6. Algebraic Expressions and Identities

Exercise 6.1

1. Question

Indetify the terms, their coeffcients for each of the following expressions.

(i) $7x^2yz - 5xy$

(ii) $x^2 + x + 1$

(iii) $3x^2y^2 - 5x^2y^2z^2 + z^2$

(iv) $9 - ab + bc - ca$

(v) $\frac{a}{2} + \frac{b}{2} - ab$

(vi) $0.2x - 0.3xy + 0.5y$

Answer

(i) $7x^2yz - 5xy$

This equation consists of two terms that are:

$7x^2yz$ and - 5xy

The coefficient of $7x^2yz$ is 7

The coefficient of – 5xy is – 5

(ii) $x^2 + x + 1$

This equation consists of three terms that are:

x^2, x, 1

The coefficient of x^2 is 1

The coefficient of x is 1

The coefficient of 1 is 1

(iii) $3x^2y^2 - 5x^2y^2z^2 + z^2$

This equation consists of three terms that are:

$3x^2y$, $-5x^2y^2z^2$ and z^2

The coefficient of $3x^2y$ is 3

The coefficient of $-5x^2y^2z^2$ is -5

The coefficient of z^2 is 1

(iv) $9 - ab + bc - ca$

The term	Coefficient
9	9
-ab	-1
bc	1
ca	-1

(v) $\frac{a}{2} + \frac{b}{2} - ab$

The term	Coefficient
$\frac{a}{2}$	1
$\frac{b}{2}$	1
-ab	1

(vi) $0.2x - 0.3xy + 0.5y$

The term	Coefficient
0.2x	0.2
-0.3xy	-0.3
0.5y	0.5

2. Question

Classify the following polynomials as monomials, binomials, trinomials. Which polynomials do not fit in any category?

(i) x+y

(ii) 1000

(iii) $x+x^2+x^3+x^4$

(iv)7+a+5b

(v) $2b-3b^2$

(vi) $2y-3y^2+4y^3$

(vii) 5x-4y+3x

(viii) $4a-15a^2$

(ix) xy+yz+zt+tx

(x) pqr

(xi) p^2q+pq^2

(xii) 2p+2q

Answer

(i) x+y

This expression contains two terms x and y

So, it is called ‘Binomial’

(ii) 1000

It contains one term 1000

So, it is called monomial

(iii) $x+x^2+x^3+x^4$

It contains four terms

So, it is not a monomial, binomial and trinomial

(iv)7+a+5b

It contains three terms

So, it is called trinomial

(v) $2b-3b^2$

It contains two terms

So, it is called binomial

(vi) $2y-3y^2+4y^3$

It contains three terms

So, it is called trinomial

(vii) 5x-4y+3x

8x – 4y

It contains two terms

So, it is called binomial

(viii) $4a-15a^2$

It contains two terms

So, it is called binomial

(ix) xy+yz+zt+tx

It contains four terms

So, it is not a monomial, binomial and trinomial

(x) pqr

It contains one term

So, it is called monomial

(xi) p^2q+pq^2

It contains two terms

So, it is called binomial

(xii) 2p+2q

It contains two terms

So, it is called monomial

Exercise 6.2

1. Question

Add the following algebraic expressions:

(i) $3a^2b, -4a^2b, 9a^2b$

(ii) $\frac{2}{3}a, \frac{3}{5}a, -\frac{6}{5}a$

(iii) $4xy^2 - 7x^2y, 12x^2y - 6xy^2, -3x^2y + 5xy^2$

(iv) $\frac{3}{2}a - \frac{5}{4}b + \frac{2}{5}c, \frac{2}{3}a - \frac{7}{2}b + \frac{7}{2}c, \frac{5}{3}a + \frac{5}{2}b - \frac{5}{4}c$

(v) $\frac{11}{2}xy + \frac{12}{5}y + \frac{13}{7}x, -\frac{11}{2}y - \frac{12}{5}x - \frac{13}{7}xy$

(vi) $\frac{7}{2}x^3 - \frac{1}{2}x^2 + \frac{5}{3}, \frac{3}{2}x^3 + \frac{7}{4}x^2 - x + \frac{1}{3}, \frac{3}{2}x^2 - \frac{5}{2}x - 2$

Answer

(i) $3a^2b$, $-4a^2b$, $9a^2b$

$= 3a^2b + (-4a^2b) + 9a^2b$

$= 3a^2b - 4a^2b + 9a^2b$

$= 3a^2b$

(ii) $\frac{2}{3}a, \frac{3}{5}a, -\frac{6}{5}a$

$= \frac{2}{3}a + \frac{3}{5}a - \frac{6}{5}a$

Taking L.C.M 3, 5 , 5 is 15

$= 11\frac{10a+9a-18a}{15}$

$= \frac{19a-18a}{15}$

$= \frac{a}{15}$

(iii) $4xy^2 - 7x^2y, 12x^2y - 6xy^2, -3x^2y + 5xy^2$

$= 4xy^2 - 7x^2y + 12x^2y - 6xy^2 - 3x^2y + 5xy^2$

$= 4x^2 + 12x^2y - 3x^2y - 7x^2y - 6xy^2 + 5xy^2$

$= 3xy^2 + 2x^2y$

(iv) $\frac{3}{2}a - \frac{5}{4}b + \frac{2}{5}c, \frac{2}{3}a - \frac{7}{2}b + \frac{7}{2}c, \frac{5}{3}a + \frac{5}{2}b - \frac{5}{4}c$

Adding all, we get

$= \frac{3}{2}a - \frac{5}{4}b + \frac{2}{5}c + \frac{2}{3}a - \frac{7}{2}b + \frac{7}{2}c + \frac{5}{3}a + \frac{5}{2}b - \frac{5}{4}c$

$= \frac{9a+4a+10a}{6} + \frac{-5b-14b+10b}{4} + \frac{8c+70c-25c}{20}$

$= \frac{23a}{6} - \frac{9b}{4} + \frac{53c}{20}$

(v) $\frac{11}{2}xy + \frac{12}{5}y + \frac{13}{7}x, -\frac{11}{2}y - \frac{12}{5}x - \frac{13}{7}xy$

Adding all, we get

$= \frac{11}{2}xy + \frac{11}{5}y + \frac{13}{7}x + \frac{-11}{2}y - \frac{12}{5}x - \frac{13}{7}xy$

$= \frac{11xy-12xy}{14} + \frac{24y-55y}{10} + \frac{65x-84x}{35}$

$= \frac{51xy}{14} - \frac{19x}{35} - \frac{31y}{10}$

(vi) $\frac{7}{2}x^3 - \frac{1}{2}x^2 + \frac{5}{3}, \frac{3}{2}x^3 + \frac{7}{4}x^2 - x + \frac{1}{3}, \frac{3}{2}x^2 - \frac{5}{2}x - 2$

Adding all, we get

$= \frac{7}{2}x^3 - \frac{1}{2}x^2 + \frac{5}{3} + \frac{3}{2}x^3 + \frac{7}{4}x^2 - x + \frac{1}{3} + \frac{3}{2}x^2 - \frac{5}{2}x - 2$

$= \frac{10}{2}x^3 + \frac{11}{4}x^2 - \frac{7x}{2} + \frac{0}{6}$

$= 5x^3 + \frac{11}{4}x^2 - \frac{7x}{2}$

2. Question

Subtract:

(i) -5xy from 12xy

(ii) $2a^2$ from $-7a^2$

(iii) 2a-b from 3a-5b

(iv) $2x^3 - 4x^2 + 3x + 5$ from $4x^3 + x^2 + x + 6$

(v) $\frac{3}{2}y^3 - \frac{2}{7}y^2 - 5$ from $\frac{1}{3}y^3 + \frac{5}{7}y^2 + y - 2$

(vi) $\frac{3}{2}x - \frac{5}{4}y - \frac{7}{2}z$ from $\frac{2}{3}x + \frac{3}{2}y - \frac{4}{3}z$

(vii) $x^2 - \frac{4}{5}xy^2 + \frac{4}{3}xy$ from $\frac{2}{3}x^2y + \frac{3}{2}xy^2 - \frac{1}{3}xy$

(viii) $\frac{ab}{7} - \frac{35}{3}bc + \frac{6}{5}ac$ from $\frac{3}{5}bc - \frac{4}{5}ac$

Answer

(i) -5xy from 12xy

After subtracting,we get

= 12xy - (- 5xy)

= 5xy + 12xy

= 17xy

(ii) $2a^2$ from $-7a^2$

After subtracting, we get

= $2a^2 + (-7a^2)$

= $-2a^2 + 7a^2$

= $-9a^2$

(iii) 2a-b from 3a-5b

After subtracting, we get

= -(2a – b)+ (3a – 5b)

= -2a + b+ 3a – 5b

= a – 4b

(iv) $2x^3 - 4x^2 + 3x + 5$ from $4x^3 + x^2 + x + 6$

After subtracting, we get

= - $(2x^3 - 4x^2 + 3x + 5) + (4x^3 + x^2 + x + 6)$

= - $2x^3 + 4x^2 - 3x - 5 + 4x^3 + x^2 + x + 6$

= $2x^3 + 5x^2 - 2x + 1$

(v) $\frac{3}{2}y^3 - \frac{2}{7}y^2 - 5$ from $\frac{1}{3}y^3 + \frac{5}{7}y^2 + y - 2$

After subtracting, we get

= $\frac{1}{3}y^2 + \frac{5}{7}y^2 + y - 2 - \frac{2}{3}y^3 + \frac{2}{7}y^2 + 5$

$= \frac{-1}{3}y^3 + \frac{7}{7}y^2 + y + 3$

$= \frac{-1}{3}y^3 + y^2 + y + 3$

(vi) $\frac{3}{2}x - \frac{5}{4}y - \frac{7}{2}z$ from $\frac{2}{3}x + \frac{3}{2}y - \frac{4}{3}z$

After subtracting, we get

$= \frac{2}{3}x + \frac{3}{2}y - \frac{4}{3}z - (\frac{3}{2}x - \frac{5}{4}y - \frac{7}{2}z)$

$= \frac{2}{3}x - \frac{3}{2}x + \frac{3}{2}y + \frac{5}{4}y - \frac{4}{3}z + \frac{7}{2}z$

$= \frac{4x-9x}{6} + \frac{6y+5y}{4} + \frac{-8z+21z}{6}$

$= \frac{-5x}{6} + \frac{11y}{4} + \frac{13z}{6}$

(vii) $x^2 - \frac{4}{5}xy^2 + \frac{4}{3}xy$ from $\frac{2}{3}x^2y + \frac{3}{2}xy^2 - \frac{1}{3}xy$

$= \frac{2}{3}x^2y + \frac{3}{2}xy^2 - \frac{1}{3}xy - (x^2 - \frac{4}{5}xy^2 + \frac{4}{3}xy)$

$= \frac{2}{3}x^2y - x^2y + \frac{3}{2}xy^2 + \frac{4}{5}xy^2 - \frac{1}{3}xy - \frac{4}{3}xy$

$= \frac{-1}{3}x^2y + \frac{25}{10}xy^2 - \frac{5}{3}xy$

(viii) $\frac{ab}{7} - \frac{35}{3}bc + \frac{6}{5}ac$ from $\frac{3}{5}bc - \frac{4}{5}ac$

After subtracting, we get

$= \frac{3}{5}bc - \frac{4}{5}ac - (\frac{ab}{7} - \frac{35}{3}bc + \frac{6}{5}ac)$

$= \frac{3}{5}bc + \frac{35}{3}bc - \frac{4}{5}ac - \frac{6}{5}ac - \frac{ab}{7}$

$= \frac{9bc+175bc}{15} + \frac{-4ac-6ac}{5} - \frac{ab}{7}$

$= \frac{184bc}{15} + \frac{-10ac}{5} - \frac{ab}{7}$

$= \frac{184bc}{15} - \frac{10ac}{5} - \frac{ab}{7}$

$= \frac{184bc}{15} - 2ac - \frac{ab}{7}$

3. Question

Take away:

(i) $\frac{6}{5}x^2 - \frac{4}{5}x^3 + \frac{5}{6} + \frac{3}{2}x$ from $\frac{x^3}{3} - \frac{5}{2}x^2 + \frac{3}{5}x + \frac{1}{4}$

(ii) $\frac{5a^2}{2} + \frac{3a^3}{2} + \frac{a}{3} - \frac{6}{5}$ from $\frac{1}{3}a^3 - \frac{3}{4}a^2 - \frac{5}{2}$

(iii) $\frac{7}{4}x^3 + \frac{3}{5}x^2 + \frac{1}{2}x + \frac{9}{2}$ from $\frac{7}{2} - \frac{x}{3} - \frac{x^2}{5}$

