05

Circular Motion

When a particle moves in two dimensions or in a plane such that its distance from a fixed (or moving) point remains constant, then its motion is called *circular motion*. This fixed point is called the *centre* and the distance is called *radius of the circular path*. When the speed of the particle performing circular motion is constant, its motion is said to be *uniform circular motion*, if the speed of the particle performing circular motion changes with respect to time, its motion is said to be *non-uniform circular motion*.

Kinematics of Circular Motion

For a particle in circular motion, following variables are required to describe its motion.

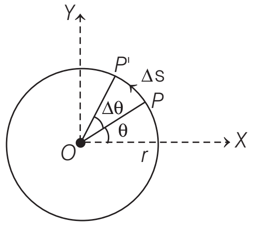
Angular Displacement

The angle between initial and final positions of particle in a given interval of time, is called angular displacement. In the given figure **OP** is the initial position and **OP′** is the final position of the particle. Then, angular displacement

$$\angle P'OP = \Delta\theta$$

It is a dimensionless quantity, as $\theta = \Delta s / r$.

Its SI unit is radian while practical unit is degree.



If a body makes n revolutions, its angular displacement $\theta = 2\pi n$ radian.

Note Angular displacement is a vector quantity, provided $\Delta\theta$ is small. Commutative law of vector addition is not valid for large $\Delta\theta$.

Angular Velocity

If the angular position of a particle changes with time, it is said to have angular velocity. If θ_1 and θ_2 are the angular positions of a particle at time t_1 and t_2 respectively, the average angular velocity ω_{av} for this time interval is defined as

$$\omega_{av} = \frac{\theta_2 - \theta_1}{t_2 - t_1} = \frac{\Delta\theta}{\Delta t}$$

and instantaneous angular velocity is defined as the limiting value of this ratio

i.e. $$\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$$

In case of uniform circular motion, $\omega = \omega_{av}$

Its unit is $rad s^{-1}$ and its dimensional formula is $[M^0 L^0 T^{-1}]$.

Important points regarding the angular velocity are given below.

(i) Average angular velocity is a scalar quantity but, instantaneous angular velocity is a vector quantity.

(ii) If a particle makes n rotations in t second.

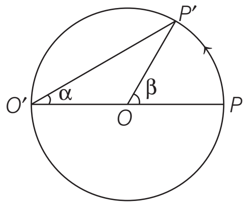
$$\omega_{av} = \left[\frac{2\pi n}{t}\right] \text{rad s}^{-1}$$

Hence, if T is the time period and f is the frequency of uniform circular motion.

$$\omega = \frac{2\pi \times 1}{T} = 2\pi f \qquad \left[\because f = \frac{1}{T}\right]$$

or $$T = 2\pi/\omega$$

(iii) Angular velocity depends on the point about which rotation is considered. For the figure given below,



$$\omega_o = \frac{\beta}{t} \text{ and } \omega_{o'} = \frac{\alpha}{t}$$

Relation between angular velocity and linear velocity

If linear velocity of particle performing circular motion is v and angular velocity is ω, then

$$v = r\omega$$

where, r is the radius of the circular path.

In vector form, $\mathbf{v} = \omega \times \mathbf{r}$

Note Linear velocity is always along the tangent to the circular path.

Relative angular velocity

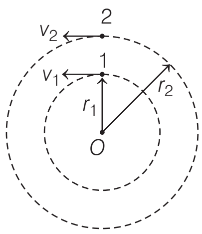
Relative angular velocity for two important cases are as follows

(i) When two particles are moving on same circle or different coplanar concentric circles in same direction with different angular velocities ω_1 and ω_2, respectively, the angular velocity of particle 2 relative to particle 1 for an observer at the centre will be

$$\omega_{rel} = \omega_2 - \omega_1$$

(ii) When two particles are moving on two different concentric circles with different angular velocities, the angular velocity of particle 2 relative to particle 1 as observed by 1 will depend on their positions and velocities.

If 1 and 2 are closest to each other and moving in the same direction, then



$$v_{rel} = v_2 - v_1$$
$$r_{rel} = r_2 - r_1$$

So, $$\omega_{rel} = \frac{v_{rel}}{r_{rel}} = \frac{v_2 - v_1}{r_2 - r_1}$$

Angular Acceleration (α)

If the angular speed of a particle is variable, the body is said to have an angular acceleration. Let ω_1 and ω_2 be the instantaneous angular speeds at times t_1 and t_2 respectively, then the *average angular acceleration* α_{av} is defined as

$$\alpha_{av} = \frac{\omega_2 - \omega_1}{t_2 - t_1} = \frac{\Delta\omega}{\Delta t}$$

The instantaneous angular acceleration is the limit of this ratio as Δt approaches to zero, *i.e.*

$$\alpha_{inst} = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t} = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$$

The SI unit of angular acceleration is rad s^{-2} and its dimensional formula is $[M^0 L^0 T^{-2}]$.

If $\alpha = 0$, circular motion is said to be uniform.

It has same characteristics as that of angular velocity.

Kinematics Equations for Circular Motion

If the angular acceleration (α) is constant, then the kinematic equations of motion are as follows:

(i) $\omega = \omega_0 + \alpha t$

(ii) $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$

(iii) $\omega^2 = \omega_0^2 + 2\alpha\theta$

(iv) $\theta = \left(\frac{\omega + \omega_0}{2}\right) t$

(v) $\theta_t = \omega_0 + \frac{1}{2}\alpha(2t - 1)$

where, ω_0 and ω are the initial and final angular velocities, respectively.

Radial and Tangential acceleration

In circular motion, acceleration of the particle can be resolved into two components

Radial acceleration It is directed along the radial direction towards the centre of the circular path. It is expressed as,

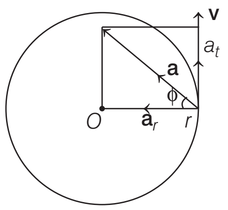
$$a_r = a_c = \frac{v^2}{r} = r\omega^2$$

It is also called centripetal acceleration or normal acceleration.

Tangential acceleration It is directed along the tangent to the circular path. Mathematically, it is equal to time rate of change of velocity.

It is expressed as, $\mathbf{a}_t = \frac{d\mathbf{v}}{dt} = \frac{d\omega}{dt} \times \mathbf{r} = \alpha \times \mathbf{r} \Rightarrow a_t = \alpha r$

Net acceleration The two components of acceleration are mutually perpendicular. So, the net acceleration is given as,



$\mathbf{a} = \frac{d\mathbf{v}}{dt} = \mathbf{a}_t + \mathbf{a}_r$, where $\mathbf{a}_t = \alpha \times \mathbf{r}$ and $\mathbf{a}_r = \omega \times \mathbf{v}$

$$\Rightarrow \qquad a = \sqrt{a_t^2 + a_r^2}$$

If a (acceleration) makes an angle ϕ with the radius, then $\tan\phi = \frac{a_t}{a_r}$.

Regarding circular motion, following possibilities exist

(a) If $a_r = 0$ and $a_t = 0$, then $a = 0$ and motion is uniform translatory.

(b) If $a_r = 0$ and $a_t \neq 0$, then $a = a_t$ and motion is accelerated translatory.

(c) If $a_r \neq 0$ but $a_t = 0$, then $a = a_r$ and motion is uniform circular.

(d) If $a_r \neq 0$ and $a_t \neq 0$, then $a = \sqrt{a_t^2 + a_r^2}$ and motion is non-uniform circular.

Example 1. *A clock has a continuously moving second's hand of 0.1 m length. The average acceleration of the tip of the hand (in ms^{-2}) is of the order of* **[JEE Main 2020]**

(a) 10^{-3}
(b) 10^{-4}
(c) 10^{-2}
(d) 10^{-1}

Sol. *(a)* Given that, radius, $R = 0.1$ m

Angular frequency, $\omega = \frac{2\pi}{T} = \frac{2\pi}{60} = 0.105$ rad/s

$\therefore$ Average acceleration, $a = \omega^2 R = (0.105)^2(0.1)$

$$= 1.102 \times 10^{-3} \text{ m/s}^2$$

and it is of the order of 10^{-3}.

Example 2. *A particle moves in a circle of radius $\left(\frac{20}{\pi}\right)$ m with constant tangential acceleration. If the speed of the particle is 80 m/s at the end of the second revolution after motion has begun, then the tangential acceleration is*

(a) $160\,\pi\ m/s^2$ (b) $40\,\pi\ m/s^2$
(c) $40\ m/s^2$ (d) $640\,\pi\ m/s^2$

Sol. (c) Given, radius $r = \frac{20}{\pi}$ m, tangential acceleration a_t = constant, velocity $v = 80$ m/s and initial angular velocity $\omega_0 = 0$,

Final angular velocity, $\omega_f = \frac{v}{r} = \frac{80}{20/\pi} = 4\pi$ rad/s

Angular displacement, $\theta = 2\pi \times 2 = 4\pi$

From 3rd equation of motion,

$$\omega^2 = \omega_0^2 + 2\alpha\theta$$

$$\Rightarrow \qquad (4\pi)^2 = 0^2 + 2 \times \alpha \times (4\pi)$$

$$\alpha = 2\pi \text{ rad/s}^2$$

Also, $a = \alpha r = 2\pi \times \frac{20}{\pi} = 40$ m/s^2

Example 3. *The speed of a particle moving in a circle of radius r = 2 m varies with time t as $v = t^2$ where, t is in second and v in ms^{-1}. The net acceleration at t = 2 s is*

(a) $\sqrt{40}\ ms^{-2}$ (b) $\sqrt{60}\ ms^{-2}$ (c) $\sqrt{80}\ ms^{-2}$ (d) $10\ ms^{-2}$

Sol. (c) Linear speed of particle at $t = 2$ s is

$$v = (2)^2 = 4 \text{ ms}^{-1}$$

$\therefore$ Radial acceleration, $a_r = \frac{v^2}{r} = \frac{(4)^2}{2} = 8 \text{ ms}^{-2}$

The tangential acceleration is

$$a_t = \frac{dv}{dt} = 2t$$

At $t = 2$ s $\qquad a_t = (2)(2) = 4 \text{ ms}^{-2}$

Net acceleration of the particle at $t = 2$ s is

$$a = \sqrt{(a_r)^2 + (a_t)^2} = \sqrt{(8)^2 + (4)^2}$$

or $\qquad a = \sqrt{80} \text{ ms}^{-2}$

Example 4. *In a circular motion of a body, which amongst the following relation between tangential acceleration and linear velocity is correct?*

(a) $a_t = \mathbf{a} \cdot \mathbf{v}$ (b) $a_t = \mathbf{v}/\mathbf{a}$ (b) $a_t = \mathbf{a}/\mathbf{v}$ (d) $a_t = \frac{\mathbf{a} \cdot \mathbf{v}}{|\mathbf{v}|}$

Sol. (d) Let velocity of the particle be, $\mathbf{v} = v_x\hat{\mathbf{i}} + v_y\hat{\mathbf{j}}$

Acceleration, $\mathbf{a} = \frac{dv_x}{dt}\hat{\mathbf{i}} + \frac{dv_y}{dt}\hat{\mathbf{j}}$

Component of $\mathbf{a}$ along $\mathbf{v}$ will be, $\frac{\mathbf{a} \cdot \mathbf{v}}{|\mathbf{v}|} = \frac{v_x \frac{dv_x}{dt} + v_y \cdot \frac{dv_y}{dt}}{\sqrt{v_x^2 + v_y^2}}$ ...(i)

Further, tangential acceleration of particle is rate of change of speed.

or $$a_t = \frac{dv}{dt} = \frac{d}{dt}\left(\sqrt{v_x^2 + v_y^2}\right)$$

or $$a_t = \frac{1}{2\sqrt{v_x^2 + v_y^2}}\left[2v_x \cdot \frac{dv_x}{dt} + 2v_y \frac{dv_y}{dt}\right]$$

or $$a_t = \frac{v_x \cdot \frac{dv_x}{dt} + v_y \cdot \frac{dv_y}{dt}}{\sqrt{v_x^2 + v_y^2}} \quad \text{...(ii)}$$

From Eqs. (i) and (ii), we can see that

$$a_t = \frac{\mathbf{a} \cdot \mathbf{v}}{|\mathbf{v}|}$$

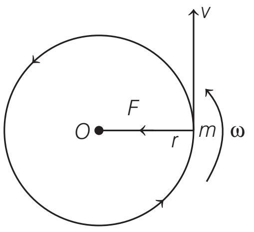
or Tangential acceleration = component of acceleration along velocity.

