

9

Chapter

WAVES

A

SINGLE CORRECT CHOICE TYPE

Each of these questions has 4 choices (a), (b), (c) and (d) for its answer, out of which ONLY ONE is correct.

- Two sound sources emitting sound each of wavelength λ are fixed at points A and B . A listener moves with velocity u from A to B . The number of beats heard by him per second is
(a) $2u/\lambda$ (b) u/λ (c) $u/3\lambda$ (d) $2\lambda u$
- If the fundamental frequency of a vibrating organ pipe is 200 Hz, then
(a) the first overtone is 200 Hz
(b) the first overtone may be 400 Hz
(c) the first overtone is 300 Hz
(d) none of the above
- Calculate the velocity of sound in a mixture of oxygen, nitrogen and argon at 0°C . The mixture consists of the gases oxygen, nitrogen and argon in the mass ratio 2 : 7 : 1. (Given $R = 8.3 \text{ J mol}^{-1} \text{ K}^{-1}$. Ratio of specific heats of the gases are argon 1.67, oxygen 1.4, nitrogen 1.4. The molecular weights of the respective gases are 40, 32 and 28.)
(a) 329.5 m/s (b) 219.0 m/s
(c) 422.0 m/s (d) 380.2 m/s
- A listener moves towards a source with a speed of 10 ms^{-1} . If the source emits a frequency 200 Hz and velocity of sound in air is 332 ms^{-1} , the wavelength of the note received by the listener is
(a) 1.685 m (b) 1.71 m (c) 1.66 m (d) 2 m
- A cylindrical hose open at both ends has a fundamental frequency f in air. The hose is dipped vertically in water so that half of it is in water. The fundamental frequency of the air column in now
(a) $f/2$ (b) $3f/4$ (c) f (d) $2f$
- A wave represented by the equation $y = a \cos(kx - \omega t)$ is superposed with another wave to form a stationary wave such that point $x = 0$ is a node. The equation for the other wave is
(a) $a \sin(kx + \omega t)$ (b) $-a \cos(kx - \omega t)$
(c) $-a \cos(kx + \omega t)$ (d) $-a \sin(kx - \omega t)$
- A train approaching a hill at a speed of 40 km/hour sounds a whistle of frequency 580 Hz when it is at a distance of 1 km from the hill. Wind is blowing in the direction of the train with a speed of 40 km/h. Find the frequency of the whistle heard by an observer on the hill: (Velocity of sound in air = 1200 km/h)
(a) 585 Hz (b) 575 Hz (c) 599 Hz (d) 589 Hz
- A siren placed at a railway platform is emitting sound of frequency 5 kHz. A passenger sitting in a moving train A records a frequency of 5.5 kHz while the train approaches the siren. During his return journey in a different train B he records a frequency of 6.0 kHz while approaching the same siren. The ratio of the velocity of train B to that of train A is
(a) 242/252 (b) 2 (c) 5/6 (d) 11/6
- A note is produced when you blow air across the top of a test tube. Two students were asked about the effect of blowing harder.
Student A : The pitch of sound would increase.
Student B : The intensity of sound would increase.
(a) A is correct, B is wrong
(b) B is correct, A is wrong
(c) Both are correct
(d) Both are wrong
- An object of specific gravity ρ is hung from a thin steel wire. The fundamental frequency for transverse standing waves in the wire is 300 Hz. The object is immersed in water so that one half of its volume is submerged. The new fundamental frequency in Hz is
(a) $300 \left(\frac{2\rho - 1}{2\rho} \right)^{1/2}$ (b) $300 \left(\frac{2\rho}{2\rho - 1} \right)^{1/2}$
(c) $300 \left(\frac{2\rho}{2\rho - 1} \right)$ (d) $300 \left(\frac{2\rho - 1}{2\rho} \right)$



**MARK YOUR
RESPONSE**

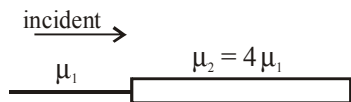
- | | | | | |
|-----------------|-----------------|-----------------|-----------------|------------------|
| 1. (a)(b)(c)(d) | 2. (a)(b)(c)(d) | 3. (a)(b)(c)(d) | 4. (a)(b)(c)(d) | 5. (a)(b)(c)(d) |
| 6. (a)(b)(c)(d) | 7. (a)(b)(c)(d) | 8. (a)(b)(c)(d) | 9. (a)(b)(c)(d) | 10. (a)(b)(c)(d) |

11. Two vibrating strings of the same material but length L and $2L$ have radii $2r$ and r respectively. They are stretched under the same tension. Both the strings vibrate in their fundamental modes, the one of length L with frequency f_1 and the other with frequency f_2 . The ratio f_1/f_2 is given by
 (a) 2 (b) 4 (c) 8 (d) 1

12. A wave disturbance in a medium is described by $y(x,t) = 0.02 \cos\left(50\pi t + \frac{\pi}{2}\right) \cos(10\pi x)$ where x and y are in metre and t is in second

- (a) A node occurs at $x = 0.15$ m
 (b) An antinode occurs at $x = 0.3$ m
 (c) The speed of wave is 5 ms^{-1}
 (d) The wavelength of wave is 0.3 m

13. String # 1 is connected with string # 2. The mass per unit length in string # 1 is μ_1 and the mass per unit length in string # 2 is $4\mu_1$. The tension in the strings is T . A travelling wave is coming from the left. What fraction of the energy in the incident wave goes into string # 2 ?



- (a) 1/8 (b) 4/9 (c) 2/3 (d) 8/9

14. An open pipe is suddenly closed at one end with the result that the frequency of third harmonic of the closed pipe is found to be higher by 100 Hz than the fundamental frequency of the open pipe. The fundamental frequency of the open pipe is

- (a) 200 Hz (b) 300 Hz
 (c) 240 Hz (d) 480 Hz

15. The extension in a string, obeying Hooke's law, is x . The speed of sound in the stretched string is v . If the extension in the string is increased to $1.5x$, the speed of sound will be
 (a) $1.22v$ (b) $0.61v$
 (c) $1.50v$ (d) $0.75v$

16. The equation $y = a \cos^2(2\pi nt - 2\pi x/\lambda)$ represents a wave with:

- (a) amplitude a , frequency n and wavelength λ
 (b) amplitude a , frequency $2n$ and wavelength 2λ
 (c) amplitude $a/2$, frequency $2n$ and wavelength λ
 (d) amplitude $a/2$, frequency $2n$ and wavelength $\lambda/2$

17. A source emitting sound of frequency f_0 is moving in a circle of radius R , having centre at the origin, with a uniform speed $= c/3$, where $c =$ speed of sound. Find the maximum and minimum frequencies heard by a stationary listener at the point $(R/2, 0)$.

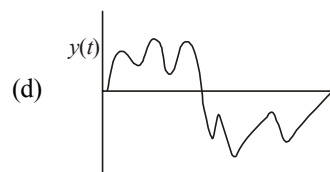
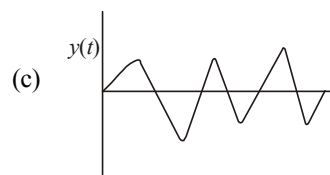
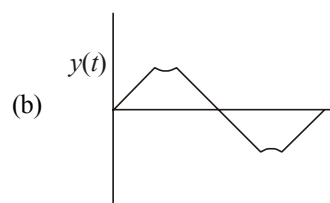
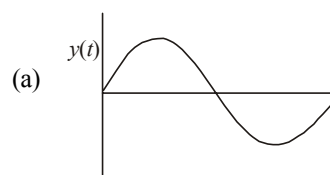
- (a) $\frac{6f_0}{5}, \frac{6f_0}{7}$ (b) $\frac{2\sqrt{3}f_0}{2\sqrt{3}-1}, \frac{2\sqrt{3}f_0}{2\sqrt{3}+1}$
 (c) $\frac{3f_0}{2}, \frac{3f_0}{5}$ (d) None of these

18. A transverse wave in a medium is described by the equation $y = A \sin^2(\omega t - kx)$. The magnitude of the maximum velocity of particles in the medium is equal to that of the wave velocity, if the value of A is

- (a) $\lambda/2\pi$ (b) $\lambda/4\pi$
 (c) λ/π (d) $2\lambda/\pi$

19. A complex wave is represented by an expression of the form:

$y(t) = 1. \sin \omega x + \frac{1}{2} \sin 3\omega x$, where $\omega = \frac{2\pi}{T}$ is the angular frequency and T is the period of the wave. Which of the following sketches best represents the wave?



**MARK YOUR
RESPONSE**

11. (a) (b) (c) (d)

12. (a) (b) (c) (d)

13. (a) (b) (c) (d)

14. (a) (b) (c) (d)

15. (a) (b) (c) (d)

16. (a) (b) (c) (d)

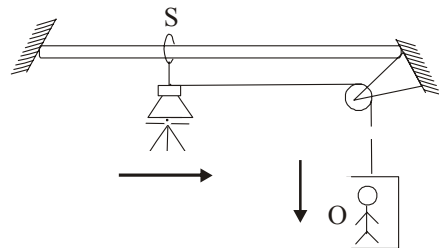
17. (a) (b) (c) (d)

18. (a) (b) (c) (d)

19. (a) (b) (c) (d)

20. A whistle giving out 450 Hz approaches a stationary observer at a speed of 33 m/s. The frequency heard by the observer in Hz is
 (a) 409 (b) 429 (c) 517 (d) 500
21. Equations of a stationary and a travelling waves are as follows $y_1 = a \sin kx \cos \omega t$ and $y_2 = a \sin (\omega t - kx)$. The phase difference between two points $x_1 = \frac{\pi}{3k}$ and $x_2 = \frac{3\pi}{2k}$ is ϕ_1 in the standing wave (y_1) and is ϕ_2 in travelling wave (y_2), then ratio $\frac{\phi_1}{\phi_2}$ is
 (a) 1 (b) 5/6 (c) 3/4 (d) 6/7
22. A travelling wave in a stretched string is described by the equation $y = A \sin (kx - \omega t)$ The maximum particle velocity is
 (a) $A\omega$ (b) ω/k (c) $d\omega/dk$ (d) x/t
23. The equation of plane progressive wave motion is $y = a \sin 2\pi/\lambda (vt - x)$. Velocity of particle is
 (a) $y \frac{dv}{dx}$ (b) $v \frac{dy}{dx}$ (c) $-y \frac{dv}{dx}$ (d) $-v \frac{dy}{dx}$
24. A train moves towards a stationary observer with speed 34 m/s. The train sounds a whistle and its frequency registered by the observer is f_1 . If the train's speed is reduced to 17 m/s, the frequency registered is f_2 . If the speed of sound is 340 m/s, then the ratio f_1/f_2 is
 (a) 18/19 (b) 1/2 (c) 2 (d) 19/18
25. Three coherent sonic sources emitting sound of single wavelength ' λ ' are placed on the x -axis at points $(-\lambda\sqrt{11}/6, 0)$, $(0, 0)$, $(\lambda\sqrt{11}/6, 0)$. The intensity reaching a point $0, 5\lambda/6$ from each source has the same value I_0 . Then the resultant intensity at this point due to the interference of the three waves will be :
 (a) $6I_0$ (b) $7I_0$ (c) $4I_0$ (d) $5I_0$
26. Two pulses on the same string are described by the following wave equations

$$y_1 = \frac{5}{(3x - 4t)^2} \quad \text{and} \quad y_2 = \frac{-5}{(3x + 4t - 6)^2}$$
 where symbols have usual meaning. Choose the incorrect statement
 (a) Pulse y_1 and pulse y_2 travel along +ve and -ve x -axis respectively
 (b) At $t = 0.75$ s, y displacement at all points on the string is zero
 (c) At $x = 1$ m, y displacement is zero for all time
 (d) Energy of string is zero at $t = 0.75$ s
27. The amplitude of a wave disturbance propagating in the positive x -direction is given by $y = \frac{1}{1 + x^2}$ at $t = 0$ and $y = \frac{1}{2 + x^2 - 2x}$ at $t = 2$ s, where x and y are in meter. Assuming that the shape of the wave disturbance does not change during the propagation, the speed of the wave is
 (a) 0.5 m/s (b) 1 m/s (c) 1.5 m/s (d) 2 m/s
28. A composite string is made up by joining two strings of different masses per unit length μ and 4μ . The composite string is under the same tension. A transverse wave pulse $Y = (6 \text{ mm}) \sin (5t + 40x)$, where ' t ' is in seconds and ' x ' is in metres, is sent along the lighter string towards the joint. The joint is at $x = 0$. The equation of the wave pulse reflected from the joint is
 (a) $Y = (2 \text{ mm}) \sin (5t - 40x)$
 (b) $Y = (4 \text{ mm}) \sin (40x - 5t)$
 (c) $Y = -(2 \text{ mm}) \sin (5t - 40x)$
 (d) $Y = (2 \text{ mm}) \sin (5t - 10x)$
29. Two monatomic ideal gases 1 and 2 of molecular masses m_1 and m_2 respectively are enclosed in separate containers kept at the same temperature. The ratio of the speed of sound in gas 1 to that in gas 2 is given by
 (a) $\sqrt{\frac{m_1}{m_2}}$ (b) $\sqrt{\frac{m_2}{m_1}}$ (c) $\frac{m_1}{m_2}$ (d) $\frac{m_2}{m_1}$
30. A sound source S, emitting a particular sound frequency can slide along a horizontal rough rod. It is tied with a string (of total length ℓ) that passes over a pulley. An observer O is in a lift which is tied to the other end of the string as shown in the figure. The system is made to move with a constant speed. Considering that ℓ is not very large, as distance between the pulley and the source S varies from ℓ to zero, the apparent frequency perceived by the observer

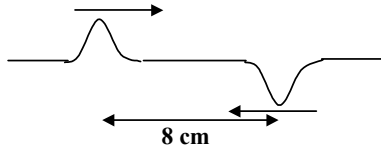


- (a) first increases, then decreases
 (b) first decreases, then increases
 (c) continuously increases
 (d) continuously decreases

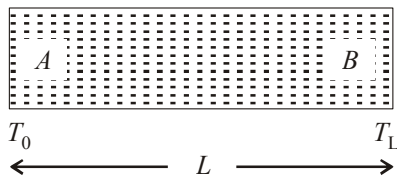


MARK YOUR RESPONSE	20. (a) (b) (c) (d)	21. (a) (b) (c) (d)	22. (a) (b) (c) (d)	23. (a) (b) (c) (d)	24. (a) (b) (c) (d)
	25. (a) (b) (c) (d)	26. (a) (b) (c) (d)	27. (a) (b) (c) (d)	28. (a) (b) (c) (d)	29. (a) (b) (c) (d)
	30. (a) (b) (c) (d)				

31. Two pulses in a stretched string whose centers are initially 8 cm apart are moving towards each other as shown in the figure. The speed of each pulse is 2 cm/s. After 2 seconds, the total energy of the pulses will be



- (a) zero
 (b) purely kinetic
 (c) purely potential
 (d) partly kinetic and partly potential
32. A stationary observer receives sound wave from two tuning forks, one of which approaches and the other recedes with the same velocity. As this takes place, the observer hears beats with frequency 2 Hz. What is the velocity of each tuning fork if their oscillation frequency is $\nu_0 = 680$ Hz and the velocity of sound in air is $\nu_s = 340$ m/s
 (a) 5 m/s (b) 0.5 m/s
 (c) 15 m/s (d) 50 m/s
33. The temperature of a mono-atomic gas in an uniform container of length L varies linearly from T_0 to T_L as shown in the figure. If the molecular weight of the gas is M_0 , then the time taken by a wave pulse in travelling from end A to end B is



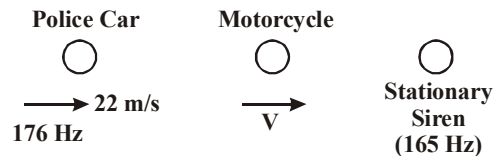
- (a) $\frac{2L}{\sqrt{T_L} \sqrt{T_0}} \sqrt{\frac{3M}{5R}}$ (b) $\sqrt{\frac{3(T_L - T_0)}{5RM_0L}}$
 (c) $\frac{L}{\sqrt{T_L} - \sqrt{T_0}} \sqrt{\frac{3M}{5R}}$ (d) $L \sqrt{\frac{M_0}{2R(T_L - T_0)}}$

34. The ends of a stretched wire of length L are fixed at $x = 0$ and $x = L$. In one experiment, the displacement of the wire is $y_1 = A \sin(\pi x/L) \sin \omega t$ and energy is E_1 and in another experiment its displacement is $y_2 = A \sin(2\pi x/L) \sin 2\omega t$ and energy is E_2 . Then
 (a) $E_2 = E_1$ (b) $E_2 = 2E_1$
 (c) $E_2 = 4E_1$ (d) $E_2 = 16E_1$
35. An object of specific gravity ρ is hung from a thin steel wire. The fundamental frequency of transverse standing waves in the wire is 300 Hz. The object is immersed in water,

so that one half of its volume is submerged. The new fundamental frequency (in Hz) is

- (a) $300 \sqrt{\left(1 - \frac{1}{2\rho}\right)}$ (b) $300 \sqrt{\left(\frac{2\rho}{2\rho - 1}\right)}$
 (c) $300 \left(\frac{2\rho}{2\rho - 1}\right)$ (d) $300 \left(\frac{2\rho - 1}{2\rho}\right)$

36. A sonometer wire resonates with a given tuning fork forming standing waves with five antinodes between the two bridges when a mass of 9 kg is suspended from the wire. When this mass is replaced by a mass M , the wire resonates with the same tuning fork forming three antinodes for the same positions of the bridges. The value of M is
 (a) 25 kg (b) 5 kg
 (c) 12.5 kg (d) 1/25 kg
37. How many frequencies below 1 kHz of natural oscillations of air column will be produced if a pipe of length 1 m is closed at one end? [velocity of sound in air is 340 m/s]
 (a) 5 (b) 6
 (c) 4 (d) 8
38. A sound source emits frequency of 180 Hz when moving towards a rigid wall with speed 5 m/s and an observer is moving away from wall with speed 5 m/s. Both source and observer moves on a straight line which is perpendicular to the wall. The number of beats per second heard by the observer will be [Speed of sound = 355 m/s]
 (a) 5 beats/s (b) 10 beats/s
 (c) 6 beats/s (d) 8 beats/s
39. A police car moving at 22 m/s, chases a motorcyclist. The police man sounds his horn at 176 Hz, while both of them move towards a stationary siren of frequency 165 Hz. Calculate the speed of the motorcycle, if it is given that he does not observe any beats.



