induced in the opposite sense.

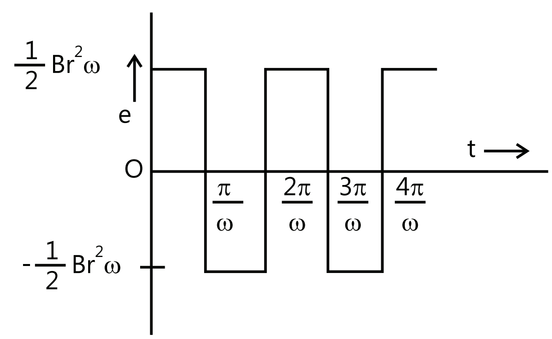


Figure 22.7

Illustration 3: Figure 22.8 shows a conducting loop placed near a long, straight wire carrying a current i as shown. If the current increases continuously, find the direction of the induced current in the loop. **(JEE MAIN)**

Sol: According to Lenz's law, the direction of the induced current is such that it opposes the cause.

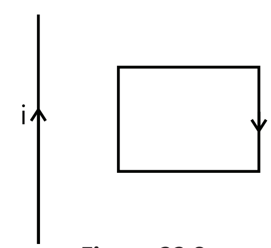


Figure 22.8

Let us put an arrow on the loop as in the Figure 22.8. The right-hand thumb rule shows that the positive normal to the loop is going into the plane of the diagram. Also, the same rule shows that the magnetic field at the site of the loop due to the current is also going into the plane of the diagram. Thus, $\vec{B}$ and $d\vec{s}$ are along the same direction everywhere so that the flux $\Phi = \int \vec{B}.d\vec{s}$ is positive. If i increases, the magnitude of Φ increases. Since Φ is positive and its magnitude increases, $\frac{d\Phi}{dt}$ is positive. Thus, E is negative and hence, the current is negative. The current is, therefore, induced in the direction opposite to the arrow.

5. THE ORIGIN OF INDUCED E.M.F.

E.M.F. is defined as the external mechanism by which work is done per unit charge to maintain the electric field in the wire so as to establish electric current in a conducting wire.

The flux $\int \vec{B}.d\vec{s}$ can be changed by

(a) Keeping the magnetic field constant as time passes and moving whole or part of the loop

(b) Keeping the loop at rest changing the magnetic field

(c) Combination of (a) and (b), that is, by moving the loop (partly or wholly) as well as by changing the field.

The mechanism by which e.m.f. is produced is different in the two basic processes (a) and (b). We now study them under the headings motional e.m.f. and induced electric field.

5.1 Motional E.M.F.

The Figure 22.9 below shows a rod PQ of length l moving in a magnetic field $\vec{B}$ with a constant velocity $\vec{V}$. The length of the rod is perpendicular to the magnetic field and the velocity is perpendicular to both the magnetic field and the rod.

The magnetic force due to the random velocity is zero on the average. Thus, the magnetic field exerts an average force $d\vec{F}_b = q\vec{v} \times \vec{B}$ on each free electron, where q=-16×10^{-19}C is the charge on the electron. This force is towards QP and hence the free electrons will move towards P. Negative charge is accumulated at P and positive charge appears at Q. An electrostatic field E is developed within the wire from Q to P. This field exerts a force $d\vec{F}_e = q\vec{E}$ on each free electron. The charge keeps on accumulating until a situation comes when $F_b = F_e$ or, $\left|q\vec{v} \times \vec{B}\right| = \left|q\vec{E}\right|$ or, $vB = E$

After this, there is no resultant force on the free electrons of the wire PQ. The potential difference between the ends Q and P is V=El=vBl

Thus, it is the magnetic force on the moving free electrons that maintains the potential difference V=vBl and hence produces an e.m.f. E=vBl

As this e.m.f. is produced due to the motion of a conductor, it is called motional e.m.f.

If the ends P and Q are connected by an external resistor Figure 22.10 (a), an electric field is produced in this resistor due to the potential difference. A current is established in the circuit. The electrons flow P to Q via the external circuit and this tries to neutralize the charges accumulated at P and Q. The magnetic force qvB on the free electrons in the wire QP, however, drives the electrons back from Q to P to maintain the potential difference and hence the current.

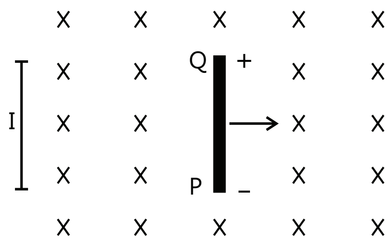


Figure 22.9 Motional emf in a conducting rod

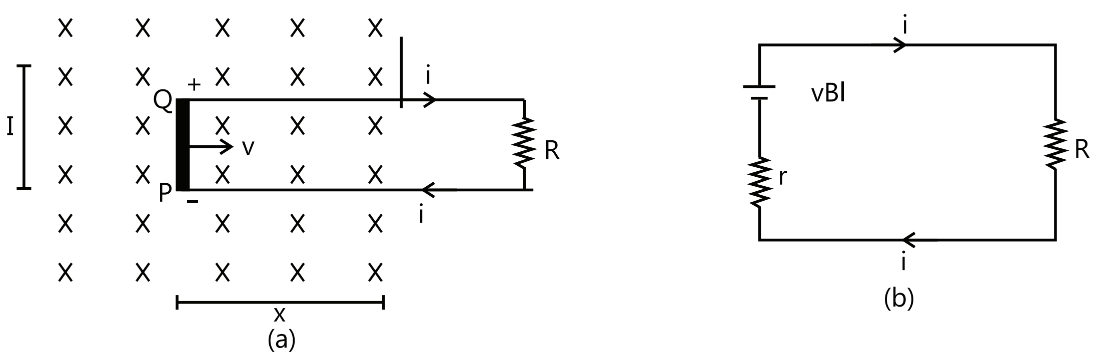


Figure 22.10: (a) Current due to motional emf (b) Equivalent circuit showing induced emf and current in the loop

Thus, we can replace the moving rod QP by battery of e.m.f. νBl with the positive terminal at Q and the negative terminal at P. The resistance r of the rod QP may be treated as the internal resistance of the battery. Figure 22.10 (b) shows the equivalent circuit.

The current is $i = \frac{\nu Bl}{R+r}$ in the clockwise direction (induced current).

We can also find the induced e.m.f. and the induced current in the loop in Fig.22.10 (a) from Faraday's law of electromagnetic induction. If x be the length of the circuit in the magnetic field at time t, the magnetic flux through the area bounded by the loop is $\Phi = Blx$.

The magnitude of the induced e.m.f. is $E = \left|\frac{d\Phi}{dt}\right| = \left|Bl\frac{dx}{dt}\right| = \nu Bl.$

The current is $i = \frac{\nu Bl}{R+r}$. The direction of the current can be worked out from Lenz's law.

Illustration 4: Figure 22.11 (a) shows a rectangular loop MNOP being pulled out of a magnetic field with a uniform velocityvby applying an external force F. The length MN is equal to l and the total resistance of the loop is R. Find (a) the current in the loop, (b) the magnetic force on the loop, (c) the external force F needed to maintain the velocity,

(JEE ADVANCED)

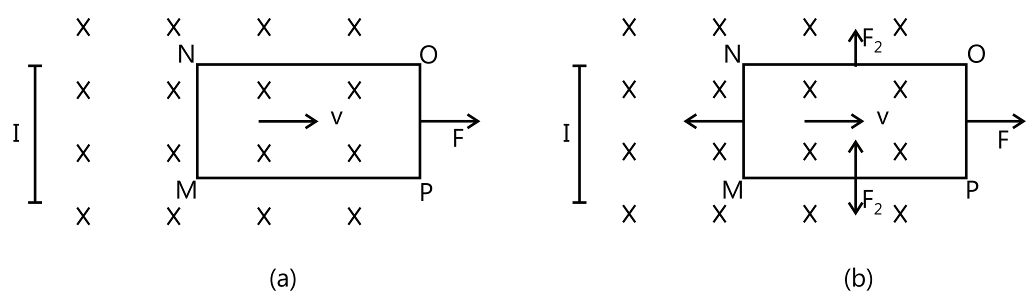


Figure 22.11

Sol: Due to the motion of the loop inside the magnetic field, the motional e.m.f. is induced in the loop. And the magnetic force acting on the loop is $\vec{F} = I\vec{\ell} \times \vec{B}$

(a) The e.m.f. induced in the loop is due to the motion of the wire MN. The e.m.f. is E=vBl with the positive end at N and the negative end at M. The current is $i = \frac{E}{R} = \frac{vBl}{R}$

inclockwise direction (see Figure 22.11 b).

(b) The magnetic force on the wire MN is $\vec{F}_1 = i\vec{l} \times \vec{B}$. The magnitude is $\vec{F}_1 = ilB = \frac{vB^2l^2}{R}$ and is opposite to the velocity on the parts of the wire NO and PM, lying in the field, cancel each other. The resultant magnetic force on the loop is, therefore, $F_1 = \frac{B^2l^2\nu}{R}$ opposite to the velocity.

(c) To move the loop at a constant velocity, the resultant force on it should be zero. Thus, one should pull the loop with a force $F = F_1 = \frac{\nu B^2l^2}{R}$

5.2 Induced Electric Field

Consider a conducting loop placed at rest in a magnetic field $\vec{B}$. Suppose, the field is constant till t=0 and then changes with time. An induced current starts in the loop at t=0.

The free electrons were at rest till t=0(we are not interested in the random motion of the electrons). The magnetic field cannot exert force on electrons at rest. Thus, the magnetic force cannot start the induced current. The electron may be forced to move only by an electric field and hence we conclude that an electric field appears at t=0. This electric field is produced by the changing magnetic field and not by charged particles according to the Coulomb's law or the Gauss's law. The electric field produced by the changing magnetic field is non-electrostatic and non-conservative in nature. We cannot define a potential corresponding to this field. We call it induced electric field. The lines of induced electric field are curves. There are no starting and terminating points of the lines.

If $\vec{E}$ be the induced electric field, the force on a charge q placed in the field is $q\vec{E}$. The work done per unit charge as the charge moves through $\vec{dl}$ is $\vec{E}\cdot\vec{dl}$.

The E.M.F. developed in the loop is,

$$\varepsilon = \oint \vec{E}.\vec{dl}.$$

Using Faraday's Law of Induction,

$$\varepsilon = -\frac{d\Phi}{dt} \text{ or } \oint \vec{E}.\vec{dl} = -\frac{d\Phi}{dt}$$

The presence of a conducting loop is not necessary to have an induced electric field. As long as $\vec{B}$ keeps changing, the induced electric field is present. If a loop is there, the free electrons start drifting and consequently an induced current results.

Note: Induced electric field is not similar to electrostatic field. The biggest difference is that electrostatic field is conservative while the other one is not.

5.3 Induction Due to Motion of a Straight Rod in the Magnetic Field

Consider a straight conducting rod CD moving velocity v towards right along a U shaped conductor in a uniform magnetic field B directed into the page. The motion of the conductor CD resulting in changing the area from CDEF to C'D'EF. It result in a change of area CDD'C' in the magnetic flux producing an increase in the magnetic flux $d\phi$ as $d\phi = B.A$

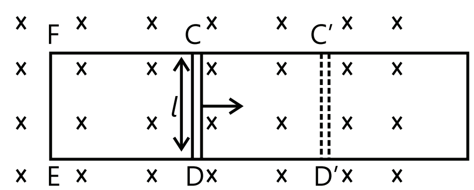


Figure 22.12 Change of flux linkage due to motion of conductor

If l is the length of rod CD, which moves with velocity v in time dt, change in area perpendicular to the field=CDD'C=l vdt.

$\therefore d\phi = Blvdt$

The magnitude of induced e.m.f. $e = \frac{d\phi}{dt} = Blv$

If R is the resistance of loop, the induced current is $I = \frac{Blv}{R}$

The direction of the induced current is given by Fleming's right hand rule.

Illustration 5: In the Figure 22.13,the arm PQ of the rectangular conductor is moved from x=0, outwards. The uniform magnetic field is perpendicular to the plane and extend from x=0 to x=b and is zero for x>b. Only the arm PQ possesses substantial resistance r. Consider the situation when the arm PQ is pulled outwards from x=0 to x=2b, and is then moved back to x=0 with constant speed v. Obtain expressions for the flux, the induced e.m.f., the force necessary to pull the arm and the power dissipated as Joule heat. Sketch the variation of these quantities with distance. **(JEE ADVANCED)**

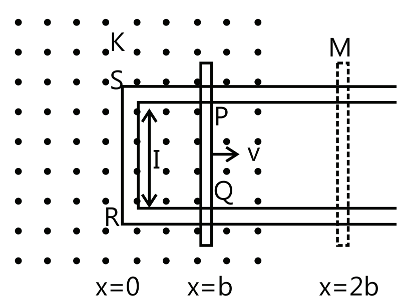


Figure 22.13

Sol: In external magnetic field, the magnetic force acting on movable part of coil is $F = \frac{B^2\ell^2 v}{r}$ and power dissipated in the circuit is given by P= I^2r.

Let us first consider the forward motion from x=0 to x=2b

The flux Φ_B linked with the circuit SPQR is $\Phi_B = B\ell x \quad 0 \le x < b \; = B\ell b \quad 0 \le b < 2b$

The induced e.m.f. is,

i $E = -\frac{d\Phi_B}{dt} = -B\ell v \quad 0 \le x < b \quad = 0 \quad 0 \le x < 2b$

When the induced e.m.f. is non-zero, the current I

is (in magnitude) $I = \frac{B\ell v}{r}$

The force required to keep the arm PQ in constant motion is $I\ell B$.

Its direction is to the left. In magnitude

$F = \frac{B^2\ell^2 v}{r} \qquad 0 \le x < b$

$= 0 \qquad 0 \le x < 2b$

The Joule heating loss is $P_J = I^2 r$

$= \frac{B^2\ell^2 v^2}{r} \qquad 0 \le x < b$

$= 0 \qquad 0 \le x < 2b$

One obtains similar expressions for the inward motion from x=2b to x=0.

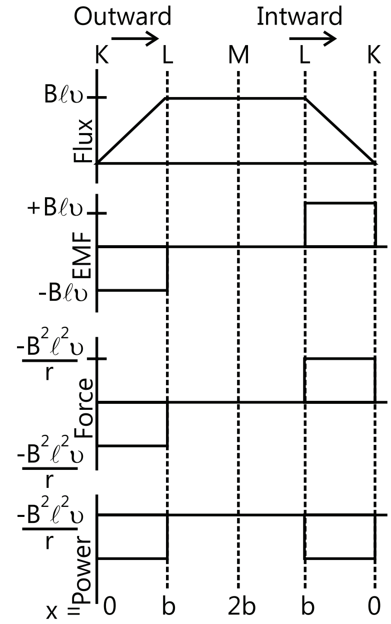


Figure 22.14

5.4 E.M.F. Due to Rotation in Magnetic Field

(a) Rod rotating in a magnetic field: If a linearly conducting rod of length l moves with a velocity v perpendicular to a magnetic field B, the induced e.m.f. $=E_o=Blv$.

If the rod of length l is rotating in a magnetic field with angular velocity ω, velocity of different parts is different and increases moving from O $\rightarrow$ A. Velocity of element at a distance x from O is ωx.

Induced e.m.f. across element of length dx.

$|dE| = (dx)\cdot(\omega x)\cdot B = \omega B \cdot (x\,dx)$

$\Rightarrow \int dE = \omega B \int_0^{\ell} x dx = \omega B\left(\frac{\ell^2}{2}\right) \Rightarrow |E| = \frac{1}{2}\omega B \ell^2$

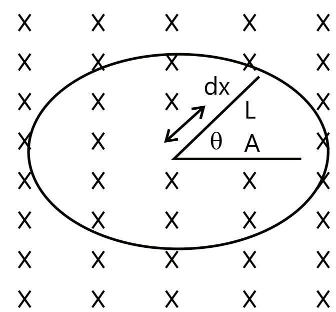


Figure 22.15: Rod rotating in uniform magnetic field

Direction of e.m.f. is given by right hand thumb rule.

Illustration 6: A wheel with 10 metallic spokes each 0.5 m long is rotated with a speed of 120 rev/min in a plane normal to the horizontal component of earth's magnetic field H_E at a place. If H_E= 0.4 G at the place, what is the induced e.m.f. between the axle and the rim of the wheel? Note that $1G=10^{-4}$ T. **(JEE MAIN)**

Sol: E.m.f. induced in a rod of length R rotating about one end in magnetic field is

$E = \frac{1}{2}\omega B R^2$

Frequency of revolution $\omega=120$ rev/s $= \frac{120 \times \pi}{60}$ m / s = 2π m/ s

$\therefore$ Induced e.m.f.$= \frac{1}{2}\omega B R^2 = \frac{1}{2} \times 2\pi \times 0.4 \times 10^{-4} \times (0.5)^2 = 6.28 \times 10^{-5}$ V

The number of spokes is immaterial because thee.m.f.'sacross the spokes are in parallel.

(b) Coil rotating in a magnetic field: If a rectangular conducting coil of area A and N turns is rotated in a uniform magnetic field B with angular velocity ω, as shown in the Figure 22.16.

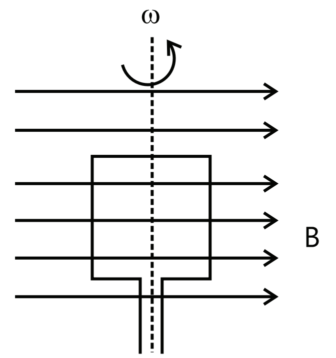


Figure 22.16: Coil rotating in uniform magnetic field

As the coil rotates, an induced e.m.f. E, is produced due to change of flux. At any instant, area vector of coil makes an angle θ with magnetic field, flux linked with coil is $\phi = NBA\cos\theta$ where $\theta = \omega t \Rightarrow \phi = NBA\cos\omega t$

$$\frac{d\phi}{dt} = -BAN\omega\sin\omega t$$

Using Faraday's Law

$e = BAN\omega\sin\omega t$ or $e = e_0\sin\omega t$

The induced e.m.f. has a sinusoidal variation with time and has a maximum value of $e_0 = NBA\omega$.

Such a coil converts mechanical energy into electrical energy. It provides the basic principle onwhich an alternating current (A.C.) generator is based.

(c) Change of area inside magnetic field changes: Let a rectangular coil of width Land length x be placed inside a magnetic field flux linked with coil,

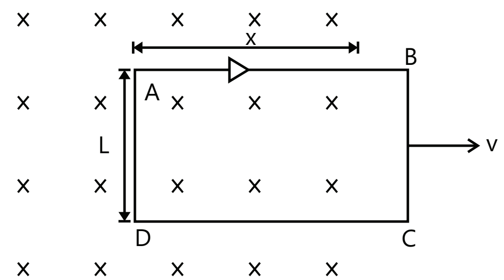


Figure 22.17: Coil moving in magnetic field

$$\phi = BA = (BL\cdot x)$$

$$\frac{d\phi}{dt} = BL\frac{dx}{dt} = (BLv)$$

According to Faraday's Law e=-LBv or |e|=LvB

The direction of induced e.m.f. is given by Lenz'sLaw.

(d) Flux linked with coil also changes when magnetic field over coil change with time.

Illustration 7: Flux associated with coil of resistance 10 Ω and number of turns 1000 is 5.5×10^{-4}. If the flux reduces to 5.5×10^{-5} wb in 0.1 s. The electromotive force and the current induced in the coil will be respectively. **(JEE MAIN)**

Sol: The induced e.m.f. in coil is $E = N\frac{d\phi}{dt} = N\frac{\phi_2 - \phi_1}{t_2 - t_1}$ where N is the number of turns in the coil.

Initial magnetic flux $\phi_1 = 5.5 \times 10^{-4}$ Wb.

Final magnetic flux $\phi_2 = 5 \times 10^{-5}$ Wb.

$\therefore$ Change in flux $\Delta\phi = \phi_2 - \phi_1 = \left(5\times10^{-5}\right) - \left(5.5\times10^{-4}\right) = -50\times10^{-5}$ Wb

Time interval for this change, $\Delta t = 0.1$ sec.

$\therefore$ Induced e.m.f. in the coil $E = -N\frac{\Delta\Phi}{\Delta t} = -1000 \times \frac{\left(-50\times10^{-5}\right)}{0.1} = 5\ \text{V}$

Resistance of the coil, R=10 Ω. Hence induced current in the coil is $i = \frac{E}{R} = \frac{5\ \text{V}}{10\ \Omega} = 0.5\ \text{A}$

6. A NEW LOOK OF ELECTRIC POTENTIAL

Electric potential has meaning only for electric fields that are produced by static charges; it has no meaning for electric fields that are produced by induction.

You can understand this statement qualitatively by considering what happens to a charged particle that makes a single journey around the circular path. It starts at a certain point and, on its return to that same point, has $\oint \vec{E}.\vec{ds} = 0$.

However, when a changing magnetic flux is present, this integral is not zero but is $-d\Phi_B/d$.Thus, assigning electric potential to an induced electric field leads us to a contradiction. We must conclude that electric potential has no meaning for electric fields associated with induction.

7. EDDY CURRENT

Consider a solid plate of metal which enters a region having a magnetic field (See Figure 22.18a). Consider a loop drawn on the plate, a part of which is in the field.

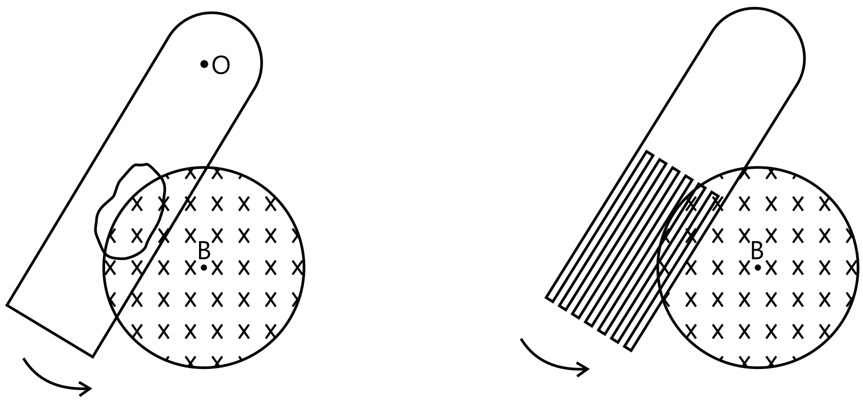


Figure 22.18: Generation of eddy current in conductor

As the plate moves, the magnetic flux through the area bounded by the loop changes and hence, a current is induced. There may be a number of such loops on the plate and hence currents are induced on the surface along a variety of paths. Such currents are called eddy currents. The basic idea is that we do not have a definite conducting loop to guide the induced current. The system itself looks for the loops on the surface along which eddy currents are induced. Because of the eddy currents in the metal plate, thermal energy is produced in it. This energy comes at the cost of the kinetic energy of the plate and the plate slows down. This is known as electromagnetic damping. To reduce electromagnetic damping, one can cut slots in the plate (See Figure 22.18 (b)). This reduces the possible paths of the eddy current considerably.

8. INDUCTORS

An inductor (symbol —ooooo—) can be used to produce a desired magnetic field. We shall consider a long solenoid (more specifically, a short length near the middle of a long solenoid) as our basic type of inductor.

If we establish a current i in the windings (turns) of the solenoid we are taking as our inductor, the current produces a magnetic flux Φ_B through the central region of the inductor. The inductance of the inductors is then $L = \frac{N\Phi_B}{i}$ (inductance defined)

In which N is the number of turns.

$N\Phi_B$ is called the magnetic flux linkage.

The inductance L is thus a measure of the flux linkage produced by the inductor per unit of current.

The SI unit of magnetic flux is the tesla-square meter (Tm^2), the SI unit of inductance is henry (H)

1 henry= $1H=1T.m^2/A$.

8.1 Potential Difference acrossan Inductor

We can find the direction of self-induced e.m.f. across an inductor from Lenz's law.

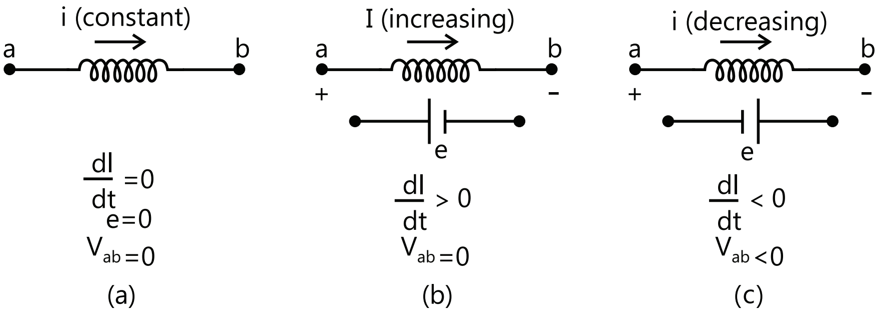


Figure 22.19: Variation of current in inductor coil

8.2 Self Induction

When the current is increased or reduced in the coil, it results in a change of magnetic flux due to which an e.m.f. is induced in the coil, andthis is called self-induced e.m.f. due to the phenomenon of self-induction. If a current I is flowing in a coil, a magnetic flux ϕ is linked to the coil which is directly proportional to the current $\therefore\ \phi \propto I$ or $\phi = LI$

Where L is a constant of proportionality and is called self-inductance of the coil or simply inductance of the coil.

$\therefore$ E.M.F. induced in the coil, $E = -\frac{d\phi}{dt} = -L\frac{dI}{dt}$

The self-inductance of a coil is the e.m.f. induced in it when the rate change of current is unity. The unit of inductance is Henry (H). One Henry is defined as the inductance of a coil in which an e.m.f. of 1 volt is produced, when the current in the coil is changing at the rate of one ampere per second (A/s). If a solenoid has n number of turns per meter and l is its length with total number of turns N=nℓ and area of cross section A, its inductance L is $L = \mu_0 n^2 A\ell = \frac{\mu_0 N^2 A}{\ell}$

The SI unit of self-inductance L is weber^{-1} or volt second ampere^{-1} (Vs/A). It is given the special name Henry and is abbreviated as H. If we have a coil or a solenoid of N turns, the flux through each turn is $\int \vec{B} \cdot \vec{ds}$. If this flux changes, an e.m.f. is induced in each turn. The net e.m.f. induced between the ends of the coil is the sum of all these.Thus, $E = -N\frac{d}{dt}\int \vec{B}.\vec{ds}$

One can compare this with the previous equation to get the inductance.

Illustration 8: The inductor shown in Figure 22.20 has inductance of 0.54 H and carries a current in the direction shown that is decreasing at a uniform rate $\frac{di}{dt} = -0.03\ A/s$.

(a) Find the self-induced e.m.f.

(b) Which end of the inductor,a or b, is at a higher potential? **(JEE MAIN)**

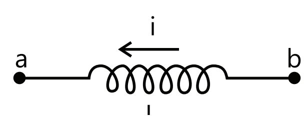


Figure 22.20

Sol: The e.m.f. induced in an inductordue to self-inductance opposesthe change in current in it. As the current decreases, the induced e.m.f. tries to increase the current, thus a will be at higher potential.

(a) Self-inducede.m.f. $E = -L\frac{dI}{dt} = (-0.54)(-0.03)V = 1.62\times10^{-2}\ V$

(b) Potential difference between two ends of inductor is $V_{ba} = L\frac{dI}{dt} = -1.62\times10^{-2}\ V$

Since V_{ba} (V_b-V_a) is negative. It implies that V_a>V_b or a is at higher potential.

Illustration 9: Consider the circuit shown in the following Figure 22.21. The sliding contact is being pulled towards the right so that the resistance in the circuit is increasing. Resistance at time instance is found to be12 Ω. Will the current be more than 0.50 A or less than it at this instant? **(JEE ADVANCED)**

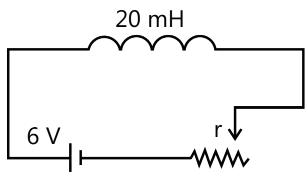


Figure 22.21

Sol: As resistance in the circuit changes, the current through the inductor also changes. Thus e.m.f. is induced in the inductor.

For change in resistance, there is equivalent change in the value of current. Then inducede.m.f. in inductor $E = -L\frac{dI}{dt}$

The net e.m.f. in the circuit is $6V - L\frac{dI}{dt}$ and hence current in circuit is $I = \frac{6V - L\frac{dI}{dt}}{12\ \Omega}$ ... (i)

Due to continuous increase in resistance, the current in the circuit decreases.

Therefore, at given time instant t, the ratio dI/dt decreases, which makes numerator of eq^n (i) higher than 6 and hence, the current in the circuit is larger than 0.5 A

Illustration 10: An average e.m.f. of 0.20V appears in a coil when the current in it is changed from 5.0 A in one direction to 5.0 A in the opposite direction in 0.20 s. Find the self-inductance of the coil. **(JEE MAIN)**

Sol: Using the formula $E = -L\frac{dI}{dt}$, we can find inductance of coil.

(i) The average change in current w.r.t. time t, $\frac{dI}{dt} = \frac{(-5.0\,A) - (5.0\,A)}{0.20s} = -50\,A/s.$

(ii) Using formula $E = -L\frac{dI}{dt}$ we get $0.2V = 50 \times L \Rightarrow L = \frac{0.2}{50} = 4.0$ mH

8.2.1 Self-Inductance in a Long Solenoid

Consider a long solenoid of radius r having n turns per unit length. Suppose a current i is passed through the solenoid. The magnetic field produced inside the solenoid is $B = \mu_0 ni$. The flux through each turn of the solenoid is $\Phi = \int \vec{B}.\vec{ds}. = (\mu_0 ni.)\pi r^2$

The e.m.f. induced in each turn is $-\frac{d\Phi}{dt} = -\mu_0 n\pi r^2 \frac{di}{dt}$

As there are nlturnsin length l of the solenoid, the net e.m.f. across a length l is $\varepsilon = -(nl)(\mu_0 n\pi r^2)\frac{di}{dt}$

Comparing with $\varepsilon = -L\frac{di}{dt}$, the self-inductance is $L = \mu_0 n^2 \pi r^2 l.$

We see that the self-inductance depends only on geometrical factors.

A coil or a solenoid made from thick wire has negligible resistance, but a considerable self-inductance. Such an element is called an ideal inductor and is indicated by the symbol —ooooo—.

The self-inductance e.m.f. in a coil opposes the change in the current that has induced it. This is in accordance with Lenz's law. If the current is increasing, the induced current will be opposite to the original current. If the current is decreasing, the induced current will be along the original current.

8.3. Inductance of a Solenoid

Let us find the inductance of a uniformly wound solenoid having N turns and length l. Assume that l is much longer than the radius of the windings and that the core of the solenoid is air. We can assume that the interior magnetic field due to a current i is uniform and is given by equation,

$B = \mu_0 ni = \mu_0 \left(\frac{N}{l}\right)i$ Where $n = \frac{N}{l}$ is the number of turns per unit length.

The magnetic flux through each turn is, $\phi_B = BS = \mu_0 \frac{NS}{l} i$. Here, S is the cross-sectional area of the solenoid.

Now, $L = \frac{N\phi_B}{i} = \frac{N}{i}\left(\frac{\mu_0 NSi}{l}\right) = \frac{\mu_0 N^2 S}{l}$ $\quad \therefore \quad L = \frac{\mu_0 N^2 S}{l}$

This result shows that L depends on dimensions (S,l) and is proportional to the square of the number of turns. $L \propto N^2$

Because N=nl, we can also express the result in the form,

$$L = \mu_0 \frac{(nl)^2}{l} s = \mu_0 n^2 Sl = \mu_0 n^2 V \quad \text{or} \quad L = \mu_0 n^2 V$$

Here, V=Sl is the volume of the solenoid.

Illustration 11: Two inductors L_1 and L_2 are placed sufficientlyapart. Find out equivalent inductance when they are connected (a) in series (b) in parallel. **(JEE MAIN)**

Sol: For inductors, when they are connected in series, the inductance of the combination should increase, while for parallel connection, the inductance of combination should decrease

(i) In series the induced current i flows in the both the inductors and the total magnetic-flux linked with them will be equal to the sum of the fluxes linked with them individually, that is, $\Phi = L_1 i + L_2 i$

If the equivalent inductance be L. then $\Phi = Li$ $\therefore$ $Li = L_1 + L_2$ or $L = L_1 + L_2$

(ii) In parallel, let the induced currents in the two coils be i_1 and i_2 Then the total induced current is $I = I_1 + I_2$

$$\therefore \frac{dI}{dt} = \frac{dI_1}{dt} + \frac{dI_2}{dt}$$

In parallel, the induced e.m.f. across each coil will be the same. Hence, $E = -L_1 \frac{dI_1}{dt} = -L_2 \frac{dI_2}{dt}$

If the equivalent inductance be L, then $E = -L\frac{di}{dt}$

$$\therefore \frac{E}{L} = -\frac{dI}{dt} = -\left(\frac{dI_1}{dt} + \frac{dI_2}{dt}\right) = \frac{E}{L_1} + \frac{E}{L_2} \text{ or } \frac{1}{L} = \frac{1}{L_1} + \frac{1}{L_2} \quad \text{or } L = \frac{L_1 L_2}{L_1 + L_2}$$

8.5 Energy Stored in an Inductor

The energy of a capacitor is stored as electric field between its plates. Similarly, an inductor has the capability of storing energy in its magnetic field.

