CHAPTER 11

Circular Curves

11.1 Introduction

- When two straight lines (in real life like highways, railway lines) intersect at some angle, then a curve is provided in between them which meets the straight lines tangentially.
- Curves are in fact arcs with some definite radius and a center i.e. they are part of a circle. It is due to the curves only that the change in direction is made possible a very smooth task.

11.2 Classification of Curves

(a) Horizontal Curve
(b) Vertical Curve

11.2.1 Horizontal Curve

Horizontal curve joins the two intersecting lines which lie in a horizontal plane. e.g. Curve provided to connect two roads meeting at an angle. Horizontal curves are further classified as:

(a) Simple circular curve
(b) Compound curve
(c) Reverse curve
(d) Transition curve
(e) Combined curve
(f) Broken-back curve

11.2.2 Vertical Curve

Vertical curve joins the two intersecting lines which lie in a vertical plane. e.g. Curve provided to connect two roads at the lower most point of a valley. Vertical curves are further classified as:

(a) Summit curve
(b) Sag curve

11.3 Horizontal Curves

11.3.1 Simple Circular Curve

- A simple circular curve has the property that it connects two straight lines with a curve of constant radius at all the points on the curve and connects the two straight lines tangentially. In Fig. 11.1, T_1TT_2 is a simple circular curve,

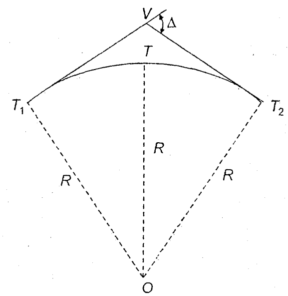


Fig. 11.1 *Simple circular curve*

11.3.2 Compound Curve

- Many a times it is not possible to provide a curve of constant radius to connect the two straight lines. In that case, we provide more than one curve of different radii to connect them.
- When all the curves turn in the same direction then the resulting curve is called as **compound curve**. In Fig. 11.2, T_1T and TT_2 are the two curves of different radii R_1 and R_2 respectively which take turn in the same direction.

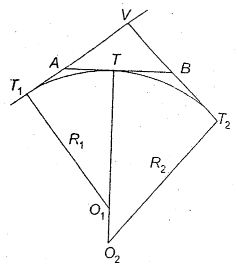


Fig. 11.2 *Compound Curve*

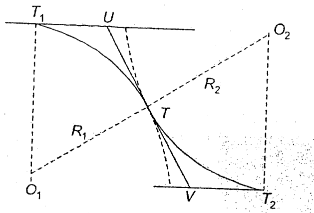


Fig. 11.3 *Reverse Curve*

11.3.3 Reverse Curve

- In this type of curve, the two curves of different radii R_1 and R_2 take turn in opposite directions as shown in Fig. 11.3.
- These types of curves are also called as **serpentine curve** or the **S curve** because of their shape.

11.3.4 Transition Curve

- While going through a straight road, if a curve is suddenly encountered, we feel a jerk. In order to avoid such a sudden jerk, it is essential that the transition from the straight line to the curve takes place gradually and NOT suddenly. Thus a curve of varying radius is provided which takes off from the straight line, turns gradually and finally meets the curve i.e. attains the same radius as that of the curve. This type of curve is called as **transition curve** as shown in Fig. 11.4. It is also called as **easement curve**.

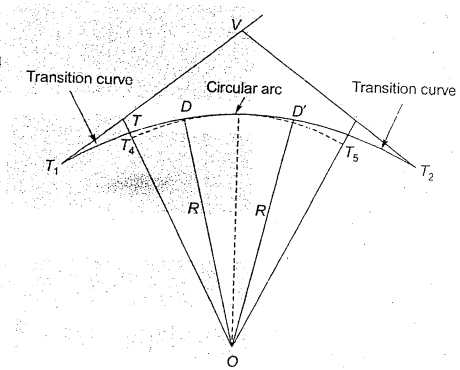


Fig. 11.4 *Transition Curve*

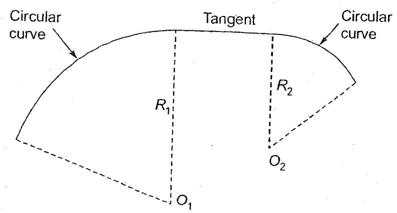


Fig. 11.5 *Broken-back curve*

11.3.5 Combined Curve

- It is a combination of simple and transition curves and is invariably provided in all types of highways and railways.

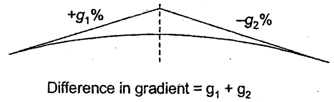
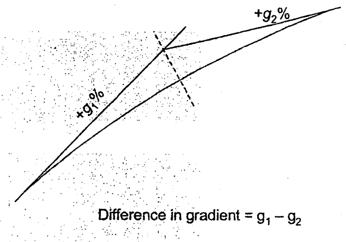
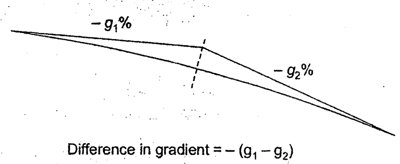
11.3.6 Broken-back Curve

- This is a very old type of curve used in the past wherein a straight line was provided in between the circular curves as shown in Fig. 11.5. This type of curve is not suitable for high speed maneuvering and is not used these days.

11.4 Vertical Curves

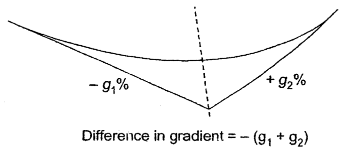
11.4.1 Summit Curve

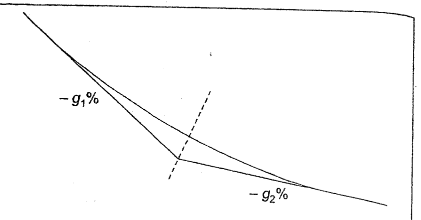
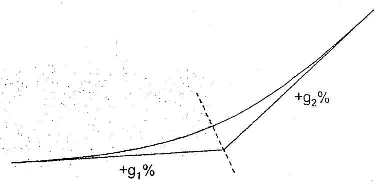
This type of vertical curve is provided in the following situations:

(a) An upgrade is followed by a downgrade as shown in **Fig. 11.6**	 Fig. 11.6 *Upgradient meeting a down gradient*
(b) A steeper upgrade is followed by a milder upgrade as shown in **Fig. 11.7.**	 Fig. 11.7 *Steeper upgradient meeting a milder upgradient*
(c) A milder downgrade is followed by a steeper downgrade as shown in **Fig. 11.8.**	 Fig. 11.8 *Milder downgradient meeting a steeper downgradient*

11.4.2 Sag Curve

This type of vertical curve is provided in the following situations:

(a) A downgrade is followed by an upgrade as shown in **Fig. 11.9.**	 Fig. 11.9 *Downgradient meeting an upgradient*

(b) A steeper downgrade is followed by a milder downgrade as shown in Fig. 11.10.	 Difference in gradient = $-(g_1 - g_2)$ **Fig. 11.10** *Steeper downgradient meeting a milder downgradient*
(c) A milder upgrade is followed by a steeper upgrade as shown in Fig. 11.11.	 Difference in gradient = $g_1 - g_2$ **Fig. 11.11** *Milder upgradient meeting a steeper upgradient*

