Chapter – 1

Sets, Relations and Functions

Ex 1.1

Question 1.
Write the following in roster form.
(i) $\{x \in N : x^2 < 121$ and x is a prime$\}$.
(ii) the set of all positive roots of the equation $(x - 1)(x + 1)(x^2 - 1) = 0$.
(iii) $\{x \in N : 4x + 9 < 52\}$.
(iv) $\{x : x-4x+2 = 3, x \in R - \{-2\}\}$

Solution:
(i) Let $A = \{ x \in N : x^2 < 121$ and x is a prime $\}$
$A = \{2, 3, 5, 7\}$

(ii) The set of positive roots of the equations
$(x - 1) (x + 1) (x^2 - 1) = 0$
$(x - 1) (x + 1) (x + 1) (x - 1) = 0$
$(x + 1)^2 (x - 1)^2 = 0$
$(x + 1)^2 = 0$ or $(x - 1)^2 = 0$
$x + 1 = 0$ or $x - 1 = 0$
$x = -1$ or $x = 1$
$A = \{ 1 \}$

(iii) Let $A = \{ x \in N : 4x + 9 < 52 \}$
When $x = 1$, $(4) \times (1) + 9 = 4 + 9 = 13$
When $x = 2$, $(4) \times (2) + 9 = 8 + 9 = 17$
When $x = 3$, $(4) \times (3) + 9 = 12 + 9 = 21$
When $x = 4$, $(4) \times (4) + 9 = 16 + 9 = 25$
When $x = 5$, $(4) \times (5) + 9 = 20 + 9 = 29$
When $x = 6$, $(4) \times (6) + 9 = 24 + 9 = 33$
When $x = 7$, $(4) \times (7) + 9 = 28 + 9 = 37$
When $x = 8$, $(4) \times (8) + 9 = 32 + 9 = 41$
When $x = 9$, $(4) \times (9) + 9 = 36 + 9 = 45$
When $x = 10$, $(4) \times (10) + 9 = 40 + 9 = 49$
$\therefore A = \{ 1, 2, 3, 4, 5, 6 ,7, 8, 9, 10 \}$

(iv) $\{x : \frac{x-4}{x+2} = 3, x \in R - \{-2\}\}$.

$$\frac{x-4}{x+2} = 3$$

$$\frac{(x-4)(x+2)}{x+2} = 3(x+2)\{\therefore x \neq -2\}$$

(i.e.) $x - 4 = 3(x + 2)$
$x - 4 = 3x + 6$
$-4 - 6 = 3x - x$
$2x = -10 \Rightarrow x = -5$
$A = \{-5\}$

Question 2.
Write the set {-1, 1} in set builder form.

Solution:
$A = \{x : x^2 - 1 = 0, x \in R\}$

Question 3.
State whether the following sets are finite or infinite.

1. $\{x \in N$: x is an even prime number$\}$
2. $\{x \in N$: x is an odd prime number$\}$
3. $\{x \in Z$: x is even and less than 10$\}$
4. $\{x \in R$: x is a rational number$\}$
5. $\{x \in N$: x is a rational number$\}$

Solution:

1. Finite set
2. Infinite set
3. Infinite
4. Infinite
5. Infinite

Question 4.
By taking suitable sets A, B, C, verify the following results:
(i) $A \times (B \cap C) = (A \times B) \cap (A \times C)$.

(if) A × (B ∪ C) = (A × B) ∪ (A × C).
(iii) (A × B) ∩ (B × A) = (A ∩ B) × (B ∩ A).
(iv) C – (B – A) = (C ∩ A) ∪ (C ∩ B).
(v) (B – A) ∩ C = (B ∩ C) – A = B ∩ (C – A).

Solution:
To prove the following results let us take U = {1, 2, 5, 7, 8, 9, 10}
A = {1, 2, 5, 7}
B = {2, 7, 8, 9}
C = {1, 5, 8, 7}

(i) Let A = {1, 2}, B = {3, 4}, C = {4, 5}
B ∩ C = {3, 4} ∩ {4, 5}
B ∩ C = {4}
A × (B ∩ C) = {1, 2} × {4}
A × (B ∩ C) = { (1,4), (2,4) } —– (1)
A × B = {1, 2} × {3, 4}
A × B = { (1,3), (1, 4), (2, 3), (2, 4)}

A × C = {1, 2} × { 4, 5 }
A × C = {(1, 4), (1, 5), (2, 4), (2, 5)}
(A × B) ∩ (A × C) = {(1, 3), (1, 4), (2, 3), (2, 4)} ∩ { (1, 4), (1, 5), (2, 4), (2, 5)}
(A × B) ∩ (A × C) = {(1, 4), (2, 4)} —- (2)
From equations (1) and (2)
A × (B ∩ C) = (A × B) ∩ (A × C)

(ii) To prove A × (B ∪ C) = (A × B) (A × C)
B = {2, 7, 8, 9}, C = {1, 5, 8, 10)
B ∪ C = {1, 2, 5, 7, 8, 9, 10}
A = {1, 2, 5, 7}

A × (B ∪ C) = {(1, 1), (1, 2), (1, 5), (1, 7), (1, 8), (1, 9), (1, 10), (2, 1), (2, 2), (2, 5), (2, 7), (2, 8), (2, 9), (2, 10), (5, 1), (5, 2), (5, 5), (5, 7), (5, 8), (5, 9), (5, 10), (7, 1), (7, 2), (7, 5), (7, 7), (7, 8), (7, 9), (7, 10)) …. (1)
A × B = {(1, 2), (1, 7), (1, 8), (1, 9), (2, 2), (2, 7), (2, 8), (2, 9), (5, 2), (5, 7), (5, 8), (5, 9),

(7, 2), (7, 7), (7, 8), (7, 9)}
A × C = {(1, 1), (1, 5), (1, 8), (1, 10), (2, 1), (2, 5), (2, 8), (2, 10), (5, 1), (5, 5),

$(5, 8), (5, 10), (7, 1), (7, 5), (7, 8), (7, 10)\}$

$(A \times B) \cup (A \times C) = (1, 1), (1, 2), (1, 5), (1, 7), (1, 8), (1,9), (1, 10), (2, 1), (2, 2), (2, 5), (2, 7), (2, 8), (2, 9), (2, 10), (5, 1), (5, 2), (5, 5), (5, 7), (5, 8), (5, 9), (5, 10), (7, 1), (7, 2), (7, 5), (7, 7), (7, 8), (7, 9), (7, 10)\}$ …… (2)
$(1) = (2) \Rightarrow A \times (B \cup C) = (A \times B) \cup (A \times C)$

(iii) Let $A = \{1, 2\}, B = \{2, 3\}$
$A \times B = \{1, 2\} \times \{2, 3\}$
$A \times B = \{(1, 2), (1, 3), (2, 2), (2, 3)\}$
$B \times A = \{2, 3\} \times \{1, 2\}$
$B \times A = \{(2, 1), (2, 2), (3, 1), (3,2)\}$
$(A \times B) \cap (B \times A) = \{(1, 2), (1, 3),(2, 2), (2, 3)\} \cap \{(2, 1), (2, 2), (3, 1),(3, 2)\}$
$(A \times B) \cap (B \times A) = \{(2, 2)\}$ ——- (1)

$A \cap B = \{1, 2\} \cap \{2, 3\}$
$A \cap B = \{2\}$
$B \cap A = \{2, 3\} \cap \{1, 2\}$
$B \cap A = \{2\}$
$(A \cap B) \times (B \cap A) = \{2\} \times \{2\}$
$(A \cap B) \times (B \cap A) = \{(2,2)\}$ ———- (2)
From equations (1) and (2)
$(A \times B) \cap (B \times A) = (A \cap B) \times (B \cap A)$

(iv) To prove $C – (B – A) = (C \cap A) \cup (C \cap B)$
$B – A = \{8, 9\}$
$C = \{1, 5, 8, 10\}$
$\therefore$ LHS $= C – (B – A) = \{1, 5, 10\}$ …… (1)
$C \cap A = \{1\}$
$U = \{1, 2, 5, 7, 8, 9, 10\}$
$B = \{2, 7, 8, 9\} \therefore B' = \{1, 5, 10\}$
$C \cap B = \{1, 5, 10\}$
R.H.S. $(C \cap A) \cup (C \cap B) = \{1\} \cup \{1, 5, 10\}$
$= \{1, 5, 10\}$ ……. (2)
$(1) = (2) \Rightarrow$ LHS = RHS

(v) Let $A = \{1, 2, 3, 4\}$, $B = \{3, 4, 5, 6\}$, $C = \{ 5, 6, 7, 8)$
$B – A = \{3, 4, 5, 6\} – \{1, 2, 3, 4\}$

$B - A = \{5, 6\}$
$(B - A) \cap C = \{5, 6\} \cap \{5, 6, 7, 8\}$
$(B - A) \cap C = \{5, 6\}$ ——- (1)
$(B \cap C) = \{3, 4, 5, 6\} \cap \{5, 6, 7, 8\}$
$B \cap C = \{5, 6\}$
$(B \cap C) - A = \{5, 6\} - \{1,2,3,4\}$
$(B \cap C) - A = \{5, 6\}$ ——- (2)
$C - A = \{5, 6, 7, 8\} - \{1, 2, 3, 4\}$
$C - A = \{5, 6, 7, 8\}$
$B \cap (C - A) = \{3, 4, 5, 6\} \cap \{5, 6, 7, 8\}$
$B \cap (C - A) = \{5, 6\}$ ——- (3)
From equations (1) , (2) and (3)
$(B - A) \cap C = (B \cap C) - A = B \cap (C - A)$

(vi) To prove $(B - A) \cup C = \{1, 5, 8, 9, 10\}$
$B - A = \{8, 9\}$,
$C = \{1, 5, 8, 10\}$
$(B - A) \cup C = \{1, 5, 8, 9, 10\}$ ……. (1)
$B \cup C = \{1, 2, 5, 7, 8, 9, 10\}$
$A - C = \{2, 7\}$
$(B \cup C) - (A - C) = \{1, 5, 8, 9, 10\}$ ……… (2)
(1) = (2)
$\Rightarrow (B - A) \cup C = (B \cup C) - (A - C)$

Question 5.
Justify the trueness of the statement.
"An element of a set can never be a subset of itself."

Solution:
"An element of a set can never be a subset of itself "
The statement is correct
Let $A = \{a, b, c, d\}$ for $a \in A$
'a' cannot be a subset of 'a'

Question 6.
If $n(P(A)) = 1024$, $n(A \cup B) = 15$ and $n(P(B)) = 32$, then find $n(A \cap B)$.

Solution:

$n(P(A)) = 1024 = 2^{10} \Rightarrow n(A) = 10$
$n(A \cup B) = 15$
$n(P(B)) = 32 = 2^5 \Rightarrow n(B) = 5$
We know $n(A \cup B) = n\{A) + n(B) - n(A \cap B)$
(i.e.) $15 = 10 + 5 - n(A \cap B)$
$\Rightarrow n(A \cap B) = 15 - 15 = 0$

Question 7.
If $n(A \cap B) = 3$ and $n(A \cup B) = 10$, then find $n(P(A(A \Delta B))$.

Solution:
Given $n(A \cap B) = 3$ and $n(A \cup B) = 10$
$A \Delta B = (A - B) \cup (B - A)$
$n(A \Delta B) = n [(A - B) \cup (B - A)]$
$n(A \Delta B) = n(A - B) + n(B - A)$ —— (1)

(Since A – B and B – A are disjoint sets)
$A \cup B = (A - B) \cup (B - A) \cup (A \cap B)$
$n(A \cup B) = n[(A - B) \cup (B - A) \cup (A \cap B)]$
$n(A \cup B) = n (A - B) + n (B - A) + n (A \cap B)$
(Since A – B, B – A and A ∩ B are disjoint sets)
$n(A \cup B) = n(A \Delta B) + n(A \cap B)$
$10 = n(A \Delta B) + 3$
$n(A \Delta B) = 10 - 3 = 7$
$\therefore n(P(A \Delta B)) = 2^7 = 128$

Question 8.
For a set A, A × A contains 16 elements and two of its elements are (1, 3) and (0, 2). Find the elements of A.

Solution:
$A \times A = 16$ elements $= 4 \times 4$
$\Rightarrow$ A has 4 elements
$\therefore A = \{0, 1, 2, 3\}$

Question 9.
Let A and B be two sets such that $n(A) = 3$ and $n(B) = 2$. If (x, 1), (y, 2), (z, 1) are in A × B, find A and B, where x, y, z are distinct elements.

Solution:
Given A and B be two sets such that $n(A) = 3$ and $n(B) = 2$.
Also given $(x, 1), (y, 2), (z, 1) \in A \times B$
$A = \{ x, y, z \}, B = \{1, 2\}$

Question 10.
If $A \times A$ has 16 elements, $S = \{(a, b) \in A \times A : a < b\}$; (-1, 2) and (0, 1) are two elements of S, then find the remaining elements of S.

Solution:
$n(A \times A) = 16 \Rightarrow n(A) = 4$
$S = \{(-1, 0), (-1, 1), (0, 2), (1, 2)\}$

Ex 1.2

Question 1.
Discuss the following relations for reflexivity, symmetricity and transitivity:

(i) The relation R defined on the set of all positive integers by "mRn if m divides n".