A changing current in an inductor causes an e.m.f. between its terminals

The work done per unit time is power.

$$P = \frac{dW}{dt} = -eI = LI\frac{dI}{dt} \quad \text{from} \quad dW = -dU \text{ or } \frac{dW}{dt} = \frac{dU}{dt}$$

We have, $\frac{dU}{dt} == -LI\frac{dI}{dt}$ or $dU = LIDI$

The total energy U supplied while the current increases from zero to a final value i is

$$U = L\int_0^i IdI = \frac{1}{2}Li^2;\ U = \frac{1}{2}Li^2$$

This is the expression for the energy stored in the magnetic field of an inductor when a current i flows through it. The source of this energy is the external source of e.m.f. that supplies the current.

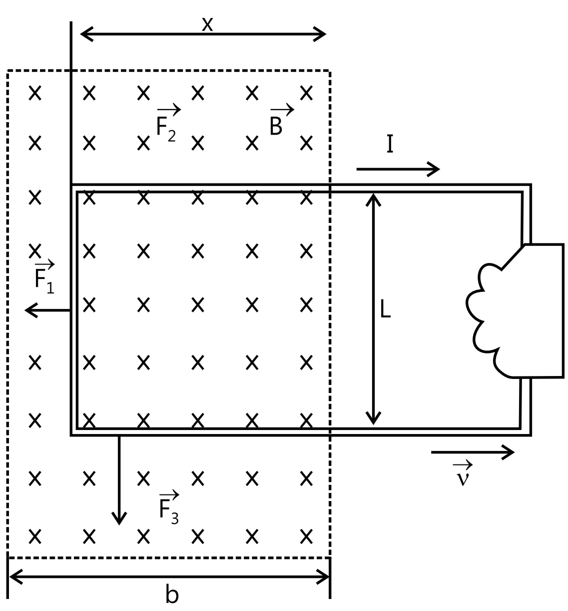


Figure 22.22: Loop pulled out of magnetic field

Energy transfer

The rate at which you do work on the loop as you pull it from the magnetic field:

$P = Fv = \frac{B^2L^2v^2}{R}$ (rate of doing work).

The rate at which thermal energy appears in the loop as you pull it along at constant speed. $P = i^2R$.

Or, $P = \left(\frac{BLv}{R}\right)^2 R = \frac{B^2L^2v^2}{R}$ (thermal energy rate), which is exactly equal to the rate at which you are doing work on the loop.

Thus, the work that you do in pulling the loop through the magnetic field appears as thermal energy in the loop.

9. L-R CIRCUITS

Consider an inductor having inductance L and a resistor R are connected in series which is connected in series to a battery of e.m.f. E in series through a two way key A,B,S as shown in the circuit diagram. When the switch S is connected to A, the current in the circuit grows from zero value. When the current starts growing through the inductance, a back e.m.f. is induced in the coil due to self-induction which opposes the rate of growth of current in the circuit. Similarly, when the switch S is connected to B by disconnecting the battery, the current begins to fall. The current, however, does not fall to zero instantaneously due to the e.m.f. induced in the coil due to self-induction which opposes and reduces the rate of decay of current in the circuit.

9.1 Growth of Current

If S is connected to A during the growth of current, let I be instantaneous current at any time in the circuit. A back e.m.f. equal to $L\frac{dI}{dt}$ will develop in the circuit so that effective e.m.f. in the circuit is $E - L\frac{dI}{dt}$ which is equal to potential drop of IR across resistor.

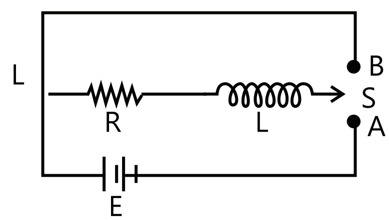


Figure 22.23: Charging of LR circuit

$\therefore E - L\frac{dI}{dt} = IR$ or $\frac{dI}{E - RI} = \frac{dt}{L}$

Integrating this equation between the limits when the current is zero at time t=0 to the instantaneous current I at time t,

$\int_0^I \frac{dI}{E - RI} = \int_0^t \frac{dt}{L}$, $I = \frac{E}{R}\left[1 - e^{\frac{-RT}{L}}\right]$

If I_0 is maximum current, so that $I_0 = \frac{E}{R}$

$I = I_0$ when $\exp\left[\frac{-Rt}{L}\right] = 0$ or $t = \infty$

Thus, current I approaches a value I_0 asymptotically and grows exponentially to a value equal to E/R. The curve for growth of the current in L-R circuit is shown in the Figure.

When $t = \frac{L}{R}$,

$I = I_0\left[1 - e^{\frac{-R}{L}\times\frac{L}{R}}\right] = I_0\left[1 - \frac{1}{e}\right] = I_0\left[\frac{e-1}{e}\right] = I_0\left[\frac{2.718 - 1}{2.178}\right] = 0.63\ I_0$

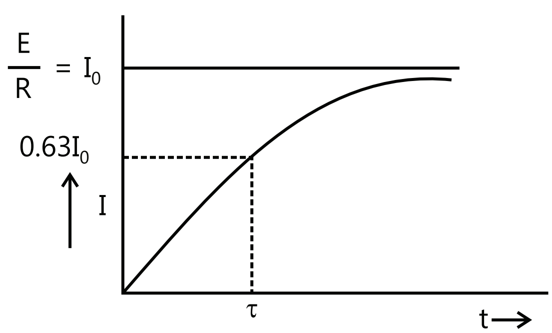


Figure 22.24: Rise of current in LR circuit

The current reaches a value which is equal to 63% of the maximum value I_0 after a time of $\tau = \frac{L}{R}$ from the beginning.

$\therefore$ Time constant of the circuit = $\tau = \frac{L}{R}$

The time constant τ of a circuit is the time during which the current rises from zero to 63% of its maximum value.

$$\therefore I = I_0\left[1 - e^{\frac{-t}{\tau}}\right]$$

PLANCESS CONCEPTS

Inductor as stabilizer:

(a) From L-R circuits, we can see that for sudden changes in voltages, there is a smooth and continuous changes in current through inductor. $I_o = E/R = \frac{10V}{100\Omega} = 0.10\text{ A}$

(b) Thus, inductor is used as a current stabilizer in circuits.

(c) Froma mathematical point of view, for any kind of voltage input (even discontinuous), current is acontinuous function.

If voltage is continuous, then current is a smooth function.

Vaibhav Gupta (JEE 2009, AIR 54)

Illustration 12: An inductor (L=20 mH), a resistor (R=100 Ω) and a battery (E=10V) are connected in series. Find (a) the time constant, (b) the maximum current and (c) the time elapsed before the current reaches 99% of the maximum value. **(JEE MAIN)**

Sol: For LR circuit the current is $I_t = I_o\left(1 - e^{-t/\tau}\right)$ where $\tau = \frac{L}{R}$ is the time constant of the circuit and maximum current $I_0 = \frac{E}{R}$

(a) The time constant is. $\tau = \frac{L}{R} = \frac{20mH}{100\Omega} = 0.20\,ms$

(b) The maximum current is

(c) when I_t=0.99I_0,then solving equation of current for time t we get

$$I_t = I_o\left(1 - e^{-t/\tau}\right) \Rightarrow 0.99\ I_0 = I_0\left(1 - e^{-t/\tau}\right) \Rightarrow e^{-t/\tau} = 0.01$$

$$\Rightarrow t = 0.2 \times \log_e\left(1 \times 10^2\right) = 0.92\ \ s$$

9.2 Decay of Current

If the S is connected to B, the battery is disconnected. The current does not fall instantaneously from I_0 to zero but decays slowly due to the current induced in the coil is in the direction opposite to that of the falling current. The induced e.m.f. in the induced will be equal to $-L\frac{dI}{dt}$ corresponding to the instantaneous current I in resistor R at that time

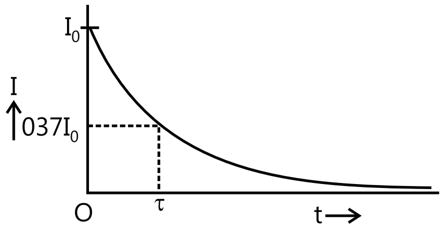


Figure 22.25: Decay of current in LR circuit

$$\therefore\ -L\frac{dI}{dt} = RI$$

Rate of decay of current $= \frac{dI}{dt} = -\left(\frac{R}{L}\right)I$ or $\frac{dI}{I} = -\left(\frac{R}{L}\right)dt$

When t=0, the current, I_0 is maximum and the current at time t is I.

$$\therefore \int_{I_0}^{I} \frac{dI}{I} = -\frac{R}{L}\int_0^t dt \qquad \therefore I = I_0\, e^{-\frac{RT}{L}} = I_0\, e^{\frac{-t}{\tau}}$$

Where $\tau = \frac{L}{R}$ is the time constant of the circuit.

When $t = \frac{L}{R}$, $I = I_0\, e^{\frac{-R}{L}\times\frac{L}{R}} = \frac{I_0}{e} = \frac{I_0}{2.718} = 0.371\, I_0$

The time constant τ is defined as the time interval during which the current decays to 37%of the maximum current during the decay. The rate of decay of the current shows an exponential decay behavior as shown in the Figure 22.27.

The energy stored in an inductor of inductance L, when the current I is passing through it, is equal to $\frac{1}{2}LI^2$ which is in the magnetic form. Such LC circuit produces harmonic oscillation in an electrical circuit in which the energy changes from the electrical to magnetic and vice versa. Such oscillations can be sustained in an electrical circuit and can continue for a long time with the sane amplitude if there is negligible resistance in the circuit.

PLANCESS CONCEPTS

The formula for current in l-r circuit is very similar to that of charge in r-c circuit.

The basic similarities are its form i.e. exponential function.

Also, listed here are some basic points about capacitor, inductor and resistor.

(a) Resistor resists flow of charge.

(b) Capacitor resists change in the charge but can hold ideally any amount of charge.

(c) Inductors do not resist charge but resist change in current and ideally it can allow any amount of current flow.

Nitin Chandrol (JEE 2012, AIR 134)

Illustration 13: A 50 mH inductor is in series with a 10Ω resistor and a battery with an e.m.f. of 25V. At t=0 the switch is closed. Find: (a) the time constant of the circuit. (b) how long it takes the current to rise to 90% of its final value;(c) the rate at which energy is stored in the inductor; (d) power dissipated in the resistor. **(JEE ADVANCED)**

Sol: For LR circuit, the current at any time instant is $I_t = I_o\left(1 - e^{t/\tau}\right)$ where $\tau = L/R$ is time constant,and the energy stored in the inductor is $U_L = \frac{1}{2}LI^2$ andpower dissipated in the circuit is $P_L = \frac{dU_L}{dt}$ & $P_R = I^2R = IV = \frac{V^2}{R}$ (a) The time constant is $\tau = L/R = 5\times10^{-3}$s.

(b) We need to find the time taken for I to reach 90% of I_0 i.e. 0.9I=0.9 E/R.

$$0.9I_0 = I_0\left(1 - e^{-t/\tau}\right)$$

From this we find that exp $\left(-t/\tau\right) = 0.1 \Rightarrow \left(-t/\tau\right) = \text{In}\left(0.1\right)$. Thus, $t = -\tau\ \text{In}\left(0.1\right) = 11.5\times10^{-3}$ s

(c) The rate at which energy is supplied to the inductor is

$$\frac{dU_L}{dt} = +LI\frac{dI}{dt};\ \frac{dI}{dt} = +E/Le^{-Rt/L}; \qquad \text{Therefore} \qquad P_L = \frac{dU_L}{dt} = I\times E\times e^{-Rt/L}$$

We now substitute for I to obtain $P_L = \frac{E^2}{R}\left[e^{-t/\tau} - e^{-2t/\tau}\right]$

(d) The power dissipated in the resistor is $P_R = I^2R = I_0^2R\left(1-2e^{-t/\tau}+e^{-2t/\tau}\right)$

From equation (iii), $E_sI_s = E_PI_P$ or $\frac{E_s}{E_P} = \frac{I_P}{I_s}$. In general, $E \propto \frac{1}{I}$

Illustration 14: (i) Calculate the inductance of an air core solenoid containing 300 turns if the length of the solenoid is 25.0 cm and its cross-sectional area is 4.00 cm^2.

(ii) Calculate the self-induced e.m.f. in the solenoid if the current through it is decreasing at the rate of 50.0 A/s.
(JEE MAIN)

Sol: For air core solenoid, inductance is calculated as $L = \frac{\mu_0 N^2 S}{l}$ and the e.m.f. induced in solenoid is $E = -L\frac{dI}{dt}$

(i) from the formula of inductance, $L = \frac{\mu_0 N^2 S}{l}$ we ..have,

$$L = \frac{\left(4\pi\times10^{-7}\right)(300)^2\left(4.00\times10^{-4}\right)}{\left(25.0\times10^{-2}\right)}\ H = 1.81\times10^{-4}\ H$$

(ii) Here, $\frac{dI}{dt} = -50.0\ A/s$ using formula of e.m.f. we get, $E = -\left(1.81\times10^{-4}\right)\left(-50.0\right) = 9.05\times10^{-3}\ V = 9.05\ mV$

10. ENERGY STORED IN A MAGNETIC FIELD

To derive a quantitative expression for that stored energy, consider a source of e.m.f. connected to a resistor R and an inductor L

If each side is multiplied by i, we obtain $\xi = L\frac{di}{dt} + iR$,

Which has the following physical interpretation in terms of work and energy: $\xi i = Li\frac{di}{dt} + i^2R$,

(a) If a differential amount of charge, dq passes through the battery of e.m.f. in time dt. The battery works on it in the amount dq. The rate at which the battery does work is (dq)/dt, or i. Thus, the left side of equation represents the rate at which the e.m.f. device delivers energy to the rest of the circuit.

(b) The term on the extreme right in the equation represents the rate at which energy appears as thermal energy in the resistor.

(c) Energy that is delivered to the circuit does not appear as thermal energy,but by the conservation-of-energy hypothesis, isstored in the magnetic field of the inductor. Because the equation represents the principle of conservation of energy for RL circuits, the middle term must represent the rate (dU_B/dt) at which magnetic potential energy U_B is stored in the magnetic field.

Thus $\frac{dU_B}{dt} = Li\frac{di}{dt}$. We can write this as $dU_B = Li\ di$.

Integrating yields $\int_0^{U_B} dU_B = \int_0^i Li\ di$ or $U_B = \frac{1}{2}Li^2$ (magnetic energy), which represents the total energy stored by inductor L carrying a current i.

Illustration 15: Calculate the energy stored in an inductor of inductance 50 mH when a current of 2.0 A is passed through it.
(JEE MAIN)

Sol: In LR circuit, magnetic energy is stored in inductor is $U_L = \frac{1}{2}L\times I^2$

The energy stored is $U = \frac{1}{2}Li^2 = \frac{1}{2}\left(50\times10^{-3}\ H\right)\left(2.0\ A\right)^2 = 0.10\ J.$

Illustration 16: What inductance would be needed to store 1.0 kWh of energy in a coil carrying a 200 A current? ($1kWh=3.6\times10^{-6}J$) **(JEE MAIN)**

Sol: In LR circuit, magnetic energy stored in inductor is $U_L = \frac{1}{2}L\times I^2$

We have, i=200 A and U=1kWh= 3.6×10^{-6} J

$\therefore$ Using formula of energy we get $L = \frac{2U}{i^2} = \frac{2\left(3.6\times10^6\right)}{\left(200\right)^2} = 180$ H

11. ENERGY DENSITY OF A MAGNETIC FIELD

Consider a length l near the middle of a long solenoid of cross-sectional area A carrying current i; the volume associated with this length is Al. The energy U_B stored by the length l of the solenoid must lie entirely within this volume because the magnetic field outside such a solenoid is approximately zero. Moreover, the stored energy must be uniformly distributed within the solenoid because magnetic field is (approximately) uniform everywhere inside.

Thus, the energy stored per unit volume of the field is $u_B = \frac{U_B}{Al}$

Or, since $U_B = \frac{1}{2}LI^2$, we have $U_B = \frac{LI^2}{2Al} = \frac{L}{l}\frac{I^2}{2A}$.

Here L is the inductance of length l of the solenoid.

Substituting for $\frac{L}{l} = \mu_0 n^2 A$, we get $u_B = \frac{1}{2}\mu_0 n^2 i^2$ where n is the number of turns per unit length. We know that

$B = \mu_0 in$, we can write this energy density as $u_B = \frac{B^2}{2\mu_0}$ (magnetic energy density).

This equation gives the density of stored energy at any point where the magnitude of the magnetic field is B. Even though we derived it by considering the special case of a solenoid, this equation holds for all magnetic fields, no matter how they are generated. This equation is comparable to $u_E = \frac{1}{2}\varepsilon_0 E^2$

Which gives the energy density (in a vacuum) at any point in an electric field. Note that both u_B and u_E are proportional to the square of the appropriate field magnitude, B or E.

Problem solving tactic

To solve the problems, one would need to learn many of the above formulae. For this, I simply advise that one should make analogy with electric field, capacitors, etc.

$\frac{1}{2}LI^2$ looks similar to $\frac{1}{2}CV^2$. A similar one for energy density formula is also available, where the electric field can be replaced with magnetic field, and absolute permittivity with absolute permeability's inverse.

13. MUTUAL INDUCTANCE

Suppose two closed circuits are placed close to each other and a current i is passed in one. It produces a magnetic field and this field has a flux Φ through the area bounded by the other circuit. As the magnetic field at a point is proportional to the current producing it, we can write Φ=MI where M is a constant depending on the geometrical shapes of the two circuits and their placing. This

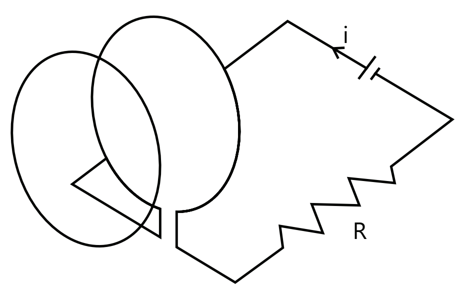


Figure 22.26: Mutual inductance of two coil

constant is called mutual inductance of the given pair of circuits. If the same current i is passed in the second circuit and the flux is calculated through the area bounded by the circuit, the same proportionality constant M appears. If there ismore than one turn in a circuit, one has to add the flux through each turn before applying the above equation.

If the current i in one circuit changes with time, the flux through the area bounded by the second circuit also changes. Thus, an e.m.f. is induced in the second circuit. This phenomenon is called mutual induction. From theabove equation, the induced e.m.f. is $E = -\frac{d\Phi}{dt} = -M\ \frac{dI}{dt}$

Illustration 17: A solenoid S_1 is placed inside another solenoid S_2 as shown in Figure 22.27. The radii of the inner and the outer solenoid are r_1 are r_2 respectively and the numbers of turns per unit length are n_1 and n_2 respectively. Consider a length l of each solenoid. Calculate the mutual inductance between them.

(JEE ADVANCED)

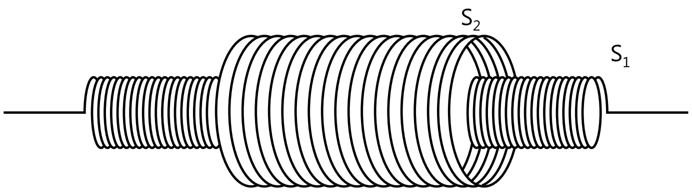


Figure 22.27

Sol: The flux linked with the secondary coil due to primary coil, is $\phi = MI$.

Suppose a current i is passed through the inner solenoid S_1. A magnetic field $B = \mu_0 n_1 i$ is produced inside S_1 where the field outside of it is zero. The flux through each turn of S_2 is $B\pi r_1^2 = \mu_0 n_1 i \pi r_1^2$

The total flux through all the turns in a length l of S_2 is

$\Phi = \left(\mu_0 n_1 I \pi r_1^2\right) n_2 l = \left(\mu_0 n_1 n_2 \pi r_1^2\, l\right) I \quad \text{Thus, } M = \mu_0 n_1 n_2 \pi r_1^2\, l.$ ... (i)

14. OSCILLATING L-C CIRCUITS

If a charged capacitor C is short-circuited through an inductor L, the charge and current in the circuit start oscillating simple harmonically. If the resistance of the circuit is zero, no energy is dissipated as heat. We also assume an idealized situation in which energy is not radiated away from the circuit. With these idealizations – zero resistance and no radiation – the oscillations in the circuit persist indefinitely and the energy is transferred from the capacitor's electric field to the inductor's magnetic field and back. The total energy associated with the circuit is constant. This analogous to the transfer of energy in an oscillating mechanical systemfrom potential energy to kinetic energy and back, with constant total energy. Later we will see that this analogy goes much further.

Let us now derive an equation for the oscillations in an L-C circuit.

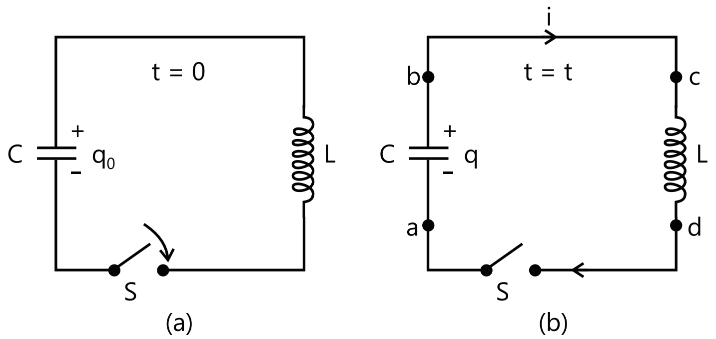


Figure 22.28: LC circuit

Refer Figure 22.28 (a): A capacitor is charged to a P.D. $V_0 = q_0 C$

Here, q_o is the maximum charge on the capacitor. At time t=0, it is connected to an inductor through a switch S. At time t=0, switch S is closed.

Refer Figure 22.28 (b): When the switch is closed, the capacitor starts discharging. Let at time t charge on the capacitor is q (<q_0) and since, it is further decreasing there is a current i in the circuit in the direction shown in

Fig.22.28 (b). Later we will see that, as the charge is oscillating there may be a situation when q will be increasing, but in that case, direction of the current is also reversed and the equation remains unchanged.

The potential difference across capacitor=potential difference across inductor, or

$$V_b - V_a = V_c - V_d \quad \therefore \quad \frac{q}{C} = L\left(\frac{di}{dt}\right) \qquad \text{... (i)}$$

Now, as the charge is decreasing, $\therefore \quad i = \left(\frac{-dq}{dt}\right)$ or $\frac{di}{dt} = -\frac{d^2q}{dt^2}$

Substituting in Eq. (i), we get $\frac{q}{C} = -L\left(\frac{d^2q}{dt^2}\right)$ or $\frac{d^2q}{dt^2} = -\left(\frac{1}{LC}\right)q$ ... (ii)

This is the standard equation of simple harmonic motion $\left(\frac{d^2x}{dt^2} = -\omega^2 x\right)$.

Here, $\omega = \frac{1}{\sqrt{LC}}$ ... (iii)

The general solution of Eq. (ii), is $q = q_0$ at $\cos(\omega t \pm \phi)$

For example in our case $\phi=0$ as $q=q_0$ at $t=0$.

Hence, $q = q_0 \cos\omega t$ ... (iv)

Thus, we can say that charge in the circuit oscillates simple harmonically with angular frequency given by Eq. (iii).

Thus, $\omega = \frac{1}{\sqrt{LC}}$, $f = \frac{\omega}{2\pi} = \frac{1}{2\pi\sqrt{Lc}}$ and $T = \frac{1}{f} = 2\pi\sqrt{Lc}$

The oscillations of the L-C circuit are electromagnetic analog to the mechanical oscillations of a block-spring system.

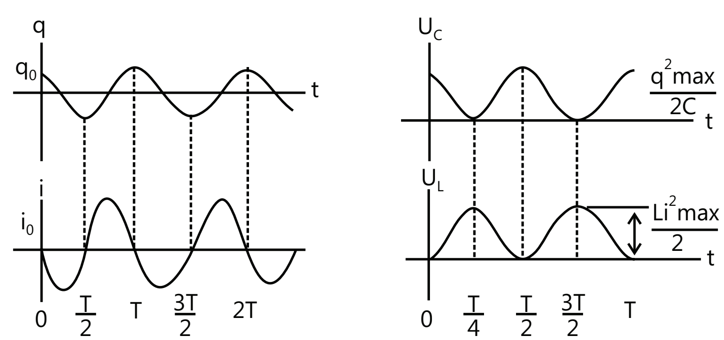


Figure 22.29: LC oscillations

Illustration 18: Two conducting loops of radii R and r are concentric and coplanar. Find the mutual inductances of the system of the two loops. Take R>>>r. **(JEE ADVANCED)**

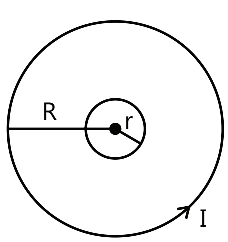


Figure 22.30

Sol: For current I in the circuit, magnetic field is produced around and at the center of the coil. The flux linked with the smallerloopis the product of the magnetic field at the center due to the bigger loop and the area of the smaller loop.

Consider a current I passing through the large loop. The magnetic field at the center of this loop due to this current is $B = \frac{\mu_0 I}{2R}$

Now since r is very small in comparison to R, value of B can be considered uniform over πr^2 area of the inner loop.

$\therefore$ The flux linked with the smaller loop is given by

$$\Phi = \frac{\mu_0 I}{2R}.\pi r^2 = \frac{\mu_0 \pi I r^2}{2R}; \qquad \therefore \qquad M = \frac{\Phi}{I} = \frac{\mu_0 \pi r^2}{2R}$$

15. ELECTROMAGNETIC WAVES

It is known that in certain situations light may be described as electromagnetic wave. The wave equation for light propagating in x-direction in vacuum is written as follows

$$E=E_0 \sin \omega \left(t - x/c\right) \qquad \text{... (i)}$$

Here E is the sinusoidally varying electric field at the position x at time t. The constant c is the speed of light in vacuum. The electric field E is in the Y-Z plane, i.e., perpendicular to the direction of propagation.

There is also a sinusoidally varying magnetic field associated with the electric field when light propagates. This magnetic field is perpendicular to the direction of propagation as well as to the electric field E. It is given by

$$B = B_0 \sin \omega \left(t - x/c\right) \qquad \text{... (ii)}$$

Such a combination of mutually perpendicular electric and magnetic field is referred to as an electromagnetic wave in vacuum.

16. MAXWELL DISPLACEMENT CURRENT

Ampere's law is stated as $\oint \vec{B} \cdot d\vec{l} = \mu_0 I$ ... (iii)

Here i is the electric current crossing a surface bounded by a closed curve and the line integral of $\vec{B}$ (circulation) is calculated along that closed curve. When the electric current at the surface does not change, this equation is valid. This law tell us that an electric current produces magnetic field and gives a method to calculate the field.

Ampere's law in this from is not valid if the electric field at the surface varies with time. As an example, consider a parallel-plate capacitor with circular plates, being charged by a battery (Figure 22.31). If we place a compass needle in the space between the plates, the needle, in general, deflects. This shows that there is a magnetic field in this region. Figure 22.31 also shows a closed curve γ which lies completely in the region between the plates. The plane surface S bounded by this curve is also parallel to the plates and lies completely inside the region between the plates.

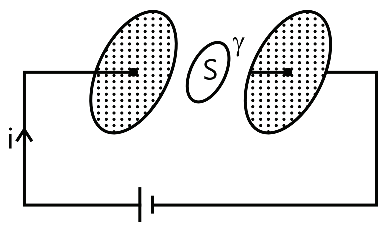


Figure 22.31

During the charging process, there is an electric current through the connecting wires. Charge is accumulated on the plates and the electric field at the points on the surface S changes. It is observed that there is a magnetic field at the points on the curve γ and the circulation $\oint \vec{B} \cdot d\vec{l}$. This equation gives a nonzero value. As no charge crosses the surface S, the current I through the surface is zero. Hence,

$$\oint \vec{B} \cdot d\vec{l} \neq \mu_0 I \qquad \text{... (iv)}$$

Now, Ampere's law (i) can be deduced from Biot-Savart law. We can calculate the magnetic field due to each current element from Biot-Savart law and then its circulation along the closed curve γ. The circulation of the magnetic field due to these current elements must satisfy equation (i). If we denotes this magnetic field by $\vec{B'}$, then

$$\oint \vec{B'} \cdot d\vec{l} = 0 \qquad \text{... (v)}$$

This shows that the actual magnetic field $\vec{B}$ is different from the field $\vec{B'}$ produced by the electric currents only. So, there must be some other source of magnetic field. This other source is nothing but the changing electric field. As the capacitor gets charged, the electric field between the plates changes and this changing electric field produces magnetic field.

It is known that a changing magnetic field produces an electric field. The relation between the two is given by Faraday's law

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt} \qquad \text{... (vi)}$$

Here, $\Phi_B = \int \vec{B} . d\vec{S}$ is the flux of the magnetic field through the area bounded by the closed curve. Along this curve the circulation of $\vec{E}$ is calculated. Now we find that a changing electric field produces a magnetic field. The relation between the changing electric field and the magnetic field resulting from it is given by

$$\oint \vec{B} \,.\, d\vec{l} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt} \qquad \text{... (vii)}$$

Here, Φ_E is the flux of the electric field through the area bounded by the closed curve along which the circulation of $\vec{B}$ is calculated. Equation (iii) gives the magnetic field resulting from an electric current due to flow of charges. Equation (vii) gives the magnetic field due to the changing electric field. If there exists an electric current as well as a changing electric field, the resultant magnetic field is given by

$$\oint \vec{B}.d\vec{l} = \mu_0 i + \mu_0 \varepsilon_0 \left(\frac{d\Phi_E}{dt}\right)$$

Or, $\oint \vec{B}.d\vec{l} = \mu_0 \left(i + i_d\right)$ ... (viii)

In the above equation $i_d = \varepsilon_0 \dfrac{d\Phi_E}{dt}$ is the displacement current.

Illustration 19: For a charging parallel plate capacitor, prove that the displacement current across an area in the region between the plates and parallel to it is equal to the conduction current in the connecting wires.

Sol: For electric flux Φ_E associated with the surface of one of the parallel plates, the displacement current in and across the area of the parallel plate is $i_d = \varepsilon_0 \dfrac{d\Phi_E}{dt}$.

The electric field between the plates is $E = \dfrac{Q}{\varepsilon_0 A}$

Where Q is the charge accumulated at the positive plate. The flux of this field through the given area is $\Phi_E = \dfrac{Q}{\varepsilon_0 A} \times A = \dfrac{Q}{\varepsilon_0}$

The displacement current is $i_d = \varepsilon_0 \dfrac{d\Phi_E}{dt} = \varepsilon_0 \dfrac{d}{dt}\left(\dfrac{Q}{\varepsilon_0}\right) = \dfrac{dQ}{dt}$

But $\dfrac{dQ}{dt}$ is the rate at which the charge is carried to the positive plate through the connecting wire. Thus, $i_d = i_c$

17 MAXWELL'S EQUATIONS AND PLANE ELECTROMAGNETIC WAVES.

We can summarize the concepts of electricity and magnetism mathematically with the help of four fundamental equations:

Gauss's law for electricity $\oint \vec{E} \cdot d\vec{S} = \dfrac{q}{\varepsilon_0}$ ... (ix)

Gauss's law for magnetism $\oint \vec{B} \cdot d\vec{S} = 0$ ... (x)

Faraday's law $\oint \vec{E} \cdot d\vec{l} = -\dfrac{d\Phi_B}{dt}$ ... (xi)

Ampere's law $\oint \vec{B} \cdot d\vec{l} = \mu_0 i + \varepsilon_0 \mu_0 \dfrac{d\Phi_E}{dt}$ ... (xii)

These equations are collectively known as Maxwell's equations.

In vacuum, there are no charges and hence no conduction currents. Faraday's law and Ampere's law take the form

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt} \qquad \text{... (xiii)}$$

and $\oint \vec{B}\cdot d\vec{l} = \mu_0\varepsilon_0 - \frac{d\Phi_E}{dt}$ ... (xiv)

Respectively.

Let us check if these equations are satisfied by a plane electromagnetic wave given by

$$\left.\begin{aligned} E = E_y &= E_0 . \sin\omega(t - x/c) \\ \text{and} \quad B = B_z &= B_0 \sin\omega(t - x/c) \end{aligned}\right] \quad \text{... (xv)}$$

The wave described above propagates along the positive x-direction, the electric field remains along the y-direction and the magnetic field along the z-direction. The magnitudes of the fields oscillate between $\pm E_0$ and $\pm B_0$ respectively. It is a linearly polarized light, polarized along the y-axis.

From the theory of the waves, we can prove the relations between electric and magnetic field represented in equation (xv) as

$E_0 = c\ B_0$. ... (xvi)

$B_0 = \mu_0\,\varepsilon_0\ c\,E_0 \Rightarrow \mu_0\,\varepsilon_0 = \frac{1}{c^2}$ ... (xvii)

Or, $c = \frac{1}{\sqrt{\mu_0\,\varepsilon_0}}$ ... (xviii)

The wave number $k = \frac{2\pi}{\lambda}$ and speed of light in vacuum is $c = \frac{\omega}{k} = f\lambda = \frac{E_o}{B_o} = \frac{1}{\sqrt{\varepsilon_o\mu_o}}$

In general the speed of electromagnetic waves in the medium of electric permittivity ε and magnetic permeability μ is $v = \frac{1}{\sqrt{\mu\,\varepsilon}}$

Illustration 20: The maximum electric field in a plane electromagnetic wave is 900 N C^{-1}. The wave is going in the x-direction and the electric field is in the y-direction. Find the maximum magnetic field in the wave and its direction.