(iv) $\frac{y^3}{3} + \frac{7}{3}y^2 + \frac{1}{2}y + \frac{1}{2}$ from $\frac{1}{3} - \frac{5}{3}y^2$

(v) $\frac{2}{3}ac - \frac{5}{7}ab + \frac{2}{3}bc$ from $\frac{3}{2}ab - \frac{7}{4}ac - \frac{5}{6}bc$

Answer

(i) $\frac{6}{5}x^2 - \frac{4}{5}x^3 + \frac{5}{6} + \frac{3}{2}x$ from $\frac{x^3}{3} - \frac{5}{2}x^2 + \frac{3}{5}x + \frac{1}{4}$

$= \frac{1}{3}x^3 - \frac{5}{2}x^2 + \frac{3}{5}x + \frac{1}{4} - (\frac{6}{5}x^2 - \frac{4}{5}x^3 + \frac{5}{6} + \frac{3}{2}x)$

$= \frac{1}{3}x^3 + \frac{4}{5}x^3 - \frac{5}{2}x^2 - \frac{6}{5}x^2 + \frac{3}{5}x - \frac{3}{2}x + \frac{1}{4} - \frac{5}{6}$

$= \frac{17}{15}x^3 - \frac{37}{10}x^2 - \frac{9x}{10} - \frac{14}{24}$

$= \frac{17}{15}x^3 - \frac{37}{10}x^2 - \frac{9}{10}x - \frac{7}{12}$

(ii) $\frac{5a^2}{2} + \frac{3a^3}{2} + \frac{a}{3} - \frac{6}{5}$ from $\frac{1}{3}a^3 - \frac{3}{4}a^2 - \frac{5}{2}$

$= \frac{1}{3}a^3 - \frac{3}{4}a^2 - \frac{5}{2} - (\frac{5}{2}a^2 + \frac{3}{2}a^2 + \frac{a}{3} - \frac{6}{5})$

$= \frac{1}{3}a^5 - \frac{3}{2}a^3 - \frac{3}{4}a^2 - \frac{5}{2}a^2 - \frac{a}{3} - \frac{5}{2} + \frac{6}{5}$

$= \frac{1}{6}(2a^3 - 9a^3) - \frac{1}{4}(3a^2 - 10a^2) - \frac{a}{3} + \frac{-25+12}{10}$

$= \frac{-7}{6}a^3 - \frac{13}{4}a^2 - \frac{a}{3} - \frac{13}{10}$

(iii) $\frac{7}{4}x^3 + \frac{3}{5}x^2 + \frac{1}{2}x + \frac{9}{2}$ from $\frac{7}{2} - \frac{x}{3} - \frac{x^2}{5}$

$= \frac{7}{2} - \frac{x}{3} - \frac{1}{5}x^2 - (\frac{7}{4}x^3 + \frac{3}{5}x^2 + \frac{1}{2}x + \frac{9}{2})$

$= \frac{-7}{4}x^3 - \frac{1}{5}x^2 - \frac{3}{5}x^2 - \frac{x}{3} - \frac{1}{2}x + \frac{7}{2} - \frac{9}{2}$

$= \frac{-7}{4}x^3 - \frac{4}{5}x^2 - \frac{2x-3x}{6} + \frac{7-9}{2}$

$= \frac{-7}{4}x^3 - \frac{4}{5}x^2 - \frac{5x}{6} - 1$

(iv) $\frac{y^3}{3} + \frac{7}{3}y^2 + \frac{1}{2}y + \frac{1}{2}$ from $\frac{1}{3} - \frac{5}{3}y^2$

$= \frac{1}{3} - \frac{5}{3}y^2 - (\frac{1}{3}y^3 + \frac{7}{3}y^2 + \frac{1}{2}y + \frac{1}{2})$

$= \frac{-1}{3}y^3 - \frac{5}{3}y^2 - \frac{7}{3}y^2 - \frac{1}{2}y + \frac{1}{3} - \frac{1}{2}$

$= \frac{-1}{3}y^3 + \frac{1}{3}(-5y^2 - 7y^2) - \frac{1}{2}y + \frac{2-3}{2}$

$= \frac{-1}{3}y^3 - \frac{12}{3}y^2 - \frac{1}{2}y - \frac{1}{2}$

(v) $\frac{2}{3}ac - \frac{5}{7}ab + \frac{2}{3}bc$ from $\frac{3}{2}ab - \frac{7}{4}ac - \frac{5}{6}bc$

$= \frac{3}{2}ab - \frac{7}{4}ac - \frac{5}{6}bc - (\frac{2}{3}ac - \frac{5}{7}ab + \frac{2}{3}bc)$

$= \frac{3}{2}ab - \frac{5}{7}ab - \frac{7}{4}ac - \frac{2}{3}ac - \frac{5}{6}bc - \frac{2}{3}bc$

$= \frac{21ab-10ab}{14} - \frac{21ac-8ac}{12} - \frac{5bc-4bc}{b}$

$= \frac{11}{14}ab - \frac{29}{12}ac - \frac{3}{2}bc$

4. Question

Subtract 3x-4y-7z from the sum of x-3y+2z and -4x+9y-11z.

Answer

The sum of x – 3y + 2z and -4x + 9y – 11z is calculated as below:

= (x – 3y + 2z) + (-4x + 9y – 11z)

= x – 4x – 3y + 9y + 2z – 11z

= -3x + 6y -9z

Now, The expression 3x- 4y -7z has to be subtracted from the resultant expression i.e. -3x + 6y -9z

= (-3x + 6y -9z) – (3x – 4y – 7z)

= -3x – 3x + 6y + 4y – 9z + 7z

= -6x + 10y – 2z

5. Question

Subtract the sum of $3l-4m-7n^2$ and $2l+3m-4n^2$ from the sum of $9l+2m-3n^2$ and $-3l+m+4n^2$.....

Answer

Subtract the sum of $3l-4m-7n^2$ and $2l+3m-4n^2$ from the sum of $9l+2m-3n^2$ and $-3l+m+4n^2$.....

Sum of $9l + 2m – 3n^2$ and $-3l + n + 4n^2$

$= 9l + 2m – 3n^2 + (-3l + m + 4n^2)$

$= 9l – 3l + 2m + m – 3n^2 + 4n^2$

$= 6l + 3m + n^2$ (i)

Sum of $3l – 4m – 7n^2$ and $2l + 5m – 4n^2$

$= 3l – 4m – 7n^2 + 2l + 5m – 4n^2$

$= 5l – m – 11n^2$ (ii)

Subtract (i) and (ii), we get

$= 6l + 3m + n^2 – (5l – m – 11n^2)$

$= 6l – 5l + 3m + m + n^2 + 12n^2$

$= l + 4m + 13n^2$

6. Question

Subtract the sum of $2x-x^2+5$ and $-4x-3+7x^2$ from 5.

Answer

As given in the question, the Sum of $2x – x^2 + 5$ and $-4x – 3 + 7x^2$ is given as:

$= 2x – x^2 + 5 – 4x – 3 + 7x^2$

$= 2x – 4x – x^2 + 7x^2 + 5 – 3$

$= -2x + 6x^2 + 2$ (i)

Now subtracting equation (i) from 5 we get,

Subtracting (ii) from (i), we get

$= 5 - (-2x + 6x^2 + 2)$

$= 5 + 2x - 6x^2 - 2$

$= 3 + 2x - 6x^2$

Therefore, the resultant expression is $3 + 2x - 6x^2$

7. Question

Simplify each of the following:

(i) $x^2 - 3x + 5 - \frac{1}{2}(3x^2 - 5x + 7)$

(ii) $[5 - 3x + 2y - (2x - y)] - (3x - 7y + 9)$

(iii) $\frac{11}{2}x^2y - \frac{9}{4}xy^2 + \frac{1}{4}xy - \frac{1}{14}y^2x + \frac{1}{15}yx^2 + \frac{1}{2}xy$

(iv) $\left(\frac{1}{3}y^2 - \frac{4}{7}y + 11\right) - \left(\frac{1}{7}y - 3 + 2y^2\right) - \left(\frac{2}{7}y - \frac{2}{3}y^2 + 2\right)$

(v) $-\frac{1}{2}a^2b^2c + \frac{1}{3}ab^2c - \frac{1}{4}abc^2 - \frac{1}{5}cb^2a^2 + \frac{1}{6}cb^2a - \frac{1}{7}c^2ab + \frac{1}{8}ca^2b.$

Answer

(i) $x^2 - 3x + 5 - \frac{1}{2}(3x^2 - 5x + 7)$

$= x^2 - \frac{1}{2}3x^2 - 3x + \frac{1}{2}5x + 5 - \frac{1}{2}7$

$= \frac{1}{2}(2x^2 - 3x^2) - \frac{1}{2}(6x + 5x) + \frac{10-7}{2}$

$= \frac{-1}{2}x^2 - \frac{x}{2} + \frac{3}{2}$

$= \frac{-1}{2}x^2 - \frac{1}{2}x + \frac{3}{2}$

(ii) $[5 - 3x + 2y - (2x - y)] - (3x - 7y + 9)$

$= 5 - 3x + 2y - 2x + y - 3x + 7y - 9$

$= - 8x + 10y - 4$

(iii) $\frac{11}{2}x^2y - \frac{9}{4}xy^2 + \frac{1}{4}xy - \frac{1}{14}y^2x + \frac{1}{15}yx^2 + \frac{1}{2}xy$

$= \frac{11}{2}x^2y + \frac{1}{15}x^2y - \frac{9}{4}xy^2 - \frac{1}{14}xy^2 + \frac{1}{4}xy + \frac{1}{2}xy$

$= \frac{1}{30}(165x^2y + 2x^2y) + \frac{1}{56}(-126xy^2 - 4xy^2) + \frac{xy+2xy}{4}$

$= \frac{167}{30}x^2y - \frac{130}{56}xy^2 + \frac{3}{4}xy$

$= \frac{167}{30}x^2y - \frac{65}{28}xy^2 + \frac{3}{4}xy$

(iv) $\left(\frac{1}{3}y^2 - \frac{4}{7}y + 11\right) - \left(\frac{1}{7}y - 3 + 2y^2\right) - \left(\frac{2}{7}y - \frac{2}{3}y^2 + 2\right)$

$= \frac{1}{3}y^2 - 2y^2 - \frac{2}{3}y^2 - \frac{4}{7}y - \frac{1}{7}y - \frac{2}{7}y + 11 + 3 - 2$

$= \frac{1}{3}(y^2 - 6y^2 + 2y^2) - \frac{1}{7}(4y - y - 2y) + 14 - 2$

$= \frac{-3}{3}y^2 - \frac{7}{7}y + 12$

$= -y^2 - y + 12$

(v) $-\frac{1}{2}a^2b^2c + \frac{1}{3}ab^2c - \frac{1}{4}abc^2 - \frac{1}{5}cb^2a^2 + \frac{1}{6}cb^2a - \frac{1}{7}c^2ab + \frac{1}{8}ca^2b.$

$= \frac{-1}{2}a^2b^2c - \frac{1}{5}a^2b^2c + \frac{1}{3}ab^2c + \frac{1}{6}ab^2c - \frac{1}{4}abc^2 - \frac{1}{7}abc^2 + \frac{1}{8}a^2bc$

$= \frac{-7}{10}a^2b^2c + \frac{1}{2}ab^2c - \frac{11}{28}abc^2 + \frac{1}{8}a^2bc$

Exercise 6.3

1. Question

Find each of the following products:

$5x^2 \times 4x^3$

Answer

$5 \times x \times x \times 4 \times x \times x \times x \times x$

$= 5 \times 4 \times x^5$

$= 20 \times x^5$

$= 20x^5$

2. Question

Find each of the following products:

$-3a^2 \times 4b^4$

Answer

$- 3 \times 4 - a^2 \times b^2$

$= -12 \times a^2 \times b^2$

$= -12a^2b^2$

3. Question

Find each of the following products:

$(-5xy) \times (-3x^2yz)$

Answer

$(-5) \times (-5) \times x \times x^2 \times y \times y \times z$

$= 15 \times x^3 \times y^2 \times z$

$= 15x^3y^2z$

4. Question

Find each of the following products:

$\frac{1}{2}xy \times \frac{2}{3}x^2yz^2$

Answer

$\frac{1}{2} \times \frac{2}{3} \times x \times x^2 \times y \times y \times z^2$

$= \frac{1}{3} \times x^3 \times y^2 \times z^2$

$= \frac{1}{3}x^3y^2z^2$

5. Question

Find each of the following products:

$\left(-\frac{7}{5}xy^2z\right) \times \left(\frac{13}{3}x^2yz^2\right)$

Answer

$\frac{-7}{5} \times \frac{13}{3} \times x \times x^2 \times y^2 \times y \times z \times z^2$

$= \frac{-91}{15} \times x^3 \times y^3 \times z^3$

$= \frac{-91}{15}x^3y^3z^3$

6. Question

Find each of the following products:

$\left(\frac{-24}{25}x^3z\right) \times \left(-\frac{15}{16}xz^2y\right)$

Answer

$\frac{-24}{25} \times \frac{-15}{16} \times x^3 \times x \times z \times z^2 \times y$

$= \frac{18}{20} \times x^4 \times z^3 \times y$

$= \frac{9}{10}x^4z^3y$

7. Question

Find each of the following products:

$\left(-\frac{1}{27}a^2b^2\right) \times \left(\frac{9}{2}a^3b^2c^2\right)$

Answer

$\frac{-1}{27} \times \frac{9}{2} \times a^2 \times a^3 \times b^2 \times b^2 \times c^2$

$= \frac{-1}{6} \times a^5 \times b^4 \times c^2$

$= \frac{-1}{6}a^5b^4c^2$

8. Question

Find each of the following products:

$(-7xy) \times \left(\frac{1}{4}x^2yz\right)$

Answer

$-7 \times \frac{1}{4} \times x \times y \times x^2 \times y \times z$

$= \frac{-7}{4} \times x3 \times y^2 \times z$

$= \frac{-7}{4}x^3y^2z$

9. Question

Find each of the following products:

$(7ab) \times (-5ab^2c) \times (6abc^2)$

Answer

$7 \times -5 \times 6 \times a \times a \times a \times b \times b^2 \times b \times c \times c^2$

$= 210 \times a^3 \times b^4 \times c^3$

$= 210a^3b^4c^3$

10. Question

Find each of the following products:

$(-5a) \times (-10a^2) \times (-2a^3)$

Answer

$(-5) \times (-10) \times (-2) \times a \times a^2 \times a^3$

$= -100 \times a^6$

$= -100a^6$

11. Question

Find each of the following products:

$(-4x^2) \times (-6xy^2) \times (-3yz^2)$

Answer

$(-4) \times (-6) - (-3) \times x^2 \times x \times y^2 \times y \times z^2$

$= -72 \times x^3 \times y^3 \times z^2$

$= -72x^3y^3z^2$

12. Question

Find each of the following products:

$\left(-\frac{2}{7}a^4\right) \times \left(-\frac{3}{4}a^2b\right) \times \left(-\frac{14}{5}b^2\right)$

Answer

$\frac{-2}{7} \times \frac{-3}{4} \times \frac{-14}{5} \times a \times a^2 \times b \times b^2$

$= \frac{-6}{10} \times a^6 \times b^3$

$= \frac{-3}{5}a^6b^3$

13. Question

Find each of the following products:

$\left(\frac{7}{9}ab^2\right) \times \left(\frac{15}{7}ac^2b\right) \times \left(-\frac{3}{5}a^2c\right)$

Answer

$\frac{7}{9} \times \frac{15}{7} \times \frac{-3}{5} \times a \times a \times a^2 \times b^2 \times b \times c^2 \times c$

$= - a^4 \times b^3 \times c^3$

$= -a^4b^3c^3$

14. Question

Find each of the following products:

$\left(\frac{4}{3}u^2vw\right) \times (-5uvw^2) \times \left(\frac{1}{3}v^2wu\right)$

Answer

$\frac{4}{3} \times -5 \times \frac{1}{3} \times u^2 \times u \times u \times v \times v \times v^2 \times w \times w^2 \times w$

$= \frac{-20}{9} \times u^4 \times v^4 \times w^4$

$= \frac{-20}{9}u^4v^4w^4$

15. Question

Find each of the following products:

$(0.5x) \times \left(\frac{1}{3}xy^2z^4\right) \times (24x^2yz)$

Answer

$0.5 \times \frac{1}{3} \times 24 \times x \times x \times y^2 \times y \times x^2 \times z^4 \times z$

$= \frac{12}{3} \times x^4 \times y^3 \times z^5$

$= 4x^4 \times y^3 \times z^5$

$= 4x^4y^3z^5$

16. Question

Find each of the following products:

$\left(\frac{4}{3}pq^2\right) \times \left(-\frac{1}{4}p^2r\right) \times (16p^2q^2r^2)$

Answer

$\frac{4}{3} \times \frac{1}{4} \times 16 \times p \times p^2 \times p^2 \times q^2 \times q^2 \times r \times r^2$

$= \frac{-16}{3} \times p^5 \times q^4 \times r^3$

$= \frac{-16}{3}p^5q^4r^3$

17. Question

Find each of the following products:

$(2.3xy) \times (0.1x) \times (0.16)$

Answer

$2.3 \times 0.1 \times o.16 \times x \times x \times y$

$= 0.0368 \times x^2 \times y$

$= 0.0368x^2y$

18. Question

Express each of the following prducts as a monomials and verify the result in each case for x=1:

$(3x) \times (4x) \times (-5x)$

Answer

$3 \times 4 \times -5 \times x \times x \times x$

$= -60 \times x^3$

$= -60x^3$

19. Question

Express each of the following prducts as a monomials and verify the result in each case for x=1:

$\left(4x^2\right) \times (-3x) \times \left(\frac{4}{5}x^3\right)$

Answer

$4 \times -3 \times \frac{4}{5} \times x^2 \times x \times x^3$

$= \frac{-48}{5} \times x^6$

$= \frac{-48}{5}x^6$

20. Question

Express each of the following prducts as a monomials and verify the result in each case for x=1:

$\left(5x^4\right) \times \left(x^2\right)^3 \times (2x)^2$

Answer

$5x^4 \times x^6 \times 4 \times x^2$

$= 5 \times 4 \times x^4 \times x^6 \times x^2$

$= 20 \times x^{12}$

$= 20x^{12}$

21. Question

Express each of the following prducts as a monomials and verify the result in each case for x=1:

$\left(x^2\right)^3 \times (2x) \times (-4x) \times (5)$

Answer

$x^6 \times 2x \times (-4x) \times 5$

$= 2 \times -4 \times 5 \times x^6 \times x \times x \times x$

$= -40 \times x^8$

$= -40\ x^8$

22. Question

Express each of the following prducts as a monomials and verify the result in each case for x=1:

Write down the product of $8x^2y^6$ and-20xy verify the product for x=2.5, y=1

Answer

$-8 \times -2 \times x^2 \times x \times y^6 \times y$

$= 16 \times x^3 \times y^7$

$= 16x^3y^7$

Verification is when, x = 2.5 and y = 1

R.H.S = $16\ (2.5)^3 \times (1)^7$

$= 16 \times 15.625$

$= 250$

L.H.S = $-8 \times 2.5^2 \times 1^6 \times -20 \times 1 \times 2.5$

$= 250$

Therefore,

L.H.S = R.H.S

23. Question

Express each of the following prducts as a monomials and verify the result in each case for x=1:

Evaluate $\left(3.2x^6y^3\right) \times \left(2.1x^2y^2\right)$ when x=1 and y=0.5

Answer

$3.2 \times 2.1 \times x^6 \times x^2 \times y^3 \times y^2$

$= 6.72 \times x^8 \times y^5$

$= 6.72x^8y^5$

Verify:

When x = 1 and y = 0.5

R.H.S = $6.72x^3y^5$

$= 6.72 \times 1^8 \times 0.5^5$

$= 0.21$

L.H.S = $3.2 \times 1^6 \times (-.5)^3 \times 2.1 \times 1^2 \times 0.5^2 = 0.21$

Therefore,

L.H.S = R.H.S

24. Question

Express each of the following prducts as a monomials and verify the result in each case for x=1:

Find the value of $\left(5x^6\right) \times \left(-1.5x^2y^3\right) \times \left(-12xy^2\right)$ when x = 1,y=0.5

Answer

$5 \times -1.5 \times -12 \times x^6 \times x^2 \times x \times y^3 \times y^2$

$= 90 \times x^9 \times y^5$

$= 90x^9y^5$

Verification:

x = 1 and y = 0.5

R.H.S = $90x^9y^5$

= $90\ (1)^9\ (05)^5$

= 2.8125

L.H.S = 2.8125

Therefore,

L.H.S = R.H.S

25. Question

Express each of the following prducts as a monomials and verify the result in each case for x=1:

Evaluate $\left(2.3a^5b^2\right) \times \left(1.2a^2b^2\right)$ when a=1 and b = 0.5

Answer

$2.3a^5b^2 \times 1.2a^2b^2$

$= 2.3 \times 1.2 \times a^5 \times a^2 \times b^2 \times b^2$

$= 2.76 \times a^7 \times b^4$

$= 2.76a^7b^4$

Verification:

a = 1 and b = 0.5

$2.76\ a^7\ b^4 = 2.76\ (1)^7\ (0.5)^4$

$= 2.76 \times 1 \times 0.0025$

= 0.1725

26. Question

Express each of the following prducts as a monomials and verify the result in each case for x=1:

Evaluate $\left(-8x^2y^6\right) \times \left(-20xy\right)$ for x = 2.5 and y=1.

Answer

$-8 \times -20 \times x^2 \times x \times y^6 \times y$

$= 160x^3y^7$

Verify:

When, x = 2.5 and y = 1

$R.H.S = 160x^3y^7$

$= 160 \times (2.5)^3 \times (1)^7$

= 2500

$L.H.S = -8 \times 2.5^2 \times 1 \times -20 \times 1 \times 2.5$

= 2500

Therefore,

L.H.S = R.H.S

27. Question

Express each of the following products as a monomials and verify the result for x=1, y= 2:

$(-xy^3) \times (yx^3) \times (xy)$

Answer

$-x \times x^3 \times x \times y^3 \times y \times y$

$= -x^5y^5$

Verify:

When x = 1 and y = 2

R.H.S = $-x^5y^5$

$= (-1)^5 \times 2^5$

$= -1 \times 32$

$= -32$

L.H.S = $(-1) \times 2^3 \times 2 \times 1^3 \times 1 \times 2$

$= - 32$

Therefore,

L.H.S = R.H.S

28. Question

Express each of the following products as a monomials and verify the result for x=1, y= 2:

$\left(\frac{1}{8}x^2y^4\right) \times \left(\frac{1}{4}x^4y^2\right) \times (xy) \times 5$

Answer

$\frac{1}{8} \times \frac{1}{4} \times 5 \times x^2 \times x^4 \times x \times y^4 \times y^2 \times y$

$= \frac{5}{32} \times x^6 \times y^6$

$= \frac{5}{32}x^6y^6$

Verification:

When x = 1 and y = 2

R.H.S = $\frac{5}{32} \times 1^6 \times 2^6$

$= \frac{5}{32} \times 64$

$= 5 \times 2$

$= 10$

L.H.S = $\frac{1}{8} \times 1^2 \times 2^4 \times \frac{1}{4} \times 1^4 \times 2^2 \times 1 \times 2 \times 5$

$= 10$

Therefore,

L.H.S = R.H.S

29. Question

Express each of the following products as a monomials and verify the result for x=1, y= 2:

$\left(\frac{2}{5}a^2b\right) \times \left(-15b^2ac\right) \times \left(-\frac{1}{2}c^2\right)$

Answer

$\frac{2}{5} \times 15 \times \frac{1}{2} \times a^2 \times a \times b \times b^2 \times c \times c^3$

$= 3\ a^3 \times b^3 \times c^3$

$= 3a^3b^3c^3$

30. Question

Express each of the following products as a monomials and verify the result for x=1, y= 2:

$\left(-\frac{4}{7}a^2b\right) \times \left(-\frac{2}{3}b^2c\right) \times \left(-\frac{7}{6}c^2a\right)$

Answer

$\frac{-4}{7} \times \frac{-2}{3} \times \frac{-7}{6} \times a^2 \times a \times b \times b^2 \times c \times c^2$

$= \frac{-4}{9} \times a^3 \times b^3 \times c^3$

$= \frac{-4}{9}a^3b^3c^3$

31. Question

Express each of the following products as a monomials and verify the result for x=1, y= 2:

$\left(\frac{4}{9}abc^3\right) \times \left(-\frac{27}{5}a^3b^3\right) \times \left(-8b^3c\right)$

Answer

$\frac{4}{9} \times \frac{-27}{5} \times -8 \times a \times a^3 \times b \times b^2 \times b^3 \times c^3 \times c$