Dynamics of Circular Motion

In circular motion of a particle, there are two kinds of forces that occur. They are described below

Centripetal Force

When a particle perform circular motion, it is acted upon by a force directed along the radius towards the centre of the circle. This force is called the centripetal force.



If m is the mass of the particle, then the centripetal force is given by

$$\mathbf{F} = -\frac{mv^2}{r}\,\hat{\mathbf{r}} = -mr\omega^2\hat{\mathbf{r}}$$

where $\hat{\mathbf{r}}$ is the unit vector acting along $\mathbf{r}$.

Important Points

- In non-uniform circular motion, the particle simultaneously possesses two forces

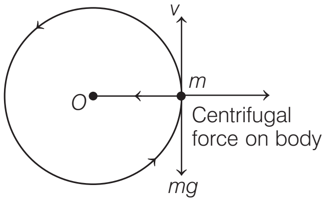
  Centripetal force, $F_c = ma_c = \frac{mv^2}{r} = mr\omega^2$

  Tangential force, $F_t = ma_t$

  $\therefore$ Net force, $F_{\text{net}} = ma = m\sqrt{a_c^2 + a_t^2}$
- If a moving particle comes to stand still, *i.e.* the particle will move along the radius towards the centre and if radial acceleration a_r is zero, the body will fly off along the tangent. So, a tangential velocity and a radial acceleration (hence force) is always present in uniform circular motion.

Centrifugal Force

It can be defined as the radially directed outward force acting on a body in circular motion, as observed by a person moving with the body.



Centrifugal force is given as, $\mathbf{F} = \frac{mv^2}{r}\,\hat{\mathbf{r}} = mr\omega^2\hat{\mathbf{r}}$

Also,

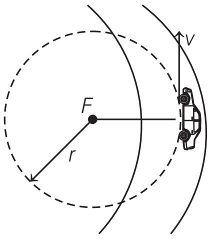
(i) In an inertial frame, the centrifugal force does not act on the object.

(ii) In non-inertial rotating frames, pseudo force arises as centrifugal force and need to be considered.

Circular Turning of Roads

When vehicles go through turns, they travel along a nearly circular arc. There must be some force which will produce the required centripetal acceleration. If the vehicles travel on a horizontal circular path, this resultant force is also horizontal. The necessary centripetal force is being provided to the vehicles by following three ways:

(i) **By friction only** For a car of mass m moving with a speed v in a horizontal circular arc of radius r, the necessary centripetal force to the car will be provided by force of friction f acting towards centre.



Thus, $$f = \frac{mv^2}{r}$$

Further, limiting value of f,

$$f_L = \mu N = \mu\, mg$$

Therefore, for a safe turn without sliding

$$\frac{mv^2}{r} \le f_L$$

or $$\frac{mv^2}{r} \le \mu\, mg$$

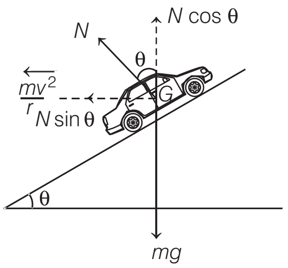
$$\mu \ge \frac{v^2}{rg} \quad \text{or} \quad v \le \sqrt{\mu rg}$$

Here, two situations may arise. If μ and r are known to us, the speed of the vehicle should not exceed $\sqrt{\mu rg}$ and if v and r are known to us, the coefficient of friction should be greater than $\frac{v^2}{rg}$.

Note If the speed of the car is too high, car starts skidding outwards. Due to this, radius of the circle increases or the necessary centripetal force is reduced $\left(\text{centripetal force} \propto \frac{1}{r}\right)$.

(ii) **By banking of roads only** Friction is not always reliable at circular turns, if high speed and sharp turns are involved. To avoid dependence on friction, the roads are banked at the turn so that the outer part of the road is somewhat raised compared to the inner part.

$\therefore$ For safe turn without sliding,

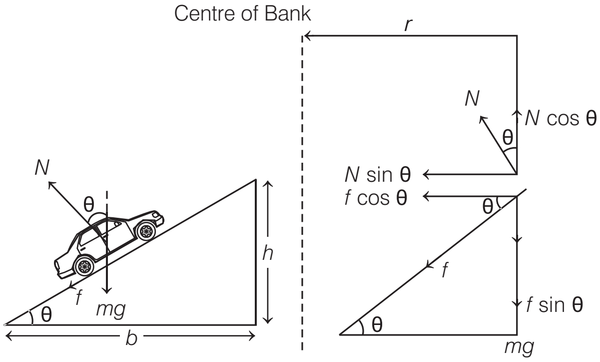


$$N\sin\theta = \frac{mv^2}{r} \text{ and } N\cos\theta = mg$$

From these two equations, we get

$$\tan\theta = \frac{v^2}{rg} \text{ or } v = \sqrt{rg\tan\theta}$$

(iii) **By friction and banking of roads both** If on a banked circular turning, there is a frictional force between car and road, then the vector sum of normal reaction force and frictional force provides the necessary centripetal force.



$$N\sin\theta + f\cos\theta = \frac{mv^2}{r} \quad \text{...(i)}$$

$$N\cos\theta = mg + f\sin\theta \quad \text{...(ii)}$$

($\because$ vertical force is balanced)

Taking limiting condition, we can write

$$f = \mu_s N \quad \text{...(iii)}$$

To obtain the value of N, Solve above three equations properly.

$$N = \frac{mg}{\cos\theta - \mu_s \sin\theta}$$

After putting the value of N in Eq. (i), we get

$\therefore$ For a car moving upward on inclined road, the maximum speed for no skidding is given as,

$$v_{\max} = \left[\frac{rg(\sin\theta + \mu_s\cos\theta)}{\cos\theta - \mu_s\sin\theta}\right]^{1/2} = \left[\frac{rg(\mu_s + \tan\theta)}{1 - \mu_s\tan\theta}\right]^{1/2}$$

Example 5. *Find the maximum speed at which a car can turn round a curve of 30 m radius on a level road, if the coefficient of friction between the tyres and the road is 0.4. (acceleration due to gravity = 10 ms^{-2})*

(a) 12 ms^{-1} (b) 10 ms^{-1} (c) 11 ms^{-1} (d) 15 ms^{-1}

Sol. (c) We know that for turning, centripetal force is provided by friction, so

$$\frac{mv^2}{r} \le f_L$$

As, $f_L = \mu N = \mu mg$ (where, $N = mg$)

Thus, $\frac{mv^2}{r} \le \mu mg$

i.e., $v \le \sqrt{\mu gr}$, so that $v_{\max} = \sqrt{\mu gr}$

Here, $\mu = 0.4, r = 30$ m

and $g = 10$ ms^{-2}

So, $v_{\max} = \sqrt{0.4 \times 30 \times 10} = 10.95 \simeq 11 \text{ ms}^{-1}$

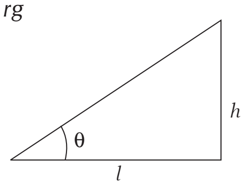
Example 6. *A train has to negotiate a curve of radius 2000 m. By how much should the outer rail be raised with respect to inner rail for a speed of 72 km h^{-1}. The distance between the rails is 1 m.*

(a) 4 cm (b) 5 cm
(c) 2 cm (d) 1 cm

Sol. (c) Given, $v = 72 \text{ km h}^{-1} = 72 \times \frac{5}{18} = 20 \text{ ms}^{-1}$

$l = 1$ m, $r = 2000$ m, $g = 10$ ms^{-2}

We have, $\tan\theta = \frac{v^2}{rg}$



Also, $\tan\theta = \frac{h}{l}$

$\Rightarrow \frac{v^2}{rg} = \frac{h}{l} \Rightarrow h = \frac{v^2 l}{rg}$

$\therefore h = \frac{(20)^2 \times 1}{2000 \times 10} = \frac{1}{50} \text{ m} = \frac{100}{50} = 2 \text{ cm}$

Example 7. *A circular race track of radius 300 m is banked at angle of 15°. If the coefficient of friction between the wheels of a race-car and the road is 0.2, then the maximum permissible speed to avoid slipping is*

(a) 28.1 ms^{-1} (b) 50 ms^{-1} (c) 38.1 ms^{-1} (d) 42 ms^{-1}

Sol. (c) Here, $r = 300$ m, $\theta = 15°$, $\mu_s = 0.2$, $g = 10$ ms^{-2},

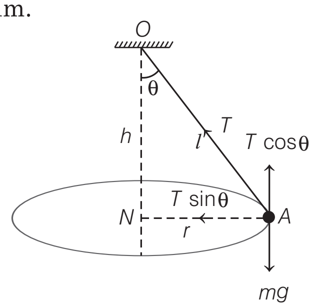
hence, maximum permissible speed to avoid slipping,

$$V_{\max} = \left[\frac{rg(\mu_s + \tan\theta)}{1-\mu_s \tan\theta}\right]^{1/2} = \left[\frac{300 \times 10[0.2 + \tan 15°]}{1 - 0.2 \tan 15°}\right]$$

We obtain, $v_{\max} = 38.1 \text{ms}^{-1}$

Conical Pendulum

It consists of a string OA, whose upper end O is fixed and bob of mass m is tied at the other free end which is whirled in a horizontal circle. As the string traces the surface of the cone, and such arrangement is called a conical pendulum.



In this case, vertical component of tension T balances the weight of the bob and horizontal component provides the necessary centripetal force.

$\therefore$ Angular speed, $\omega = \sqrt{\dfrac{g\tan\theta}{r}}$

and time period of pendulum,

$$\tau = 2\pi\sqrt{\frac{l\cos\theta}{g}} = 2\pi\sqrt{\left(\frac{h}{g}\right)}$$

where, $h = ON = l\cos\theta$

Example 8. *A hemispherical bowl of radius R is rotating about its axis of symmetry which is kept vertical. A small ball kept in the bowl rotates with the bowl without slipping on its surface. If the surface of the bowl is smooth and the angle made by the radius through the ball with the vertical is α, find the angular speed at which bowl is rotating.*

(a) $\omega = \sqrt{\dfrac{R}{g\cos\alpha}}$ (b) $\omega = \sqrt{\dfrac{g}{\cos\alpha}}$

(c) $\sqrt{\dfrac{g}{R\cos\alpha}}$ (d) None of these

Sol. (c) Let ω be the angular speed of rotation of the bowl. Two forces are acting on the ball.