Sol: The magnetic field is found using the relation $E_0 = c\ B_0$

We have $B_0 = \frac{E_0}{c} = \frac{900\ NC^{-1}}{3\times10^8\ ms^{-1}} = 3\times10^{-6}$ T.

As $\vec{E},\vec{B}$ and the direction of propagation are mutually perpendicular, $\vec{B}$ should be along the z-direction.

18 ENERGY DENSITY AND INTENSITY IN ELECTROMAGNETIC WAVE

The electric and magnetic field in a plane electromagnetic wave are given by

$E = E_0 \sin\omega(t - x/c)$ and $B = B_0\ \sin\omega(t - x/c)$.

In any small volume dV, the energy of the electric field is $U_E = \frac{1}{2}\varepsilon_0\,E^2 dV$ ... (xix)

And the energy of the magnetic field is $U_B = \frac{1}{2\mu_0}B^2 dV$ ... (xx)

Thus, the total energy is $U = \frac{1}{2}\varepsilon_0\,E^2 dV + \frac{1}{2\mu_0}B^2 dV$ ... (xxi)

The energy density is $u = \frac{1}{2}\varepsilon_0 E^2 + \frac{1}{2\mu_0}B^2 = \frac{1}{2}\varepsilon_0\,E_0^2 \sin^2\omega(t - x/c) + \frac{1}{2\mu_0}B_0^2\ \sin^2\omega(t - x/c)$ ... (xxii)

If we take the average over a long time, the sin^2 terms have an average value of ½ Thus,

$$u_{au} = \frac{1}{4}\varepsilon_0 E_0^2 + \frac{1}{4\mu_0}B_0^2 \qquad \text{... (xxiii)}$$

From equations (xvi) and (xx)

$$E_0 = cB_0 \text{ and } \mu_0\varepsilon_0 = \frac{1}{c^2} \text{so that, } \frac{1}{4\mu_0}B_0^2 = \frac{\varepsilon_0 c^2}{4}\left(\frac{E_0}{c}\right)^2 = \frac{1}{4}\varepsilon_0 E_0^2$$

Thus, the electric energy density is equal to the magnetic energy density in average.

Or, $u_{av} = \frac{1}{4}\varepsilon_0 E_0^2 + \frac{1}{4}\varepsilon_0 E_0^2 = \frac{1}{2}\varepsilon_0 E_0^2$ ... (xxiv)

Also, $u_{av} = \frac{1}{4\mu_0}B_0^2 + \frac{1}{4\mu_0}B_0^2 = \frac{1}{2\mu_0}B_0^2.$ ... (xxv)

Illustration 21: The electric field in an electromagnetic wave is given by $E = (50\ NC^{-1})\sin\omega(t - x/c)$. Find the energy contained in a cylinder of cross-section 10 cm^2 and length 50 cm along the x-axis.

Sol: The energy of electric field is given by $U_E = \frac{1}{2}V\varepsilon_0 E^2$ where V is the volume of the cylinder

The energy density is $u_{av} = \frac{1}{2}\varepsilon_0 E_0^2 = \frac{1}{2}\times(8.55\times10^{-12}\ C^2N^{-1}m^{-2})\times(50\ NC^{-1})^2 = 1\cdot1\times10^{-8}\ Jm^{-3}$

The volume of the cylinder is V=10 cm^2 x 50 cm=5 x 10^{-4} m^3.

The energy contained in this volume is $U = (1\cdot1\times10^{-8}\ Jm^{-3})\times(5\times10^{-4}\ m^3) = 5\cdot5\times10^{-12}\ J$

Intensity

The energy crossing per unit area per unit time perpendicular to direction of propagation is called the intensity of a wave.

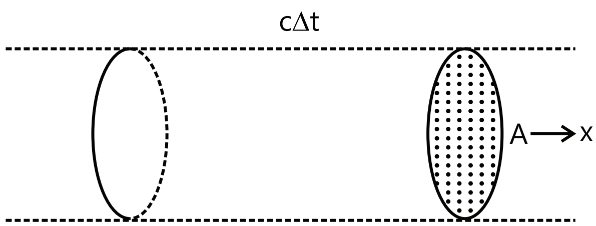


Figure 22.32

Consider a cylindrical volume with area of cross-section A and length $c\Delta t$ along the X-axis (See Figure 22.32). The energy contained in this cylinder crosses the area A in time Δt as the wave propagates at speed c. The energy contained is

$U = u_{av}(c\Delta t)A$.

The intensity is of the wave is $I = \frac{U}{A\Delta t} = u_{av}\,c.$

In terms of maximum electric field, the intensity is written as $I = \frac{1}{2}\varepsilon_0 E_0^2 c$ ... (xxvi)

Illustration 22: Find the intensity of the wave discussed in Illustration 3

Sol. The intensity of the wave in terms of electric field is given by $I = \frac{1}{2}\varepsilon_0 E_0^2 c$. The intensity is $I = \frac{1}{2}\varepsilon_0 E_0^2 c = (1.1\times10^{-8}\ Jm^{-2})\times(3\times10^8\ ms^{-1}) = 3.3\ Wm^{-2}$.

19. MOMENTUM

The propagating electromagnetic wave also carries linear momentum with it. The linear momentum carried by the portion of wave having energy U is given by $p = \frac{U}{c}$ ... (xxvii)

Thus, if the wave incident on a material surface is completely absorbed, it delivers energy U and momentum p=U/c to the surface. If the wave is totally reflected, the momentum delivered to the surface of the material is 2U/c because the momentum of the wave changes from p to –p. It follows that electromagnetic waves incident on a surface exert a force on the surface.

20. ELECTROMAGNETIC SPECRUM AND RADIATION IN ATMOSPHERE

Maxwell's equations are applicable for electromagnetic waves of all wavelengths. Visible light has wavelengths roughly in the range 380 nm to 780 nm. Today we are familiar with electromagnetic waves having wavelengths as small as 30 fm (1 fm=10^{-15} m) to as large as 30 km. Figure 22.33 shows the electromagnetic spectrum we are familiar with.

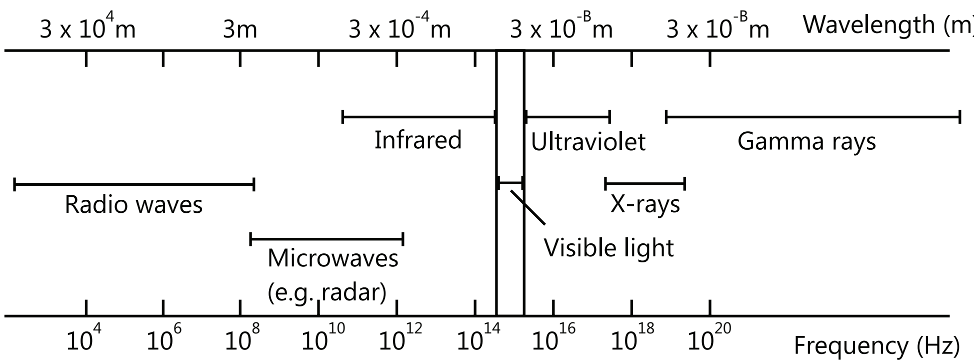


Figure 22.33

The accelerated charge is the basic source of electromagnetic wave. This produces changing electric field and changing magnetic field which constitute the wave. Among the electromagnetic waves, visible light is most familiar to us. This is emitted by atoms under suitable conditions. An atom contains electrons and the light emission is related to the acceleration of an electron inside the atom. The mechanism of emission of ultraviolet radiation is similar to that for visible light.

PROBLEM SOLVING TACTIC

You can remember a single point that when uniform field is into the paper and the rod is moving to the right, i.e. moving out of magnetic field, then higher potential is at the upper end with a difference of Bvl. By remembering this single point you can change it whenever required according to actual situation by just reversing the sign. (E.g. if field is out of the paper and all other conditions are same, then multiply a negative sign.)

FORMULAE SHEET

(a) Flux of magnetic field through a surface: $\Phi_B = \int \vec{B} \cdot d\vec{s}$

(b) Faraday's law of electromagnetic induction

(i) in coil of single loop $E = -\frac{d\Phi_B}{dt}$ (ii) in coil of N loops $E = -\frac{N \cdot d\Phi_B}{dt}$ where E is induced E.M.F.

(c) Motional E.M.F. $E = -\int \vec{E} \cdot d\vec{\ell} = \int (\vec{v} \times \vec{B}) \cdot \vec{\ell} = vB\ell$

(d) The magnitude of induced current is $I = \frac{vB\ell}{R}$

(e) Electric field induced due to changing magnetic field $\int \vec{E} \cdot d\vec{\ell} = -\frac{d\Phi_B}{dt}$

(f) Power $P = F \times v = \frac{(\ell vB)^2}{R}$

(g) Self-inductance of a coil is $L = \frac{N\Phi_B}{I}$

(h) For infinitely long solenoid, self-inductance per unit length $L_{\text{unit length}} = \mu_0 n^2 \pi r^2$

(i) Self-Induced e.m.f. $E = -L\frac{dI}{dt}$

(j) Series Inductors: $L = L_1 + L_2 + \ldots..$

(k) Parallel Inductors: $\frac{1}{L} = \frac{1}{L_1} + \frac{1}{L_2} + \ldots..$

(l) For LR circuit

(i) Sourcee.m.f. is $E = L\frac{dI}{dt} + IR$ (ii) Growth of current is $I = \frac{E}{R}(1 - e^{-t/\tau})$

(iii) Decay of current is $i = \frac{E}{R}(e^{-t/\tau})$ (iv) Time constant $\tau = \frac{L}{R}$

(m) Energy stored in an Inductor is $U = \frac{1}{2} LI^2$

(n) Energy density in magnetic field is $u_B = \frac{U}{V} = \frac{B^2}{2\mu_o}$

(o) In LC circuit

(i) The p.d. across each component is $V = \frac{q}{C} = L\left(\frac{di}{dt}\right)$ (ii) Charge in capacitor $q = q_0 \cos(\omega t \pm \phi)$

(iii) Frequency of oscillation $\omega = \frac{1}{\sqrt{LC}}$

(p) E.m.f. due to Mutual Induction $E_1 = -M\frac{di_2}{dt}$ $E_2 = -M\frac{di_1}{dt}$

(q) Speed of electromagnetic wave: $c = \frac{\omega}{k} = f\lambda = \frac{E_o}{B_o} = \frac{1}{\sqrt{\varepsilon_o \mu_o}}$

(r) Energy density in electromagnetic wave $u_{av} = \frac{1}{2}\varepsilon_0 E_0^2 = \frac{1}{2\mu_0} B_0^2$

(s) Intensity of wave in terms of maximum electric field is $I = \frac{1}{2}\varepsilon_0 E_0^2 c$

Solved Examples

JEE Main/Boards

Example1: A coil made up of inductance L=50 μH and resistance r=0.2 Ω is connected to a battery of e.m.f.=5.0 V. A resistance R=10 Ω is connected parallel to the coil. Now at some instant the connection of the battery is switched off. Find the amount of heat generated in the coil after switching off the battery.

Sol: In LR circuit, the magnetic energy is stored in inductor and is $U_L = \frac{1}{2} L \times I^2$

Given: (i) L= 50 μH, (ii) r=0.2 Ω,

(iii) R=10 Ω

We want to find the fraction of energy lost by the inductor in the form of heat.

Total energy stored in the inductor is

$$U_L = \frac{1}{2} L i_0^2 = \frac{1}{2} L \left(\frac{V}{r}\right)^2$$

$\therefore$ Fraction of energy lost across inductor as heat

$$= U_L \bullet \frac{r}{(R+r)} = \frac{LV^2}{2r(R+r)} = \frac{50 \times 10^{-6} \times 5^2}{2 \times 0.2 (10 + 0.2)} = 3.1 \times 10^{-4} J$$

Example 2: A square loop ACDE of area 20 cm² and resistance 5 Ω is rotated in a magnetic field B=2T through 180^0

Find the magnitude of E, i and Δq after time

(a) 0.01s and (b) in 0.02s.

Sol: When the loop is rotated in external magnetic field, the change in flux linked with the loop induces e.m.f. in it.

Let $\vec{S}$ be the area vector of loop. Before rotation $\vec{S}$ is in direction to $\vec{B}$. After rotating loop by $180^o \vec{S}$ is in opposite direction to $\vec{B}$.

Hence, flux through the loop before rotation is

$$\phi_i = BS\cos 0^0 = 2 \times 20 \times 10^{-4} = 4.0 \times 10^{-3} \text{ Wb} \quad \text{... (i)}$$

& flux passing through the loop when it is rotated by 180^0,

$$\phi_f = BS\cos 180^0 = -1 \times 2 \times 20 \times 10^{-4} = 4.0 \times 10^{-3} \text{ Wb} \quad \text{... (i)}$$

Therefore, change in flux,

$$\Delta\phi_B = \phi_f - \phi_i = -8.0 \times 10^{-3} \text{ Wb}$$

Using formula $E = \frac{d\Phi_B}{dt}$, $I = \frac{E}{R}$ & $dq = I \times dt$

When Δt=0.01 s $|E| = \left| -\frac{\Delta\phi_B}{\Delta t} \right| = \frac{8 \times 10^{-3}}{0.01} = 0.8$ V

$$I = \frac{|E|}{R} = \frac{0.8}{5} = 0.16 \text{ A}$$

& $\Delta q = I \times \Delta t = 0.16 \times 0.01 = 1.6 \times 10^{-3}$ C

When Δt=0.01 s

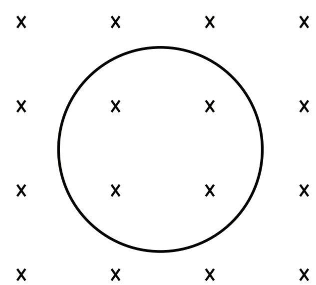
$$|E| = \left| -\frac{\Delta\phi_B}{\Delta t} \right| = \frac{8.0 \times 10^{-3}}{0.02} = 0.4 \text{ V}$$

$$I = \frac{|E|}{R} = \frac{0.4}{5} = 0.8 \text{ A}$$

& $\Delta q = I \times \Delta t = (0.08)(0.02) = 1.6 \times 10^{-3}$ C

Example 3: A coil of area 2 m² is placed in magnetic fieldwhich varies as $B = (2t^2 + 2)$ T with area vector in the direction of B. What is the magnitudeof E.M.F.at t=2s?

Sol: The rate of change of magnetic flux linked with the coil is equal to the induced e.m.f. in the coil $E = -\frac{d\phi}{dt}$



We want to find E.M.F. through the coil when t=2 s. If we find the rate of change of flux, we have E.M.F.

For $\theta = 0^o$, $\phi = BA\cos\theta = BA\cos 0$

Differentiating the above equation, we get $\frac{d\phi}{dt} = \frac{dB}{dt}.A$

$$\Rightarrow |E| = A.\frac{dB}{dt} = A(4t + 4) \left(\because |E| = \frac{d\Phi_B}{dt} \right)$$

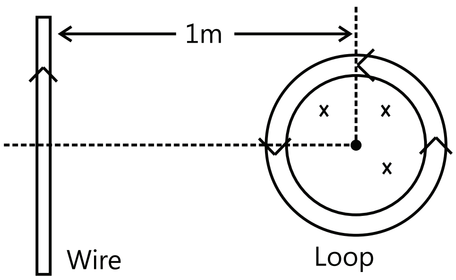
for $A = 2$; $|E| = 8t + 8$

When t=2 s, |E|=16+8=24 V

Example 4: A current $i=(3+2t)\times10^{-2}$ Aincreases at a steady rate in a long straight wire. A small circular loop of radius 10^{-3} m has its plane parallel to the wire and placed at a distance of 1m from the wire. The resistance of the loop is 8 mΩ. Find the magnitude and the direction of the induced current in the loop.

Sol: As the circular loop is small, the magnetic field through it can be assumed to be uniform, having magnitude equal to that of the field at the center of the circular loop, and flux associated with loop is $\phi = B\pi r^2$.

The emf induced in loop is $E = \frac{d\phi}{dt}$.



The arrangement is shown in Figure. The field due to straight wire at the center of loop is:

$$B = \frac{\mu_0}{4\pi}\frac{2I}{d} = 10^{-7}\times\frac{2I}{1} = 2I\times10^{-7}\ T$$

& flux linked with the loop is

$$\phi = BA = B\times\pi r^2 = 2I\times10^{-7}\times\pi\times\left(10^{-3}\right)^2\ Wb$$

(Area of coil is very small so B over it can be taken to be constant)

E.M.F. Einduced in the loop due to change of current is

$$|e| = \frac{d\phi}{dt} = 2\pi\times10^{-13}\frac{dI}{dt}$$

$$\because I = \left(3+2t\right)\times10^{-2}$$

$$\text{So, } \frac{dI}{dt} = 2\times10^{-2}\ As^{-1}$$

And hence, $e = 2\pi\times10^{-13}\times2\times10^{-2} = 1.26\times10^{-14}\ V$

Induced current in the loop

$$I = \frac{E}{R} = \frac{1.26\times10^{-14}}{8\times10^{-4}} = 1.6\times10^{-11}\ A$$

Due to an increase in the current in the wire, the flux linked with the loop will increase.So in accordance with Lenz's law, the direction of the current induced in the loop will be opposite of that in the wire, i.e., anticlockwise.

Example 5: What inductance would be needed to store 1.0kWh of energy in a coil carrying a 200Acurrent? $\left(1kWh = 3.6\times10^6\ J\right)$

Sol: The inductance in the coil is $L = \frac{2U}{i^2}$

Given: (i) energy stored in inductor U_L=1 kWh=3.6 MJ, (ii) Current =200 A.

We want to find inductance of coil.

The energy stored in inductor is $U_L = \frac{1}{2}Li^2$

The inductance is

$$\therefore L = \frac{2U}{i^2} = \frac{2\times3.6\times10^6}{(200)^2} = 180\ H$$

Example 6: The two rails of a railway track insulated from each other and the ground are connected to a millivolt-meter. What is the reading of the voltmeter when a train travels at a speed of 108 kmh^{-1} along the track? Given the vertical component of earth's magnetic field $= 2\times10^{-4}$ T& separation between the rails= 1m.

Sol: Here the train can be considered to move perpendicular to the earth's magnetic field. Due to motion of the train, motional e.m.f. is induced in the axle of train, given by $E = -\frac{d\phi}{dt} = B\ell v\sin\theta$

The train moves in a direction perpendicular to the component of the earth's magnetic field. So the flux associated with the axle of train changes such that the induced E.M.F. in axle is given by

$$E = -\frac{d\phi}{dt} = B\ell v\sin\theta \qquad \text{... (i)}$$

As $(\vec{v}\times\vec{B})$ is parallel to $\vec{\ell}$, $\theta = 0°$

$$\therefore\ E = -Blv \qquad \text{... (ii)}$$

where l=1m, $B_V = 2\times10^{-4}$

$$\&\ v = \frac{180\times1000}{60\times60} = 50\,ms^{-1} \qquad \text{... (iii)}$$

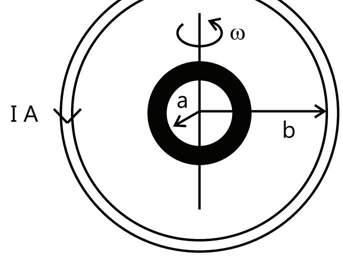
From (i),(ii) & (iii)

$$|E| = 2\times10^{-4}\times1\times50 = 10\times10^{-3}\ mV$$

$\therefore$ Milli-voltmeter will read 10 mV when the train passes with a speed of 108 km/h.

Example 7: A very small circular loop of area 5 cm^2 &resistance 2 Ω, and negligible inductance is initially coplanar and concentric, with a much larger fixed circular loop of radius 10cm. A constant current of 1

Ais passed in the bigger loop and the smaller loop is rotated with angular velocity ωrad/s about a diameter. Calculate (a) the flux linked with the smaller loop (b) induced e.m.f. and current in the smaller loop as a function of time.



Sol: Current in the larger loop produces magnetic field at the center of the loop. Magnetic flux is linked with the smaller loop. When the smaller loop is rotated, flux linked with it changes, and thus e.m.f. is induced in it.

(a) The Figure represents the arrangement of coils. When current passes through the larger loop, the field at the center of larger loop is,

$$B_1 = \frac{\mu_0 I}{2R} = \frac{\mu_0}{4\pi}\frac{2\pi\times I}{R} = 10^{-7}\times\frac{2\pi\times 1}{0.1} = 2\pi\times 10^{-6}\ \frac{Wb}{m^2}$$

is normal to the area of smaller loop.

The smaller loop is rotating at angular velocityω. Therefore the angle of rotation is $\theta = \omega t$ w.r.to B

The flux linked with the smaller loop at time t,

$$\phi_2 = B_1 S_2 \cos\theta = \left(2\pi\times 10^{-6}\right)\left(5\times 10^{-4}\right)\cos(\omega t)$$

$$\text{i.e., } \phi_2 = \pi\times 10^{-9}\cos(\omega t)\ \text{Wb}$$

(b) The induced e.m.f. in the smaller loop,

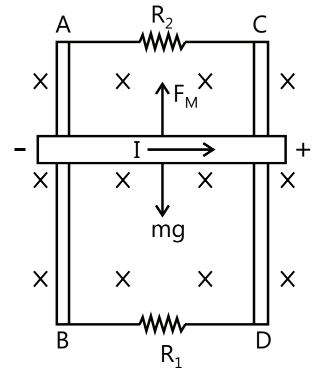
$$E_2 = -\frac{d\phi_2}{dt} = -\frac{d}{dt}\left(\pi\times 10^{-9}\ \cos\omega t\right)$$

$$\text{i.e., } E_2 = \pi\times 10^{-9}\omega\sin\omega t$$

And induced current in the smaller loop,

$$I_2 = \frac{E_2}{R} = \frac{1}{2}\pi\omega\times 10^{-9}\ \sin\omega t\ \text{A.}$$

Example 8: Two parallel vertical metallic rails AB and CD are separated by 1 m. They are connected at the two ends by resistances R_1 and R_2 as shown in Figure 22.40. A horizontal metallic bar of mass 0.2 kg slides without friction, vertically down the rails under the action of gravity. There is a uniform horizontal magnetic field of 0.6 T perpendicular to the plane of the rails. It is observed that when the terminal velocity is attained, the power dissipated in R_1 and R_2 are 0.76 W and 1.2 W respectively. Find the terminal velocity of the bar and the values of R_1 and R_2.



Sol: The motional e.m.f. induced in the bar is E=ℓBv. The direction of induced current in the bar is as shown in Figure. By Fleming's left hand rule the ampere force on the bar will be vertically upwards.

The bar falling freely under action of gravity will acquire terminal velocity only when its motion is opposed by magnetic force F_M=Bil,

Such that Bil=mg

$$\text{i.e., } I = \frac{0.2\times 9.8}{0.6\times 1} = \frac{9.8}{3}\ \text{A}$$

The total power dissipated in the circuit if E is the E.M.F. linked with the coil is

$$E\times I = P = P_1 + P_2$$

$$\Rightarrow E = \frac{(0.76+1.20)}{(9.8/3)} = 0.6\ \text{V}$$

The E.M.F. E=I•Bv$_T$ $\quad \therefore v_T = \frac{E}{Bl} = \frac{0.6}{0.6\times 1} = 1\ \text{ms}^{-1}$

Using the formula of power $P = \frac{V^2}{R}$ i.e., $R = \frac{V^2}{P}$

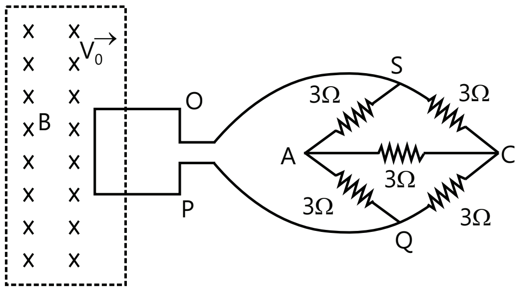
For constant potential drop $V_1 = V_2 = E$

$$R_1 = \frac{E^2}{P_1} = \frac{(0.6)^2}{0.76} = \frac{9}{19}\ \Omega\ \&,$$

$$R_2 = \frac{E^2}{P_2} = \frac{(0.6)^2}{1.20} = 0.3\ \Omega$$

∴ The terminal velocity of the rod is 1m/s & R_1=0.47 Ω & R_2=0.3 Ω

Example 9: A square metal wire loop of side 10 cm and resistance 1 Ω is moved with a constant velocity V_0 in a uniform magnetic field of induction B=2 Wbm^{-2}. The magnetic field lines are perpendicular to the plane of the loop directed into the paper. The loop is connected to a network of resistors, each of 3 Ω. The resistances of lead wire OS and PQ are negligible. What should be the speed of the loop so as to have a steady current 1 mA in the loop? Give the direction of current in the loop.



Sol: The network of resistors is a balanced wheatstone bridge. The induced e.m.f. in the loop is E=BlV, where l is one side of square loop, moving with speed v in the magnetic field.

The network mesh ASCQ is a balanced Wheatstone. So there is no current through branch AC.

Let R be the effective resistance of mesh ASCQ

$$\therefore R = \frac{6\times 6}{6+6} = 3\Omega$$

Resistance of loop OSCQP =3+1=4 Ω

Let speed of loop through the field be V_0

∴ The induced E.M.F. in the loop is $E = BlV_0$

$E = 2\times 0.1\times V_0 = 0.2V_0$

& using Ohm's law the current in the circuit is

$$I = \frac{E}{R} = \frac{BlV_0}{R} = \frac{0.2V_0}{4}$$

$$\because I = 10^{-3}A \Rightarrow V_0 = \frac{4\times 10^{-3}}{0.2} = 2\times 10^{-2} ms^{-1}$$

According to Fleming's right hand rule direction of induced current in the loop is in clockwise direction.

Example 10: A power transformer is used to step up an alternating e.m.f. from 230 V to 4.6kV to transmit 6.9KW of power. If primary coil has 1000 turns, find

(a) no. of turns in the secondary

(b) the current rating of the secondary coil.

Sol: For coil of transformer $E \propto N$ where E is induced E.M.F. and N is number of turns in the coil.

For transformer the $\frac{N_S}{N_P} = \frac{E_S}{E_P}$

$$N_s = \left(\frac{E_s}{E_p}\right)N_p = \frac{4.6\times 1000\times 1000}{230} = 20{,}000$$

If I_p is current in primary, than the power in primary coil is

$$P_P = I_P \times E_P = 6.9 \text{ kW}$$

$$\therefore I_p = \frac{6.9\times 10^3}{230} = 30 \text{ A};$$

$$\& \frac{I_s}{I_p} = \frac{N_p}{N_s} = \frac{1000}{20000} = \frac{1}{20}$$

$$\because I_s = \frac{1}{20}\times I_p = \frac{30}{20} = 1.5 \text{ A};$$

∴ Current rating of the secondary coil is 1.5

Example 11: An infinitesimally small bar magnet of dipole moment M is pointing and moving with the speed v in the x-direction. A small closed circular conducting loop of radius 'a' and of negligible self-inductance lies in the y-z plane with its center at x=0, and its axis coinciding with the x-axis. Find the force opposing the motion of the magnet, if the resistance of the loop is R. Assume that the distance x of the magnet from the center of the loop is much greater than a.

Sol: The flux linked with loop due to magnetic field of bar magnet will decrease as the bar moves away from the loop. The current induced in the loop will oppose its cause i.e. will create a magnetic field at the location of bar magnet such that the bar magnet is attracted towards the loop, thus bar magnet is decelerated.

Field due to the bar magnet at distance x (near the loop) $B = \frac{\mu_0}{4\pi}\frac{2M}{x^3}$

Flux linked with the loop:

$$\phi = BA = \pi a^2 \times \frac{\mu_0}{4\pi}\frac{2M}{x^3}$$

e.m.f. induced in the loop:

$$E = -\frac{d\phi}{dt} = \frac{\mu_0}{4\pi}\frac{6\pi\times Ma^2}{x^4}\frac{dx}{dt} = \frac{\mu_0}{4\pi}\frac{6\pi Ma^2}{x^4}v$$

∴ Induced current:

$$I = \frac{E}{R} = \frac{\mu_0}{2\pi}\times\frac{3\pi Ma^2}{Rx^4}\cdot v = \frac{3\mu_0 Ma^2}{2Rx^4}\cdot v$$

(B) Find the opposing force

The induced current develops field around it. As coil is moving in the external field it will be opposed by the force which is equal to heat dissipated in the coil due to resistive force.

Heat dissipated in coil= Resistive force acting on coil while it is in motion.

$\therefore Fv = I^2R$; (Dimension of power)

$$\Rightarrow F = \frac{I^2R}{v} = \left(\frac{3\mu_0 Ma^2}{2Rx^4}\right)^2 \times v^2 \times \frac{R}{v} = \frac{9}{4}\frac{\mu_0^2 M^2 a^4 v}{Rx^8}.$$

Example 12: In an L-C circuit L=3.3 H and C=840 pF. At t=0 charge on the capacitor is 105μC and maximum. Compute the following quantities at t=2.0 ms:

(a) The energy stored in the capacitor.

(b) The energy stored in the inductor.

(c) The total energy in the circuit.

Sol: In LC circuit, the energy stored in inductor is $\frac{1}{2}Li^2$ and energy stored in capacitor is $\frac{q^2}{2C}$.

Given, L=3.3 H, $C=840\times10^{-12}$ F and $q_{max}=105\times10^{-6}$C

The circuit when connected to AC supply, oscillated and the angular frequency of oscillations of circuit which is,

$$\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{3.3\times840\times10^{-12}}} = 1.9\times10^4 \text{ rad/s}$$

Charge stored in the capacitor at any time instant t is given by, $q = q_0 \cos\omega t$

(a) At $t = 2\times10^{-3}$ s; charge in capacitor is

$$q = \left(105\times10^{-6}\right)\cos\left[1.9\times10^4\right]\left[2\times10^{-3}\right]$$
$$= 100.3\times10^{-6}\text{ C} = 100\ \mu\text{C}$$

$\therefore$ Energy stored in the capacitor is

$$U_C = \frac{1}{2}\frac{q^2}{C} = \frac{\left(100.3\times10^{-6}\right)^2}{2\times840\times10^{-12}} = 5.99\text{ J}$$

(c) Total energy in the circuit

$$U = \frac{1}{2}\frac{q_0^2}{C} = \frac{\left(105\times10^{-6}\right)^2}{2\times840\times10^{-12}} = 6.56\text{ J}$$

(b) Energy stored in inductor in the given time

=total energy in circuit – energy stored in capacitor

=6.56-6=0.56 J

Example 13: A light beam travelling in the x - direction is described by the electric field $E_y = 300\sin\omega(t - x/v)$. An electron is constrained to move along the y-direction with the speed of 2.0×10^7 m/s. Find the maximum electric force and the maximum magnetic force on the electron.

Sol: The maximum force exerted by the wave is $F=F_E+F_B$= qE + qvB.

(i) Maximum electric field $E_0 = 300$ V/m

$\therefore$ Maximum electric force $F_E=qE_0$

$= (1.6\times10^{-19})(300) = 4.8\times10^{-17}$ N

(ii) From the equation, $c = \frac{E_0}{B_0}$

Maximum magnetic field $B_0 = \frac{E_0}{c}$

Or $B_0 = \frac{300}{3.0\times10^8} = 10^{-6}$ T

$\therefore$ Maximum magnetic force $F_B = B_0 qv \sin 90° = B_0 qv$

Substituting the values we have,

Maximum magnetic force $= (10^{-6})(1.6\times10^{-19})(2.0\times10^7)$

$= 3.2\times10^{-18}$ N

Hence total force is F= $(4.8 + 0.32)\times10^{-17}$ N

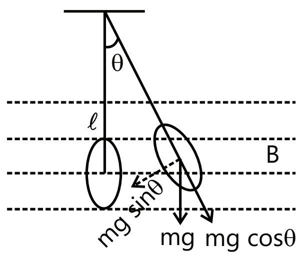
$= 5.12\times10^{-17}$ N

JEE Advanced/Boards

Example 1: A wire frame of area 3.92×10^{-4}m and resistance 20Ω is suspended from a 0.392 m long thread. There is a uniform magnetic field of 0.784 T and the plane of wire-frame is perpendicular to the magnetic field. The frame is made to oscillate under gravity by displacing it through 2×10^{-2}m from its initial position along the direction of magnetic field. The plane of the frame is always along the direction of the thread and does not rotate about it. What is the induced e.m.f. in a wire-frame as a function of time? Also find the maximum current in the frame.

Sol: As the wire frame oscillates in the magnetic field, the angle between the area vector and the magnetic field continuously varies. Thus, the flux linked with the frame changes and e.m.f.and current is induced in the frame. As the magnetic field is uniform, the net magnetic force on the frame will be zero.

The instantaneous flux through the frame when it is displaced through an angle θ is given by $\Phi = BA\cos\theta$



Instantaneous induced e.m.f. to the coil is

$$E = -\frac{d\Phi}{dt} = BA\sin\theta\,\frac{d\theta}{dt}$$

since θ is very small

$$E = BA\theta\,\frac{d\theta}{dt}\ \left(\because \sin\theta = \theta\right) \qquad \text{... (i)}$$

(B) Find the equation of motion & its solution

The force acting on the coil when it is displaced by small angle θ

$$m\frac{dx^2}{dt^2} = -mg\sin\theta \quad \text{or} \quad \frac{d^2x}{dt^2} = -g\sin\theta$$

From Figure 22.43 the displacement of the coil is $\theta = \frac{x}{\ell} \Rightarrow x = \ell\theta$

$$\because \frac{d^2x}{dt^2} = -g\theta \Rightarrow \frac{d^2\theta}{dt^2} = -\frac{g\theta}{l}$$

Putting $\omega = \sqrt{(g/l)}$, we get

$$\frac{d^2\theta}{dt^2} + \omega^2\theta = 0 \qquad \text{... (ii)}$$

This is the equation of S.H.M.