$= \frac{-96}{5} \times a^4 \times b^6 \times c^4$

$= \frac{-96}{5}a^4b^6c^4$

32. Question

Evaluate each of the following when x=2, y -1

$(2xy) \times \left(\frac{x^2y}{4}\right) \times \left(x^2\right) \times \left(y^2\right)$

Answer

$2 \times \frac{1}{4} \times x \times x^2 \times x^2 \times y \times y^2 \times y$

$= \frac{1}{2}x^5y^5$

$= \frac{1}{2}x^5y^5$

Verification:

When x = 2 and y = 1

R.H.S $= \frac{1}{2}x^5y^5$

$= \frac{1}{2}(2)^5 \times (-1)^5$

$= \frac{1}{2} \times 32 \times -1$

$= -16$

Therefore,

L.H.S = R.H.S

33. Question

Evaluate each of the following when x=2, y -1

$\left(\frac{3}{5}x^2y\right) \times \left(-\frac{15}{4}xy^2\right) \times \left(\frac{7}{9}x^2y^2\right)$

Answer

$\frac{3}{5} \times \frac{-15}{4} \times \frac{7}{9} \times x^2 \times x \times x^2 \times y \times y^2 \times y^2$

$= \frac{-7}{4} \times x^5 \times y^5$

$= \frac{7}{4}x^5y^5$

Verification:

When x = 2 and y = -1

R.H.S $= \frac{-7}{4}x^5y^5$

$= \frac{-7}{4}(2)^5\ (-1)^5$

$= \frac{-7}{4} \times 32 \times -1$

$= 56$

Therefore,

L.H.S = R.H.S

Exercise 6.4

1. Question

Find the following products:

$2a^3(3a + 5b)$

Answer

$2a^3\ (3a + 5b)$

$= 2a^3 \times 3a + 2a^2 \times 5b$

$= 6 \times a^4 + 10a^3b$

2. Question

Find the following products:

$-11a(3a + 2b)$

Answer

-11a (3a + 2b)

= (-11a × 3a) + (-11a × 2b)

$= -33a^2 - 2 \times 11 \times a \times b$

$= -33a^2 - 22ab$

3. Question

Find the following products:

$-5a(7a - 2b)$

Answer

-5a (7a – 2b)

$= -5a \times 7a - (-5a) \times 2b$

$= -5 \times 7 \times a \times a + 5 \times 2 \times a \times b$

$= -35a^2 + 10ab$

4. Question

Find the following products:

$-11y^2(3y + 7)$

Answer

$-11y^2$ (3y + 7)

$= -11y^2 \times 3y - 11y^2 \times 7$

$= -11 \times 3 \times y^2 \times y - 11y^2 \times 7$

$= -33y^3 - 77y^2$

5. Question

Find the following products:

$\frac{6x}{5}(x^3 + y^3)$

Answer

$\frac{6}{5}x$ (x^3 + y^3)

$= \frac{6}{5}x \times x^3 + \frac{6}{5}x \times y^3$

$= \frac{6}{5}x^4 + \frac{6}{5}xy^3$

6. Question

Find the following products:

$xy(x^3 - y^3)$

Answer

xy (x^3 – y^3)

$= xy \times x^3 - xy \times y^3$

$= x^4y - xy^4$

7. Question

Find the following products:

$0.1y\left(0.1x^5+0.1y\right)$

Answer

$0.1y\ (0.1x^5 + 0.1y)$

$= 0.1y \times 0.1x^5 + 0.1y \times 0.1y$

$= 0.01 \times x^5 \times y + 0.01 \times y^2$

$= 0.01x^5y + 0.01y^2$

8. Question

Find the following products:

$\left(-\frac{7}{4}ab^2c-\frac{6}{25}a^2c^2\right)\left(-50a^2b^2c^2\right)$

Answer

$(\frac{-7}{4}ab^2c - \frac{6}{25}a^2c^2)\ (-50a^2b^2c^2)$

$= \frac{-7}{4}ab^2c \times -50a^2b^2c^2 - \frac{6}{25}a^2c^2 \times -50a^2b^2 \times c^2$

$= \frac{7}{4} \times 50 \times a^3 \times b^4 \times c^3 - \frac{6}{25} \times -50 \times a^4 \times b^2 \times c^4$

$= \frac{350}{4}a^3b^4c^3 + 12a^4b^2c^4$

$= \frac{175}{2}a^3b^4c^3 + 12a^4b^2c^4$

9. Question

Find the following products:

$-\frac{8}{27}xyz\left(\frac{3}{2}xyz^2-\frac{9}{4}xy^2z^3\right)$

Answer

$\frac{-8}{27}xyz\ (\frac{3}{2}xyz^2 - \frac{9}{4}xy^2z^3)$

$= \frac{-8}{27}xyz \times \frac{3}{2}xyz^2 - \frac{8}{27}xyz \times \frac{-9}{4}xy^2z^3$

$= \frac{-4}{9} \times x^2 \times y^2 \times z^3 + \frac{2}{3} \times x^2 \times y^3 \times z^4$

$= \frac{-4}{9}x^2y^2z^3 + \frac{2}{3}x^2y^3z^4$

10. Question

Find the following products:

$-\frac{4}{27}xyz\left(\frac{9}{2}x^2yz-\frac{3}{4}xyz^2\right)$

Answer

$\frac{-4}{7}xyz\ (\frac{9}{2}x^2yz - \frac{3}{4}xyz^2)$

$= \frac{-4}{27}xyz \times \frac{9}{2}x^2yz - \frac{4}{7}xyz \times \frac{-3}{4}xyz^2$

$= \frac{-2}{3} \times x^3 \times y^2 \times z^2 + 9 \times x^2 \times y^2 \times z^3$

$= \frac{-2}{3}x^3y^2z^2 + 9x^2y^2z^3$

11. Question

Find the following products:

$1.5x\left(10x^2y - 100xy^2\right)$

Answer

$1.5x\ (10x^2y - 100xy^2)$

$= 1.5x \times 10x^2y - 1.5x \times 100xy^2$

$= 15 \times x^3 \times y - 150 \times x^2 \times y^2$

$= 15x^3y - 150x^2y^2$

12. Question

Find the following products:

$4.1xy\left(1.1x - y\right)$

Answer

$4.1xy\ (1.1x - y)$

$= 4.1xy \times 1.1x - 4.1xy \times y$

$= 4.51x^2y - 4.1xy^2$

13. Question

Find the following products:

$250.5xy\left(xz + \frac{y}{10}\right)$

Answer

$250 \times 5\ (x^2yz + \frac{xy*y}{10}$

$= 250\ (5x^2yz + \frac{x*y*y}{2})$

$= 250 \times 5x^2yz + 125xy^2$

14. Question

Find the following products:

$\frac{7}{5}x^2y\left(\frac{3}{5}xy^2 + \frac{2}{5}x\right)$

Answer

$\frac{7}{5}\ (\frac{3}{5}x^3y^3 + \frac{2}{5}x^3y)$

$= \frac{21}{25}x^3y^3 + \frac{14}{25}x^3y$

15. Question

Find the following products:

$\frac{4}{5}a\left(a^2 + b^2 - 3c^2\right)$

Answer

$\frac{4}{5}(a^3 + ab^2 - 3ac^2)$

$= \frac{4}{5}a^3 + \frac{4}{5}ab^2 - \frac{12}{5}ac^2$

16. Question

Find the product $24x^2(1-2x)$ and evaluate its value for x=3

Answer

$24x^2 (1 - 2x)$

$= 24x^2 - 48x^3$

According to question,

When x = 3

$= 24x^2 - 48x^3$

$= 24 (3)^2 - 48 (3)^3$

$= 24 (9) - 48 (27)$

$= 216 - 1296$

$= - 1080$

17. Question

Find the product $-3y (xy+y^2)$ and find its value for x = 4 and y = 5

Answer

$- 3y (xy + y^2)$

$= - 3xy^2 - 3y^3$

According to question:

When x = 4 and y = 5

$= - 3xy^2 - 3y^3$

$= - 3 (4) (5)^2 - 3 (5)^3$

$= - 300 - 375$

$= - 675$

18. Question

Multiply $-\frac{3}{2}x^2y^3bx(2x-y)$ and verify the answer for x = 1 and y = 2

Answer

$-\frac{3}{2}x^2y^3bx(2x-y)$

$= - 3x^3y^3bx + \frac{3}{2}x^2y^4bx$

$= -3x^4y^3b + \frac{3}{2}x^3y^4b$

According to question:

When x = 1 and y = 2

$= -3(1)^4(2)^3 b + \frac{3}{2}(1)^3(2)^4 b$

$= -3(8)b + 3(8)b$

$= 0$

19. Question

Multiply the monomial by the binomial and find the value of each for x=-1, y=0.25 and z=0.005:

(i) $15y^2(2-3x)$

(ii) $-3x(y^2+z^2)$

(iii) $z^2(x-y)$

(iv) $xz(x+y^2)$

Answer

(i) $15y^2(2-3x)$

$= 30y^2 - 45xy^2$

Putting x = -1, y = $\frac{25}{100}$ and z = $\frac{5}{1000}$

$= 30(\frac{25}{100})^2 - 45(-1)(\frac{25}{100})^2$

$= 30(\frac{1}{16}) + 45(\frac{1}{16})$

$= \frac{15}{8} + \frac{45}{16}$

$= \frac{30+45}{16}$

$= \frac{75}{16}$

(ii) $-3x(y^2+z^2)$

Putting x = - 1, y = $\frac{25}{100}$ and z = $\frac{5}{1000}$

$= -3(-1)(\frac{25}{100})^2 - 3(-1)(\frac{5}{1000})^2$

$= \frac{3*25*25}{100*100} + \frac{3*5*5}{1000*1000}$

$= \frac{3}{16} + \frac{3}{40000}$

$= \frac{39}{200}$

(iii) $z^2(x-y)$

Putting x = - 1, y = $\frac{25}{100}$ and z = $\frac{5}{1000}$

$z^2(x - y)$

$= (\frac{5}{1000})^2(-1 - \frac{25}{100})$

$= (\frac{1}{40000})(\frac{-5}{4})$

$= \frac{-1}{32000}$

(iv) $xz(x+y^2)$

Putting $x = -1$, $y = \frac{25}{100}$ and $z = \frac{5}{1000}$

$= (-1)^2 (\frac{5}{1000}) + (-1) (\frac{25}{100})^2 (\frac{5}{1000})$

$= \frac{1}{200} - \frac{1}{16} (\frac{1}{200})$

$= \frac{16-1}{3200}$

$= \frac{3}{640}$

20. Question

Simplify:

(i) $2x^2(x^3 - x) - 3x(x^4 + 2x) - 2(x^4 - 3x^2)$

(ii) $x^3y(x^2 - 2x) + 2xy(x^3 - x^4)$

(iii) $3a^2 + (a + 2) - 3a(2a + 1)$

(iv) $x(x + 4) + 3x(2x^2 - 1) + 4x^2 + 4$

(v) $a(b - c) - b(c - a) - c(a - b)$

(vi) $a(b - c) + b(c - a) + c(a - b)$

(vii) $4ab(a - b) - 6a^2(b - b^2) - 3b^2(2a^2 - a) + 2ab(b - a)$

(viii) $x^2(x^2 + 1) - x^3(x + 1) - x(x^3 - x)$

(ix) $2a^2 + 3a(1 - 2a^3) + a(a + 1)$

(x) $a^2(2a - 1) + 3a + a^3 - 8$

(xi) $\frac{3}{2}x^2(x^2 - 1) + \frac{1}{4}x^2(x^2 + x) - \frac{3}{4}x(x^3 - 1)$

(xii) $a^2b(a - b^2) + ab^2(4ab - 2a^2) - a^3b(1 - 2b)$

(xiii) $a^2b(a^3 - a + 1) - ab(a^4 - 2a^2 + 2a) - b(a^3 - a^2 - 1)$

Answer

(i) $2x^2(x^3 - x) - 3x(x^4 + 2x) - 2(x^4 - 3x^2)$

$= 2x^5 - 2x^3 - 3x^5 - 6x^2 - 2x^4 + 6x^2$

$= -x^5 - 2x^4 - 2x^3$

(ii) $x^3y(x^2 - 2x) + 2xy(x^3 - x^4)$

$= x^5y - 2x^4y + 2x^4y - 2x^5y$

$= -x^5y$

(iii) $3a^2 + (a + 2) - 3a(2a + 1) = 3a^2 + a + 2 - 6a^2 - 34$

$= -3a^2 - 2a + 2$

(iv) $x(x + 4) + 3x(2x^2 - 1) + 4x^2 + 4$

$= x^2 + 4x + 6x^3 - 3x + 4x^2 + 4$

$= 6x^3 + 5x^2 + x + 4$

(v) $a(b - c) - b(c - a) - c(a - b)$

$= ab - ac - bc + ab - ca + bc$

= 2ab – 2ac

(vi) a(b – c) + b(c – a) + c(a – b)