Solution:
Let $Z = \{1, 2, 3, \ldots\ldots\ldots\}$
R is a relation defined on the set of all positive integers by m R n if m divides n
$R = \{ (m, n) : m/n \text{ for all } m, n \in Z \}$ n

(a) Reflexive:
m divides m for all $m \in Z$
$\therefore (m, m) \in R$ for all $m \in Z$
Hence R is reflexive

(b) Symmetric:
Let $(m, n) \in R \Rightarrow$ m divides n
$\Rightarrow n = km$ for some integers k
But km need not divide m, ie. n need not divide m
$\therefore (n, m) \notin R$
Hence R is not symmetric.

(c) Transitive:
Let (m, n), (n, r) ∈ R
Then m divides n ⇒ n = km and
n divides r ⇒ r = k_1n
r = k_1(km) = (k_1k) m
m divides r
∴ (m, r) ∈ R
Hence R is transitive.

(ii) Let P denote the set of all straight lines in a plane. The relation R defined by "lRm if l is perpendicular to m".

Solution:
P = {set of all straight lines in a plane}
lRm ⇒ l is perpendicular to m

(a) lRl ⇒ l is not perpendicular to l
⇒ It is not reflexive

(b) lRm ⇒ l is perpendicular to m
mRl ⇒ m is perpendicular to l
It is symmetric

(c) l perpendicular to m ⇒ m perpendicular to n ⇒ l is parallel to n It is not transitive

(iii) Let A be the set consisting of all the members of a family. The relation R defined by "aRb if a is not a sister of b".

Solution:
Let F = Father,
M = Mother
G = Male child
H = Female child
A = { F, M, G, H }
The relation R is defined by
a R b if a is not a sister of b.

R = {(F, F), (F, M), (F, G), (F, H), (M, F), (M, M), (M, G), (M, H), (G, F), (G, M), (G, G), (G, H), (H, F), (H, M), (H, H)}

(a) Reflexive:
(F, F) , (M , M), (G, G), (H, H) $\in$ R
$\therefore$ R is reflexive.

(b) Symmetric:
For (G, H) $\in$ R, we have (H, G) $\notin$ R
$\therefore$ R is not symmetric.

(c) Transitive:
Suppose in a family if we take mother M , male child-G and female child-H.
H is not a sister of M $\Rightarrow$ HRM, (H, M) $\in$ R
M is not a sister of G $\Rightarrow$ MRG, (M, G) $\in$ R
But H is a sister of G $\Rightarrow$ HRG, (H, G) $\notin$ R
Thus, for (H, M), (M, G) $\in$ R
we have (H, G) $\notin$ R
$\therefore$ R is transitive.

(iv) Let A be the set consisting of all the female members of a family. The relation R defined by "aRb if a is not a sister of b".

Solution:
A = {set of all female members of a family}

(a) aRa $\Rightarrow$ a is a sister of a
It is reflexive

(b) aRb $\Rightarrow$ a is a sister of b
bRa $\Rightarrow$ b is the sister of a
$\Rightarrow$ It is symmetric

(c) aRb $\Rightarrow$ a is a sister of b bRc $\Rightarrow$ b is a sister of c aRc $\Rightarrow$ a can be the sister of c
It is not transitive.

(v) On the set of natural numbers the relation R defined by "xRy if x + 2y = 1".

Solution:
x + 2y = 1 for x, y ∈ N
There is no x , y ∈ N satisfying x + 2y = 1
∴ The relation R is an empty relation.
An empty relation is symmetric and transitive.
∴ R is symmetric and transitive.
R is not reflexive

Question 2.
Let X = {a, b, c, d} and R = {(a, a), {b, b), (a, c)}. Write down the minimum number
of ordered pairs to be included to R to make it
(i) reflexive
(ii) symmetric
(iii) transitive
(iv) equivalence

Solution:
X = {a, b, c, d}
R = {(a, a), (b, b), (a, c)}

(i) To make R reflexive we need to include (c, c) and (d, d)
(ii) To make R symmetric we need to include (c, a)
(iii) R is transitive
(iv) To make R reflexive we need to include (c, c)

To make R symmetric we need to include (c, c) and (c, a) for transitive
∴ The relation now becomes
R = {(a, a), (b, b), (a, c), (c, c), (c, a)}
∴ R is equivalence relation.

Question 3.
Let A = {a, b, c} and R = {(a, a), (b, b), (a, c)}. Write down the minimum number of ordered pairs to be included to R to make it
(i) reflexive
(ii) symmetric
(iii) transitive

(iv) equivalence

Solution:
Given A = {a, b, c }
R = { (a, a), (b, b),(a, c) }

(i) The minimum ordered pair to be included to R in order to make it reflexive is (c, c).

(ii) The minimum ordered pair to be included to R in order to make it symmetrical is (c, a).

(iii) R is transitive. We need not add any pair.

(iv) After including the ordered pairs (c, c),(c, a) to R the new relation becomes
R_1 = { (a, a), (b, b), (c, c) , (a, c) , (c, a) }
R_1 is reflexive symmetric and transitive.
∴ R_1 is an equivalence relation.

Question 4.
Let P be the set of all triangles in a plane and R be the relation defined on P as aRb if a is similar to b. Prove that R is an equivalence relation.

Solution:
P = {set of all triangles in a plane}
aRb ⇒ a similar to b

(a) aRa ⇒ every triangle is similar to itself
∴ aRa is reflexive

(b) aRb ⇒ if a is similar to b ⇒ b is also similar to a.
⇒ It is symmetric

(c) aRb ⇒ bRc ⇒ aRc
a is similar to b and b is similar to c
⇒ a is similar to a

⇒ It is transitive
∴ R is an equivalence relation

Question 5.
On the set of natural numbers let R be the relation defined by aRb if 2a + 3b = 30. Write down the relation by listing all the pairs. Check whether it is
(i) reflexive
(ii) symmetric
(iii) transitive
(iv) equivalence

Solution:
N = {set of natural numbers}
R ={(3, 8), (6, 6), (9, 4), (12, 2)}

(a) (3, 3) ∉ R ⇒ R is not reflexive
2a + 3b = 30
3b = 30 – 2a

$$b = \frac{30-2a}{3}$$

(b) (3, 8) ∈ R(8, 3) ∉ R
⇒ R is not symmetric

(c) (a, b) (b, c) ∉ R ⇒ R is transitive
∴ It is not an equivalence relation.

Question 6.
Prove that the relation "friendship" is not an equivalence relation on the set of all people in Chennai.

Solution:
If a is a friend of b and b is a friend of c, then a need not be a friend of c.
a R b and b R c does not imply a R c.
∴ R is not transitive.
∴ The relation is not an equivalence relation.

Question 7.
On the set of natural numbers let R be the relation defined by aRb if $a + b \leq 6$. Write down the relation by listing all the pairs. Check whether it is
(i) reflexive
(ii) symmetric
(iii) transitive
(iv) equivalence

Solution:
Set of all natural numbers aRb if $a + b \leq 6$
R= {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (2, 1), (2, 2), (2, 3), (2, 4), (3, 1), (3, 2), (3, 3), (4, 1), (4, 2), (5, 1)}
(i) $(5, 1) \in R$ but $(5, 5) \notin R$
It is not reflexive

(ii) aRb ⇒ bRa ⇒ It is symmetric

(iii) $(4, 2), (2, 3) \in R \Rightarrow (4, 3) \notin R$
∴ It is not transitive

(iv) ∴ It is not an equivalence relation

Question 8.
Let A = {a, b, c}. What is the equivalence relation of smallest cardinality on A? What is the equivalence relation of largest cardinality on A?

Solution:
A = { a, b, c }
Let R_1 = { (a, a),(b, b),(c, c) }
Clearly, R_1 is reflexive, symmetric, and transitive.
Thus R_1 is the equivalence relation on A of smallest cardinality, $n(R_1) = 3$
Let R_2 = { (a, a), (b, b), (c, c), (a, b), (b, a), (b, c), (c, b), (c, a), (a, c)}

(i) Reflexive:
(a, a) , (b, b) , (c, c) ∈ R
∴ R_2 is reflexive.

(ii) Symmetric:

$(a, b) \in R_2$ we have $(b, a) \in R_2$
$(b, c) \in R_2$ we have $(c, b) \in R_2$
$(c, a) \in R_2$ we have $(a, c) \in R_2$
$\therefore R_2$ is symmetric.

(iii) Transitive:
$(a, b), (b, c) \in R_2 \Rightarrow (a, c) \in R_2$
$(b, c), (c, a) \in R_2 \Rightarrow (b, a) \in R_2$
$(c, a), (a, b) \in R_2 \Rightarrow (c, b) \in R_2$
$\therefore R_2$ is transitive and R_2 is an equivalence relation of largest cardinality.
$n(R_2) = 9$

Question 9.
In the set Z of integers, define mRn if m – n is divisible by 7. Prove that R is an equivalence relation.

Solution:
mRn if m – n is divisible by 7
(a) mRm = m – m = 0
0 is divisible by 7
∴ It is reflexive

(b) mRn = {m – n) is divisible by 7
nRm = (n – m) = – {m – n) is also divisible by 7
It is symmetric

(c) $m\text{R}n \Rightarrow (m-n)$ is divisible by $7 = \frac{(m-n)}{7} = \frac{k}{7}$

$n\text{R}r \Rightarrow (n-r)$ is divisible by $7 = \frac{(n-r)}{7} = \frac{l}{7}$

$$\Rightarrow m\text{R}r = m - r = \left(\frac{k}{7} + n\right) - \left(n - \frac{l}{7}\right)$$

$$m - r = \frac{k}{7} + n - n + \frac{l}{7}$$

$$(m-r) = \frac{1}{7}(k+l) \text{ is divisible by } 7$$

It is transitive
mRn if m – n is divisible by 7
∴ R is an equivalence relation.

Ex 1.3

Question 1.
Suppose that 120 students are studying in 4 sections of eleventh standard in a school. Let A denote the set of students and B denote the set of the sections. Define a relation from A to B as "x related toy if the student x belongs to the section y". Is this relation a function? What can you say about the inverse relation? Explain your answer.

Solution:
(i) A = {set of students in 11^{th} standard}
B = {set of sections in 11sup>th standard}
R : A → B ⇒ x related to y
⇒ Every students in eleventh Standard must in one section of the eleventh standard.
⇒ It is a function.
Inverse relation cannot be a function since every section of eleventh standard cannot be related to one student in eleventh standard.

Question 2.
Write the values of f at – 4, 1, -2, 7, 0 if

$$f(x) = \begin{cases} -x+4 & \text{if } -\infty < x \le -3 \\ x+4 & \text{if } -3 < x < -2 \\ x^2 - x & \text{if } -2 \le x < 1 \\ x - x^2 & \text{if } 1 \le x < 7 \\ 0 & \text{otherwise} \end{cases}$$

Solution:
$f(-4) = -(-4) + 4 = 8$
$f(1) = 1 - 1^2 = 0$
$f(-2) = (-2)^2 - (-2) = 4 + 2 = 6$
$f(7) = 0$
$f(0) = 0$

Question 3.
Write the values of f at -3, 5, 2, -1, 0 if

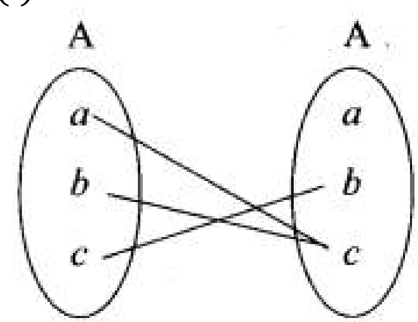
$$f(x) = \begin{cases} x^2 + x - 5 & \text{if } x \in (-\infty, 0) \\ x^2 + 3x - 2 & \text{if } x \in (3, \infty) \\ x^2 & \text{if } x \in (0, 2) \\ x^2 - 3 & \text{otherwise} \end{cases}$$

Solution:
$f(-3) = (-3)^2 - 3 - 5 = 9 - 8 = 1$
$f(5) = (5)^2 + 3(5) - 2 = 25 + 15 - 2 = 38$
$f(2) = 4 - 3 = 1$
$f(-1) = (-1)^2 + (-1) - 5 = 1 - 6 = -5$
$f(0) = 0 - 3 = -3$

Question 4.
State whether the following relations are functions or not. If it is a function check for one-to-oneness and ontoness. If it is not a function, state why?

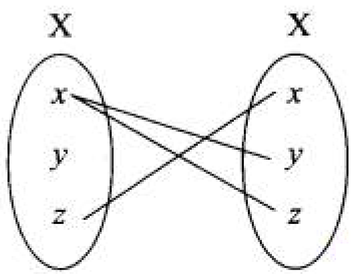
(i) If A = {a, b, c] and/= {(a, c), (b, c), (c, b)};(f: A → A).
(ii) If X = {x, y, z} and/= {(x, y), (x, z), (z, x)}; (f: X → X).

Solution:
(i) f : A → A



It is a function but it is not 1 – 1 and not onto function.

(ii) f : X → X

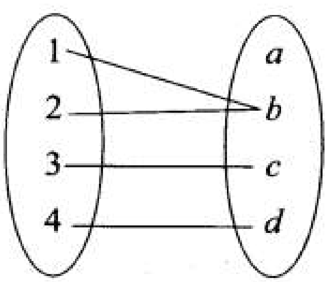


x ∈ X (Domain) has two images in the co-domain x. It is not a function.