(C) Solve equation (i) to get E_{max} and I_{max}

Solution of equation (ii) is given by $\theta = \theta_0 \sin\omega t$

Substituting the value of θ in equation (i), we get

$$E = BA\left(\theta_0 \sin\omega t\right)\frac{d}{dt}\left(\theta_0 \sin\omega t\right)$$

$$= BA\theta_0 \sin\omega t\ \omega\theta_0 \cos\omega t$$

$$E = BA\omega\theta_0^2 \sin 2\omega t \qquad \text{... (iii)}$$

$$\text{Here } \omega = \sqrt{\left(\frac{g}{l}\right)} = \sqrt{\left(\frac{9.8}{0.392}\right)} = 5\ \text{rad s}^{-1}$$

$$\text{And } \theta_0 = \frac{x_0}{l} = \frac{2\times10^{-2}}{0.392} = 5\times10^{-2}\ \text{rad}$$

Substituting the values, we get

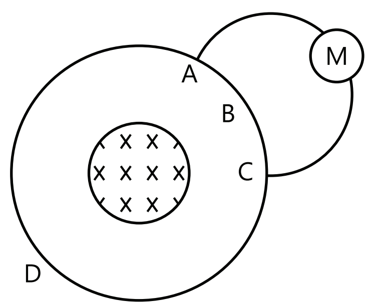
$$E = (0.784)\times\left(3.92\times10^{-4}\right)\times5\times\left(5\times10^{-2}\right)^2 \sin 10t$$

$$= 4\times10^{-6}\ \sin 10t$$

$$\Rightarrow E_{max} = 4\times10^{-6}\ \text{V and}$$

$$I_{max} = \frac{E_{max}}{R} = \frac{4\times10^{-6}}{20} = 2\times10^{-7}\,\text{A}$$

Example 2: A variable magnetic field creates a constant e.m.f. E in a conductor ABCDA. The resistance of the portions ABC, CDA and AMC are R_1, R_2 and R_3, respectively. What current will be recorded by the meter M? The magnetic field is concentrated near the axis of the circular conductor.

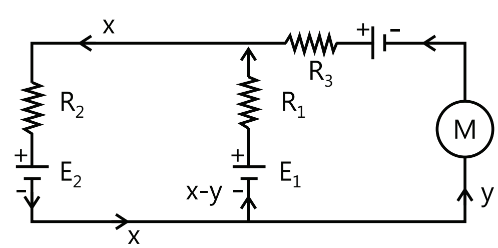


Sol: Due to variable magnetic field, e.m.f. and current are induced in the coil ABCDA.

Let E_1 and E_2 be the e.m.f.s developed in ABC and CDA, respectively. Then $E_1 + E_2 = E$.

There is no net e.m.f. in the loop AMCBA as it does not enclose the magnetic field. If E_3 is the e.m.f. in AMC then $E_1 - E_3 = 0$. The equivalent circuit and distribution of current is shown in Figure.

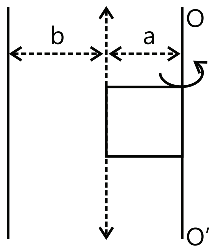
By the loop rule $R_1(x-y) + R_2 x = E_1 + E_2 = E$



And $R_3 y - R_1(x-y) = E_3 - E_1 = 0$

Solving for y, $y = \frac{ER_1}{R_1R_2 + R_2R_3 + R_3R_1}$.

Example 3: A square loop of side 'a' and a straight, infinite conductor are placed in the same plane with two sides of the square parallel to the conductor. The inductance and resistance are equal to L and R respectively. The frame is turned through 180^0 about the axis OO'. Find the electric charge that flows in the square loop.



Sol: For LR circuit, the total E.M.F. is $E = iR + L\frac{di}{dt}$. And the charge in the coil is $q = \int I\,dt$.

By circuit equation $iR = \left(\varepsilon - L\frac{di}{dt}\right)$ where

ε=induced e.m.f. and $L\frac{di}{dt}$ = self-induced e.m.f.

Integrating above equation w.r.t time we get

$$\int Ri\,dt = \int \varepsilon dt - \int L\frac{di}{dt}dt$$

$$\Rightarrow Rq = \int -\frac{d\phi}{dt}dt - L[i]_i^f = \phi_i - \phi_f$$

$$\left(\because i_{initial} = 0,\ i_{final} = 0\right)$$

$$\Rightarrow\ q = \left(\phi_i - \phi_f\right)/R$$

Consider a strip at a distance x in the initial position. Then $B = \frac{\mu_0 I}{2\pi x}$ along the inward normal to the plane.

$$\therefore\ d\phi_i = \frac{\mu_0 I}{2\pi x}a\,dx\cos 0 = \frac{\mu_0 Ia}{2\pi}\frac{dx}{x}$$

$$\Rightarrow \phi_i = \frac{\mu_0 Ia}{2\pi}\int_b^{a+b}\frac{dx}{x} = \frac{\mu_0 Ia}{2\pi}\ln\frac{a+b}{b}$$

Similarly $\phi_f = -\frac{\mu_0 Ia}{2\pi}\ln\frac{2a+b}{a+b}$ $\therefore \left|\phi_i - \phi_f\right| = \frac{\mu_0 Ia}{2\pi}\ln\frac{2a+b}{b}$

$$\therefore\ |q| = \frac{\mu_0 Ia}{2\pi R}\ln\frac{2a+b}{b}$$

Example 4: A straight solenoid has 50 turns per cm in primary and 200 turns in the secondary. The area of cross-section of the solenoid is 4 cm². Calculate the mutual inductance.

Sol: If n_2 is the number of turns in secondary and ϕ_2 is the flux linked through one turn, then the flux linked through the secondary is $n_2\phi_2$.

Magnetic field inside any point of solenoid $B = \mu_0 n_1 i_1$ where n_1 is no turns in primary and i_1 is current in primary.

Flux through secondary having turns n_2 is

$$n_2\phi_2 = n_2\left(BA\right) = \mu_0 n_1 n_2 i_1 A$$

$$\Rightarrow\ M = \frac{n_2\phi_2}{i_1} = \mu_0 n_1 n_2\ S$$

$$= \frac{4\pi\times10^{-7}\times50\times200\times4\times10^{-4}}{10^{-2}} = 5\times10^{-4}\ \text{H}.$$

Example 5: A rectangular conducting loop in the vertical x-z plane has length L, width W, mass M and resistance R. It is dropped lengthwise from rest. At t=0 the bottom of the loop is at a height h above the horizontal x-axis. There is a uniform magnetic field B perpendicular to the x-z plane, below the x-axis. The bottom and top of the loop cross this axis at $t=t_1$ and t_2 respectively. Obtain the expression for the velocity of the loop for time $t_1 \le t \le t_2$.

Sol: The motional e.m.f. induces in the loop as it moves in the magnetic field. The direction of induced current will be such that the ampere force on the width of the loop will be vertically upwards.

For time t_1, the loop is freely falling under gravity, so velocity attained by loop at $t=t_1$

$$v_1 = gt_1 = \sqrt{2gh}$$

During the time $t_1 \le t \le t_2$, flux linked with the loop is changing, so induced e.m.f.

$$E = -\frac{d\phi}{dt} = -BvW$$

and induced current $I = -\frac{BvW}{R}$ clockwise

Magnetic force $F = WIB = -\frac{B^2vW^2}{R}$

So, $m\frac{dv}{dt} = mg - \frac{B^2vW^2}{R}$

$$dt = \frac{mdv}{\left[\left(mg - \frac{B^2W^2v}{R}\right)\right]}\ \text{Integrating,}$$

$$t = -\frac{mR}{B^2W^2}\log_e\left[mg - \frac{B^2vW^2}{R}\right] + A$$

At $t = t_1,\ v = v_1 = gt_1$

$$A = t_1 + \frac{mR}{B^2W^2}\log_e\left[mg - \frac{B^2v_1W^2}{R}\right]$$

Substituting for A,

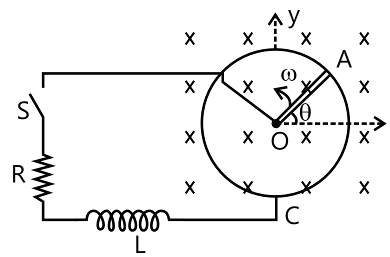
$$e^{-\frac{B^2W^2}{mR}(t-t_1)} = \log_e\left[\frac{mg - \frac{B^2vW^2}{R}}{mg - \frac{B^2v_1W^2}{R}}\right]$$

Gives the expression for velocity of the loop in the interval $t_1 \le t \le t_2$.

Example 6: A metal rod OA of mass m and length l is kept rotating with a constant angular speed ω in a vertical plane about a horizontal axis at the end O. The free end A is arranged to slide without friction along a fixed conducting circular ring in the same plane as that of rotation. A uniform and constant magnetic induction B is applied perpendicular and into the plane of rotation as shown in Figure. An inductor L and an external resistance R are connected through a switch S between the point O and a point C on the ring to from an electrical circuit. Neglect the resistance of the ring the rod. Initially, the switch is open.

(a) What is the induced e.m.f. across the terminals of the switch?

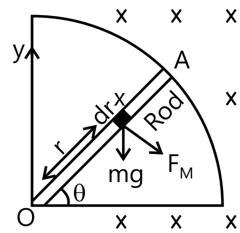
(b) The switch S is closed at time t=0



(i) Obtain an expression for the current as a function of time

(ii) In the steady state, obtained the time dependence of the torque required to maintain the constant angular speed, given that the rod OA was along the positive x-axis at t=0.

Sol: As the rod rotates in uniform magnetic field, motional e.m.f. is induced in it. When the switch is closed, induced current flows in the coil. The direction of current will be such that the torque on the rod due to ampere force will oppose the motion of the rod. The torque, due to weight of the rod, and the torque due to ampere force should be balanced by the net torque of the external agent which is maintaining constant angular velocity of the rod.



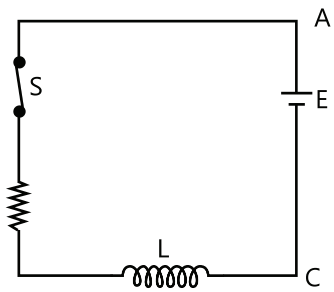
(a) As the terminals of the switch S are connected between the points O and C, so the e.m.f. across the switch is same as across the ends of the rod. Now to calculate the e.m.f. across the rod, consider an element of the rod of length dr at a distance r from O, then

$$dE = Bv\,dr = Br\omega\,dr \quad (\text{as } v = r\omega)$$

$$\text{so } E = \int_0^l B\omega r\,dr = \frac{1}{2}B\omega l^2 \ ....(i)$$

And in accordance with Fleming's right hand rule the direction of current in the will be from A to Oand so O will be at a higher potential (as inside a source of e.m.f. current flows from lower to higher potential)

(b)(i) Treating the ring and rod rotating in the field as a source of e.m.f. E given by equation (i), the equivalent circuit (when the switch S is closed) is as shown in Figure.



Applying Kirchhoff's loop rule to it, keeping in mind that current in the circuit is increasing, we get

$$E - IR - L\frac{dI}{dt} = 0; \text{or} \frac{dI}{(E - IR)} = \frac{1}{L}dt$$

which on integration with initial condition I=0 at t=0 yields

$$I = I_0\left(1 - e^{-t/\tau}\right) \text{ with } I_0 = \frac{E}{R} \text{ and } \tau = \frac{L}{R}$$

So substituting the value of E from Eqn. (i) we have

$$I = \frac{B\omega l^2}{2R}\left[1 - e^{-(R/L)t}\right] \quad \text{... (ii)}$$

As in steady state I is independent of time, i.e., $e^{-t/\tau} \to 0 \Rightarrow t \to \infty$, so

$$I_{steady\ state} = I_{max} = \frac{B\omega l^2}{2R} \quad \text{... (iii)}$$

Now as the rod is rotating in a vertical plane so for the situation shown in Figure 22.48 it will experience torque in clockwise sense due to its own weight and also due to the magnetic force on it. So the torque on element dr, $d\tau = (mg) \times r\cos\theta + F_M \times r$

$$\text{i.e. } d\tau = \frac{M}{l}(dr) g \times r\cos\theta + BI\,dr \times r \left[\text{ as } m = \frac{M}{l}dr \text{ and } F_M = BI\,dr\right]$$

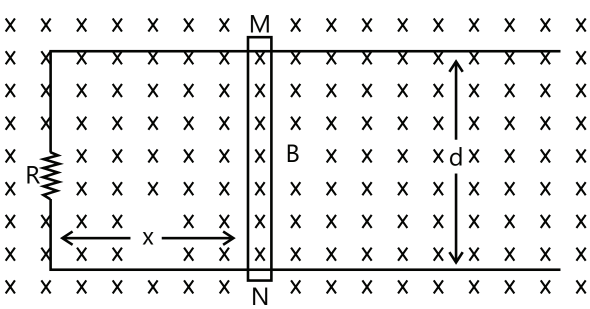
So total torque acting on the rod

$$\tau = \left[\frac{M}{l}g\cos\theta + BI\right]\int_0^l r\,dr = \frac{Mgl}{2}\cos\theta + BI\frac{l^2}{2}$$

But as rod is rotating at constant angular velocity ω, $\theta = \omega t$ and from equation (iii) $I = \left(B\omega l^2 / 2R\right)$

$$\text{So, } \tau = \frac{Mgl}{2}\cos\omega t + \frac{B^2\omega l^4}{4R} \quad \text{... (iv)}$$

And hence the rod will rotate at constant angular velocity ω if a torque having magnitude equal to that given by equation is applied to it in anticlockwise sense.



Example 7: Two long parallel horizontal rails at distance d apart and each having a resistance λ per unit length, are joined at one end by a resistance R. A perfectly conducting rod MN of mass m is free to slide on rails without friction. There is a uniform magnetic field of induction B normal to the plane of the paper and direct into the paper. A variable force F is applied to the rod MN such that, as the rod moves, a constant current flows through R.

(a) Find the velocity of the rod and the applied force F as function of the distance x of the rod from R.

(b) What fraction of the work done per second by F is converted into heat?

Sol: As the rod moves in the magnetic field, motional e.m.f. is induced in it. The current in the rod will be such that the ampere force on it will be opposite to the direction of motion. As the rod moves the resistance of path increases. So to maintain constant current the motional e.m.f. should also increase. So in turn, the velocity of the rod should increase.

Let F be the instantaneous force acting on the rod MN at any instant t when the rod is at a distance x. The instantaneous flux ϕ is given by $\phi = B \times d \times x$ The instantaneously induced e.m.f. is given by

$$E = -\frac{d\phi}{dt} = -Bd\left(\frac{dx}{dt}\right)$$

The instantaneous total resistance of the circuit=R+2λx

Current in the circuit is

$$i = \frac{E}{R} = \frac{Bd}{(R+2\lambda x)}\left(\frac{dx}{dt}\right) \Rightarrow \frac{dx}{dt} = \frac{i(R+2\lambda x)}{Bd}$$

$$\text{i.e., velocity} = \frac{i(R+2\lambda x)}{Bd}$$

The instantaneous acceleration

$$a = \frac{d^2x}{dt^2} = \frac{2i\lambda}{Bd}\left(\frac{dx}{dt}\right)$$

$$= \frac{2i\lambda}{Bd}\left[\frac{i(R+2\lambda x)}{Bd}\right] = \frac{2i^2\lambda(R+2\lambda x)}{B^2d^2}$$

$\therefore$ Instantaneous applied force

$$F = ma = \frac{2i^2\lambda(R+2\lambda x)}{B^2d^2} \times m$$

From this equation $i^2 = \dfrac{FB^2d^2}{2m\lambda(R+2\lambda x)}$

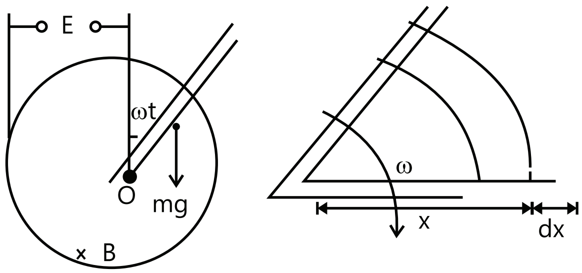
Heat produced per second = $i^2(R+2\lambda x) = \dfrac{FB^2d^2}{2m\lambda}$

Power W= F. v= $F \times \dfrac{i(R+2\lambda x)}{Bd}$

$$\text{Therefore, } \frac{\text{Heat product}}{\text{work done}} = \frac{H}{W} = \frac{FB^2d^2}{2m\lambda} \times \frac{Bd}{Fi(R+2\lambda x)}$$

$$= \frac{B^3d^3}{2m\lambda(R+2\lambda x)}$$

Example 8: A metal rod of mass m can rotate about a horizontal axis O, sliding along a circular conductor of radius a. The arrangement is located in a horizontal and uniform magnetic field of induction B directed perpendicular to the ring plane. The axis and the ring are connected to an e.m.f. source to form a circuit of resistance R. Deduce the relation according to which the source e.m.f. must vary to make the rod rotate with a constant angular velocity ω. Neglect the friction, circuit inductance and ring resistance.



Sol: As current flows in the rod due to the source e.m.f., it experiences torque due to ampere forces and starts rotating. The torque due to weight of the rod should balance the torque due to ampere force to maintain constant angular velocity. As torque due to weight of the rod varies with angular position the torque due to ampere force should also vary. So in turn, the current and thus source e.m.f. should also vary.

Inductance e.m.f. across the ends of the rod

$$E=\int dE=\int_0^a B\omega x\,dx=\frac{1}{2}B\omega a^2$$

Force on the rod if a current I flow through it:

$F= IaB$

If the angular velocity is constant so that torque about O must vanish. Hence

$$mg\frac{a}{2}\sin\omega t=\frac{1}{2}I\,a^2 B$$

$\therefore$ Current required through the rod

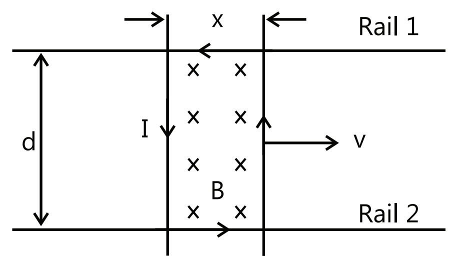
$$I=\frac{mg\sin\omega t}{a\,B}$$

This must be equal be to the current due to total e.m.f. in the circuit

$$I=\frac{E-\frac{1}{2}B\omega a^2}{R}=\frac{mg\,\sin\omega t}{aB};$$

$$\therefore E=\frac{1}{2Ba}\left(2\,mgR\sin\,\omega t+B^2\omega a^2\right)$$

Example 9: Two long wires are placed on a pair of parallel rails perpendicular to the wires. The spacing between the rails d is large compared with x, the distance between the wires. Both wires and rails are made of a material of resistivity ρ per unit length. A magnetic flux of density B applied perpendicular to the rectangle made by the wires and rails. One wire is moved along the rails with a uniform speed v while the other is held stationary. Determine how the force on the stationary wire varies with x and show that it vanishes for a value of x approximately equal to $\frac{\mu_0 v}{4\pi\rho}$.



Sol: Due to motional e.m.f. current will be induced in rectangular loop. The stationary wire will be attracted by the moving wire, as well as it will experience a force due to the uniform magnetic field.

Let at any instant t, during the motion of second wire, the second wire is at a distance x. The area of the rectangle between the two wires is xd. Rate of change of magnetic flux through the rectangle

$$\frac{d\phi}{dt}=\frac{d}{dt}\left(B\times d\right)=Bd\frac{dx}{dt}=Bvd$$

$\therefore$ Induced e.m.f.

$$e=-\frac{d\phi}{dt}=-Bvd$$

So, the current induced in the rectangle I is given by

$$I=\frac{E}{R}=-\frac{Bvd}{2\left(d+x\right)\rho}$$

The force between the two wires due to current flow

$$F=\frac{\mu_0 i_1 i_2}{2\pi x}.d=\frac{\mu_0}{4\pi}\times\frac{2I^2 d}{x}$$

$$=\frac{\mu_0}{4\pi}\left(\frac{2d}{x}\right)\left[\frac{Bvd}{2\left(d+x\right)\rho}\right]^2$$

The force F′, due to magnetic field on the stationary wires

$$F'=BId=Bd\left[\frac{Bvd}{2\left(d+x\right)\rho}\right]=\frac{B^2d^2v}{2\left(d+x\right)\rho}$$

The former force on stationary wire will be directed towards left hand side because opposite currents repel each other while the force due to magnetic field will be directed toward right hand according to Fleming's left hand rule.

$$\therefore F_{resultant} = F' - F$$

$$= \frac{B^2d^2v}{2(d+x)\rho} - \frac{\mu_0}{4\pi}\left(\frac{2d}{x}\right)\left[\frac{Bdv}{2(d+x)\rho}\right]^2$$

$$= \frac{B^2d^2v}{2(d+x)\rho}\left[1 - \frac{\mu_0 dv}{4\pi(d+x)\rho}\right]$$

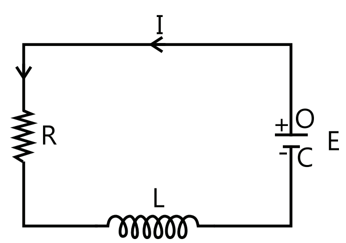
The force will be zero, when

$$\frac{\mu_0 dv}{4\pi(d+x)\rho} = 1 \text{ or } x = \frac{\mu_0 v}{4\pi\rho}$$

(Neglecting x in comparison with d).

Example 10: An inductance L and a resistance R are connected in series with a battery of e.m.f. . Find the maximum rate at which the energy is stored in the magnetic field.

Sol: Substitute the expression for instantaneous current in the LR series circuit in the formula for energy stored in the inductor.



The energy in the magnetic field at time t is,

$$U = \frac{1}{2}Li^2 = \frac{1}{2}Li_0^2\left(1 - e^{-t/\tau}\right)^2$$

The rate at which the energy is stored is

$$P = \frac{dU}{dt} = Li_0^2\left(1 - e^{-t/\tau}\right)\left(-e^{-t/\tau}\right)\left(-\frac{1}{\tau}\right)$$

$$= \frac{Li_0^2}{\tau}\left(e^{-t/\tau} - e^{-2t/\tau}\right) \ldots (i)$$

This rate will be maximum when

$$\frac{dP}{dt} = 0 \Rightarrow \frac{Li_0^2}{\tau}\left(-\frac{1}{\tau}e^{-t/\tau} + \frac{2}{\tau}e^{-2t/\tau}\right) = 0$$

$$\Rightarrow - e^{-t/\tau} = \frac{1}{2}$$

Putting in (i)

$$P_{max} = \frac{Li_0^2}{\tau}\left(\frac{1}{2} - \frac{1}{4}\right) = \frac{LE^2}{4R^2(L/R)} = \frac{E^2}{4R}$$

Example 11: A parallel-plate capacitor having plate area A and plate separation d is joined to a battery of emf V and internal resistance 2R, at t=0. Consider a plane surface of area A, parallel to the plates and situated symmetrically between them. Find the displacement current through this surface as a function of time. [The charge on the capacitor at time t is given by q=CV $(1 - e^{-t/\tau})$ .where τ =CR]

Sol: $i_d = \varepsilon_0 \frac{d\Phi_E}{dt}$ is the displacement current, φ_E is the flux of the electric field between the plates of the capacitor.

Given, q=CV $(1 - e^{-t/\tau})$

$\therefore$ Surface charge density $\sigma = \frac{q}{A} = \frac{CV}{A}(1 - e^{-t/\tau})$

Electric field between the plates of capacitor,

$$E = \frac{\sigma}{\varepsilon_0} = \frac{CV}{\varepsilon_0 A}(1 - e^{-t/\tau})$$

Electric flux from the given area,

$$\phi_E = EA = \frac{CV}{\varepsilon_0}(1 - e^{-t/\tau})$$

Displacement current, $i_d = \varepsilon_0 \frac{d\phi_E}{dt}$

Or, $i_d = \varepsilon_0 \frac{d}{dt}\left[\frac{CV}{\varepsilon_0}(1 - e^{-t/\tau})\right] = \frac{CV}{\tau}e^{-t/\tau}$

Substituting, $\tau = CR'$ where $R' = 2R$

We have, $i_d = \frac{V}{2R}e^{-t/2CR}$

Again substituting, $C = \frac{\varepsilon_0 A}{d}$

$$i_d = \frac{V}{2R}e^{\frac{td}{2\varepsilon_0 AR}}$$

JEE Main/Boards

Exercise 1

Q.1 Can a person sitting in a moving train measure the potential difference between the ends of the axle by a sensitive voltmeter?

Q.2 A coil of mean area 500 cm^2 and having 1000 turns is held perpendicular to a uniform field of 4×10^{-4} T. The coil is turned through 180^0 in $\frac{1}{10}$ s. Calculate the average induced e.m.f..

Q.3 The self-inductance of an inductance coil having 100 turns is 20 mH. Calculate the magnetic flux through the cross-section of the coil corresponding to a current of 4 mA. Also find the total flux.

Q.4 A rectangular loop of wire is being withdrawn out of the magnetic field with velocity v. The magnetic field is perpendicular to the plane of paper. What will be the direction of induced current, in the loop?

Q.5 A solenoidal coil has 50 turns per centimeter along its length and cross sectional area of 4×10^{-4} m^2. 200 turns of another wire is wound round the first solenoid coaxially. The two coils are electrically insulted from each other. Calculate the mutual inductance between the two coils.

Q.6 Calculate the mutual inductance between two coils, when a current of 4.0 A changes to 8.0 A in 0.5 second and induces an e.m.f. of 50 m V in the secondary coil.

Q.7 In a car spark coil, an e.m.f. of 40,000 V is induced in the secondary coil when the primary coil current changes from 4 A to 0 A in 10μs. Calculate the mutual inductance between the primary secondary windings of this spark coil.

Q.8 A current of 10 A is flowing in a long straight wire situated near a rectangular coil. The two sides, of the coil, of length 0.2 m are parallel to the wire. One of them is at a distance of 0.05m and the other is at a distance of 0.10 m from the wire. The wire is in the plane of the coil. Calculate the magnetic flux through the rectangular coil. If the current decays uniformly to zero in 0.02s, find the e.m.f. induced in the coil and indicate the direction in which the induced current flows.

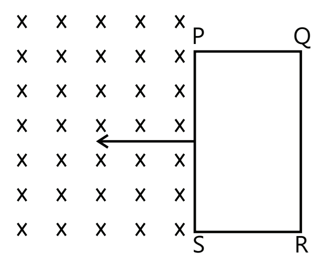
Q.9 A square copper coil of each side 8 cm consists of 100 turns. The coil is initially in vertically plane, such that the plane of coil is normal to the uniform magnetic field of induction 0.4 weber m^{-2}. The coil is turned through 180^0 about a horizontal axis in 0.2s. Find the induced e.m.f.

Q.10 A 5 H inductor carries a steady current of 2 A. How can a 50 V self-induced e.m.f. be made to appear in the inductor?

Q.11 A conducting wire of 100 turns is wound over 1 cm near the center of a solenoid of 100 dm length and 2 cm radius having 1000 turns. Calculate coefficient of mutual inductance of the two solenoids.

Q.12 If the self-inductance of an air core inductor increases from 0.01 mH to 10 mH on introducing an iron core to it, what is the relative permeability of the core used?

Q.13 State Lenz's law. The closed loop PQRS is moving into uniform magnetic field acting at right angle to the plane of the paper as shown in the Figure. State the direction in which the induced current flows in the loop.



Q.14 A solenoid with an iron core and a bulb are connected to a D.C. source. How does the brightness of the bulb change, when the iron core is removed from the solenoid?

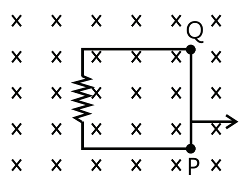
Q.15 What is induced e.m.f.? Write faraday's law of electromagnetic induction. Express it mathematically.

A conducting rod of length 'l', with one end pivoted, is rotated with a uniform angular speed 'ω' in a vertical plane, normal to a uniform magnetic field 'B'. Deduce an expression for the e.m.f. induced in this rod.

Q.16 A circular coil of radius 8 cm and 20 turns rotates about its vertical diameter with an angular speed of 50 s^{-1} in a uniform horizontal magnetic field of magnitude 3×10^{-2} T. Find the maximum and average value of the e.m.f. induced in the coil.

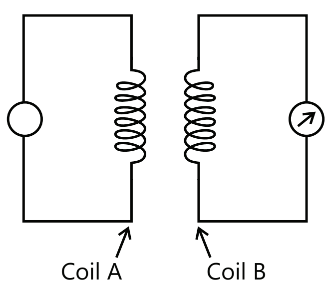
Q.17 Define self-inductance and give its S.I. unit. Derive an expression for self-inductance of a long, air-cored solenoid of length l, radius r, and having N number of turns.

Q.18 A 0.5 m long metal rod PQ completes the circuit as shown in the Figure. The area of the circuit is perpendicular to the magnitude field of flux density 0.15 T. If the resistance of the total circuit is 3 Ω, calculate the force needed to move the rode in the direction as indicated with a constant speed of 2 ms^{-1}.



Q.19 What are eddy currents? How are these produced? In what sense are eddy currents considered undesirable in a transformer and how are these reduced in such a device?

Q.20 The circuit arrangement given below shows that when an a.c. passes through the coil A, the current starts flowing in the coil B.



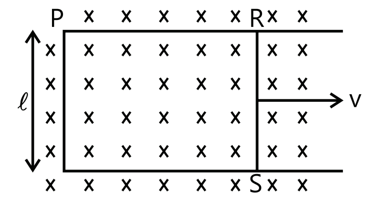
(i) State the underlying principle involved.

(ii) Mention two factors on which the current produced in the coil B depends.

Q21 (i) State faraday's law of electromagnetic induction.

(ii) A jet plane is travelling towards west at a speed of 1800 km/h. what is the voltage difference developed between the ends of the wing having a span of 25m, if the earth's magnetic field at the location has a magnitude of 5×10^{-4} T and the dip angle is 30^0?

Q.22 (a) Write the two applications of eddy currents. (b) Figure 22.57 shows a rectangular conducting lop PQSR in which arm RS of length 'ℓ' is movable. The loop is kept in a uniform magnetic field 'B' directed downward perpendicular to the plane of the loop. The arm RS is moved with a uniform speed 'v'.



Deduce an expression for

(i) The e.m.f. induced across the arm 'RS',

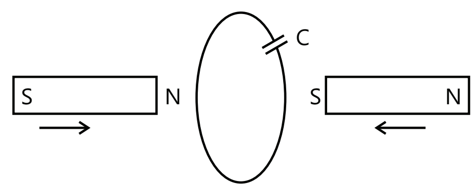
(ii) The external force required to move the arm, and

(iii) The power dissipated as heat.

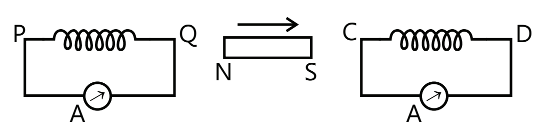
Q.23 Define self-inductance of a coil. Write its S.I. units.

Q 24 The identical loops, one of copper and the other of aluminum, are rotated with the same angular speed in the same magnetic field. Compare (i) the induced e.m.f. and (ii) the current produced in the two coils. Justify your answer.

Q.25 Two bar magnets are quickly moved towards a metallic loop connected across a capacitor 'C' as shown in the Figure. Predict the polarity of the capacitor.



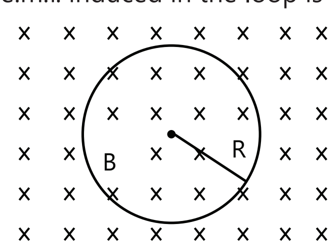
Q.26 A bar magnetic is moved in the direction indicated by the arrow between two coils PQ and CD. Predict the directions of induced current in each coil.