= ab – ac + bc – ab + ac – bc

= 0

(vii) $4ab(a – b) – 6a^2(b – b^2) – 3b^2(2a^2 – a) + 2ab(b – a)$

$= 4a^2b – 4ab^2 – 6a^2b + 6a^2b^2 – 6a^2b^2 + 3ab^2 + 2ab^2 – 2a^2b$

$= 3ab^2$

(viii) $x^2(x^2 + 1) – x^3(x + 1) – x(x^3 – x)$

$= x^4 + x^2 – x^4 – x^3 – x^4 + x^2$

$= 2x^2 – 2x^3$

(ix) $2a^2 + 3a (1 – 2a^3) + a(a + 1)$

$= 2a^2 + 3a – 6a^4 + a^2 + a$

$= -6a^4 + 3a^2 + 4a$

(x) $a^2(2a – 1) + 3a + a^3 – 8$

$= 2a^3 – a^2 + 3a + a^3 – 8$

$$(xi)\ \frac{3}{2}x^2(x^2-1)+\frac{1}{4}x^2(x^2+x)-\frac{3}{4}x(x^3-1)$$

$$= 3a^3 – a^2 + 3a – 8 = \frac{3}{2}x^4-\frac{3}{2}x^2+\frac{1}{4}x^4+\frac{1}{4}x^3+\frac{3}{4}x^4+\frac{3}{4}x$$

$$=x^4-\frac{3}{2}x^2+\frac{1}{4}x^3+\frac{3}{4}x$$

(xii) $a^2b(a – b^2) + ab^2(4ab – 2a^2) – a^3b(1 – 2b)$

$= a^3b – a^2b^3 + 4a^2b^3 – 2a^3b^2 – a^3b + 2a^3b^2$

$= -a^2b^3 + 4a^2b^3$

$= 3a^2b^3$

(xiii) $a^2b(a^3 – a + 1) – ab(a^4 – 2a^2 + 2a) – b(a^3 – a^2 – 1)$

$= a^5b – a^3b + a^2b – a^5b + 2a^3b – 2a^2b – ba^3 + a^2b + b$

= b

Exercise 6.5

1. Question

Multiply:

$(5x + 3)\text{by}(7x + 2)$

Answer

(5x + 3) × (7x + 2)

= 5x (7x + 2) + 3 (7x + 2)

$= 35x^2 + 10x + 21x + 6$

$= 35x^2 + 31x + 6$

2. Question

Multiply:

$(2x + 8) \text{ by } (x - 3)$

Answer

$(2x + 8) \times (x - 3)$

$= 2x (x - 3) + 8 (x - 3)$

$= 2x^2 - 6x + 8x - 24$

$= 2x^2 - 2x - 24$

3. Question

Multiply:

$(7x + y) \text{ by } (x + 5y)$

Answer

$(7x + y) \times (x + 5y)$

$= 7x (x + 5y) + y (x + 5y)$

$= 7x^2 + 35xy + xy + 5y^2$

$= 7x^2 + 36xy + 5y^2$

4. Question

Multiply:

$(a - 1) \text{ by } (0.1a^2 + 3)$

Answer

$(a - 1) \times (0.1a^2 + 3)$

$= a (0.1a^2 + 3) - 1 (0.1a^2 + 3)$

$= 0.1a^3 + 3a - 0.1a^2 - 3$

5. Question

Multiply:

$(3x^2 + y^2) \text{ by } (2x^2 + 3y^2)$

Answer

$(3x^2 + y^2) \times (2x^2 + 3y^2)$

$= 3x^2 (2x^2 + 3y^2) + y^2 (2x^2 + 3y^2)$

$= 6x^4 + 9x^2y^2 + 2x^2y^2 + 3y^4$

$= 6x^4 + 11x^2y^2 + 3y^4$

6. Question

Multiply:

$\left(\frac{3}{5}x + \frac{1}{2}y\right) \text{ by } \left(\frac{5}{6}x + 4y\right)$

Answer

$(\frac{3}{5}x + \frac{1}{2}y) \times (\frac{5}{6}x + 4y)$

$= \frac{3}{5}x\ (\frac{5}{6}x + 4y) + \frac{1}{2}y\ (\frac{5}{6}x + 4y)$

$= \frac{1}{2}x^2 + \frac{12}{5}xy + \frac{5}{12}xy + 2y^2$

$= \frac{1}{2}x^2 + \frac{169}{60}xy + 2y^2$

7. Question

Multiply:

$(x^6 - y^6)$ by $(x^2 + y^2)$

Answer

$(x^6 - y^6) \times (x^2 + y^2)$

$= x^6 (x^2 + y^2) - y^6 (x^2 + y^2)$

$= x^8 + x^6y^2 - x^2y^6 - y^8$

8. Question

Multiply:

$(x^2 - y^2)$ by $(3a + 2b)$

Answer

$(x^2 - y^2) \times (3a + 2b)$

$= x^2 (3a + 2b) - y^2 (3a + 2b)$

$= 3ax^2 + 2bx^2 - 3ay^2 - 2by^2$

9. Question

Multiply:

$[-3d + (7f)]$ by $(5f + f)$

Answer

$(- 3d + 7f) \times (5d + f)$

$= -3d (5d + f) + 7f (5d + f)$

$= - 15d^2 - 3df + 35df + 7f^2$

$= - 15d^2 + 32df + 7f^2$

10. Question

Multiply:

$(0.8a - 0.5b)$ by $(1.5a - 3b)$

Answer

$(0.8a - o.5b) \times (1.5a - 3b)$

$= 0.8a (1.5a - 3b) - 0.5b (1.5a - 3b)$

$= 1.2a^2 - 2.4ab - 7.5ab + 1.5b^2$

$= 1.2a^2 - 9.9ab + 1.5b$

11. Question

Multiply:

$\left(2x^2y^2 - 5xy^2\right)$ by $\left(x^2 - y^2\right)$

Answer

$(2x^2y^2 - 5xy^2) \times (x^2 - y^2)$

$= 2x^2y^2\,(x^2 - y^2) - 5xy^2\,(x^2 - y^2)$

$= 2x^4y^2 - 2x^2y^4 - 5x^3y^2 + 5xy^4$

12. Question

Multiply:

$\left(\frac{x}{7} + \frac{x^2}{2}\right)$ by $\left(\frac{2}{5} + \frac{9x}{4}\right)$

Answer

$(\frac{x}{7} + \frac{x*x}{2}) \times (\frac{2}{5} + \frac{9x}{4})$

$= \frac{x}{7}(\frac{2}{5} + \frac{9x}{4}) + \frac{x*x}{2}(\frac{2}{5} + \frac{9x}{4})$

$= \frac{2x}{35} + \frac{9x*x}{28} + \frac{x*x}{5} + \frac{9x*x*x}{8}$

$= \frac{2x}{35} + \frac{73}{140}x^2 + \frac{9}{8}x^3$

13. Question

Multiply:

$\left(-\frac{a}{7} + \frac{a^2}{9}\right)$ by $\left(\frac{b}{2} - \frac{b^2}{3}\right)$

Answer

$(\frac{-a}{7} + \frac{a*a}{9}) \times (\frac{b}{2} - \frac{b*b}{3})$

$= \frac{-a}{7}(\frac{b}{2} - \frac{b*b}{3}) + \frac{a*a}{9}(\frac{b}{2} - \frac{b*b}{3})$

$= \frac{-ab}{14} + \frac{a*b*b}{21} + \frac{a*a*b}{18} - \frac{a*a*b*b}{27}$

14. Question

Multiply:

$\left(3x^2y - 5xy^2\right)$ by $\left(\frac{1}{5}x^2 + \frac{1}{3}y^2\right)$

Answer

$(3x^2y - 5xy^2) \times (\frac{1}{5}x^2 + \frac{1}{3}y^2)$

$= 3x^2y\,(\frac{1}{5}x^2 + \frac{1}{3}y^2) - 5xy^2\,(\frac{1}{5}x^2 + \frac{1}{3}y^2)$

$= \frac{3}{5}x^4y + 3x^2y^3 - x^3y^2 + \frac{5}{3}xy^4$

15. Question

Multiply:

$(2x^2 - 1)$ by $(4x^3 + 5x^2)$

Answer

$(2x^2 - 1) \times (4x^3 + 5x^2)$

$= 2x^2 (4x^3 + 5x) - 1 (4x^3 + 5x^2)$

$= 8x^5 + 10x^3 - 4x^3 - 5x^2$

$= 8x^5 + 6x^3 - 5x^2$

16. Question

Multiply:

$(2xy + 3y^2)(3y^2 - 2)$

Answer

$(2xy + 3y^2) \times (3y^2 - 2)$

$= 2xy (3y^2 - 2) + 3y^2 (3y^2 - 2)$

$= 6xy^3 - 4xy + 3y^4 - 6y^2$

17. Question

Find the following products and verify the result for x=-1, y=-2:

$(3x - 5y)(x + y)$

Answer

$(3x - 5y) \times (x + y)$

$= x (3x - 5y) + y (3x - 5y)$

$= 3x^2 - 5xy + 3xy - 5y^2$

$= 3x^2 - 2xy - 5y^2$

Putting x = - 1 and y = - 2, we have

$[3 (-1) - 5 (-2)] [(1) + (-2)] = 3 (-1)^2 - 2 (-1) (-2) - 5 (-2)^2$

$(-3 + 10) (-1 - 2) = 3 - 4 - 20$

$- 21 = - 21$

Therefore,

L.H.S = R.H.S

Hence, verified

18. Question

Find the following products and verify the result for x=-1, y=-2:

$(x^2y - 1)(3 - 2x^2y)$

Answer

$x^2y (3 - 2x^2y) - 1 (3 - 2x^2y)$

$= 3x^2y - 2x^4y^2 - 3 + 2x^2y$

$= 2x^4y^2 + 5x^2y - 3$

Putting x = -1 and y = -2, we have

$= [(-1)^2 (-2) - 1] [3 - 2 (-1)^2 (-2) = [-2 (-1)^4 (-2)^2 + 5 (-1)^2 (2) - 3]$

$= (-2 - 1) (3 + 4) = - 8 - 10 - 3$

$-21 = - 21$

Therefore,

L.H.S = R.H.S

Hence, verified

19. Question

Find the following products and verify the result for x=-1, y=-2:

$\left(\frac{1}{3}x - \frac{y^2}{5}\right)\left(\frac{1}{3}x + \frac{y^2}{5}\right)$

Answer

$(\frac{1}{3}x)^2 - (\frac{y*y}{5})^2$

$= (\frac{1}{3}x - \frac{y*y}{5}) (\frac{1}{3}x + \frac{y*y}{5})$

$= \frac{1}{9}x^2 - \frac{1}{25}y^4$

Putting x = -1 and y = -2, we have

$(\frac{1}{3}(-1) - \frac{(-2)(-25)}{5}) = (\frac{1}{9}(-1)^2 - \frac{-2*-2*-2*-2}{25})$

$= (\frac{-1}{3} - \frac{4}{5}) (\frac{-1}{3} + \frac{4}{5}) = (\frac{1}{9} - \frac{16}{25})$

$= (\frac{-17}{15}) (\frac{7}{15}) = \frac{-119}{225}$

$= \frac{-119}{225} = \frac{-119}{225}$

Therefore,

L.H.S = R.H.S

Hence, verified

20. Question

Simplify:

$x^2 (x + 2y)(x - 3y)$

Answer

$x^2 (x^2 - 3xy + 2xy - 3y^2)$

$= x^2 (x^2 - xy - 6y^2)$

$= x^4 - x^3y - 6x^2y^2$

21. Question

Simplify:

$(x^2 - 2y^2)(x + 4y)x^2y^2$

Answer

$(x^3 + 4x^2y - 2xy^2 - 8y^3) \times x^2y^2$

$= x^5y^2 + 4x^4y^3 - 2x^3y^4 - 8x^2y^5$

22. Question

Simplify:

$a^2b^2(a+2b)(3a+b)$

Answer

$a^2b^2 (3a^2 + ab + 6ab + 2b^2)$

$= a^2b^2 (3a^2 + 7ab + 2b^2)$

$= 3a^4b^2 + 7a^3b^3 + 2a^2b^4$

23. Question

Simplify:

$x^2(x-y)y^2(x+2y)$

Answer

$x^2y^2 (x - y) (x + 2y)$

$= x^2y^2 (x^2 + 2xy - xy - 2y^2)$

$= x^2y^2 (x^2 + xy - 2y^2)$

$= x^4y^2 + x^3y^3 - 2x^2y^4$

24. Question

Simplify:

$(x^3 - 2x^2 + 5x - 7)(2x - 3)$

Answer

$2x^4 - 4x^3 + 4x^2 - 14x - 3x^3 + 6x^2 - 6x + 21$

$= 2x^4 - 7x^3 + 10x^2 - 20x + 21$

25. Question

Simplify:

$(5x+3)(x-1)(3x-2)$

Answer

$(5x^2 - 2x - 3) (3x - 2)$

$= 15x^3 - 6x^2 - 9x - 10x^2 + 4x + 6$

$= 15x^3 - 16x^2 - 5x + 6$

26. Question

Simplify:

$(5-x)(6-5x)(2-x)$

Answer

$(x^2 - 7x + 10) (6 - 5x)$

$= -5x^3 + 35x^2 – 50x + 6x^2 – 42x + 60$

$= -5x^2 + 41x^2 – 92x + 60$

27. Question

Simplify:

$(2x^2 + 3x – 5)(3x^2 – 5x + 4)$

Answer

$6x^4 + 9x^3 – 15x^2 – 10x^3 – 15x^2 + 25x + 8x^2 + 12x – 20$

$= 6x^4 – x^3 – 22x^2 + 37x - 20$

28. Question

Simplify:

$(3x – 2)(2x – 3) + (5x – 3)(x + 1)$

Answer

$6x^2 – 9x – 4x + 6 + 5x^2 + 5x – 3x – 3$

$= 11x^2 – 11x + 3$

29. Question

Simplify:

$(5x – 3)(x + 2) – (2x + 5)(4x – 3)$

Answer

$5x^2 + 10x – 3x – 6 – 8x^2 + 6x – 20x + 15$

$= -3x^2 – 7x + 9$

30. Question

Simplify:

$(3x + 2y)(4x + 3y) – (2x – y)(7x – 3y)$

Answer

$12x^2 + 9xy + 8xy$

$= 12x^2 + 9xy + 8xy + 6y^2 – 14x^2 + 6xy + 7xy – 3y^2$

$= -2x^2 + 30xy + 3y^2$

31. Question

Simplify:

$(x^2 – 3x + 2)(5x – 2) – (3x^2 + 4x – 5)(2x – 1)$

Answer

$5x^4 – 15x^2 + 10x – 2x^3 + 6x – 4 – (6x^3 + 8x^2 – 10x – 3x^2 – 4x + 5)$

$= 5x^4 – 15x^2 – 2x^3 + 16x – 4 – 6x^3 – 5x^2 + 14x – 5$

$= 5x^4 – 8x^3 – 20x^2 + 30x - 9$

32. Question

Simplify:

$(x^3 - 2x^2 + 3x - 4)(x - 1) - (2x - 3)(x^2 - x + 1)$

Answer

$x^4 - 2x^3 + 3x^2 - 4x - x^3 + 2x^2 - 3x + 4 - (2x^3 - 2x^2 + 2x - 3x^2 + 3x - 3)$

$= x^4 - 3x^3 + 5x^2 - 7x + 4 - 2x^3 + 5x^2 - 5x + 3$

$= x^4 - 5x^3 + 10x^2 - 12x + 7$

Exercise 6.6

1. Question

Write the following squares of binomials as trinomias:

(i) $(x + 2)^2$

(ii) $(8x + 3b)^2$

(iii) $(2m + 1)^2$

(iv) $\left(9a + \frac{1}{6}\right)^2$

(v) $\left(x + \frac{x^2}{2}\right)^2$

(vi) $\left(\frac{x}{4} - \frac{y}{3}\right)$

(vii) $\left(3x - \frac{1}{3x}\right)^2$

(viii) $\left(\frac{x}{y} - \frac{y}{x}\right)^2$

(ix) $\left(\frac{3a}{2} - \frac{5b}{4}\right)^2$

(x) $(a^2b - bc^2)^2$

(xi) $\left(\frac{2a}{3b} + \frac{2b}{3a}\right)^2$

(xii) $(x^2 - ay)^2$

Answer

(i) $(x+2)^2$

$x^2 + 2\,(x)\,(2) + 2^2$

$= x^2 + 4x + 4$

(ii) $(8x+3b)^2$

$(8x)^2 + 2\,(8x)\,(3b) + (3b)^2$

$= 16x^2 + 48xb + 9b^2$

(iii) $(2m+1)^2$

$(2m)^2 + 2\,(2m)\,(1) + 1^2$

$= 4m^2 + 4m + 1$

(iv) $\left(9a + \frac{1}{6}\right)^2$

$(9a)^2 + 2\,(9a)\,(\frac{1}{6}) + (\frac{1}{6})^2$

$= 81a^2 + 3a + \frac{1}{36}$

(v) $\left(x + \frac{x^2}{2}\right)^2$

$(x)^2 + 2\,(x)\,(\frac{x*x}{2}) + (\frac{x*x}{2})^2$

$= x^2 + x^3 + \frac{1}{4}x^4$

(vi) $\left(\frac{x}{4} - \frac{y}{3}\right)$

$(\frac{x}{4})^2 - 2\,(\frac{x}{4})\,(\frac{y}{3}) + (\frac{y}{3})^2$

$= \frac{1}{16}x^2 - \frac{xy}{6} + \frac{1}{9}y^2$

(vii) $\left(3x - \frac{1}{3x}\right)^2$

$(3x)^2 - 2\,(3x)\,(\frac{1}{3x}) + (\frac{1}{3x})^2$

$= 9x^2 - 2 + \frac{1}{9*x*x}$

(viii) $\left(\frac{x}{y} - \frac{y}{x}\right)^2$

$(\frac{x}{y})^2 - 2\,(\frac{x}{y})\,(\frac{y}{x}) + (\frac{y}{x})^2$

$= \frac{x*x}{y*y} - 2 + \frac{y*y}{x*x}$

(ix) $\left(\frac{3a}{2} - \frac{5b}{4}\right)^2$

$(\frac{3a}{2})^2 - 2\,(\frac{3a}{2})\,(\frac{5b}{4}) + (\frac{5b}{4})^2$

$= \frac{9}{4}a^2 - \frac{15}{4}ab + \frac{25}{16}b$

(x) $\left(a^2b - bc^2\right)^2$

$(a^2b)^2 - 2\,(a^2b)\,(bc^2) + (bc^2)^2$

$= a^4b^2 - 2a^2b^2c^2 + b^2c^4$

(xi) $\left(\frac{2a}{3b} + \frac{2b}{3a}\right)^2$

$(\frac{2a}{3b})^2 + 2\,(\frac{2a}{3b})\,(\frac{2b}{3a}) + (\frac{2b}{3a})^2$

$= \frac{4*a*a}{9*b*b} + \frac{8}{9}a + \frac{4*b*b}{9*a*a}$

(xii) $(x^2 - ay)^2$

$(x^2)^2 - 2(x^2)(ay) + (ay)^2$

$= x^4 - 2x^2ay + a^2y^2$

2. Question

Find the product of the following binomials:

(i) (2x + y) (2x + y)

(ii) (a + 2b) (a – 2b)

(iii) $(a^2 + bc)(a^2 - bc)$

(iv) $\left(\frac{4x}{5} - \frac{3y}{4}\right)\left(\frac{4x}{5} + \frac{3y}{4}\right)$

(v) $\left(2x + \frac{3}{y}\right)\left(2x - \frac{3}{4}\right)$

(vi) $(2a^3 + b^3)(2a^3 - b^3)$

(vii) $\left(x^4 + \frac{2}{x^2}\right)\left(x^4 - \frac{2}{x^2}\right)$

(viii) $\left(x^3 + \frac{1}{x^3}\right)\left(x^3 - \frac{1}{x^3}\right)$

Answer

(i) (2x + y) (2x + y)

2x (2x + y) + y (2x + y)

$= 4x^2 + 2xy + 2xy + 3y$

$= 4x^2 + 4xy + 3y$

(ii) (a + 2b) (a – 2b)

a (a – 2b) + 2b (a – 2b)

$= a^2 - 2ab + 2ab - 4b^2$

$= a^2 - 4b^2$

(iii) $(a^2 + bc)(a^2 - bc)$

$a^2(a^2 - bc) + bc(a^2 - bc)$

$= a^4 - a^2bc + bca^2 - b^2c^2$

$= a^4 - b^2c^2$

(iv) $\left(\frac{4x}{5} - \frac{3y}{4}\right)\left(\frac{4x}{5} + \frac{3y}{4}\right)$

$\frac{4x}{5}\left(\frac{4x}{5} + \frac{3y}{4}\right) - \frac{3y}{4}\left(\frac{4x}{5} + \frac{3y}{4}\right)$

$= \frac{16}{25}x^2 + \frac{12}{20}yx - \frac{12xy}{20} - \frac{9*y*y}{16}$

$= \frac{16}{25}x^2 - \frac{9}{16}y^2$

(v) $\left(2x + \frac{3}{y}\right)\left(2x - \frac{3}{4}\right)$

$2x\ (2x - \frac{3}{y}) + \frac{3}{y}\ (2x - \frac{3}{y})$

$= 4x^2 - \frac{6x}{y} + \frac{6x}{y} - \frac{9}{y*y}$

$= 4x^2 - \frac{9}{y*y}$

(vi) $(2a^3 + b^3)(2a^3 - b^3)$

$2a^3\ (2a^3 - b^3) + b^3\ (2a^3 - b^3)$

$= 4a^6 - 2a^3b^3 + 2a^3b^3 - b^6$

$= 4a^6 - b^6$

(vii) $\left(x^4 + \frac{2}{x^2}\right)\left(x^4 - \frac{2}{x^2}\right)$

$x^4\ (x^4 - \frac{2}{x*x}) + \frac{2}{x*x}\ (x^4 - \frac{2}{x*x})$

$= x^8 - 2x^2 + 2x^2 - \frac{4}{x*x*x*x}$

$= (x^8 - \frac{4}{x*x*x*x})$

(viii) $\left(x^3 + \frac{1}{x^3}\right)\left(x^3 - \frac{1}{x^3}\right)$

$x^3\ (x^3 - \frac{1}{x*x*x}) + \frac{1}{x*x*x}\ (x^3 - \frac{1}{x*x*x})$

$= x^6 - 1 + 1 - \frac{1}{x*x*x*x*x*x}$

$= x^6 - \frac{1}{x*x*x*x*x*x}$

3. Question

Using the formula for squaring a binomial, evaluate the following:

(i) $(102)^2$

(ii) $(99)^2$

(iii) $(1001)^2$

(iv) $(999)^2$

(v) $(703)^2$

Answer

(i) $(102)^2$

This can be written as:

$(100 + 2)^2$

$= (100)^2 + 2\ (100)\ (2) + 2^2$

$= 10000 + 400 + 4$

$= 10404$

(ii) $(99)^2$

This can be written as:

$(100 - 1)^2$

$= (100)^2 - 2\ (100)\ (1) + 1^2$

$= 10000 - 200 + 1$

$= 9801$

(iii) $(1001)^2$

This can be written as:

$(1000 + 1)^2$

$= (1000)^2 + 2\ (1000)\ (1) + 1^2$

$= 1000000 + 2000 + 1$

$= 1002001$

(iv) $(999)^2$

This can be written as:

$(1000 - 1)^2$

$= (1000)^2 - 2\ (1000)\ (1) + 1^2$

$= 1000000 - 2000 + 1$

$= 998001$

(v) $(703)^2$

This can be written as:

$(700 + 3)^2$

$= (700)^2 + 2\ (700)\ (3) + 3^2$

$= 490000 + 4200 + 9$

$= 494209$

4. Question

Simplify the following using the formula: $(a-b)(a+b) = a^2 - b^2$:

(i) $(82)^2 - (18)^2$

(ii) $(467)^2 - (33)^2$

(iii) $(79)^2 - (69)^2$

(iv) 197×203

(v) 113×87

(vi) 95×105

(vii) 1.8×2.2

(viii) 9.8×10.2

Answer

(i) $(82)^2 - (18)^2$

Using formula:

$(a – b)(a + b) = a^2 – b^2$, we get

$= (82 – 18)(82 + 18)$

$= 64 \times 100$

$= 6400$

(ii) $(467)^2 – (33)^2$

Using formula:

$(a – b)(a + b) = a^2 – b^2$, we get

$= (467 – 33)(467 + 33)$

$= (434)(500)$

$= 217000$

(iii) $(79)^2 – (69)^2$

Using formula:

$(a – b)(a + b) = a^2 – b^2$, we get

$= (79 + 69)(79 – 69)$

$= (148)(10)$

$= 1480$

(iv) 197×203

Using formula:

$(a – b)(a + b) = a^2 – b^2$, we get

$= (200 – 3)(200 + 3)$

$= (200)^2 – (3)^2$

$= 40000 – 9$

$= 39991$

(v) 113×87

Using formula:

$(a – b)(a + b) = a^2 – b^2$, we get

$= (100 + 3)(100 – 3)$

$= (100)^2 – (3)^2$

$= 10000 – 9$

$= 9991$

(vi) 95×105

Using formula:

$(a – b)(a + b) = a^2 – b^2$, we get

$= (100 – 5)(100 + 5)$

$= (100)^2 – (5)^2$

= 10000 – 25

= 9975

(vii) 1.8×2.2

Using formula:

(a – b) (a + b) = $a^2 – b^2$, we get

= (2- 0.2) (2 + 0.2)

= $(2)^2 – (0.2)^2$

= 4 – 0.04

= 3.96

(viii) 9.8×10.2

Using formula:

(a – b) (a + b) = $a^2 – b^2$, we get

= (10 – 0.2) (10 + 0.2)

= $(10)^2$ - $(0.2)^2$

= 100 – 0.04

= 90.96

5. Question

Simplify the following using the indentities:

(i) $\frac{58^2 - 42^2}{16}$

(ii) $178 \times 178 - 22 \times 22$

(iii) $\frac{198 \times 198 - 102 \times 102}{96}$

(iv) $1.73 \times 1.73 - 0.27 \times 0.27$

(v) $\frac{8.63 \times 8.63 - 1.37 \times 1.37}{0.726}$

Answer

(i) $\frac{58^2 - 42^2}{16}$

$\frac{(58-42)(58+42)}{4*4}$

$= \frac{16(100)}{16}$

= 100

(ii) $178 \times 178 - 22 \times 22$

$(178)^2 – (22)^2$

= (178 + 22) (178 – 22)

= 200 × 156

= 31200

(iii) $\frac{198 \times 198 - 102 \times 102}{96}$

$\frac{(198-102)(198+102)}{96}$

$= \frac{96*300}{96}$

= 300

(iv) $1.73 \times 1.73 - 0.27 \times 0.27$

(1.73) – (0.27)

= (1.73 + 0.27) (1.73 – 0.27)

= 2 (1.46)

= 2.92

(v) $\frac{8.63 \times 8.63 - 1.37 \times 1.37}{0.726}$

$\frac{(8.63+1.37)(8.63-1.37)}{0.726}$

$= \frac{10*7.26}{0.726}$

= 100

6. Question

Find the value of x, if:

(i) $4x = (52)^2 - (48)^2$

(ii) $14x = (47)^2 - (33)^2$

(iii) $5x = (50)^2 - (40)^2$

Answer

(i) $4x = 52^2 - 48^2$

4x = (52 – 48) (52 + 48)

4x = 4 × 100

4x = 400

x = 100

(ii) $14x = (47)^2 - (33)^2$

14x = (47 – 33) (47 + 33)

14x = 14 × 80

x = 80

(iii) $5x = (50)^2 - (40)^2$

Using formula:

$a^2 - b^2 = (a - b)(a + b)$, we get

5x = (50 – 40) (50 + 40)

5x = 10 × 90

5x = 900

x = 180

7. Question

If $x+\frac{1}{x}$=20, find the value of $x^2+\frac{1}{x^4}$.

Answer

Given that,

$x+\frac{1}{x}=20$

Squaring both sides, we get

$(x+\frac{1}{x})^2=(20)^2$

$x^2+2\times x\times\frac{1}{x}+(\frac{1}{x})^2=400$

$x^2+2+\frac{1}{x*x}=400$

$x^2+\frac{1}{x*x}=398$

8. Question

If $x-\frac{1}{x}$=3, find the values of $x^2+\frac{1}{x^2}$ and $x^4+\frac{1}{x^4}$.

Answer

(i) Given that,

$x-\frac{1}{x}=3$

Squaring both sides, we get

$(x-\frac{1}{x})^2=(3)^2$

$x^2-2\times x\times\frac{1}{x}+(\frac{1}{x})^2=9$

$x^2-2+\frac{1}{x*x}=9$

$x^2+\frac{1}{x*x}=11$

(ii) Squaring both sides, we get

$(x^2+\frac{1}{x*x})^2=(11)^2$

$(x^2)^2+2\times x^2\times\frac{1}{x*x}+(\frac{1}{x*x})^2=121$

$x^4+2+\frac{1}{x*x*x*x}=121$

$x^4+\frac{1}{x*x*x*x}=119$

9. Question

If $x^2+\frac{1}{x^2}$= 18, find the values of $x+\frac{1}{x}$ and $x-\frac{1}{x}$.

Answer

$x^2 + \frac{1}{x*x} = 18$

Adding 2 on both sides, we get

$x^2 + \frac{1}{x*x} + 2 = 18 + 2$

$x^2 + \frac{1}{x*x} + 2 \times x \times \frac{1}{x} = 20$

$(x + \frac{1}{x})^2 = 20$

$x + \frac{1}{x} = 2\sqrt{5}$

Given that,

$x^2 + \frac{1}{x*x} = 18$

Subtracting 2 from both sides, we get

$x^2 + \frac{1}{x*x} - 2 \times x \times \frac{1}{x} = 18 - 2$

$(x - \frac{1}{x})^2 = 16$

$x - \frac{1}{x} = 4$

10. Question

If x+y = 4 and xy=2, find the value of x^2+y^2

Answer

Given that,

x + y = 4 and xy=2

We take the equation: x + y = 4 and on squaring both sides, we get

$(x + y)^2 = 4^2$

$x^2 + y^2 + 2xy = 16$

$x^2 + y^2 + 2 (2) = 16$ (Because xy=2 is given)

$x^2 + y^2 + 4 = 16$

$x^2 + y^2 = 16 - 4$

$x^2 + y^2 =12$

Therefore, the value of $x^2 + y^2$ is 12

11. Question

If x- y = 7 and xy = 9, find the value fo x^2+y^2

Answer

Given that, x – y = 7

Squaring both sides, we get

$(x - y)^2 = (7)^2$

$x^2 + y^2 - 2xy = 49$

Its given that xy = 9,

$x^2 + y^2 - 2(9) = 49$

$x^2 + y^2 = 49 + 18$

$x^2 + y^2 = 67$

12. Question

If $3x+5y = 11$ and $xy = 2$, find the value of $9x^2+25y^2$

Answer

Given that,

$3x + 5y = 11$

Squaring both sides, we get

$(3x + 5y)^2 = (11)^2$

$(3x)^2 + (5y)^2 + 2(3x)(5y) = 121$

$9x^2 + 25y^2 + 30xy = 121$

$9x^2 + 25y^2 + 30(2) = 121$

$9x^2 + 25y^2 = 121 - 60$

$9x^2 + 25y^2 = 61$

13. Question

Find the values of the following expressions:

(i) $16x^2 + 24x + 9, \text{when } x = \frac{7}{4}$

(ii) $64x^2 + 81y^2 + 144xy, \text{when } x = 11 \text{ and } y = \frac{4}{3}$

(iii) $81x^2 + 16y^2 - 72xy, \text{when } x = \frac{2}{3} \text{ and } y = \frac{3}{4}$

Answer

(i) $16x^2 + 24x + 9, \text{when } x = \frac{7}{4}$

$(4x)^2 + 2(4x)(3) + 3^2$

$= (4x + 3)^2$

Putting $x = \frac{7}{4}$

$= [4(\frac{7}{4}) + 3]^2$

$= (7 + 3)^2$

$= 100$

(ii) $64x^2 + 81y^2 + 144xy, \text{when } x = 11 \text{ and } y = \frac{4}{3}$

$(8x)^2 + 2(8x)(9y) + (9y)^2$

$= (8x + 9y)^2$

Putting $x = 11$ and $y = \frac{4}{3}$

$= [8 (11) + 9 (\frac{4}{3})]^2$

$= (88 + 12)^2$

$= (100)^2$

$= 10000$

(iii) $81x^2 + 16y^2 - 72xy, \text{when } x = \frac{2}{3} \text{ and } y = \frac{3}{4}$

$(9x)^2 + (4y)^2 - 2 (9x) (4y)$

$= (9x - 4y)^2$

Putting $x = \frac{2}{3}$ and $y = \frac{3}{4}$

$= [9 (\frac{2}{3}) - 4 (\frac{3}{4})]^2$

$= (6 - 3)^2$

$= 3^2$

$= 9$

14. Question

If $x + \frac{1}{x} = 9$, find the value of $x^4 + \frac{1}{x^4}$

Answer

Given that,

$x + \frac{1}{x} = 9$

Squaring both sides, we get

$(x + \frac{1}{x})^2 = 9^2$

$x^2 + \frac{1}{x*x} + 2 = 81$

$x^2 + \frac{1}{x*x} = 79$

Again,

Squaring both sides, we get

$(x^2 + \frac{1}{x*x})^2 = 79^2$

$x^4 + \frac{1}{x*x*x*x} + 2 = 6241$

$x^4 + \frac{1}{x*x*x*x} = 6239$

15. Question

If $x + \frac{1}{x} = 12$, find the value of $x - \frac{1}{x}$.

Answer

Given that,

$x + \frac{1}{x} = 12$

Squaring both sides, we get

$(x + \frac{1}{x})^2 = 12^2$

$x^2 + (\frac{1}{x})^2 + 2 \times x \times \frac{1}{x} = 144$

$x^2 + \frac{1}{x^2} = 142$

Subtract 2 from both sides, we get

$x^2 + \frac{1}{x*x} - 2 \times x \times \frac{1}{x} = 142 - 2$

$(x - \frac{1}{x})^2 = 140$

$x - \frac{1}{x} = \pm \sqrt{140}$

16. Question

If 2x+3y=14 and 2x-3y=2, find value of xy. [Hint: Use $(2x+3y)^2 - (2x-3y)^2 = 24xy$]

Answer

Given that,

2x + 3y = 14.....(1)

2x – 3y = 2.....(2)

Now, on squaring both the equation and subtracting (2) from (1), we get,

$(2x + 3y)^2 - (2x - 3y)^2 = (14)^2 - (2)^2$

$4x^2 + 9y^2 + 12xy - 4x^2 - 9y^2 + 12xy = 196 - 4$

(The positive and negative terms gets cancelled)

24 xy = 192

xy = 8

Therefore, the value of "xy"is 8.

17. Question

if $x^2+y^2 = 29$ and xy = 2, find the value of

(i) x+y

(ii) x-y

(iii) x^4+y^4

Answer

(i) x+y

Given that,

$x^2 + y^2 = 29$

$x^2 + y^2 + 2xy - 2xy = 29$

$(x + y)^2 - 2 (2) = 29$

$(x + y)^2 = 29 + 4$

$x + y = \pm\sqrt{33}$

(ii) x-y

$x^2 + y^2 = 29$

$x^2 + y^2 + 2xy - 2xy = 29$

$(x - y)^2 + 2(2) = 29$

$(x - y)^2 + 4 = 29$

$(x - y)^2 = 25$

$(x - y) = \pm 5$

(iii) x^4+y^4

$x^2 + y^2 = 29$

Squaring both sides, we get

$(x^2 + y^2)^2 = (29)^2$

$x^4 + y^4 + 2x^2y^2 = 841$

$x^4 + y^4 + 2(2)^2 = 841$

$x^4 + y^4 = 841 - 8$

$x^4 + y^4 = 833$

18. Question

What must be added each of the following expression to make it a whole square?