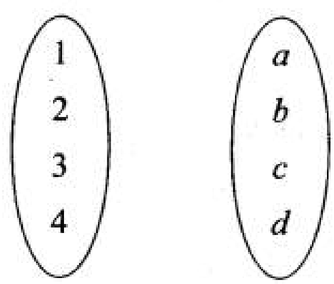
Question 5.
Let A = {1, 2, 3, 4} and B = {a, b, c, d}. Give a function from A → B for each of the following:
(i) neither one-to-one nor onto.
(ii) not one-to-one but onto.
(iii) one-to-one but not onto.
(iv) one-to-one and onto.

Solution:
A = {1, 2, 3, 4}
B = {a, b, c, d}.

(i)



(ii)

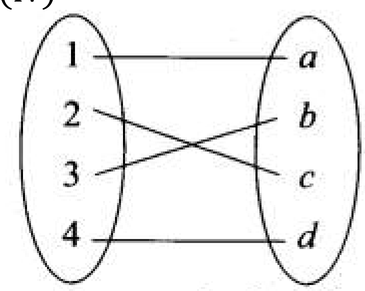


Not possible

R = {(1, b) (2, b) (3, c) (4, d)} is not 1-1 and not onto

(iii) Not possible

(iv)



1 – 1 and onto

Question 6.

Find the domain of $\frac{1}{1-2\sin x}$

Solution:

The Denominator $\neq 0 \Rightarrow 1 - 2\sin x \neq 0$

$\Rightarrow \sin x \neq \frac{1}{2}$ (i.e.,) $x \neq \frac{\pi}{6}$

So domain $= \mathrm{R} - \{n\pi + (-1)^n \frac{\pi}{6}\}, n \in \mathrm{Z}$

Question 7.

Find the largest possible domain of the real valued function f(x) = $\frac{\sqrt{4-x^2}}{\sqrt{x^2-9}}$

Solution:

$$f(x) = \frac{\sqrt{4-x^2}}{\sqrt{x^2-9}}$$

Here $\sqrt{4-x^2} > 0$	Here $\sqrt{x^2-9} > 0$
$4 - x^2 > 0$	$x^2 - 9 > 0$
$\therefore x^2 > 4$	$\therefore x^2 > 9$
$x < 2$ and $x > -2$	$x > 3$ and $x < -3$

∴ No largest possible domain

The domain is null set

Question 8.

Find the range of the function $\frac{1}{2\cos x - 1}$

Solution:

The range of cos x is – 1 to 1

$$-1 \le \cos x \le 1$$

$$(\times \text{ by } 2)\ -2 \le 2\cos x \le 2$$

adding –1 throughout

$$-2 - 1 \le 2\cos x - 1 \le 2 - 1$$

$$(i.e.,)\ -3 \le 2\cos x - 1 \le 1$$

$$\text{so } 1 \le \frac{1}{2\cos x - 1} \le \frac{-1}{3}$$

The range is outside $\frac{-1}{3}$ and 1

i.e., range is $(-\infty, \frac{-1}{3}] \cup [1, \infty)$

Question 9.

Show that the relation xy = -2 is a function for a suitable domain. Find the domain and the range of the function.

Solution:

(i) Let f: R → R defined as f: x → $-\frac{2}{x}$ then

f(x) = $-\frac{2}{x}$ or y = $-\frac{2}{x}$

⇒ xy = – 2

f (x) is not a function since f(x) is not defined for x = 0

(ii) Let f: R – {0} → R defined as f(x) = $-\frac{2}{x}$

⇒ y = $-\frac{2}{x}$ = xy = – 2

f is one – one but not onto because 0 has no preimage.

f : R – {0} → R {0} is a function which is one- one and onto

Domain = R – {0}

Range = R – {0}

Question 10.
If f, g : R → R are defined by f(x) = |x| + x and g(x) = |x| – x, find gof and fog.

Solution:

$$f(x) = \begin{cases} 0 & x < 0 \\ 2x & x > 0 \end{cases} \text{ and } g(x) = \begin{cases} -2x & x < 0 \\ 0 & x > 0 \end{cases}$$

Now $(fog)(x) = 0$ and $gof(x) = 0$

Question 11.
If f, g, h are real-valued functions defined on R, then prove that
(f + g)oh = foh + goh. What can you say about fo(g + h)? Justify your answer.

Solution:
Let f + g = k
Now $k \odot h = k(h(x))$
= (f + g((h(x))
= f[h(x)] + g [h(x)]
= foh + goh
(i.e.,)(f + g)(o)h = foh + goh
fo(g + h) is also a function

Question 12.
If f: R → R is defined by f(x) = 3x – 5, prove that f is a bijection and find its inverse.

Solution:
P(x) = 3x – 5
Let y = 3x – 5 ⇒ 3x = y + 5

$$x = \frac{y+5}{3}$$

$$\text{Let } g(y) = \frac{y+5}{3}$$

$$\text{Now } g \circ f(x) = g[(f(x)] = g(3x-5)$$

$$= \frac{3x-5+5}{3} = x$$

$$\text{also } f \circ g(y) = f[g(y)] = f\left[\frac{y+5}{3}\right]$$

$$= 3\left(\frac{y+5}{3}\right) - 5 = y + 5 - 5 = y$$

$$\text{Thus } g \circ f = I_x \text{ and } f \circ g = I_y$$

f and g are bijections and inverse to each other. Hence f is a bijection and $f^{-1}(y) = \frac{y+5}{3}$

Replacing y by x we get $f^{-1}(x) = \frac{x+5}{3}$

Question 13.
The weight of the muscles of a man is a function of his bodyweight x and can be expressed as W(x) = 0.35x. Determine the domain of this function.

Solution:
Given W(x) = 0.35x
W(0) = W(1) = 0.35, W(2) = 0.7 …………. W (∞) = ∞
Since x. denotes the bodyweight of a man, it will take only positive integers.
That is x > 0.
W(x) : (0, ∞) → (0, ∞)
Domain = (0, ∞) , Range = (0, ∞)

Question 14.
The distance of an object falling is a function of time t and can be expressed as $s(t) = -16t^2$. Graph the function and determine if it is one-to-one.

Solution:
$s(t) = -16t^2$
Suppose $S(t_1) = S(t_2)$

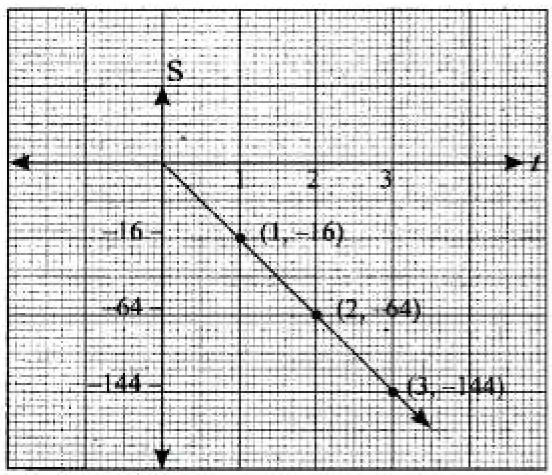
$$-16t_1^2 = -16t_2^2$$

$$t_1 = t_2 \text{ (or) } t_1 = -t_2$$

since time cannot be negative, we to take $t_1 = t_2$

Hence it is one-one.

t	0	1	2	3
s	0	-16	-64	-144



Question 15.
The total cost of airfare on a given route is comprised of the base cost C and the fuel surcharge S in rupee. Both C and S are functions of the mileage m; $C(m) = 0.4m + 50$ and $S(m) = 0.03m$. Determine a function for the total cost of a ticket in terms of the mileage and find the airfare for flying 1600 miles.

Solution:
Given the cost of airfare function and fuel surcharge functions are as follows.
$C(m) = 0.4\ m + 50$ ———- (1)
$S\ (m) = 0.03\ m$ ———- (2)
Total cost of a ticket = $C(m) + S(m)$
$f(x) = 0.4\ m + 50 + 0.03\ m$
$f(x) = 0.43\ m + 50$
Given $m = 1600$ miles
The cost of Airfare for flying 1600 miles
$f(1600) = 0.43 \times 1600 + 50$
$= 688 + 50$
$= 738$
∴ Airfare for flying 1600 miles is Rs. 738.

Question 16.
A salesperson whose annual earnings can be represented by the function $A(x) = 30,000 + 0.04x$, where x is the rupee value of the merchandise he sells. His son is also in sales and his earnings are represented by the function $S(x) = 25,000 + 0.05x$. Find $(A + S)(x)$ and determine the total family income if they each sell Rupees 1,50,00,000 worth of merchandise.

Solution:
$A(x) = 30,000 + 0.04x$, where x is merchandise rupee value
$S(x) = 25000 + 0.05\,x$
$(A + S)\,(x) = A(x) + S(x)$
$= 30000 + 0.04x + 25000 + 0.05\,x$
$= 55000 + 0.09x$
$(A + S)\,(x) = 55000 + 0.09x$
They each sell $x = 1,50,00,000$ worth of merchandise
$(A + S)\,x = 55000 + 0.09\,(1,50,00,000)$
$= 55000 + 13,50,000$
$\therefore$ Total income of family = ₹ 14,05,000

Question 17.
The function for exchanging American dollars for Singapore Dollar on a given day is $f(x) = 1.23x$, where x represents the number of American dollars. On the same day, the function for exchanging Singapore Dollar to Indian Rupee is $g(y) = 50.50y$, where y represents the number of Singapore dollars. Write a function which will give the exchange rate of American dollars in terms of the Indian rupee.

Solution:
$f(x) = 1.23x$ where x is number of American dollars.
$g(y) = 50.50y$ where y is number of Singapore dollars.
$gof(x) = g(f(x))$
$= g(1.23x)$
$= 50.50\,(1.23x)$
$= 62.115\,x$

Question 18.
The owner of a small restaurant can prepare a particular meal at a cost of Rupees 100. He estimates that if the menu price of the meal is x rupees, then the number of customers who will order that meal at that price in an evening is given by the function D(x) = 200 – x. Express his day revenue, total cost and profit on this meal as functions of x.

Solution:
Number of customers = 200 – x
Cost of one meal = Rs. 100
Cost of (200 – x) meals = (200 – x) × 100
Menu price of the meal = Rs. x
∴ Total menu price of (200 – x) meals = (200 – x) x
Profit = Menu price – Cost
= (200 – x) x – (200 – x) 100
Profit = (200 – x) (x – 100)

Question 19.
The formula for converting from Fahrenheit to Celsius temperatures is y = 5x/9 – 160/9

Find the inverse of this function and determine whether the inverse is also a function.

Solution:

$$y = \frac{5x}{9} - \frac{160}{9}$$

$$y = \frac{5x - 160}{9}$$

$$9y = 5x - 160$$

$$9y + 160 = 5x$$

$$x = \frac{9y + 160}{5}$$

$$\therefore\ y = \frac{9x + 160}{5} \quad \text{(or)} \quad f^{-1}(x) = \frac{9x}{5} + 32$$

Yes it is also a function.

Question 20.
A simple cipher takes a number and codes it, using the function f(x) = 3x – 4. Find the inverse of this function, determine whether the inverse is also a function and verify the symmetrical property about the line y = x (by drawing

the lines).

Solution:

f(x) = 3x – 4
Let y = 3x – 4

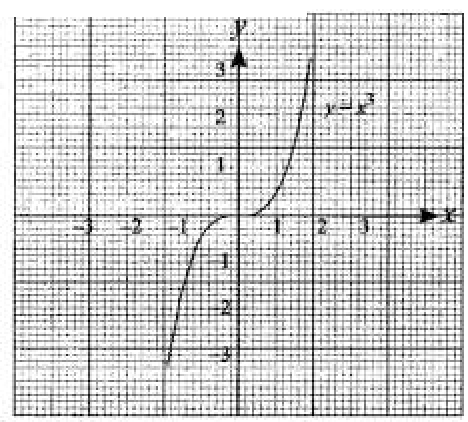
$$\Rightarrow \quad x \quad = \quad \frac{y+4}{3}$$

$$\text{So now } f\left(\frac{y+4}{3}\right) = 3\left(\frac{y+4}{3}\right) - 4 = y$$

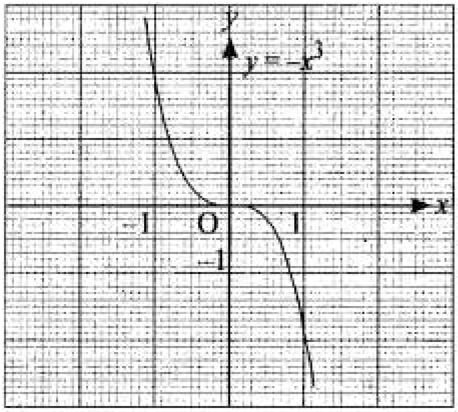
$$\text{So } f^{-1}(x) \quad = \quad \frac{x+4}{3} \text{ which is also a function.}$$

Ex 1.4

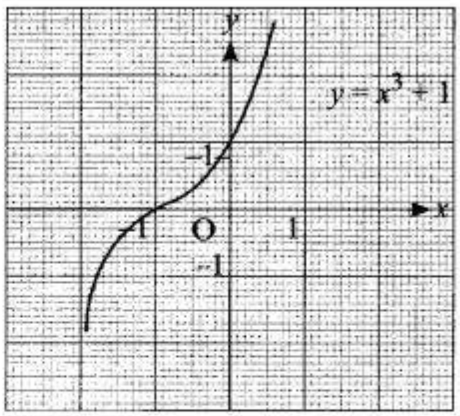
Question 1.
For the curve $y = x^3$ given in Figure, draw
(i) $r = -x^3$
(ii) $y = x^3 + 1$
(iii) $y = x^3 - 1$
(iv) $y = (x + 1)^3$ with the same scale.