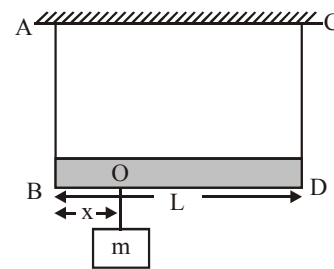
- (a) 33m/s (b) 22m/s
 (c) zero (d) 11m/s
40. A string of length 0.3 m and mass 10^{-2} kg is clamped at both of its ends. The tension in the string is 1.2 N. When a pulse travels along the string, the shape of the string is found to be the same at times t and $t + \Delta t$. The minimum value of Δt is
 (a) 0.1 sec (b) 0.2 sec
 (c) 0.3 sec (d) 0.4 sec



MARK YOUR
RESPONSE

31. (a)(b)(c)(d)	32. (a)(b)(c)(d)	33. (a)(b)(c)(d)	34. (a)(b)(c)(d)	35. (a)(b)(c)(d)
36. (a)(b)(c)(d)	37. (a)(b)(c)(d)	38. (a)(b)(c)(d)	39. (a)(b)(c)(d)	40. (a)(b)(c)(d)

41. A plane progressive simple harmonic sound wave of angular frequency 680 rad/s moves with speed 340 m/s in the direction which makes equal angle with each x , y and z -axis. The phase difference ($\phi_1 - \phi_2$) between the oscillations of the particle in the medium located at the positions $(\sqrt{3}, \sqrt{3}, \sqrt{3})$ and $(2\sqrt{3}, 2\sqrt{3}, 2\sqrt{3})$ is (assume $\cos \theta > 0$)
 (a) 2 radian (b) 3 radian
 (c) 4 radian (d) 6 radian
42. A pipe of length ℓ_1 , closed at one end is kept in a chamber of gas of density ρ_1 . A second pipe open at both ends is placed in a second chamber of gas of density ρ_2 . The compressibility of both the gases is equal. Calculate the length of the second pipe if frequency of first overtone in both the cases is equal
 (a) $\frac{4}{3}\ell_1\sqrt{\frac{\rho_2}{\rho_1}}$ (b) $\frac{4}{3}\ell_1\sqrt{\frac{\rho_1}{\rho_2}}$
 (c) $\ell_1\sqrt{\frac{\rho_2}{\rho_1}}$ (d) $\ell_1\sqrt{\frac{\rho_1}{\rho_2}}$
43. An organ pipe of 3.9π m long, open at both ends is driven to third harmonic standing wave. If the amplitude of pressure oscillation is 1 % of mean atmospheric pressure ($\rho_0 = 10^5$ N/m²). The maximum displacement of particle from mean position will be [given $v =$ velocity of sound = 200 m/s and $\rho_0 \Rightarrow$ density of air 1.3 kg/m³)
 (a) 2.5 cm (b) 5 cm (c) 1 cm (d) 2 cm
44. A progressive wave is travelling in a medium such that frequency of oscillation and displacement amplitude of the particles of the medium are f and A respectively. The ratio of their acceleration, amplitude and velocity amplitude is
 (a) $2\pi f$ (b) πf
 (c) $\frac{2\pi f}{A}$ (d) $\frac{\pi f}{A}$
45. In a resonance tube with tuning fork of frequency 512 Hz, first resonance occurs at water level equal to 30.3 cm and second resonance occurs at 63.7 cm. The maximum possible error in the speed of sound is
 (a) 51.2 cm/s (b) 102.4 cm/s
 (c) 204.8 cm/s (d) 153.6 cm/s
46. A tube of length L_1 is open at both ends. A second tube of length L_2 is closed at one end and open at the other end. Both tubes have the same fundamental frequency of vibration of air in it. What is the value of L_2 ?
 (a) $4L_1$ (b) $2L_1$ (c) $\frac{L_1}{2}$ (d) $\frac{L_1}{4}$
47. In a standing wave formed as a result of reflection from a surface, the ratio of the amplitude at an antinode to that at node is x . The fraction of energy that is reflected is
 (a) $\left[\frac{x-1}{x}\right]^2$ (b) $\left[\frac{x}{x-1}\right]^2$
 (c) $\left[\frac{x-1}{x+1}\right]^2$ (d) $\left[\frac{1}{x}\right]^2$
48. An open pipe is in resonance in 2nd harmonic with frequency f_1 . Now one end of the tube is closed and frequency is increased to f_2 such that the resonance again occurs in n th harmonic. Choose the correct option
 (a) $n = 3, f_2 = \frac{3}{4}f_1$ (b) $n = 3, f_2 = \frac{5}{4}f_1$
 (c) $n = 5, f_2 = \frac{3}{4}f_1$ (d) $n = 5, f_2 = \frac{5}{4}f_1$
49. A string of length L is clamped at each end and vibrates in a standing wave pattern. The wavelengths of the constituent traveling waves cannot be
 (a) $3L$ (b) $2L$ (c) $2L/3$ (d) $L/2$
50. A loop of a string of mass per unit length μ and radius R is rotated about an axis passing through centre perpendicular to the plane with an angular velocity ω . A small disturbance is created in the loop having the opposite sense of rotation. The linear speed of the disturbance for a stationary observer is
 (a) ωR (b) $2\omega R$ (c) $3\omega R$ (d) zero
51. A massless rod of length L is suspended by two identical strings AB and CD of equal length. A block of mass m is suspended from point O such that BO is equal to ' x '. Further it is observed that the frequency of 1st harmonic in AB is equal to 2nd harmonic frequency in CD . ' x ' is
 (a) $\frac{L}{5}$ (b) $\frac{4L}{5}$
 (c) $\frac{3L}{4}$ (d) $\frac{L}{4}$

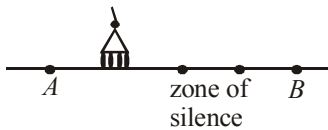


MARK YOUR RESPONSE	41. (a) (b) (c) (d)	42. (a) (b) (c) (d)	43. (a) (b) (c) (d)	44. (a) (b) (c) (d)	45. (a) (b) (c) (d)
	46. (a) (b) (c) (d)	47. (a) (b) (c) (d)	48. (a) (b) (c) (d)	49. (a) (b) (c) (d)	50. (a) (b) (c) (d)
	51. (a) (b) (c) (d)				

52. A parachutist jumps from the top of a very high tower with a siren of frequency 800 Hz on his back. Assume his initial velocity to be zero. After falling freely for 12s, he observes that the frequency of sound heard by him reflected from level ground below him is differing by 700Hz w.r.t. the original frequency. What was the height of tower. Velocity of sound in air is 330 m/s, and $g = 10 \text{ m/s}^2$.

- (a) 511.5m. (b) 1057.5m.
(c) 757.5m. (d) 1215.5m.

53. There are cases when an explosion at a point A will be heard at point B that is far away from A while in a certain region located much closer to A than to B , the explosion is not heard due to obstruction. This will be possible



- (a) if air temperature increases with altitude
(b) if air temperature decreases with altitude
(c) if air is blowing from B to A
(d) if air is blowing from A to B

54. In the experiment to determine the speed of sound using a resonance column,

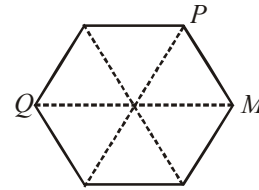
- (a) prongs of the tuning fork are kept in a vertical plane
(b) prongs of the tuning fork are kept in a horizontal plane
(c) in one of the two resonances observed, the length of the resonating air column is close to the wavelength of sound in air
(d) in one of the two resonances observed, the length of the resonating air column is close to half of the wavelength of sound in air

55. A piece of wire is cut into two pieces A and B , and stretched to the same tension and mounted between two rigid walls. Segment A is longer than segment B . Which of the following quantities will always be larger for waves on A than for waves on B ?

- (a) Amplitude of the wave
(b) Frequency of the fundamental mode
(c) Wave velocity
(d) Wavelength of the fundamental mode

56. Two identical point like sound sources emitting sound in same phase of wavelength 1m are located at points P and Q as shown in figure. All sides of the polygon are equal and of

length 1m. The intensity of sound at M due to source P above is I_0 . What will be the intensity of sound at point M when both the sources are on?

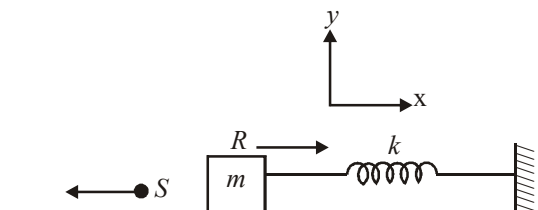


- (a) $4I_0$ (b) $\frac{3I_0}{2}$
(c) $\frac{9}{4}I_0$ (d) None of these

57. A sound source S emitting a sound of frequency 500Hz and receiver R of mass m are at the same point. R is performing SHM with the help of a spring of force constant k . At a time $t = 0$, R is at mean position and moving toward right as shown in figure. At the same time, source starts moving away from the R with an acceleration 18.75m/s^2 . Find the frequency (in Hz) registered by receiver at a time $t = 10\text{s}$.

Given that $\frac{m}{k} = \frac{100}{\pi^2}$ and amplitude of oscillation of

$$R = \frac{150}{\pi} \text{ m, } v_{\text{sound}} = 300 \text{ m/s.}$$



- (a) 320 Hz (b) 220 Hz
(c) 420 Hz (d) 350 Hz

58. Sound waves of frequency 16 kHz are emitted by two coherent point sources of sound placed 2m apart at the centre of a circular train track of large radius. A person riding the train observes 2 maxima per second when the train is running at a speed of 36 km/h. Calculate the radius of the track. [Velocity of sound in air 320 m/s]

- (a) $\frac{1000}{\pi}$ m (b) $\frac{500}{\pi}$ m
(c) $\frac{250}{\pi}$ m (d) $\frac{700}{\pi}$ m



**MARK YOUR
RESPONSE**

52. (a) (b) (c) (d)

53. (a) (b) (c) (d)

54. (a) (b) (c) (d)

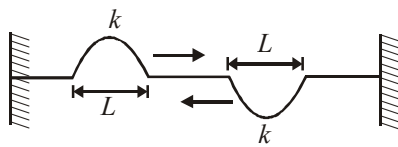
55. (a) (b) (c) (d)

56. (a) (b) (c) (d)

57. (a) (b) (c) (d)

58. (a) (b) (c) (d)

59. The fundamental frequency of a sonometer wire of length ℓ is n_0 . A bridge is now introduced at a distance of $\Delta\ell$ ($\ll \ell$) from the centre of the wire. The lengths of wire on the two sides of the bridge are now vibrated in their fundamental modes. Then, the beat frequency nearly is
- (a) $n_0\Delta\ell/\ell$ (b) $8n_0\Delta\ell/\ell$
 (c) $2n_0\Delta\ell/\ell$ (d) $n_0\Delta\ell/2\ell$
60. Two identical pulses move in opposite directions with same uniform speeds on a stretched string. The width and kinetic energy of each pulse is L and k respectively. At the instant they completely overlap, the kinetic energy of the width L of the string is



- (a) k (b) $2k$
 (c) $4k$ (d) $8k$
61. A siren placed at a railway platform is emitting sound of frequency 5 kHz. A passenger sitting in a moving train A records a frequency of 5.5 kHz while the train approaches the siren. During his return journey in a different train B he records a frequency of 6.0 kHz while approaching the same siren. The ratio of the velocity of train B to that train A is
- (a) 242/252 (b) 2
 (c) 5/6 (d) 11/6

62. **Statement 1 :** Due to the motion of listener, the frequency of the sound waves (as received by listener) emitted by stationary source is affected.

Statement 2 : Due to the motion of source, wavelength of the sound waves (emitted by source) as received by stationary listener is affected.

Statement 3 : If receiver and source both are moving, the observed frequency must be different from the original frequency of source.

Treat motion of source or listener always along a line joining them for all above cases. Choose correct option.

- (a) All the three statements are correct
 (b) Only all three statements are wrong
 (c) Only statements 1 and 2 are correct
 (d) Only statements 2 and 3 are correct

63. A thin string is held at one end and oscillated vertically, so that $y(x=0, t) = 8 \sin 4t$ cm. Neglect the gravitational force. The string's density is 0.2 kg/m and its tension is 1N. The string passes through a bath filled with 1 kg water. Due to friction, heat is transferred to the bath. The heat transfer efficiency is 50%. Calculate how much time (approximately) passes before the temperature of the bath rises one degree Kelvin.

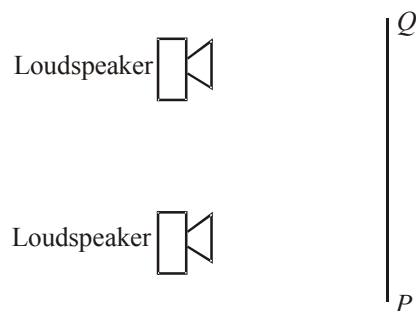
- (a) 5.2 days (b) 4.2 days
 (c) 3.2 days (d) 6.2 days

64. In the experiment for the determination of the speed of sound in air using the resonance column method, the length of the air column that resonates in the fundamental mode, with a tuning fork is 0.1 m. When this length is changed to 0.35 m, the same tuning fork resonates with the first overtone. Calculate the end correction.

- (a) 0.012 m (b) 0.025 m
 (c) 0.05 m (d) 0.024 m

65. In the diagram below (not to scale), each of the loudspeakers emits a continuous sound of the same frequency.

A microphone moved along the line PQ detects a series of maximum and minimum sound intensities. Which one of the following actions on its own, will lead to an increase in the distance between the maxima of sound intensity ?



- (a) Decreasing the frequency of the sound emitted by the loudspeakers.
 (b) Increasing the frequency of the sound emitted by the loudspeakers.
 (c) Increasing the separation of the loudspeakers
 (d) Decreasing the distance of the loudspeakers from the line PQ .



MARK YOUR RESPONSE	59. (a) (b) (c) (d)	60. (a) (b) (c) (d)	61. (a) (b) (c) (d)	62. (a) (b) (c) (d)	63. (a) (b) (c) (d)
	64. (a) (b) (c) (d)	65. (a) (b) (c) (d)			

COMPREHENSION TYPE

B

This section contains groups of questions. Each group is followed by some multiple choice questions based on a paragraph. Each question has 4 choices (a), (b), (c) and (d) for its answer, out of which ONLY ONE is correct.

PASSAGE-1

There is a sinusoidal standing wave created in a string of length 150 cm and mass 1.5 kg tied at both ends. String has tension of 36 N. There are 4 nodes in the string other than the ends. Maximum displacement of particle at a distance 35 cm from one end is 2mm.

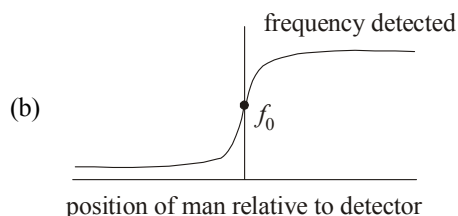
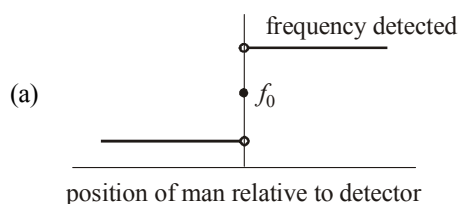
- Circular natural frequency of the wave is
 - $20\pi\sqrt{10}$ per sec
 - $20\pi\sqrt{\frac{2}{3}}$ per sec
 - 20π per sec
 - 200π per sec
- Maximum displacement of mid point of the string is
 - 8 mm
 - 4 mm
 - 2 mm
 - 1 mm
- Speed of a point of string at a distance 25 cm from one end, when the displacement of the midpoint of the string is zero
 - 40π mm/s
 - 80π mm/s
 - $40\pi\sqrt{\frac{2}{3}}$ mm/s
 - $80\pi\sqrt{\frac{2}{3}}$ mm/s

PASSAGE-2

A man of mass 50 kg is running on a plank of mass 150 kg with speed of 8 m/s relative to plank as shown in the figure (both were initially at rest and the velocity of man with respect to ground any how remains constant). Plank is placed on smooth horizontal surface. The man, while running whistles with frequency f_0 . A detector (D) placed on plank detects frequency. The man jumps off with same velocity from point D and slides on the smooth horizontal surface (Assume coefficient of friction between man and horizontal surface is zero). The speed of sound in still medium is 330 m/s.



- The frequency of sound detected by detector D, before man jumps of the plank is
 - $\frac{332}{324}f_0$
 - $\frac{330}{322}f_0$
 - $\frac{328}{336}f_0$
 - $\frac{330}{338}f_0$
- The frequency of sound detected by detector D, after man jumps of the plank is
 - $\frac{332}{324}f_0$
 - $\frac{330}{322}f_0$
 - $\frac{328}{336}f_0$
 - $\frac{330}{338}f_0$
- Choose the correct plot between the frequency detected by detector VS position of the man relative to detector.



**MARK YOUR
RESPONSE**

1. (a) (b) (c) (d)

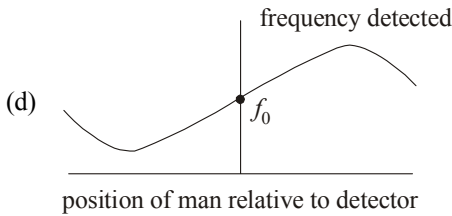
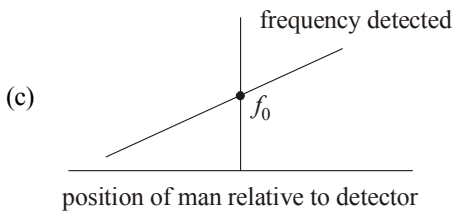
2. (a) (b) (c) (d)

3. (a) (b) (c) (d)

4. (a) (b) (c) (d)

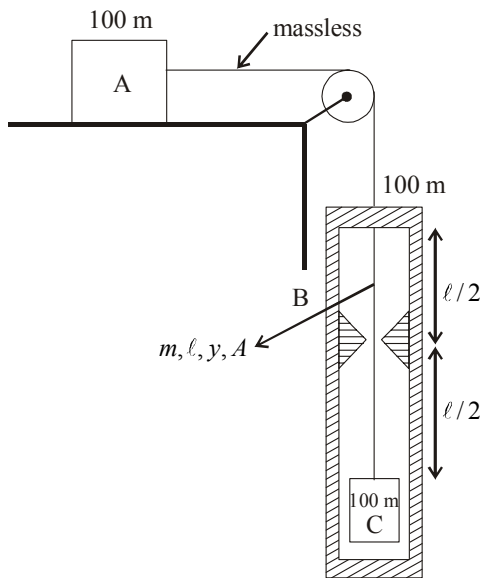
5. (a) (b) (c) (d)

6. (a) (b) (c) (d)



PASSAGE-3

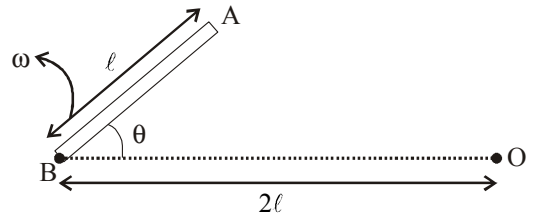
Figure shows an arrangement of 3 masses A, B, C of mass 100 m each and 2 strings. Block B is hollow having C hung with rope of mass m inside it. Upper string is massless. Lower string has mass m , length L , Young's modulus Y , and area of cross section A . All surfaces are frictionless.



7. Find acceleration of system
- (a) $\frac{2g}{3}$ (b) $\frac{201}{301}g$
 (c) g (d) $g/2$
8. What among the following is the closest value of velocity of sound wave in lower string ?
- (a) $\sqrt{100g\Delta\ell}$ (b) $\sqrt{\frac{YA\Delta\ell}{m}}$
 (c) $\sqrt{\frac{10000g\Delta\ell}{301}}$ (d) none of these
9. Wavelength corresponding to first harmonic of lower string is
- (a) ℓ (b) $\ell/2$
 (c) 2ℓ (d) $\ell/4$

PASSAGE-4

A sound source is attached to one end A of a rod of length ℓ , which is performing uniform circular motion with angular velocity ω about the other end B.