(i) Normal reaction, N (ii) Weight, mg

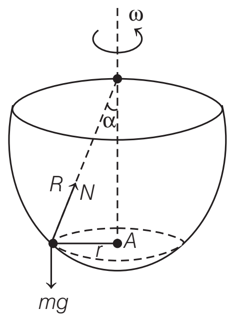
The ball is rotating in a circle of radius, $r = (R\sin\alpha)$ with centre at A with an angular speed, ω

Hence, $N\sin\alpha = mr\omega^2$

$\Rightarrow$ $N\sin\alpha = m(R\sin\alpha)\omega^2$ $[\because r = R\sin\alpha]$

$\Rightarrow$ $N = mR\omega^2$ ...(i)

and $N\cos\alpha = mg$ ...(ii)



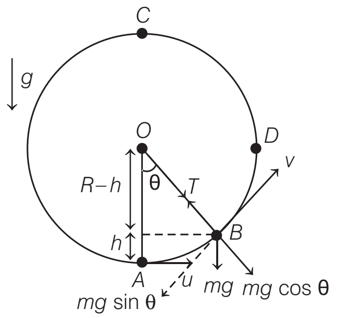
Dividing Eq. (i) by Eq. (ii), we get

$$\frac{1}{\cos\alpha} = \frac{\omega^2 R}{g}$$

$$\therefore \quad \omega = \sqrt{\frac{g}{R\cos\alpha}}$$

Motion in a Vertical Circle

This is an example of non-uniform circular motion. Consider a particle of mass m attached to a string of length R to be whirled in a vertical circle about a fixed point O.



(i) **Velocity at any point on vertical loop**

At the lowest point A, it is imparted a velocity u in the horizontal direction.

Let v be its velocity at any point B at height h, is given by

$$v = \sqrt{u^2 - 2gh} = \sqrt{u^2 - 2gR(1-\cos\theta)}$$

(ii) **Tension at any point on the vertical loop**,

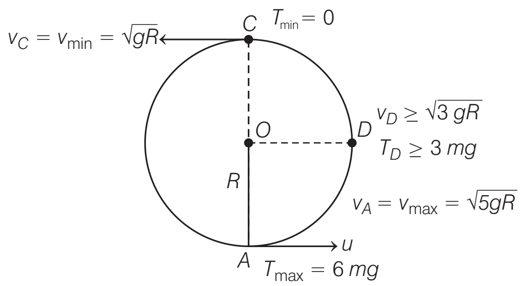
At any point B, the necessary centripetal force is provided by the resultant of tension T and $mg\cos\theta$, so

$$T_B = mg\cos\theta + \frac{mv^2}{R} = \frac{m}{R}(u^2 + gR - 3gh)$$

(iii) **Tension at the lowest point A and highest point C**,

$$T_A = T_{\max}$$
$$T_C = T_{\min} = 0$$

Velocity at the highest point C



(iv) The particle will move on the circular path only and only if,

$$T_{min} > 0 \quad \text{and} \quad T_{max} \geq 6\ mg$$

So, for looping the circle or completing the loop, velocity at the lowest point should be $v_A \geq \sqrt{5gR}$ and at the highest point $v_C \geq \sqrt{gR}$

Note If $T_{min} \leq 0$, the string will slack and the particle will fall down instead of moving on the circle.

(v) In case of looping the circle for horizontal position of string,

$$v_D \geq \sqrt{3gR} \quad \text{and} \quad T_D \geq 3\ mg$$

(vi) **For oscillations,**

$$\frac{v_A^2}{2g} < \frac{v_A^2 + gR}{3g} \quad \text{or} \quad v_A < \sqrt{2gR}$$

So, if the velocity of the particle at the lowest point is $0 > v_A < \sqrt{2gR}$, then the particle will oscillate about the lowest position.

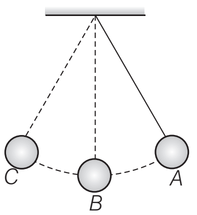
(vii) **For leaving the circle,**

$$\frac{v_A^2 + gR}{3g} < \frac{v_A^2}{3g} \quad \text{or} \quad v_A > \sqrt{2gR}$$

So, if the velocity of the particle at the lowest point is $\sqrt{2gR} < v_A > \sqrt{5gR}$, then the particle will move along the circle for $\theta > 90°$ and will not reach the point C but will leave the circle somewhere between $90° < \theta < 180°$.

Note (i) Instead of string, if a light rod is used, then the particle will complete the circular path if the velocity given at the lowest position is $\sqrt{4gR}$.

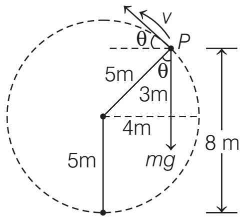
(ii) Oscillation of a pendulum is a part of circular motion. At points A and C, since velocity is zero, net centripetal force will be zero. Only tangential force is present. From A to B or C to B, speed of the bob increases. Therefore, tangential force is parallel to velocity. From B to A or B to C, speed of the bob decreases. Hence, tangential force is antiparallel to velocity.



Example 9. *A particle is suspended from a fixed point by a string of length 5 m. It is projected from the equilibrium position with such a velocity that the string slackens after the particle has reached a height 8 m above the lowest point. Find the velocity of the particle, just before the string slackens.*

(a) 5.42 m/s (b) 6.24 m/s (c) 10.26 m/s (d) 9.28 m/s

Sol. (a) At P,



$$T = 0$$

$\therefore \quad mg\cos\theta = \frac{mv^2}{R}$

or $\quad g\cos\theta = \frac{v^2}{R}$

or $\quad (9.8)\left(\frac{3}{5}\right) = \frac{v^2}{5}$

$\therefore \quad v = 5.42\ \text{m/s}$

Example 10. *A stone of mass 1 kg tied to a light in extensible string of length $L = \frac{10}{3}$ m is whirling in a circular path of radius L in a vertical plane. The ratio of the maximum tension in the string to the minimum tension in the string is 4 and if g is taken to be 10 m/s^2. The speed of stone at the highest point of the circular is* **[JEE Main 2019]**

(a) 20 m/s (b) $10\sqrt{3}$ m/s (c) $5\sqrt{2}$ m/s (d) 10 m/s

Sol. (d) Since the maximum tension T_B in the string moving in the vertical circle is at the bottom and minimum tension T_T is at the top.

$\therefore \quad T_B = \frac{mv_B^2}{L} + mg$

and $\quad T_T = \frac{mv_T^2}{L} - mg$

$\therefore \quad \frac{T_B}{T_T} = \frac{\frac{mv_B^2}{L} + mg}{\frac{mv_T^2}{L} - mg} = \frac{4}{1}$

or $\quad \frac{v_B^2 + gL}{v_T^2 - gL} = \frac{4}{1}$

or $\quad v_B^2 + gL = 4v_T^2 - 4gL$

but $\quad v_B^2 = v_T^2 + 4gL$

$\therefore \quad v_T^2 + 4gL + gL = 4v_T^2 - 4gL$

$\Rightarrow \quad 3v_T^2 = 9gL$

$\therefore \quad v_T^2 = 3 \times g \times L = 3 \times 10 \times \frac{10}{3}$

or $\quad v_T = 10\ \text{m/s}$

Practice Exercise

ROUND I Topically Divided Problems

Kinematics of Circular Motion

1. A wheel rotates with a constant angular velocity of 300 rpm. The angle with which the wheel rotates in one second is

(a) π rad (b) 5π rad
(c) 10π rad (d) 20π rad

2. A fan is making 600 revolutions per minute. If after some time, it makes 1200 revolution per minute, then increase in its angular velocity is

(a) 10π rad/s (b) 20π rad/s
(c) 40π rad/s (d) 60π rad/s

3. The radius of the Earth's orbit around the Sun is 1.5×10^{11} m. The linear velocity of the Earth is

(a) 1.99×10^{-7} ms^{-1} (b) 2.99×10^{-7} ms^{-1}
(c) 1.99×10^{4} ms^{-1} (d) 2.99×10^{4} ms^{-1}

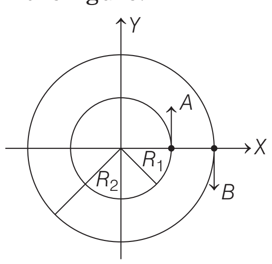
4. The length of second's hand in a watch is 1 cm. The change in velocity of its tip in 15 s is

(a) zero (b) $\frac{\pi}{30\sqrt{2}}$ cm/s
(c) $\frac{\pi}{30}$ cm/s (d) $\frac{\pi\sqrt{2}}{30}$ cm/s

5. A particle is moving along a circular path with a constant speed of 10 ms^{-1}. What is the magnitude of the change in velocity of the particle, when it moves through an angle of 60° around the centre of the circle? **[JEE Main 2019]**

(a) $10\sqrt{2}$ m/s (b) 10 m/s
(c) $10\sqrt{3}$ m/s (d) Zero

6. Two particles A and B are moving on two concentric circles of radii R_1 and R_2 with equal angular speed ω. At $t = 0$, their positions and direction of motion are shown in the figure.



The relative velocity $\mathbf{v}_A - \mathbf{v}_B$ at $t = \frac{\pi}{2\omega}$ is given by **[JEE Main 2019]**

(a) $\omega(R_1 + R_2)\hat{\mathbf{i}}$ (b) $-\omega(R_1 + R_2)\hat{\mathbf{i}}$
(c) $\omega(R_1 - R_2)\hat{\mathbf{i}}$ (d) $\omega(R_2 - R_1)\hat{\mathbf{i}}$

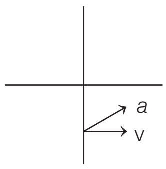
7. A car-wheel is rotated to uniform angular acceleration about its axis, initially its angular velocity is zero. It rotates through an angle θ_1 in the first 2 s, in the next 2 s, it rotates through an additional angle θ_2, the ratio of $\frac{\theta_2}{\theta_1}$ is

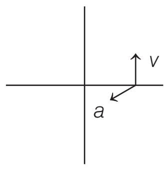
(a) 1 (b) 2 (c) 3 (d) 5

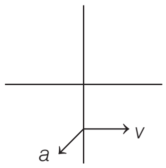
8. The speed of revolution of a particle moving round a circle is doubled and its angular speed is halved. Then, centripetal acceleration will be

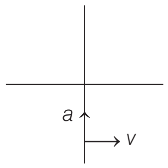
(a) unchanged
(b) halved
(c) doubled
(d) 4 times

9. A particle is undergoing uniformly accelerated circular motion with angular retardation π rad/s^2. If the angular velocity of the particle at $t = 0$ is 2π rad/s, the velocity and acceleration vectors of the body at $t = 0$ s are best represented by

(a)


(b)


(c)


(d)