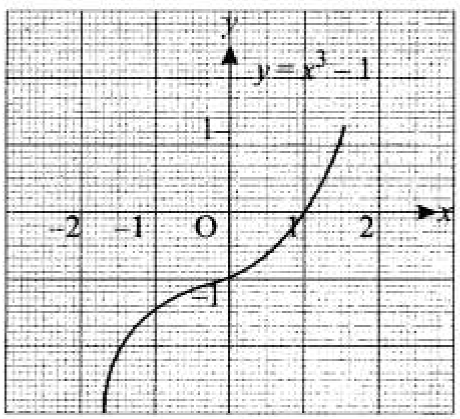
(*i*)



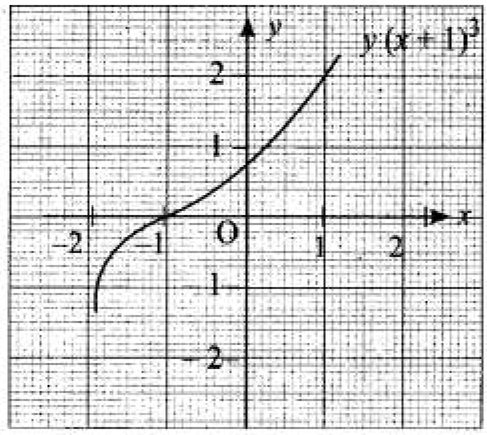
(*ii*)



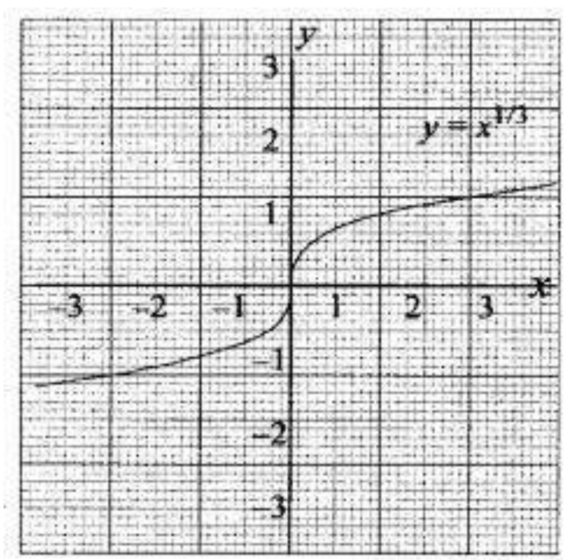
(*iii*)



(iv)



Solution:
(i) It is the reflection on y axis
(ii) The graph of $y = x^3 + 1$ is shifted upward to 1 unit.
(iii) The graph of $y = x^3 - 1$ is shifted downward to 1 unit.
(iv) The graph of $y = (x + 1)^3$ is shifted to the left for 1 unit.



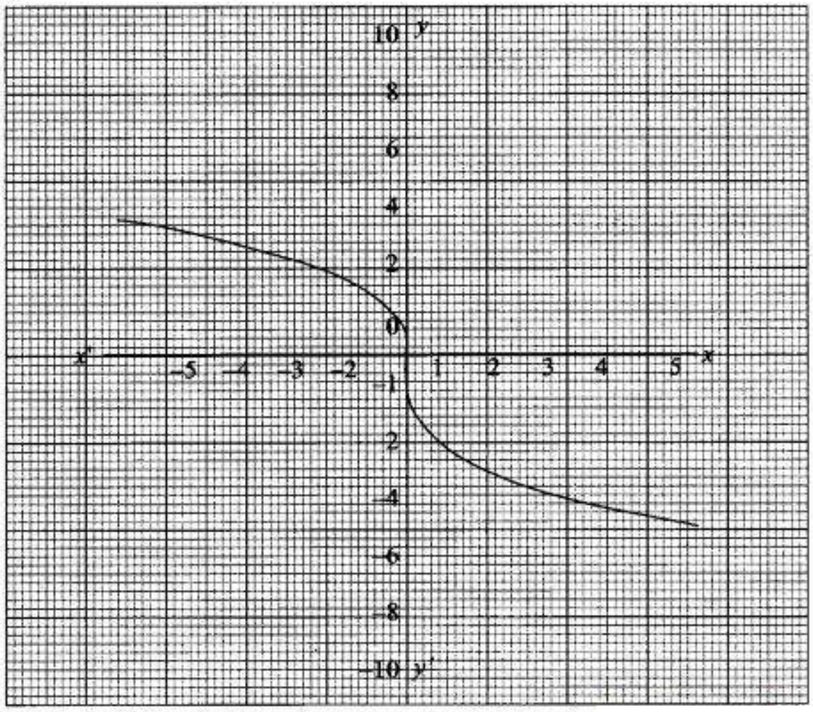
Question 2.

(i) $y = -x^{\left(\frac{1}{3}\right)}$ **(ii)** $y = x^{\left(\frac{1}{3}\right)} + 1$

(iii) $y = x^{\left(\frac{1}{3}\right)} - 1$ **(iv)** $y = (x+1)^{\left(\frac{1}{3}\right)}$

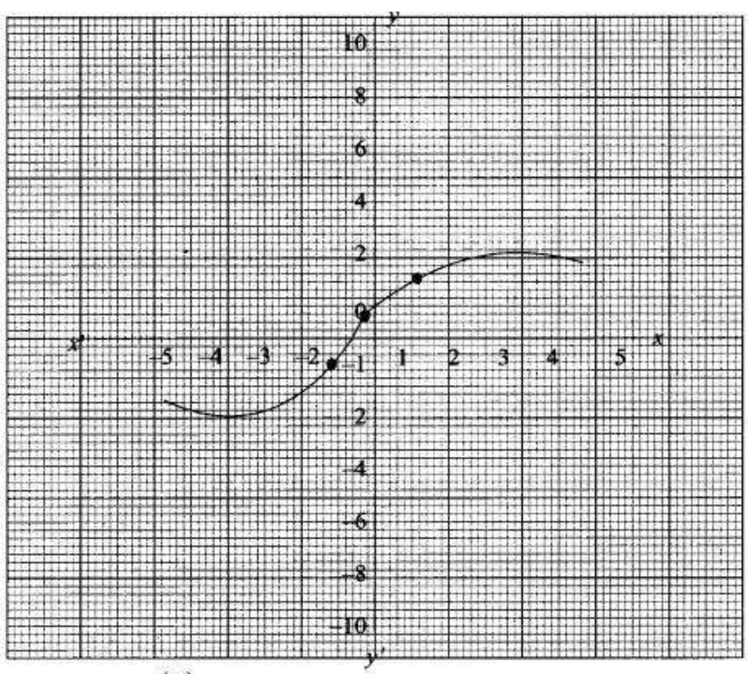
Solution:

(i) Let $y=-x^{\left(\frac{1}{3}\right)}$



Then $y=-x^{\frac{1}{3}}$ is the reflection of the graph of $y=x^{\left(\frac{1}{3}\right)}$ about the x-axis

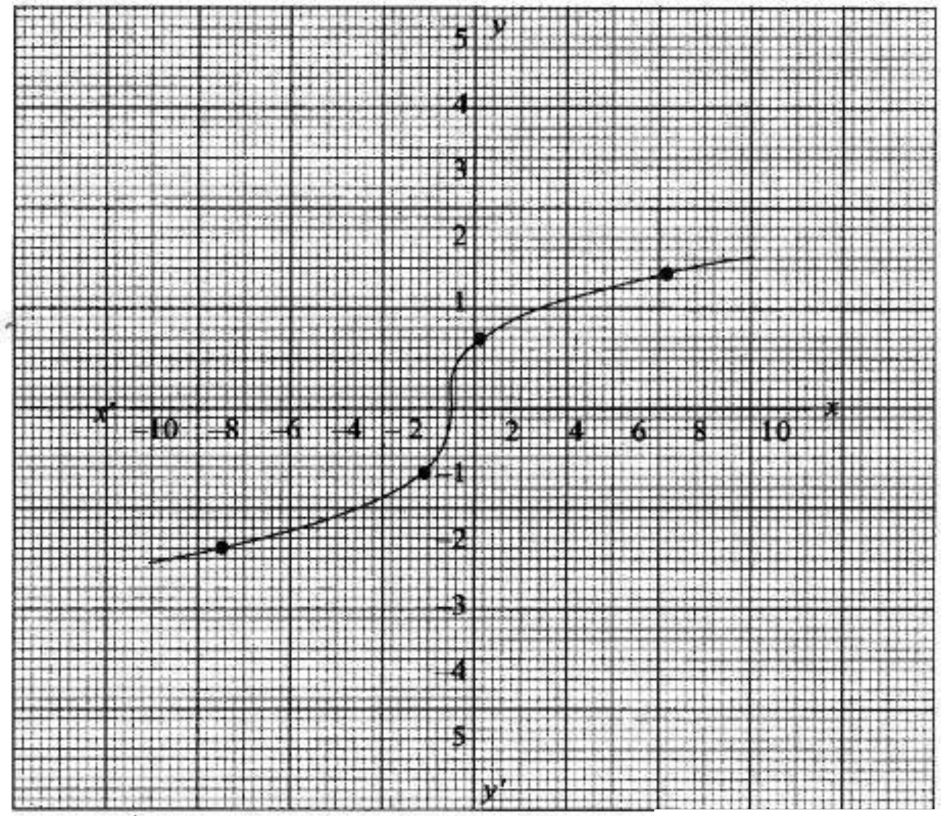
(ii) Let $y=x^{\left(\frac{1}{3}\right)}+1$



Then $y=x^{\left(\frac{1}{3}\right)}+1$ is the x graph of $y=x^{\left(\frac{1}{3}\right)}$ shifts to the upward for one unit.

(iii) $y = x^{\left(\frac{1}{3}\right)} - 1$

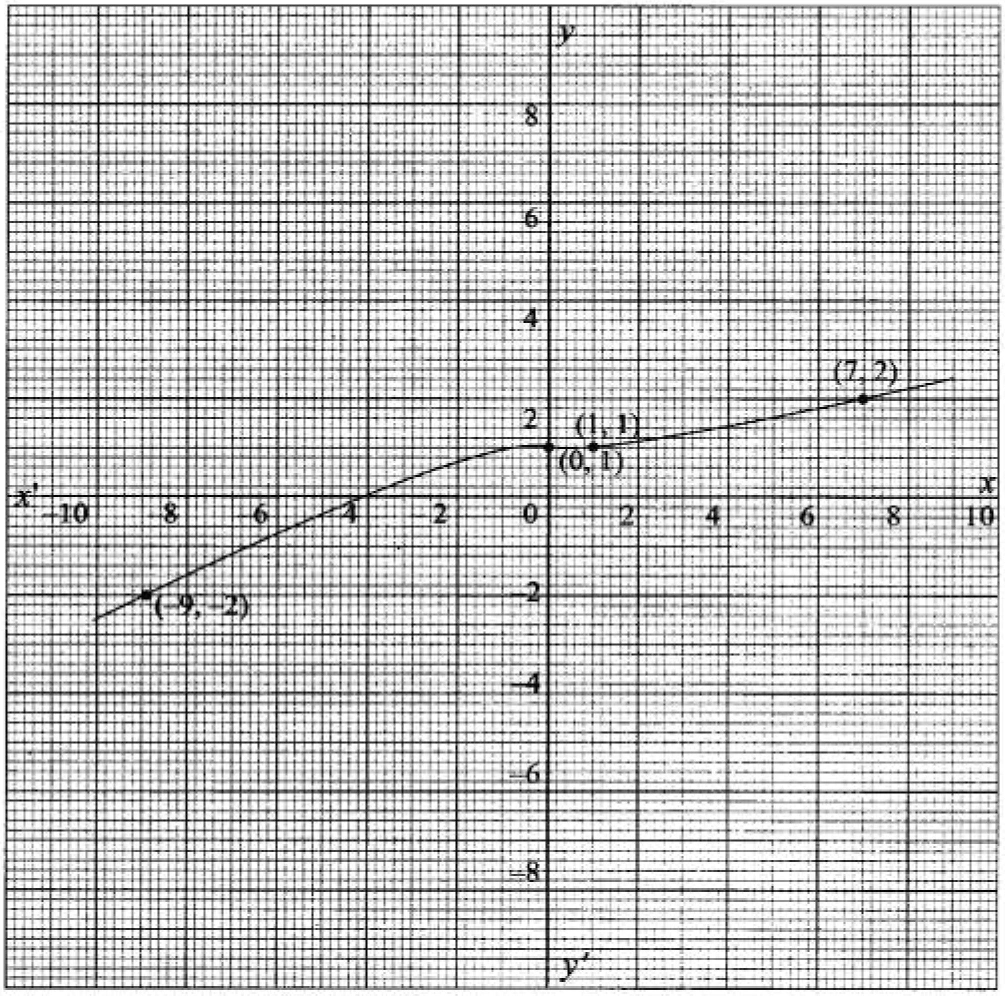
Let $y = -x^{\left(\frac{1}{3}\right)}$



Then $y = x^{\left(\frac{1}{3}\right)} - 1$ is the graph of $x^{\frac{1}{3}}$ shifts to the upward for one unit.