An observer O is stationary at a distance 2ℓ from B in the plane of circular motion. Angle ABO at any time is represented by θ . $\ell = 30\text{m}$, $v_{\text{sound}} = 300\text{ m/s}$, $\omega = \pi\text{ rad/s}$, speed of source is \ll velocity of sound.

10. Find the least time difference between two natural (original) frequency perceptions by O.
- (a) 0.8 s (b) 1.2 s
 (c) 1 s (d) 2 s
11. Distance traveled by the source during the time between a least frequency perception and the immediate next highest frequency perception by the observer.
- (a) $20\pi\text{m}$ (b) $30\pi\text{m}$
 (c) $40\pi\text{m}$ (d) $10\pi\text{m}$



MARK YOUR RESPONSE

7. (a) (b) (c) (d)

8. (a) (b) (c) (d)

9. (a) (b) (c) (d)

10. (a) (b) (c) (d)

11. (a) (b) (c) (d)

12. What will be value of θ at an instant, when the observer records least frequency. (Tick the closest answer)
- (a) $\pi/2$ (b) $\pi/4$
 (c) $2\pi/3$ (d) π

PASSAGE-5

Waves $y_1 = A \cos(0.5\pi x - 100\pi t)$ and $y_2 = A \cos(0.46\pi x - 92\pi t)$ are travelling along x -axis. (Here x is in m and t is in second)

13. Find the number of times intensity is maximum in time interval of 1 sec.
- (a) 4 (b) 6
 (c) 8 (d) 10
14. The wave velocity of louder sound is
- (a) 100 m/s (b) 192 m/s
 (c) 200 m/s (d) 96 m/s
15. The number of times $y_1 + y_2 = 0$ at $x = 0$ in 1 sec is
- (a) 100 (b) 46
 (c) 192 (d) 96

PASSAGE-6

A wave described by the equation $y = A \sin\left(ax + bt + \frac{\pi}{2}\right)$

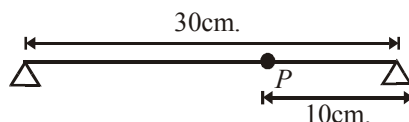
is reflected by an obstacle at $x = 0$. The intensity of the reflected wave is 64 % of the incident wave. As a result, the incident and reflected wave superpose to give a resultant wave

$$y = -1.6 A \sin ax \sin bt + cA \cos (bt + ax)$$

16. The amplitude of reflected wave is
- (a) 0.2 A (b) 0.8 A
 (c) 0.6 A (d) 0.4 A
17. The value of c is
- (a) 0.2 (b) 0.3
 (c) 0.4 (d) 0.6
18. The maximum particle velocity is
- (a) 1.36 bA (b) 1.66 bA
 (c) 1.8 bA (d) 2 bA

PASSAGE-7

Figure shows a clamped metal string of length 30cm. and linear mass density 0.1 kg/m. Which is taut at a tension of 40N. A small rider (piece of paper) is placed on string at point P as shown. An external vibrating tuning fork is brought near the string and oscillations of rider are carefully observed.



19. At which of the following frequencies of tuning fork, rider will not vibrate at all
- (a) $\frac{100}{3}$ Hz (b) 50 Hz
 (c) 200 Hz (d) None of these
20. At which of the following frequencies the point P on string will have maximum oscillation amplitude among all points on string
- (a) $\frac{200}{3}$ Hz (b) 100 Hz
 (c) 200 Hz (d) None of these
21. Now if the tension in the string is made 160N, at which of the following frequencies of tuning fork, rider will not vibrate at all
- (a) $\frac{100}{3}$ Hz (b) 50 Hz
 (c) 200 Hz (d) None of these

PASSAGE-8

A sinusoidal wave is propagating in negative x -direction in a string stretched along x -axis. A particle of string at $x=2m$ is found at its mean position and it is moving in positive y -direction at $t = 1$ sec. The amplitude of the wave, the



**MARK YOUR
RESPONSE**

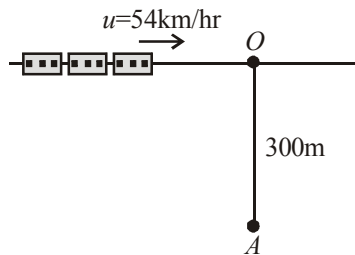
12. (a) (b) (c) (d)	13. (a) (b) (c) (d)	14. (a) (b) (c) (d)	15. (a) (b) (c) (d)	16. (a) (b) (c) (d)
17. (a) (b) (c) (d)	18. (a) (b) (c) (d)	19. (a) (b) (c) (d)	20. (a) (b) (c) (d)	21. (a) (b) (c) (d)

wavelength and the angular frequency of the wave are 0.1 meter, $\pi/2$ meter and 2π rad/sec respectively.

22. The equation of the wave is
 (a) $y = 0.1 \sin(4\pi(t-1) + 8(x-2))$
 (b) $y = 0.1 \sin((t-1) - (x-2))$
 (c) $y = 0.1 \sin(2\pi(t-1) + 4(x-2))$
 (d) None of these
23. The speed of particle at $x = 2\text{m}$ and $t = 1$ sec is
 (a) 0.2π m/s (b) 0.6π m/s
 (c) 0.4π m/s (d) 0
24. The instantaneous power transfer through $x = 2\text{m}$ and $t = 1.25$ sec, is
 (a) 10 J/s (b) $4\pi/3$ J/s
 (c) $2\pi/3$ J/s (d) 0

PASSAGE-9

A stationary observer is situated at A , which is at a distance of 300m from a railway crossing O . A train moving with constant speed 54 km/h on a straight track perpendicular to OA passes O by blowing horn continuously. The frequency of sound is 400 Hz. The speed of sound on that particular day was $V = 300$ m/s.



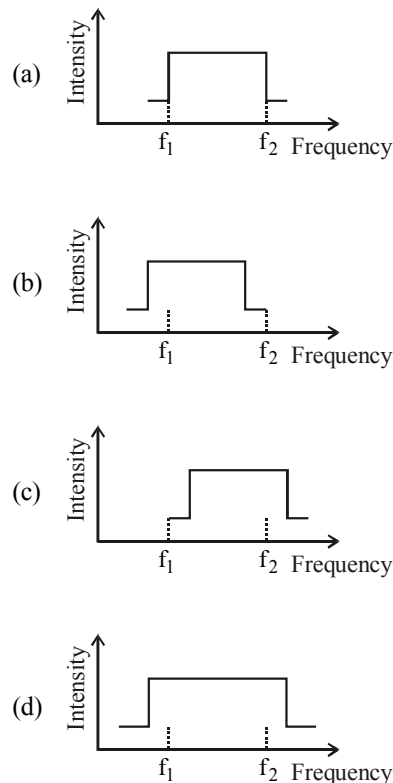
25. When the train crosses the junction O , the frequency observed by the observer at A approximately is
 (a) 400 Hz (b) 399 Hz
 (c) 421 Hz (d) 401 Hz
26. The distance of the train from O at the time the observer hears actual frequency is
 (a) zero (b) 15m right of O
 (c) 15m left of O (d) 30m right of O
27. As the train approaches the crossing at O and crosses it, the frequency heard by the observer at A will
 (a) first decrease and then increase
 (b) decrease continuously
 (c) first increase and then decrease
 (d) increase continuously

PASSAGE-10

Two trains A and B moving with speeds 20 m/s and 30 m/s respectively in the same direction on the same

straight track, with B ahead of A . The engines are at the front ends. The engine of train A blows a long whistle. Assume that the sound of the whistle is composed of components varying in frequency from $f_1 = 800$ Hz to $f_2 = 1120$ Hz, as shown in the figure. The spread in the frequency (highest frequency – lowest frequency) is thus 320 Hz. The speed of sound in still air is 340 m/s.

28. The speed of sound of the whistle is
 (a) 340 m/s for passengers in A and 310 m/s for passengers in B
 (b) 360 m/s for passengers in A and 310 m/s for passengers in B
 (c) 310 m/s for passengers in A and 360 m/s for passengers in B
 (d) 340 m/s for passengers in both the trains
29. The distribution of the sound intensity of the whistle as observed by the passengers in train A is best represented by



30. The spread of frequency as observed by the passengers in train B is
 (a) 310 Hz (b) 330 Hz
 (c) 350 Hz (d) 290 Hz

MARK YOUR RESPONSE

22. (a) (b) (c) (d)	23. (a) (b) (c) (d)	24. (a) (b) (c) (d)	25. (a) (b) (c) (d)	26. (a) (b) (c) (d)
27. (a) (b) (c) (d)	28. (a) (b) (c) (d)	29. (a) (b) (c) (d)	30. (a) (b) (c) (d)	

PASSAGE-11

You are provided with three similar, but slightly different, tuning forks, when A and B are both struck, a beat frequency of f_{AB} is heard. When A and C are both struck, a beat frequency of f_{AC} is heard. It was noticed that $f_{AB} < f_{AC}$. This experiment is repeated after coating tuning fork A with a little wax. Now it is observed that values of both f_{AB} and f_{AC} increase.

31. Which tuning fork has the highest frequency ?
 (a) A
 (b) B
 (c) C
 (d) The answer cannot be determined from the information given
32. Which tuning fork has the middle frequency ?
 (a) A
 (b) B
 (c) C
 (d) The answer cannot be determined from the information given

PASSAGE-12

A uniform string of length ℓ is fixed at both ends such that tension T is produced in it. The string is excited to vibrate with maximum displacement amplitude a_0 .

33. The maximum kinetic energy of the string for its fundamental tone is
 (a) $\frac{a_0^2 \pi^2 T}{4\ell}$ (b) $\frac{a_0^2 \pi^2 T}{\ell}$
 (c) $\frac{a_0^2 \pi^2 T}{2\ell}$ (d) $\frac{a_0^2 \pi^2 T}{3\ell}$
34. The maximum kinetic energy of the string for its first overtone is
 (a) $\frac{a_0^2 \pi^2 T}{4\ell}$ (b) $\frac{a_0^2 \pi^2 T}{\ell}$
 (c) $\frac{a_0^2 \pi^2 T}{2\ell}$ (d) $\frac{a_0^2 \pi^2 T}{3\ell}$



MARK YOUR RESPONSE	31. (a) (b) (c) (d)	32. (a) (b) (c) (d)	33. (a) (b) (c) (d)	34. (a) (b) (c) (d)
---------------------------	---------------------	---------------------	---------------------	---------------------

REASONING TYPE

C

In the following questions two Statements (1 and 2) are provided. Each question has 4 choices (a), (b), (c) and (d) for its answer, out of which ONLY ONE is correct. Mark your responses from the following options:

- (a) Both Statement-1 and Statement-2 are true and Statement-2 is the correct explanation of Statement-1.
 (b) Both Statement-1 and Statement-2 are true and Statement-2 is not the correct explanation of Statement-1.
 (c) Statement-1 is true but Statement-2 is false.
 (d) Statement-1 is false but Statement-2 is true.

1. **Statement - 1 :** Sound travels faster in solids than gases.
Statement - 2 : Solids possess greater density than gases.
2. **Statement - 1 :** Speed of wave $\frac{\text{wavelength}}{\text{time period}}$
Statement - 2 : Wavelength is the distance between two nearest particles in phase.
3. **Statement - 1 :** When a beetle moves along the sand within a few tens of centimeters of a sand scorpion, the scorpion immediately turns towards the beetle and dashes towards it.
Statement - 2 : When a beetle disturbs the sand, it sends pulses along the sand's surface. One set of pulses is longitudinal while the other set in transverse.
4. **Statement - 1 :** When pressure of an ideal gas is increased, the speed of sound in gas must increase.
Statement - 2 : The speed of sound in ideal gas is directly proportional to square root of pressure of the gas.
5. **Statement - 1 :** The pitch of wind instruments rises and that of string instruments falls as an orchestra warms up.
Statement - 2 : When temperature rises, speed of sound increases but speed of wave in a string fixed at both ends decreases.



MARK YOUR RESPONSE	1. (a) (b) (c) (d)	2. (a) (b) (c) (d)	3. (a) (b) (c) (d)	4. (a) (b) (c) (d)	5. (a) (b) (c) (d)
---------------------------	--------------------	--------------------	--------------------	--------------------	--------------------

6. **Statement - 1 :** Two sound waves of same intensity in a particular medium will have displacement amplitude in ratio of 2 : 1 if they have frequency in the ratio 1 : 2.

Statement - 2 : Two wave of same velocity amplitude in a particular medium have equal intensity.

7. **Statement - 1 :** Two identical pulses travel on a string in the opposite direction as shown. When the pulses overlap, the entire energy is potential.



Statement - 2 : The velocity of any point can be found by superposition principle

$$\vec{v}_p = \vec{v}_{p1} + \vec{v}_{p2}$$

8. **Statement - 1 :** When a closed organ pipe vibrates, the pressure of the gas at the closed end remains constant.

Statement - 2 : In a stationary wave system, displacement nodes are pressure antinodes, and displacement antinodes are pressure nodes.

9. **Statement - 1 :** When a wave propagates from a heavier string to a lighter string amplitude of transmitted wave must be less than the amplitude of incident wave.

Statement - 2 : Power of incident wave is divided into reflected and transmitted waves.



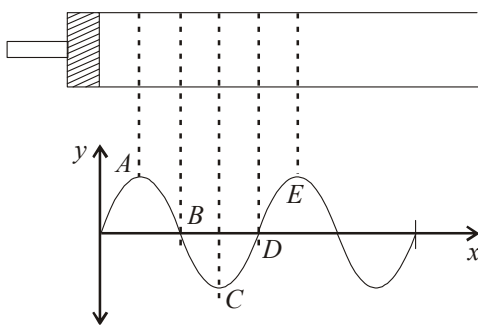
MARK YOUR RESPONSE	6. (a) (b) (c) (d)	7. (a) (b) (c) (d)	8. (a) (b) (c) (d)	9. (a) (b) (c) (d)
---------------------------	--------------------	--------------------	--------------------	--------------------

D

MULTIPLE CORRECT CHOICE TYPE

Each of these questions has 4 choices (a), (b), (c) and (d) for its answer, out of which ONE OR MORE is/are correct.

1. Displacement (y) of air column at position x from its mean position at any instant is given by graph shown in the figure. We can conclude from graph that



- (a) Density of air is maximum at B
- (b) Density of air is minimum at D
- (c) Density is maximum at D
- (d) Density is minimum at A

2. A wave equation which gives the displacement along the y-direction is given by $y = 10^{-4} \sin(60t + 2x)$ where x and y are in metres and t is time in seconds. This represents a wave

- (a) travelling with a velocity of 30 m/s in the negative x direction
- (b) of wavelength π m
- (c) of frequency $30/\pi$ hertz
- (d) of amplitude 10^{-4} m traveling along the negative x-direction

3. A solid is attached to the free end of a sonometer wire. The wire has a frequency of 500Hz. Then the solid is immersed in water and it is found that the wire has a frequency of 460Hz. When the solid is immersed in a liquid the frequency of the wire is 480Hz. Choose the correct options

- (a) The specific gravity of the solid is 6.51
- (b) The specific gravity of the liquid is 0.51
- (c) The specific gravity of the solid is 0.51
- (d) The specific gravity of the liquid is 6.51



MARK YOUR RESPONSE	1. (a) (b) (c) (d)	2. (a) (b) (c) (d)	3. (a) (b) (c) (d)		
---------------------------	--------------------	--------------------	--------------------	--	--

4. Velocity of sound in air is 320 m/s. A pipe closed at one end has a length of 1 m. Neglecting end corrections, the air column in the pipe can resonate for sound of frequency :
- (a) 80 Hz (b) 240 Hz
(c) 320 Hz (d) 400 Hz
5. An air column in a pipe, which is closed at one end, will be in resonance with a vibrating tuning fork of frequency 264 Hz if the length of the column in cm is :
- (a) 31.25 (b) 62.50
(c) 93.75 (d) 125
6. In a resonance-column experiment to measure the velocity of sound, the first resonance is obtained at a length ℓ_1 and the second resonance at a length ℓ_2 . Then which of the following is (are) incorrect
- (a) $\ell_2 = 3\ell_1$
(b) $\ell_2 = 3\ell_1$
(c) $\ell_2 = 3\ell_1$
(d) May be any of the above, depending on the frequency of the tuning fork used
7. A wave pulse moving to the right along the x -axis is represented by the wave function $y(x, t) = \frac{2.0}{(x - 3.0t)^2 + 1}$, where x and y are in centimeters and t is in seconds. (The maximum pulse height is defined as maximum displacement along y -axis). Then
- (a) The maximum pulse height is decreasing with time
(b) The maximum pulse height is constant with time
(c) The speed of the pulse is 3.0 cm/s
(d) The speed of the pulse is 0.33 cm/s
8. Standing waves can be produced
- (a) on a string clamped at both the ends.
(b) on a string clamped at one end free at the other
(c) when incident wave gets reflected from a wall
(d) when two identical waves with a phase difference of π are moving in the same direction
9. As a wave propagates,
- (a) the wave intensity remains constant for a plane wave
(b) the wave intensity decreases as the inverse of the distance from the source for a spherical wave
(c) the wave intensity decreases as the inverse square of the distance from the source for a spherical wave
(d) total intensity of the spherical wave over the spherical surface centred at the source remains constant at all times.
10. A string is stretched along the x -axis. There is a transverse perturbation along the string :
- $$\phi(x, t) = 4 \sin\left(\frac{\pi x}{3}\right) \cos(10\pi t)$$
- where x is measured in centimeters and t in seconds. Both waves moving towards each other create this disturbance. Assume the velocities and amplitudes are equal for the two waves.
- Choose the correct options
- (a) amplitude of each waves is 2cm.
(b) velocity of each waves is 40cm/sec
(c) distance between two adjacent junctions is 2cm.
(d) distance between two adjacent junctions is 3cm.
11. A transverse sinusoidal wave of amplitude a , wavelength λ and frequency f is travelling on a stretched string. The maximum speed of any point on the string is $v/10$, where v is the speed of propagation of the wave. If $a = 10^{-3}$ m and $v = 10$ m s $^{-1}$, then λ and f are given by
- (a) $\lambda = 2\pi \times 10^{-2}$ m (b) $\lambda = 10^{-3}$ m
(c) $f = 10^3$ Hz / (2 π) (d) $f = 10^4$ Hz
12. $y(x, t) = 0.8/[4x+5t]^2+5$ represents a moving pulse, where x and y are in meter and t in second. Then
- (a) pulse is moving in + x direction
(b) in 2 s it will travel a distance of 2.5 m
(c) its maximum displacement is 0.16 m
(d) it is a symmetric pulse
13. A metallic rod of length 1m is rigidly clamped at its midpoint. Longitudinal stationary waves are set up in the rod in such a way that there are two nodes on either side of the midpoint. The amplitude of an antinode is 2×10^{-6} m. (Young's modulus = 2×10^{11} Nm $^{-2}$ and density = 8000 kg m $^{-3}$)
- Choose the correct options
- (a) The equation of motion at a point 2cm. from the midpoint is $y = 2 \times 10^{-6} \sin 5\pi x \cos 25 \times 10^3 \pi t$
(b) The equation of one of the constituent waves in the rod is $y_1 = (1 \times 10^{-6}) \sin (5\pi x - 25 \times 10^3 \pi t)$
(c) The equation of one of the constituent waves in the rod is $(1 \times 10^{-6}) \sin (5\pi x + 25 \times 10^3 \pi t)$
(d) The equation of motion at a point 2cm. from the midpoint is $y = 2 \times 10^{-6} \sin 10\pi x \cos 25 \times 10^3 \pi t$