Exercise 2

Single Correct Choice Type

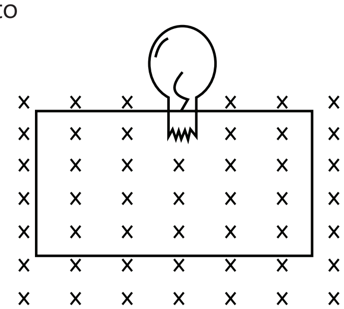
Q.1. A conducting loop of radius R is present in a uniform magnetic field B perpendicular to the plane of the ring. If radius R varies as a function of time't', as $R=R_0+t$. The e.m.f. induced in the loop is



(A) $2\pi(R_0+t)B$ clockwise (B) $\pi(R_0+t)B$ clockwise

(C) $2\pi(R_0+t)B$ anticlockwise (D) zero

Q.2 A square wire loop of 10.0 cm side lies at right angle to a uniform magnetic field of 20T. A 10V light bulb is in a series with the loop as shown in the Figure. The magnetic field is decreasing steadily to zero over a time interval Δt. The bulb will shine full brightness if Δt is equal to



(A) 20 ms (B) 0.02 ms

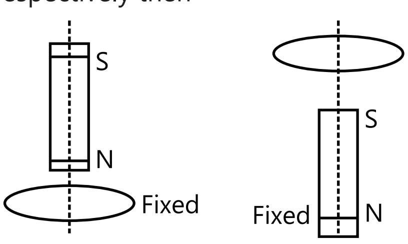
(C) 2 ms (D) 0.2 ms

Q.3 The dimensions of permeability of free space can be given by

(A) $\left[MLT^{-2}A^{-2}\right]$ (B) $\left[MLA^{-2}\right]$

(C) $\left[ML^{-3}T^{2}A^{2}\right]$ (D) $\left[MLA^{-1}\right]$

Q.4 A vertical magnet is dropped from position on the axis of a fixed metallic coil as shown in Figure, figure (i). In figure (ii) the magnet is fixed and horizontal coil is dropped. The acceleration of the magnet and coil are a_1 and a_2 respectively then



(i) (ii)

(A) $a_1 > g, a_2 > g$ (B) $a_1 > g, a_2 < g$

(C) $a_1 < g, a_2 < g$ (D) $a_1 < g, a_2 > g$

Q.5 Two identical coaxial circular loops carry a current I each circulating in the same direction. If the loops approach each other

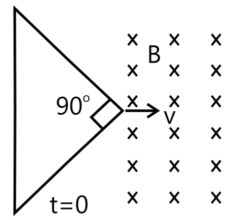
(A) The current in each will decrease

(B) The current in each will increase

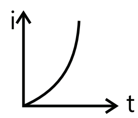
(C) The current in each will remain the same

(D) The current in one will increase and in other will decrease

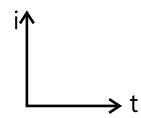
Q.6 The Figure shows an isosceles triangle wire frame with apex angle equal to $\pi/2$. The frame starts entering into the uniform magnetic field B with Constant velocity v at t=0. The longest side of the frame is perpendicular to the direction of velocity. If i is the instantaneous current through the frame then choose the alternative showing the correct variation of i with time.



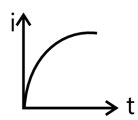
(A)



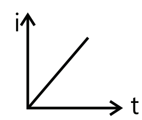
(B)



(C)



(D)



Q.7 A thin wire of length 2 m is perpendicular to the xy plane. It is moved with velocity $\vec{v} = \left(2\hat{i}+3\hat{j}+\hat{k}\right)m/s$ through a region of magnetic induction $B = \left(\hat{i}+2\hat{j}\right)$ Wb/ m^2. Then potential difference induced between the ends of the wire:

(A) 2 V (B) 4 V

(C) 0 V (D) none of these

Q.8 A long metal bar of 30 cm length is aligned along a north south line and moves eastward at a speed of 10 ms^{-1}. A uniform magnetic field of 4.0 T points vertically downwards. If the south end of the bar has a potential of 0 V, the induced potential at the end of the bar is

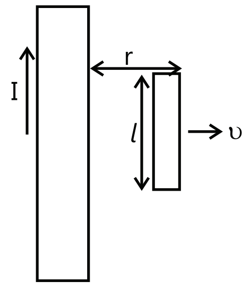
(A) +12 V

(B) -12 V

(C) 0 V

(D) Cannot be determined since there is not closed circuit

Q.9 A conducting rod moves with constant velocity v perpendicular to the long, straight wire carrying a current I as shown compute that the e.m.f. generated between the ends of the rod.



(A) $\frac{\mu_0 \nu Il}{\pi r}$ (B) $\frac{\mu_0 \nu Il}{2\pi r}$

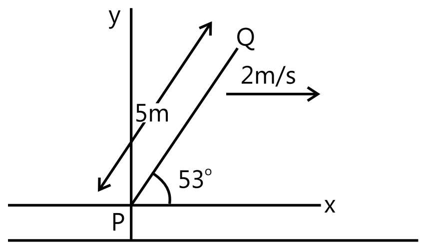
(C) $\frac{2\mu_0 \nu Il}{\pi r}$ (D) $\frac{\mu_0 \nu Il}{4\pi r}$

Q.10 There is a uniform field B normal to the xy plane. A conductor ABC has length AB=l_1, parallel to the x-axis, and length BC=l_2, parallel to the y-axis. ABC moves in the xy plane with velocity $v_x\hat{i}+v_y\hat{j}$. The potential difference between A and C is proportional to

(A) $V_x l_1 + V_y l_2$ (B) $V_x l_2 + V_y l_1$

(C) $V_x l_2 - V_y l_1$ (D) $V_x l_1 - V_y l_2$

Q.11 A conducting rod PQ of length 5 m oriented as shown in Figure is moving with velocity $2\,\hat{i}$ m/s without any rotation in a uniform magnetic field $\left(3\hat{j}+4\hat{k}\right)$T. e.m.f. induced in the rod is



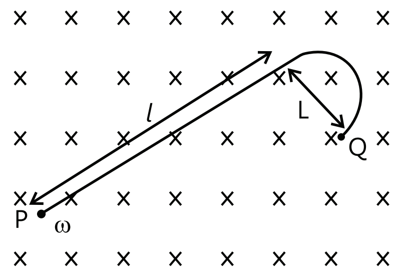
(A) 32 V (B) 40 V (c) 50 V (D) none

Q.12 The magnetic field in a region is given by $B = B_0\left|1+\frac{x}{a}\right|\hat{k}$. A square loop of edge length d is placed with its edge along x & y axis. The loop is moved with constant velocity $\vec{V} = V_0\,\hat{i}$. The e.m.f. induced in the loop is

(A) $\frac{V_0 B_0 d^2}{a}$ (B) $\frac{V_0 B_0 d^2}{2a}$

(C) $\frac{V_0 B_0 a^2}{d}$ (D) none

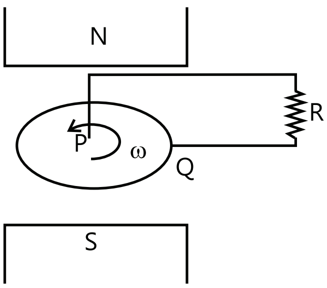
Q.13 When a 'J' shaped conducting rod is rotating in its own plane with constant angular velocity ω, about one of its end P, in a uniform magnetic field B (directed normally into the plane of paper) then magnitude of e.m.f. induced across it will be



(A) $B\omega\sqrt{L^2+l^2}$ (B) $\frac{1}{2}B\omega L^2$

(C) $\frac{1}{2}B\omega\left(L^2+l^2\right)$ (D) $\frac{1}{2}B\omega l^2$

Q.14 A metal disc rotates freely, between the poles of a magnet in the direction indicated. Brushes P and Q make contact with the edge of the disc and the metal axle. What current, if any, flows through R?



(A) A current from P to Q

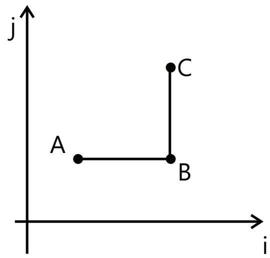
(B) A current from Q to P

(C) No current, because the e.m.f. induced in one side of the disc is opposed by the back e.m.f.

(D) No current, because the e.m.f. induced in one side of disc is opposed by the e.m.f. induced in the other side

(E) No current, because no radial e.m.f. is induced in the disc

Q.15 A rectangular coil of single turn, having area A, rotates in a uniform magnetic field B with an angular velocity ω about an axis perpendicular to the field. If initially the plane of coil is perpendicular to the field, then the average induced e.m.f. when it has rotated through 90^0 is

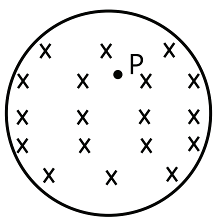


(A) $\frac{\omega BA}{\pi}$ (B) $\frac{\omega BA}{2\pi}$ (C) $\frac{\omega BA}{4\pi}$ (D) $\frac{2\omega BA}{\pi}$

Q.16 A copper rod AB of length L, pivoted at one end A, rotates at constant angular velocity ω, at right angle to a uniform magnetic field of induction B. The e.m.f. developed between the midpoint C to of the rod and end B is

(A) $\frac{B\omega l^2}{4}$ (B) $\frac{B\omega l^2}{2}$ (C) $\frac{3B\omega l^2}{4}$ (D) $\frac{3B\omega l^2}{8}$

Q. 17 Figure 22.70shows a uniform magnetic field B confined to a cylindrical volume and is increasing at a constant rate. The instantaneous acceleration experienced by an electron placed at P is



(A) Zero (B) Towards right

(C) Towards left (D) Upwards

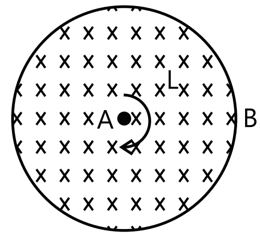
Q.18 A small coil of radius r is placed at the center of a large coil of radius R, where R>>r. The coils are coplanar. The coefficient of mutual inductance between the coils is

(A) $\frac{\mu_0 \pi r}{2R}$ (B) $\frac{\mu_0 \pi r^2}{2R}$ (C) $\frac{\mu_0 \pi r^2}{2R^2}$ (D) $\frac{\mu_0 \pi r}{2R^2}$

Q.19 A long straight wire is placed along the axis of circular ring of radius R. The mutual inductance of this system is

(A) $\frac{\mu_0 R}{2}$ (B) $\frac{\mu_0 \pi R}{2}$ (C) $\frac{\mu_0}{2}$ (D) 0

Q.20 Two identical circular loops of metal wire are lying on a table without touching each other. Loop-A carries a current which increases with time. In response, the loop-B



(A) Remains stationary

(B) Is attracted by the loop-A

(C) Is repelled by the loop-A

(D) Rotates about its CM, with CM fixed

Q.21 A circular loop of radius R, carrying current I, lies in x-y plane with its center at origin. The total magnetic flux through x-y plane is

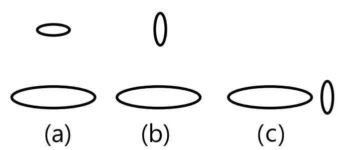
(A) Directly proportional to I

(B) Directly proportional to R

(C)Directly proportional to R^2

(D) Zero

Q.22 Two circular coils can be arranged in any of the three situations in the Figure 22.72. Their mutual inductance will be



(A) Maximum in situation (a)

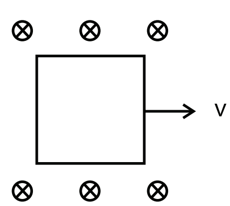
(B) Maximum in situation (b)

(C) Maximum in situation (c)

(D) The same in all situations

Previous Years' Questions

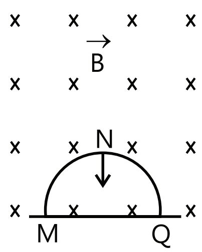
Q.1 A conducting square loop of side L and resistance R moves in its plane with a uniform velocity v perpendicular to one of its sides. A magnetic induction B, constant in time and space, pointing perpendicular to and into the plane of the loop exists everywhere. The current induced in the loop is ***(1989)***



(A) BLv/R clockwise (B) BLv/R anticlockwise

(C) 2BLv/R anticlockwise (D) Zero

Q.2 A thin semicircular conducting ring of radius R is falling with its plane vertical in a horizontal magnetic induction $\vec{B}$. At the position MNQ the speed of the ring is v and the potential difference developed across the ring is ***(1996)***



(A) Zero

(B) $Bv\pi R^2/2$ and M is at higher potential

(C) πBRv and Q is at higher potential

(D) $2RBv$ and Q is at higher potential

Q.3 A metal rod moves at a constant velocity in a direction perpendicular to its length. A constant magnetic field exist in space in a direction perpendicular to the rod as well as its velocity. Select the correct statement (s) from the following. ***(1998)***

(A) The entire rod is at the same electric potential

(B) There is an electric field in the rod

(C) The electric potential is higher at the center of the rod and decrease towards its ends

(D) The electric potential is lowest at the center of the rod and increase towards its ends

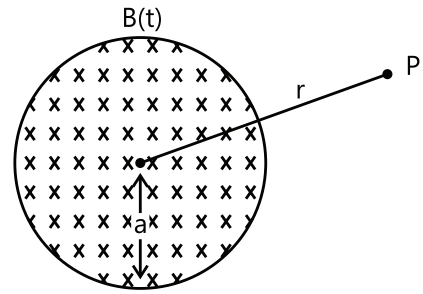
Q.4 A small square loop of wire of side l is placed inside a large square of wire of side L (L>>l). The loops are coplanar and their centers coincide. The mutual inductance of the system is proportional to ***(1998)***

(A) l/L (B) l^2/L (C) L/l (D) L^2/l

Q.5 A coil of inductance 8.4 mH and resistance 6 Ω is connected to a 12 Ω battery. The current in the coil is 1A at approximately the time ***(1999)***

(A) 500s (B) 20s (C) 35 ms (D) 1 ms

Q.6 A uniform but time-varying magnetic field B (t) exists in a circular region a and is directed into the plane of the paper as shown. The magnitude of the induced electric field at point P at a distance r from the center of the circular region ***(2000)***



(A) is zero (B) decreases as 1/r

(C) increases as r (D) decreases as $1/r^2$

Q.7 A coil of wire having finite inductance and resistance has a conducting ring placed co-axially within it. The coil is connected to a battery at time t=0, so that a time dependent current I_1 (t) starts flowing through the coil. I_2 (t) is the current induced in the ring and B (t) is the magnetic field at the axis of the coil due to I_1 (t) then as a function of time (t>0), the product I_2 (t) B(t) ***(2000)***

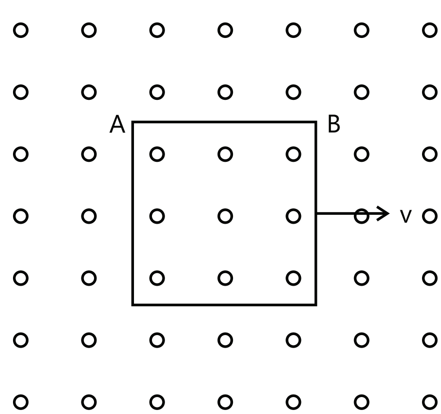
(A) Increases with time

(B) Decreases with time

(C) Does not vary with time

(D) Passes through a maximum

Q.8 A metallic square loop ABCD is moving in its own plane with velocity v in a uniform magnetic field perpendicular to its plane as shown in the Figure 22.84. Electrical field is induced ***(2001)***



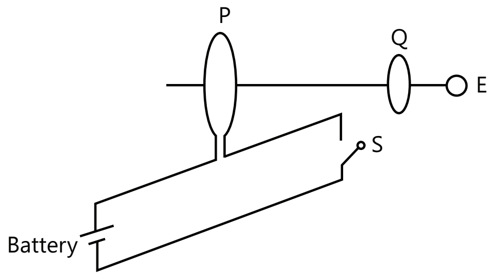
(A) In AD, but not in BC

(B) In BC, but not in AD

(C) Neither in AD nor in BC

(D) In both AD and BC

Q.9 As shown in the Figure, P and Q are two coaxial conducting loops separate by some distance. When the switch S is closed, a clockwise current I_P flows in P (as seen by E) and an induced current I_{Q1} flows in Q. The switch remains closed for a long time. When S is opened, a current I_{Q2} flows in Q. Then the direction I_{Q1} and I_{Q2} (as seen by E) are ***(2002)***



(A) Respectively clockwise and anticlockwise

(B) Both clockwise

(C) Both anticlockwise

(D) Respectively anticlockwise and clockwise

Q.10 A short-circuited coil is placed in a time varying magnetic field. Electric power is dissipated due to the current induced in the coil. If the number of turns were to be quadrupled (four time) and the wire radius halved, the electrical power dissipated would be ***(2002)***

(A) Halved (B) The same

(C) Doubled (D) Quadrupled

Q.11 An electromagnetic wave in vacuum has the electric and magnetic fields $\vec{E}$ and $\vec{B}$, which are always perpendicular to each other. The direction of polarization is given by $\vec{X}$ and that of wave propagation by $\vec{k}$. Then : **(2012)**

(A) $\vec{X}\|\vec{B}$ and $\vec{k}\|\vec{B}\times\vec{E}$ (B) $\vec{X}\|\vec{E}$ and $\vec{k}\|\vec{E}\times\vec{B}$

(C) $\vec{X}\|\vec{B}$ and $\vec{k}\|\vec{E}\times\vec{B}$ (D) $\vec{X}\|\vec{E}$ and $\vec{k}\|\vec{B}\times\vec{E}$

Q.12 A coil is suspended in a uniform magnetic field, with the plane of the coil parallel to the magnetic lines of force. When a current is passed through the coil it starts oscillating; it is very difficult to stop. But if an aluminium plate is placed near to the coil, it stops. This is due to : ***(2012)***

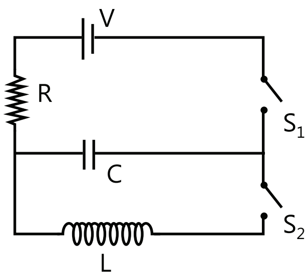
(A) development of air current when the plate is placed.

(B) induction of electrical charge on the plate

(C) shielding of magnetic lines of force as aluminium is a paramagnetic material.

(D) electromagnetic induction in the aluminium plate giving rise to electromagnetic damping.

Q.13 In an LCR circuit as shown below both switches are open initially. Now switch S_1 is closed, S_2 kept open. (q is charge on the capacitor and $\tau = RC$ is capacitive time constant). Which of the following statement is correct? ***(2013)***



(A) At $t=\tau$, $q=CV/2$

(B) At $t=2\tau$, $q=CV(1-e^{-2})$

(C) At $t=\frac{\tau}{2}$, $q=CV(1-e^{-1})$

(D) Work done by the battery is half of the energy dissipated in the resistor.

Q.14 A circular loop of radius 0.3 cm lies parallel to a much bigger circular loop of radius 20 cm. The centre of the small loop is on the axis of the bigger loop. The distance between their centres is 15 cm. If a current of 2.0 A flows through the smaller loop, then the flux linked with bigger loop is ***(2013)***

(A) 6×10^{-11} weber (B) 3.3×10^{-11} weber

(C) 6.6×10^{-9} weber (D) 9.1×10^{-11} weber

Q.15 The magnetic field in a travelling electromagnetic wave has a peak value of 20 nT. The peak value of electric field strength is : ***(2013)***

(A) 6 V/m (B) 9 V/m (C) 12 V/m (D) 3 V/m

Q.16 Match List-I (Electromagnetic wave type) with List-II (Its association / application) and select the correct option from the choices given below the lists: ***(2014)***

	List - I		List - II
(a)	Infrared waves	(i)	To treat muscular strain
(b)	Radio waves	(ii)	For broadcasting
(c)	X-rays	(iii)	To detect fracture of bones
(d)	Ultraviolet rays	(iv)	Absorbed by the ozone layer of the atmosphere

(A) (a) $\rightarrow$ (iii), (b) $\rightarrow$ (ii), (c) $\rightarrow$ (i), (d) $\rightarrow$ (iv)

(B) (a) $\rightarrow$ (i), (b) $\rightarrow$ (ii), (c) $\rightarrow$ (iii), (d) $\rightarrow$ (iv)

(C) (a) $\rightarrow$ (iv), (b) $\rightarrow$ (iii), (c) $\rightarrow$ (ii), (d) $\rightarrow$ (i)

(D) (a) $\rightarrow$ (i), (b) $\rightarrow$ (ii), (c) $\rightarrow$ (iv), (d) $\rightarrow$ (iii)

Q.17 During the propagation of electromagnetic waves in a medium: ***(2014)***

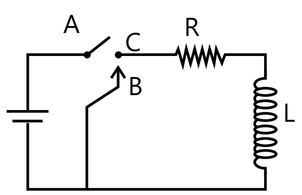
(A) Electric energy density is equal to the magnetic energy density.

(B) Both electric and magnetic energy densities are zero.

(C) Electric energy density is double of the magnetic energy density.

(D) Electric energy density is half of the magnetic energy density.

Q.18 In the circuit shown here, the point 'C' is kept connected to point 'A' till the current flowing through the circuit becomes constant. Afterward, suddenly, point 'C' is disconnected from point 'A' and connected to point 'B' at time t=0. Ratio of the voltage across resistance and the inductor at t=L/R will be equal to : ***(2014)***



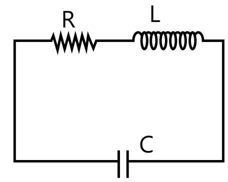
(A) -1 (B) $\frac{1-e}{e}$ (C) $\frac{e}{1-e}$ (D) 1

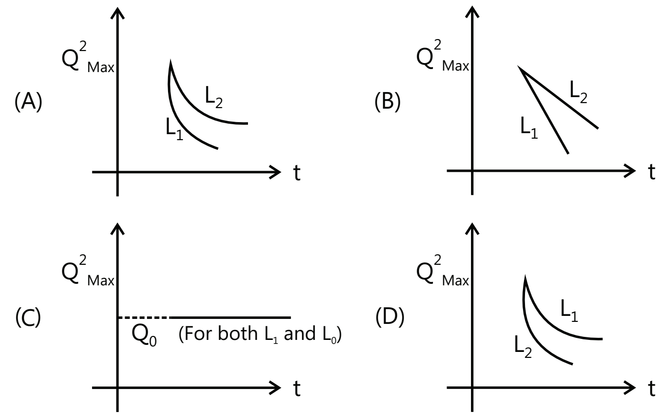
Q.19 An inductor (L=0.03 H) and a resistor (R=0.15 kΩ) are connected in series to a battery of 15 V EMF in a circuit shown. The key K_1 has been kept closed for a long time. Then at t=0, K_1 is opened and key K_2 is closed simultaneously. At t=1 ms, the current in the circuit will be ($e^5 \cong 150$) **(2015)**

(A) 67 mA (B) 6.7 mA

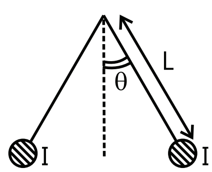
(C) 0.67 mA (D) 100 mA

Q.20 An LCR circuit is equivalent to a damped pendulum. In an LCR circuit the capacitor is charged to Q_0 and then connected to the L and R as shown. If a student plots graphs of the square of maximum charge (Q^2_{Max}) on the capacitor with time (t) for two different values L_1 and L_2 ($L_1 > L_2$) of L then which of the following represents this graph correctly? (Plots are schematic and not drawn to scale) **(2015)**





Q.21 Two long current carrying thin wires, both with current I, are held by insulating threads of length L and are in equilibrium as shown in the figure, with threads making an angle 'θ' with the vertical. If wires have mass λ per unit length then the value of I is: (g=gravitational acceleration) ***(2015)***



(A) $2\sin\theta\sqrt{\frac{\pi\lambda gL}{\mu_0\cos\theta}}$ (B) $2\sqrt{\frac{\pi gL}{\mu_0}\tan\theta}$

(C) $\sqrt{\frac{\pi\lambda gL}{\mu_0}\tan\theta}$ (D) $\sin\theta\sqrt{\frac{\pi\lambda gL}{\mu_0\cos\theta}}$

Q.22 Two identical wires A and B, each of length 'ℓ', carry the same current I. Wire A is bent into a circle of radius R and wire B is bent to form a square of side 'a'. If B_A and B_B are the values of magnetic field at the centres of the circle and square respectively, then the ratio $\frac{B_A}{B_B}$ is: ***(2016)***

(A) $\frac{\pi^2}{16\sqrt{2}}$ (B) $\frac{\pi^2}{16}$ (C) $\frac{\pi^2}{8\sqrt{2}}$ (D) $\frac{\pi^2}{8}$

Q.23 Arrange the following electromagnetic radiations per quantum in the order of increasing energy : ***(2016)***

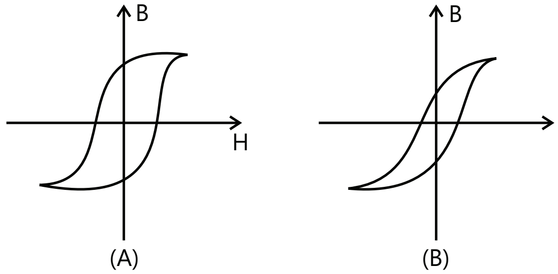
(1) : Blue light (2) : Yellow light

(3) : X-ray (4) : Radiowave

(A) (1), (2), (4), (3) (B) (3), (1), (2), (4)

(C) (2), (1), (4), (3) (D) (4), (2), (1), (3)

Q.24 Hysteresis loops for two magnetic materials A and B are given below :



These materials are used to make magnets for electric generators, transformer core and electromagnet core. Then it is proper to use: **(2016)**

(A) A for electromagnets and B for electric generators

(B) A for transformers and B for electric generators

(C) B for electromagnets and transformers

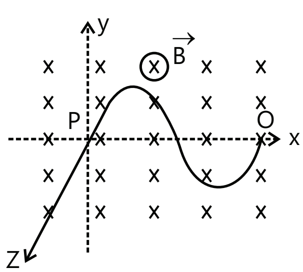
(D) A for electric generators and transformers

JEE Advanced/Boards

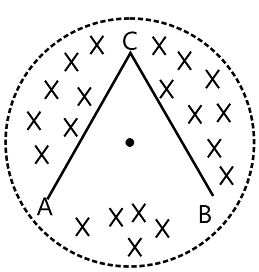
Exercise 1

Q.1 The horizontal component of the earth's magnetic field at a place is 3×10^{-4} T and the dip is $\tan^{-1}$ (4/3). A metal rod of length 0.25 m placed in the north- south position is moved at a constant speed of 10 cm/s towards the east. Find the e.m.f. induced in the rod.

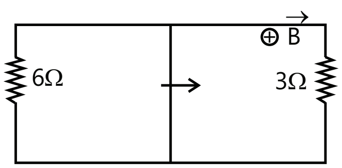
Q.2 A wire forming one cycle sine curve is moved in x-y plane with velocity $\vec{V} = V_x\hat{i}+V_y\hat{j}$. There exist a magnetic field is $\vec{B} = -B_0\hat{k}$. Find the motional e.m.f. develop across the ends PQ of wire.



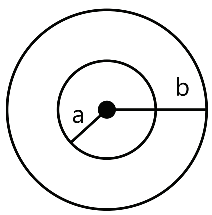
Q.3 A conducting circular loop is placed in a uniform magnetic field of 0.02 T, with its plane perpendicular to the field. If the radius of the loop starts shrinking at a constant rate of 1.0 mm/s, then find the e.m.f. induced in the loop, at the instant when the radius is 4 cm.



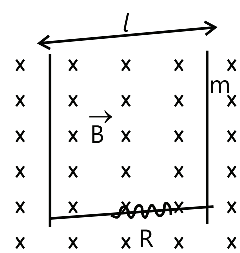
Q.4 A rectangular loop with a sliding connector of length l=1.0 m is situated in a uniform magnetic field B=2T perpendicular to the plane of loop. Resistance of connector is r= 2 Ω. Two resistances of 6 Ω and 3 Ω are connected as shown in Figure. Find the external force required to keep the connector moving with a constant velocity V=2m/s.



Q.5 Two concentric and coplanar circular coils have radii a and b(>>a) as shown in Figure. Resistance of the inner coil is R. Current in the outer coil is increased from 0 to i, then find the total charge circulating the inner coil.



Q.6 A horizontal wire is free to slide on the vertical rails of a conducting frame as shown in Figure. The wire has a mass m and length l and the resistance of the circuit is R. If a uniform magnetic field B is directed perpendicular to the frame, then find the terminal speed of the wire as it falls under the force of gravity.

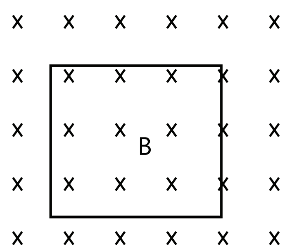


Q.7 A metal rod of resistance 20Ω is fixed along a diameter of a conducting ring of radius 0.1 m and lies on x-y plane. There is a magnetic field B= $(50T)\hat{K}$. The ring rotates with an angular velocity ω=20 rad/s about its axis. An external resistance of 10 Ω is connected across the center of the ring and rim. Find the current through external resistance.

Q.8 A triangular wire frame (each side=2m) is placed in a region of time variant magnetic field

Having dB/dt= $\sqrt{3}$ T/s. The magnetic field is perpendicular to the plane of the triangle. The base of the triangle AB has a resistance 1 Ω while the other two sides have resistance 2 Ω each. The magnitude of potential difference between the points A and B will be.

Q.9 A uniform magnetic field of 0.08 T is directed into the plane of the page and perpendicular to it as shown in the Figure. A wire loop in the plane of the page has constant area 0.010m^2. The magnitude of magnetic field decrease at a constant rate 3×10^{-4} Ts^{-1}. Find the magnitude and direction of the induced e.m.f. in the loop.

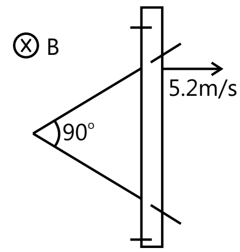


Q.10 There exists a uniform cylindrically symmetric magnetic field directed along the axis of a cylinder but varying with time as B=kt. If an electron is released from rest in this filed at a distance 'r' from the axis of cylinder, its acceleration, just after it is released would be (e and m are the electronic charge and mass respectively)

Q.11 A uniform but time varying magnetic field B=Kt–C; $(0 \le t \le C/K)$, where K and C are constants and t is time, is applied perpendicular to the plane of the circular loop of radius 'a' and resistance R. Find the total charge that will pass around the loop.

Q.12 A charged ring of mass m=50gm, charge 2 coulomb and radius R=2m is placed on a smooth horizontal surface. A magnetic field varying with at a rate of (0.2t) T/ s is applied on to the ring in a direction normal to the surface of ring. Find the angular speed attained in a time t_1=10 s.

Q.13 Two straight conducting rails form a right angle where their ends are joined. A conducting bar contact with the rails starts at vertex at the time t=0 & moves symmetrically with a constant velocity of 5.2 m/s to the right as shown in Figure. A 0.35 T magnetic field points out of the page. Calculate:

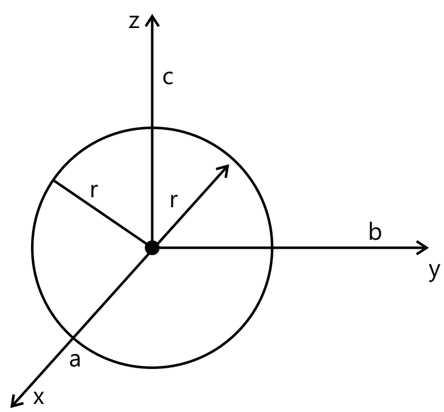


(i) The flux through the triangle by the rails & bar at t=3.0s

(ii) The e.m.f. around the triangle at that time.

(iii) In what manner does the e.m.f. around the triangle vary with time?

Q.14A wire is bent into 3 circular segments of radius r=10cm as shown in Figure. Each segment is a quadrant of a circle, ab lying in the xy plane, bclying in the yz plane & ca lying in the zx plane.



(i) If a magnetic field B points in the positive x direction, what is the magnitude of the e.m.f. developed in the wire, when B increases at the rate of 3 mT/s?

(ii) What is the direction of the current in the segment bc.

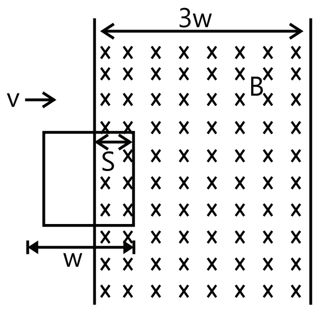
Q.15 Consider the possibility of a new design for an electric train. The engine is driven by the force due to the vertical component of the earth's magnetic field on a conducting axle. Current is passed down one coil, into a conducting wheel through the axle, through another conducting wheel & then back to the source via the other rail.

(i) What current is needed to provide a modest 10-KN force? Take the vertical component of the earth's field be 10 μT & the length of axle to be 3.0 m.

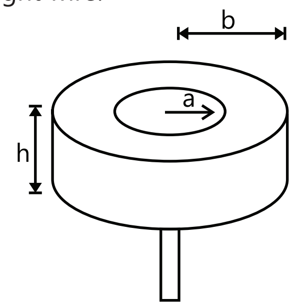
(ii) How much power would be lost for each Ω of resistivity in the rails?

(iii) Is such a train realistic?

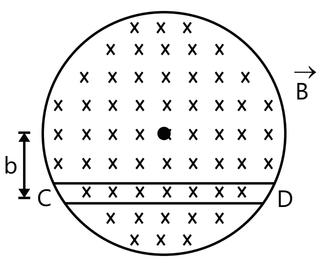
Q.16 A rectangular loop of dimensions l& w and resistance R moves with constant velocity V to the right as shown in the Figure. It continues to move with same speed through a region containing a uniform magnetic field B directed into the plane of the paper &extending a distance 3 W. sketch the flux, induced e.m.f. & external force acting on the as a function of the distance.



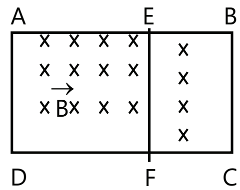
Q.17 A long straight wire is arranged along the symmetry a toroidal coil of rectangular cross-section, whose dimensions are gives in the Figure. The number of turns on the coil is N, and relative permeability of the surrounding medium is unity. Find the amplitude of the e.m.f. induced in this coil, if the current $i=i_m \cos\omega t$ flows along the straight wire.