10. An aircraft executes a horizontal loop of radius 1 km with a speed of 900 km/h, then the ratio of its centripetal acceleration and the acceleration due to gravity will be

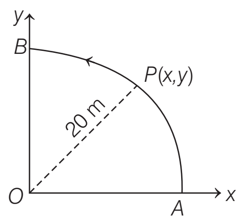
(a) 6 (b) 7
(c) 8 (d) 5

11. A stone tied to the end of a string 80 cm long is whirled in a horizontal circle with a constant speed. If the stone makes 14 revolutions in 25 s, what is the magnitude and direction of acceleration of the stone?
(a) $9.9\ m/s^2$ along the tangent
(b) $7.9\ m/s^2$ along the radius
(c) $9.9\ m/s^2$ along the radius
(d) None of the above

12. Read the following statements and choose the correct option given below.
(i) The net acceleration of a particle in the circular motion is always along the radius of the circle towards the centre.
(ii) The velocity vector of a particle at a point is always along the tangent to the path of the particle at that point.
(iii) The acceleration vector of a particle in uniform circular motion averaged over one cycle is a null vector.
(a) (i) and (iii) (b) (ii) and (iii)
(c) (iii) Only (d) All the three

13. A car is moving along a circular path of radius 500 m with a speed of $30\ ms^{-1}$. If at some instant, its speed increases at the rate of $2\ ms^{-1}$, then at that instant, the magnitude of resultant acceleration will be
(a) $4.7\ ms^{-2}$ (b) $3.8\ ms^{-2}$
(c) $3\ ms^{-2}$ (d) $2.7\ ms^{-2}$

14. A point P moves in counter-clockwise direction on a circular path as shown in the figure. The movement of P is such that it sweeps out a length $s = t^3 + 5$, where s is in metre and t is in second. The radius of the path is 20 m. The acceleration of P when $t = 2$ s is nearly

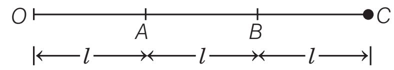


(a) $13\ ms^{-2}$ (b) $12\ ms^{-2}$
(c) $7.2\ ms^{-2}$ (d) $14\ ms^{-2}$

Dynamics of Circular Motion

15. A coin placed on a rotation turn table stops when it is placed at a distance of 9 cm from the centre. If the angular velocity of the turn table is trippled. It will just slip, if its distance from the centre is
(a) 27 cm (b) 9 cm
(c) 3 cm (d) 1 cm

16. Three identical particles are joined together by a thread as shown in figure. All the three particles are moving in a horizontal plane. If the velocity of the outermost particle is v_0, then the ratio of tensions in the three sections of the string is



(a) 3 : 5 : 7 (b) 3 : 4 : 5 (c) 7 : 11 : 6 (d) 3 : 5 : 6

17. Two particles of equal mass are connected to a rope AB of negligible mass such that one is at end A and other dividing the length of rope in the ratio 1 : 2 from B. The rope is rotated about end B in a horizontal plane. Ratio of tensions in the smaller part to the other is (ignore the effect of gravity)
(a) 4 : 3 (b) 1 : 4 (c) 1 : 2 (d) 1 : 3

18. A coin is placed on a gramophone record rotating at a speed of 45 rpm. It flies away when the rotational speed is 50 rpm. If two such coins are placed over the other on the same record, both of them will fly away when rotational speed is
(a) 100 rpm (b) 25 rpm (c) 12.5 rpm (d) 50 rpm

19. A body moves along a circular path of radius 5 m. The coefficient of friction between the surface of path and the body is 0.5. The angular velocity, in rad/s, with which the body should move so that it does not leave the path is (Take, $g = 10\ ms^{-2}$)
(a) 4 (b) 3 (c) 2 (d) 1

20. A motorcycle moving with a velocity of $72\ kmh^{-1}$ on a flat road takes a turn on the road at a point where the radius of curvature of the road is 20 m. The acceleration due to gravity is $10\ ms^{-2}$. In order to avoid skidding, he must not bent with respect to the vertical plane by an angle greater than
(a) $\theta = \tan^{-1}(2)$ (b) $\theta = \tan^{-1}(6)$
(c) $\theta = \tan^{-1}(4)$ (d) $\theta = \tan^{-1}(25.92)$

21. **Statement I** A cyclist is moving on an unbanked road with a speed of $7\ kmh^{-1}$ and takes a sharp circular turn along a path of radius of 2m without reducing the speed. The static friction coefficient is 0.2. The cyclist will not slip and pass the curve (Take, $g = 9.8\ m/s^2$)

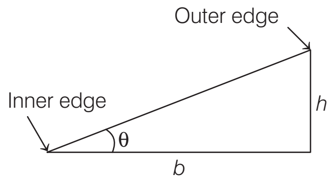
Statement II If the road is banked at an angle of 45°, cyclist can cross the curve of 2m radius with the speed of $18.5\ kmh^{-1}$ without slipping.

In the light of the above statements, choose the correct answer from the options given below.
[JEE Main 2021]
(a) Statement I is incorrect and statement II is correct.
(b) Statement I is correct and statement II is incorrect.
(c) Both statements I and II are false.
(d) Both statements I and II are true.

22. A curved road of 50 m radius is banked at correct angle for a given speed. If the speed is to be doubled keeping the same banking angle, the radius of curvature of the road should be changed to

(a) 25 m (b) 100 m
(c) 150 m (d) 200 m

23. A vehicle is moving with a velocity v on a curved road of width b and radius of curvature R. For counteracting the centrifugal force on the vehicle, the difference in elevation required in between the outer and inner edges of the rod is

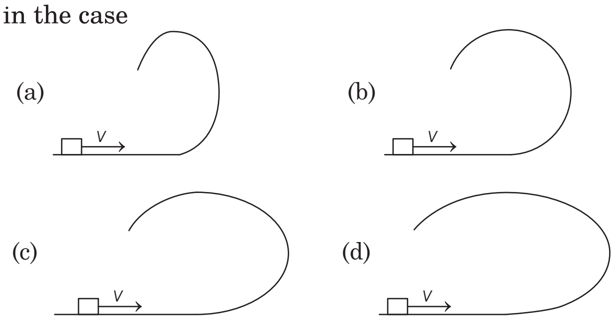


(a) v^2b/Rg (b) vb/Rg
(c) vb^2/Rg (d) vb/R^2g

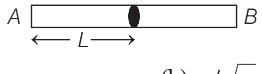
24. An object is being weighed on a spring balance moving around a curve of radius 100 m at a speed 7 ms^{-1}. The object has a weight of 60 kg-wt. The reading registered on the spring balance would be

(a) 60.075 kg-wt (b) 60.125 kg-wt
(c) 60.175 kg-wt (d) 60.225 kg-wt

25. A small block is shot into each of the four tracks shown below. Each of the tracks rises to the same height. The speed with which the block enters the track is the same in all cases. At the highest point of the track, the normal reaction is the maximum in the case

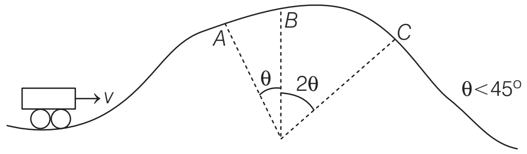


26. A long horizontal rod has a bead, which can slide along its length and initially placed at a distance L from one end A of the rod. The rod is set in angular acceleration α. If the coefficient of friction, between the rod and the bead is μ and gravity is neglected, then the time after which the bead starts slipping is



(a) $\sqrt{\mu/\alpha}$ (b) $\mu/\sqrt{\alpha}$
(c) $1/\sqrt{\mu\alpha}$ (d) $\mu\alpha$

27. A self-propelled vehicle (assume it as a point mass) runs on a track with constant speed v. It passes through three positions A, B and C on the circular part of the track. Suppose N_A, N_B and N_C are the normal forces exerted by the track on the vehicle when it is passing through points A, B and C respectively, then



(a) $N_A = N_B = N_C$
(b) $N_B > N_A > N_C$
(c) $N_C > N_A > N_B$
(d) $N_B > N_C > N_A$

28. A car moving on a circular path and takes a turn. If R_1 and R_2 be the reactions on the inner and outer wheels respectively, then

(a) $R_1 = R_2$ (b) $R_1 < R_2$
(c) $R_1 > R_2$ (d) $R_1 \geq R_2$

Conical Pendulum and Motion in Vertical Circle

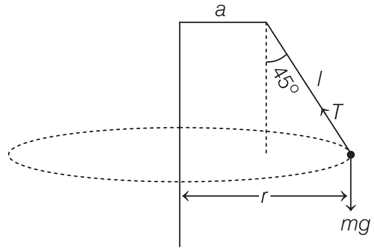
29. A sphere of mass 0.2 kg is attached to an inextensible string of length 0.5 m whose upper end is fixed to the ceilling. The sphere is made to describe a horizontal circle of radius 0.3 m. The speed of the sphere will be

(a) 1.5 ms^{-1} (b) 2.5 ms^{-1}
(c) 3.2 ms^{-1} (d) 4.7 ms^{-1}

30. A particle describes a horizontal circle in a conical funnel whose inner surface is smooth with speed of 0.5 m/s. What is height of the plane of circle from vertex of the funnel?

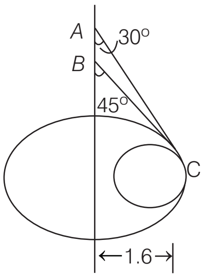
(a) 0.25 m (b) 2 cm
(c) 4 cm (d) 2.5 cm

31. How many revolutions per minute must the apparatus shown in figure make about a vertical axis, so that the cord makes an angle of 45° with the vertical? (Given, $l = \sqrt{2}$ m, $a = 0.2$ m, $m = 5$ kg)



(a) 6 rpm (b) 27.3 rpm
(c) 29 rpm (d) 12.6 rpm

32. Two wires AC and BC are tied at C of small sphere of mass 5 kg, which revolves at a constant speed v in the horizontal circle of radius 1.6 m. The minimum value of v is



(a) 3.01 ms^{-1} (b) 4.01 ms^{-1}
(c) 8.2 ms^{-1} (d) 3.96 ms^{-1}

33. The string of a pendulum of length l is displaced through 90° from the vertical and released. Then, the minimum strength of the string in order to withstand the tension as the pendulum passes through the mean position is

(a) mg (b) $6mg$ (c) $3mg$ (d) $5mg$

34. An insect is at the bottom of a hemispherical ditch of radius 1 m. It crawls up the ditch but starts slipping after it is at height h from the bottom. If the coefficient of friction between the ground and the insect is 0.75, then h is (Take, $g = 10\,ms^{-2}$)

[JEE Main 2020]

(a) 0.20 m (b) 0.45 m (c) 0.60 m (d) 0.80 m

35. A fighter plane is pulling out for a dive at 900 kmh^{-1} in a vertical circle of radius 2 km. Its mass is 5000 kg. Find the force exerted by the air on it at the lowest point.