11.5 Simple Circular Curve-A Detailed Overview

11.5.1 Elements of Simple Circular Curve

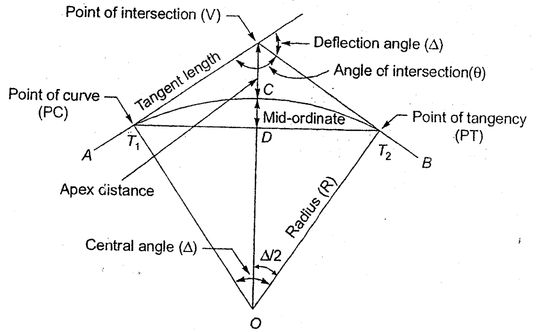


Fig. 11.12 *Elements of a circular curve*

- As shown in **Fig. 11.12**, let two straight lines *AV* and *BV* intersect at point *V* which is called as the **point of intersection** being abbreviated as *PI*. The external angle between these two intersecting lines is referred to as the **deflection angle,** Δ. The internal angle between these two intersecting lines is called as the **intersection angle** or the **angle of intersection**, θ. Thus obviously,

$$\Delta + \theta = 180° \qquad ...(11.1)$$

- The point from where the curve begins (T_1) is called as the **point of curve** being abbreviated as *PC* and the point where the curve ends (T_2) is called as the **point of tangency** being abbreviated as *PT*.
- **Tangent Length:** The lengths VT_1 and VT_2 are called as **tangent lengths** which are in fact being equal in lengths. This is due to the fact that:
  "Tangents drawn from an external point to a circle are equal in lengths."
- **Length of arc:** The arc length T_1CT_2 are called as the length of the circular arc where point *C* is the summit of the curve and it lies on the angle bisector of the angle T_1OT_2 and also bisects the curve T_1CT_2.
- **Long chord:** The line joining the point of curve (T_1) and the point of tangency (T_2) is called as long chord.
- **Mid ordinate:** The line joining the summit of the curve (*C*) and mid-point of the long chord (*D*) is called as the mid ordinate i.e. the length *CD* as shown in **Fig. 11.12.**
- **Central angle:** The angle subtended at the center '*O*' i.e. $\angle T_1OT_2$ is called as the central angle. The central angle is always equal to the deflection angle (Δ) i.e.
  $$\angle T_1OT_2 = \Delta$$
- **Normal chord:** If there are two stations at the end of a chain which forms the chord of a curve then this chord is called as the normal chord or the full chord.
- **Sub-chord:** Any other chord which is shorter in length than the normal chord is called as the sub-chord. It generally occurs at the start and the end of a curve.
- **Apex distance:** The distance from the point of intersection (*V*) and the summit point (*C*) is called as the apex distance.

11.5.2 Degree of Curve

- The two important parameters of a circle (or the circular curve) are its **center** and the **radius**.
- Often the center of the curve which is to be laid out on a highway or for a railway is inaccessible and in such cases, the radius of the curve has no importance.
- In order to set out a curve in such situations, we define the degree of curve. This degree of curve is defined either on the basis of arc or the chord.
- **Arc definition of degree of curve:** According to this, the degree of curve is the central angle subtended by an arc of length 20 m or 30 m as the case may be.

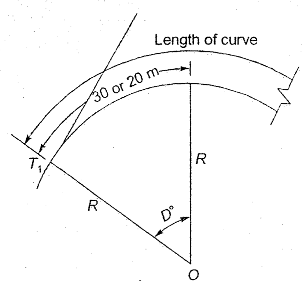


Fig. 11.13 *Arc definition of degree of curve*

Let *R* = Radius of the curve
D = Degree of the curve as per arc definition for a 30 m arc length i.e. a 30 m arc length subtends an angle of $D°$ at the center.

Now, $$D° = D\frac{\pi}{180} \text{ radians}$$

and $$\theta(\text{in radians}) = \frac{\text{Arc}}{\text{Radius}} = \frac{30}{R}$$

$$\Rightarrow \quad \frac{RD\pi}{180} = 30$$

$$\Rightarrow \quad R = \frac{30\times180}{D\pi} = \frac{1718.9}{D} \approx \frac{1719}{D} \Rightarrow D = \frac{1719}{R} \qquad ...(11.2)$$

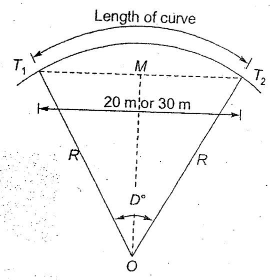
If however, degree of curve (D) is defined with 20 m arc length then,

$$\Rightarrow \quad R = \frac{20\times180}{D\pi} = \frac{1145.9}{D} \approx \frac{1146}{D} \Rightarrow D = \frac{1146}{R} \qquad ...(11.3)$$

- **Chord definition of degree of curve:** According to this, the degree of curve is the central angle subtended by a chord of length 20 m or 30 m as the case may be.

Let R = Radius of the curve

D = Degree of the curve as per chord definition for a 30 m chord length i.e. a 30 m chord length subtends an angle of $D°$ at the center.



D = Degree of curve

Fig. 11.14 *Chord definition of degree of curve*

Now, in ΔT_1OM as shown in Fig. 11.14

$$\sin\frac{D}{2} = \frac{MT_1}{OT_1} = \frac{15}{R}$$

$$\Rightarrow \quad R = \frac{15}{\sin\frac{D}{2}}$$

Now for very small angle θ, $\sin\theta \approx \theta$

where θ is in radians.

Here angle D is very small and so,

$$\sin\frac{D}{2} \approx \frac{D}{2} \quad \text{(where } D \text{ is in radians)}$$

But in the present case, angle D is in degrees.

So,
$$R = \frac{15}{\sin\frac{D}{2}} \approx \frac{15}{\frac{D}{2}\times\frac{\pi}{180}} = \frac{15\times2\times180}{D\pi} = \frac{1718.9}{D} \approx \frac{1719}{D}$$

$$\Rightarrow \quad D = \frac{1719}{R} \qquad \text{[Same as Eq.(11.2)]}$$

Similarly for a 20 m chord length,

$$R = \frac{10}{\sin\frac{D}{2}} \approx \frac{10}{\frac{D}{2}\times\frac{\pi}{180}} = \frac{10\times2\times180}{D\pi} = \frac{1145.9}{D} \approx \frac{1146}{D}$$

$$\Rightarrow \quad D = \frac{1146}{R} \qquad \text{[Same as Eq.(11.3)]}$$

11.5.3 Mathematical Expressions for Elements of Simple Circular Curve

Length of the curve: The length of the curve T_1CT_2 is denoted as 'l' where,

$$l = R\Delta \qquad \text{where } \Delta \text{ is in radians}$$

$$= \frac{R\Delta\pi}{180} \qquad \text{where } \Delta \text{ is in degrees}$$

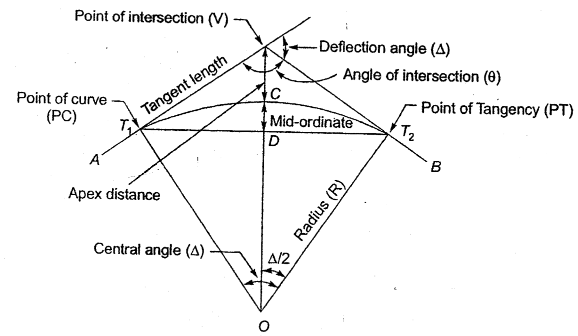


Fig. 11.15 *Elements of a circular curve*

Now for 30 m arc or chord definition,

$$R = \frac{1719}{D}$$

Thus,
$$l = \frac{1719}{D}\times\frac{\Delta\pi}{180} = \frac{30\Delta}{D} \qquad ...(11.4)$$

Similarly for 20 m arc or chord definition,

$$l = \frac{1146}{D}\times\frac{\Delta\pi}{180} = \frac{20\Delta}{D} \qquad ...(11.5)$$

- **Length of the tangent:**

$$\text{Tangent length } (T) = T_1V = T_2V = R.\tan\left(\frac{\Delta}{2}\right) \qquad ...(11.6)$$

- **Length of long chord:**

$$\text{Length of long chord } (L) = T_1DT_2 = 2T_1D$$

From ΔDT_1O,

$$\sin\left(\frac{\Delta}{2}\right) = \frac{T_1D}{R}$$

$$\Rightarrow \quad T_1D = R\sin\left(\frac{\Delta}{2}\right)$$

$$\Rightarrow \quad L = 2T_1D = 2R\sin\left(\frac{\Delta}{2}\right) \qquad ...(11.7)$$

- **Apex distance:**

$$\text{Apex distance } (VC) = VO - CO = R.\sec\left(\frac{\Delta}{2}\right) - R = R\left[\sec\left(\frac{\Delta}{2}\right) - 1\right] \qquad ...(11.8)$$

- **Mid-ordinate:**

$$\text{Mid-ordinate } (O_o) = CD = CO - DO = R - R.\cos\left(\frac{\Delta}{2}\right) = R\left(1 - \cos\left(\frac{\Delta}{2}\right)\right) \qquad ...(11.9)$$

NOTE: Earlier, $(1-\cos\theta)$ was referred to as vers θ i.e. versed sine of angle θ.