(iv) $y = (x+1)^{\left(\frac{1}{3}\right)}$

x	0	1	7	-9
y	1	1	2	-2



$y=(x+1)^{\left(\frac{1}{3}\right)}$ causes the graph of $x^{\frac{1}{3}}$ on the same co-ordinate plane. Fine *fog* and graph it on the plane as well. Explain your results.

Question 3.

Graph the functions f(x) = x^3 and g(x) = $\sqrt[3]{x}$ on the same coordinate plane. Find fog and graph it on the plane as well. Explain your results.

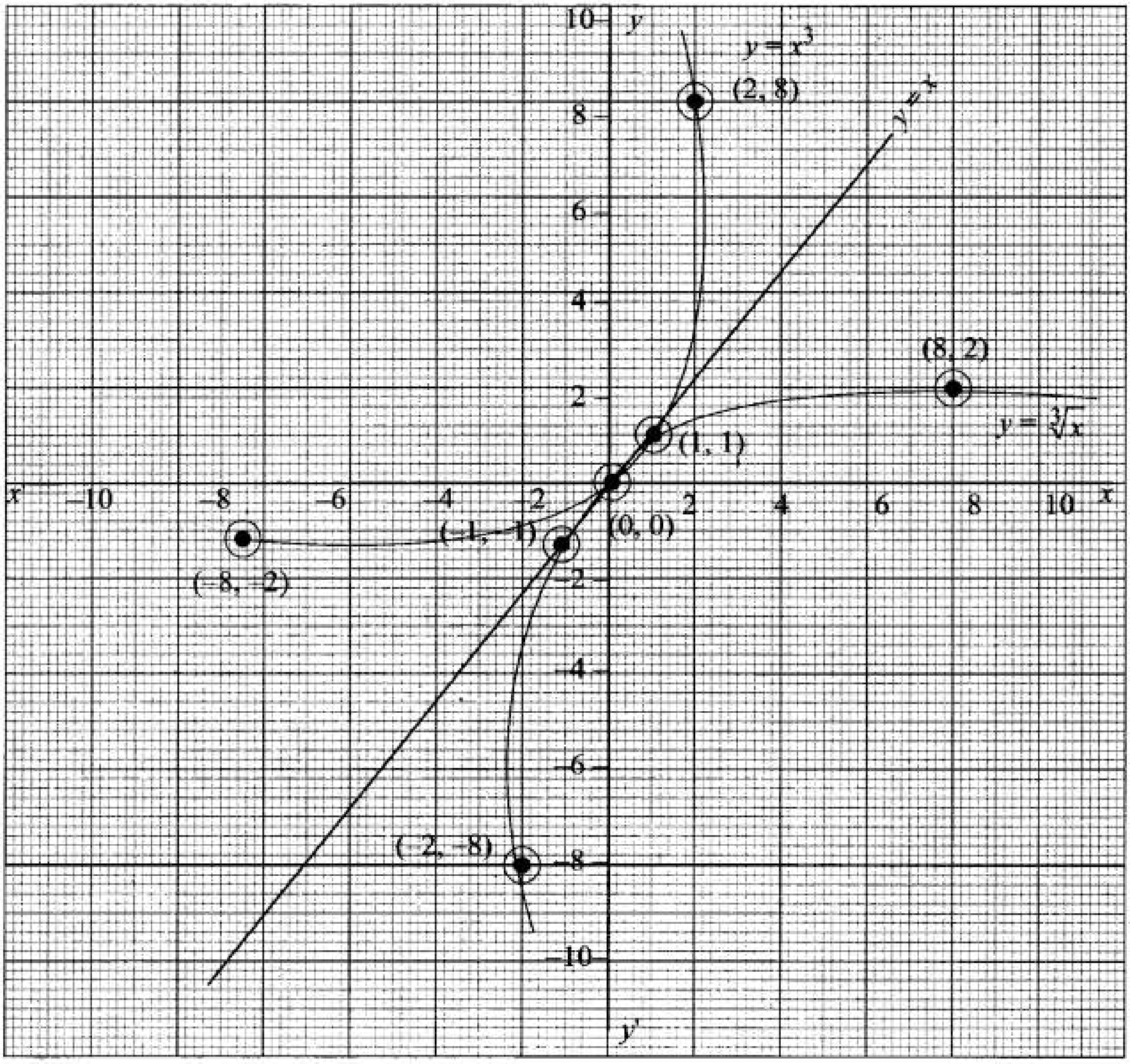
Solution:

$y=x^3$

x	0	1	2	–1	–2
y	0	1	8	–1	–8

$y = \sqrt[3]{x}$

x	0	1	–1	8	–8
y	0	1	–1	2	–2



Now $f \circ g\,(x) = f[g(x)] = f[\sqrt[3]{x}\,] = (\sqrt[3]{x}\,)^3 = x$

(*i.e.,*) $y = x$

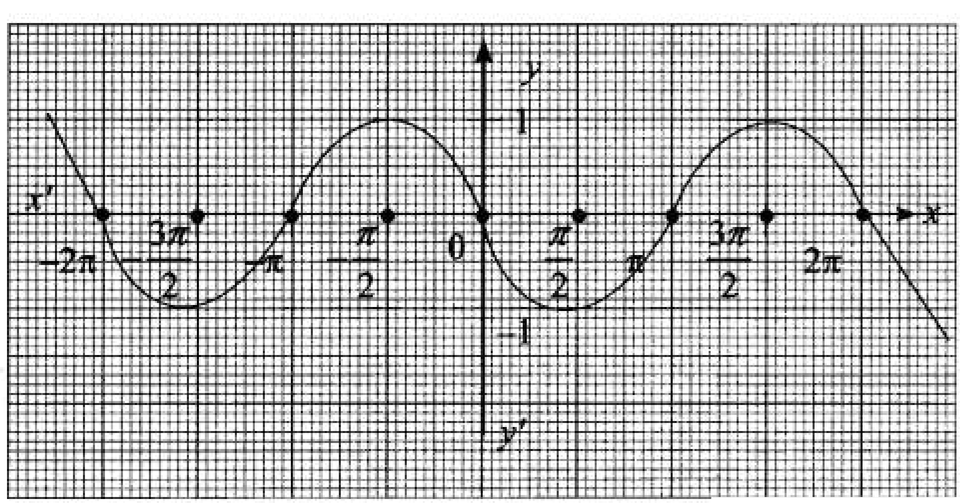
Question 4.
Write the steps to obtain the graph of the function $y = 3(x - 1)^2 + 5$ from the graph $y = x^2$.

Solution:
Draw the graph of $y = x^2$
To get $y = (x - 1)^2$ we have to shift the curve 1 unit to the right.
Then we have to draw the curve $y = 3(x - 1)^2$ and finally, we have to draw $y = 3(x - 1)^2 + 5$

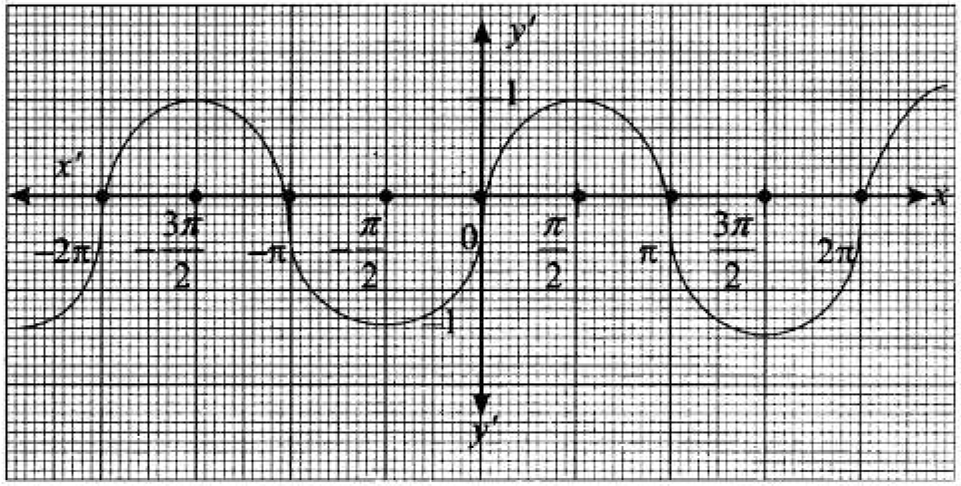
Question 5.
From the curve $y = \sin x$, graph the functions
(i) $y = \sin(-x)$
(ii) $y = -\sin(-x)$
(iii) $y = \sin(\frac{\pi}{2} + x)$ which is $\cos x$
(iv) $y = \sin(\frac{\pi}{2} - x)$ which is also $\cos x$ (refer trigonometry)

Solution:
First we have to draw the curve $y = \sin x$

(i) $y = \sin(-x) = -\sin x = f(x)$

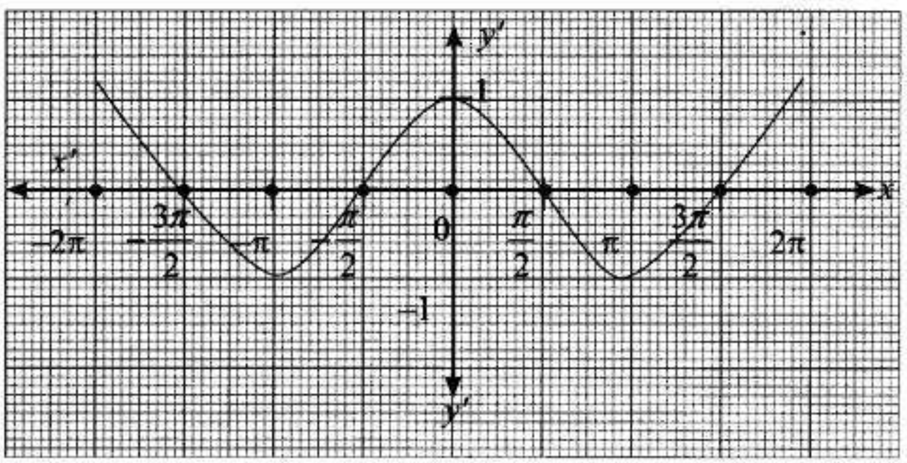


(ii) $y = -\sin(-x) = -[-\sin x] = \sin x$

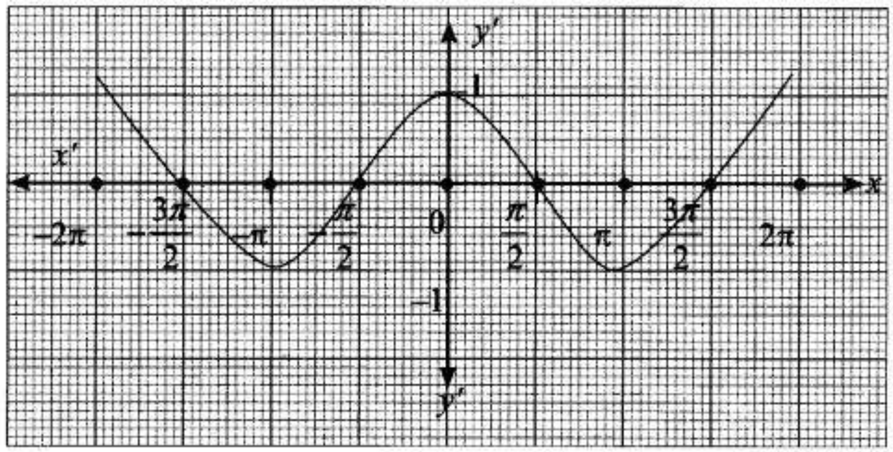


it is the same as $y = \sin x$ by shifting the graph to $\frac{\pi}{2}$ to the left.

(*iii*) $y = \sin\left(\frac{\pi}{2} + x\right)$ is the graph which is obtained from $y = \sin x$ by shifting the graph to $\frac{\pi}{2}$ to the left.

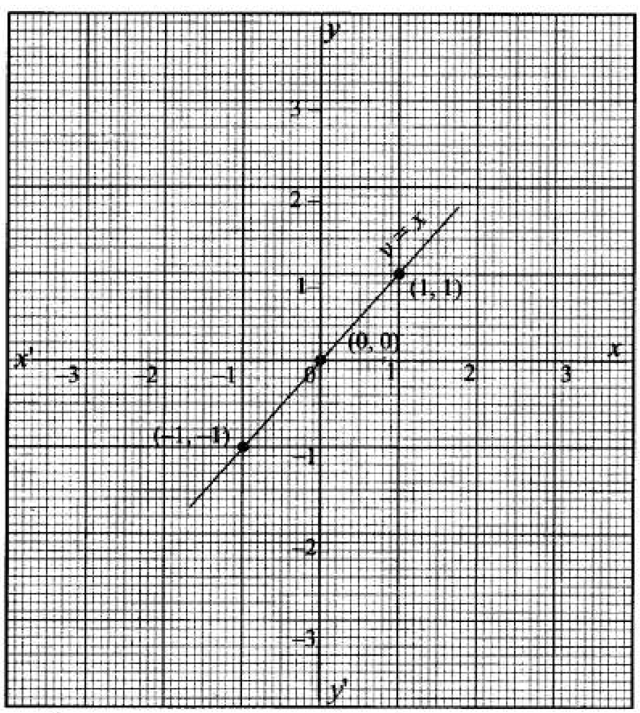


(*iv*) $y = \sin\left(\frac{\pi}{2} - x\right)$ is the graph obtained from $y = \sin x$ by shifting the graph to $\frac{\pi}{2}$ to the right.