(a) 2.0625×10^4 N upward
(b) 2.0625×10^5 N downward
(c) 2.0625×10^5 N upward
(d) 2.0625×10^4 N downward

36. A stone of mass m is tied to a string and is moved in a vertical circle of radius r making n revolution per minute. The total tension in the string when the stone is at its lowest point is

(a) mg (b) $m(g + \pi n r^2)$
(c) $m(g + \pi n r)$ (d) $m\{g + (\pi^2 n^2 r)/900\}$

37. A 2 kg stone at the end of a string 1 m long is whirled in a vertical circle at a constant speed. The speed of the stone is 4 m/s. The tension in the string will be 52 N, when the stone is

(a) at the top of the circle
(b) at the bottom of the circle
(c) halfway down
(d) None of the above

38. A body crosses the topmost point of a vertical circle with critical speed. What will be its acceleration when the string is horizontal?

(a) g (b) $2g$
(c) $3g$ (d) $6g$

39. A particle is moving in a vertical circle. The tensions in the string when passing through two positions at angles 30° and 60° from vertical (lowest position) are T_1 and T_2 respectively.

(a) $T_1 = T_2$
(b) $T_2 > T_1$
(c) $T_1 > T_2$
(d) Tension in the string always remains the same

40. A stone tied to a string of length L is whirled in a vertical circle, with the other end of the string at the centre. At a certain instant of time, the stone is at its lowest position, and has a speed u. The magnitude of change in its velocity as it reaches a position, where the string is horizontal is

(a) $\sqrt{u^2 - 2gL}$ (b) $\sqrt{2gL}$
(c) $\sqrt{u^2 - gL}$ (d) $\sqrt{2(u^2 - gL)}$

41. A stone of mass 1 kg is tied to a string 4 m long and is rotated at constant speed of 40 ms^{-1} in a vertical circle. The ratio of the tension at the top and the bottom is

(a) 11 : 12 (b) 39 : 41
(c) 41 : 39 (d) 12 : 11

42. An object is tied to a string and rotated in a vertical circle of radius r. Constant speed is maintained along the trajectory. If $T_{max}/T_{min} = 2$, then v^2/rg is

(a) 1 (b) 2
(c) 3 (d) 4

ROUND II Mixed Bag

Only One Correct Option

1. A block of 200 g mass moves with a uniform speed in a horizontal circular groove, with vertical side walls of radius 20 cm. If the block takes 40 s to complete one round, the normal force by the side walls of the groove is **[JEE Main 2021]**

(a) 0.0314 N (b) 9.859×10^{-2} N
(c) 6.28×10^{-3} N (d) 9.859×10^{-4} N

2. The distance r from the origin of a particle moving in xy-plane varies with time as $r = 2t$ and the angle made by the radius vector with positive X-axis is $\theta = 4t$. Here, t is in second, r in metre and θ in radian. The speed of the particle at $t = 1$ s is

(a) 10 ms^{-1} (b) 16 ms^{-1}
(c) 20 ms^{-1} (d) 12 ms^{-1}

3. A particle moves along a circle of radius $\left(\frac{20}{\pi}\right)$ m with constant tangential acceleration. If the velocity of the particle is 80 ms^{-1}, at the end of seconds revolution after motion has begun, the tangential acceleration is

(a) 40 ms^{-2} (b) 640 π ms^{-2}
(c) 1609 π ms^{-2} (d) 40 π ms^{-2}

4. A particle moves such that its position vector $\mathbf{r}(t) = \cos \omega t \hat{\mathbf{i}} + \sin \omega t \hat{\mathbf{j}}$, where ω is a constant and t is time. Then, which of the following statements is true for the velocity $\mathbf{v}(t)$ and acceleration $\mathbf{a}(t)$ of the particle? **[JEE Main 2020]**

(a) $\mathbf{v}$ and $\mathbf{a}$ both are parallel to $\mathbf{r}$.
(b) $\mathbf{v}$ is perpendicular to $\mathbf{r}$ and $\mathbf{a}$ is directed away from the origin.
(c) $\mathbf{v}$ and $\mathbf{a}$ both are perpendicular to $\mathbf{r}$.
(d) $\mathbf{v}$ is perpendicular to $\mathbf{r}$ and $\mathbf{a}$ is directed towards the origin.

5. For a particle in uniform circular motion, the acceleration $\mathbf{a}$ at a point $P(R, \theta)$ on the circle of radius R is (here θ is measured from the X-axis)

(a) $-\frac{v^2}{R}\cos\theta\,\hat{\mathbf{i}} + \frac{v^2}{R}\sin\theta\,\hat{\mathbf{j}}$ (b) $-\frac{v^2}{R}\sin\theta\,\hat{\mathbf{i}} + \frac{v^2}{R}\sin\theta\,\hat{\mathbf{j}}$

(c) $-\frac{v^2}{R}\cos\theta\,\hat{\mathbf{i}} - \frac{v^2}{R}\sin\theta\,\hat{\mathbf{j}}$ (d) $\frac{v^2}{R}\hat{\mathbf{i}} + \frac{v^2}{R}\hat{\mathbf{j}}$

6. When a ceiling fan is switched on, it makes 10 rotations in the first 4 s. How many rotations will it make in the next 4 s? (Assuming uniform angular acceleration).

(a) 10 (b) 20
(c) 40 (d) 30

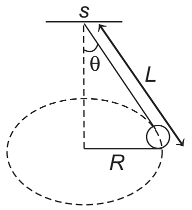
7. A particle is moving in a circle of radius R in such a way that at any instant the normal and tangential components of its acceleration are equal. If its speed at $t = 0$ is v_0, the time taken to complete the first revolution is

(a) $\frac{R}{v_0}$ (b) $\frac{R}{v_0}(1 - e^{-2\pi})$ (c) $\frac{R}{v_0}e^{-2\pi}$ (d) $\frac{2\pi R}{v_0}$

8. An aeroplane flying at a velocity of 900 kmh^{-1} loops the loop. If the maximum force pressing the pilot against the seat is five times its weight, the loop radius should be

(a) 1594 m (b) 1402 m
(c) 1315 m (d) 1167 m

9. A string of length L is fixed at one end and the string makes $\frac{2}{\pi}$ rev/s around the vertical axis through the fixed and as shown in the figure, then tension in the string is



(a) ML (b) 2 ML
(c) 4 ML (d) 16 ML

10. An object of mass 10 kg is whirled round a horizontal circle of radius 4 m by a revolving string inclined 30° to the vertical. If the uniform speed of the object is 5 ms^{-1}, the tension in the string (approximately) is

(a) 720 N (b) 960 N
(c) 114 N (d) 125 N

11. When the road is dry and coefficient of friction is μ, the maximum speed of a car in a circular path is 10 ms^{-1}. If the road becomes wet and $\mu' = \mu/2$, what is the maximum speed permitted?

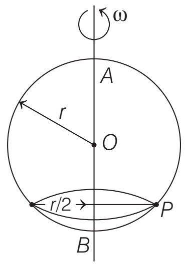
(a) 5 ms^{-1} (b) 10 ms^{-1}
(c) $10\sqrt{2}$ ms^{-1} (d) $5\sqrt{2}$ ms^{-1}

12. A heavy small-sized sphere is suspended by a string of length l. The sphere rotates uniformly in a horizontal circle with the string making an angle θ with the vertical, then the time period of this conical pendulum is

(a) $t = 2\pi\sqrt{\frac{l}{g}}$ (b) $t = 2\pi\sqrt{\frac{l\sin\theta}{g}}$

(c) $t = 2\pi\sqrt{\frac{l\cos\theta}{g}}$ (d) $t = 2\pi\sqrt{\frac{l}{g\cos\theta}}$

13. A smooth wire of length $2\pi r$ is bent into a circle and kept in a vertical plane. A bead can slide smoothly on the wire. When the circle is rotating with angular speed ω about the vertical diameter AB, as shown in figure, the bead is at rest with respect to the circular ring at position P as shown. Then, the value of ω^2 is equal to **[JEE Main 2019]**



(a) $\frac{\sqrt{3}g}{2r}$ (b) $2g/(r\sqrt{3})$

(c) $(g\sqrt{3})/r$ (d) $2g/r$

Numerical Value Questions

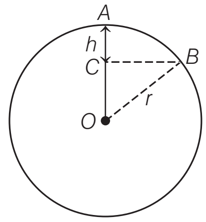
14. One end of a string of length 1.5 m is tied to the stone of mass 0.4 kg and the other end to a small pivot on a smooth vertical board. What is the minimum speed (in m/s) of the stone required at its lowermost point so that the string does not slack at any point in its motion along the vertical circle?

15. A weightless thread can bear tension upto 3.7 kg-wt. A stone of mass 500 g is tied to it and revolved in a circular path of radius 4 m in a vertical plane. If $g = 10\,\text{ms}^{-2}$, then the maximum angular velocity (in rad/s) of the stone will be ....

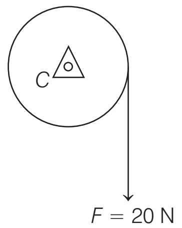
16. A circular disc rotates at 60 rpm. A coin of 18 g is placed at a distance of 8 cm, from the centre. The centrifugal force on the coin becomes $q \times 10^{-2}$ N. The value of q is ....

17. A stone of mass 2 kg is tied to a string of length 0.5 m. If the breaking tension of the string is 900 N, then the maximum angular velocity (in rad/s), that stone can have in uniform circular motion is ....

18. In figure, a particle is placed at the highest point A of a smooth sphere of radius r. It is given slight push, and it leaves the sphere at B, at a depth h vertically below A such that h is equal to r/n, where the value of n is ....



19. Consider a 20 kg uniform circular disc of radius 0.2 m. It is pin supported at its centre and is at rest initially. The disc is acted upon by a constant force $F = 20$ N through a massless string wrapped around its periphery as shown in the figure.



Suppose the disc makes n number of revolutions to attain an angular speed of $50\,\text{rad s}^{-1}$. The value of n, to the nearest integer, is .... (Given, in one complete revolution, the disc rotates by 6.28 rad) **[JEE Main 2021]**

20. The angular speed of truck wheel is increased from 900 rpm to 2460 rpm in 26 s. The number of revolutions by the truck engine during this time is .... (Assuming the acceleration to be uniform) **[JEE Main 2021]**

Answers

Round I

1. (c)	2. (b)	3. (d)	4. (d)	5. (b)	6. (d)	7. (c)	8. (a)	9. (b)	10. (a)
11. (c)	12. (b)	13. (d)	14. (d)	15. (d)	16. (d)	17. (a)	18. (d)	19. (d)	20. (a)
21. (d)	22. (d)	23. (a)	24. (a)	25. (a)	26. (a)	27. (b)	28. (b)	29. (a)	30. (d)
31. (b)	32. (d)	33. (c)	34. (a)	35. (d)	36. (d)	37. (b)	38. (c)	39. (c)	40. (d)
41. (b)	42. (c)								

Round II

1. (d)	2. (b)	3. (a)	4. (d)	5. (c)	6. (d)	7. (b)	8. (a)	9. (d)	10. (d)
11. (d)	12. (c)	13. (b)	14. 8.6	15. 4	16. 5.689	17. 30	18. 3	19 20	20. 728

Solutions

Round I

1. Frequency of wheel, $\nu = \frac{300}{60} = 5$ rps. Angle rotated by wheel in one rotation= 2π rad. Therefore, angle rotated by wheel in 1 s $= 2\pi \times 5$ rad $= 10\pi$ rad.