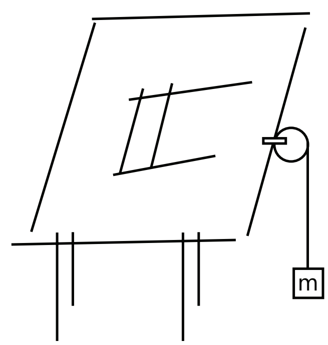
Q.18 A uniform magnetic field B fills a cylindrical volume radius R. A metal rod CD of length l is placed inside the cylinder along a chord of circular cross-section as shown in the Figure. If the magnitude of magnetic field increases in the direction of field at a constant rate dB/dt, find the magnitude and direction of the E.M.F. induced in the rod.



Q.19 A rectangular frame ABCD made of a uniform metal wire has a straight connection between E & F made of the same wire as shown in the figure. AEFD is a square of side 1m & EB= FC=0.5 m. The entire circuit is placed in a steadily increasing uniform magnetic field directed into the place of the paper & normal to it. The rate change of the magnetic field is 1T/s, the resistance per unit length of the wire is 1 Ω/m. Find the current in segments AE, BE &EF.



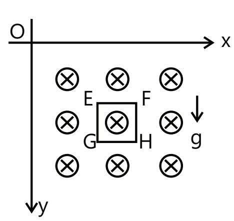
Q.20 A pair o parallel horizontal conducting rails of negligible resistance shorted at one end is fixed on a table. The distance between the rails is L. A conducting massless rod of resistance R can slide on the rails frictionally. The rod is tied to a massless string which passes over a pulley fixed to the edge of the table. A mass m, tied to the other end of the string hangs vertically. A constant magnetic field B exists perpendicular to the table. If the system is released from rest, calculate:



(i) The terminal velocity achieved by the rod.

(ii) The acceleration of the mass at the instant when the velocity of the rod is half the terminal velocity.

Q.21 A magnetic field $B=(B_0 y/a)\hat{k}$ is into the plane of paper in the +z direction. B_0 and a are positive constants. A square loop EFGH of side a, mass m and resistance R, in x-y plane, starts falling under the influence of gravity. Note the directions of x and y axes in the Figure. Find

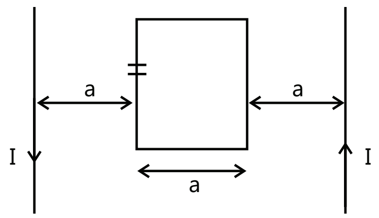


(i) The induced current in the loop and indicated its direction,

(ii) The total Lorentz force acting on the loop and indicated its direction,

(iii) An expression for the speed of the loop, $v(t)$ and its terminal value.

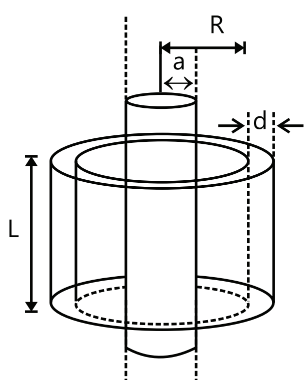
Q.22 A square loop of 'a' with a capacitor of capacitor C is located between two current carrying long parallel wires as shown. The value of I is given as $I=I_0 \sin \omega t$.



(a) Calculate maximum current in the square loop.

(b) Draw a graph between charge on the lower plate of the capacitor v/s time.

Q.23 A long solenoid of radius a and number of turns per unit length n is enclosed by cylindrical shell of radius R, thickness d(d<<R) and length L. A variable current $i=i_0 \sin \omega t$ flows through the coil. If the resistivity of the material of cylindrical shell is ρ, find the induced current in the shell.



Exercise 2

Single Correct Choice Type

Q.1 An electron is moving in a circular orbit of radius R with an angular acceleration α. At the center of the orbit is kept a conducting loop of radius r,(r<<R). The e.m.f. induced in the smaller loop due to the motion of the electron is

(A) Zero, since charge on electron in constant

(B) $\frac{\mu_0 e r^2}{4R}\alpha$

(C) $\frac{\mu_0 e r^2}{4\pi R}\alpha$ (D) none of these

Q.2 A closed planar wire loop of area A and arbitrary shape is placed in a uniform magnetic field of magnitude B, with its plane perpendicular to magnitude to magnetic field. The resistance of the wire loop is R. The loop is now turned upside down by 180° so that its plane again becomes perpendicular to the magnetic field. The total charge that must have flowed through the wire in the process is

(A) <AB/R (B) =AB/R (C) =2AB/R (D) None

Q.3 A square loop of side a and resistance R is moved in the region of uniform magnetic field B (loop remaining completely inside field), with a velocity v through a distance x. The work done is:

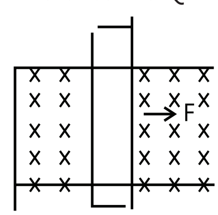
(A) $\frac{B\ell^2 vx}{R}$ (B) $\frac{2B^2\ell^2 vx}{R}$ (C) $\frac{4B^2\ell^2 vx}{R}$ (D) None

Q.4 A metallic rod of length L and mass M is moving under the action of two unequal forces F_1 and F_2 (directed opposite to each other) acting at its ends along its length. Ignore gravity and any external magnetic field. If specific charge of electrons is (e/m), then the potential difference between the ends of the rod is steady state must be

(A) $|F_1-F_2|mL/eM$ (B) $(F_1-F_2)mL/eM$

(C) $[mL/eM] \ln [F_1/F_2]$ (D) None

Q.5 A rod closing the current (shown in Figure) moves along a U shaped wire at a constant speed v under the action of the force F. The circuit is in a uniform magnetic perpendicular to the plane. Calculate F if the rate of heat generation in the circuit is Q.

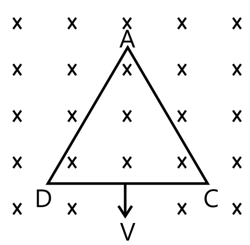


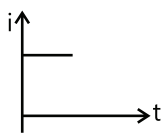
(A) F=Qv (B) $F=\frac{Q}{v}$ (C) $F=\frac{v}{Q}$ (D) $F=\sqrt{Qv}$

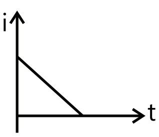
Q.6 Two parallel long straight conductors lie on a smooth surface. Two other parallel conductors rest on them at right angles so as to form a square side a initially. A uniform magnetic field B exists at right angles to the plane containing the conductors. They all start moving out with a constant velocity v. If r is the resistance per unit length of the wire the current in the circuit will be

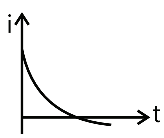
(A) $\frac{Bv}{r}$ (B) $\frac{Br}{v}$ (C) Bvr (D) Bv

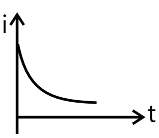
Q.7 An equilateral triangle loop ADC of some finite B as shown in the Figure. At time t=0, side DC of loop is at edge of the magnetic field. Magnetic field is perpendicular to the paper inwards (or perpendicular to the plane of the coil). The induced current versus time graph will be as



(A) 

(B) 

(C) 

(D) 

Q.8 A ring of resistance 10 Ω, radius 10cm and 100 turns is rotated at a rate 100 rev/s about its diameter is perpendicular to a uniform magnetic field of induction 10mT. The amplitude of the current in the loop will be nearly (take: $\pi^2 = 10$)

(A) 200A (B) 2A

(C) 0.002 A (D) None of these

Q.9 A long solenoid of N turns has a self-inductance L and area of cross section A. When a current I flows through the solenoid, the magnetic field inside it has magnetic B. the current I is equal to:

(A) BAN/L (B) BANL

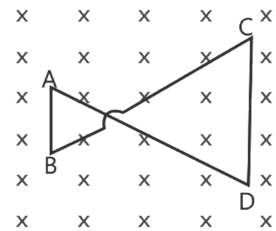
(C) BN/AL (D) B/ANL

Q.10 A small square loop of wire of side l is placed inside a large square loop of wire of side L (L>>l). The loop are co-planner & their centers coincide. The mutual inductance of the system is proportional to:

(A) $\frac{l}{L}$ (B) $\frac{l^2}{L}$ (C) $\frac{L}{l}$ (D) $\frac{L^2}{l}$

Multiple Correct Choice Type

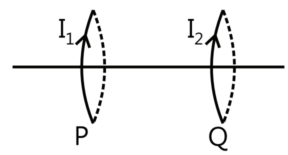
Q.11 A conducting wire is placed in a magnetic field which is directed into the paper. The magnetic field is increasing at a constant rate. The directions of induced currents in wire AB and CD are



(A) B to A and D to C (B) A to B and C to D

(C) A to B and D to C (D) B to A and C to D

Q. 12 Two circular coils P& Q are fixed coaxially & carry currents I_1 and I_2 respectively



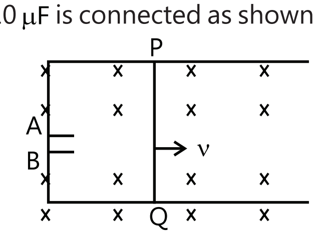
(A) If I_2=0& P moves towards Q, a current in the same direction as I_1 is induced in Q

(B) If I_1=0 & Q moves towards P, a current in the opposite direction to that of I_2 is induced in P.

(C) When $I_1 \neq 0$ and $I_2 \neq 0$ are in the same direction then the two coils tend to move apart.

(D) When $I_1 \neq 0$ and $I_2 \neq 0$ are in opposite directions then the coils tends to move apart.

Q.13 A conducting rod PQ of length L= 1.0 m is moving with a uniform speed v=20 m/s in a uniform magnetic field B=4.0T directed into the paper. A capacitor of capacity C= 10 μF is connected as shown in Figure. Then



(A) q_A=+800 μC and $q_B = -800\mu C$

(B) $q_A = -800\mu C$ and $q_B = +800\mu C$

(C) $q_A = 0 = q_B$

(D) charged stored in the capacitor increases exponentially with time

Q.14 The e.m.f. induced in a coil of wire, which is rotating in a magnetic field, does not depend on

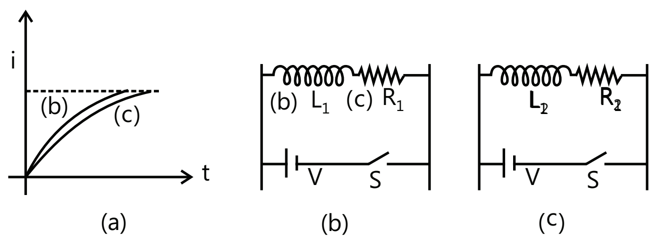
(A) The angular speed of rotation

(B) The area of the coil

(C) The number of turns on the coil

(D) The resistance of the coil

Q.15 Current growth in two L-R circuit (b) and (c) as shown in Figure (a). Let L_1, L_2, R_2 and R_2 be the corresponding value in two circuits, then



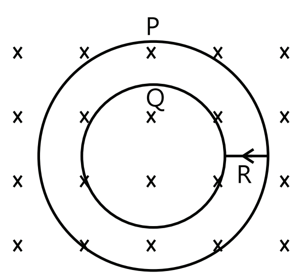
(A) $R_1 > R_2$ (B) $R_1 = R_2$ (C) $L_1 > L_2$ (D) $L_1 < L_2$

Q.16 The dimension of the ratio of magnetic flux and the resistance is equal to that of:

(A) Induced e.m.f. (B) Charge

(C) Inductance (D) Current

Q.17 Figure 22.73 shows a plane figure made of a conductor located in a magnetic field along the inward normal to the plane of the figure. The magnetic field starts diminishing. Then the induced current



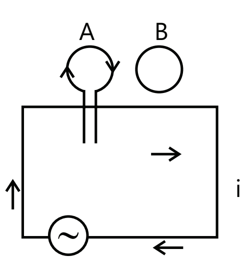
(A) At point P is clockwise

(B) At point Q is anticlockwise

(C) At point Q is clockwise

(D) At point R is zero

Q.18 Two circular coils A and B are facing each other as shown in Figure. The current I through A can be altered



(A) There will be repulsion between A and B if i is increased

(B) There will be attraction between A and B if i is increased

(C) There will be neither attraction nor repulsion when i is changed

(D) Attraction or repulsion between A and B depends on the direction of current. It does not depend whether the current is increased or decreased.

Q.19 A bar magnet is moved along the axis of copper ring placed far away from the magnet. Looking from the side of the magnet, an anticlockwise current is found to be induced in the ring. Which of the following may by true?

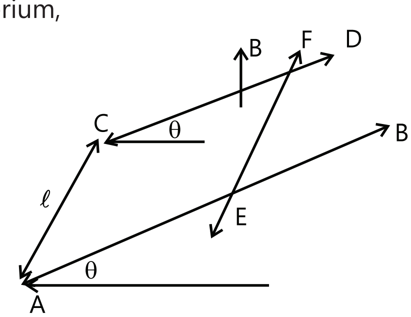
(A) The south pole faces the ring and the magnet moves towards it.

(B) The north pole faces the ring and the magnet moves towards it.

(C) The south pole faces the ring and the magnet moves away from it.

(D) The north pole faces the ring and the magnet moves away from it.

Q.20 AB and CD are smooth parallel rails, separated by a distance l, and inclined to the horizontal at an angel θ. A uniform magnetic field of magnitude B, directed vertically upwards, exists in the region. EF is a conductor of mass m, carrying a current i. For EF to be in equilibrium,



(A) i must flow from E to F (B) Bil=mg tanθ

(C) Bil=mg sinθ (D) Bil=mg

Q.21 In the previous question, if B is normal to the plane of the rails

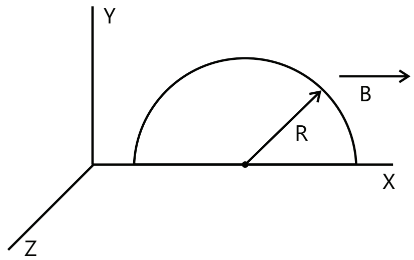
(A) Bil=mg tanθ

(B) Bil=mg sinθ

(C) Bil=mg cosθ

(D) equilibrium cannot be reached

Q.22 A semicircle conducting ring of radius R is placed in the xy plane, as shown in the Figure. A uniform magnetic field is set up along the x-axis. No net e.m.f., will be induced in the ring. If



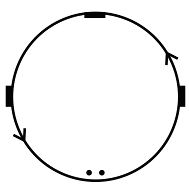
(A) It moves along the x-axis

(B) It moves along the y-axis

(C) It moves along the z-axis

(D) It remains stationary

Q.23 In the given diagram, a line of force of a particular force field is shown. Out of the following options, it can never represent



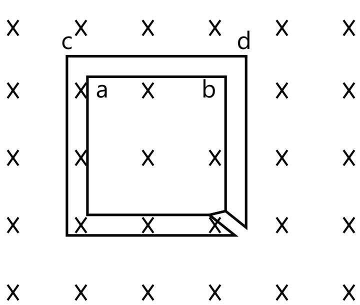
(A) An electrostatic field

(B) A magnetic field

(C) A gravitation field of mass at rest

(D) An induced electric field

Q.24 The Figure shows certain wire segments joined together to form a coplanar loop. The loop is placed in a perpendicular magnetic field in the direction going into the plane of the figure. The magnitude of the field increases with time.I_1 and I_2 are the currents in the segments ab and cd. Then,



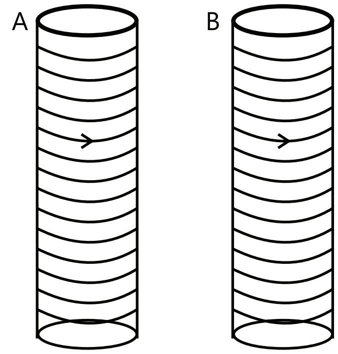
(A) $I_1 > I_2$

(B) $I_1 < I_2$

(C) I_1 is in the direction ba and I_2 is in the direction cd

(D) I_1 is in the direction ab and I_2 is in the direction dc

Q.25 Two metallic rings A and B, identical in shape and size but having different resistivities ρ_A and ρ_B, are kept on top of two identical solenoids as shown in the Figure. When current I is switched on in both the solenoids in identical manner, the rings A and B jump to heights h_A and h_B respectively, with $h_A > h_B$. The possible relation (s) between their resistivity and their masses m_A and m_B is (are)

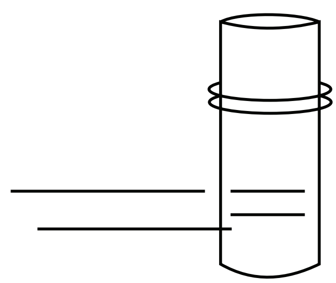


(A) $\rho_A > \rho_B$ and $m_A = m_B$ (B) $\rho_A < \rho_B$ and $m_A = m_B$

(C) $\rho_A > \rho_B$ and $m_A > m_B$ (D) $\rho_A < \rho_B$ and $m_A < m_B$

Assertion Reasoning Type

Q.26 Statement-I: A vertical iron rod has a coil of wire wound over it at the bottom end. An alternating current flows in the coil. The rod goes through a conducting ring as shown in the Figure. The ring can float at a certain height above the coil because



Statement-II: In the above situation, a current is induced in the ring which interacts with the horizontal component of the field to produce an average force in the upward direction.

(A) Statement-I is true, statement-II is true; statement-II is a correct explanation for statement-I.

(B) Statement-I is true, statement-II is true; statement-II is not a correct explanation for statement-I.

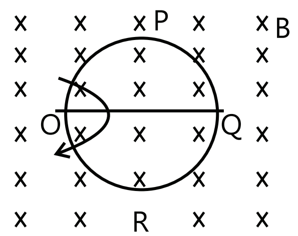
(C) Statement-I is true, statement-II is false

(D) Statement-I is false, statement-II is true

Comprehension Type

Comprehension-I

A conducting ring of radius a is rotated about a point O on its periphery as shown in the Figure on a plane perpendicular to uniform magnetic field B which exists everywhere. The rotational velocity is ω.



Q.27 choose the correct statement (s) related to the potential of the points P, Q and R

(A) $V_P - V_0 > 0$ and $V_R - V_0 < 0$

(B) $V_P = V_R > V_0$

(C) $V_0 > V_P = V_Q$

(D) $V_Q - V_P = V_P - V_0$

Q.28 Choose correct statement (s) related to the magnitude of potential differences

(A) $V_R - V_0 = \frac{1}{2}B\omega a^2$ (B) $V_P - V_Q = \frac{1}{2}B\omega a^2$

(C) $V_Q - V_O = 2B\omega a^2$ (D) $V_P - V_R = 2B\omega a^2$

Q.29 Choose the correct statement(s) related to the induced current in the ring

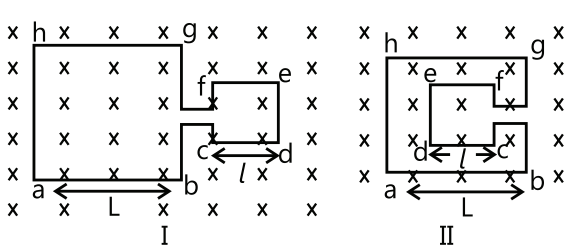
(A) Current flows from $Q \to P \to O \to R \to Q$ (B) Current flows from $Q \to R \to O \to P \to Q$

(C) Current flows from

$Q \to P \to O$ and $Q \to R \to O$

(D) No current flows

Comprehension–II The adjoining Figure 22.80 shows two different arrangements in which two square wire frames of same resistance are placed in a uniform constantly decreasing magnetic field B.



Q.30 The value of magnetic flux in each case is given by

(A) Case I : $\Phi = \pi(L^2 + \ell^2)B$

Case II : $\Phi = \pi(L^2 - \ell^2)B$

(B) Case I : $\Phi = \pi(L^2 + \ell^2)B$

Case II : $\Phi = \pi(L^2 + \ell^2)B$

(C) Case I : $\Phi = (L^2 + \ell^2)B$

Case II : $\Phi = (L^2 - \ell^2)B$

(D) Case I : $\Phi = (L + \ell)^2 B$

Case II : $\Phi = \pi(L - \ell)^2 B$

Q.31 The direction of induced current in the case I is

(A) From a to b and from c to d

(B) From a to b and from f to e

(C) From b to a and from d to c

(D) From b to a and from e to f

Q.32 The direction of induced current in the case II is

(A) From a to b and from c to d

(B) From b to a and from f to e

(C) From b to a and from c to d

(D) From a to b and from d to c

Q.33 If I_1 and I_2 are the magnitudes of induced current in the cases I and II, respectively, then

(A) $I_1 = I_2$ (B) $I_1 > I_2$

(C) $I_1 < I_2$ (D) Nothing can be said

Q.34 Match the Following Columns

	Column 1		Column 2
(A)	Dielectric ring uniform charged	(P)	Time independent electrostatic field out of system
(B)	Dielectric ring uniform charged Rotating with angular velocity.	(Q)	Magnetic field
(C)	Constant current i_0 in ring	(R)	Induced electric filed
(D)	Current $i = i_0 \cos \omega t$ in ring	(S)	Magnetic moment

Previous Years' Questions

Q.1 An infinitely long cylinder is kept parallel to a uniform magnetic field B directed along positive z-axis. The direction of induced as seen from the z-axis will be ***(2005)***

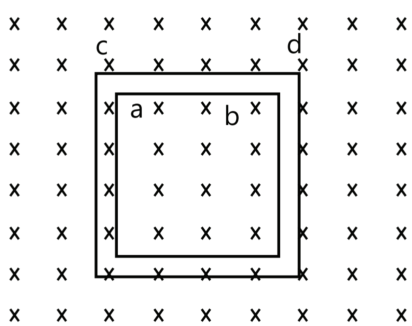
(A) Clockwise of the+ ve z-axis

(B) Anticlockwise of the + ve z-axis

(C) Zero

(D) Along the magnetic field

Q.2 The Figure shows certain wire segment joined together to from a coplanar loop. The loop is placed in a perpendicular magnetic field in the direction going into the plane of the figure. The magnitude of the field. The magnitude of the field increases with time. I_1 and I_2 are the currents in the segments ab and cd. Then, ***(2009)***



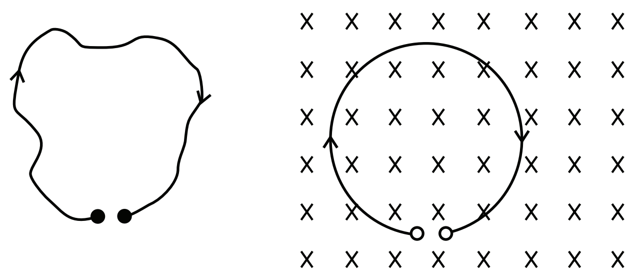
(A) $I_1 > I_2$

(B) $I_1 > I_2$

(C) I_1 is in the direction ba and I_2 is in the direction cd

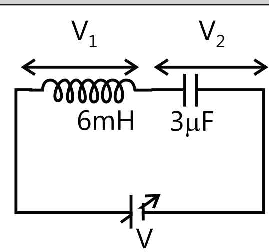
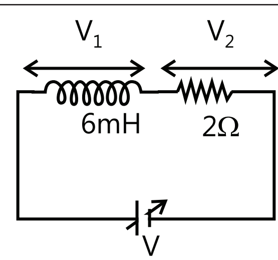
(D) I_1 is in the direction ab and I_2 is in the direction dc

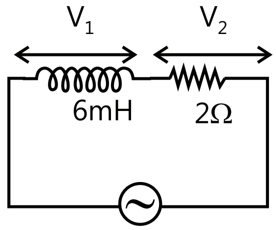
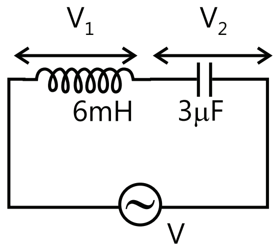
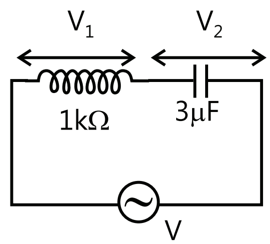
Q.3 A thin flexible wire of length L is connected to two adjacent fixed points and carries a current I in the clockwise direction, as shown in the Figure. When the system is put in a uniform magnetic field of straight B going into the plane of the paper, the wire takes the shape of a circle. The tension in the wire is ***(2010)***



(A) IBL (B) $\frac{IBL}{\pi}$ (C) $\frac{IBL}{2\pi}$ (D) $\frac{IBL}{4\pi}$

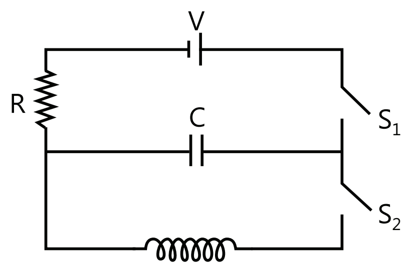
Q.4 You are given many resistances, capacitors and inductors. These are connected to a variable DC voltage source (the first two circuits) or an AC voltage source of 50Hz frequency (the next three circuits) in different ways as shown in column II. When a current I (steady state for DC or rms for AC) flows through the circuit, the corresponding voltage V_1 and V_2 (indicated in circuits) are related as shown in column I. ***(2010)***

Column I	Column I
(A) $I \neq 0, V_1$ is proportional to I	(p) 
(B) $I \neq 0, V_2 > V_1$	(q) 

(C) $V_1 = 0, V_2 = V$	(r) 
(D) $I \neq 0, V_1$ is proportional to I	(s) 
	(s) 

Passage I

The capacitor of capacitance C and be charged (with the help of a resistance R) by a voltage source V, by closing switch S_2 open. The capacitor can be connected in series with an inductor L by closing switch S_2 and opening S_1 (See fig.).



Q.5 Initially, the capacitor was uncharged. Now, switch S_1 is closed and S_2 is kept open. If time constant of this circuit is τ, then ***(2006)***

(A) After time interval τ, charge on the capacitor is CV/2

(B) After time interval 2τ, Charge on the capacitor is CV $(1-e^{-2})$

(C) The work done by the voltage source will be half of the heat dissipated when the capacitor is fully charged

(D) After time interval 2τ, charge on the capacitor is CV $(1-e^{-1})$

Q.6 After the capacitor gets fully charged, S_1 is opened and S_2 is closed that the inductor in series with the capacitor. Then, ***(2006)***

(A) At t=0, energy stored in the circuit is purely in the form of magnetic energy

(B) At any time t>0, current in the circuit is in the same direction

(C) At t>0, there is no exchange of energy between the inductor and capacitor

(d) At any time t>0, maximum instantaneous current in the circuit may be $V\sqrt{\frac{C}{L}}$

Q.7 If the total charge stored in the LC circuit is Q_0, then for $t \geq 0$ ***(2006)***

(A) The charge on the capacitor is $Q = Q_0 \cos\left(\frac{\pi}{2} + \frac{t}{\sqrt{Lc}}\right)$

(B) The charge on the capacitor is $Q = Q_0 \cos\left(\frac{\pi}{2} - \frac{t}{\sqrt{Lc}}\right)$

(C) The charge on the capacitor is $Q = -LC\frac{d^2Q}{dt^2}$

(D) The charge on the capacitor is $Q = -\frac{1}{\sqrt{LC}}\frac{d^2Q}{dt^2}$

Q.8 Two different coils have self-inductances L_1=8 mH and L_2 =2 mH. The current in one coil is increased at a constant rate. The current in the second coil is also increased at the same constant rate. At a certain instant of time, the power given to the coils is the same, At that time, the current the induced voltage and the energy stored in the first soil are i_1,V_1 and W_1 respectively. Corresponding value for the second coil at the same instant are i_2, V_2 and W_2 respectively. ***(1994)***

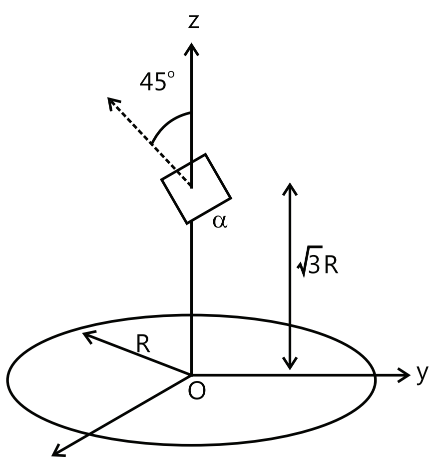
Then

(A) $\frac{i_1}{i_2}=\frac{1}{4}$ (B) $\frac{i_1}{i_2}=4$ (C) $\frac{W_1}{W_2}=\frac{1}{4}$ (D) $\frac{V_1}{V_2}=4$

Q.9 A series R-C circuit is connected to AC voltage source. Consider two cases; (A) when C is without a dielectric medium and (B) when C is filled C is filled with dielectric of constant 4. The current I_R through the resistor and V_C across the capacitor are compared in the two cases. Which of the following is/are true? ***(2011)***

(A) $I_R^A > I_R^B$ (B) $I_R^A < I_R^B$ (C) $V_C^A > I_C^B$ (D) $V_C^A < I_C^B$

Q.10 A circular wire loop of radius R is placed in the x-y plane centered at the origin O. A square loop of side a (a < < R) having two turns is placed with its centre at $z=\sqrt{3}R$ along the axis of the circular wire loop, as shown in figure. The plane of the square loop makes an angle of 45° with respect to the z-axis. If the mutual inductance between the loops is given by $\frac{\mu_0 a^2}{2^{1/2}R}$, then the value of p is ***(2012)***



Q.11 A current carrying infinitely long wire is kept along the diameter of a circular wire loop, without touching it. The correct statement(s) is (are) ***(2012)***

(A) The emf induced in the loop is zero if the current is constant.