Question 6.
From the curve y = x, draw
(i) y = -x
(ii) y = 2x
(iii) y = x + 1
(iv) y = ½ x + 1
(v) 2x + y + 3 = 0. 2

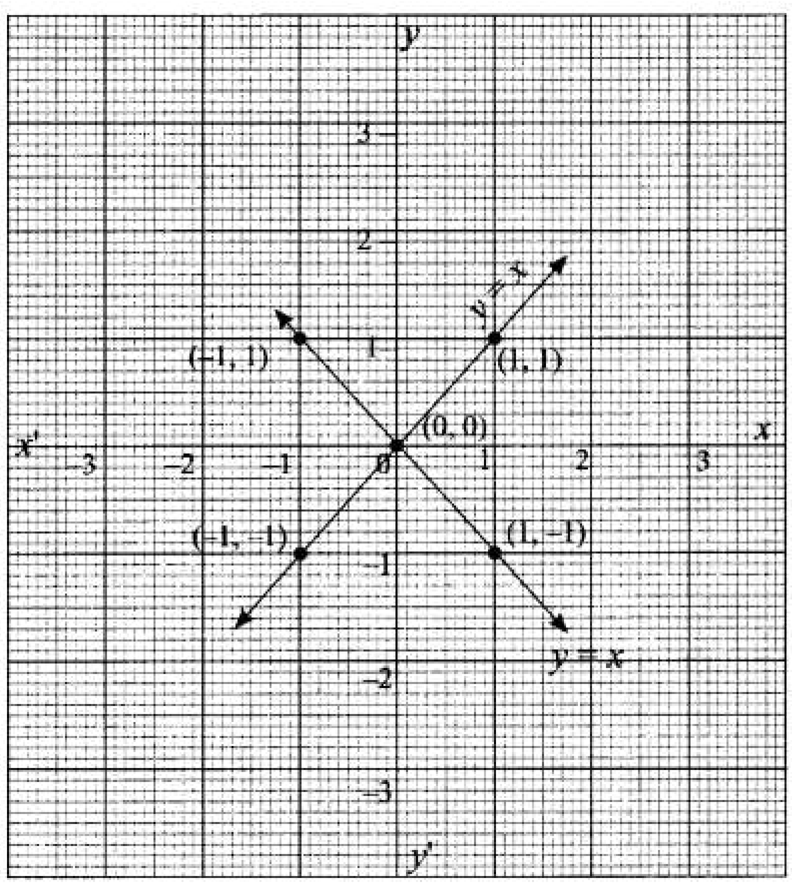
Solution:
y = x



x	−1	0	1
y	−1	0	1

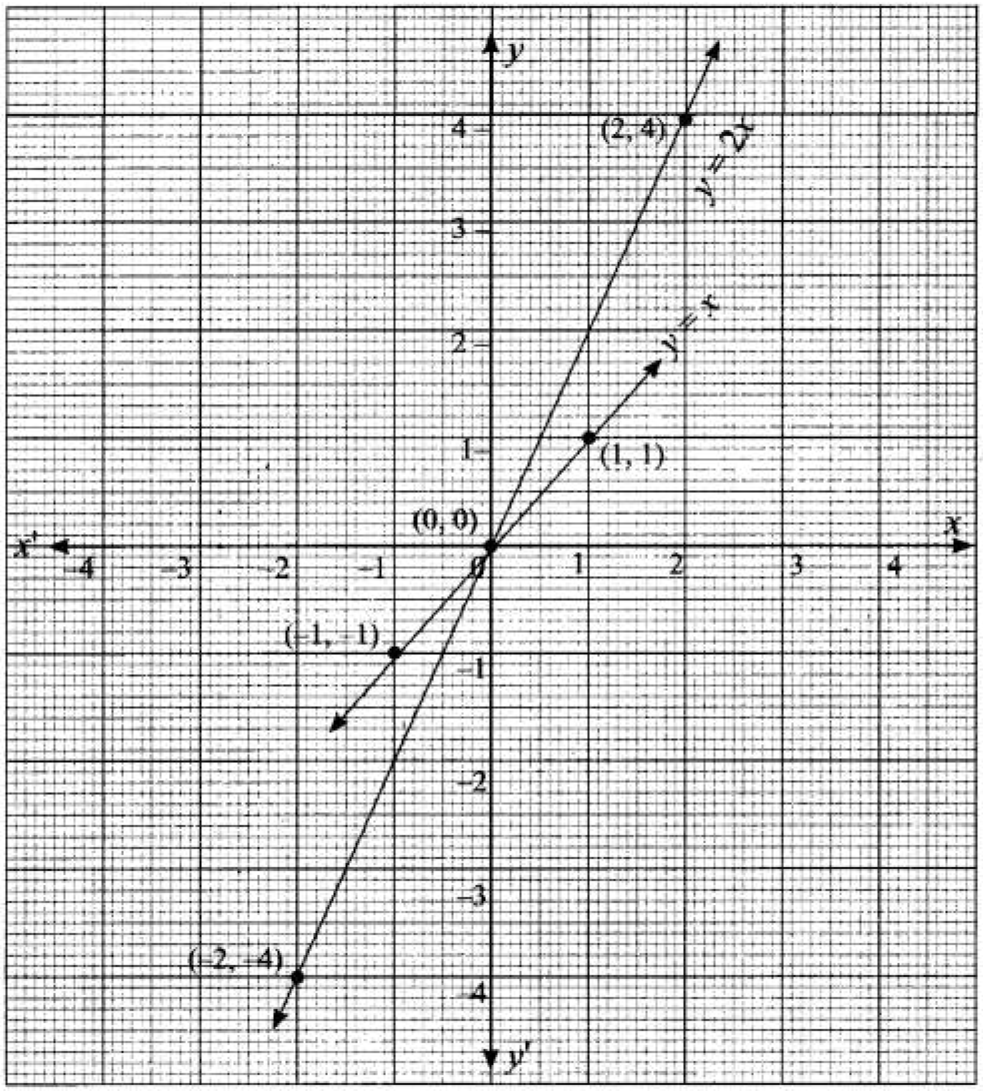
(i) y = -x

x	–1	0	1
y	1	0	–1



(ii) y = 2x

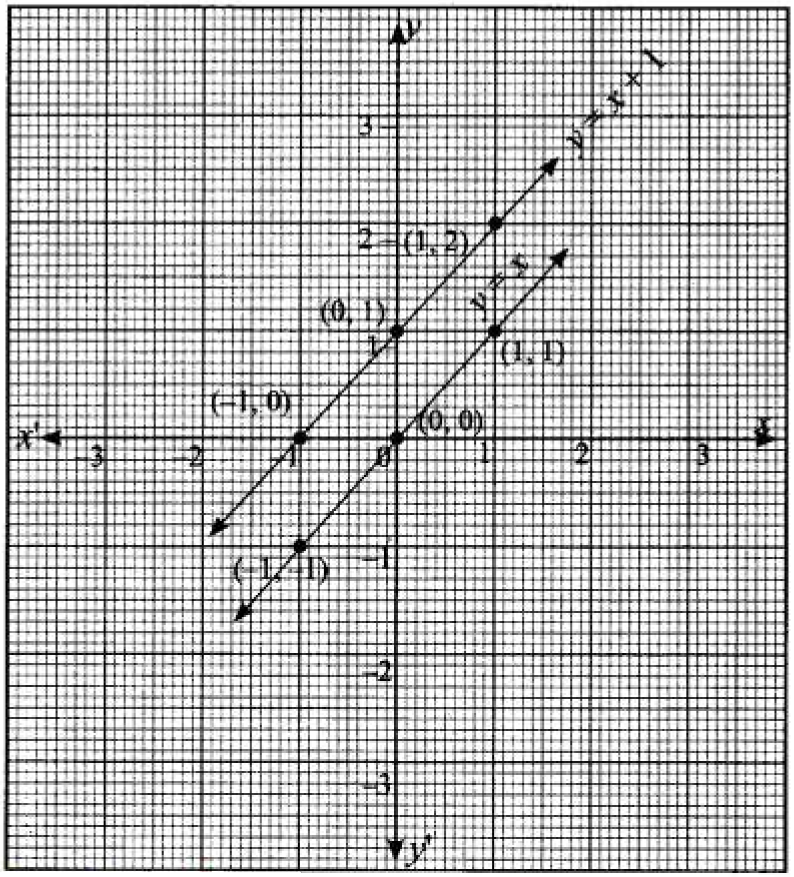
x	–2	0	2
y	–4	0	4



y = 2x the graph moves away from the x-axis, as multiplying factor is 2 which is greater than one.

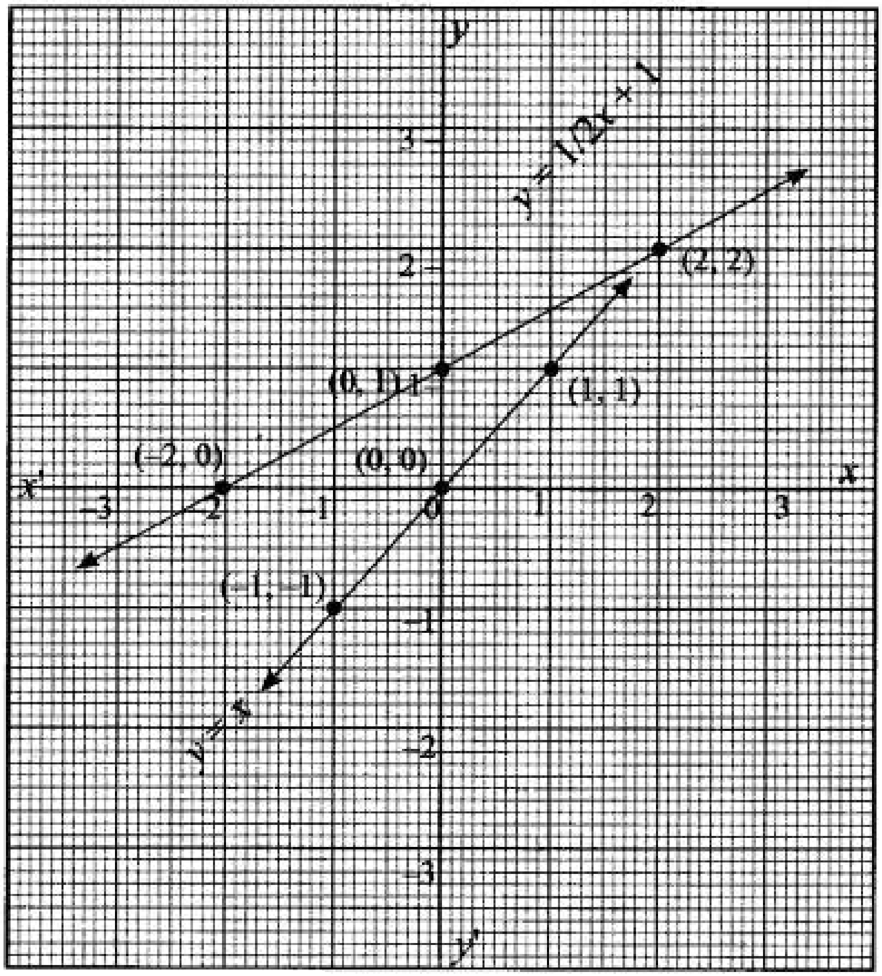
(iii) y = 2x + 1

x	–1	0	1
y	0	1	2



(iv) y = 1/2x + 1

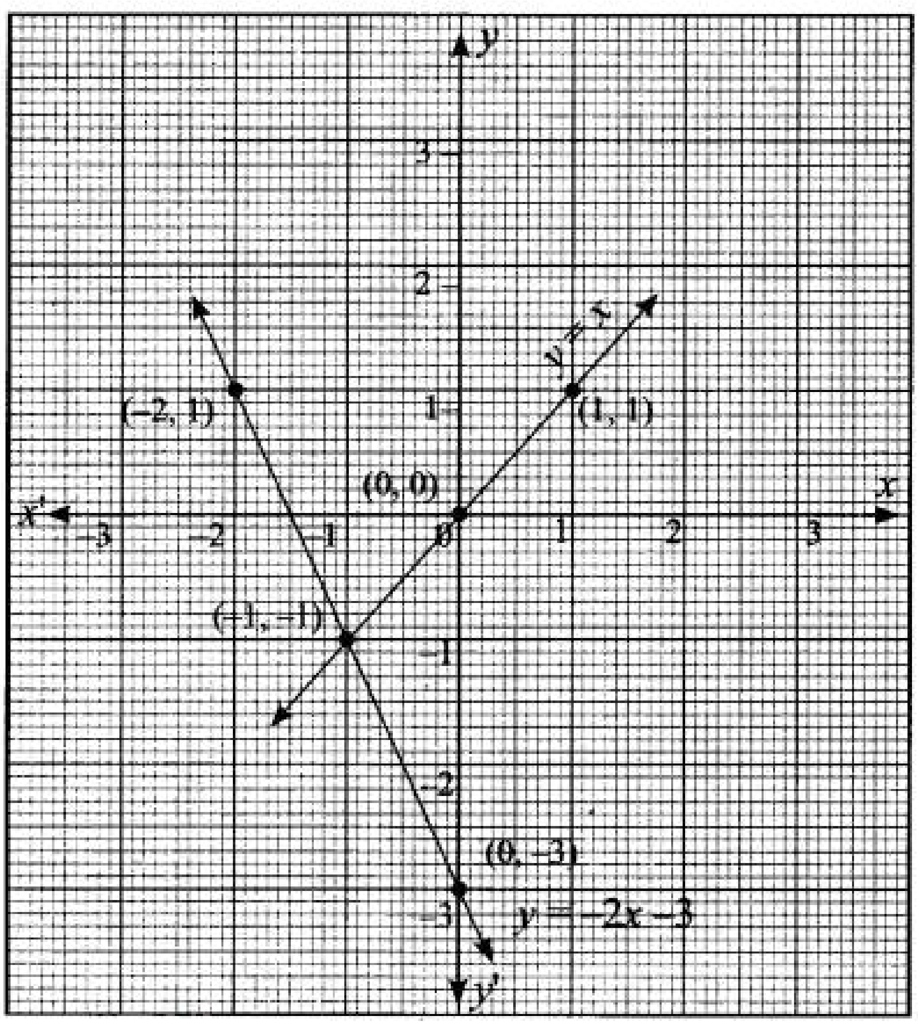
x	–2	0	2
y	0	1	2



y = ½ x moves towards x – axis by a side factor 1/2 which is less than y = ½ x + 1 upwards by 1 unit.