MARK YOUR RESPONSE	4. (a)(b)(c)(d)	5. (a)(b)(c)(d)	6. (a)(b)(c)(d)	7. (a)(b)(c)(d)	8. (a)(b)(c)(d)
	9. (a)(b)(c)(d)	12. (a)(b)(c)(d)	11. (a)(b)(c)(d)	12. (a)(b)(c)(d)	13. (a)(b)(c)(d)

14. Two identical straight wires are stretched so as to produce 6 beats per second when vibrating simultaneously. On changing the tension slightly in one of them, the beat frequency remains unchanged. Denoting by T_1 , T_2 the higher and the lower initial tension in the strings, then it could be said that while making the above changes in tension,
- (a) T_2 was decreased (b) T_2 was increased
 (c) T_1 was decreased (d) T_1 was increased
15. The (x, y) co-ordinates of the corners of a square plate are $(0, 0)$, $(L, 0)$, (L, L) and $(0, L)$. The edges of the plate are clamped and transverse standing waves are set up in it. If $u(x, y)$ denotes the displacement of the plate at the point (x, y) at some instant of time, the possible expression(s) for u is (are) ($a =$ positive constant)
- (a) $a \cos(\pi x/2L) \cos(\pi y/2L)$
 (b) $a \sin(\pi x/L) \sin(\pi y/L)$
 (c) $a \sin(\pi x/L) \sin(2\pi y/L)$
 (d) $a \cos(2\pi x/L) \sin(\pi y/L)$
16. In a standing wave on a string rigidly fixed at both ends.
- (a) all the particles must be at their positive extremes simultaneously once in half of the time period
- (b) all the particles must be at their positive extremes simultaneously once in a time period
 (c) in one time period all the particles are simultaneously at rest twice
 (d) all the particles are never at rest simultaneously
17. In a wave motion $y = a \sin(kx - \omega t)$, y can represent
- (a) electric field (b) magnetic field
 (c) displacement (d) pressure
18. A sound wave of frequency f travels horizontally to the right. It is reflected from a large vertical plane surface moving to left with a speed v . The speed of sound in medium is c
- (a) The number of wave striking the surface per second is $f \frac{(c - v)}{c}$
- (b) The wavelength of reflected wave is $\frac{c(c - v)}{f(c + v)}$
- (c) The frequency of the reflected wave is $f \frac{(c + v)}{(c - v)}$
- (d) The number of beats heard by a stationary listener to the left of the reflecting surface is $\frac{vf}{c - v}$



MARK YOUR RESPONSE

14. (a)(b)(c)(d)

15. (a)(b)(c)(d)

16. (a)(b)(c)(d)

17. (a)(b)(c)(d)

18. (a)(b)(c)(d)

MATRIX-MATCH TYPE

E

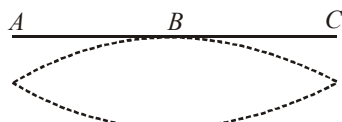
Each question contains statements given in two columns, which have to be matched. The statements in Column-I are labeled A, B, C and D, while the statements in Column-II are labelled p, q, r, s and t. Any given statement in Column-I can have correct matching with ONE OR MORE statement(s) in Column-II. The appropriate bubbles corresponding to the answers to these questions have to be darkened as illustrated in the following example: If the correct matches are A-p, s and t; B-q and r; C-p and q; and D-s and t; then the correct darkening of bubbles will look like the given.

	p	q	r	s	t
A	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input checked="" type="checkbox"/>
B	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
C	<input checked="" type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
D	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input checked="" type="checkbox"/>

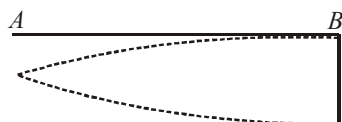
1. Match the column

Column I

(A) Graphical representation of pressure variation in both end open organ pipe.



(B) Graphical representation of pressure variation in one end closed organ pipe.



Column II

(p) Maximum kinetic energy at B

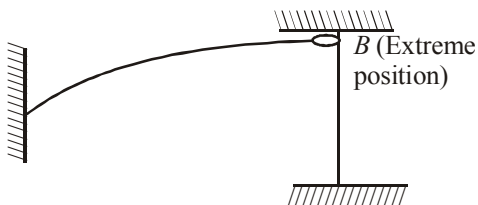
(q) Maximum potential energy at B

(C) Snapshot of string fixed at both ends



(r) Maximum particle velocity at B

(D) Snapshot of a string fixed at one end and connected to a smooth massless ring that is constrained to move vertically.



(s) Maximum particle acceleration at B

2. Each of the properties of sound listed in the column A primarily depends on one of the quantities in column B. Write down the matching pairs from the two columns.

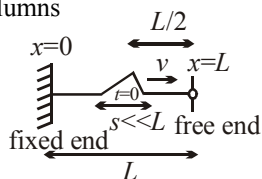
Column A

- (A) pitch
- (B) quality
- (C) loudness

Column B

- p. Waveform
- q. frequency
- r. intensity

3. A small pulse travelling with speed v in a string is shown at $t=0$, moving towards free end. Select the shape of string from column II at moments shown in column I. Match the columns



Column I

- (A) $t = \frac{L}{v}$
- (B) $t = \frac{2L}{v}$
- (C) $t = \frac{3L}{v}$

Column II

- (p)
- (q)
- (r)
- (s)



MARK YOUR RESPONSE	1. <table style="display: inline-table; border-collapse: collapse;"> <tr><td></td><td style="text-align: center;">p</td><td style="text-align: center;">q</td><td style="text-align: center;">r</td><td style="text-align: center;">s</td></tr> <tr><td style="text-align: center;">A</td><td style="border: 1px solid black; border-radius: 50%; text-align: center;">p</td><td style="border: 1px solid black; border-radius: 50%; text-align: center;">q</td><td style="border: 1px solid black; border-radius: 50%; text-align: center;">r</td><td style="border: 1px solid black; border-radius: 50%; text-align: center;">s</td></tr> <tr><td style="text-align: center;">B</td><td style="border: 1px solid black; border-radius: 50%; text-align: center;">p</td><td style="border: 1px solid black; border-radius: 50%; text-align: center;">q</td><td style="border: 1px solid black; border-radius: 50%; text-align: center;">r</td><td style="border: 1px solid black; border-radius: 50%; text-align: center;">s</td></tr> <tr><td style="text-align: center;">C</td><td style="border: 1px solid black; border-radius: 50%; text-align: center;">p</td><td style="border: 1px solid black; border-radius: 50%; text-align: center;">q</td><td style="border: 1px solid black; border-radius: 50%; text-align: center;">r</td><td style="border: 1px solid black; border-radius: 50%; text-align: center;">s</td></tr> <tr><td style="text-align: center;">D</td><td style="border: 1px solid black; border-radius: 50%; text-align: center;">p</td><td style="border: 1px solid black; border-radius: 50%; text-align: center;">q</td><td style="border: 1px solid black; border-radius: 50%; text-align: center;">r</td><td style="border: 1px solid black; border-radius: 50%; text-align: center;">s</td></tr> </table>		p	q	r	s	A	p	q	r	s	B	p	q	r	s	C	p	q	r	s	D	p	q	r	s	2. <table style="display: inline-table; border-collapse: collapse;"> <tr><td></td><td style="text-align: center;">p</td><td style="text-align: center;">q</td><td style="text-align: center;">r</td></tr> <tr><td style="text-align: center;">A</td><td style="border: 1px solid black; border-radius: 50%; text-align: center;">p</td><td style="border: 1px solid black; border-radius: 50%; text-align: center;">q</td><td style="border: 1px solid black; border-radius: 50%; text-align: center;">r</td></tr> <tr><td style="text-align: center;">B</td><td style="border: 1px solid black; border-radius: 50%; text-align: center;">p</td><td style="border: 1px solid black; border-radius: 50%; text-align: center;">q</td><td style="border: 1px solid black; border-radius: 50%; text-align: center;">r</td></tr> <tr><td style="text-align: center;">C</td><td style="border: 1px solid black; border-radius: 50%; text-align: center;">p</td><td style="border: 1px solid black; border-radius: 50%; text-align: center;">q</td><td style="border: 1px solid black; border-radius: 50%; text-align: center;">r</td></tr> </table>		p	q	r	A	p	q	r	B	p	q	r	C	p	q	r	3. <table style="display: inline-table; border-collapse: collapse;"> <tr><td></td><td style="text-align: center;">p</td><td style="text-align: center;">q</td><td style="text-align: center;">r</td><td style="text-align: center;">s</td></tr> <tr><td style="text-align: center;">A</td><td style="border: 1px solid black; border-radius: 50%; text-align: center;">p</td><td style="border: 1px solid black; border-radius: 50%; text-align: center;">q</td><td style="border: 1px solid black; border-radius: 50%; text-align: center;">r</td><td style="border: 1px solid black; border-radius: 50%; text-align: center;">s</td></tr> <tr><td style="text-align: center;">B</td><td style="border: 1px solid black; border-radius: 50%; text-align: center;">p</td><td style="border: 1px solid black; border-radius: 50%; text-align: center;">q</td><td style="border: 1px solid black; border-radius: 50%; text-align: center;">r</td><td style="border: 1px solid black; border-radius: 50%; text-align: center;">s</td></tr> <tr><td style="text-align: center;">C</td><td style="border: 1px solid black; border-radius: 50%; text-align: center;">p</td><td style="border: 1px solid black; border-radius: 50%; text-align: center;">q</td><td style="border: 1px solid black; border-radius: 50%; text-align: center;">r</td><td style="border: 1px solid black; border-radius: 50%; text-align: center;">s</td></tr> </table>		p	q	r	s	A	p	q	r	s	B	p	q	r	s	C	p	q	r	s
		p	q	r	s																																																											
	A	p	q	r	s																																																											
	B	p	q	r	s																																																											
C	p	q	r	s																																																												
D	p	q	r	s																																																												
	p	q	r																																																													
A	p	q	r																																																													
B	p	q	r																																																													
C	p	q	r																																																													
	p	q	r	s																																																												
A	p	q	r	s																																																												
B	p	q	r	s																																																												
C	p	q	r	s																																																												

Answerkey

A SINGLE CORRECT CHOICE TYPE

1	(a)	11	(d)	21	(d)	31	(b)	41	(d)	51	(a)	61	(b)
2	(b)	12	(c)	22	(a)	32	(b)	42	(b)	52	(b)	62	(c)
3	(a)	13	(d)	23	(d)	33	(a)	43	(a)	53	(a)	63	(b)
4	(b)	14	(a)	24	(d)	34	(c)	44	(a)	54	(a)	64	(b)
5	(c)	15	(a)	25	(b)	35	(a)	45	(c)	55	(d)	65	(a)
6	(c)	16	(d)	26	(d)	36	(a)	46	(c)	56	(c)		
7	(c)	17	(a)	27	(a)	37	(a)	47	(c)	57	(d)		
8	(b)	18	(b)	28	(c)	38	(a)	48	(d)	58	(a)		
9	(b)	19	(b)	29	(b)	39	(b)	49	(a)	59	(b)		
10	(a)	20	(d)	30	(d)	40	(a)	50	(d)	60	(c)		

B COMPREHENSION TYPE

1	(c)	7	(b)	13	(a)	19	(c)	25	(d)	31	(c)
2	(b)	8	(b)	14	(c)	20	(d)	26	(b)	32	(b)
3	(a)	9	(a)	15	(d)	21	(c)	27	(b)	33	(a)
4	(a)	10	(a)	16	(b)	22	(c)	28	(b)	34	(b)
5	(c)	11	(c)	17	(a)	23	(a)	29	(a)		
6	(a)	12	(a)	18	(c)	24	(d)	30	(a)		

C REASONING TYPE

1	(b)	3	(a)	5	(a)	7	(a)	9	(d)
2	(a)	4	(d)	6	(a)	8	(d)		

D MULTIPLE CORRECT CHOICE TYPE

1	(a, b)	5	(a, c)	9	(a, c, d)	13	(a, b, c)	17	(a, b, c, d)
2	(a, b, c, d)	6	(a, b, d)	10	(a, b, d)	14	(b, c)	18	(b)
3	(a, b)	7	(b, c)	11	(a, c)	15	(b, c)		
4	(a, b, d)	8	(a, b, c)	12	(b, c, d)	16	(b, c)		

E MATRIX-MATCH TYPE

1. A-q; B-q; C-q; D-s 2. A-q; B-p; C-r 3. A-p; B-s; C-r

F NUMERIC/INTEGER ANSWER TYPE

1	9	2	0.75	3	1328	4	0.12	5	3
6	175	7	1650	8	110	9	27	10	600
11	0.11	12	4	13	70	14	0.075	15	11
16	1.5	17	336	18	30	19	336		

Solutions

A

SINGLE CORRECT CHOICE TYPE

1. (a)
$$\begin{matrix} A & \text{observer} & u & B \\ x & \dots\dots\dots & & x \\ v_A & & & v_B \end{matrix}$$

The observed frequency from A,

$$v_A \frac{v(v_s - u)}{v_s}$$

The observed frequency from B,

$$v_B \frac{v(v_s + u)}{v_s}$$

$$\therefore \text{Number of beats} = v_B - v_A = 2 \cdot \frac{v}{v_s} \cdot u = \frac{2u}{\lambda}$$

2. (b) A vibrating organ pipe can be closed or open. For a closed organ pipe fundamental frequency

$$f_c = \frac{v}{4l} = 200\text{Hz}$$

$$\text{Its first overtone} = \frac{3v}{4l} = 3f_c = 600\text{Hz}$$

For an open organ pipe, fundamental frequency

$$f_0 = \frac{v}{2l} = 200\text{Hz}$$

$$\text{Its first overtone} = \frac{2v}{2l} = 2f_0 = 400\text{Hz}$$

3. (a) The relation for the velocity of sound in a gas

$$v = \sqrt{\frac{\gamma RT}{M}}$$

Considering the mixture of gas while all the constituents of the mixture occupy the same volume their masses vary. Let m_O, m_N, m_A be the fractions of masses of the respective gases and M_O, M_N, M_A be their respective molecular weights. Now the velocity of sound in the mixture can be given by the relation,

$$v = \sqrt{RT} \left[\frac{\gamma_O m_O}{M_O} + \frac{\gamma_N m_N}{M_N} + \frac{\gamma_A m_A}{M_A} \right]^{1/2}$$

$$= \left[8.3 \times 273 \left(\frac{1.4 \times \frac{2}{10}}{32 \times 10^{-3}} + \frac{1.4 \times \frac{7}{10}}{28 \times 10^{-3}} + \frac{1.67 \times \frac{1}{10}}{40 \times 10^{-3}} \right) \right]^{1/2}$$

$$= [8.3 \times 273 \times 1000 (8.75 \times 10^{-3} + 35 \times 10^{-3} + 4.175 \times 10^{-3})]^{1/2}$$

$$= [8.3 \times 273 \times 47.925]^{1/2} = 329.5 \text{ m/s}$$

4. (b) The apparent frequency heard by the listener

$$n' = n \left(\frac{v - v_0}{v} \right) = 200 \left(\frac{332 - 10}{332} \right) = 194 \text{ Hz}$$

$$\text{but } n' = \frac{v}{\lambda}$$

So, wavelength of the note received by the listener

$$\lambda = \frac{v}{n'} = \frac{332}{194} = 1.71 \text{ m}$$

5. (c) Frequency of the open organ pipe $n = f$.
Initial length of the pipe = l

$$\text{Final length} = \frac{l}{2}$$

We know that frequency of open organ pipe is

$$f_1 = \frac{v}{2l}$$

and when it is dipped in water, it behaves as a closed end pipe

$$\therefore \text{Its frequency } f_2 = \frac{v}{4l_2} = \frac{v}{4l/2} = \frac{v}{2l} = f$$

6. (c) Stationary wave is produced when two waves travel in opposite direction.

$$\text{Now, } y = a \cos(kx - \omega t) - a \cos(kx + \omega t)$$

$\therefore y = 2a \sin kx \sin \omega t$ is equation of stationary wave which gives a node at $x = 0$.

7. (c) The air is blowing in the direction of train i.e., in the direction of motion of sound. The frequency of sound heard by observer

$$v' = v \frac{(v + w) - v_0}{(v + w) - v_s}$$

$$\text{Here, } v = 580 \text{ Hz}$$

$$v + w = 1200 + 40 = 1240 \text{ km/h}$$

$$v_s = 40 \text{ km/h}$$

$$v_0 = 0$$

$$\text{So, } v' = 580 \left(\frac{1240 - 0}{1240 - 40} \right) = 599 \text{ Hz}$$

$$8. (b) f_A - \frac{v}{v_A} f, \text{ or } v_A = \frac{v}{f} f_A - f$$

$$f_B - \frac{v}{v_B} f, \text{ or } v_B = \frac{v}{f} f_B - f$$

$$\therefore \frac{v_B}{v_A} = \frac{f_B - f}{f_A - f} = \frac{6.0 - 5}{5.5 - 5} = \frac{1}{0.5} = 2$$

9. (b) Pitch is related with frequency ($f = v/\lambda = \text{const.}$) But intensity is related with particle energy which increases.