(B) The emf induced in the loop is finite if the current is constant

(C) The emf induced in the loop is zero if the current decreases at a steady rate

(D) The emf induced in the loop is finite if the current decreases at a steady rate

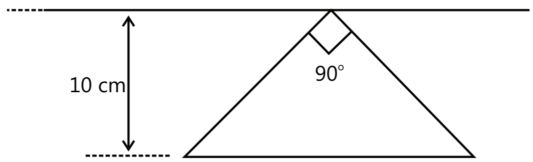
Q.12 If the direct transmission method with a cable of resistance $0.4\ \Omega\ km^{-1}$ is used, the power dissipation (in %) during transmission is ***(2013)***

(A) 20 (B) 30 (C) 40 (D) 50

Q.13 In the method using the transformers, assume that the ratio of the number of turns in the primary to that in the secondary in the step-up transformer is 1 : 10. If the power to the consumers has to be supplied at 200 V, the ratio of the number of turns in the primary to that in the secondary in the step-down transformer is ***(2013)***

(A) 200 : 1 (B) 150 : 1 (C) 100 : 1 (D) 50 : 1

Q.14 A conducting loop in the shape of a right angled isosceles triangle of height 10 cm is kept such that the 90° vertex is very close to an infinitely long conducting wire (see the figure). The wire is electrically insulated from the loop. The hypotenuse of the triangle is parallel to the wire. The current in the triangular loop is in counterclockwise direction and increased at a constant rate of 10 A s^{-1}. Which of the following statement(s) is(are) true? **(2016)**



(A) The magnitude of induced emf in the wire is $\left(\frac{\mu_0}{\pi}\right)$ volt

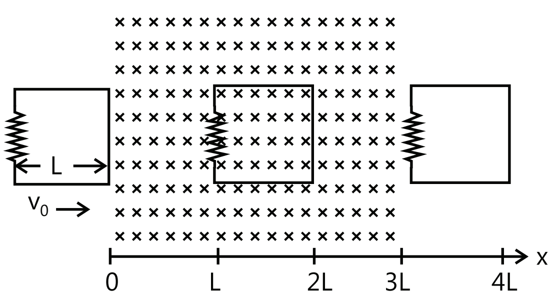
(B) If the loop is rotated at a constant angular speed about the wire, an additional emf of $\left(\frac{\mu_0}{\pi}\right)$ volt is induced in the wire

(C) The induced current in the wire is in opposite direction to the current along the hypotenuse

(D) There is a repulsive force between the wire and the loop

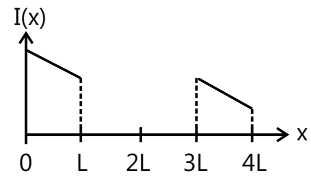
Q.15 Two inductors L_1 (inductance 1 mH, internal resistance 3 Ω) and L_2 (inductance 2 mH, internal resistance 4 Ω), and a resistor R (resistance 12 Ω) are all connected in parallel across a 5V battery. The circuit is switched on at time t=0. The ratio of the maximum to the minimum current (I_{max}/I_{min}) drawn from the battery is ***(2016)***

Q.16 A rigid wire loop of square shape having side of length L and resistance R is moving along the x-axis with a constant velocity v_0 in the plane of the paper. At t=0, the right edge of the loop enters a region of length 3L where there is a uniform magnetic field B_0 into the plane of the paper, as shown in the figure. For sufficiently large v0, the loop eventually crosses the region. Let x be the location of the right edge of the loop. Let v(x), I(x) and F(x) represent the velocity of the loop, current in the loop, and force on the loop, respectively, as a function of x. Counter-clockwise current is taken as positive. ***(2016)***

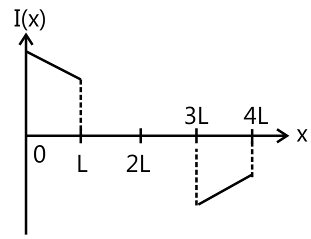


Which of the following schematic plot(s) is(are) correct? (Ignore gravity)

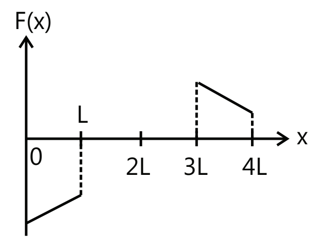
(A)



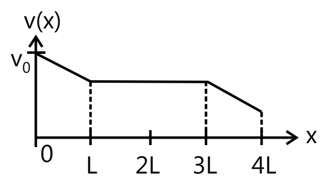
(C)



(B)



(D)



PlancEssential Questions

JEE Main/Boards

Exercise 1

Q.5 Q.8 Q.9

Q.18

Exercise 2

Q. 1 Q.2 Q.8

Q.11 Q.14

JEE Advanced/Boards

Exercise 1

Q.4 Q.7 Q.9

Q.13 Q.14 Q.19

Q.20

Exercise 2

Q.3 Q.4 Q.7

Q.13 Q. 15 Q.24

Q.25 Q.26

Answer Key

JEE Main/Boards

Exercise 1

Q.2 0.4 V

Q.3 8×10^{-5} Wb. 8×10^{-3} Wb

Q.5 5.03×10^{-4} H

Q.6 6.25×10^{-3} H

Q.7 0.1 H

Q.8 Clockwise Direction

Q.9 2.56 V

Q.10 By decreasing current from 2 A to zero in 0.28s

Q.11 1.58×10^{-4} H

Q.12 $\mu_r = 1000$

Q.13 Along PSRQP

Q.16 e_{max}=0.6032 V and e_{av}=0

Q.18 F=0.00375 N

Q.20 (i) Mutual inductance

(ii) The current product in coil B depends on:

(a) Number of turns in the coil

(b) Natural of material

(c) geometry of coil

Q.21 (ii) $\frac{625}{\sqrt{3}}\times10^{-4}$ V

Q.24 (i) Same

(ii) Current in copper loop is more than aluminum loop

Exercise 2

Single correct choice type

Q.1 C	**Q.2** A	**Q.3** A	**Q.4** C	**Q.5** A	**Q.6** D
Q.7 A	**Q.8** A	**Q.9** B	**Q.10** C	**Q.11** A	**Q.12** A
Q.13 C	**Q.14** A	**Q.15** D	**Q.16** D	**Q.17** B	**Q18** B
Q.19 D	**Q.20** C	**Q.21** D	**Q.22** A		

Previous Years' Question

Q.1 D	**Q.2** D	**Q.3** B	**Q.4** B	**Q.5** D	**Q.6** B
Q.7 D	**Q.8** D	**Q.9** D	**Q.10** D	**Q.11** C	**Q.12** D
Q.13 B	**Q.14** D	**Q.15** A	**Q.16** B	**Q.17** A	**Q.18** D
Q.19 C	**Q.20** D	**Q.21** A	**Q.22** C	**Q.23** D	**Q.24** C

JEE Advanced/Boards

Exercise 1

Q.1 10 μV

Q.2 $\lambda V_y B_0$

Q.3 5.0 μV

Q.4 2N

Q.5 $\frac{\mu_0 i a^2 \pi}{2Rb}$

Q.6 $\frac{Rmg}{B^2\ell^2}$

Q.7 $\frac{1}{3}A$

Q.8 0.4V

Q.9 $3\,\mu V$,clockwise

Q.10 $\frac{erk}{2m}$ directed along tangent to the circleof radius r, whose center lies on the axis of cylinder

Q.11 $\frac{\pi a^2 C}{R}$

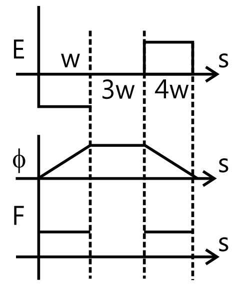
Q.12 200 rad/s

Q.13 (i) 85.22 Tm^2; (ii)56.8 V (iii) Linearly

Q.14 (i) 2.4×10^{-5} V (ii) from c to b

Q.15 (i) 3.3×10^{8} A, (ii) 4.1×10^{7} W, (iii) totally unrealistic

Q.16



Q.17 $\frac{\mu_0 ih\omega i_m N}{2\pi}\ln\frac{b}{a}$

Q.18 $\frac{\ell}{2}\frac{dB}{dt}\sqrt{R^2-\frac{\ell^2}{4}}$

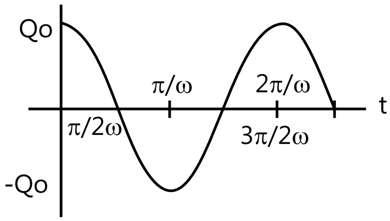
Q.19 $i_{EA}=\frac{7}{22}Al;\ i_{BE}=\frac{3}{11}A;\ i_{FE}=\frac{1}{22}A$

Q.20 (i) $E=\frac{1}{2}B\omega r^2$ (ii) $I=\frac{B\omega r^2\left[1-e^{-Rt/L}\right]}{2R}$

Q.21 (i) $V_{terminal}=\frac{mgR}{B^2L^2}$; (ii) $\frac{g}{2}$

Q.22 (a) $I_{max}=\frac{\mu_0 a}{\pi}CI_0\omega^2\ln 2,$

(b)



Q.23 $I=\frac{(\mu_0 n i_0\cos\omega t)\pi a^2(Ld)}{\rho 2\pi R}$

Exercise 2

Single Correct Choice Type

Q.1 B **Q.2** C **Q.3** D **Q.4** A **Q.5** B **Q.6** A

Q.7 B **Q.8** B **Q.9** A **Q.10** B

Multiple Correct Choice Type

Q.11 A **Q.12** B, D **Q.13** A **Q.14** D **Q.15** B, D **Q.16** B

Q.17 A, B, D **Q.18** A **Q.19** B, C **Q.20** A, B **Q.21** B **Q.22** A, B, C, D

Q.23 A, C **Q.24** D **Q.25** B, D

Assertion Reasoning Type

Q.26 C

Comprehension Type

Q.27 B, D **Q.28** A, C **Q.29** D **Q.30** C **Q.31** C **Q.32** B

Q.33 B

Match the Column Type

Q.34 A → P; B → P, Q, S; C → Q, S; D → Q, R, S

Previous Years' Questions

Q.1 C **Q.2** D **Q.3** C **Q.5** B **Q.6** D **Q.7** C

Q.8 A, C, D **Q.9** B, C **Q.10** 7 **Q.11** A, C **Q.12** B **Q.13** A

Q.14 A, D **Q.15** 8 **Q.16** C, D

Solutions

JEE Main/Boards

Exercise 1

Sol 1: No, as the voltmeter also gets induced emf.

Sol 2: E.m.f.= $\frac{d\phi}{dt} = \frac{nB\,dA}{dt}$

($\Delta A=2A$ as it turned through 180°)

$= \frac{4\times10^{-4}\times10^{3}\times500\times10^{-4}\times2}{\frac{1}{10}} = 0.4V$

Sol 3: $\frac{d\phi}{dt} = L\frac{di}{dt}$

$\phi=Li$; $\phi=20\times10^{-3}\times4\times10^{-3}=80\ \mu\,Wb$

Total flux=$\eta\phi=100\times80\ \mu Wb=8000\ \mu Wb$

Sol 4: Field is perpendicular outwards the paper. As the loop area increases, net flux increases, so induced current tries to reduce flux. So it flow clock wise.

Sol 5: $B=\mu_0 ni$

$\varepsilon=-\frac{dB}{dt}A=-\mu_0 nA\frac{di}{dt}$

$N\in=-M\frac{di}{dt}$

$\mu_0 nNA=M$

$M=4\pi\times10^{-7}\times50\times10^{2}\times200\times4\times10^{-4}=5.03\times10^{-4}$ H

Sol 6 : $\varepsilon=M\frac{\Delta I}{\Delta T}$

$50\times10^{-3}=M.\frac{4}{\frac{1}{2}}$

$M=\frac{50\times10^{-3}}{8}=6.25\times10^{-3}H$

Sol 7: $\varepsilon=L\frac{\Delta I}{\Delta T}$

$4\times10^{4}=L.\frac{4}{10\times10^{-6}}$

L=0.1 Henry

Sol 8: $B=\frac{\mu_0 i}{2\pi r}$

$d\phi=B.dA=B\ell.dr$

$d\phi=\frac{\mu_0 i\ell}{2\pi r}dr$

$\phi = \frac{\mu_0 i\ell}{2\pi}\ln\frac{r_2}{r_1} = \frac{4\times\pi\times10^{-7}\times10\times0.2}{2\pi}\ln\frac{0.1}{0.05}$

$\varepsilon = \frac{d\phi}{dt} = \frac{2.77\times10^{-7}}{2\times10^{-2}} = 1.39\times10^{-5}\text{ v} = 2.77\times10^{-7}\text{Wb}.$

Current will be in clockwise direction.

Sol 9: $\varepsilon = Bn\frac{\Delta A}{\Delta T}$ ($\Delta A = 2A$ as it turns 180º)

$= \frac{Bn2A}{t} = \frac{0.4\times100\times2\times(8\times10^{-2})^2}{0.2} = 2.56\text{V}$

Sol 10: $\varepsilon = -L\frac{\Delta i}{\Delta t}$

$50 = -5\frac{(-2)}{\Delta t}$

T=0.28

Current should reduce to 0 in 0.28.

Sol 11: $B=\mu_0 n_1 i$

$\varepsilon = -L\frac{di}{dt}$

$\varepsilon = n_2\left(-\frac{d}{dt}B.A\right) = n_2\left(-\frac{d}{dt}\mu_0 i n_1 \pi r^2\right) = n_2\mu_0 n_1\pi r^2\frac{di}{dt}$

$\Rightarrow M = \mu_0 n_1 n_2 \pi r^2 = 4\pi\times10^{-7}\times\frac{1000}{100\times10^{-1}}\times100\times\pi(2\times10^{-2})^2$

$= 1.58\times10^{-4}$ H

Sol 12: $L \propto \mu$

$\frac{L_2}{L_1} = \frac{\mu_2}{\mu_1}$

$\mu_1 = \mu_0$, $\mu_2 = \mu_r\mu_0$

$\Rightarrow \mu_r = \frac{L_2}{L_1} = \frac{10}{0.01} = 1000$

$\therefore \mu_r = 1000$

Sol 13: It flows anti-clock wise to increase flux along outside the plane. Hence it flow PSRQP.

Sol 14: $\varepsilon = -L\frac{di}{dt}$

Solenoid tries to go back to initial state i.e. If an action produce a change $\Delta\varepsilon_1$, solenoid tries to produce a change $\Delta\varepsilon_2$ such that $\Delta\varepsilon_2$ is in Opposite direction of $\Delta\varepsilon_1$.

When you remove iron core, L keeps decreasing

$\Rightarrow \Delta i$ increases. i increases

$\therefore$ bulb becomes brighter

After completely removing, the current again decreases as steady state current is $I_0 = \frac{V}{r}$, which was also initial current

Sol 15: The voltage induced across a conductor when it is exposed to a varying magnetic field in called inducedemf.

$\varepsilon = -\frac{d\phi}{dt}$

$\Rightarrow dV = -B\,v(d\,\ell)$

$\Rightarrow dV = -B\omega\,r\,dr$

$V = \frac{B\omega L^2}{2}$

Sol 16: $A = A_0\cos\omega t$

$\phi = BnA = nB.A_0\cos\omega t$

$e = -\frac{d\phi}{dt} = nBA_0\,w\sin\omega t$

$= 20\times3\times10^{-2}\times\pi(8\times10^{-2})^2\times50 = 0.6$ V

$\Rightarrow e_{av} = 0$ as in one complete rotatior, $\Sigma e = 0$

Sol 17: If a current i in a coil changes with time on e.m.f. is induced in the coil. The self – induced rmf is $\epsilon_L = -L\frac{di}{dt}$

$B = \mu_0$ in

$\varepsilon = -\mu_0 n\pi r^2\frac{di}{dt}$ $\therefore L = \mu_0 n\pi r^2$

$\pi = \frac{N}{\ell}$ $\therefore L = \frac{\mu_0 N\pi r^2}{\ell}$

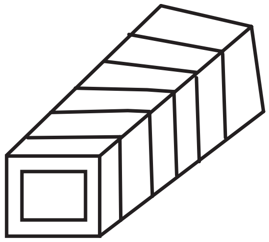
Sol 18: $\varepsilon = B\ell v$

$i = \frac{B\ell V}{R}$

$F = i\ell B = \frac{B^2\ell^2V}{R} = \frac{(0.15)^2(0.5)^2(2)}{3} = 3.75\times10^{-3}$ N

Sol 19: The currents induced in a solid conducting body as it passes through a magnetic field is called eddy current.

Eddy currents lead to heating up of Transformer core. Eddy current is reduced by making transformer with thin slabs.



Sol 20: (i) The Principal involved is mutual inductance

(ii) The current produced in coil B depends on

(a) number of turns in the coil,

(b) Nature of material

(c) geometry of coil

Sol 21: (i) Faraday's law of electromagnetic induction An emf is induced in the loop when the number of magnetic field lines that pass through the loop is changing.

(ii) $\varepsilon = B\ell v \tan\theta$

$= 5\times10^{-4}\times25\times1800\times\dfrac{5}{18}\times\dfrac{1}{\sqrt{3}}$

$= \dfrac{625}{\sqrt{3}}\times10^{-4}$ V

Sol 22: (a) The current induced in a solid conducting body as it passes through a magnetic field is called eddy current. It is used in induction stove, water heaters, etc.

(b) (i) $\in = B\ell v$

(ii) $i = \dfrac{\in}{R} = \dfrac{B\ell v}{R}$; $F = i\ell B = \dfrac{B^2\ell^2 v}{R}$

(iii) Power dissipated $P = \dfrac{\in^2}{R} = \dfrac{B^2\ell^2 v^2}{R}$

Sol 23: If a current i in a coil charges with time, an emf is induced in the coil. The self-inducedemf is $\in_L = -L\dfrac{di}{dt}$
S.I unit Henry–H.

Sol 24: (i) Inducedemf is same

$\varepsilon = 2\pi r^2 w B$

(ii) Current in copper is more, as its resistance is less.

Sol 25: It induces current in opposite direction.

Sol 26: Emf induces Anticlockwise as seen from north. Both Magnets produce current in same direction.

Exercise 2

Sol 1 : (C) $A = \pi R^2 = \pi(R_0 + t)^2$

$\dfrac{dA}{dt} = 2\pi(R_0 + t)$

$\varepsilon = \dfrac{-BdA}{dt} = -2\pi B(R_0 + t)$

$\therefore 2\pi(R_0 + t)B$ is induced anticlockwise.

Note: To have clarity about clockwise or anticlockwise, remember as flux increases, it tries to reduce net magnetic field B. Hence voltage is induced. It leads to current in direction of voltage, which reduces magnetic field.

Sol 2 : (A) $E = \dfrac{BA}{\Delta t}$

$10 = \dfrac{20\times(0.1)^2}{\Delta t}$

$\therefore \Delta t = 20$ ms

Sol 3 : (A) $[MA^{-1}T^{-2}]$

Now $B = \dfrac{\mu_0 i}{2r}$ (for circular wire)

$\Rightarrow [\mu_0] = \dfrac{[B][r]}{[i]} = \dfrac{[MA^{-1}T^{-1}][L]}{[A]} = MLA^{-2}T^{-2}$

Sol 4 : (C) Induced emf tries to push the coil upward in case II and magnet in case-I, to present sudden change in net flux.

$\therefore a_1, a_2 < g$

Sol 5 : (A) For a circular loop B at center is greater than B at any point along the axis.

When both the loops approach each other, magnetic field (B) starts increasing at center. To compensate it, Current decreasing.

Sol 6 : (D) Let the triangle travel a distance x along $\vec{v}$ in time t.

Area of triangle in magnetic field

$A = \dfrac{1}{2}x(2x) = x^2$

$A = v^2t^2$

$E = \dfrac{-BdA}{dt}$

$iR = B\frac{d}{dt}(v^2t^2)$

$i = \frac{2Bv^2}{R}t$

$\therefore i \propto t$

Sol 7 : (A) $E = \left(\vec{v} \times \vec{B}\right).\ell$

$= \left[\left(2\hat{i} + 3\hat{j} + \hat{K}\right) \times \left(\hat{i} + 2\hat{j}\right)\right]\left(2\hat{k}\right) = \left[-2\hat{i} + \hat{j} + \hat{k}\right]\left[2\hat{k}\right]$

E=2V

Sol 8 : (A) $E = \left(\vec{V} \times \vec{B}\right).\ell = \left[10\hat{i} \times \left(4\hat{k}\right)\right]\left(0.3\hat{j}\right) = 12\text{ V}$

Sol 9 : (B) $E = \left(\bar{V} \times \bar{B}\right).\ell$

$B = \frac{\mu_0 I}{2\pi r}$

$E = \frac{\mu_0 I}{2\pi r}.V.\ell$

Sol 10 : (C) $B = B\vec{K}$; $V = v_x\hat{i} + v_y\hat{j}$; $L = \ell_1\hat{i} + \ell_2\hat{j}$

$E = (v \times B).(\ell) = \left[\left(V_x\hat{i} + V_y\hat{j}\right) \times B\hat{k}\right].\left[-\ell\hat{i} + \ell_2\hat{j}\right]$

$= +V_x B\ell,\ \hat{j} + v_y Bl_2\hat{i}$

$\Rightarrow V_A - V_B = V_y B\ell_2$

$V_C - V_B = V_x B\ell_1$

$V_A - V_C = V_y B\ell_2 - V_x B\ell_1$

$\therefore V_A - V_C \propto (V_x\ell_2 - V_y\ell_1)$

Sol 11 : (A) V =2i

$\ell = 5\cos\theta\,\hat{i} + 5\sin\theta\,\hat{j} = 3\hat{i} + 4\hat{j}$

$E = \left(\vec{V} \times \vec{B}\right).\ell = \left[2\hat{i} \times (3\hat{j} + 4\hat{k})\right]\left[3\hat{i} + 4\hat{j}\right]$

$= \left[6\hat{k} - 8\hat{j}\right]\left[3\hat{i} + 4\hat{j}\right] = 32$ Volts

Sol 12 : (A) $\phi = B.dA$

$\phi = B_0\left|1 + \frac{x}{a}\right|d^2$

$\frac{d\phi}{dt} = \frac{d\phi}{dv}.\frac{dx}{dt}$

$= V_0.\frac{B_0 d^2}{a}$

Sol 13 : (C) Let displacement of PQ be x.dx be small displacement along dv

dE=vBdx

$v = x\omega$

$\therefore dE = \omega B\,xdx$

$\Rightarrow E = \frac{WB}{2}x^2\Big|_0^{x_0} \Rightarrow E = \frac{\omega Bx_0^2}{2}$

$x_0^2 = \ell^2 + L^2$

$\therefore E = \frac{\omega B\left(L^2 + \ell^2\right)}{2}$

Sol 14 : (A) Current is from P to Q

(A)

Sol 15 : (D) $\omega t = \frac{\pi}{2} \Rightarrow t = \frac{\pi}{2\omega}$

Avg. E.m.f $= \frac{BDA}{Dt} = \frac{BA}{\frac{\pi}{2}\omega} = \frac{2\omega BA}{\pi}$

Sol 16 : (D) $d\varepsilon = vB\,d\ell$

$V = \ell\omega$

$\therefore d\epsilon = B\omega\ell d\ell$

$E = B\omega\int \ell d\ell = \frac{B\omega\ell^2}{2}\Big|_{\ell_1}^{\ell_2}$

$\ell_2 = L$; $\ell_1 = \frac{L}{2}$

$\therefore \hat{I} = \frac{B\omega}{2}\left(L^2 - \left(\frac{L}{2}\right)^2\right) = \frac{3B\omega\ell^2}{8}$

Sol 17 : (B) Electric field is induced to left

$\therefore$ it accelerates to right (B)

Sol 18 : (B) $B = \frac{\mu_0 i}{2R}$

$\frac{dB}{dt} = \frac{\mu_0}{2R}\frac{di}{dt}$

$E = \frac{-\mu_0}{2R}\frac{di}{dt}.\pi r^2$

$E = -L\frac{di}{dt}$

$\Rightarrow L = \frac{\mu_0\pi r^2}{2R}$

Sol 19 : (D) $E=-\frac{d}{dt}B.dA$

B.dA=0 Q E=0 $\Rightarrow$ L=0

Note: Simply we can say. The magnetic field vectors will be along the plant.

B.dA=0

$\therefore$ E=0

$\Rightarrow$ L=0

Sol 20 : (C) Current increases

$\Rightarrow$ magnetic field increases at a giver point. Magnetic field also decreases radially. Hence to nullify the increases magnetic field, loop B repels.

Sol 21 : (D) $\phi=\int B.dA$ =K inside loop, – K outside loos

$\therefore$ total ϕ=0

Sol 22 : (A) In (a), magnetic field in perpendicular to plane others along plane,

$\therefore$ in others it is minimum, maximum in (a)

Previous Years' Questions

Sol 1 : (D) Net change in magnetic flux passing through the coil is zero.

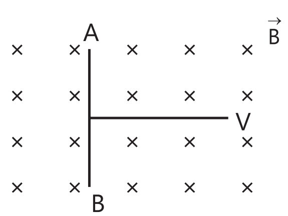
$\therefore$ Current (or emf) induced in the loop is zero

Sol 2 : (D) Induced motional emf in MNQ is equivalent to the motional emf in an imaginary wire MQ i.e.,

$e_{MNQ}=e_{MQ}=Bvl=Bv(2R)$ [ℓ=MQ=2R]

Therefore, potential difference developed across the ring is 2RBv with Q at higher potential.

Sol 3 : (B) A motional emf, e=Blv is induced in the rod. Or we can say a potential difference is induced between the two ends of the rod. AB with A at higher potential and B at lower potential. Due to this potential difference, there is an electric field in the rod.



Sol 4: (B) Magnetic field produced by a current I in a large square loop at its centre,

$B\propto\frac{i}{L}$

Say $B=K\frac{i}{L}$

$\therefore$ Magnetic flux linked with smaller loop

ϕ=B.S

$\phi=\left(K\frac{i}{L}\right)(\ell^2)$

Therefore, the mutual inductance

$M=\frac{\phi}{i}=K\frac{\ell^2}{L}$ or $M\propto\frac{\ell^2}{L}$

Note Dimensions of self-inductance (L) or mutual inductance (M) are:

[Mutual inductance]=[Self-inductance]=[μ_0][length]

Similarly dimensions of capacitance are :

[capacitance]=[ε_0][length]

From this point of view options (b) and (d) may be correct

Sol 5 : (D) The current-time (i – t) equation in L-R circuit is given by [Growth of current in L-R circuit]

$i=i_0(1-e^{-t/t_L})$ ... (i)

where $i_L=\frac{V}{R}=\frac{12}{6}=2$ A

and $i_{0=}\frac{L}{R}=\frac{8.4\times10^{-3}}{6}=1.4\times10^{-3}$ S

and i=1A(given), t=?

Substituting these values in Eq. (i), we get

$t=0.97\times10^{-3}$ s

or t=0.97 ms $\Rightarrow$ t=1 ms

Sol 6 : (B)

$\int\vec{E}\,d\vec{\ell}=\left|\frac{d\varphi}{dt}\right|=S\left|\frac{dB}{dt}\right|$

or $E(2\pi r)=\pi a^2\left|\frac{dB}{dt}\right|$ for $r\geq a$

$\therefore E=\frac{a^2}{2r}\left|\frac{dB}{dt}\right|$

Induced electric field $\propto\frac{1}{r}$

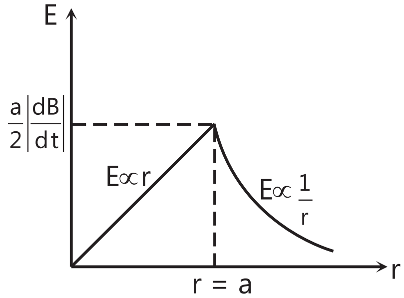
For $r \le a$

$$E(2\pi r)=\pi r^2\left|\frac{dB}{dt}\right|$$

Or $E=\frac{r}{2}\left|\frac{dB}{dt}\right|$ or $E \propto r$

At r=a, $E=\frac{a}{2}\left|\frac{dB}{dt}\right|$

Therefore, variation of E with r (distance from centre) will be as follows



Sol 7: (D) The equations of $\ell_1(t)$, $\ell_2(t)$ and B(t) will take the following form :

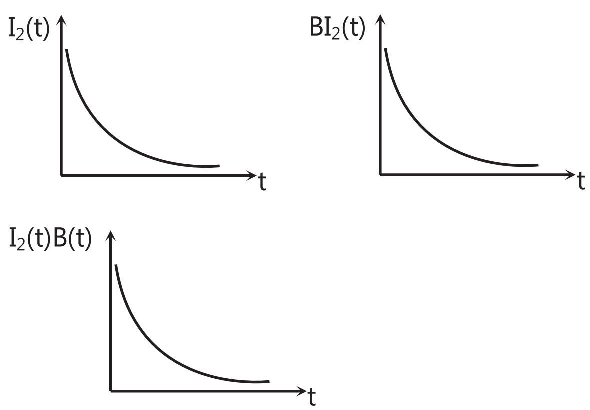
$I_1(t)=K_1(1-e^{-k_2t}) \rightarrow$ current growth in L-R circuit

$B(t)=K_3(1-e^{-k_2t}) \rightarrow (t) \propto I_1(t)$

$I_2(t)=K_4e^{-k_2t}$

$$\left[I_2(t)=\frac{e_2}{R} \text{ and } e_2 \propto \frac{dI_1}{dt} e_2=-m\frac{dI_1}{dt}\right]$$

Therefore, the product $I_2(t)B(t)=K_5e^{-k_2t}(1-e^{-k_2t})$. The value of this product is zero at t=0 and t=∞. Therefore, the product will pass through a maximum value. The corresponding graphs will be as follows :



Sol 8 : (D) Electric field will be induced in both AD and BC.

Sol 9: (D) When switch S is closed magnetic field lines passing through Q increases in the direction from right to left. So according to Lenz's law induced current in Q i.e., I_{Q_1} will flow in such a direction, so that the magnetic field lines duet to I_{Q_1} passes from left to right through Q. This is possible when I_{Q_1} flows in anticlockwise direction as seen by E. Opposite is the case when switch S is opened i.e., I_{Q_2} will be clockwise as seen by E.

Sol 10 : (D) Power $P=\frac{e^2}{R}$

Here, e=induced emf=$-\left(\frac{d\varphi}{dt}\right)$

where ϕ=NBA

$E=-NA\left(\frac{dB}{dt}\right)$

Also, $R \propto \frac{1}{r^2}$

Where R= resistance, r=radius,

ℓ=length.

$\therefore P \propto N^2r^2$

$\therefore \frac{P_2}{P_1}=4$

Sol 11: (C) Direction of polarization is parallel to magnetic field,

$\therefore \quad \vec{X} \parallel \vec{B}$

and direction of wave propagation is parallel to $\vec{E}\times\vec{B}$

$\therefore \quad \vec{K} \parallel \vec{E}\times\vec{B}$

Sol 12 : (D) Oscillating coil produces time variable magnetic field. It cause eddy current in the aluminium plate which causes anti–torque on the coil, due to which is stops.

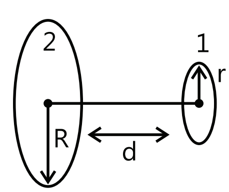
Sol 13 : (B) Charge on the capacitor at any time 't' is

$q=CV(1-e^{-t/\tau})$

At $t=2\tau$

$q=CV(1-e^{-2})$

Sol 14 : (D)



Let M_{12} be the coefficient of mutual induction between loops

$\phi = M_{12}i_2$

$$\Rightarrow \quad \frac{\mu_0 i_2 R^2}{2(d^2+R^2)^{3/2}}\pi r^2 = M_{12}i_2$$

$$\Rightarrow \quad M_{12} = \frac{\mu_0 R^2 \pi r^2}{2(d^2+R^2)^{3/2}}$$

$\phi_2 = M_{12}i_1 \quad \Rightarrow \quad \phi_2 = 9.1\times10^{-11}$ weber

Sol 15 : (A) $E_0 = CB_0 = 3\times10^8 \times 20\times10^{-9} = 6\ V/m$

Sol 16 : (B)

Infrared waves $\rightarrow$ To treat muscular strain

radio waves $\rightarrow$ for broadcasting

X-rays $\rightarrow$ To detect fracture of bones

Ultraviolet rays $\rightarrow$ Absorbed by the ozone layer of the atmosphere;

Sol 17 : (A) Energy is equally divided between electric and magnetic field.

Sol 18 : (D) Since resistance and inductor are in parallel, so ratio will be 1.

Sol 19 : (C) When K_1 is closed and K_2 is open,

$$I_0 = \frac{E}{R}$$

when K_1 is open and K_2 is closed, current as a function of time 't' in L.R. circuit.

$$I = I_0 e^{-R\frac{t}{L}} = \frac{1}{10}e^{-5} = \frac{1}{1500} = 0.67\text{ mA}$$

Sol 20 : (D)

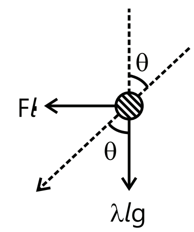
As $L_1 > L_2$, therefore $\frac{1}{2}L_1 i^2 > \frac{1}{2}L^2 > \frac{1}{2}L_2 i^2$,

$\therefore$ Rate of energy dissipated through R from L1 will be slower as compared to L_2.

Sol 21 : (A)

$$\tan\theta = \frac{E\ell}{\lambda\ell g} = \frac{\left(\frac{\mu_0 I^2}{4\pi L\sin\theta}\right)\ell}{\lambda\ell g}$$

$$\Rightarrow \quad I = 2\sin\theta\sqrt{\frac{\pi\lambda Lg}{\mu_0\cos\theta}}$$



Sol 22 : (C)

$$B_A = \frac{\mu_0}{4\pi}\frac{2\pi i}{(\ell/2\pi)}$$

$$B_B = \left[\frac{\mu_0}{4\pi}\frac{i}{\ell/8}(\sin 45^o + \sin 45^o)\right]\times 4$$

$$\frac{B_A}{B_B} = \frac{\pi^2}{8\sqrt{2}}$$

Sol 23 : (D)

Radiation energy per quantum is

E=hϖ

As per EM spectrum, the increasing order of frequency and hence energy is

Radio wave < Yellow light < Blue light < X Ray

Sol 24 : (C) For electromagnet and transformer, the coercivity should be low to reduce energy loss.

JEE Advanced/Boards

Exercise 1

Sol 1 : $e=B\tan\theta \times v.\ell = 3\times10^{-4}\times\frac{4}{3}\times0.1\times0.23$

$=10^{-5}$. V=10μV

Sol 2 : $E = (\overline{V}\times\overline{B}).\ell$

$V= v_x\hat{i} + v_y\hat{i}$

$B=-B_0\hat{k}$

Here $\ell=\lambda\hat{i}$ we us are taking a cross PQ

$\therefore E=[(v_x\hat{i} + v_y\hat{j})\times B_0\hat{k}]\,(\ell i)=\lambda V_y B_0$

Sol 3 : $A=\pi r^2$

$$\frac{dA}{dt} = \frac{dA}{dr}.\frac{dr}{dt} = 2\pi r.\frac{dr}{dt}$$

$$E \Rightarrow = -\frac{-BdA}{dt}$$

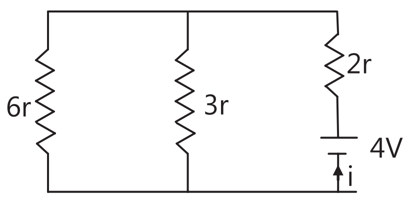
$= - B2\pi r \cdot \frac{dr}{dt} = (0.02) \cdot 2\pi\,(4 \times 10^{-2}) \cdot (1 \times 10^{-3}) = 5\ \mu V$

Sol 4 : Consider the sum as two loops, one with 6r and other 3r.

E=BLV =2 × 1 × 2=4V

Similarly in loop with 3r also 4 V is induced.

hence the circuit can be shown as



$\Rightarrow 1 = \frac{4}{2 + \frac{1}{\frac{1}{6} + \frac{1}{3}}} = 1A$

$\Rightarrow F = i\ell b = 1 \times 1 \times 2 = 2\ N$

Sol 5 : $B = \frac{\mu_0 i}{2r} = \frac{\mu_0 i}{2b}$

Area of small coil $A_i = \pi a^2$

$\in = \frac{d}{dt} BA = \frac{d}{dt} \frac{\pi a^2 . \mu_0 i}{2b}$

$\in = \frac{\pi a^2 \mu_0}{2b} \frac{di}{dt}$

$\in = iR = R\frac{dQ}{dt}$

$\Rightarrow R\frac{dQ}{dt} = \frac{\pi a^2 \mu_0}{2b} \frac{di}{dt}$

$\Rightarrow \Delta Q = \frac{\pi a^2 \mu_0}{2bR} \Delta i = \frac{\pi a^2 \mu_0}{2bR} i$

Sol 6 : Let terminal velocity be V

$E = - B\ell v$

$I = - \frac{b\ell V}{R}$

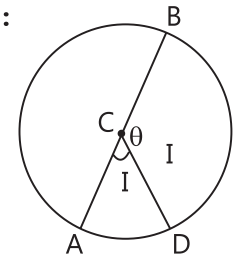
Force due to magnetic field $fi = iLB = - \frac{B^2 \ell^2 v}{R}$

Force due to gravity $(f_2) = mg$

$f_1 + f_2 = 0$

$\Rightarrow mg - \frac{B^2 \ell^2 v}{R} = 0 \Rightarrow v = \frac{Rmg}{B^2 \ell^2}$

Sol 7 :



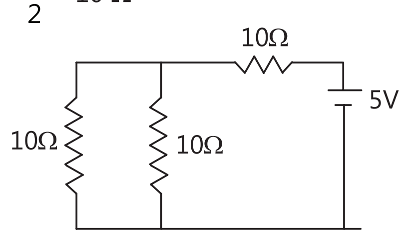
Let AB be diameter rod, CD be external resistor CD is fixed

area of part I be $A_1 = \frac{R^2}{2}\theta$

$E = B\frac{dA}{dt} = \frac{BR^2}{2} \frac{d\theta}{dt}$

$E = \frac{BR^2 w}{2} = \frac{500 \times (0.1)^2 \times 20}{2} = 5\ V$

$R_{AC} = \frac{R_{AB}}{2} = \frac{20}{2} = 10\ \Omega$



$i = \frac{5}{10 + \frac{1}{\frac{1}{10} + \frac{1}{10}}} = \frac{1}{3} A$

Current through external resistance is $\frac{1}{3}A$

Sol 8 : $E = A\frac{dB}{dt} = \frac{\sqrt{3}}{4} a^2 . \frac{dB}{dt} = \frac{\sqrt{3}}{4} (a) 2 . \sqrt{3}$

E=3V

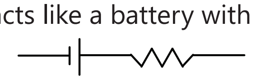
Emf induced is 3V

Current induced $I = \frac{E}{8} = \frac{3}{5} = 0.6\ B$

Voltage induced in each side $V_1 = \frac{E}{3}$

$V_1 = 1\ V$

Now each side acts like a battery with a resists



$\therefore V_{AB} = V1 = iR_{AB} = 1 - 0.6\,(1) = 0.4\ V$

Sol 9 : $\frac{dB}{dt} = - 3 \times 10^{-4}$ (here is taken positive)

$E = - \frac{AdB}{dt}$

$E = -10^{-2} \times (-3 \times 10^{-4}) = 3\ \mu V$

It is induced clock wise.