So, mid-ordinate $(O_o) = R.\text{vers}\left(\frac{\Delta}{2}\right)$ ...(11.10)

11.6 Setting Out a Simple Circular Curve

The chainage of point of curve (*P.C.*) i.e. T_1 may not be a full chain multiple and mostly it will be some full chain multiple and a fraction of a full chain length. The next point after T_1 will be at full chain length and so the all other points except the last point i.e point of tangent (*P.T.*) or T_2. Thus the first and the last chord of the curve whose lengths are lesser than the full chain length are called as sub-chord.

The methods of setting a simple circular curve are broadly classified as:

(a) Linear methods: In this method, only a chain or a tape is used and no angle measuring device/instrument is used.

(b) Angular methods: In this method, angle measuring device like theodolite is used with or without a chain or tape. Angular methods are more precise and thus are always preferred but they are time consuming.

- Before setting out a curve in the field firstly the point of intersection of the straight lines (*P.I.*), the point of curve (*P.C.*) and the point of tangency (*P.T.*) are located. Among these, firstly *P.I.* is located and a theodolite is set up at *P. V.* and levelled. The telescope of the theodolite is directed towards one of the straight line and transited by 180°. The telescope is then swung towards the other straight line. The deflection angle (Δ) is being noted from the horizontal scale reading of the theodolite. The length of the tangent is calculated from Eq. (11.6).
- Points T_1 and T_2 are then established at the tangent length (T) from the point of intersection, (*P. I.*)

11.6.1 Linear Methods of Setting a Simple Circular Curve

(a) Offsets from the long chord
(b) Perpendicular offsets from the tangent
(c) Radial offsets from the tangent
(d) Successive bisection of arcs or chords
(e) Offsets from the chord produced

11.6.1.1 Offsets from the Long Chord

As shown in **Fig. 11.6**, two straights T_1V and T_2V intersect at point of intersection, V. It is required to set out a curve of radius R between the points T_1 and T_2. O_o is mid-ordinate and O_x is the offset at point P at a distance x from the mid-point M of the long chord.

Now from ΔOMT_1,

$$OT_1^2 = OM^2 + MT_1^2 \quad \text{(Pythagoras theorem)}$$

$$\Rightarrow \quad OM = \sqrt{OT_1^2 - MT_1^2}$$

$$\Rightarrow \quad OM = \sqrt{R^2 - \left(\frac{L}{2}\right)^2}$$

Also,

$$CM = OC - OM$$

$$\Rightarrow \quad O_o = R - \sqrt{R^2 - \left(\frac{L}{2}\right)^2}$$

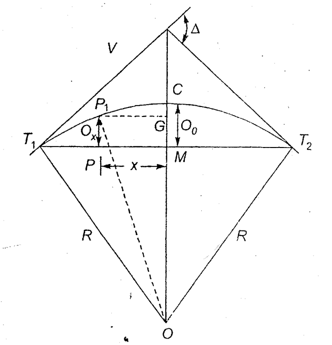


Fig. 11.16 *Offsets from the long chord*

Thus offset PP_1 is given by,

$$PP_1 = MG = OG - OM$$

$$\Rightarrow \quad O_x = \sqrt{R^2 - x^2} - (R - O_o) \quad \text{...(11.11)}$$

The Eq. (11.11) gives the exact expression for the required offset at a distance x from mid-point M on the long chord.

This relationship can be approximated also by expanding the square root term as,

$$O_x = \sqrt{R^2 - x^2} - (R - O_o)$$

$$\Rightarrow \quad O_x = R\left(1 - \frac{x^2}{R^2}\right)^{\frac{1}{2}} - R + O_o$$

$$\Rightarrow \quad O_x = R\left(1 - \frac{x^2}{2R^2} + \ldots\right) - R + O_o$$

$$\Rightarrow \quad O_x = \left(R - \frac{x^2}{2R} + \ldots\right) - R + O_o \quad \text{(neglecting higher order terms)}$$

$$\Rightarrow \quad O_x = O_o - \frac{x^2}{2R} \quad \text{...(11.12)}$$

Thus by putting different values of x, offset O_x can be found out both exactly and approximately as per requirement from Eqs. (11.11) and (11.12).

11.6.1.2 Perpendicular Offsets from the Tangent

This method of setting out a simple circular curve is particularly suitable for curve of small radius and small deflection angle. Let there is a point P on the initial tangent T_1V at a distance x from T_1. PP_1 is the perpendicular offset from point P. Obviously point P_1 is required to lie on the curve.

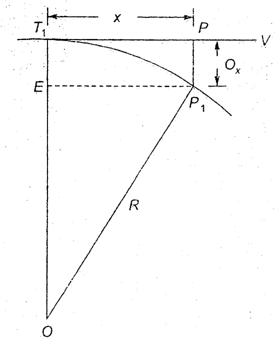


Fig. 11.17 *Perpendicular offsets from the tangent*

Now from ΔP_1EO,

$$P_1O^2 = P_1E^2 + EO^2$$

$$\Rightarrow \quad R^2 = x^2 + (R - O_x)^2$$

$$\Rightarrow \quad R - O_x = \sqrt{R^2 - x^2}$$

$$\Rightarrow \quad O_x = R - \sqrt{R^2 - x^2}$$

(Exact expression for perpendicular offset) ...(11.13)

Also

$$O_x = R - R\left(1 - \frac{x^2}{R^2}\right)^{\frac{1}{2}}$$

$$\Rightarrow \quad O_x = R - R\left(1 - \frac{x^2}{2R^2} + \ldots\right)$$

$$\Rightarrow \quad O_x \approx \frac{x^2}{2R} \quad \text{(Approximate expression for perpendicular offset) ... (11.14)}$$

Thus by substituting different values of x, perpendicular offsets can be found out.

Note: *If the versed sine of curve (mid-ordinate) is less than 1/8th of chord then the curve approximately very close to circle.*

NOTE: If the curve is set out using the approximate expressions, then the resultant curve will be a parabola and not a circular curve. But for very flat curves whose radius is very large, then approximate expressions will give circular curve only.

11.6.1.3 Radial Offsets from the Tangent

As shown in Fig. 11.18, PP_1 is the radial offset where the point P is located at a distance x from the point of curve T_1.

Now from ΔPT_1O,

$$PO^2 = PT_1^2 + T_1O^2$$

$$(R + O_x)^2 = x^2 + R^2$$

$$\Rightarrow \quad O_x = \sqrt{R^2 + x^2} - R$$

(Exact expression for radial offset) ...(11.15)

Now, $$O_x = \sqrt{R^2 + x^2} - R$$

$$\Rightarrow \quad O_x = R\left(1+\frac{x^2}{R^2}\right)^{\frac{1}{2}} - R$$

$$\Rightarrow \quad O_x \approx \frac{x^2}{2R}$$

(Approximate expression for radial offset) ...(11.16)

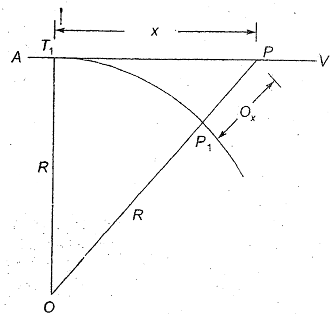


Fig. 11.18 *Radial offsets from the tangent*

11.6.1.4 Successive Bisection of Arcs or Chords

- As shown in Fig. 11.19, the long chord T_1T_2 is bisected at point M. Also mid-ordinate is equal to $R(1-\cos(\Delta/2))$. Thus point C gets established which is located at a distance equal to mid-ordinate from the point M. T_1C and T_2C are joined and bisected at points M_1 and M_2 respectively.
- Now the perpendicular offsets C_1M_1 and C_2M_2 will be equal to $R(1-\cos(\Delta/4))$. These offsets give points C_1 and C_2 on the curve. Now join T_1C_1, C_1C, CC_2 and C_2T_2 and bisect term. Determine new mid-ordinates which will give points on the curve. All these points when joined will give the required curve in the field.