10. (a) $f = \frac{1}{2\ell} \sqrt{\frac{T}{m}}$;
 In air : $T = mg = \rho Vg$
 $\therefore f = \frac{1}{2\ell} \sqrt{\frac{\rho Vg}{m}}$... (i)
 In water : $T = mg - \text{upthrust}$
 $= V\rho g - \frac{V}{2}\rho_{\omega}g = \frac{Vg}{2}(2\rho - \rho_{\omega})$

$$\therefore f' = \frac{1}{2\ell} \sqrt{\frac{\frac{Vg}{2}(2\rho - \rho_{\omega})}{m}}$$

$$= \frac{1}{2\ell} \sqrt{\frac{Vg\rho}{m}} \sqrt{\frac{2\rho - \rho_{\omega}}{2\rho}}$$

$$\frac{f'}{f} = \sqrt{\frac{2\rho - \rho_{\omega}}{2\rho}}$$

$$f' = f \left(\frac{2\rho - \rho_{\omega}}{2\rho} \right)^{1/2}$$

$$= 300 \left[\frac{2\rho - 1}{2\rho} \right]^{1/2} \text{ Hz}$$

11. (d) Fundamental frequency $f = \frac{1}{2\ell} \sqrt{\frac{T}{\mu}}$

$$\therefore \frac{f_1}{f_2} = \frac{\ell_1}{\ell_2} \sqrt{\frac{\mu_2}{\mu_1}} \text{ (since tension is same)}$$

$$\frac{2L}{L} \sqrt{\frac{\pi r^2 \rho}{\pi \cdot 4r^2 \rho}},$$

(since the wires are of same material)

= 1

12. (c) Comparing it with $y(x, t) = A \cos(\omega t + \pi/2) \cos kx$
 If $kx = \pi/2$, a node occurs;
 $\therefore 10\pi x = \pi/2 \Rightarrow x = 0.05 \text{ m}$
 If $kx = \pi$, an antinode occurs
 $\Rightarrow 10\pi x = \pi \Rightarrow x = 0.1 \text{ m}$
 Also speed of wave
 $= \frac{\omega}{k} = \frac{50\pi}{10\pi} = 5 \text{ m/s}$ and
 $\lambda = 2\pi/k = 2\pi/10\pi = 0.2 \text{ m}$

13. (d) $\frac{E_r}{E_i} = \left(\frac{A_r}{A_i} \right)^2 = \left(\frac{V_2 - V_1}{V_1 + V_2} \right)^2 = 1/9$

Therefore, $\frac{E_t}{E_i} = 8/9$

14. (a) Fundamental frequency of open pipe is $f_0 = \frac{v}{2\ell}$

Third harmonic of the closed pipe $f_c = 3 \left(\frac{v}{4\ell} \right)$

Given : $3 \frac{v}{4\ell} = \frac{v}{2\ell} \cdot 100$ or $\frac{v}{2\ell} = 200 \text{ Hz}$

15. (a) According to Hooke's law $F_R = x$
 [Restoring Force $F_R = T$, tension of spring]

Velocity of sound by a stretched string $v = \sqrt{\frac{T}{m}}$

where m is the mass per unit length

$\therefore v \propto \sqrt{T}$

Hence $v \propto \sqrt{T}$

or, $\frac{v}{v'} = \sqrt{\frac{T}{T'}}$

or $v' = v \sqrt{\frac{T'}{T}} = v \sqrt{\frac{1.5x}{x}} = 1.22v$

16. (d) $y = a \cos^2 \left(2\pi nt - \frac{2\pi x}{\lambda} \right)$ can be written as

$$= a \left[\frac{1 + \cos 2 \left\{ 2\pi nt - \frac{2\pi x}{\lambda} \right\}}{2} \right]$$

$$= \frac{a}{2} + \frac{a}{2} \cos \left[4\pi nt - \frac{4\pi x}{\lambda} \right]$$

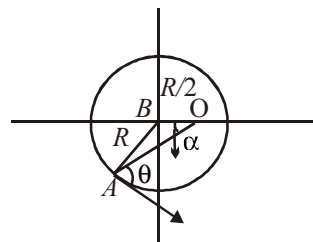
Comparing with standard equation of harmonic wave $y = A \cos(\omega t - x)$, we get

$A = \frac{a}{2}$, $\omega = 4\pi n = 2\pi f$, where $f = 2n$

Also $k = \frac{4\pi}{\lambda} = \frac{2\pi}{(\lambda/2)} = \frac{2\pi}{\lambda}$; $\therefore \lambda' = \lambda/2$

17. (a) For f to be greatest, θ should be minimum ΔABC , using the rule

$$\frac{\sin(90^\circ - \theta)}{R/2} = \frac{\sin \alpha}{R} \Rightarrow \cos \theta = \frac{\sin \alpha}{2}$$

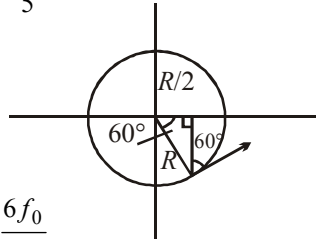


For θ minimum $\sin \alpha$ should be max. $\Rightarrow \alpha = 90^\circ$
 $\Rightarrow \theta = 60^\circ$

$$\therefore \text{approach velocity} = \left(\frac{c}{3}\right) \cos 60 = \frac{c}{6}$$

$$\therefore f_{\max} = f_0 \left(\frac{c}{c - \frac{c}{6}}\right) = \frac{6f_0}{5}$$

$$\text{and } f_{\min} = f_0 \left(\frac{c}{c + \frac{c}{6}}\right) = \frac{6f_0}{7}$$



18. (b) The equation of the given transverse wave is given by the displacement of the particle y.

$$y = A \sin^2(\omega t - kx)$$

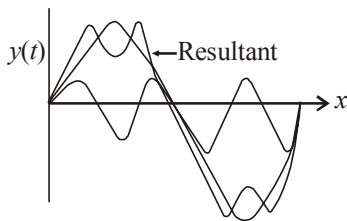
$$\text{Velocity of the particle, } \frac{dy}{dt} = 2A\omega \sin(\omega t - kx)$$

$$\text{Maximum velocity} = 2A\omega$$

$$\text{But the velocity of the wave} = \frac{\omega}{k} = \frac{2\pi\nu}{2\pi/\lambda}$$

$$\text{If } 2A\omega = \frac{\omega}{k}, \quad 2A = \frac{1}{k} = \frac{\lambda}{2\pi} \Rightarrow A = \frac{\lambda}{4\pi}$$

19. (b) Draw a sketch graph showing the two terms and use the principle of superposition to find the final waveform.



20. (d) $v_s = 0.33 \text{ m/s}$



$$v' = v \left[\frac{v}{v - v_s} \right]$$

$$450 \left[\frac{330}{330 - 33} \right] = 500 \text{ Hz}$$

21. (d) x_1 and x_2 are in successive loops of stationary waves.
so, $\phi_1 = \pi$

$$\text{and } \phi_2 = k(\Delta x) = k \left(\frac{3\pi}{2k} - \frac{\pi}{3k} \right) = \frac{7\pi}{6} \Rightarrow \frac{\phi_1}{\phi_2} = \frac{6}{7}$$

22. (a) $v = \frac{dy}{dt} = -A\omega \cos(kx - \omega t)$

$$\therefore v_{\max} = A\omega$$

23. (d) Velocity of particle

$$\frac{dy}{dt} = a \cos \left\{ \frac{2\pi}{\lambda}(vt - x) \right\} \frac{2\pi v}{\lambda} \quad \dots(1)$$

Slope of curve,

$$\frac{dy}{dx} = a \cos \left\{ \frac{2\pi}{\lambda}(vt - x) \right\} \left\{ \frac{-2\pi}{\lambda} \right\} \quad \dots(2)$$

By equation (1) and (2)

$$\frac{dy/dt}{dy/dx} = -v \quad \therefore \frac{dy}{dt} = -v \frac{dy}{dx}$$

24. (d) $f_1 = n_0 \frac{340}{340 - 34} = \frac{10}{9} f_0$;

$$f_2 = n_0 \frac{340}{340 - 17} = \frac{20}{19} f_0$$

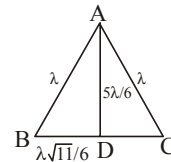
$$\frac{f_1}{f_2} = \frac{10}{9} \times \frac{19}{20} = \frac{19}{18}$$

25. (b) $AB = \sqrt{AD^2 + BD^2} = \lambda AC$

So sound reaching from B and C will be in same phase

$$\text{Now } AD = 5\lambda/6$$

$$\Delta\phi = \frac{2\pi}{\lambda} \times \frac{5\lambda}{6} - \frac{10\pi}{6} = \frac{5\pi}{3}$$



$$A = \sqrt{(2A)^2 + (A)^2} = 2 \times 2A \times A \times \cos(\pi/3)$$

$$\sqrt{5A^2 + 2A^2} = A\sqrt{7} \quad \therefore I = 7I_0$$

26. (d) A travelling wave equation $y = f(x \pm vt)$

At $t = 0.75 \text{ s}$,

$$y_1 = \frac{5}{(3x - 3)^2 - 2}$$

$$y_2 = \frac{-5}{(3x - 3)^2 - 2}$$

$$y = y_1 + y_2 = 0 \text{ for any value of } x$$

At $x = 1 \text{ m}$,

$$y_1 = \frac{5}{(3 - 4t)^2 - 2}, \quad y_2 = \frac{-5}{(4t - 3)^2 - 2}$$

$$y = y_1 + y_2 = 0, \text{ for any value of } t.$$

At $t = 0.75 \text{ s}$, $y = 0$ for all x i.e. string is in its natural length hence its elastic potential energy is zero but kinetic energy is not zero.

Interference of waves does not destroy energy, it remains conserved.

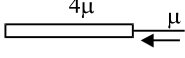
27. (a) At $t=0$, $y = \frac{1}{1+x^2}$ or $x = \sqrt{\frac{1-y}{y}} x_1$

At $t=2s$, $y = \frac{1}{2+x^2-2x} = \frac{1}{1+(x-1)^2}$

or $x-1 = \sqrt{\frac{1-y}{y}}$ or $x = 1 + \sqrt{\frac{1-y}{y}} x_2$

∴ Speed of the wave

$v = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{1}{2-0} = 0.5 \text{ m/s}$

28. (c) $v_1 = \sqrt{\frac{T}{\mu}}$; $v_2 = \sqrt{\frac{T}{4\mu}}$ 

$v_2 < v_1$
⇒ 2nd medium is denser

∴ The wave reflected from the denser medium has phase change of π .

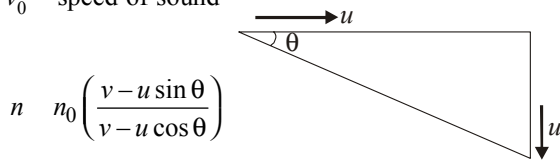
⇒ $A_r = \frac{v_2 - v_1}{v_2 + v_1} \times 6 = \frac{\frac{v_1}{2} - v_1}{\frac{v_1}{2} + v_1} \times 6 = -2 \text{ mm}$

⇒ eqⁿ of reflected wave pulse is
 $Y = -(2 \text{ mm}) \sin(5t - 40x)$

29. (b) $C_{\text{rms}} = \sqrt{\left(\frac{\gamma RT}{M}\right)}$ Here $C_{\text{rms}} = \sqrt{\frac{1}{m}}$;

∴ $\frac{C_{\text{rms}_1}}{C_{\text{rms}_2}} = \sqrt{\left(\frac{m_2}{m_1}\right)}$

30. (d) $v_0 =$ speed of sound

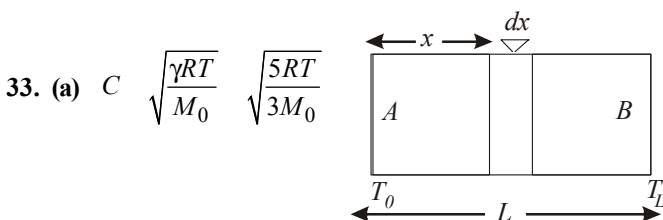


As θ varies from zero to 90° , n decreases continuously.

31. (b) 

After two seconds pulses will overlap each other. According to superposition principle the string will not have any distortion and will be straight. Hence there will be no P.E. The total energy will be only kinetic.

32. (b) Beats are produced due to the difference in apparent frequency of the two tuning forks.



$dx = C dt \sqrt{\frac{5R}{3M_0} \left[T_0 \left(\frac{T_L - T_0}{L} \right) \right] dt}$

$t = \frac{2L}{\sqrt{T_L} \sqrt{T_0}} \sqrt{\frac{3M}{5R}}$

34. (c) $E = A^2 v^2$ where $A =$ amplitude and $v =$ frequency.

Also $\omega = 2\pi v \Rightarrow \omega \propto v$

In case 1 : Amplitude = A and $v_1 = v$

In case 2 : Amplitude = A and $v_2 = 2v$

∴ $\frac{E_2}{E_1} = \frac{A^2 v_2^2}{A^2 v_1^2} = 4$

⇒ $E_2 = 4E_1$

35. (a) The expression for fundamental frequency is

$v = \frac{1}{2\ell} \sqrt{\frac{T}{\mu}}$

In air, $T = mg = V\rho g$

∴ $v = \frac{1}{2\ell} \sqrt{\frac{V\rho g}{\mu}}$

When the object is half immersed in water
 $T' = mg - \text{upthrust}$

$= V\rho g - (V/2)\rho_w g = (V/2)g(2\rho - \rho_w)$

The new fundamental frequency is

$v' = \frac{1}{2\ell} \sqrt{\frac{T'}{\mu}} = \frac{1}{2\ell} \sqrt{\frac{Vg}{2} \frac{(2\rho - \rho_w)}{\mu}}$

$\frac{v'}{v} = \sqrt{\frac{2\rho - \rho_w}{2\rho}} \Rightarrow v' = 300 \sqrt{\left(1 - \frac{1}{2\rho}\right)} \text{ Hz}$

36. (a) $f_0 = \frac{5}{2\ell} \sqrt{\frac{9g}{\mu}} = \frac{3}{2\ell} \sqrt{\frac{Mg}{\mu}}$

⇒ $M = 25 \text{ kg}$

Using the formula of a vibrating string,

$f = \frac{p}{2\ell} \sqrt{\frac{T}{\mu}}$ where $p =$ number of loops.

In each case, the wire vibrates, in resonance with the same tuning fork. Frequency of wire remains same while p and T change.

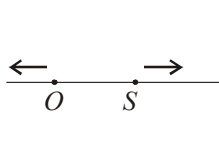
∴ $\frac{p_1}{2\ell} \sqrt{\frac{T_1}{\mu}} = \frac{p_2}{2\ell} \sqrt{\frac{T_2}{\mu}}$ or $p_1 \sqrt{T_1} = p_2 \sqrt{T_2}$

or $\sqrt{\frac{T_2}{T_1}} = \frac{p_1}{p_2}$

$\sqrt{\frac{M \times g}{9 \times g}} = \frac{5}{3}$ or $M = \frac{5 \times 5 \times 9}{3 \times 3}$ or $M = 25 \text{ kg}$.

37. (a) $L = (2n - 1)\frac{\lambda}{4}$ and $v = \frac{v}{\lambda} \Rightarrow v = (2n - 1)85\text{Hz}$

38. (a)



Frequency heard by observer directly coming from

$$\text{source} = \frac{355 - 5}{355 + 5} \times 180 = 175 \text{ Hz.}$$

$f_2 \rightarrow$ frequency heard by observer after reflection

$$= \left[\frac{355}{355 - 5} \times \left[\frac{355 - 5}{355} \right] \right] \times 180 = 180 \text{ Hz}$$

$$f_2 - f_1 = 5 \text{ Hz}$$

39. (b) $f_1 =$ frequency of the police car heard by motorcyclist,
 $f_2 =$ frequency of the siren heard by motorcyclist.

$$f_1 = \frac{330 - v}{330 - 22} \times 176; f_2 = \frac{330}{330} \times 165;$$

$$\therefore f_1 - f_2 = 0 \Rightarrow v = 22 \text{ m/s}$$

40. (a) $v = \sqrt{\frac{T}{\mu}} = 6 \text{ m/s}$

$$\therefore \Delta t = \frac{2 \times 0.3}{6} = 0.1 \text{ sec}$$

41. (d) $y(\vec{r}, t) = A \sin(\omega t - \vec{k} \cdot \vec{r})$

$$\hat{k} = \frac{2\pi}{\lambda} (\cos\alpha \hat{i} + \cos\beta \hat{j} + \cos\gamma \hat{k}) = \frac{k}{\sqrt{3}} (\hat{i} + \hat{j} + \hat{k})$$

$$\phi_1 - \phi_2 = 2(3) = 6 \text{ rad.}$$

42. (b) Frequency of first overtone in closed pipe,

$$v = \frac{3v}{4\ell_1} \sqrt{\frac{P}{\rho_1}} \quad \dots (i)$$

Frequency of first overtone in open pipe,

$$v' = \frac{1}{\ell_2} \sqrt{\frac{P}{\rho_2}} \quad \dots (ii)$$

From equation (i) and (ii)

$$\Rightarrow \ell_2 = \frac{4}{3} \ell_1 \sqrt{\frac{\rho_1}{\rho_2}}$$

43. (a) $y(x, t) = A \sin \omega t \cos kx$ [Assuming $t = 0 \Rightarrow y = 0$]

$$P(x) = B \frac{\delta y}{\delta x} = kBA \sin \omega t \cos kx$$

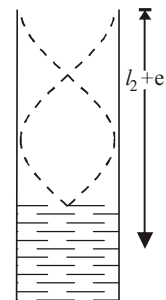
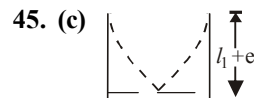
$$[B \text{ is Bulk modulus of elasticity, } v = \sqrt{\frac{B}{\rho_0}}]$$

$$P_{\max} = \frac{2\pi}{\lambda} \rho_0 v^2 A = \frac{3\pi v^2 \rho_0 A}{L}$$

$$A = \frac{10^3 \times 3.9\pi}{3\pi \times 4 \times 10^4 \times 1.3} = 0.025 \text{ m} = 2.5 \text{ cm}$$

44. (a) $v_{\max} = (2\pi f)A$
 $a_{\max} = (2\pi f)^2 A$

$$\frac{a_{\max}}{v_{\max}} = (2\pi f)$$



For first resonance

$$\ell_1 + e = \frac{\lambda}{4}$$

$$\text{But } v = v\lambda$$

$$\therefore v = v \frac{4}{3} (\ell_2 + e)$$

... (i)

$$\therefore v = v 4(\ell_1 + e)$$

... (ii)

Subtracting (i) and (ii),

$$v = 2v(\ell_2 - \ell_1)$$

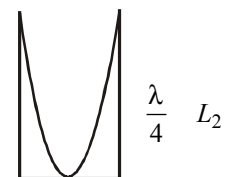
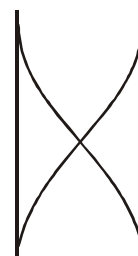
$$\therefore \Delta v = 2v(\Delta \ell_2 + \Delta \ell_1)$$

$$= 2 \times 512 \times (0.1 + 0.1) \text{ cm/s}$$

$$= 204.8 \text{ cm/s}$$

46. (c) Fundamental frequency for a tube open at both ends, v_1

$$= \frac{v}{2L_1}$$



Fundamental frequency for a tube closed at one end,

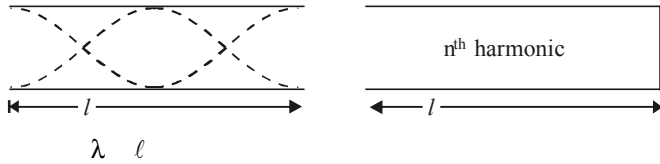
$$v_2 = \frac{v}{4L_2}$$

$$\therefore v_1 = v_2$$

$$\therefore 2L_1 = 4L_2 \Rightarrow L_1 = 2L_2$$

47. (c) $\frac{A_1}{A_1 - A_2} \frac{A_2}{x} ; \frac{A_2}{A_1} \frac{x-1}{x-1} ; \text{Energy } A^2$
 $\Rightarrow \left(\frac{x-1}{x-1}\right)^2$

48. (d)



$\therefore f_1 = \frac{v}{\lambda} = \frac{v}{l/n}$

... (i)

$\lambda = \frac{4\ell}{n}$

$\therefore f_2 = \frac{v}{\lambda} = \frac{nv}{4\ell}$

... (ii)

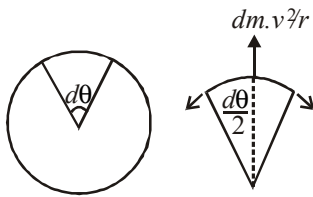
Here n is an odd number. From (i) and (ii)

$f_2 = \frac{n}{4} f_1$

For first resonance, $n = 5, f_2 = \frac{5}{4} f_1$

49. (a) Cannot be greater than $2L$.
 For fundamental case ($\lambda/2 = L$)

50. (d) $dm \cdot \omega^2 R = 2T \sin \frac{d\theta}{2}$



$\mu R d\theta \omega^2 R = 2T \frac{d\theta}{2}$

$\Rightarrow \mu \omega^2 R^2 = T \Rightarrow v_w = \sqrt{\frac{T}{\mu}} = \sqrt{\omega^2 R^2} = \omega R$

also speed of string is ωR

\therefore The velocity of disturbance w.r.t. ground
 $= \omega R - \omega R = 0$

51. (a) Frequency of 1st harmonic of AB

$= \frac{1}{2\ell} \sqrt{\frac{T_{AB}}{m}}$

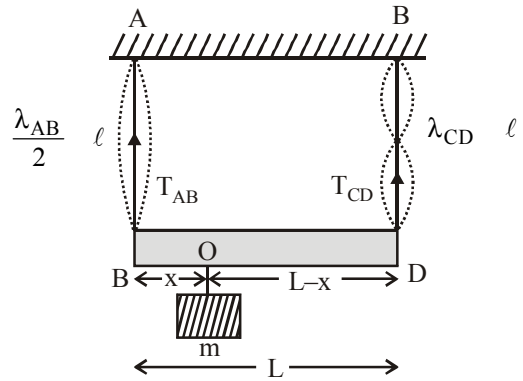
Frequency of 2nd harmonic of CD = $\frac{1}{\ell} \sqrt{\frac{T_{CD}}{m}}$

Given that the two frequencies are equal.