Note: It you get confused with direction, remember the induced emf produces current, which produces magnetic field. This field will be opposite to direction of change. i.e, if $B_1 = B - \Delta B_1$, then induced B will produces ΔB_2 such that it opposite sign of ΔB_1.

Sol 10 : The electron experiences the force tangentially, along the circular paths of induced emf.

$\Rightarrow E = \pi r^2 . \frac{dB}{dt}$

$E = \pi r^2 K$

$E = 2\pi r . E$

$F = qE$

Acceleration $a = \frac{F}{m}$

$a = \frac{qE}{M}$

$\Rightarrow E = \frac{E}{2\pi r} = \frac{rK}{2}$

$a = \frac{q}{m} E = \frac{q}{m}\frac{rK}{2} = \frac{erK}{2m}$

(charge of electron is e)

Sol 11 : $e = -A\frac{dB}{dt}$

$R\frac{dQ}{dt} = -A\frac{dB}{dt}$

$\Rightarrow \Delta Q = \frac{-A}{R}\Delta B = -\left(\frac{\pi a^2}{R}(-c)\right)$

$\Delta Q = \frac{\pi a^2 C}{R}$

Sol 12 : $e = \pi r^2 . \frac{dB}{dt}$

$E = \frac{E}{2\pi r} = \frac{\pi r^2}{2\pi r} . \frac{dB}{dt}$

$e = \frac{r}{2}\left(\frac{dB}{dt}\right)$

$F = qE = \frac{qr}{2}\left(\frac{dB}{dt}\right)$ (F is tangential at every point)

$F = I\alpha$

$F . r = mr^2 . \alpha$

$\alpha = \frac{F}{mr} = \frac{qr}{2}\left(\frac{dB}{dt}\right) . \frac{1}{mr}$

$\Rightarrow \alpha = \frac{q}{2m}\left(\frac{dB}{dt}\right)$

$\Rightarrow \alpha = \frac{q}{2m}(0.2t)$

$\Rightarrow \frac{d\omega}{dt} = \frac{q}{2m}(0.2t)$

$a = \frac{0.1q}{m}\int_0^t t.d\,t = \frac{0.1q}{m}\frac{t^2}{2} = \frac{0.1 \times 2 \times 10^2}{50 \times 10^{-3} \times 2} = 200$ rad/sec.

Sol 13 : Let perpendicular distance of bar from vertex be x

$x = vt$

Area of triangle $A = \frac{1}{2}x(2x) = x^2$

$A = V^2t^2$

(i) These $\phi = BA$

$\phi(t) = BV^2t^2 = 0.35 \times (5.2)^2 t^2$

$\phi(3) = 9.464\ (3)^2 = 85.22\ Tm^2$

(ii) emf $e = -\frac{d\phi}{dt}$

$C(t) = -2BV^2t = -18.93\ t$

$e(3) = -18.93\ (3) = 56.8\ V$

$|e(3)| = 56.8V$

(iii) $e(t) = -2BV^2t$

$E(t) = Kt$

It varies linearly

Sol 14 : (i) $A = \frac{\pi r^2}{4}$

$\frac{dB}{dt} = 3 \times 10^{-4}$

$e = -A\frac{dB}{dt} = \frac{\pi r^2}{4}\frac{dB}{dt} = \frac{\pi}{4} \times (0.1)^2 . 3 \times 10^{-4}$

$e = 2.4 \times 10^{-5}\ V$

Induced emf is $2.4 \times 10^{-5}V$

(ii)It flows from c to b, to reduce the increasing emf.

Sol 15 : (i) $f = i\ell b$

$10 \times 10^3 = i \times 3 \times 10 \times 10^{-6}$

$i = 3.3 \times 10^8 A$

(ii) $P=i^2R$

$\frac{P}{R}=i^2= 4.1 \times 10^7$ W

(iii) Totally unrealistic

Sol 16 : $\phi=B\ell vt \qquad \left(0<t<\frac{w}{v}\right)$

(downwards positive)

$=B\ell w \qquad \left(\frac{w}{v}<t<\frac{3w}{v}\right)$

$=B\ell v\left(\frac{4w}{v}-t\right) \qquad \left(\frac{3w}{v}<t<\frac{4w}{v}\right)$

$e=\frac{d\phi}{dt}$

$\Rightarrow e=-Blv \qquad \left(0<t<\frac{w}{v}\right)$

$=0 \qquad \left(\frac{w}{v}<t<\frac{3w}{v}\right)$

$=B\ell v \qquad \left(\frac{3\omega}{v}<t<\frac{3\omega}{v}\right)$

$F=i\ell B=\frac{\in}{R}\vec{\ell}B$

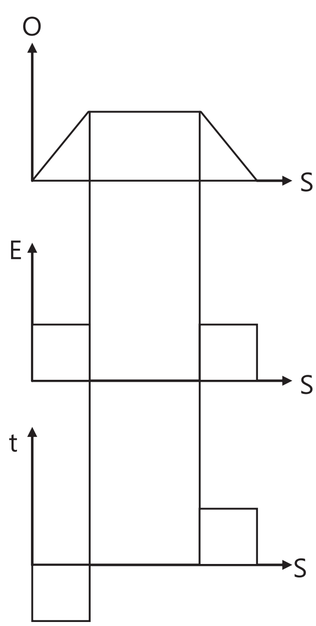
$\Rightarrow E=\frac{-\ell^2B^2V}{R} \qquad \left(0<t<\frac{w}{v}\right)$

$= 0 \qquad \left(\frac{w}{v}<t<\frac{3w}{v}\right)$

$=\frac{-\ell^2B^2v}{R} \qquad \left(\frac{3w}{v}<t<\frac{4w}{v}\right)$

(here $\ell=-\ell$)

x=vt



$e=-B\ell v$ 0< x < w=0 w < x < 3w=$B\ell v$w < x < 4w

Sol 17 : $\int B.ds=\mu_0 ienc$

$B.2\pi r=\mu_0 im$ in $\cos\omega t$

$B=\frac{\mu_0 Nim\cos\omega t}{2\pi r}$

$d\phi=B.dA=B.\ h.dr$

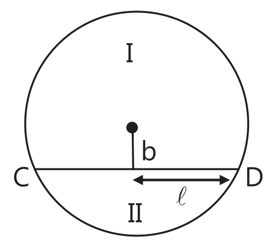
$d\phi=\frac{\mu_0 N\ im\cos\omega t}{2\pi}\ h\frac{dr}{r}$

$\Rightarrow \phi=\frac{\mu_0 N\ im\cos\omega t}{2\pi}\ h\ /\ n\frac{b}{a}$

$\in=\frac{-d\phi}{dt}=\frac{w\mu_0 N\ imh\sin\omega t}{2\pi}\ln\frac{b}{a}$

Amplitude$=\frac{\mu_0 N\omega hi_m}{2\pi}\ \ln\frac{b}{a}$

Sol 18 :



$e=A\frac{dB}{dt}$

$\Rightarrow e_1=A_1\frac{dB}{dt}$

$\Rightarrow e_2=A_2\frac{dB}{dt}$

$\Rightarrow e_1$ is along CD and $\Rightarrow_2$ along DC

$\therefore e\ (A_1-A_2)\frac{dB}{dt}$ along CD

$A_1-A_2=\frac{\ell}{2}\sqrt{R^2-\frac{\ell^2}{4}}$

$\therefore e=\frac{\ell}{2}\sqrt{R^2-\frac{\ell^2}{4}}\frac{dB}{dt}$

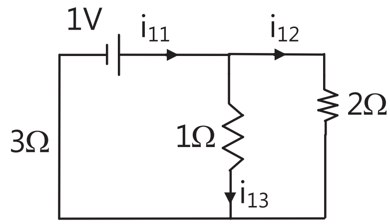
Sol 19 : Take loop AEFD

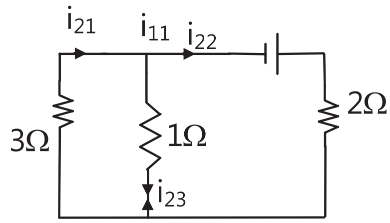
$\Rightarrow e_1=A_1.\frac{DB}{DT}=1\times 1=1V$

Take loop EBCT

$\Rightarrow e_2=A_2\frac{DB}{Dt}=\frac{1}{2}\times 1=0.5$ V

Lets use superposition of current





$i_{AE}=i_{11} + i_{21} i_{EF}=i13 - i_{23} i_{BE}= i_{12} + i_{22}$

$$i_{11}= \frac{e_1}{3+\frac{1}{1+\frac{1}{2}}} = \frac{3}{11}A$$

$$i_{13}=\frac{2}{2+1} i_{11}=\frac{2}{11}A$$

$$i_{12}=i_{11}-i_{13}=\frac{1}{11}A$$

$$i_{22}=\frac{e_2}{2+\frac{1}{1+\frac{1}{3}}}=\frac{2}{11}A$$

$$i_{23}=\frac{3}{3+1} i_{22}=\frac{3}{22}A$$

$$i_{21}=i_{22}-i_{23}=\frac{1}{22}A$$

$$i_{AE}=\frac{3}{11}+\frac{1}{22}=\frac{7}{22}A$$

$$i_{Ef}=\frac{2}{11}-\frac{3}{22}=\frac{1}{22}A$$

$$i_{EB}=\frac{1}{11}+\frac{2}{11}=\frac{3}{11}A$$

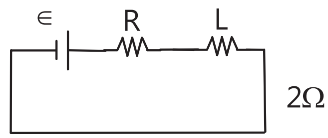
Sol 20 : (i)$d\in=Bvdr$

$de=Br\omega dr$

$$e=\frac{B\omega r^2}{2}$$

Emfacross the terminals of switch is $\frac{B\omega r^2}{2}$

(ii)



$$I=I_0\left(1-e^{\frac{-Rt}{L}}\right)$$

I_0=steady state current

$$=\frac{e}{R}=\frac{B\omega r^2}{2R}$$

$$I=\frac{B\omega r^2}{2R}\left(1-e^{\frac{-Rt}{L}}\right)$$

$dT_m=rdE$

(T_m=torque due to magnetic field)

$dF_m=Bid\ell$

Bidr

(t_m=magnetic force)

$d.T_m=Birdr$

$$\Rightarrow T_m=\frac{Bir^2}{2}$$

$$\Rightarrow T_m=\frac{\omega B^2 g^4}{4R}\left(1-e^{-\frac{Rt}{L}}\right)$$

$f_g=mg\cos\theta$ (f_g=force of graving)

$$T_g=\frac{fgr\ell}{2}=mg\cos\theta\left(\frac{r}{2}\right)=\frac{mgr\cos(\theta)}{2}$$

$$\therefore T=\frac{mgr\cos\theta}{2}+\frac{\omega B^2 r^4}{4R}\left(1-e^{-\frac{Rt}{L}}\right)$$

Sol 21 : e=BLV(V is terminal velocity)

$$i=\frac{e}{R}=\frac{BLV}{R}$$

$F_m=iLB$ (f_m=force due to magnetic field)

$$=\frac{B^2L^2V}{R}$$

$F_g=mg$ (fg=force due to grouts)

$$mg=\frac{B^2L^2V}{R}$$

$\Rightarrow v=\frac{mgR}{B^2L^2}$

(ii) $F_m=\frac{B^2L^2}{R}\left(\frac{V}{2}\right)=\frac{mg}{2}$

$f_g=mg$

$f=f_g-f_m=mg-\frac{mg}{2}=\frac{mg}{2}$

F=ma

$\therefore a=\frac{g}{2}$

$\therefore$acceleration of the mass is $\frac{g}{2}$

(iii) (a) $d\phi=B.dy=\frac{B_0y}{a}$. a. dy

$\phi=\frac{B}{2}\left(y_2^2-y_1^2\right)$

(y_1, y_2 are instantaneous heights of edges parallel to x-axis)

$\phi=\frac{B_0a}{2}(y_2+y_1)\ (\because\ y_2-y_1=a)$

$\frac{d\phi}{dt}=\frac{B_0d}{2}\frac{d}{dt}\ (y_2+y_1)$

$=\frac{B_0a}{2}(2V)\ (v=\frac{dy}{dt}=\frac{dy_1}{dt}=\frac{dy_2}{dt})$

$\frac{d\phi}{dt}=B_0aV$

$i=\frac{\varepsilon}{R}=\frac{d\phi}{dt}.\frac{1}{R}=\frac{B_0aV}{R}$

(b) $f_m=\Sigma i\ell B$ (f_n is magnetic force)

$=i\,a\left(\frac{B_0y_2}{a}-\frac{B_0y_1}{a}\right)=\frac{B_0av}{R}.a\left[\frac{B_0}{a}(y_2-y_1)\right]$

$F_m=\frac{B_0^2a^2v}{R}$

(c)$a=\frac{f_g-f_m}{m}$

$\frac{dv}{dt}=\frac{mg-\frac{B_0^2a^2v}{R}}{m}$

$\frac{dv}{g-\frac{B_0^2a^2v}{mR}}=at$

Integrating on both sides.

$\frac{-mR}{B_0^2a^2}\ln\frac{g-\frac{B_0av}{mR}}{g-\frac{B_0^2a^2v_0}{mR}}=t$

$\Rightarrow v_0=0$ (initially dropped fromrest)

$\ln\frac{g-\frac{B_0^2a^2v}{mR}}{g}=-\frac{B_0^2a^2t}{mR}$

$\Rightarrow\frac{B_0^2a^2v}{mR}=g\left(1-e^{\frac{-B_0^2a^2t}{mR}}\right)$

$\Rightarrow v=\frac{mgR}{B_0^2a^2}\left(1-e^{\frac{-B_0^2a^2t}{mR}}\right)$

Sol 22 : (a) $B=\frac{\mu_0i}{2\pi r}$

Let magnetic field due to upward current be B,

$B_1=\frac{\mu_0I}{2\pi r}$

Force due to it be ϕ,

$d\phi,=\frac{\mu_0I}{2\pi r}$.a.dr

$\phi_1=\frac{\mu_0Ia}{2\pi}\ln\frac{2a}{a}$

$\phi_1=\frac{\mu_0Ia}{2\pi r}\ln 2$

Similarly $\phi_2=\frac{\mu_0Ia}{2\pi}\ln 2$ (ϕ_2 is by downward current)

$\therefore\phi=\phi_1+\phi_2=\mu_0Ia\ln 2$

$\phi=\frac{\mu_0I_0a.\ln2\sin\omega t}{\pi}$

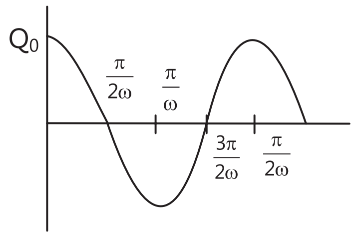
$e=-\frac{d\phi}{dt}=-\frac{\mu_0\omega I_0a\ln2}{\pi}\cos\omega t$

Q=CV

$i=\frac{dQ}{dt}=C\frac{dv}{dt}=C\frac{(\mu_0\omega^2I_0a\ln2)}{\pi}\sin\omega t$

$I_{more}=\frac{\mu_0a}{\pi}CI_0\omega^2\ln2$

(b) $Q=CV=\frac{-\mu_0\omega I_0ac\ln2}{\pi}\cos\omega t$



$Q_0 = \frac{\mu_0 \omega I_0 a.c\ln 2}{\pi}$

Sol 23 : $B = \mu_0$ in

$\phi = B.A = \mu_0$ in πa^2

$\phi = \mu_0 n\pi a^2 i_0 \sin \omega t$

$\in = \frac{-d\phi}{dt} = -\mu_0 n\pi a^2 \omega I_0 \cos \omega t$

Resistance of shell, $r_5 = \frac{e.\ell}{A} = \frac{e.2\pi R}{L.d}$

$i = \frac{\in}{r_s}$

$\therefore I = \frac{(\mu_0 n i_0 \omega \cos \omega t)\pi a^2 (Ld))}{\rho.2\pi R}$

Exercise 2

Sol 1 : (B) Let angular velocity be ω.

$i = -\frac{\omega e}{2\pi}$

$B = \frac{\mu_0 i}{2R} = \frac{-\mu_0 \omega e}{4\pi R}$

$\phi = B.A = \frac{-\mu_0 \omega e}{\ell \pi R} . \pi r^2 = \frac{-\mu_0 \omega e r^2}{4R}$

$\in = \frac{-d\phi}{dt} = \frac{\mu_0 e r^2}{4R} \frac{dw}{dt} = \frac{\mu_0 e r^2 \alpha}{4R}$

Sol 2 : (C) $e = B\frac{dA}{dt} \Rightarrow iR = \frac{BdA}{dt}$

$R . \frac{dQ}{dt} = \frac{BdA}{dt} \Rightarrow dQ = \frac{B}{R} dA$

$\Rightarrow DQ = \frac{B}{R} DA$

DA=2A (as it is rotated by 180º)

$\therefore DQ = \frac{2AB}{R}$

Sol 3 : (D) Work done is zero as magnetic fields is uniform

Sol 4 : (A) Let voltage induced be V.

Total charge $a = \frac{eN}{m}$

Electric field $E = \frac{V}{L}$

$QE = |f_1 - f_2|$

$\frac{eM}{M} . \frac{V}{L} = |f_1 - f_2|$

$V = |f_1 - f_2|$ ML/eM

Sol 5 : (B) Here power Supplied=Heat generated as no other element is using I,

$\Rightarrow F . V = Q \Rightarrow F = \frac{Q}{V}$

Sol 6 : (A) area of loop $A = a^2$

$\frac{dA}{dt} = 2a\frac{da}{dt} = 2a\ (2V) = 4av$

$B\frac{dA}{dt} = iR$

$\Rightarrow i = \frac{B}{R} = (4aV)$

$\Rightarrow R = 4ar$

$\Rightarrow i = \frac{Bv}{r}$

Sol 7 : (B) Let height of triangle be a at time t, area inside the magnetic field

$A = \frac{1}{2}(a - vt)\ \frac{2}{\sqrt{3}}(a - vt) = \frac{(a - vt)^2}{\sqrt{3}}$

$\in = -B\frac{dA}{dt}$

$iR = -B\frac{d}{dt}\frac{(a - vt)^2}{\sqrt{3}}$

$i = \frac{B}{R} . \frac{2}{\sqrt{3}}(a - vt).v$

$i = \frac{2BV}{R\sqrt{3}}(a - vt) = \frac{2BVa}{R\sqrt{3}} - \frac{2BV^2}{R\sqrt{3}}$

$i = C_1 - C_2 t$

Sol 8 : (B) $A=A_0 \cos\theta$

$\theta=\omega t$

$\phi=nBA$

$\phi=nBA_0 \cos \omega t$

$e=\frac{-d\phi}{dt}=n\omega BA_0 \sin \omega t$

$i=\frac{e}{R}=\frac{n\omega BA_0}{R}\sin \omega t$

$\text{Amplitude}=\frac{n\omega BA_0}{R}$

$\omega=100\ (2\pi)=200\ \pi$

$\Rightarrow$Amplitude

$=\frac{100\times 200\pi\times 10\times 10^{-3}\times\pi\times(10^{-1})^2}{10}=2A$

Sol 9 : (A) $e=-L\frac{di}{dt}$

$\Rightarrow -\int e.dt=Li$

$\Rightarrow e=-\frac{d}{dt}BAN$

$\int e\ dt=BAN$

$\Rightarrow Li=BAN$

$i=BAN/L$

Sol 10 : (B) $B\propto\frac{1}{L}$

$A\propto\ell^2$

$\therefore L\propto\frac{\ell^2}{L}$

Sol 11 : (A) Induced current is along DC for loop DC. For loop AB it should be along AB but since area of CD loop is greater than AB loop, hence current is along BA.

(A)

$e=\frac{-dB}{dt}\left(A_{CD}-A_{AB}\right)$

$\therefore$A cos DC

Sol 12 : (B, D) Opposite currents (anti parallel currents) repel

Hence (D)

I_2 induces opposite current to oppose the increase flux

(B)

Sol 13 : (A) $a=B\ell v$

$\frac{Q}{c}=-BLv$

$Q=BlVC=4\times 1\times 20\times 10\times 10^{-6}=800\ \mu C$

Phas greater potential than Q as

[V × B] is directed towards P

Hence q_A is the

$\therefore q_A=+800\ \mu c\ q_B=-800\ \mu c$

Sol 14 : (D) it is independent or resistance

$e=qvB$

$e=q.r\omega B$

Sol 15 : (B, D) $i=i_0\left(1-e^{-\frac{Rt}{L}}\right)$

$i_0=\frac{V}{R}$

i_0 is same,

$\Rightarrow R_1=R_2$

Time constant $t=\frac{L}{R}$

$t_C>t_B$

$\Rightarrow L_2>L_1$

Sol 16 : (B) $\frac{\phi}{R}=\frac{\phi}{t}.\frac{t}{R}=\frac{\in}{R}\times t=it=Q$

Here charge (B)

Sol 17 : (A, B, D) For both P, Q it is induced inward hence clockwise.

iR=0 which is obvious

Sol 18 : (A) If i increases B increases, to reduce B, they repel

Sol 19 : (B, C) Antilockwise means field should increase into plane.

Sol 20 : (A, B) Magnetic force $f_m=i\ell B$

Gravity force $f_g=mg$

$f_m \cos\theta=mg \sin\theta$

$ilB=mg\ \text{Tan}\ \theta$

is show from E to F.

Sol 21 : (B) f_m would be along rats

$f_m = mg \sin\theta$

$\therefore B\ell = mg \sin\theta$

Sol 22 : (A, B, C, D)

B is along the plane of ring Hence, it cannot t be induced, irrespective of directions of motion

Sol 23 : (A, C)

Its common knowledge Reason will be taught in higher classes.

Sol 24 : (D) Induced current is anti-clockwise hence i_2 along dc, i, along ab

$i_1 = i_1$ since there are in same wire

Sol 25 : (B, D) Assume $m_A = m_B$

Then $i_A > i_B$ $(h_A > h_B)$

$\Rightarrow P_A > P_B$

Now is $m_A < m_B$ and $P_A > P_B$ then surely $h_A > h_B$

Sol 26 : (C) Opposite current will induce in the upper ring and it will get repelled by the coil at the bottom

Sol 27 : (B, D)

V α horizontal displacement

$\therefore V_{QP} = V_{PO}, V_{PO} = V_{RO}$

$\therefore V_Q = -V_P = V_P - V_O, V_P = V_R > V_0$

Sol 28 : (A, C) $d\in = Bvdr$

$de = B\omega\, rdr$

$\Rightarrow e = \dfrac{B\omega r^2}{2}$

$V_P - V_0 = \dfrac{B\omega a^2}{2}$

$V_Q - V_0 = \dfrac{Bw(2a)^2}{2} = 2B\omega a^2$

$V_p - V_R = 0$

Sol 29 : (D) No current flows. As it doesn't form a closed circuit.

Sol 30 : (C) $\phi = \int B.dA$

Case I : $A = L^2 + \ell^2$

$\therefore \phi_1 = (L^2 + \ell^2)B$

Case II : $A = L^2 - \ell^2$

$\therefore d = (L^2 - \ell^2)B$

Sol 31 : (C) Clockwise current is induced

Current from ℓ to c and b to a

Sol 32 : (B) Clockwise overall current

$\therefore$ f toe, b to a

Sol 33 : (B) $\phi_2 < \phi_1$

$\therefore I_2 < I_1$

Sol 34 : (A) Di-electric ring which is uniformly charged has stationary any charges. Hence time independent electrostatic field out of system

$A \rightarrow P$

(B) Rotating charge produce magnetic field within the system, and hence induced electric field. But outside remains unchanged as it is

Di-electric

$B \rightarrow P, Q, S$

(C) Current in a ring produces magnetic field hence induced electric field

$C \rightarrow Q, S$

(D) Current carrying ring has magnetic field and induced electric field.

$\mu = I\pi r^2$

$\mu = \pi r^2 I_0 \cos\omega t$

μ charges with time,

$\Rightarrow$ magnetic moment charge Q, R, S

Previous Years' Questions

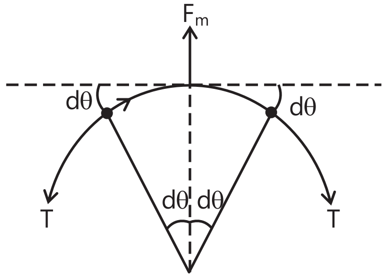
Sol 1 : (C) In uniform magnetic field, change in magnetic flux is zero. Therefore, induced current will be zero.

$\therefore$ correct answer is (c)

Sol 2 : (D)

Cross $\otimes$ magnetic field passing from the closed loop is increasing. Therefore, from Lenz's law induced current will produce dot $\odot$ magnetic field. Hence, induced current is anticlockwise.

Sol 3 : (C)



$L=2\pi R$

$\therefore R=\frac{L}{2\pi}$

$2T\sin(d\theta)=F_m$

From small angles, $\sin(d\theta)=d\theta$

$\therefore 2T(d\theta)=I(dL)B\sin 90°= I(2R.\ d\theta)\ .\ B$

$\therefore\ T=IRB=\frac{ILB}{2\pi}$

$\therefore$Correct option is (c)

Sol 4 : A → r, s, t ; B → q, r, s, t ;

C → q, p ; D → q, r, s, t

In circuit (p) : I can't be non-zero in steady state.

In circuit (q)

$V_1=0$ and $V_2=2I=V$ (also)

In circuit (r): $V_1=X_LI=(2\pi fL)\ I$

$= (2\pi \times 50 \times 6 \times 10^{-3})\ I=1.88\ I$

$V_2=2I$

In circuit (s): $V_1=X_LI=1.88\ I$

$V_2=X_CI=\left(\frac{1}{2\pi fC}\right)I=\left(\frac{1}{2\pi\times 50\times 3\times 10^{-3}}\right)I=(1061)\ I$

In circuit (t):

$V_1=IR=(1000)\ I$

$V_2=X_CI=(1061)\ I$

Therefore the correct options are as under

(A) → r, s, t ; (B) → q, r, s, t ;

(C) → q, p; (D) → q, r, s, t

Sol 5 : (B)

Charge on capacitor at time t is

$q=q_0(1-e^{-t/\tau})$

Here, $q_0=CV$ and $t=2\tau$

$\therefore\ q=CV(1-e^{-2\tau/\tau})=CV(1-e^{-2})$

Sol 6 : (D)

From conservation of energy,

$\frac{1}{2}LI_{max}^2=\frac{1}{2}CV^2$

$\therefore\ I_{max}=V\sqrt{\frac{C}{L}}$

Sol 7 : (C)

Comparing the LC oscillations with normal SHM, we get

$\frac{d^2Q}{dt^2}=-\omega^2Q$

Here, $\omega^2=\frac{1}{LC}\ \ \therefore\ Q=-LC\frac{d^2Q}{dt^2}$

Sol 8 : (A, C, D) From Faraday's law, the induced voltage

$V\propto L$, if rate of change of current is constant $\left(V=-L\frac{di}{dt}\right)$

$\therefore \frac{V_2}{V_1}=\frac{L_2}{L_1}=\frac{2}{8}=\frac{1}{4}$ or $\frac{V_1}{V_2}=4$

Power given to the two coils is same, i.e.

$V_1i_1=V_2i_2$ or $\frac{i_1}{i_2}=\frac{V_2}{V_1}=\frac{1}{4}$

Energy stored $W=\frac{1}{2}Li^2$

$\therefore \frac{W_2}{W_1}=\left(\frac{L_2}{L_1}\right)\left(\frac{i_2}{i_1}\right)^2=\left(\frac{1}{4}\right)(4)^2$or $\frac{W_1}{W_2}=\frac{1}{4}$

Sol 9 : (B, C)

$Z=\sqrt{R^2+X_C^2}=\sqrt{R^2+\left(\frac{1}{\omega C}\right)^2}$

In case (b) capacitance C will be more. Therefore, impedance Z will be less. Hence, current will be more.

$\therefore$ Option (B) is correct

Further,

$V_C=\sqrt{V^2-V_R^2}=\sqrt{V^2-(IR)^2}$

In case (b), since current I is more.

Therefore, V_C will be less.

$\therefore$ Option (C) is correct

$\therefore$Correct options are (B) and (C)

Sol 10 : (7)

Assume circular wire loop as primary and square loop as secondary coil

$$\phi_{secondary} = \frac{2\mu_0 iR^2}{2(3R^2 + R^2)3/2} \times a^2 \times \cos 45^o$$

$$= \frac{\mu_0 iR^2}{2 \times 8R^3} \times a^2 \times \frac{2}{\sqrt{2}}$$

$$M = \frac{\phi_{secondary}}{i} = \frac{\mu_0 a^2}{2^3 \times 2^{1/2} R}$$

$$M = \frac{\mu_0 a^2}{2^{7/2} R}$$

Sol 11 : (A, C)

Total flux associate with loop=0

Therefore emf=0 in any case.

Sol 12 : (B)

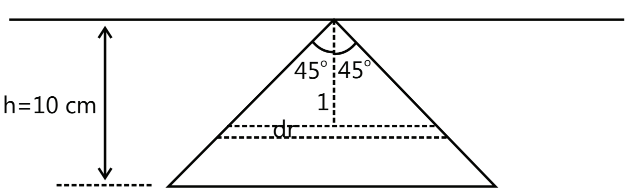
For direct transmission

$$P = i^2R = (150)^2 (0.4 \times 20) = 1.8 \times 10^5\,W$$

Fraction (in %) $= \frac{1.8 \times 10^5}{6 \times 10^5} \times 100 = 30\%$

Sol 13 : (A) $\frac{40000}{200} = 200$

Sol 14 : (A, D)



$$\phi_{\ell w} = \int_0^h \frac{\mu_0 I}{2\pi r} 2rdr = \frac{\mu_0 Ih}{\pi}$$

So, Mutual inductance $M_{\ell w} = \frac{\mu_0 h}{\pi}$

$$\therefore \quad \varepsilon_w = \frac{\mu_0 h}{\pi}\frac{di}{dt} = \frac{\mu_0}{\pi}$$

Due to rotation there is no change in flux through the wire, so there is no extra induced emf in the wire. From Lenz's Law, current in the wire is rightward so repulsive force acts between the wire and loop.

Sol 15 : (8)

At t=0, current will flow only in $12\,\Omega$ resistance

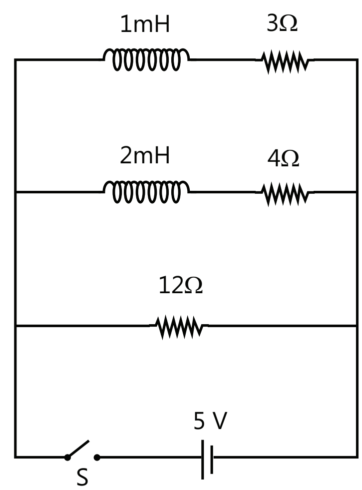
$$\therefore \quad I_{min} = \frac{5}{12}$$

At $t \to \infty$ both L_1 and L_2 behave as conducting wires

$$\therefore \quad R_{eff} = \frac{3}{2}$$

$$I_{max} = \frac{10}{3}$$

$$\frac{I_{max}}{I_{min}} = 8$$



Sol 16 : (C, D)

For right edge of loop from x=0 to x=L

$$i = +\frac{vBL}{R}$$

$$F = iLB = \frac{vB^2L^2}{R} \text{ (leftwards)}$$

$$-mv\frac{dv}{dx} = \frac{vB^2L^2}{R}$$

$$\therefore \quad v(x) = v_0 - \frac{B^2L^2}{mR}x$$

$$i(x) = \frac{v_0 BL}{R} - \frac{B^3L^3}{mR^2}x$$

$$F(x) = \frac{v_0 B^2L^2}{R} - \frac{B^4L^4}{mR^2}x \text{ (leftwards)}$$