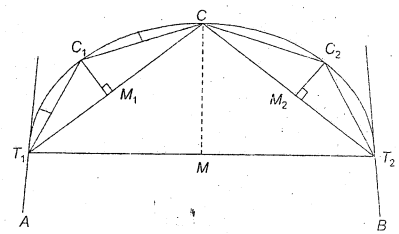


Fig. 11.19 *Successive bisection fo arcs or chords*

11.6.1.5 Offsets from the Chord Produced

- It is generally used for setting out the highway curves when theodolite is not available.
- As shown in Fig. 11.20, T_1a is the first sub-chord denoted as C_1.
- From the point of curve, T_1a length equal to first sub-chord ($C_1 = T_1a'$) is marked on the tangent through T_1.
- From point a', draw perpendicular to get point 'a' on the curve. T_1a is joined and produced by a distance C_2 (which is equal to full chord length).
- Set out perpendicular offset $b'b$ from point b' to get point 'b' on the curve. Points a and b are joined and produced by a distance C_3 (equal to full chord length) and the process goes on.

Now, $\angle a'T_1a = \delta_1$ = Deflection angle of the first chord

Thus,
$$O_1 = a'a = (T_1a)\delta_1 = C_1\delta_1$$
$$\angle T_1Oa = 2\angle aT_1a' = 2\delta_1$$
$$T_1a = R\cdot(2\delta_1)$$
$$\delta_1 = \frac{T_1a}{2R} = \frac{C_1}{2R}$$

Putting this value of δ_1 in the expression for O_1,

$$O_1 = C_1\left(\frac{C_1}{2R}\right) = \left(\frac{C_1^2}{2R}\right)$$

Now,
$$ab' = ab = C_2$$
$$O_2 = b'b = b'Q + Qb$$
$$b'Q = O_2'$$
$$Qb = O_2''$$
$$O_2'' = C_2\cdot\delta_2 = C_2\left(\frac{C_2}{2R}\right) = \left(\frac{C_2^2}{2R}\right)$$
$$O_2' = C_2\delta_1 = C_2\left(\frac{C_1}{2R}\right)$$

Thus
$$O_2 = O_2' + O_2''$$
$$= \frac{C_1C_2}{2R} + \frac{C_2^2}{2R} = \frac{C_2}{2R}(C_1 + C_2)$$

Similarly, $$O_3 = \frac{C_3}{2R}(C_2 + C_3)$$

And the last sub-chord is given by,

$$O_n = \frac{C_3}{2R}(C_2 + C_3)$$

$$O_n = \frac{C_n}{2R}(C_n + C_{n-1}) \qquad ...(11.17)$$

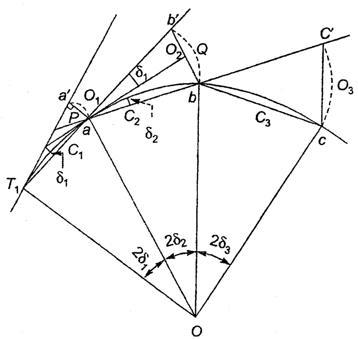


Fig. 11.20 *Offsets from chords produced*

11.6.2 Angular Methods of Setting a Simple Circular Curve

(a) Rankine's method of deflection angle

(b) Two theodolite method

(c) Tacheometric method

11.6.2.1 Rankine's Method of Deflection Angle

- This method is quite suitable for setting out a circular curve of large radius and large length.
- From geometric theorem on circle, the deflection angle to any point on curve is the angle subtended at the PC (T_1) between the tangent and the chord from PC to that Point. This deflection angle is one-half of the angle subtended by the arc at the center.

In **Fig. 11.21**, point of curve (*PC*) is T_1 and a, b, c.... are the point on the curve with $\delta_1, \delta_2, \delta_3$... the respective deflection angles between chord and respective tangents at T_1, *a*, *b* ... etc. Δ_1, Δ_2 ... etc. are the total deflection angles to the points *a*, *b*, *c* ... etc.

Thus, $\angle T_1Oa = 2\angle VT_1a = 2\delta_1$

Now, $R(2\delta_1) = \text{Curve } T_1a = c_1$

$$\therefore \quad \delta_1 = \frac{c_1}{2R} \text{ radians}$$

$$\Rightarrow \quad \delta_1 = \frac{c_1}{2R} \times \frac{180}{\pi} \text{ degrees}$$

$$\Rightarrow \quad \delta_1 = 1718.87\frac{c_1}{R} \text{ minutes}$$

Similarly, $$\delta_2 = 1718.87\frac{c_2}{R} \text{ minutes}$$

$$\delta_n = 1718.87\frac{c_n}{R} \text{ minutes}$$

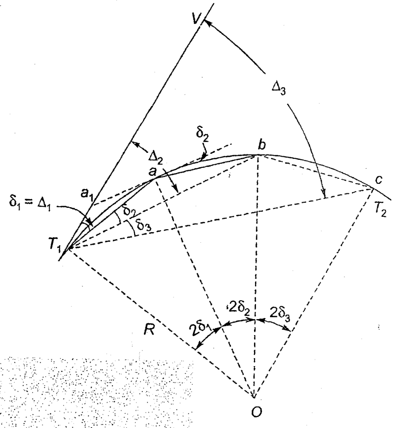


Fig. 11.21 *Rankine's method of deflection angle*

For the first chord T_1a, deflection angle = Δ_1 = Tangential angle δ_1

For point '*b*' on curve, $\Delta_2 = \angle VT_1b$

Angle subtended by chord 'ab' at on

$$T_1V = \angle aT_1b = \delta_2$$

Thus, $$\Delta_2 = \angle VT_1b = \angle VT_1a + \angle aT_1b$$

$$= \delta_1 + \delta_2 = \Delta_1 + \delta_2$$

$$\Delta_n = \Delta_{n-1} + \delta_n$$

As a check, for the last point T_2,

$$\Delta_n = \angle VT_1T_2 = \frac{\Delta}{2}$$

11.6.2.2 Two Theodolite Method

- It is the most preferred method of setting out a circular curve when the ground is undulating, rough and is also not suitable for the linear measurements.
- In this, no linear measurements are done. Instead two theodolites are used which gives more precision. It is based on the principle that:

  "The angle subtended by a tangent and a chord at a point on a circle is equal to the angle subtended by the chord in the opposite segment."

Thus in the **Fig. 11.22**

$$\angle VT_1a = \delta_1 = \angle aT_2T_1$$

$$\angle VT_1b = \delta_2 = \angle bT_2T_1$$

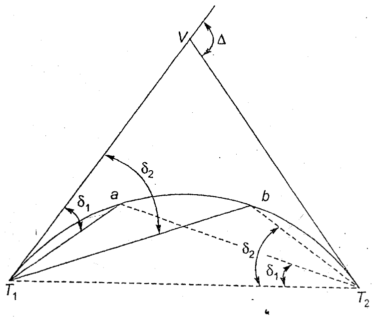


Fig. 11.22 *Two-theodolite method*

Procedure:

Step 1. Set up the theodolite at point of curve, T_1 and other theodolite at point of tangent, T_2.

Step 2. Set the Vernier *A* of both the theodolites to zero.

Step 3. Direct the theodolite at T_1 towards *V* and theodolite at T_2 towards T_1.

Step 4. Set angle δ_1 in both the theodolites so as to direct the line of sights towards T_1a and T_2a thereby locating the point '*a*' on the curve. The point '*a*' is the point of intersection of the two lines of sight.

Step 5. Similarly establish point '*b*' by setting an angle δ_2 in both the theodolites.

Step 6. Repeat the above steps with different values of δ to get different points on the curve.

Drawback : It is expensive and time consuming but at the same time it is the most precise method as well. All the points are established independently and thus error in locating a particular point is not carried forward to other points.