$\therefore \frac{1}{2\ell} \sqrt{\frac{T_{AB}}{m}} = \frac{1}{\ell} \sqrt{\frac{T_{CD}}{m}}$

$\Rightarrow \frac{T_{AB}}{4} = T_{CD}$

$\Rightarrow T_{AB} = 4T_{CD}$... (i)



For rotational equilibrium of massless rod, taking torque about point O.

$T_{AB} \times x = T_{CD} (L-x)$... (ii)

For translational equilibrium,

$T_{AB} + T_{CD} = mg$... (iii)

On solving, (i) and (iii), we get

$T_{CD} = \frac{mg}{5}$;

$\therefore T_{AB} = \frac{4mg}{5}$

Substituting these values in (ii), we get

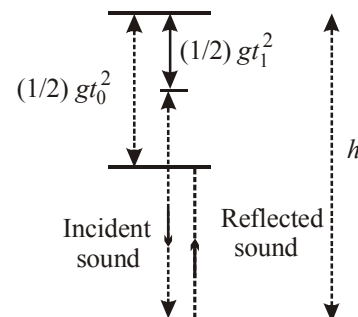
$\frac{4mg}{5} \times x = \frac{mg}{5} (L-x)$

$4x = L-x$

$\Rightarrow x = \frac{L}{5}$

52. (b) Let the sound observed by the parachutist at $t_0 = 12$ s be produced at t_1 s. Velocity of source at the instant of sound = gt_1 and velocity of observer at the instant of observing same sound = gt_0 . Hence the relation between apparent frequency f' and original frequency f will be

$f' = f \left(\frac{v - gt_0}{v - gt_1} \right)$



Here $f = 800 \text{ Hz}$, $g = 10 \text{ m/s}^2$, $v = 330 \text{ m/s}$, $t_0 = 12 \text{ s}$ and $f' = 800 + 700 = 1500 \text{ Hz}$

Putting these, we get, $t_1 = 9 \text{ s}$

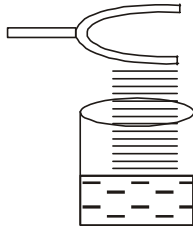
Now the distance travelled by sound in $(t_0 - t_1)$ sec is

$$v(t_0 - t_1) = \left(h - \frac{1}{2}gt_0^2 \right) + \left(h - \frac{1}{2}gt_1^2 \right)$$

Putting the values, we get, $h = 1057.5 \text{ m}$.

53. (a) This concept is same as that of mirage i.e. total internal reflection of sound wave due to varying density of air of atmosphere.

54. (a) As shown in the figure, the prongs of the tuning fork are kept in a vertical plane.



55. (d) For fundamental mode, $= \frac{\lambda}{2} = L$, $\lambda = 2L$

High L assures high λ .

56. (c) Wave from P reaches M in same phase as originated
wave from Q also reaches M in same phase as originated
Hence both are in same phase at M and thus constructive interference takes place.

$$I = (\sqrt{I_1} + \sqrt{I_2})^2 = \left(\sqrt{I_0} + \sqrt{\frac{I_0}{4}} \right)^2 = \frac{9}{4}I_0$$

57. (d) Time period of oscillation of R

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi \times \frac{10}{\pi} = 20 \text{ s}$$

At a time $t = 10 \text{ s}$, R will be at mean position and moving along negative x -axis.

$$v_R = A\omega = 15 \text{ m/s}$$

The second which is received at $t = 10 \text{ s}$, is emitted at $t = t_0 \text{ s}$.

$$\frac{1}{2}at_0^2 = v(10 - t_0)$$

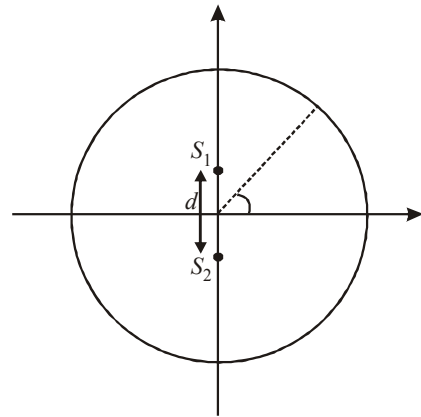
$$\Rightarrow \frac{1}{2} \times 18.75t_0^2 = 300(10 - t_0) \Rightarrow t_0 = 8 \text{ s}$$

$$v_s = at = 18.75 \times 8 = 150 \text{ m/s}$$

$$f' = 500 \left(\frac{300 + 15}{300 - 150} \right) = 350 \text{ Hz}$$

58. (a) The wavelength of the sound wave in air is

$$\lambda = \frac{320}{16 \times 10^3} = 2 \times 10^{-2} \text{ m}$$



The positions of maxima on the circumference of the circular track will be given by

$$d \sin \theta = n\lambda$$

When d is the separation between the sources and θ is the angular position of n^{th} maximum as shown in the figure

$$2 \sin \theta = n(2 \times 10^{-2}) \Rightarrow \sin \theta = \frac{n}{100}$$

Since $\sin \theta$ lies between 0 and 1 there are 400 maxima on the entire circle.

These 400 maxima will be heard by the person in the time

$$t = \frac{400}{2} = 200 \text{ s}$$

$$\text{Speed of the train} = 36 \text{ km/h} = 36 \times \frac{5}{18} = 10 \text{ m/s}$$

From the obtained values so far we get length of the track

$$\ell = (10 \text{ m/s})(200 \text{ s}) = 2000 \text{ m}$$

$$\text{So radius of the track} = \frac{2000}{2\pi} = \frac{1000}{\pi} \text{ m}$$

59. (b) $n_0 = \frac{v}{2\ell}$; $n_1 = \frac{v}{2(\ell/2 - \Delta\ell)}$, $n_2 = \frac{v}{2(\ell/2 + \Delta\ell)}$

Beat frequency = $n_1 - n_2$

$$\Rightarrow v \left[\frac{1}{\ell - 2\Delta\ell} - \frac{1}{\ell + 2\Delta\ell} \right]$$

$$= v \left[\frac{(\ell + 2\Delta\ell) - (\ell - 2\Delta\ell)}{\ell^2 - 4\Delta\ell^2} \right]$$

$$v \frac{4\Delta\ell}{\ell^2 - 4\Delta\ell^2} = \frac{8}{\ell} \frac{\Delta\ell v}{2\ell} = \frac{8\Delta\ell v_0}{\ell}$$

60. (c) The velocity profile of each elementary section of the pulse is shown in figure 1 and figure 2.

When both the pulses completely overlap, the velocity profiles of both the pulses in overlap region are identical. By superposition, velocity of each elementary section doubles. Therefore, KE of each section becomes four times. Hence the K.E. in the complete width of overlap becomes four times.

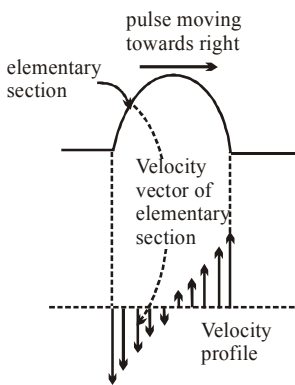


Figure-1

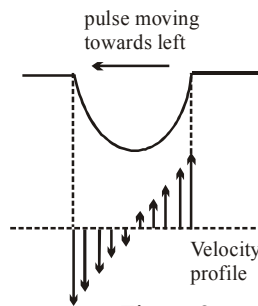


Figure-2

61. (b) Using the formula $n' = n \left(\frac{v_A + v}{v} \right)$

$$\frac{v_A + v}{v} = \frac{5.5}{5} \text{ and } \frac{V_B + V}{V} = \frac{6}{5}$$

$$\Rightarrow \frac{v_B}{v_A} = 2$$

62 (c) $f' = \left(\frac{V + V_0}{V + V_s} \right) f$; $f_1 = \left(\frac{V + V_0}{V} \right) f$

$f_2 = \left(\frac{V}{V + V_s} \right) S$; $\lambda_2 = \left(\frac{V + V_s}{f} \right)$

$V_0 =$ velocity of listener, $V_s =$ velocity of source.

But if $\vec{V}_s = \vec{V}_0$ then $f' = f$

Thus statement-3 is not always correct.

63. (b) The wave resulting from the oscillation of the end of the string is defined by :

$$y(x, t) = 8 \sin \left(4t - \sqrt{\frac{\mu}{T}} 4x \right) \text{ cm} \quad \dots (1)$$

Using eq. (1), we can immediately write : $\omega = 4\text{s}^{-1}$

$$k = \sqrt{\frac{\mu}{T}} \omega = \frac{4}{\sqrt{5}} \text{ m}^{-1}$$

The wave power is :

$$P = F_y v_y = -T \frac{\partial y}{\partial x} \frac{\partial y}{\partial t} = T \omega k A^2 \cos^2(\omega t - kx) \dots (2)$$

The average power over a period is :

$$\bar{P} = \frac{1}{2} T \omega k A^2 = \frac{1}{2} \underbrace{\mu \omega^2 A^2 v}_{\text{the energy density}} \dots (3)$$

where $v = \frac{\omega}{k}$ is the wave velocity. Note that eq. (3) was derived from the known result that the average over a whole period of $\cos^2 x$ is $\frac{1}{2}$. Substitution of the numerical

values yield $\bar{P} = 0.0229 \text{ J/s}$

The power transferred to the bath, P , is

$$P = 0.5 \bar{P} = 0.01145 \text{ J/s} \quad \dots (4)$$

The temperature difference caused by the heat transfer ΔQ to a substance with heat capacity C is

$$\Delta T = \frac{1}{C} \Delta Q = \frac{1}{C} (P \Delta t) \quad \dots (5)$$

In our case : $\frac{C}{\Delta T} = \frac{1 \text{ kcal}}{1 \text{ K}}$

Therefore, $\Delta t = 3.6 \times 10^5 \text{ s} \approx 4.2 \text{ day} \quad \dots (6)$

64. (b) $\ell_1 = x = \frac{\lambda}{4}$

or, $\lambda = 4(\ell_1 - x)$

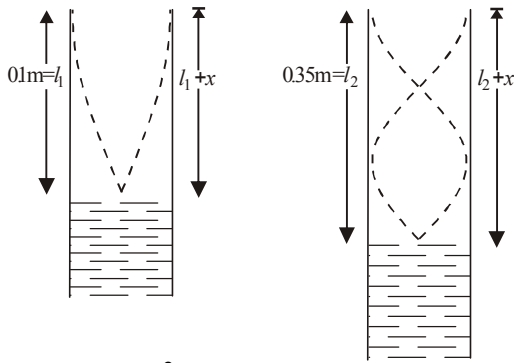
$(\ell_2 - x) = \frac{3\lambda}{4}$

$\Rightarrow \lambda = \frac{4}{3}(\ell_2 - x)$

$\therefore v_1 = \frac{v}{\lambda_1} = \frac{v}{4(\ell_1 - x)}$

$\therefore v_2 = \frac{v}{\lambda_2} = \frac{3v}{4(\ell_2 - x)}$

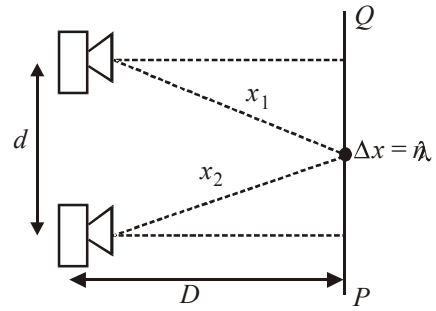
Given $v_1 = v_2$



$$\text{or, } \frac{v}{4(l_1 + x)} = \frac{3v}{4(l_2 + x)}$$

$$\text{or, } x = 0.025 \text{ m}$$

65. (a)



$$\Delta x \leq d \Rightarrow n\lambda \leq d$$

$$n \leq \frac{d}{\lambda} \quad \text{or} \quad n \leq \left(\frac{d}{v}\right) f$$

$$\text{or } n_{\max} = \frac{df}{v}$$

by decreasing f , number of maxima will decrease, thus, their separation will increase.

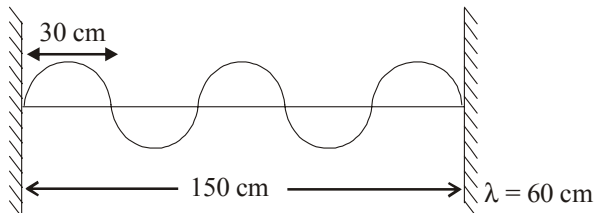
B

COMPREHENSION TYPE

1. (c); 2. (b); 3. (a)

$$v = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{36}{1}} = 6 \text{ m/s}$$

$$\omega = \frac{2\pi v}{\lambda} = 20\pi \text{ per sec.}$$



4. (a); 5. (c); 6. (a)

Let velocity of plank is v and velocity of man is $(v - 8)$

$$0 = 50 \times (v - 8) + 150v$$

$$v = 2 \text{ m/s}$$

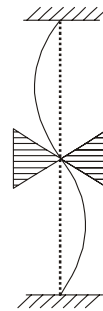
$f_1 \Rightarrow$ frequency of sound detected by detector D before man jumps off the plank

$$\frac{330 - 2}{330 - 6} f_0 = \frac{332}{324} f_0$$

$f_2 \Rightarrow$ frequency of sound detected by detector D after man jump off the plank

$$\frac{330 - 2}{330 - 6} f_0 = \frac{328}{336} f_0$$

7. (b); 8. (b); 9. (a)



$$\frac{201mg}{301m} = \frac{201g}{301}$$

$$v = \sqrt{\frac{Y}{\rho}} = \sqrt{\frac{YA\ell}{m}}$$

$$\lambda = \ell$$

10. (a); 11. (c); 12. (a)

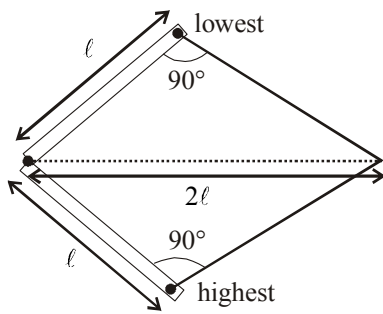
Original frequencies will be produced, when AB lies along BO. So time difference

$$\Rightarrow \frac{\pi}{\omega} - \frac{2\ell}{v} = 1 - \frac{2 \times 30}{300} = 0.8 \text{ s}$$

Least and highest frequency perceptions will be as shown in the figure.

Distance traveled

$$\Rightarrow \frac{2}{3} \times 2\pi\ell = \frac{4\pi}{3} \ell = 40\pi \text{ m}$$

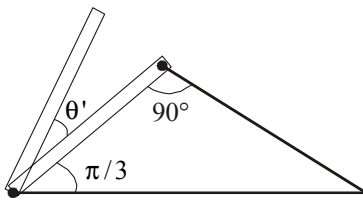


Additional θ in the time that sound takes to reach from A to O.

$$\theta' = \omega \frac{\left(\frac{\sqrt{3}}{2} 2\ell\right)}{v}$$

$$\frac{\pi \times \sqrt{3} \times 30}{300} = \frac{\sqrt{3}}{10} \pi$$

$$= 0.1732 \pi$$



$$\theta = \theta' \frac{\pi}{3} = \left(\frac{1}{3} \frac{\sqrt{3}}{10}\right) \pi$$

$$= 0.5065 \pi$$

13. (a); 14. (c); 15. (d)

The equations are $y_1 = A \cos(0.5 \pi x - 100 \pi t)$ and $y_2 = A \cos(0.46 \pi x - 92 \pi t)$ represents two progressive wave travelling in the same direction with slight difference in the frequency. This will give the phenomenon of beats.

Comparing it with the equation

$$y = A \cos(kx - \omega t), \text{ we get}$$

$$\omega_1 = 100 \pi \Rightarrow 2\pi f_1 = 100 \pi \Rightarrow f_1 = 50 \text{ Hz and}$$

$$K_1 = 0.5 \pi \Rightarrow \frac{2\pi}{\lambda_1} = 0.5\pi \Rightarrow \lambda_1 = 4 \text{ m}$$

Wave velocity = $\lambda_1 f_1 = 200 \text{ m/s}$ [Alternatively use $v = \frac{\omega}{K}$]

$$\omega_2 = 92 \pi \Rightarrow 2\pi f_2 = 92 \pi \Rightarrow f_2 = 46 \text{ Hz}$$

Therefore beat frequency = $f_1 - f_2 = 4 \text{ Hz}$ and

$$K_2 = 0.46 \pi \Rightarrow \frac{2\pi}{\lambda_2} = 0.46\pi \Rightarrow \lambda_2 = \frac{200}{46}$$

Wave velocity = $\frac{200}{46} \times 46 = 200 \text{ m/s}$

Wave velocity is same because it depends on the medium in which the wave is travelling.

Now, at $x=0$,

$$y_1 + y_2 = (A \cos 10 \pi t) + (A \cos 92 \pi t) = 0$$

$$\Rightarrow \cos 100 \pi t = -\cos 92 \pi t = \cos(-92 \pi t)$$

$$= \cos[(2n+1)\pi - 92 \pi t]$$

$$\Rightarrow t = \frac{2n-1}{192}$$

when $t=0$, $n = -\frac{1}{2}$ and when $t=1$,

$$n = \frac{191}{2} = 95.2$$

\Rightarrow net amplitude is zero for $n = 96$ times (the nearest answer).

16. (b) $I \propto A^2$

\therefore Intensity of reflected wave is 64% of incident wave

$$\therefore A_r = 0.8 A.$$

17. (a) $y_i = A \sin(ax - bt) \frac{\pi}{2} = A \cos(ax + bt)$

$$\text{and } y_r = -0.8 A \cos(ax - bt)$$

$$\therefore y = y_i + y_r = -1.6 A \sin ax \sin bt + 0.2 A \cos(ax + bt)$$

$$\therefore c = 0.2.$$

18. (c) $\left(\frac{dy}{dt}\right)_{\max} = 1.8 \text{ bA.}$

19. (c); 20. (d); 21. (c)

Wave velocity in string is

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{40}{0.1}} = 20 \text{ m/s}$$

Fundamental frequency of string oscillations is

$$n_0 = \frac{v}{2\ell} = \frac{20}{0.6} = \frac{100}{3} \text{ Hz}$$

Thus string will be in resonance with a tuning fork of frequency,

$$n_f = \frac{100}{3} \text{ Hz}, \frac{200}{3} \text{ Hz}, 100 \text{ Hz}, \frac{400}{3} \text{ Hz}, \dots$$

Here rider will not oscillate at all only if it is at a node of stationary wave in all other cases of resonance and non-resonance. It will vibrate at the frequency of tuning fork. At a distance $\ell/3$ from one end node will appear at 3rd, 6th, 9th or similar higher harmonics i.e. at frequencies 100Hz, 200Hz,

If string is divided in odd no. of segments, these segments

can never resonate simultaneously, hence at the location of rider, antinode is never obtained at any frequency.