(v) y = -2x – 3

x	–2	–1	0
y	1	–1	–3



Question 7.
From the curve y = |x|, draw
(i) y= |x – 1| + 1
(ii) y = |x + 1| – 1
(iii) y = |x + 2| – 3.

Solution:
Given, y = |x|
If y = x

x	0	1	2
y	0	1	2

If y = -x

x	0	–1	–2
y	0	1	2

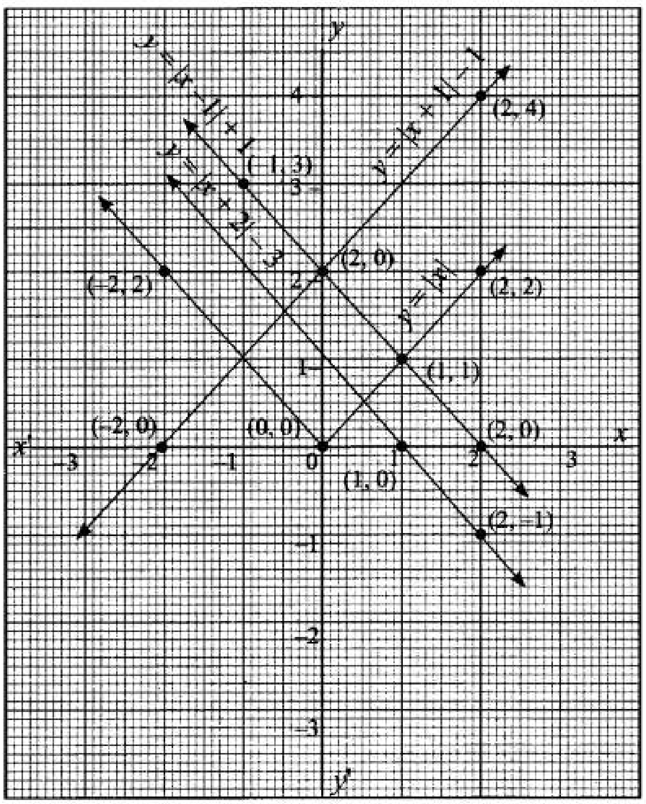
(i) y = |x – 1| + 1

y = x – 1 + 1

y = -x + 1 + 1 = x

x	–1	0	1
y	3	2	1

(For $y = 2 - x$)



(ii) y = |x + 1| – 1

y = x + 1 – 1 = x

y = -x -1 – 1

y = -x – 2

y = – (x + 2)

x	0	1	2
y	–2	–3	4

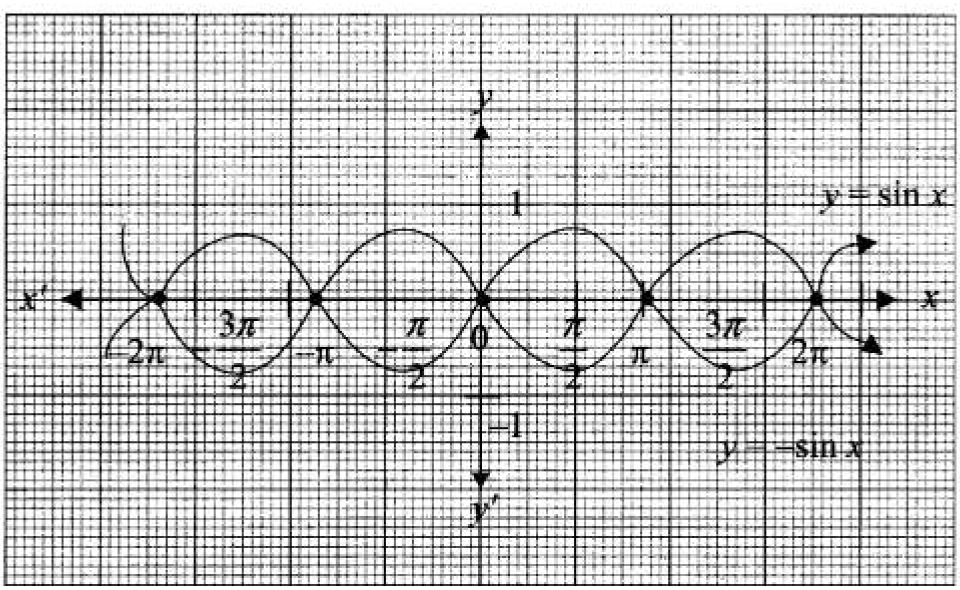
(iii) y = |x + 2| – 3
y = x + 2 – 3 ⇒ x – 1
y = -x – 2 + 3 = 1 – x
y = -(x – 1)

x	0	1	2	3
y	1	0	–1	–2

Question 8.
From the curves = sin x, draw y = sin |x| (Hint: sin(-x) = -sin x.)

Solution:
y = sin |x|
∴ y = sin x
∴ y = sin (-x) = – sin x
y = – sin x

x	-2π	$-\pi$	0	π	2π
y	0	0	0	0	0

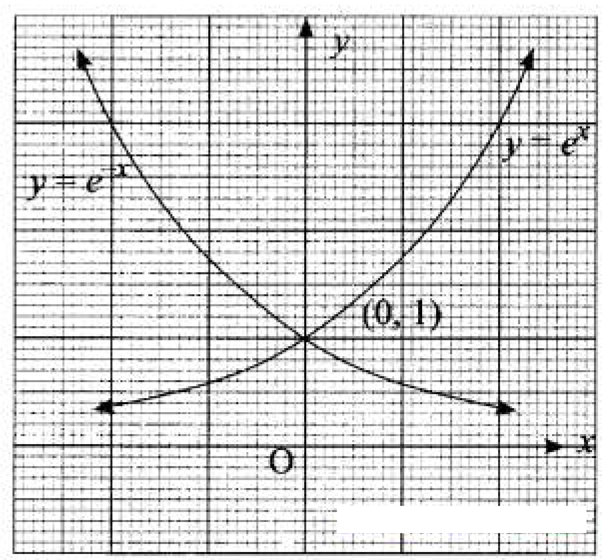


Ex 1.5

Choose the correct or the most suitable answer.

Question 1.
If A = {(x, y) : y = e^x ; x ∈ R } and B = {(x, y) : y = e^{-x}, x ∈ R } then n(A ∩ B)
(a) Infinity
(b) 0
(c) 1
(d) 2

Solution:
(c) 1
Hint.
A∩B = (0, 1)
n(A∩B) = 1



Question 2.
IfA {(x, y) : y = sin x, x ∈ R) and 8= (x, y) : y = cos x, x ∈ R) then A∩B contains ……..
(a) no element
(b) infinitely many elements
(c) only one element
(d) cannot be determined.

Solution:
(b) infinitely many elements

Question 3.
The relation R defined on a set A = {0, -1, 1, 2} by xRy if $|x^2 + y^2| \leq 2$, then which one of the following is true?
(a) R = {(0, 0), (0, -1), (0, 1), (-1, 0), (-1, 1), (1, 2), (1, 0)}
(b) R = {(0, 0), (0, -1), (0, 1), (-1, 0), (1, 0)
(c) Domain of R is {0, -1, 1, 2}

Solution:
(a) Range of R is {0, -1, 1}
Hint.

Given A = {0, -1, 1, 2 }
the relation R is given by x R y = $|x^2 + y^2| \leq 2$
∴ x, y must be 0 or 1
∴ Range of R is {0, – 1 , 1 }

Question 4.

If f(x) = |x – 2| + |x + 2|, x ∈ R, then

$$(a) \quad f(x) = \begin{cases} -2x & \text{if } x \in (-\infty, -2] \\ 4 & \text{if } x \in (-2, 2] \\ 2x & \text{if } x \in (2, \infty) \end{cases}$$

$$(b) \quad f(x) = \begin{cases} 2x & \text{if } x \in (-\infty, -2] \\ 4x & \text{if } x \in (-2, 2] \\ -2x & \text{if } x \in (2, \infty) \end{cases}$$

$$(c) \quad f(x) = \begin{cases} -2x & \text{if } x \in (-\infty, -2] \\ -4x & \text{if } x \in (-2, 2] \\ 2x & \text{if } x \in (2, \infty) \end{cases}$$

$$(d) \quad f(x) = \begin{cases} -2x & \text{if } x \in (-\infty, -2] \\ 2x & \text{if } x \in (-2, 2] \\ 2x & \text{if } x \in (2, \infty) \end{cases}$$

Solution:

(a) $$f(x) = \begin{cases} -2x & \text{if } x \in (-\infty, -2] \\ 4 & \text{if } x \in (-2, 2] \\ 2x & \text{if } x \in (2, \infty) \end{cases}$$

Hint.

f(x) = |x – 2| + |x + 2|

$$f(x) = \begin{cases} -(x-2)-(x+2) = -2x, x \in (-\infty, -2] \\ -(x-2)+(x+2) = 4, x \in (-2, 2] \\ x-2+x+2 = 2x, x \in (2, \infty] \end{cases}$$

$$\therefore f(x) = \begin{cases} -2x, & x \in (-\infty, -2] \\ 4, & x \in (-2, 2] \\ 2x, & x \in (2, \infty] \end{cases}$$

Question 5.

Let R be the set of all real numbers. Consider the following subsets of the plane R x R: S = {(x, y): y = x + 1 and 0 < x < 2} and T = {(x, y) : x – y is an integer} Then which of the following is true?

(a) T is an equivalence relation but S is not an equivalence relation.
(b) Neither S nor T is an equivalence relation
(c) Both S and T are equivalence relations
(d) S is an equivalence relation but T is not an equivalence relation.

Solution:

(a) T is an equivalence relation but S is not an equivalence relation.
Hint.

Given R is the set of all real numbers
S = { (x, y): y = x + 1 and 0 < x < 2}
T = { (x, y): x – y is an integer} are subsets of R × R
s = { (x, y): y = x + 1 and 0 < x < 2} for x ∈ R,
x = x + 1 is not possible. ∴ (x , x) ∉ S

Hence S does not satisfy the reflexive property

∴ S is not an equivalence relation
T = {(x, y): x – y is an integer}

Reflexive:
For $x \in R$, we have $x - x = 0$ is an integer.
∴ $(x,x) \in T$ forall $X \in R$
Hence T satisfies reflexive property

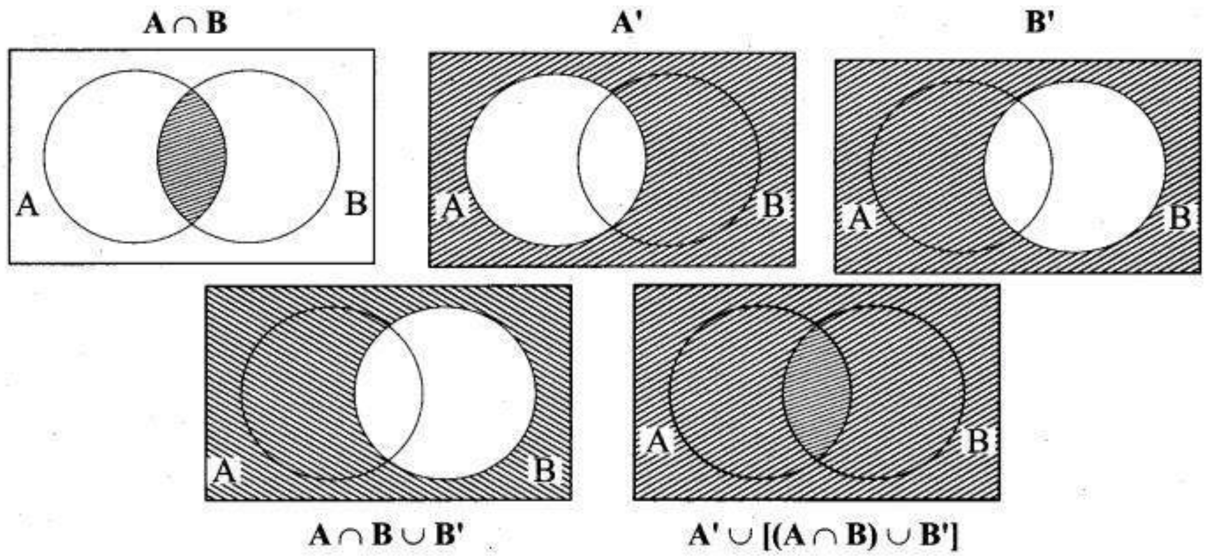
Symmetric:
Let $(x, y) \in T$, then $x - y$
⇒ $-(x - y)$ is an integer
⇒ $y - x$ is an integer
⇒ $(y, x) \in T$
∴ T satisfies the symmetric property

Transitive:
Let $(x, y), (y, z) \in T$ then $x - y$ and $y - z$ are integers.
⇒ $x - y + y - z$ is an integer
⇒ $x - z$ is an integer
⇒ $(x, z) \in T$

∴ T satisfies the transitive property
we have proved T is reflexive, symmetric, and transitive.
Thus T is an equivalence relation.