11.6.2.3 Tacheometric Method

- This method is very similar to Rankine's method of deflection angle.
- The theodolite at T_1 is used as a tacheometer and tacheometric observations are made.
- It is less precise than Rankine's method but here in this method, the chaining operation is completely eliminated.
- Point on the curve is established by setting the deflection angle in the tacheometer and measuring the distance of the point on the curve by placing a staff on it.
- The points *a*, *b*, *c*, ... So obtained are joined by T_1 and the chord lengths are given by:

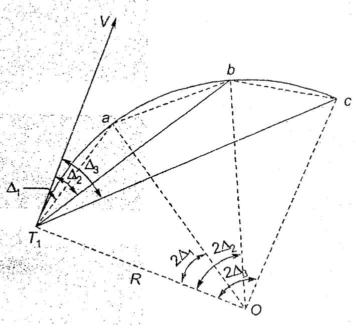


Fig. 11.23 *Tacheometric method*

$$T_1a = L_1 = 2R\sin\Delta_1$$

$$T_1b = L_2 = 2R\sin\Delta_2$$

$$T_1c = L_3 = 2R\sin\Delta_3$$

.

$$T_1T_2 = L_n = 2R\sin\Delta_n = 2R\sin(\Delta/2) = \text{Length of the long chord}$$

- The required staff intercepts for measuring the distances are obtained from the tacheometric formula as:

$$D = \left(\frac{f}{i}\right)s + (f+d) \quad \text{(for horizontal line of sight)}$$

$$D = \left(\frac{f}{i}\right)s \cdot \cos^2\theta + (f+d)\cos\theta \quad \text{(for inclined line of sight)}$$

11.7 Curve Passing Through a Fixed Point

Many a times it becomes indispensable to pass a curve from a fixed point say *P* as shown in **Fig. 11.24**. Let the point *P* be at a distance *l* from the point of intersection *V* and at an angle θ from the initial tangent

Now in ΔIPO,

$$\alpha = \theta - \angle T_1VO$$

$$\alpha = \theta - \left(90° - \frac{\Delta}{2}\right)$$

$$OV = R\sec\left(\frac{\Delta}{2}\right) \quad ...(11.18)$$

Now

$$\frac{OV}{\sin\beta} = \frac{OP}{\sin\alpha}$$

$$\frac{R\sec(\Delta/2)}{\sin\beta} = \frac{R}{\sin\alpha}$$

$$\sin\beta = \sin\alpha\sec\left(\frac{\Delta}{2}\right) \quad ...(11.19)$$

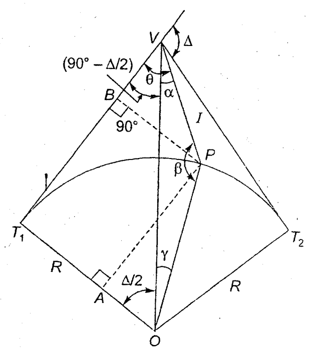


Fig. 11.24 *Curve passing through a fixed point*

Thus from the known values of α and Δ, β can be determined.

Now in ΔVOP, $\gamma = 180° - (\alpha + \beta)$

In ΔOAP, $OA = R\cos\left(\frac{\Delta}{2} + \gamma\right)$

In ΔPBV, $PB = l\sin\theta$

Thus, $OA = OT_1 - T_1A = OT_1 - PB$

$OA = R - l\sin\theta$

Therefore, $R\cos(\Delta/2 + \gamma) = R - l\sin\theta$

Now, θ, γ and l are known and thus R can be determined.

11.8 Compound Curve

- A compound curve is the combination of two or more simple circular curves of different radii.
- Thus because of two different radii, the two centers will also be different.
- Because of different length of tangents, compound curve allows the fitting of location of a topography with enhanced refinement as compared to simple circular curve.
- As far as possible, where simple circular curve can be used there compound curve should not be used.

11.9 Elements of Compound Curve

As shown in Fig. 11.25 the two straights AV and BV intersect at point of intersection V. T_1DT_2 is the compound curve having two circular arcs of radii R_1 and R_2. The two compound curves meet at the point of compound curvature D. MN is the common tangent to the two curves making angles of Δ_1 at M and Δ_2 at N.

Thus, deflection angle $(\Delta) = \Delta_1 + \Delta_2$

In ΔVMN,

$$\frac{VM}{\sin\Delta_2} = \frac{VN}{\sin\Delta_1} = \frac{MN}{\sin[180^0 - (\Delta_1 + \Delta_2)]}$$

Thus,

$$VM = \frac{MN\sin\Delta_2}{\sin(\Delta_1+\Delta_2)} = \frac{MN\sin\Delta_2}{\sin\Delta} \quad ...(11.20)$$

and

$$VN = \frac{MN\sin\Delta_1}{\sin(\Delta_1+\Delta_2)} = \frac{MN\sin\Delta_1}{\sin\Delta} \quad ...(11.21)$$

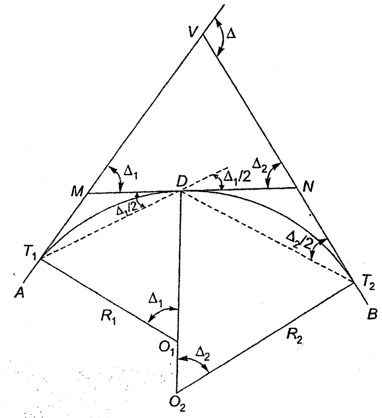


Fig. 11.25 *Compound curve*

Common Tangent MN

$$MD = R_1\tan\frac{\Delta_1}{2} = MT_1$$

$$ND = R_2\tan\frac{\Delta_2}{2} = NT_1$$

Thus,

$$MN = R_1\tan\frac{\Delta_1}{2} + R_2\tan\frac{\Delta_2}{2}$$

$$= R_1\tan\frac{\Delta_1}{2} + R_2\tan\frac{\Delta_2}{2} \quad ...(11.22)$$

Lengths of tangents VT_1 and VT_2

$$VT_1 = MT_1 + MV$$

$$VT_1 = R_1\tan\frac{\Delta_1}{2} + \frac{MN\sin\Delta_2}{\sin\Delta} \quad ...(11.23)$$

$$VT_2 = NT_2 + NV$$

$$VT_2 = R_2\tan\frac{\Delta_2}{2} + \frac{MN\sin\Delta_1}{\sin\Delta} \quad ...(11.24)$$

In general, in a compound curve, there are a total of seven elements viz. Δ_1, Δ_2, Δ, R_1, R_2, VT_1 and VT_2. If any of the four elements are known (with at least one angle and two lengths), the three other unknowns can be determined from the above derived equations.

11.10 Reverse Curve

Reverse curve consists of two curves with their centres on opposite side of the common tangent at the Point of Reverse Curvature (PRC). The radii of the two curves may be same or different.

Uses: Reverse curve is used in the following situations:

(a) When the two straight lines are parallel to each other.

(b) When the angle between the two straight lines is very small.

11.10.1 Elements of Reverse Curve

Total straight VA and VC include a total deviation angle of Δ. O_1 and O_2 are centres of two curves with corresponding radii R_1 and R_2 respectively. BE is common tangent which is perpendicular to O_1O_2.

Join T_1T_2 and draw $O_1G \perp T_1T_2$ and $O_2F \perp T_1T_2$.

In ΔBVE $\quad \Delta_1 = \Delta + \Delta_2$

$\Rightarrow \quad \Delta = \Delta_1 - \Delta_2 \quad ...(11.25)$

In ΔT_1VT_2 $\quad \delta_1 = \Delta + \delta_2$

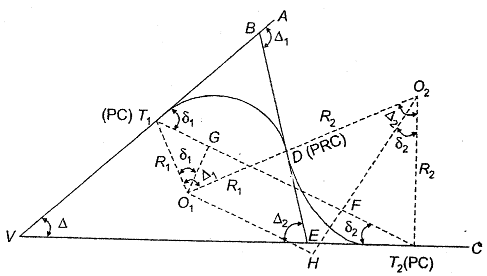


Fig. 11.26 *Reverse curve*

$\Rightarrow \quad \Delta = \delta_1 - \delta_2$

From Eqs. (11.25) and (11.26) ...(11.26)

$$\Delta_1 - \Delta_2 = \delta_1 - \delta_2 \quad ...(11.27)$$

Now, $O_1G \parallel O_2F$

$\therefore \quad \Delta_1 - \delta_1 = \Delta_2 - \delta_2$

$\Rightarrow \quad \Delta_1 - \Delta_2 = \delta_1 - \delta_2$ which is same as (11.27)