22. (c) The equation of wave moving in negative x -direction, assuming origin of position at $x = 2$ and origin of time (i.e. initial time) at $t = 1$ sec.

$$y = 0.1 \sin(2\pi t + 4x)$$

Shifting the origin of position to left by 2m, that is, to $x = 0$. Also shifting the origin of time backwards by 1 sec, that is to $t = 0$ sec.

$$y = 0.1 \sin(2\pi(t-1) + 4(x-2))$$

23. (a) As given the particle at $x = 2$ is at mean position at $t = 1$ sec.

$$\therefore \text{Its velocity } v = \omega A = 2\pi \times 0.1 = 0.2\pi \text{ m/s.}$$

24. (d) Time period of oscillation

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{2\pi} = 1 \text{ sec.}$$

Hence, $t = 1.25$ sec, that is, at $T/4$ seconds after $t = 1$ second, the particle is at rest at extreme position.

Hence instantaneous power at $x = 2$ at $t = 1.25$ sec. is zero.

25. (d) The sound heard by observer must have been emitted sometime earlier at B so that by the sound reached at A , train also reacted at O .

$$\frac{BO}{BA} = \frac{u}{v} = \cos \theta = \frac{15}{300} = \frac{1}{20}$$

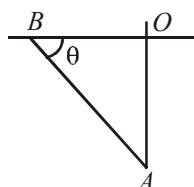
The observed frequency

$$f = f_0 \frac{v}{v - u \cos \theta}$$

$$= 400 \times \frac{300}{300 - 15 \times \frac{1}{20}} = \frac{400 \times 300}{300 - \frac{3}{4}}$$

$$= 400 \left(1 - \frac{1}{400}\right)^{-1} \text{ Hz}$$

$$= 400 \left(1 + \frac{1}{400}\right) \text{ Hz} = 401 \text{ Hz}$$



26. (b) There will be no Doppler effect when sound is emitted at O . By the time sound reaches A , the train will travel $S = 15 \times 1 = 15$ m.

27. (b) Decrease continuously

28. (b) The speed of sound depends on the frame of reference of the observer.

29. (a) Since all the passengers in train A are moving with a velocity of 20 m/s therefore the distribution of sound

intensity of the whistle by the passengers in train A is uniform.

30. (a)
$$v' = v_1 \left[\frac{v - v_0}{v - v_s} \right] = 800 \left[\frac{340 - 30}{340 - 20} \right] = 800 \times \frac{31}{32}$$

$$v'' = v_2 \left[\frac{v - v_0}{v - v_s} \right] = 1120 \times \frac{31}{32}$$

$$\therefore v'' - v' = (1120 - 800) \times \frac{31}{32} = 320 \times \frac{31}{32} = 310 \text{ Hz.}$$

31. (c), 32. (b)

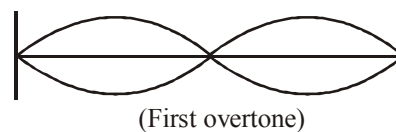
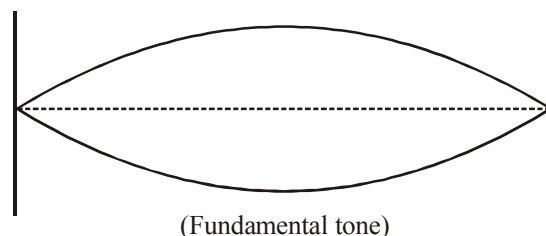
Coating tuning fork A will decrease its frequency, beat frequency increases means $f_B > f_A$ and $f_C > f_A$

Given $(f_B - f_A) < (f_C - f_A)$

$$f_C > f_B > f_A$$

33. (a), 34. (b)

When a string oscillates, nodes are produced at its ends. In case of fundamental tone, it vibrates in single loop. Hence, wavelength of fundamental tone, $\lambda_0 = 2\ell$ and in case of first overtone it vibrates in two loops as shown in figure. Hence wavelength of first overtone is $\lambda_1 = \ell$.



When stationary performs SHM and displacement amplitude at a point distant x from one end is given by

$$a = a_0 \sin\left(\frac{2\pi x}{\lambda}\right) \dots\dots\dots (1)$$

where a_0 is maximum displacement amplitude which occurs at antinode. Since, tension in string is T , therefore, velocity

of transverse wave is given by $v = \sqrt{\frac{T}{m}}$ where m is mass per unit length of string.

Fundamental tone : Since, frequency is $n = \frac{v}{\lambda}$, therefore, frequency of fundamental tone of the string,

$$n_0 = \frac{1}{2\ell} \sqrt{\frac{T}{m}}$$

Considering an elemental length dx of string at a distance x from left end, its mass = $m dx$.

$$\begin{aligned} \text{Its oscillation energy} &= \frac{1}{2} (m dx) a^2 (2\pi n_0)^2 \\ &= \frac{a_0^2 \pi^2 T}{2\ell^2} \sin^2 \left(\frac{2\pi x}{2\ell} \right) dx \end{aligned}$$

\therefore Total oscillation energy of the string

$$= \frac{a_0^2 \pi^2 T}{2\ell^2} \int_0^\ell \sin^2 \left(\frac{\pi x}{\ell} \right) dx = \frac{a_0^2 \pi^2 T}{4\ell}$$

Since, maximum kinetic energy of a particle performing SHM is equal to its oscillation energy, therefore, maximum kinetic

$$\text{energy of the string in its fundamental tone} = \frac{a_0^2 \pi^2 T}{4\ell}$$

$$\text{First overtone : Frequency, } n_1 = \frac{v}{\lambda_1} = \frac{1}{\ell} \sqrt{\frac{T}{m}}$$

Considering an elemental length dx of string at a distance from left end, its mass = $m dx$.

$$\begin{aligned} \text{Its oscillation energy} &= \frac{1}{2} (m dx) a_0^2 \cdot \sin^2 \left(\frac{2\pi x}{\ell} \right) (2\pi n_1)^2 \\ &= \frac{2\pi^2 a_0^2 T}{\ell^2} \sin^2 \left(\frac{2\pi x}{\ell} \right) dx \end{aligned}$$

\therefore Total oscillation energy of the string

$$= \frac{2\pi^2 a_0^2 T}{\ell^2} \int_0^\ell \sin^2 \left(\frac{2\pi x}{\ell} \right) dx$$

or maximum kinetic energy of string in its overtone

$$= \frac{a_0^2 \pi^2 T}{\ell}$$

C

REASONING TYPE

- (b) Sound travels faster in solids than gases. It is because the elasticity of solid is more than that of gases. Solids possess greater density than gases. Though density has effect on the velocity of sound in the medium as follows

$$v \propto \frac{1}{\sqrt{\rho}}$$

In case of solid, its elasticity far exceeds that of gas so its effect far exceeds the effect of density.
- (a) Since wavelength is distance between two nearest particles in phase and time period is time required by a wave to cover this distance.

So, speed of wave = $\frac{\text{wave length}}{\text{time period}}$
- (a) When beetle moves along the sand it sends two sets of pulses, one longitudinal and the other transverse. Scorpion has the capacity to intercept the waves. By getting a sense of time interval between receipt of these two waves, it can determine the distance of beetle also.
- (d) If temperature of gas is constant change in pressure will not effect the speed of sound. Hence statement-1 is false.
- (a) Pitch is related to frequency and $f \propto \frac{v}{\lambda}$
- (a) Statement 1 : $I \propto A^2$ and $I \propto f^2$

$$\frac{I_1}{I_2} = \left(\frac{A_1}{A_2} \right)^2 \left(\frac{f_1}{f_2} \right)^2 = \left(\frac{2}{1} \right)^2 = 4$$

Statement 2 : Velocity amplitude : $A_w \propto \sqrt{I}$

therefore if velocity amplitude is same then intensity will also be same.
- (a) Both pulses overlap each other, as they are travelling in opposite directions.

\therefore Using relation $-v \frac{dy}{dx} = \frac{dy}{dt} = v_p$

We see that velocity of all particle is zero.

\therefore Net energy is potential.
- (d) Closed end is pressure antinode therefore pressure is not constant. Statement 2 is true.
- (d) $A_t = \frac{2\sqrt{\mu_1}}{\sqrt{\mu_1} + \sqrt{\mu_2}}$

$\mu_1 > \mu_2 \Rightarrow A_t > A_i$

\therefore Statement 1 is false.

1. (a, b) The displacement of air column to the right of *B* is negative (i.e. towards *B*) while displacement of air column to the left of *B* is positive (i.e., towards *B*) so density is maximum at *B*, similarly density is minimum at *D*.

2. (a, b, c, d) $y = 10^{-4} \sin(60t + 2x)$
Comparing the given equation with the standard wave equation travelling in negative *x*-direction $y = a \sin(\omega t + kx)$

we get amplitude $a = 10^{-4} \text{m}$

Also, $\omega = 60 \text{ rad/s}$

$$\therefore 2\pi f = 60$$

$$\Rightarrow f = \frac{30}{\pi} \text{ Hz}$$

Also, $k = 2$

$$\Rightarrow \frac{2\pi}{\lambda} = 2$$

$$\Rightarrow \lambda = \pi \text{ m}$$

We know that $v = f\lambda = \frac{30}{\pi} \times \pi = 30 \text{ m/s}$

3. (a, b) The solid in air, in water and in liquid have three different weights due to the buoyancy.

Let its weight in air be W_1 , that in water be W_2 and that in liquid be W_3 .

The frequency of the wire is given by $f = \frac{1}{2L} \sqrt{\frac{T}{m}}$

and in the present case $f = \text{a constant} \times \sqrt{T}$

or, $T = kf^2$ (k is a constant)

Now, $W_1 = k(500)^2$, $W_2 = k(460)^2$, $W_3 = k(480)^2$

From the principle of Archimedes we have the specific gravity of solid

$$= \frac{\text{Weight of solid in air}}{\text{Loss of weight in water}} = \frac{W_1}{W_1 - W_2}$$

$$= \frac{k(500)^2}{k(500)^2 - k(460)^2} = \frac{500^2}{500^2 - 460^2} = \frac{250000}{38400} = 6.51$$

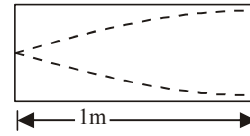
Specific gravity of liquid

$$= \frac{\text{Loss of weight of solid in liquid}}{\text{Loss of weight of solid in water}}$$

$$\frac{k(500)^2 - k(480)^2}{k(500)^2 - k(460)^2}$$

$$\frac{500^2 - 480^2}{500^2 - 460^2} = 0.51$$

4. (a, b, d)



In general, we can write for a closed end pipe that

$$v = \frac{(2n-1)c}{4\ell} \text{ where } n = 1, 2, 3, \dots$$

$$(2n-1) \frac{\lambda}{4} = \ell \Rightarrow \lambda = \frac{4\ell}{(2n-1)}$$

$$\therefore v = \frac{c}{4\ell}, \frac{3c}{4\ell}, \frac{5c}{4\ell}, \dots = 80, 240, 400, \dots$$

5. (a, c) The wavelengths possible in an air column in a pipe which has one closed end is

$$\lambda = \frac{4\ell}{(2n-1)}$$

So, $c = v\lambda$

$$300 = 264 \times \frac{4\ell}{2n-1}$$

$v = 264 \text{ Hz}$ as it is in resonance with a vibrating tuning fork of frequency 264 Hz.

$$\ell = \frac{330 \times (2n-1)}{264 \times 4}$$

For $n = 1$, $\ell = 0.3125 \text{ m} = 31.25 \text{ cm}$

For $n = 2$, $\ell = 0.9375 \text{ m} = 93.75 \text{ cm}$

6. (a, b, d)

$\lambda \rightarrow$ wavelength

$\delta \rightarrow$ end correction

$$\ell_1 = \frac{\lambda}{4} + \delta; \ell_2 = \frac{3\lambda}{4} + \delta$$

$$\text{or, } \ell_2 = 3(\ell_1 - \delta) + \delta = 3\ell_1 - 2\delta$$

7. (b, c)

The maximum pulse height is 2.0 cm. when $x - 3.0t = 0$. It is constant.

Speed of the pulse

$$= \frac{\text{coefficient of } t}{\text{coefficient of } x} = 3.0 \text{ cm/s}$$

8. (a, b, c)

Standing waves are produced by two similar waves superposing while travelling in opposite direction. This can happen in case (a), (b) and (c).

9. (a,c,d) For a plane wave, intensity (energy crossing per unit area per unit time) is constant at all points. But for a spherical wave, intensity at a distance r from a point source of power (P), energy transmitted per unit area per unit time is given by

$$I = \frac{P}{4\pi r^2}$$

$$\Rightarrow I \propto \frac{1}{r^2}$$

But the **total intensity** of the spherical wave over the spherical surface centered at the source remains constant at all times.

For line source $I \propto \frac{1}{r}$

10. (a,b,d) If we use the trigonometric identity,

$$\sin x + \sin y = 2 \sin \left(\frac{y+x}{2} \right) \cos \left(\frac{y-x}{2} \right) \dots (1)$$

The transverse perturbation can be decomposed into two transverse waves which move in different directions :

$$\begin{cases} \phi_1(x,t) = A \sin(kx - \omega t) \\ \phi_2(x,t) = A \sin(kx + \omega t) \end{cases} \dots (2)$$

Since $2A = 4$, we immediately obtain $A = 2$ cm. The velocity of each wave is

$$v = \frac{\omega}{k} = \frac{10\pi}{\pi/3} = 40 \text{ cm/s} \dots (3)$$

The distance between two adjacent junctions can be derived from the condition $\sin(kx) = 0$, which leads to:

$$kx = n\pi, n = 0, 1, 2, \dots \dots (4)$$

$$\text{or } x = \frac{\pi}{k} n = \frac{\lambda}{2} n \dots (5)$$

The wavelength can be calculated by considering the given $\phi(x,t)$:

$$\lambda = \frac{2\pi}{k} = \frac{2\pi}{\pi/3} = 6 \text{cm.} \dots (6)$$

Therefore, the distance between two adjacent junctions

$$(n=1) \text{ is } \ell = \frac{\lambda}{2} = 3 \text{ cm.}$$

11. (a,c) For a transverse sinusoidal wave travelling on a string, the maximum velocity is $a\omega$.

But maximum velocity is $\frac{v}{10} = \frac{10}{10} = 1 \text{ m/s}$

$$\therefore a\omega = 1$$

$$\Rightarrow 10^{-3} \times 2\pi v = 1$$

$$\Rightarrow v = \frac{1}{2\pi \times 10^{-3}} = \frac{10^3}{2\pi} \text{ Hz}$$

The velocity $v = v\lambda$

$$\therefore \lambda = \frac{v}{\nu} = \frac{10}{10^3 / 2\pi} = 2\pi \times 10^{-2} \text{ m}$$

12. (b,c,d) $y = \frac{0.8}{(4x - 5t)^2 + 5}$

$$\frac{0.8}{16 \left[x - \frac{5}{4}t \right]^2 + 5} \dots (1)$$

We know that equation of moving pulse is $y = f(x + vt)$... (2)

On comparing (1) and (2), we get

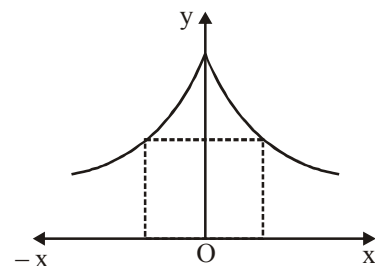
$$v = \frac{5}{4} \text{ ms}^{-1} = \frac{2.5}{2} \text{ ms}^{-1}$$

So, the wave will travel a distance of 2.5 m in 2 sec.

$$\therefore y = \frac{0.8}{4x - 5t^2 + 5}$$

At $x = 0, t = 0, y = \frac{0.8}{5} = 0.16 \text{ m}$

\therefore maximum displacement is 0.16 m



The graph for the given equation is drawn. This is symmetric about y-axis.

13. (a, b, c) The equation of standing wave can be written as $y = 2A \sin kx \cos \omega t$

where $k = \frac{2\pi}{\lambda}$ and $\omega = \frac{2\pi v}{\lambda}$

The standing wave is obtained by adding the equation of two identical progressive waves travelling in opposite directions

$$y_1 = A \sin(kx - \omega t), \quad y_2 = A \sin(kx + \omega t)$$

In the present problem the length L of the rod = 1 metre.

i.e., $L = \frac{5\lambda}{2}$ or $\lambda = \frac{2}{5}$ metre.



Velocity of longitudinal wave is given by

$$v = \sqrt{\frac{Y}{\rho}} = \sqrt{\frac{2 \times 10^{11}}{8000}} = 5 \times 10^3 \text{ ms}^{-1}$$

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{2/5} = 5\pi \text{ metre}^{-1}$$

$$\omega = \frac{2\pi v}{\lambda} = \frac{2\pi \times 5 \times 10^3}{2/5} = (25 \times 10^3 \pi) \text{ s}^{-1}$$

Hence, equation of standing wave is

$$y = 2 \times 10^{-6} \sin 5\pi x \cos 25 \times 10^3 \pi t$$

Equations of component waves are

$$y_1 = (1 \times 10^{-6}) \sin(5\pi x - 25 \times 10^3 \pi t)$$

$$y_2 = (1 \times 10^{-6}) \sin(5\pi x + 25 \times 10^3 \pi t)$$

14. (b,c) As, $f = \frac{1}{2\pi} \sqrt{\frac{T}{\mu}}$

$$\therefore f \propto \sqrt{T}$$

Given that $T_1 = T_2$

$$\therefore f_1 = f_2$$

Initially beat frequency $f_1 - f_2 = 6$.

The beat frequency remains unchanged which is possible when f_2 increases and f_1 decreases. Thus T_2 increases and T_1 decreases.

15. (b,c) Due to the clamping of the square plate at the edges, its displacements along the x and y axes will individually be zero at the edges.

Only the choices (b) and (c) predict these displacements correctly. This is because $\sin 0 = 0$.

Option (a) :

$$u(x, y) = 0 \text{ at } x = 0, y = 0 \text{ and } x = L, y = L$$

$$u(x, y) \neq 0 \text{ at } x = 0, y = L$$

Option (b) :

$$u(x, y) = 0 \text{ at } x = 0, y = 0 \because \sin 0 = 0$$

$$u(x, y) = 0 \text{ at } x = L, y = L \because \sin \pi = 0$$

Option (c) :

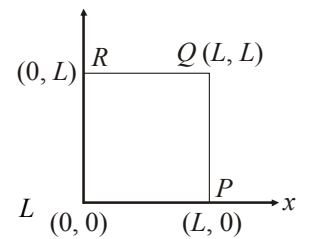
$$u(x, y) = 0 \text{ at } x = 0, y = 0 \because \sin 0 = 0$$

$$u(x, y) = 0 \text{ at } x = L, y = L \because \sin \pi = 0, \sin 2\pi = 0$$

Option (d) :

$$u(x, y) = 0 \text{ at } y = 0, y = L \because \sin 0 = 0, \sin \pi = 0$$

$$u(x, y) \neq 0 \text{ at } x = 0, x = L \because \cos 0 = 1, \cos 2\pi = 1$$



16. (b, c) $y = 2A \sin kx \cdot \sin \omega t$.

$$V_y = \frac{dy}{dt} = 2A \sin kx \cdot \cos \omega t$$

$$V_y = 0 \Rightarrow t = T/4, 3T/4 \left(T = \frac{2\pi}{\omega} \right)$$

Hence, all the particles will be at rest at $T/4$ and $3T/4$, i.e., two times in one time period.