2. Increase in angular velocity, $\omega = 2\pi\,(n_2 - n_1)$

$$\omega = 2\pi\,(1200 - 600)\,\frac{\text{rad}}{\text{min}}$$
$$= \frac{2\pi \times 600}{60}\,\frac{\text{rad}}{\text{s}} = 20\pi\,\frac{\text{rad}}{\text{s}}$$

3. Here, $r = 1.5 \times 10^{11}$ m; time period of revolution of Earth around the Sun is 1 yr, *i.e.*

$$T = 1\text{ yr} = 365 \times 24 \times 60 \times 60\text{ s}$$

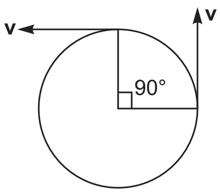
$\therefore$ Angular velocity, $\omega = \frac{2\pi}{T}$

$$= \frac{2 \times (22/7)}{365 \times 24 \times 60 \times 60}$$
$$= 1.99 \times 10^{-7}\text{ rad s}^{-1}$$

Linear velocity, $v = \omega r$

$$= 1.99 \times 10^{-7} \times 1.5 \times 10^{11}$$
$$= 2.99 \times 10^{4}\text{ ms}^{-1}$$

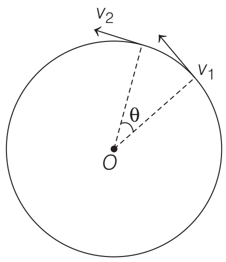
4. In 15 s, second's hand rotate through 90°

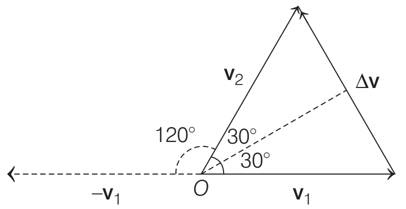


$\therefore$ Change in velocity ,

$$|\Delta \mathbf{v}| = 2v\sin\left(\frac{\theta}{2}\right)$$
$$= 2\,(r\omega)\sin\left(\frac{90^\circ}{2}\right)$$
$$= 2 \times 1 \times \frac{2\pi}{T} \times \frac{1}{\sqrt{2}}$$
$$= \frac{4\pi}{60\sqrt{2}} = \frac{\pi\sqrt{2}}{30}\,\frac{\text{cm}}{\text{s}} \qquad [\text{As, } T = 60\text{ s}]$$

5. Let v_1 be the velocity of the particle moving along the circular path initially, v_1 and v_2 be the velocity when it moves through an angle of 60° as shown below.





From the figure,

$\because \quad \Delta\mathbf{v} = \mathbf{v}_2 - \mathbf{v}_1 = \mathbf{v}_2 + (-\mathbf{v}_1)$

$\therefore \quad |\Delta\mathbf{v}|^2 = v_1^2 + v_2^2 + 2v_1v_2\cos 120^\circ$

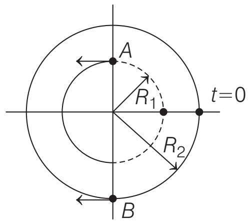
$$= v^2 + v^2 + 2v \times v \times \left(-\frac{1}{2}\right)$$

$\Rightarrow \quad |\Delta\mathbf{v}| = v = 10$ m/s

6. Angle covered by each particle in time duration 0 to $\frac{\pi}{2\omega}$ is

$$\theta = \omega \times t = \omega \times \frac{\pi}{2\omega} = \frac{\pi}{2}\text{ rad}$$

So, positions of particles at $t = \frac{\pi}{2\omega}$ is as shown below;



Velocities of particles at $t = \frac{\pi}{2\omega}$ are

$$\mathbf{v}_A = -\omega R_1\hat{\mathbf{i}} \quad \text{and} \quad \mathbf{v}_B = -\omega R_2\hat{\mathbf{i}}$$

The relative velocity of particles is

$$\mathbf{v}_A - \mathbf{v}_B = -\omega R_1\hat{\mathbf{i}} - (-\omega R_2\hat{\mathbf{i}})$$
$$= -\omega(R_1 - R_2)\hat{\mathbf{i}} = \omega(R_2 - R_1)\hat{\mathbf{i}}$$

7. $\alpha = \frac{\omega}{t}$ and $\omega = \frac{\theta}{t}$

$\therefore \quad \alpha = \frac{\theta}{t^2}$

but $\quad \alpha =$ constant

So, $\quad \frac{\theta_1}{\theta_1 + \theta_2} = \frac{(2)^2}{(2+2)^2}$

or $\quad \frac{\theta_1}{\theta_1 + \theta_2} = \frac{1}{4}$

or $\quad \frac{\theta_1 + \theta_2}{\theta_1} = \frac{4}{1}$

or $\quad 1 + \frac{\theta_2}{\theta_1} = \frac{4}{1}$

$\therefore \quad \frac{\theta_2}{\theta_1} = 3$

8. Centripetal acceleration $a_c = \dfrac{v^2}{r} = v\omega$

When v is doubled, ω is halved, $v\omega$ shall remain unchanged.

9. Radial acceleration must be towards the centre of the circle. Therefore, option (c) is incorrect. Since, tangential acceleration $\mathbf{a}_t$ must be opposite to velocity $\mathbf{v}$, hence options (a) and (d) are also incorrect.

10. Radius of horizontal loop $(r) = 1$ km $= 1000$ m

Speed of aircraft $(v) = 900$ km/h

$$= 900 \times \frac{5}{18} \text{ m/s} \quad \left(\because 1 \text{ km/h} = \frac{5}{18} \text{ m/s}\right)$$

$$= 250 \text{ m/s}$$

Centripetal acceleration of the aircraft

$$a = \frac{v^2}{r} = \frac{(250)^2}{1000} = \frac{62500}{1000} = 62.5 \text{ m/s}^2$$

Acceleration due to gravity $(g) = 9.8$ m/s^2

$$\therefore \frac{\text{Centripetal acceleration } (a)}{\text{Acceleration due to gravity } (g)} = \frac{62.5}{9.8} = 6.38 \simeq 6$$

11. Radius of the horizontal circle = Length of the string

$$= 80 \text{ cm} = 0.80 \text{ m}$$

Frequency of revolution $(n) = \dfrac{14}{25}$ s^{-1}

Angular speed of the revolution of the stone

$$\omega = 2\pi n$$

$$= 2 \times \frac{22}{7} \times \frac{14}{25} = \frac{88}{25} \text{ rad/s}$$

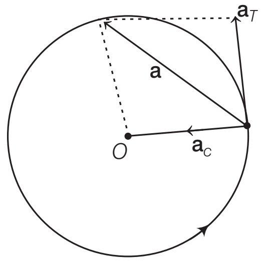
Centripetal acceleration of the stone,

$$a = r\omega^2 = 0.80 \times \left(\frac{88}{25}\right)^2$$

$$= 0.80 \times \frac{88}{25} \times \frac{88}{25} = 9.91 \text{ m/s}^2$$

The direction of the acceleration is towards the centre of the horizontal circle, *i.e.* along its radius.

12. (i) False, because in uniform circular motion, the centripetal acceleration is along the radius of the circle towards the centre, but in non-uniform circular motion, the direction of the resultant acceleration is not along the radius of the circle.



(ii) True, the velocity vector of a particle is always along the tangent to the path of the particle either it is in rectilinear, circular or curvilinear motion.

(iii) True, the direction of acceleration vector in uniform circular motion is directed towards the centre of the circular path, which is continuously changing with time. Therefore, the resultant of all these vectors over one cycle will be a null vector.

13. Centripetal acceleration, $a_c = \dfrac{v^2}{r} = \dfrac{(30)^2}{500} = 1.8 \text{ ms}^{-2}$

Tangential acceleration, $a_t = 2 \text{ ms}^{-1}$

$\therefore$ Resultant acceleration, $a = \sqrt{a_t^2 + a_c^2}$

$$= \sqrt{(2)^2 + (1.8)^2} = 2.7 \text{ ms}^{-2}$$

14. Given, $s = t^3 + 5$

$\therefore$ Speed, $v = \dfrac{ds}{dt} = 3t^2$

and rate of change of speed, $a_t = \dfrac{dv}{dt} = 6t$

$\therefore$ Tangential acceleration at $t = 2$ s,

$$a_t = 6 \times 2 = 12 \text{ ms}^{-2}$$

and at $t = 2$ s, $v = 3(2)^2 = 12 \text{ ms}^{-1}$

$\therefore$ Centripetal acceleration, $a_c = \dfrac{v^2}{R} = \dfrac{144}{20} \text{ ms}^{-2}$

$\therefore$ Net acceleration $= \sqrt{a_t^2 + a_c^2} \approx 14 \text{ ms}^{-2}$

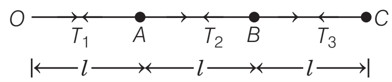
15. In the given condition friction provides the required centripetal force and that is constant. *i.e.*

$$m\omega^2 r = \text{constant}$$

$$\Rightarrow \quad r \propto \frac{1}{\omega^2}$$

$$\therefore \quad r_2 = r_1 \left(\frac{\omega_1}{\omega_2}\right)^2 = 9 \times \left(\frac{1}{3}\right)^2 = 1 \text{ cm}$$

16. Let ω be the angular speed of revolution.



$$T_3 = m\omega^3 3l$$

$$T_2 - T_3 = m\omega^2 2l$$

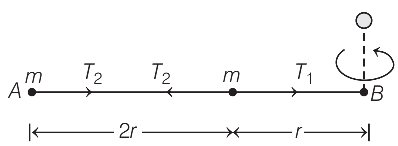
$$\Rightarrow \quad T_2 = m\omega^2 5l$$

$$T_1 - T_2 = m\omega^2 l$$

$$\Rightarrow \quad T_1 = m\omega^2 6l$$

$$\Rightarrow \quad T_3 : T_2 : T_1 = 3 : 5 : 6$$

17. Tension in the respective parts are shown in figure.



Let ω be angular velocity, then

$$T_1 - T_2 = m\omega^2 \times r \quad \text{...(i)}$$

and

$$T_2 = m\omega^2(r + 2r)$$

$$T_2 = 3m\omega^2 r \quad \text{...(ii)}$$

From Eqs. (i) and (ii), we get

$$T_1 = 4m\omega^2 r$$

$$\Rightarrow \quad \frac{T_1}{T_2} = \frac{4}{3}$$

18. A coin flies off when centrifugal force just exceeds the force of friction, *i.e.* $mr\omega^2 \geq \mu mg$

or $$\omega \geq \sqrt{\frac{\mu g}{r}}$$

Thus, ω does not depend upon mass and will remain the same (*i.e.* 50 rpm).

19. Here, $r = 5$ m, $\mu = 0.5$, $\omega = ?$, $g = 10\ \text{ms}^{-2}$

As, $mr\omega^2 = F = \mu R = \mu\, mg$

$$\omega = \sqrt{\frac{\mu g}{r}} = \sqrt{\frac{0.5 \times 10}{5}} = 1\ \text{rad s}^{-1}$$

20. Using the formula for motor cycle not to skid,

$$\theta = \tan^{-1}\left(\frac{v^2}{rg}\right)$$

where, $r = 20$ m

$$v = 72\ \text{km h}^{-1} = 72 \times \frac{5}{18} = 20\ \text{ms}^{-1}$$

$$\therefore \quad \theta = \tan^{-1}\left(\frac{20 \times 20}{20 \times 10}\right)$$

or $$\theta = \tan^{-1}(2)$$

21. **Statement I**

$$v_{\max} = \sqrt{\mu Rg} = \sqrt{(0.2) \times 2 \times 9.8}$$

$$v_{\max} = 1.97\ \text{m/s}$$

$$7\ \text{km/h} = 1.944\ \text{m/s}$$

Speed is lower than $v_{\max}$, hence it can take safe turn.