(i) $4x^2 - 12x + 7$

(ii) $4x^2 - 20x + 20$

Answer

(i) $4x^2 - 12x + 7$

$(2x)^2 - 2(2x)(3) + 3^2 - 3^2 + 7$

$= (2x - 3)^2 - 9 + 7$

$= (2x - 3)^2 - 2$

Hence, 2 must be added to the expression in order to make a whole square

(ii) $4x^2 - 20x + 20$

$(2x)^2 - 2(2x)(5) + 5^2 - 5^2 + 20$

$= (2x - 5)^2 - 25 + 20$

$= (2x - 5)^2 - 5$

Hence, 5 must be added to the expression in order to make it a whole square

19. Question

Simplify:

(i) $(x - y)(x + y)(x^2 + y^2)(x^4 + y^4)$

(ii) $(2x - 1)(2x + 1)1(4x^2 + 1)(16x^4 + 1)$

(iii) $(7m - 8n)^2 + (7m + 8n)^2$

(iv) $(2.5p - 1.5q)^2 - (1.5p - 2.5q)^2$

(v) $(m^2 - n^2m)^2 + 2m^3n^2$

Answer

(i) $(x - y)(x + y)(x^2 + y^2)(x^4 + y^4)$

$(x^2 - y^2)(x^2 + y^2)(x^4 + y^4)$

$= [(x^2)^2 - (y^2)^2](x^4 + y^4)$

$= (x^4 - y^4)(x^4 - y^4)$

$= [(x^4)^2 - (y^4)^2]$

$= x^8 - y^8$

(ii) $(2x - 1)(2x + 1)1(4x^2 + 1)(16x^4 + 1)$

$[(2x)^2 - (1)^2](4x^2 + 1)(16x^4 + 1)$

$= (4x^2 - 1)(4x^2 + 1)(16x^4 + 1)\,1$

$= [(4x^2)^2 - (1)^2](16x^4 + 1)\,1$

$= (16x^4 - 1)(16x^4 + 1)\,1$

$= [(16x^4)^2 - (1)^2]\,1$

$= 256x^8 - 1$

(iii) $(7m - 8n)^2 + (7m + 8n)^2$

$(7m)^2 + (8n)^2 - 112mn + (7m)^2 + (8n)^2 + 112mn$

$= 49m^2 + 64n^2 + 49m^2 + 64n^2$

$= 98m^2 + 64n^2 + 64n^2$

$= 98m^2 + 128n^2$

(iv) $(2.5p - 1.5q)^2 - (1.5p - 2.5q)^2$

$(2.5p)^2 + (1.5q)^2 - 2(2.5p)(1.5q) - (1.5p)^2 - (2.5q)^2 + 2(1.5p)(2.5q)$

$= 6.25p^2 + 2.25q^2 - 2.25p^2 - 6.25q^2$

$= 4p^2 - 6.25q^2 + 2.25q^2$

$= 4p^2 - 4q^2$

$= 4(p^2 - q^2)$

(v) $(m^2 - n^2m)^2 + 2m^3n^2$

$(m^2)^2 - 2(m^2)(n^2)(m) + (n^2m)^2 + 2m^3n^2$

$= m^4 - 2m^3n^2 + (n^2m)^2 + 2m^3n^2$

$= m^4 + n^4m^2 - 2m^3n^2 + 2m^3n^2$

$= m^4 + m^2n^4$

$= m^2 (m^2 + n^4)$

20. Question

Show that:

(i) $(3x+7)^2 - 84x = (3x-7)^2$

(ii) $(9a-5b)^2 + 180ab = (9a+5b)^2$

(iii) $\left(\frac{4m}{3} - \frac{3n}{4}\right)^2 + 2mn = \frac{16m^2}{9} + \frac{9n^2}{16}$

(iv) $(4pq+3q)^2 - (4pq-3q)^2 = 48pq^2$

(v) $(a-b)(a+b) + (b-c)(b+c) + (c-a)(c+a) = 0$

Answer

(i) $(3x+7)^2 - 84x = (3x-7)^2$

L.H.S = $(3x + 7)^2 - 84x$

$= (3x)^2 + (7)^2 + 2 (3x) (7) - 84x$

$= (3x)^2 + (7)^2 + 42x - 84x$

$= (3x)^2 + (7)^2 - 42x$

$= (3x)^2 + (7)^2 - 2 (3x) (7)$

$= (3x - 7)^2$

= R.H.S

Hence, proved

(ii) $(9a-5b)^2 + 180ab = (9a+5b)^2$

L.H.S = $(9a - 5b)^2 + 180ab$

$= (9a)^2 + (5b)^2 - 2 (9a) (5b) + 180ab$

$= (9a)^2 6 (5b)^2 - 90ab + 180ab$

$= (9a)^2 + (5b)^2 + 9ab$

$= (9a)^2 + (5b)^2 + 2 (9a) (5b)$

$= (9a + 5b)^2$

= R.H.S

Hence, proved

(iii) $\left(\frac{4m}{3} - \frac{3n}{4}\right)^2 + 2mn = \frac{16m^2}{9} + \frac{9n^2}{16}$

L.H.S = $(\frac{4m}{3} - \frac{3n}{4})^2 + 2mn$

$= (\frac{4m}{3})^2 + (\frac{3n}{4})^2 - 2mn + 2mn$

$= (\frac{4m}{3})^2 + (\frac{3n}{4})^2$

$= \frac{16}{9}m^2 + \frac{9}{16}n^2$

= R.H.S

Hence, verified

(iv) $(4pq+3q)^2-(4pq-3q)^2=48pq^2$

L.H.S = $(4pq + 3q)^2 – (4pq – 3q)^2$

= $(4pq)^2 + (3q)^2 + 2\ (4pq)\ (3q) – (4pq)^2 – (3q)^2 + 24pq^2$

= $24pq^2 + 24pq^2$

= $48pq^2$

Hence, proved

(v) $(a-b)(a+b)+(b-c)(b+c)+(c-a)(c+a)=0$

L.H.S = (a – b) (a + b) + (b – c) (b + c) + (c – a) (c + a)

Using identity:

(a – b) (a + b) = $a^2 – b^2$

We get,

= $(a^2 – b^2) + (b^2 – c^2) + (c^2 – a^2)$

= $a^2 – b^2 + b^2 – c^2 + c^2 – a^2$

= 0

= R.H.S

Hence, verified

Exercise 6.7

1. Question

Find the following products:

(i) $(x+4)(x+7)$ (ii) $(x-11)(x+4)$

(iii) $(x+7)(x-5)$ (iv) $(x-3)(x-2)$

(v) $(y^2-4)(y^2-3)$ (vi) $\left(x+\frac{4}{3}\right)\left(x+\frac{3}{4}\right)$

(vii) $(3x+5)(3x+11)$ (viii) $(2x^2-3)(2x^2+5)$

(ix) $(z^2+2)(z^2-3)$ (x) $(3x-4y)(2x-4y)$

(xi) $(3x^2-4xy)(3x^2-3xy)$ (xii) $\left(x+\frac{1}{5}\right)(x+5)$

(xiii) $\left(z+\frac{3}{4}\right)\left(z+\frac{4}{3}\right)$ (xiv) $(x^2+4)(x^2+9)$

(xv) $(y^2+12)(y^2+6)$ (xvi) $(y^2 + \frac{5}{7})\ (y^2 - \frac{14}{5})$

(xvii) $(p^2+16)\left(p^2-\frac{1}{4}\right)$

Answer

(i) $(x+4)(x+7)$

x (x + 7) + 4 (x + 7)

$= x^2 + 7x + 4x + 28$

$= x^2 + 11x + 28$

(ii) $(x-11)(x+4)$

x (x + 4) – 11 (x + 4)

$= x^2 + 4x – 11x – 44$

$= x^2 – 7x - 44$

(iii) $(x+7)(x-5)$

x (x – 5) + 7 (x – 5)

$= x^2 – 5x + 7x – 35$

$= x^2 + 2x - 35$

(iv) $(x-3)(x-2)$

x (x – 2) – 3 (x – 2)

$= x^2 – 2x – 3x + 6$

$= x^2 – 5x + 6$

(v) $(y^2-4)(y^2-3)$

$y^2 (y^2 – 3) – 4 (y^2 – 3)$

$= y^4 – 3y^2 – 4y^2 + 12$

$= y^4 – 7y^2 + 12$

(vi) $\left(x+\frac{4}{3}\right)\left(x+\frac{3}{4}\right)$

$x (x + \frac{3}{4}) + \frac{4}{3} (x + \frac{3}{4})$

$= x^2 + \frac{3x}{4} + \frac{4x}{3} + \frac{12}{12}$

$= x^2 + \frac{3x}{4} + \frac{4x}{3} + 1$

$= x^2 + \frac{25x}{12} + 1$

(vii) $(3x+5)(3x+11)$

3x (3x + 11) + 5 (3x + 11)

$= 9x^2 + 33x + 15x + 55$

$= 9x^2 + 48x + 55$

(viii) $(2x^2-3)(2x^2+5)$

$2x^2 (2x^2 – 5) – 3 (2x^2 – 5)$

$= 4x^4 – 10x^2 – 6x^2 + 15$

$= 4x^4 – 16x^2 + 15$

(ix) $(z^2+2)(z^2-3)$

$z^2 (z^2 – 3) + 2 (z^2 – 3)$

$= z^4 – 3z^2 + 2z^2 – 6$

$= z^4 – z^2 - 6$

(x) $(3x – 4y)(2x – 4y)$

$3x (2x – 4y) – 4y (2x – 4y)$

$= 6x^2 – 12xy – 8xy + 16y^2$

$= 6x^2 – 20xy + 16y^2$

(xi) $(3x^2 – 4xy)(3x^2 – 3xy)$

$3x^2 (3x^2 – 3xy) – 4xy (3x^2 – 3xy)$

$= 9x^4 – 9x^3y – 12x^3y + 12x^2y^2$

$= 9x^4 – 21x^3y + 12x^2y^2$

(xii) $\left(x + \frac{1}{5}\right)(x + 5)$

$x (x + \frac{1}{5}) + 5 (x + \frac{1}{5})$

$= x^2 + \frac{x}{5} + 5x + 1$

$= x^2 + \frac{26}{5}x + 1$

(xiii) $\left(z + \frac{3}{4}\right)\left(z + \frac{4}{3}\right)$

$z (z + \frac{4}{3}) + \frac{3}{4} (z + \frac{4}{3})$

$= z^2 + \frac{4}{3}z + \frac{3}{4}z + \frac{12}{12}$

$= z^2 + \frac{4}{3}z + \frac{3}{4}z + 1$

$= z^2 + \frac{25}{12}z + 1$

(xiv) $(x^2 + 4)(x^2 + 9)$

$x^2 (x^2 + 9) + 4 (x^2 + 9)$

$= x^4 + 9x^2 + 4x^2 + 36$

$= x^4 + 13x^2 + 36$

(xv) $(y^2 + 12)(y^2 + 6)$

$y^2 (y^2 + 6) + 12 (y^2 + 6)$

$= y^4 + 6y^2 + 12y^2 + 72$

$= y^4 + 18y^2 + 72$

(xvi) $(y^2 + \frac{5}{7}) (y^2 - \frac{14}{5})$

$y^2 (y^2 - \frac{14}{5}) + \frac{5}{7} (y^2 - \frac{14}{5})$

$= y^4 - \frac{14}{5}y^2 + \frac{5}{7}y^2 - 2$

$= y^4 - \frac{73}{35}y^2 - 2$

(xvii) $\left(p^2 + 16\right)\left(p^2 - \frac{1}{4}\right)$

$p^2 (p^2 - \frac{1}{4}) + 16 (p^2 - \frac{1}{4})$

$= p^4 - \frac{1}{4}p^2 + 16p^2 - 4$

$= p^4 - \frac{63}{4}p^2 - 4$

2. Question

Evaluate the following:

(i) 102×106

(ii) 109×107

(iii) 35×37

(iv) 53×55

(v) 103×96

(vi) 34×36

(vii) 994×1006

Answer

(i) 102×106

(100 + 2) (100 + 6)

= 100 (100 + 6) + 2 (100 + 6)

= 10000 + 600 + 200 + 12

= 10812

(ii) 109×107

This can be written as:

(100 + 9) (100 + 7)

= 100 (100 + 7) + 9 (100 + 7)

= 10000 + 700 + 900 + 63

= 11663

(iii) 35×37

This can be written as:

(30 + 5) (30 + 7)

= 30 (30 + 7) + 5 (30 + 7)

= 900 + 210 + 150 + 35

= 1295

(iv) 53×55

This can be written as:

(50 + 3) (50 + 5)

= 50 (50 + 5) + 3 (50 + 5)

= 2500 + 250 + 150 + 15

= 2915

(v) 103×96

This can be written as:

(100 + 3) (100 - 4)

= 100 (100 - 4) + 3 (100 - 4)

= 10000 - 400 + 300 - 12

= 10000 – 112

= 9888

(vi) 34×36

This can be written as:

(30 + 4) (30 + 6)

= 30 (30 + 6) + 4 (30 + 6)

= 900 + 180 + 120 + 24

= 1224

(vii) 994×1006

This can be written as:

(1000 - 6) (1000 + 6)

= 1000 (1000 + 6) - 6 (1000 + 6)

= 1000000 + 6000 - 6000 - 36

= 999964