17. (a,b,c,d) In the wave motion $y = a \sin(kx - \omega t)$, y can represent electric and magnetic fields in electromagnetic waves and displacement and pressure in sound waves.

18. (b) For such a case, we may consider that the observer is standing along the stationary surface. Then

$$f' = f \left(\frac{c + v}{c} \right)$$

Frequency of reflected wave is $f'' = f \left(\frac{c - v}{c - v} \right)$

$$\Rightarrow \text{Beat freq.} = f'' - f = \frac{2v}{c - 1}$$

Wavelength of reflected wave = $\frac{c}{f''} = \frac{c(c - v)}{f(c - v)}$

E

MATRIX-MATCH TYPE

1. A-q; B-q; C-q; D-s

(A) and (B) B is displacement node
 $\Rightarrow a=0, v=0$ and $E=0$ and deformation maximum \Rightarrow
 PE max.

(D) B is displacement antinode
 $\Rightarrow a=\max, KE=0, PE=0, V=0$

2. A-q; B-p; C-r

(A) Pitch q. frequency
 (B) quality p. waveform
 (C) loudness r. intensity

3. A-p; B-s; C-r

At $t = \frac{L}{v}$, pulse position will be same as at $t=0$ but v
 will be in opposite direction (not inverted).

At $t = \frac{2L}{v}$, pulse position will be same as at $t=0$ but
 inverted due to reflection at rigid end.

At $t = \frac{3L}{v}$, inverted moving towards fix end.

F

NUMERIC/INTEGER ANSWER TYPE

1. 9

As 54 waves reaches the shore per minute

$$f = \frac{54}{60} = \frac{9}{10} \text{ Hz}$$

And as wavelength of waves is 10 m

$$v = f\lambda = \frac{9}{10} \times 10 = 9 \text{ m/s}$$

2. 0.75

For propagation of sound in liquid

$$v = \sqrt{B/\rho}, \quad \text{i.e.,} \quad B = v^2\rho$$

But by definition $B = -V \frac{\Delta P}{\Delta V} \Rightarrow -V \frac{\Delta P}{\Delta V} = v^2\rho,$

$$\text{i.e.,} \quad \Delta V = \frac{V(-\Delta P)}{\rho v^2}$$

Here $\Delta P = H_2\rho g - H_1\rho g = (75 - 200) \times 13.6 \times 981$
 $= -1.667 \times 10^6 \text{ dynes/cm}^2$

$$\text{So } \Delta V = \frac{6 \times 10^3 \cdot 1.667 \times 10^6}{0.81 \times 1.280 \times 10^5} = 0.75 \text{ cc}$$

3. 1328

The velocity of sound in air is given by

$$v = \sqrt{\frac{\gamma P}{\rho}}$$

In terms of density and pressure

$$\frac{v_H}{v_{air}} = \sqrt{\frac{P_H}{\rho_H} \times \frac{\rho_{air}}{P_{air}}} = \sqrt{\frac{\rho_{air}}{\rho_H}} \quad [\text{as } P_{air} = P_H]$$

$$\text{or } v_H = v_{air} \times \sqrt{\frac{\rho_{air}}{\rho_H}} = 332 \times \sqrt{\frac{16}{1}} = 1328 \text{ m/s}$$

4. 0.12

Tension at the lower end, $T_1 = 2g \text{ N}$,

Tension at the upper end, $T_2 = 8g \text{ N}$

If v_1 and v_2 are the speeds at the lower and the upper ends,
 respectively, then

$$\frac{v_2}{v_1} = \sqrt{\frac{T_2}{T_1}} = \sqrt{\frac{8}{2}} = 2$$

Further, if λ_1 and λ_2 are the wavelength at the lower and
 the upper ends, respectively, then, since the frequency is
 constant,

$$\frac{\lambda_2}{\lambda_1} = \frac{v_2}{v_1} = 2$$

or $\lambda_2 = 2\lambda_1 = 2 \times 0.06 = 0.12 \text{ m}$

5. 3

The situation is shown in figure. Suppose the detector is
 placed at a distance of x metre from the sources. The wave
 received from the source after reflection from the wall has
 traveled a distance of $2[(2)^2 - x^2/4]^{1/2}$ metre. The difference

between the two waves is $\Delta \left\{ 2 \left[(2)^2 - \frac{x^2}{4} \right]^{1/2} - x \right\}$ metre.

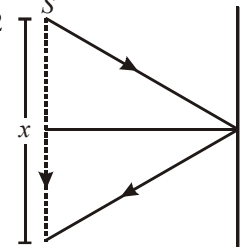
Constructive interference will take place when $\Delta = \lambda, 2\lambda, \dots$

The minimum distance x for a maximum corresponds to

$$\Delta = \lambda \quad \dots(i)$$

The wavelength is $\lambda = \frac{u}{v} = \frac{360 \text{ m/s}}{180 \text{ s}^{-1}} = 2 \text{ m}$

Thus, by (i), $2 \left[(2)^2 + \frac{x^2}{4} \right]^{1/2} - x = 2$



or $\left[4 + \frac{x^2}{4} \right]^{1/2} = 1 + \frac{x}{2}$

or $4 + \frac{x^2}{4} = 1 + \frac{x^2}{4} + x$ or $3 = x$

Thus, the detector should be placed at a distance of **3m** from the sources. Note that there is no abrupt phase change.

6. 175

From the law of length of stretched string, we have

$$n_1 \ell_1 = n_2 \ell_2 = n_3 \ell_3$$

Here $n_1 : n_2 : n_3 = 1 : 3 : 15$

$$\therefore \frac{\ell_1}{\ell_2} = \frac{n_2}{n_1} = \frac{3}{1} \text{ and } \frac{\ell_1}{\ell_3} = \frac{n_3}{n_1} = \frac{15}{1}$$

$$\ell_2 = \frac{\ell_1}{3} \text{ and } \ell_3 = \frac{\ell_1}{15}$$

The total length of the wire is 105 cm

Therefore, $\ell_1 + \ell_2 + \ell_3 = 105$

$$\text{or } \ell_1 + \frac{\ell_1}{3} + \frac{\ell_1}{15} = 105 \text{ or } \frac{21\ell_1}{15} = 105$$

$$\ell_1 = \frac{105 \times 15}{21} = 75 \text{ cm } \therefore \ell_2 = \frac{\ell_1}{3} = \frac{75}{3} = 25 \text{ cm } ; \ell_3 = \frac{\ell_1}{15}$$

$$= \frac{75}{15} = 5 \text{ cm}$$

Hence the bridge should be placed at 75 cm and $(75 + 25) = 100 \text{ cm}$ from one end.

\therefore Required sum = $75 + 100 = \mathbf{175 \text{ cm}}$

7. 1650

As diaphragm C is a node, A and B will be antinode (as in an organ pipe either both ends are antinode or one end node and the other antinode), i.e., each part will behave as closed end organ pipe so that

$$f_H = \frac{v_H}{4L_H} = \frac{1100}{4 \times 0.5} = 550 \text{ Hz}$$

$$\text{and } f_0 = \frac{v_0}{4L_0} = \frac{300}{4 \times 0.5} = 150 \text{ Hz}$$

As the two fundamental frequencies are different, the system will vibrate with a common frequency f such that

$$f = n_H f_H = n_0 f_0 \text{ i.e., } \frac{n_H}{n_0} = \frac{f_0}{f_H} = \frac{150}{550} = \frac{3}{11}$$

i.e., the third harmonic of hydrogen and 11th harmonic of oxygen or 6th harmonic of hydrogen and 22nd harmonic of oxygen will have same frequency. So the minimum common frequency

$$f = 3 \times 550 = \mathbf{1650 \text{ Hz}}$$

8. 110

Let the frequency of the tuning fork be $n \text{ Hz}$

Then frequency of air column at $15^\circ\text{C} = n + 4$

Frequency of air column at $10^\circ\text{C} = n + 3$

According to $v = n\lambda$, we have

$$v_{15} = (n + 4)\lambda \text{ and } v_{10} = (n + 3)\lambda$$

$$\therefore \frac{v_{15}}{v_{10}} = \frac{n + 4}{n + 3}$$

The speed of sound is directly proportional to the square-root of the absolute temperature.

$$\therefore \frac{v_{15}}{v_{10}} = \sqrt{\frac{15 + 273}{10 + 273}} = \sqrt{\frac{288}{283}}$$

$$\therefore \frac{n + 4}{n + 3} = \sqrt{\frac{288}{283}} = \left(1 + \frac{5}{283} \right)^{1/2} \Rightarrow 1 + \frac{1}{n + 3} = 1 + 1/2 \times$$

$$\frac{5}{283} = 1 + \frac{5}{566}$$

$$\Rightarrow \frac{1}{n + 3} = \frac{5}{566} \Rightarrow n + 3 = 113 \Rightarrow n = \mathbf{110 \text{ Hz}}$$

9. 27

$$n \text{ (fundamental frequency of the string)} = \frac{1}{2\ell} \sqrt{\frac{T}{m}}$$

$$\text{or } n = \frac{1}{2 \times 0.25} \sqrt{\frac{T}{10^{-2}}} = 20\sqrt{T}$$

The fundamental frequency of a closed pipe $n' = \frac{c}{4\ell}$

$$\therefore n' = \frac{320}{4 \times 0.40} = 200 \text{ Hz}$$

The frequency of the first overtone of the string

$$= 2n = 40\sqrt{T}$$

Since there are 8 beats/second, $2n - n' = 8$ or $40\sqrt{T} - 200 = 8$

Since on decreasing the tension, the beat frequency decreases,

$2n$ is definitely greater than n' .

$$\therefore 40\sqrt{T} - 200 = 8 \text{ or } T = \mathbf{27 \text{ N}}$$

10. 600

According to Doppler's effect, the apparent frequency when both source and observer move along the same direction is

$$n' = \frac{(v + w) - v_0}{(v + w) - v_s}$$

Velocity of observer $v_0 = 0 \quad \therefore n' = \frac{(v + w)}{v + w - v_s} n$

Given $v = 1200$ km/hr, $w = 40$ km/hr, $v_s = 40$ km/hr. and $n = 580$ Hz

$$\therefore n' = \frac{1200 + 40}{(1200 + 40) - 40} \times 580 = 599.33 \text{ Hz} = \mathbf{600 \text{ Hz}}$$

11. 0.11

For interference at $A : S_2$ is behind of S_1 by a distance of

$$100\lambda \frac{\lambda}{4} \text{ (equal to phase difference } \pi/2).$$

Further S_2 lags S_1 by $\pi/2$. Hence the waves from S_1 and S_2 interference at A with a phase difference of $200.5\pi + 0.5\pi = 201\pi = \pi$.

Hence the net amplitude at A is $2a - a = a$

For interference at $B : S_2$ is ahead of S_1 by a distance of

$$100\lambda \frac{\lambda}{4} \text{ (equal to phase difference } \frac{\pi}{2}).$$

Further S_2 lags S_1 by $\frac{\pi}{2}$. Hence the waves from S_1 and S_2

interference at B with a phase difference of

$$200.5\pi - 0.5\pi = 200\pi = 0\pi.$$

Hence the net amplitude at A is $2a + a = 3a$.

$$\text{Hence } \left(\frac{I_A}{I_B}\right) \left(\frac{a}{3a}\right)^2 \frac{1}{9} = \mathbf{0.11}$$

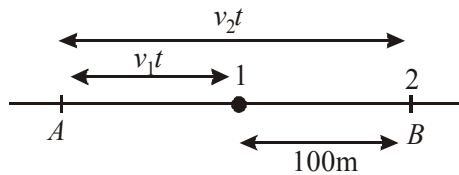
12. 4

Let the velocities of car 1 and car 2 be v_1 m/s and v_2 m/s

\therefore Apparent frequencies of sound emitted by car 1 and car 2 as detected at end point are

$$f_1 = f_0 \frac{v}{v - v_1}, \quad f_2 = f_0 \frac{v}{v - v_2}$$

$$\Rightarrow 330 = 300 \frac{330}{330 - v_1}; \quad 360 = 300 \frac{330}{330 - v_2}$$



$$\Rightarrow v_1 = 30 \text{ m/s} \quad \text{and} \quad v_2 = 100 \text{ m/s}$$

The distance between both the cars just when the 2nd car reaches point B (as shown in figure is)

$$100\text{m} = v_2 t - v_1 t \Rightarrow t = \mathbf{4 \text{ sec}}$$

13. 70

Thermal stress = $\gamma\alpha\Delta\theta$

$$= 1.3 \times 10^{11} \times 1.7 \times 10^{-5} \times 20 = 44.2 \times 10^6$$

$$\text{speed} = \sqrt{\frac{T}{m}} = \sqrt{\frac{T}{dAL/\ell}}$$

$$= \sqrt{\frac{T}{dA}} = \sqrt{\frac{\text{Thermal stress}}{\text{density}}}$$

$$= \sqrt{\frac{4.42 \times 10^7}{9 \times 10^3}} \approx \mathbf{70 \text{ m/s}}$$

14. 0.075

Mass per unit length of sonometer wire,

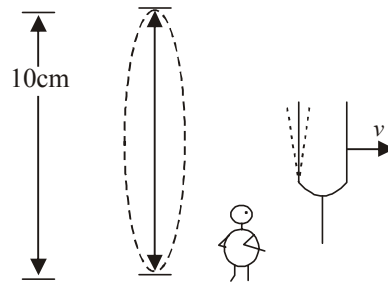
$$\mu = \frac{m}{\ell} = \frac{0.001}{0.1} = 0.01 \text{ kg/m}$$

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{64}{(0.01)^2}} \frac{8}{0.01} = 8 \times 100$$

$$\text{Also, } \frac{\lambda}{2} = 0.1 \Rightarrow \lambda = 0.2$$

$$\therefore f = \frac{v}{\lambda} = \frac{8 \times 100}{0.2} = 4000 \text{ Hz}$$

Since tuning fork is in resonance, therefore, frequency of tuning fork is 4 Hz. The observer is hearing one beat per second when the tuning fork is moved away with a constant speed b .



The frequency of tuning fork as heard by the observer standing stationary near sonometer wire can be found with the help of Doppler effect.

$$v' = v \left[\frac{c - v_0}{c - v_s} \right] = \frac{vc}{c - v_s} \quad v_0 = 0 \text{ m/s}$$

$$\therefore v' = \frac{4000 \times 300}{300 - v_s}$$

Since the beat frequency is 1 and as the tuning fork is going away from the observer, its apparent frequency is (normal frequency - 1)

$$= 4000 - 1 = 3999$$

$$\therefore 3999 = \frac{4000 \times 300}{300 v_s}$$

$$\Rightarrow 3999 \times 300 + 3999 v_s = 4000 \times 300$$

$$\Rightarrow v_s = \mathbf{0.075 \text{ m/s}}$$

15. 11

$$\Delta \ell = \ell \alpha \times \Delta T$$

$$\text{Also, } Y = \frac{\text{stress}}{\text{strain}} \quad \therefore Y = \frac{T/A}{\Delta \ell / \ell}$$

$$\therefore Y = \frac{T}{A} \times \frac{\ell}{\Delta \ell} = \frac{T}{A} \times \frac{\ell}{\ell \alpha \Delta T} = \frac{T}{A \alpha \Delta T}$$

$$\therefore T = YA \alpha \Delta T$$

The frequency of the fundamental mode of vibration.

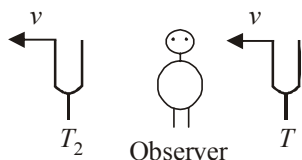
$$v = \frac{1}{2\ell} \sqrt{\frac{T}{m}} = \frac{1}{2\ell} \sqrt{\frac{YA \alpha \Delta T}{m}}$$

$$= \frac{1}{2 \times 1} \sqrt{\frac{2 \times 10^{11} \times 10^{-6} \times 1.21 \times 10^{-5} \times 20}{0.1}}$$

$$= \mathbf{11 \text{ Hz}}$$

16. 1.5

The apparent frequency from tuning fork T_1 as heard by the observer will be



$$v_1 = \frac{c}{c-v} \times v \quad \dots (i)$$

The apparent frequency from tuning fork T_2 as heard by the observer will be

$$v_2 = \frac{c}{c-v} \times v \quad \dots (ii)$$

Given $v_1 - v_2 = 3$

$$\therefore c \times v \left[\frac{1}{c-v} - \frac{1}{c-v} \right] = 3 \quad c v \left[\frac{c+v-c}{c^2-v^2} \right]$$

$$\Rightarrow 3 = \frac{c \times v \times 2v}{c^2 - v^2}$$

Since, $v \ll c$

$$\therefore 3 = \frac{c \times v \times 2v}{c^2}$$

$$\Rightarrow v = \frac{3 \times c^2}{c \times v \times 2}$$

$$\therefore v = \frac{3 \times 340 \times 340}{340 \times 340 \times 2} = \mathbf{1.5 \text{ m/s}}$$

17. 336

$$(\ell = 0.6r) \frac{\lambda}{4} = \frac{v}{4f}$$

$$\Rightarrow v = 4f (\ell = 0.6r) = \mathbf{336 \text{ m/s}}$$

18. 30

Let v be the actual frequency of the whistle. By Doppler's effect

$$v' = v \frac{v_s}{v_s - v_t}$$

where v_s = Speed of sound = 300 m/s (given)

$$v' = 2.2 \text{ kHz} = 2200 \text{ Hz (given)}$$

$$\therefore 2200 = v \frac{300}{300 - v_t} \quad \dots (i)$$

WHILE THE TRAIN IS RECEDING

$$v'' = v \frac{v_s}{v_s + v_t}$$

Here, $v'' = 1.8 \text{ kHz} = 1800 \text{ Hz (given)}$

$$\therefore 1800 = v \frac{300}{300 + v_t} \quad \dots (ii)$$

Dividing (i) and (ii)

$$\frac{2200}{1800} = \frac{300}{300 - v_t} \times \frac{300 + v_t}{300}$$

$$\Rightarrow \frac{11}{9} = \frac{300 + v_t}{300 - v_t}$$

$$\Rightarrow 3300 - 11v_t = 2700 = 9v_t$$

$$\Rightarrow 600 = 20v_t$$

$$\Rightarrow v_t = \mathbf{30 \text{ m/s}}$$

19. 336

The smallest resonating length ℓ_1 , corresponds to the fundamental mode. The diameter D , of the pipe is not very small compared to the resonating length ℓ_1 . So one should account for the end correction which is generally taken to be in the range of $0.29 D$ to $D/3$. Taking the end correction to be $0.3 D$.

$$\ell_1 = 0.3D + \frac{\lambda}{4}; \text{ where } \lambda \text{ is the wavelength of the}$$

fundamental mode. Then $\lambda = 70 \text{ cm}$ and the velocity of sound = $\mathbf{336 \text{ m/s}}$