Statement II

$$V_{\max} = \sqrt{Rg\left(\frac{\tan\theta + \mu}{1 - \mu\tan\theta}\right)}$$

$$= \sqrt{2 \times 9.8\left(\frac{1 + 0.2}{1 - 0.2}\right)} = 5.42\ \text{m/s}$$

$$18.5\ \text{km/h} = 5.14\ \text{m/s}$$

Speed is lower than $v_{\max}$, hence it can take safe turn.

22. As, $\tan\theta = \frac{v^2}{rg}$, therefore, when speed v is doubled; r must be made 4 times, if θ remains the same.

$\therefore$ New radius of curvature,

$$r' = 4r = 4 \times 50\ \text{m} = 200\ \text{m}$$

23. As, $$\tan\theta = \frac{v^2}{Rg}$$

From the given figure, we get

$$\frac{h}{b} = \frac{v^2}{Rg} \Rightarrow h = \frac{v^2 b}{Rg}$$

24. Here, $r = 100$ m, $v = 7\ \text{ms}^{-1}$, $m = 60$ kg

Reading registered = resultant force = ?

Two forces are acting as weight (mg) and centripetal force, $\left(\frac{mv^2}{r}\right)$ are at 90° to each other.

$$\therefore \text{ Resultant force} = \sqrt{(mg)^2 + \left(\frac{mv^2}{r}\right)^2}$$

$$= mg\left[1 + \left(\frac{v^2}{rg}\right)^2\right]^{1/2}$$

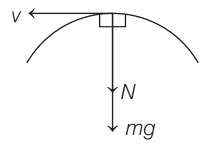
$$= 60 \times 9.8\left[1 + \left(\frac{7 \times 7}{100 \times 9.8}\right)^2\right]^{1/2}$$

$$= 60.075 \times 9.8\ \text{N}$$

$$= 60.075\ \text{kg-wt}$$

25. The blocks will have the same speed, say, equal to v, at the highest point of each track, as they all rise to the same height. If R be the radius of curvature of a track and N be the normal reaction of the track at the highest point of the track, then

$$\text{centripetal force} = N + mg = \frac{mv^2}{R}$$



$\Rightarrow N$ will be the maximum when R is the minimum. This occurs when the track is most sharply curved, *i.e.* in option (a).

26. Tangential acceleration, $a = L\alpha$

$\therefore$ Normal reaction, $N = Ma = ML\alpha$

$\therefore$ Frictional force, $F = mN = \mu\, ML\alpha$

For no sliding along the length, frictional force ≥ centripetal force.

i.e. $$\mu\, ML\alpha \geq ML\omega^2$$

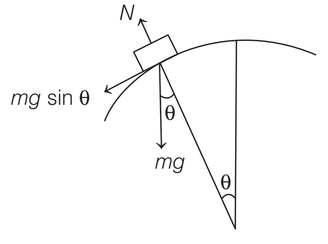
As $$\omega = \omega_0 + \alpha t = \alpha t$$

$$\therefore \quad \mu\, ML\alpha \geq ML(\alpha t)^2$$

$$\Rightarrow \quad t = \sqrt{\frac{\mu}{\alpha}}$$

27. As we know, $mg\cos\theta - N = \frac{mv^2}{R}$

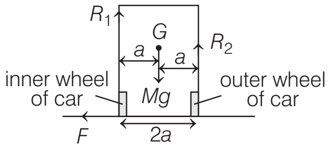
$$\Rightarrow \quad N = mg\cos\theta - \frac{mv^2}{R}$$



Hence, N decrease as θ increases.

So $N_B > N_A > N_C$

28. The given situation is shown below.



Since, $R_1 + R_2 = Mg$

If F be the horizontal force which provides the necessary centripetal force for motion in a circle, then

$$F = \frac{mv^2}{r}$$

Taking moment about point G, there should be no resultant turning effect about the centre of gravity.

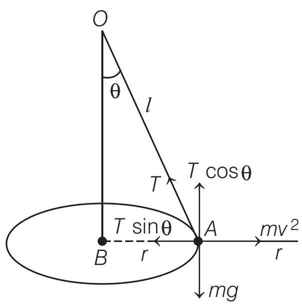
$\therefore \quad Fh + R_1 a = R_2 a$

$\therefore \quad R_2 = \frac{Fh + R_1 a}{a}$

$$R_2 = R_1 + \frac{Fh}{a}$$

$\therefore$ Clearly, $R_2 > R_1$

29. In figure, $T \sin\theta = \frac{mv^2}{r}$; $T\cos\theta = mg$;

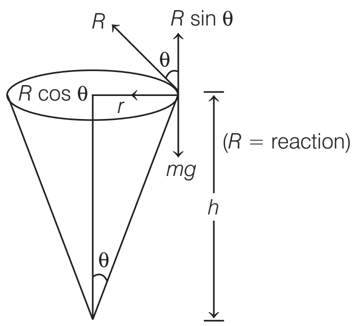


So, $\quad \tan\theta = \frac{v^2}{rg} = \frac{r}{\sqrt{l^2 - r^2}}$

$$v = \left[\frac{r^2 g}{(l^2 - r^2)^{1/2}}\right]^{1/2} = \left[\frac{0.09 \times 10}{(0.25 - 0.09)^{1/2}}\right]^{1/2}$$

$$= 1.5 \text{ ms}^{-1}$$

30. The particle is moving in circular path as shown below



From the figure, $\quad mg = R\sin\theta \quad$ ...(i)

$$\frac{mv^2}{r} = R\cos\theta \quad \text{...(ii)}$$

From Eqs. (i) and (ii), we get

$$\tan\theta = \frac{rg}{v^2}$$

and $\quad \tan\theta = \frac{r}{h}$

$$h = \frac{v^2}{g} = \frac{(0.5)^2}{10} = 0.25 \text{ m} = 2.5 \text{ cm}$$

31. $r = a + l\sin 45° = (0.2) + (\sqrt{2})\left(\frac{1}{\sqrt{2}}\right) = 1.2$ m

Now, $\quad T\cos 45° = mg \quad$ ...(i)

and $\quad T\sin 45° = mr\omega^2 \quad$ ...(ii)

From Eqs. (i) and (ii), we have $\omega = 2n\pi = \sqrt{\frac{g}{r}}$

$\therefore \quad n = \frac{1}{2\pi}\sqrt{\frac{g}{r}} = \frac{60}{2\pi}\sqrt{\frac{9.8}{1.2}}$ rpm $= 27.3$ rpm

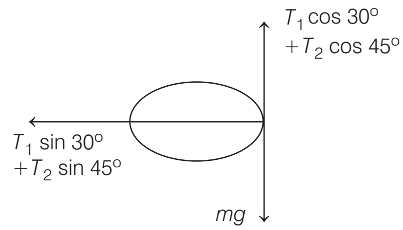
32. From free body diagram shown in figure,

$$T_1 \cos 30° + T_2 \cos 45° = mg \quad \text{...(i)}$$

$$T_1 \sin 30° + T_2 \sin 45° = \frac{mv^2}{r} \quad \text{...(ii)}$$

After solving Eqs. (i) and (ii), we get

$$T_1 = \frac{mg - \frac{mv^2}{r}}{\left(\frac{\sqrt{3} - 1}{2}\right)}$$



But $\quad T_1 \geq 0$

$$\therefore \quad \frac{mg - \frac{mv^2}{r}}{\frac{\sqrt{3} - 1}{2}} \geq 0$$

or $\quad mg \geq \frac{mv^2}{r}$

or $\quad v \leq \sqrt{rg}$

$\therefore \quad v_{max} = \sqrt{rg} = \sqrt{1.6 \times 9.8} = 3.96 \text{ ms}^{-1}$

33. Velocity at the lowest point,

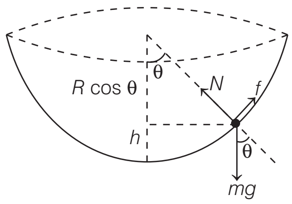
$$v = \sqrt{2\,gl}$$

At the lowest point, the tension in the string,

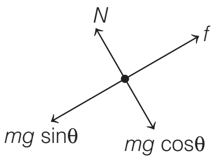
$$T = mg + \frac{mv^2}{l}$$

$$= mg + \frac{m}{l}(2\,gl) = 3\,mg$$

34. Let h be maximum height up to which insect crawls up the ditch. The free body diagram is as shown



Resolving the components of force along tangential and radial direction



For balancing, $mg\cos\theta = N$

$$mg\sin\theta = f_{\max} = \mu N$$

$$\Rightarrow \quad mg\sin\theta = \mu mg\cos\theta$$

$$\frac{\sin\theta}{\cos\theta} = \frac{\mu mg}{mg}$$

$$\tan\theta = \mu = 0.75 = \frac{3}{4}$$

$$\Rightarrow \quad \cos\theta = \frac{4}{5}$$

From diagram, $h = R - R\cos\theta = R - R\left(\frac{4}{5}\right) = \frac{R}{5}$

or $$h = \frac{1}{5} = 0.20 \text{ m}$$

35. Force, $F = \frac{mv^2}{r} + mg = \frac{5\times10^3\times(250)^2}{2\times10^3} + 5\times10^4$

$$= 2.0625\times10^4 \text{ N downward}$$

36. Here, $T = mg + m\omega^2 r = m\,(g + 4\pi^2 n^2 r)$

$$= m\left\{g + \left(4\pi^2\left(\frac{n}{60}\right)^2 r\right)\right\} = m\left\{g + \left(\frac{\pi^2 n^2 r}{900}\right)\right\}$$

37. Here, $mg = 2\times10 = 20$ N and $\frac{mv^2}{r} = \frac{2\times(4)^2}{1} = 32$ N

It is clear that, 52 N tension will be at the bottom of the circle because we know that

$$T_{\text{bottom}} = mg + \frac{mv^2}{r}$$

38. The body crosses the topmost position of a vertical circle with critical velocity, so the velocity at the lowest point of vertical circle, $u = \sqrt{5\,gr}$.