In ΔT_1GO_1, $\quad T_1G = R_1 \sin\delta_1, \quad O_1G = R_1 \cos\delta_1$

In ΔT_2FO_2, $\quad T_2F = R_2 \sin\delta_2, \quad O_2F = R_2 \cos\delta_2$

$\therefore \quad GF = O_1H = (R_1 + R_2)\sin(\Delta_2 - \delta_2)$

Thus,

$$T_1T_2 = T_1G + GF + FT_2$$
$$= R_1 \sin\delta_1 + (R_1 + R_2)\sin(\Delta_2 - \delta_2) + R_2 \sin\delta_2$$

Also,

$$O_2H = O_2F + FH$$
$$= O_2F + FH$$
$$= O_2F + O_1G$$
$$= R_2\cos\delta_2 + R_1\cos\delta_1 \quad (11.28)$$

In ΔO_1O_2H $\quad O_2H = (R_1 + R_2)\cos(\Delta_2 - \delta_2)$ ...(11.29)

From Eqs. (11.28) and (11.29)

$$R_2\cos\delta_2 + R_1\cos\delta_1 = (R_1 + R_2)\cos(\Delta_2 - \delta_2)$$

$$\Rightarrow \quad \cos(\Delta_2 - \delta_2) = \frac{R_1\cos\delta_1 + R_2\cos\delta_2}{(R_1 + R_2)} \quad ...(11.30)$$

Similarly,

$$\cos(\Delta_1 - \delta_1) = \frac{R_1\cos\delta_1 + R_2\cos\delta_2}{(R_1 + R_2)} \quad ...(11.31)$$

Illustrative Examples

Example 11.1 A vertical curve has an up gradient of +1.45% which is followed by a down gradient of –1.15%. The rate of change of is 0.35% per chain length of 20 m. What is the length of vertical curve?

Solution:

Given

$$g_1 = +1.45\%$$
$$g_2 = -1.15\%$$
$$r = 0.35\% \text{ per chain length of 20 m}$$

$\therefore$ Length of vertical curve, $L = \dfrac{g_1 - g_2}{r}$

$$= \frac{1.45 - (-1.15)}{0.35} = 7.429 \text{ chains of 20 m length}$$
$$= 7.429 \times 20 \text{ m} = 148.58 \text{ m}$$

Example 11.2 A parabolic vertical curve of length L is formed by joining on uphill gradient of $+p$% with a downhill gradient of $-q$%. After a few years, for renovation purpose, the uphill gradient was reduced to r% and downhill gradient was increased to s%, but as far as possible, the original curve is retained. Show that length of new vertical curve, is $\dfrac{L(r+s)}{(p+q)}$

Solution:

Let $\quad L_1$ = New length of vertical curve

The equation of parabola is, $y = kx^2$

It is assumed that chord length is equal to curve length

Now $\quad \dfrac{dy}{dx} = 2kx$

$\Rightarrow \quad x = \dfrac{1}{2k}\dfrac{dy}{dx}$

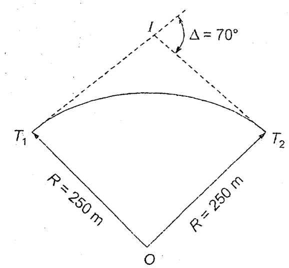
Thus, $\quad L = \dfrac{1}{2k}(p+q)$ and $L' = \dfrac{1}{2k}(r+s)$

$\therefore \quad \dfrac{L'}{L} = \dfrac{(r+s)}{(p+q)}$

$\Rightarrow \quad L' = \dfrac{L(r+s)}{(p+q)}$ Hence proved.

Example 11.3 A circular curve of radius 250 m is to be inserted between two straight meeting at a deflection angle of 70°. What is the degree of curve by arc definition and by chord definition? Also find the length of curve, tangent length, length of long chord, apex distance and mid-ordinate.

Solution:



Length of curve by arc definition

$$RD = 30 \text{ m}$$

(Assuming 30 m arc length)

$$\Rightarrow \quad \frac{RD\pi}{180} = 30 \text{ m}$$

(Converting D in radians to D in degrees)

$$\Rightarrow \quad D = \frac{180 \times 30}{\pi R} \text{degrees}$$

$$\Rightarrow \quad D = 6.875 \text{ degrees}$$

Length of arc by chord definition

$$\sin\left(\frac{D}{2}\right) = \frac{15}{R} \quad \text{(Assuming 30 m chord length)}$$

$$\Rightarrow \quad R = \frac{15}{\sin\left(\frac{D}{2}\right)} = \frac{15}{\frac{D}{2} \times \frac{\pi}{180}}$$ (For small angles, $\sin\theta \simeq \theta$ where θ is in radians)

$$\Rightarrow \quad D = \frac{30}{\frac{\pi R}{180}} = 6.875 \text{degrees}$$

$$\text{Length of the curve } (L) = R\Delta \quad (\Delta \text{ in radians})$$

$$= \frac{R\Delta\pi}{180} (\Delta \text{ indegrees})$$

$$= 250 \times 70 \times \frac{\pi}{180} = 305.43 \text{ m}$$

$$\text{Tangent length } (T) = R\tan\left(\frac{\Delta}{2}\right) = 250\tan\left(\frac{70}{2}\right) = 175.05 \text{ m}$$

$$\text{Length of long chord } (L) = 2R\sin\left(\frac{\Delta}{2}\right) = 2 \times 250 \times \sin\left(\frac{70}{2}\right) = 286.79 \text{ m}$$

$$\text{Apex distance} = R\left(\sec\left(\frac{\Delta}{2}\right) - 1\right) = R\left(\sec\left(\frac{70}{2}\right) - 1\right) = 55.19 \text{ m}$$

$$\text{Mid-ordinate} = R\left(1 - \cos\left(\frac{\Delta}{2}\right)\right) = 250\left(1 - \cos\left(\frac{70}{2}\right)\right) = 45.21 \text{ m}$$

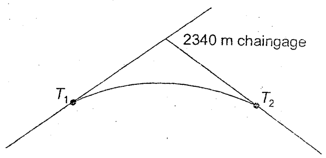
Example 11.4 A vertical curve of parabolic profile is required to join two grades of 0.8% and −1.4%. The rate of change of grade is −0.2% per 100 m. Find the reduced level of pegs if R.L. of point of intersection is 190.50 m. The chainage of point of intersection is 2340 m.

Solution:

Length of the curve is given by,

$$L = \frac{g_2 - g_1}{\Delta g} \times 100$$

$$= \frac{-1.4 - 0.8}{-0.2} \times 100 = 1100 \text{ m}$$



$\therefore$ Length of curve on either side of summit $= \frac{1100}{2} = 550$ m

$\therefore$ Chainage of first tangent point $T_1 = 2340 - 550 = 1790$ m

Chainage of second tangent point $T_2 = 1790 + 1100 = 2890$ m

$$\therefore \quad RL \text{ of } T_1 = 190.5 - \frac{0.8}{100} \times 550 = 186.1 \text{ m}$$

and

$$RL \text{ of } T_2 = 190.5 - \frac{1.4}{100} \times 550 = 182.8 \text{ m}$$

Equation of vertical curve is,

$$y = \left(\frac{g_2 - g_1}{200L}\right)x^2 + \frac{g_1}{100}x$$

$$\Rightarrow \quad y = \frac{-1.4 - 0.8}{200 \times 1100}x^2 + \frac{0.8}{100}x$$

$$\Rightarrow \quad y = -1 \times 10^{-5}x^2 + 8 \times 10^{-3}x$$

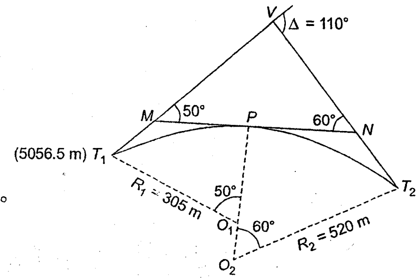
$\therefore$ RL of any point on vertical curve = RL of $T_1 + y$

x(m)	y(m)	RL(m)
30	0.231	186.331
60	0.444	186.544
90	0.639	185.929
120	0.816	186.916
150	0.975	187.075
180	1.116	187.216
210	1.239	187.339
240	1.344	187.444
270	1.431	187.531
300	1.5	187.600
330	1.551	187.651
360	1.584	187.684
390	1.599	187.699
⋮		
1100	−3.3	187.8

Example 11.5 A compound curve is composed of two arcs of radii 305 m and 520 m. The resulting deflection angle due to the combined curve is 110° and that due to first arc of radius 305 m is 50°. If chainage of first tangent point is 5056.5 m then find the chainages of other salient points.