Question 6.
Let A and B be subsets of the universal set N, the set of natural numbers. Then $A' \cup [(A \cap B) \cup B']$ is ………
(a) A
(b) A’
(c) B
(d) N

Solution:
(d) N
Hint.



Question 7.
The number of students who take both the subjects Mathematics and Chemistry is 70. This represents 10% of the enrollment in Mathematics and 14% of the enrollment in Chemistry. How many students take at least one of these two subjects?
(a) 1120
(b) 1130
(c) 1100
(d) insufficient data

Solution:

(b) 1130
Hint.
Let M denotes Mathematics students
C denotes Chemistry students
Given $n(M \cap C) = 70$

10 % of the enrolement in Mathematics
Out of 100 enrolement 10 students take mathematics
$\therefore$ Number of Mathematics students $n(M) = 100/10 \times 70$
$n(M) = 700$

Number of Chemistry students $n(C) = 100/14 \times 70$
$n(C) = 500$

$\therefore n(M \cup C) = n(M) + n(C) - n(M \cap C)$
$= 700 + 500 - 70$
$= 1200 - 70$
$= 1130$

The number of students take atleast one of the subject mathematics or Chemistry = 1130

Question 8.
If $n[(A \times B) \cap (A \times C)] = 8$ and $n(B \cap C) = 2$, then $n(A)$ is
(a) 6
(b) 4
(c) 8
(d) 16

Solution:
(b) 4

Question 9.
If $n(A) = 2$ and $n(B \cup C) = 3$, then $n[(A \times B) \cup (A \times C)]$ is …….
(a) 2^3
(b) 3^2
(c) 6
(d) 5

Solution:
(c) 6
Hint.
Given $n(A) = 2$ and $n(B \cup C) = 3$
$n[(A \times B) \cup (A \times C)] = n[A \times (B \cup C)]$
$A \times (B \cup C) = (A \times B) \cup (A \times C)$
$= n(A) . n(B \cup C)$
$= 2 \times 3 = 6$

Question 10.
If two sets A and B have 17 elements in common, then the number of elements common to the set $A \times B$ and $B \times A$ is
(a) 2^{17}
(b) 17^2

(c) 34
(d) insufficient data

Solution:
(b) 17^2
Hint.
$n(A \cap B) = 17$
So $n[(A \times B) \cap (B \times A)]$
$= n(A \cap B) \times n(B \cap A) = 17 \times 17 = 17^2$

Question 11.
For non-empty sets A and B, if $A \subset B$ then $(A \times B) \cap (B \times A)$ is equal to ……….
(a) $A \cap B$
(b) $A \times A$
(c) $B \times B$
(d) None of these

Solution:
(b) $A \times A$
Hint.
Given $A \subset B$, take $A = \{1, 2\}$ and $B = \{1, 2, 3\}$
$A \times B = \{1, 2\} \times \{1, 2, 3\}$
$A \times B = \{(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3)\}$
$B \times A = \{1, 2, 3\} \times \{1, 2\}$
$B \times A = \{(1, 1), (1, 2), (2, 1), (2, 2), (3, 1), (3, 2)\}$
$(A \times B) \cap (B \times A) = \{(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3)\} \cap \{(1, 1), (1, 2),$
$(2, 1),$
$(2, 2), (3, 1), (3, 2)\}$
$(A \times B) \cap (B \times A) = \{(1, 1), (1, 2), (2, 1), (2, 2)\}$
$A \times A = \{1, 2\} \times \{1, 2\}$
$A \times A = \{(1, 1), (1, 2), (2, 1), (2, 2)\}$
$(A \times B) \times (B \times A) = A \times A$

Question 12.
The number of relations on a set containing 3 elements is
(a) 9
(b) 81
(c) 512
(d) 1024

Solution:
(c) 512
Hint.
Number of relations = $2^{n2} = 2^{32} = 2^9 = 512$

Question 13.
Let R be the universal relation on a set X with more than one element. Then R is
(a) Not reflexive
(b) Not symmetric
(c) Transitive
(d) None of the above

Solution:
(c) Transitive

Question 14.
Let X = {1, 2, 3, 4} and R = {(1, 1), (1, 2), (1, 3), (2, 2), (3, 3), (2, 1), (3, 1), (1,4), (4, 1)}. Then R is ……..
(a) Reflexive
(b) Symmetric
(c) Transitive
(d) Equivalence

Solution:
(b) Symmetric
Hint.
x = {1, 2, 3, 4}
R = { (1, 1) , (1, 2), (1, 3), (2, 2), (3, 3), (2, 1), (3, 1), (1, 4), (4, 1)}

Question 15.
The range of the function $\frac{1}{1-2\sin x}$ is ……………………… .
(a) $(-\infty,-1)\cup\left(\frac{1}{3},\infty\right)$
(b) $\left(-1,\frac{1}{3}\right)$
(c) $\left[-1,\frac{1}{3}\right]$
(d) $(-\infty,-1]\cup\left[\frac{1}{3},\infty\right)$

Solution:

$-1 \leq \sin x \leq 1$

So $-2 \leq 2 \sin x \leq 2$

$\Rightarrow 2 \geq -2\sin x \geq -2$

$2+1 \geq -2\sin x + 1 \geq -2+1$

(i.e.) $3 \geq 1-2\sin x \geq -1$

$$\Rightarrow \frac{1}{3} \leq \frac{1}{1-2\sin x} \leq -1$$

$$\Rightarrow \text{The range is } (-\infty, -1] \cup \left[\frac{1}{3}, \infty\right)$$

Question 16.

The range of the function $f(x) = |\lfloor x \rfloor - x|, x \in R$ is ....

Solution:

(c) [0, 1)

Hint. $f(x) = |\lfloor x \rfloor - x|$ $f(x) = \lfloor x \rfloor - x$

$$f(0) = 0 - 0 = 0$$

$$f(6.5) = 6 - 6.5 = -.5$$

$$f(-7.2) = 8 - 7.2 = .8$$

$\therefore$ Range is (0, 1)

Question 17.

The rule $f(x) = x^2$ is a bijection if the domain and the co-domain are given by ….

(a) R,R

(b) R, (0, ∞)

(c) (0, ∞), R

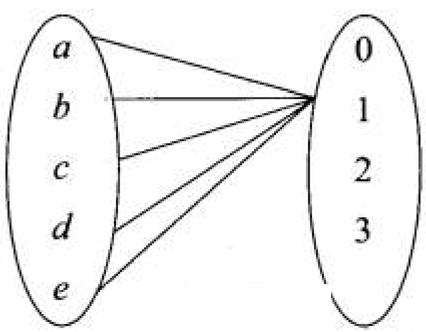
(d) [0, ∞), [0, ∞)

Solution:

(d) [0, ∞), [0, ∞)

Question 18.
The number of constant functions from a set containing m elements to a set containing n elements is
(a) mn
(b) m
(c) n
(d) m + n

Solution: (c) n



Question 19.
The function f: [0, 2π] → [-1, 1] defined by f(x) = sin x is
(a) One to one
(b) Onto
(c) Bijection
(d) Cannot be defined

Solution:
(b) Onto

For $x = \pi/4$, $x = 3\pi/4$, $f(x) = \frac{1}{\sqrt{2}}$

So it is not one-to-one
So it is an onto function

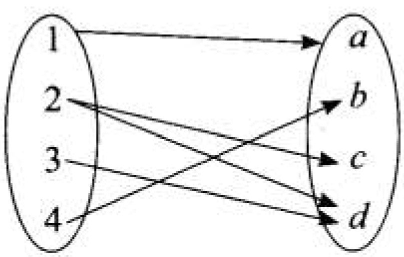
Question 20.
If the function f : [-3, 3] → S defined by f(x) = x^2 is onto, then S is ....
(a)[-9, 9]
(b) R
(c) [-3, 3]

(d) [0, 9]

Solution:
(d) [0, 9]

Question 21.
Let X = {1, 2, 3, 4}, Y = {a, b, c, d) and f = {(1, a), (4, b), (2, c), (3, d) (2, d)}.
Then f is ………
(a) An one-to-one function
(b) An onto function
(c) A function which is not one-to-one
(d) Not a function

Solution:
(d) Not a function
Hint.



Since the element 2 has two images, it is not a function

Question 22.

The inverse of $f(x) = \begin{cases} x & ; \quad x < 1 \\ x^2 & ; \quad 1 \le x \le 4 \\ 8\sqrt{x} & ; \quad x > 4 \end{cases}$ is …………..

(a) $f^{-1}(x)=\begin{cases} x & ; \quad x<1 \\ \sqrt{x} & ; \quad 1\le x\le 16 \\ \dfrac{x^2}{64} & ; \quad x>16 \end{cases}$

(b) $f^{-1}(x)=\begin{cases} -x & ; \quad x<1 \\ \sqrt{x} & ; \quad 1\le x\le 16 \\ \dfrac{x^2}{64} & ; \quad x>16 \end{cases}$

(c) $f^{-1}(x)=\begin{cases} x^2 & ; \quad x<1 \\ \sqrt{x} & ; \quad 1\le x\le 16 \\ \dfrac{x^2}{64} & ; \quad x>16 \end{cases}$

(d) $f^{-1}(x)=\begin{cases} 2x & ; \quad x<1 \\ \sqrt{x} & ; \quad 1\le x\le 16 \\ \dfrac{x^2}{8} & ; \quad x>16 \end{cases}$

Solution:

(a) $f^{-1}(x)=\begin{cases} x & ; \quad x<1 \\ \sqrt{x} & ; \quad 1\le x\le 16 \\ \dfrac{x^2}{64} & ; \quad x>16 \end{cases}$

Hint. $f(x)=\begin{cases} x & x<1 \\ x^2 & 1\le x\le 4 \\ 8\sqrt{x} & x>4 \end{cases}$

$y=x$ then $x=y \Rightarrow f^{-1}(x)=x$

When $y=x^2$, $x=\sqrt{y}$. So $f^{-1}(x)=\sqrt{x}$

When $y=8\sqrt{x}$

$\sqrt{x}=\dfrac{y}{8}\,(or)\,x=\dfrac{y^2}{64}$

$\therefore f^{-1}(x)=\dfrac{x^2}{64}$

$\therefore \quad f^{-1}(x)=\begin{cases} x & x<1 \\ \sqrt{x} & 1\le x\le 16 \\ \dfrac{x^2}{64} & x>16 \end{cases}$

Question 23.
Let f : R → R be defined by f(x) = 1 – |x|. Then the range of f is ....
(a) R
(b) (1, ∞)
(c) (-1, ∞)
(d) (-∞, 1]

Solution:
(d) (-∞, 1]
Hint.
f(x) = 1 – |x|
When x = 0, f(0) = 1 – 0 = 1
When x = – 2 , f(-2) = 1 – |- 2| = 1 – 2 = -1
When x = – 5 , f(-5) = 1 – |- 5| = 1 – 5 = -4
∴ Range of f is (- ∞, 1]

Question 24.
The function f : R → R is defined by f(x) = sin x + cos x is ......
(a) An odd function
(b) Neither an odd function nor an even function
(c) An even function
(d) Both odd function and even function

Solution:
(b) Neither an odd function nor an even function

Question 25.
The function f: R → R is defined by $f(x) = \dfrac{(x^2+\cos x)(1+x^4)}{(x-\sin x)(2x-x^3)} + e^{-|x|}$ is ....
(a) An odd function
(b) Neither an odd function nor an even function
(c) An even function
(d) Both odd function arid even function

Solution:
(c) An even function

$$f(x)=\frac{(x^2+\cos x)(1+x^4)}{(x-\sin x)(2x-x^3)}+e^{-|x|}$$

$$f(-x)=\frac{x^2+\cos(-x)\left(1+(-x)^4\right)}{[-x-\sin(-x)]\left[-2x-(-x)^3\right]}+e^{-|-x|}$$

$$=\frac{(x^2+\cos x)(1+x^4)}{(-x+\sin x)(-2x+x^3)}+e^{-|x|}$$

$$=\frac{(x^2+\cos x)(1+x^4)}{-(x-\sin x)(-1)(2x-x^3)}+e^{-|x|}$$

$$=\frac{(x^2+\cos x)(1+x^4)}{(x-\sin x)(2x-x^3)}+e^{-|x|}=f(x)$$

$\Rightarrow f(x)$ is an even function