Solution:

In ΔVMN



$$\text{exp } \angle\Delta = \angle VMN + \angle VNM$$

$$\Rightarrow \quad 110° = 50° + \angle VNM$$

$$\Rightarrow \quad \angle VNM = 60°$$

In ΔT_1MPO_1

$$\angle T_1MP = 180° - \angle V_1MP$$

$$= 180° - 50° = 130°$$

$$\angle O_1T_1M = \angle MPO_1 = 90°$$

Now

$$\angle O_1T_1M + \angle T_1MP + \angle MPO_1 + \angle PO_1T_1 = 360°$$

$$\Rightarrow \quad 90° + 130° + 90° + \angle PO_1T_1 = 360°$$

$$\Rightarrow \quad \angle PO_1T_1 = 50°$$

Similarly, In ΔT_2NPO_2

$$\angle PO_2T_2 = 60°$$

Tangent length $MT_1 = MP = R_1\tan(50°/2) = 305\tan 25° = 142.22$ m

Tangent length $NT_2 = NP = R_2\tan(60°/2) = 520\tan 30° = 300.22$ m

From sine law in ΔVM_1N

$$\frac{VM}{\sin 60°} = \frac{VN}{\sin 50°} = \frac{MN}{\sin(180° - \Delta)}$$

$$\Rightarrow \quad \frac{VM}{\sin 60°} = \frac{VN}{\sin 50°} = \frac{MP + PN}{\sin(180° - 110°)}$$

$$\therefore \quad VM = \frac{(MP + PN)}{\sin 70°}\sin 60° = \frac{(142.22 + 300.22)\sin 60°}{\sin 70°} = 407.75 \text{ m}$$

$$VN = \frac{(MP + PN)\sin 50°}{\sin 70°} = \frac{(142.22 + 300.22)\sin 50°}{\sin 70°} = 360.68 \text{ m}$$

Given chainage of $T_1 = 5056.5$ m

$\therefore$ Chainage of V = chainage of $T_1 + T_1V$

$= 5056.5 + (T_1M + MV)$

$= 5056.5 + (142.22 + 407.75) = 5606.47$ m

$$\text{Length of arc } T_1P = 305 \times \left(50° \times \frac{\pi}{180°}\right) = 266.16 \text{ m}$$

$$\text{Length of arc } PT_2 = 520 \times \left(60° \times \frac{\pi}{180°}\right) = 544.54 \text{ m}$$

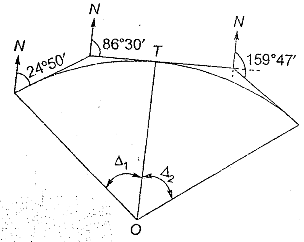
Chainage of P = Chainage of T_1 + Length of arc T_1P

$= 5056.5 + 266.16 = 5322.66$ m

$$\text{Chainage of } T_2 = \text{Chainage of } P + \text{Length of arc } PT_2$$
$$= 5322.66 + 544.54$$
$$= 5867.2 \text{ m}$$

Example 11.6 The WCB of lines *AB*, *BC* and *CD* are 24°50′. 86°30′ and 159°47′ respectively. If length of line BC is 550 m then find the radius of curve which is tangential to three lines and also find the tangent lengths.

Solution:



Deflection angle $\Delta_1 = 86°30' - 24°50'$
$= 86.5° - 24.83° = 61.67°$

Deflection angle $\Delta_2 = 159°47' - 86°30'$
$= 159.78° - 86.5° = 73.28°$

Tangent length $AB = BT = R\tan(\Delta_1/2)$
$= R\tan(61.67°/2) = 0.59695\,R$

Tangent length $TC = CD = R\tan(\Delta_2/2)$
$= R\tan(73.28°/2) = 0.74375\,R$

Now given $BC = 550$ m

$\Rightarrow$ $BT + TC = 550$

$\Rightarrow$ $0.59695\,R + 0.74375\,R = 550$

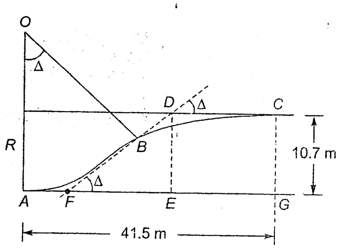
$\Rightarrow$ $R = 410.23$ m

$\therefore$ $AB = BT = 0.59695(410.23) = 244.89$ m

$TC = CD = 0.74375(410.23) = 305.11$ m

Example 11.7 Two straight parallel roads distant 10.7 m c/c apart are to be connected by a reverse curve of same radius such that maximum distance between tangent point is 41.5 m. What is the allowable radius?

Solution:



$AF + FE + EG = 41.5$ m

Now $AF = FB = \text{Tangent length } (T)$
$= R\tan(\Delta/2)$

and $DB = DC = \text{Tangent length } (T)$
$= R\tan(\Delta/2)$

$\therefore$ $AF + FE + EG = 41.5$ m

$\Rightarrow$ $T + T\cos\Delta + T\cos\Delta + T = 41.5$

$\Rightarrow$ $2T(1 + \cos\Delta) = 41.5$

$\Rightarrow$ $R\tan(\Delta/2)(1 + \cos\Delta) = \dfrac{41.5}{2} = 20.75$

$\Rightarrow$ $R\tan\left(\dfrac{\Delta}{2}\right)2\cos^2\left(\dfrac{\Delta}{2}\right) = 20.75$

$\Rightarrow$ $2R\sin\left(\dfrac{\Delta}{2}\right)\cos\left(\dfrac{\Delta}{2}\right) = 20.75$ ...(i)

$\Rightarrow$ $R\sin\Delta = 20.75$

Also $DE = DF\sin\Delta = 2T\sin\Delta$ ... (i) (a)

$\Rightarrow$ $10.7 = 2T\sin\Delta = 2\sin\Delta\, R\tan\left(\dfrac{\Delta}{2}\right)$

$= R4\sin\left(\dfrac{\Delta}{2}\right)\cos\left(\dfrac{\Delta}{2}\right)\dfrac{\sin\left(\dfrac{\Delta}{2}\right)}{\cos\left(\dfrac{\Delta}{2}\right)} = 4R\sin^2\left(\dfrac{\Delta}{2}\right)$

$\Rightarrow$ $R\sin^2\left(\dfrac{\Delta}{2}\right) = \dfrac{10.7}{4} = 2.675$ ...(ii)

Dividing equation (ii) by equation (i),

$\dfrac{\tan\left(\dfrac{\Delta}{2}\right)}{2} = \dfrac{2.675}{20.75}$

$\Rightarrow$ $\tan\left(\dfrac{\Delta}{2}\right) = 0.25783$

$\Rightarrow$ $\Delta = 28.92°$

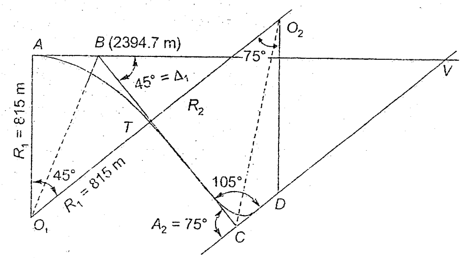
Also $R\sin\Delta = 20.75$ [From equation (i) (a)]

$\Rightarrow$ $R\sin 28.92° = 20.75$

$\Rightarrow$ $R = 42.91$ m

Example 11.8 There are two straights *AB* and *CD* which intersect at *V*. The $\angle VBC = 45°$ and $\angle BCV = 105°$. A reverse curve is proposed to connect the two straights such that the two reverse curves meet on line *BC*. The length of line *BC* is 803 m, radius of curve for straight *AB* is 815 m, chainage of *B* is 2394.7 m. Compute the radius of curve for straight *CD*, both the lengths of arcs and chainage of point *D*.

Solution:



Let radius of curve for straight
$AB = R_1 = 815$ m
and radius of curve for straight
$CD = R_2$

Now $BT = R_1\tan\left(\dfrac{\Delta_1}{2}\right)$

$= 815\tan\left(\dfrac{45°}{2}\right) = 337.58$ m

$\therefore$ $AB = BT = 337.58$ m

$\therefore$ $TC = BC - BT = 803 - 337.58 = 465.42$ m

But $TC = CD = R_2\tan\left(\dfrac{\Delta_2}{2}\right)$

$\Rightarrow$ $465.42 = R_2\tan\left(\dfrac{75°}{2}\right)$

$\Rightarrow$ $R_2 = 606.55$ m

Length of curve $AT = R_1\Delta_1 = \dfrac{815 \times 45° \times \pi}{180°} = 640.099 \text{ m} \simeq 640 \text{ m}$

Length of curve $TD = R_2\Delta_2$

$$= \frac{606.55 \times 75° \times \pi}{180°} = 793.972 \text{ m} \simeq 794 \text{ m}$$

$\therefore$ Total length of curve = 640 + 794 = 1434 m

Chainage of A = Chainage of $B - AB$

= 2394.7 − 337.58 = 2057.12 m

$\therefore$ Chainage of D = Chainage of A + Arc length of A + Arc length TD

= 2057.12 + 640 + 794 = 3491.12 m

Example 11.9 Two straights PQ and QR meet each other at Q at an angle of 120°. What is the radius of curve required if it has to pass through a point M which is 55 m away from the point of intersection Q and subtends and angle of 28° with straights PQ at Q?

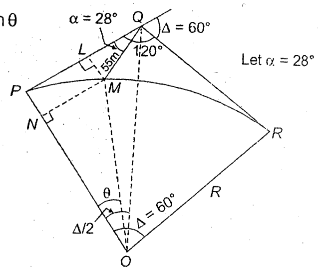
Solution:

Join M and O. From M, draw a perpendicular ML on PQ meeting PQ at L. Similarly draw another perpendicular from M as MN meeting PO at N.

Now, $\tan\alpha = \tan 28° = \dfrac{ML}{LQ} = \dfrac{ML}{(PQ - PL)} = \dfrac{NP}{(PQ - PL)}$

$$= \frac{R - R\cos\theta}{R\tan\left(\frac{\Delta}{2}\right) - NM} = \frac{R(1-\cos\theta)}{R\tan\left(\frac{\Delta}{2}\right) - R\sin\theta}$$

$$= \frac{1-\cos\theta}{\tan\left(\frac{\Delta}{2}\right) - \sin\theta}$$



Now, $\angle MQO = \left(90° - \dfrac{\Delta}{2}\right) - \alpha = 90° - \left(\alpha + \dfrac{\Delta}{2}\right)$

$\therefore$ $\angle QMO = 180° - \angle MQO - \angle QOM$

$$= 180° - \left(90° - \left(\alpha + \frac{\Delta}{2}\right)\right) - \left(\frac{\Delta}{2} - \theta\right)$$

$$= 90° + \alpha + \frac{\Delta}{2} - \frac{\Delta}{2} + \theta$$

$$= 90° + (\alpha + \theta)$$

From sine law in ΔOMQ

$$\frac{OM}{\sin\angle MQO} = \frac{OQ}{\sin\angle OMQ}$$

$\Rightarrow$ $$\frac{R}{\sin\left[90° - \left(\alpha + \frac{\Delta}{2}\right)\right]} = \frac{R\sec\left(\frac{\Delta}{2}\right)}{\sin[90° + (\alpha + \theta)]}$$

$\Rightarrow$ $$\cos(\alpha + \theta) = \cos\left(\alpha + \frac{\Delta}{2}\right)\cdot\sec\left(\frac{\Delta}{2}\right)$$

Also $PO = PN + NO = LM + NO = QM\sin\alpha + R\cos\theta$

$\Rightarrow$ $R = QM\sin\alpha + R\cos\theta$

$\Rightarrow$ $QM = \dfrac{R(1-\cos\theta)}{\sin\alpha}$

or $R = \dfrac{QM\cdot\sin\alpha}{(1-\cos\theta)} = \dfrac{55\sin 28°}{(1-\cos\theta)}$

From $\cos(\alpha + \theta) = \cos\left(\alpha + \dfrac{\Delta}{2}\right)\cdot\sec\left(\dfrac{\Delta}{2}\right)$

$\Rightarrow$ $\cos(\theta + 28°) = \cos(28° + 30°)\sec 15° = 0.54861$

$\therefore$ $\theta + 28° = \cos^{-1}(0.54861) = 56.73°$

$\Rightarrow$ $\theta = 28.73°$

$\therefore$ $R = \dfrac{55\sin 28°}{1-\cos 28.73°} = 209.75$ m

Objective Brain Teasers

Q.1 The ratio of radius and apex distance of a curve of radius R deflecting through Δ is:

(a) $1-\sec\left(\frac{\Delta}{2}\right)$ (b) $\sec\left(\frac{\Delta}{2}\right)-1$

(c) $\tan\left(\frac{\Delta}{2}\right)-1$ (d) None of these

Q.2 For a curve of 100 m radius and normal chord 10 m, the deflection angle by Rankine's formula is:

(a) 2° 51.53′ (b) 1° 21.53′

(c) 2° 30′ (d) 1°21.65′

Q.3 The radius of a simple circular curve is 400 m and 120° deflection angle the mid-ordinate is:

(a) 400 m (b) 600 m

(c) 200 m (d) 800 m

Q.4 The length of first sub-chord is S_1 and that of second chord is S_2. The deflection angle of second chord in minutes is:

(a) $\dfrac{1718.87}{R}S_1(S_1 + S_2)$

(b) $\dfrac{1718.87}{R}(S_1 + S_2)$

(c) $\dfrac{1718.87}{R}(S_2 - S_1)$

(d) Data insufficient

Q.5 The length of long chord and tangent of a circular curve of radius R will be equal if angle of deflection is:

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{6}$

(c) $\frac{2\pi}{3}$ (d) $\frac{\pi}{4}$

Q.6 The total deflection angle for a simple circular curve is $\frac{\pi}{2}$ radians. A tangent is drawn which has a length of 150 m between the two tangents and this makes an angle of $\frac{\pi}{4}$ radians with the back tangent. The radius of the curve is:

(a) 305 m (b) 450 m

(c) 225 m (d) 181 m

Q.7 A curve which is tangential to four sight lines and consists of area of different radii. This curve is known as:

(a) two centered compound curve

(b) four centered compound curve

(c) one centered compound curve

(d) three centered compound curve

Q.8 The radius of a circular curve is five times the length of transition curve. The spiral angle is:

(a) 0.1 rad. (b) 0.2 rad.
(c) 0.05 rad. (d) 0.02 rad.

Q.9 The chainage of P.T. is 1435 m. A simple circular curve of 400 m radius was set out with 20 m chain by the method of deflection distances. The offset required at last chord is:

(a) 0.66 m (b) 0.61 m
(c) 0.68 m (d) 0.50 m

Q.10 Overturning of vehicles can be avoided by:

(a) Providing a compound curve
(b) Providing a transition curve
(c) Providing a reverse curve
(d) Data not sufficient

Answers

1. (b)	2. (a)	3. (c)	4. (b)	5. (c)
6. (d)	7. (d)	8. (a)	9. (a)	10. (b)

Student's Assignments

Ex.1 Two straight T_1V and T_2V intersect at V with a chainage of (390 + 14). The angle of deflection is 115°. For a right hand circular curve of 380 m radius, compute the chainage of tangent points if 20 m chain was used.

Ans. (360 + 31.6) (398 + 45.15)

Ex.2 Two straight intersect at a chainage of 4305.42 m. The angle of intersection is 132°. Compute all the necessary data to set out a 5° simple circular curve to connect the two straights. Use different method of setting out a simple circular Use 20 m and 30 m chord length.

Ex.3 A circular curve passes through a point A which is 15 m from point of intersection of two tangents. 3540 is the chainage of point of intersection with corresponding intersection angle of 28°. Determine the radius of curve and chainages of tangent points.

Ans. 490 m, 3417.83 m, 3657.29 m