Velocity of the body when string is horizontal, is

$$v^2 = u^2 - 2\,gr = 5\,gr - 2\,gr = 3\,gr$$

$\therefore$ Centripetal acceleration, $a_c = \frac{v^2}{r} = \frac{3\,gr}{r} = 3\,g$

39. Tension, $T = \frac{mv^2}{r} + mg\cos\theta$

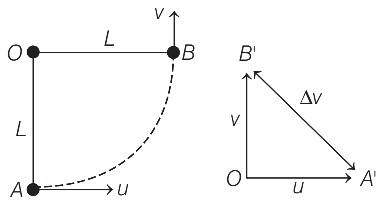
For $\theta = 30°$, $\quad T_1 = \frac{mv^2}{r} + mg\cos30°$

For $\theta = 60°$, $\quad T_2 = \frac{mv^2}{r} + mg\cos60°$

Since, $\cos30° > \cos60°$

$\therefore \quad T_1 > T_2$

40. The velocity at B is v, where $v^2 = u^2 - 2\,gL$, in vertically upward direction.



$$\therefore \quad \Delta v = \sqrt{u^2 + v^2} = \sqrt{u^2 + (u^2 - 2\,gL)} = \sqrt{2\,(u^2 - gL)}$$

41. As, $\quad T_{\text{top}} = \frac{mv^2}{r} - mg \quad$ ...(i)

and $\quad T_{\text{bottom}} = \frac{mv^2}{r} + mg \quad$ ...(ii)

From Eqs. (i) and (ii), we get

$$\Rightarrow \quad \frac{T_{\text{top}}}{T_{\text{bottom}}} = \frac{\frac{v^2}{r} - g}{\frac{v^2}{r} + g} = \frac{\frac{40\times40}{4} - 10}{\frac{40\times40}{4} + 10}$$

$$= \frac{400-10}{400+10} = \frac{390}{410} = \frac{39}{41}$$

42. At the lowest point, $\frac{mv^2}{r} = T_L - mg \quad$ ...(i)

At the highest point, $\frac{mv^2}{r} = T_H + mg \quad$ ...(ii)

As $$\frac{T_{\max}}{T_{\min}} = \frac{T_L}{T_H} = 2$$

$\therefore \quad T_L = 2\,T_H$

From Eqs. (i) and (ii), we have

$$2\,T_H - mg = T_H + mg$$

$$T_H = 2\,mg$$

From Eq. (ii), $\quad \frac{mv^2}{r} = 3\,mg \quad$ or $\quad \frac{v^2}{rg} = 3$

Round II

1. $N = m\omega^2 R$

$$N = m\left[\frac{4\pi^2}{T^2}\right]R \quad \text{...(i)}$$

Given, $m = 0.2$ kg, $T = 40$ s, $R = 0.2$ m

Putting the values in Eq. (i), we get

$$N = 9.859\times10^{-4} \text{ N}$$

2. Here, $r = 2\,t, \theta = 4\,t$

As, $\quad l = r\theta = (2\,t)\,(4\,t) = 8\,t^2$

$$\therefore \quad v = \frac{dl}{dt} = \frac{d}{dt}(8\,t^2) = 16\,t$$

$\Rightarrow \quad v_t = 16 \text{ ms}^{-1} \quad$ (at $t = 1$ s)

3. Initial angular velocity, $\omega_0 = 0$.

Final angular velocity,

$$\omega = \frac{v}{r} = \frac{80}{(20/\pi)} = 4\pi \text{ rads}^{-1}$$

Angle described, $\theta = 4\pi$ rad

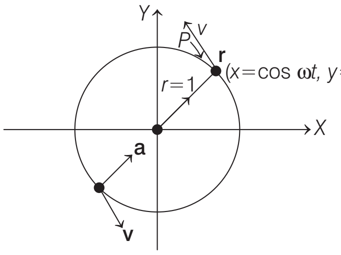
$\therefore$ Angular acceleration, $\alpha = \dfrac{\omega^2 - \omega_0^2}{2\theta}$

$$= \frac{(4\pi)^2 - 0}{2 \times 4\pi} = 2\pi \text{ rads}^{-2}$$

Tangential acceleration,

$$a = \alpha r = 2\pi \times \frac{20}{\pi} = 40 \text{ ms}^{-2}$$

4. Position vector of a particle moving around a unit circle ($r = 1$) in xy-plane is given by



$$\mathbf{r}(t) = \cos\omega t\,\hat{\mathbf{i}} + \sin\omega t\,\hat{\mathbf{j}} \quad \text{...(i)}$$

Velocity, $\mathbf{v}(t) = \dfrac{d\mathbf{r}(t)}{dt} = \omega(-\sin\omega t\,\hat{\mathbf{i}} + \cos\omega t\,\hat{\mathbf{j}})$

$\Rightarrow \quad \mathbf{v}(t) = \omega[\cos(\omega t + \pi/2)\,\hat{\mathbf{i}} + \sin(\omega t + \pi/2)\,\hat{\mathbf{j}}] \quad \text{...(ii)}$

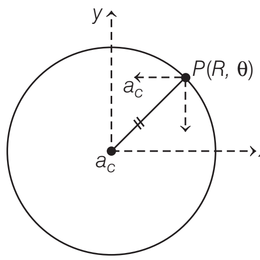
$$\mathbf{a}(t) = \frac{d\mathbf{v}(t)}{dt} = -\omega^2\cos\omega t\,\hat{\mathbf{i}} - \omega^2\sin\omega t\,\hat{\mathbf{j}}$$

$$= -\omega^2(\cos\omega t\,\hat{\mathbf{i}} + \sin\omega t\,\hat{\mathbf{j}})$$

$$\mathbf{a}(t) = -\omega^2\,\mathbf{r}(t) \quad \text{...(iii)}$$

From Eqs. (i), (ii) and (iii), it is clear that $\mathbf{v}$ is perpendicular to $\mathbf{r}$ and $\mathbf{a}$ is directed towards the origin.

5. For a particle in uniform circular motion,



$$\mathbf{a} = \frac{v^2}{R} \text{ towards centre of circle}$$

$$\mathbf{a} = \frac{v^2}{R}(-\cos\theta\,\hat{\mathbf{i}} - \sin\theta\,\hat{\mathbf{j}})$$

or $\quad \mathbf{a} = -\dfrac{v^2}{R}\cos\theta\,\hat{\mathbf{i}} - \dfrac{v^2}{R}\sin\theta\,\hat{\mathbf{j}}$

6. As, $\theta = 2\pi n = \omega_0 t + \dfrac{1}{2}\alpha t^2$

$\Rightarrow \quad 2\pi \times 10 = \dfrac{1}{2}\alpha(4)^2$

or $\quad \alpha = \dfrac{40\pi}{16}$

Let it makes N rotations in first 8s, then

$$2\pi N = \frac{1}{2}\alpha 8^2$$

$\Rightarrow \quad N = 40 \quad \left(\text{as } \alpha = \dfrac{40\pi}{16}\right)$

$\therefore$ Required number of rotations $= 40 - 10 = 30$

7. According to question, in circular motion of particle, at any instant,

radial acceleration = tangential acceleration

$\Rightarrow \quad \dfrac{v^2}{R} = \dfrac{dv}{dt}$

Hence, $\quad \dfrac{R}{v^2}dv = dt$

$\Rightarrow \quad \displaystyle\int \frac{R}{v^2}dv = dt$

$\Rightarrow \quad \dfrac{-R}{v} = t + c \quad \text{...(i)}$

When $t = 0$, $v = v_0$

$\therefore \quad \dfrac{-R}{v_0} = c \quad \text{...(ii)}$

Hence, from Eqs. (i) and (ii), we get

$$\frac{-R}{v} = t - \frac{R}{v_0}$$

$\Rightarrow \quad t = \dfrac{R}{v_0} - \dfrac{R}{v}$

$\Rightarrow \quad t = \dfrac{R}{v_0} - \dfrac{R}{(ds/dt)} \quad \left[\because v = \dfrac{ds}{dt}\right]$

$\Rightarrow \quad ds\left(\dfrac{R}{v_0} - t\right) = Rdt$

$\Rightarrow \quad \displaystyle\int \frac{ds}{R} = \int \frac{dt}{\frac{R}{v_0} - t}$

$\Rightarrow \quad \dfrac{s}{R} = -\ln\left(\dfrac{R}{v_0} - t\right) + c$

When $t = 0$, $s = 0$,

$\therefore \quad c = \dfrac{\ln R}{v_0}$

$\therefore \quad \dfrac{s}{R} = \dfrac{\ln R}{v_0} - \ln\left(\dfrac{R}{v_0 - t}\right) = \ln\left[\dfrac{R}{R - v_0 t}\right]$

For complete revolution, $t = T$ and $s = 2\pi R$

$\therefore \quad \dfrac{2\pi R}{R} = \ln\left(\dfrac{R}{R - v_0 T}\right)$

$\Rightarrow \quad T = \dfrac{R}{v_0}(1 - e^{-2\pi})$

8. Here, $\quad v = 900 \text{ km h}^{-1} = \dfrac{900 \times 1000}{60 \times 60} \text{ms}^{-1} = 250 \text{ ms}^{-1}$

Maximum force is at bottom of the vertical circle

$$F_{max} = \frac{mv^2}{r} + mg = 5\,mg$$

$\therefore$ $$v^2 = 4\,gr$$

or $$r = \frac{v^2}{4\,g} = \frac{250 \times 250}{4 \times 9.8} = 1594.4 \simeq 1594 \text{ m}$$

9. From figure in question,

$$T \sin\theta = M\omega^2 R \qquad [\because R = L \sin\theta]$$

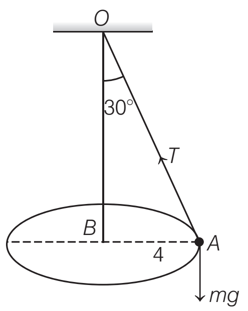
$$\Rightarrow \quad T \sin\theta = M\omega^2 L \sin\theta$$

$$\Rightarrow \quad T = M\omega^2 L = M \cdot 4\pi^2 n^2 L$$

$$= M \cdot 4\pi^2 \left(\frac{2}{\pi}\right)^2 L$$

$$= 16\,ML$$

10. In figure, $\sin 30° = \frac{AB}{OA}$



or $$OA = \frac{AB}{\sin 30°} = \frac{4}{1/2} = 8 \text{ m}$$

$$\frac{T}{AO} = \frac{F}{AB} = \frac{mg}{OB}$$

$$F = \frac{AO}{AB} \times F$$

$$= \frac{AO}{AB} \frac{mv^2}{r}$$

$$= \frac{8}{4} \times 10 \times \frac{5^2}{4} = 125 \text{ N}$$

11. The maximum speed without skidding is

$$v = \sqrt{\mu\, rg}$$

$\therefore$ $$\frac{v_2}{v_1} = \sqrt{\frac{\mu_1}{\mu_1}} = \sqrt{\frac{\mu/2}{\mu}} = \frac{1}{\sqrt{2}} \quad (\text{for } rg = \text{constant})$$

$$v_2 = \frac{v_1}{\sqrt{2}} = 5\sqrt{2} \text{ ms}^{-1} \qquad (\because v_1 = 10 \text{ ms}^{-1})$$

12. Radius of circular path in the horizontal plane

$$r = l \sin\theta$$

Resolving T along the vertical and horizontal directions, we get,

$$T \cos\theta = Mg \qquad \ldots\text{(i)}$$

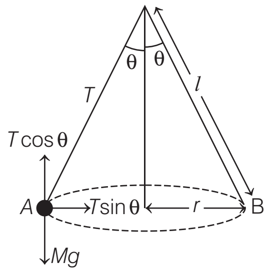
$$T \sin\theta = Mr\omega^2 = M\,(l \sin\theta)\,\omega^2 \qquad [\because r = l \sin\theta]$$

or $$T = Ml\omega^2 \qquad \ldots\text{(ii)}$$

Dividing Eq. (ii) by Eq. (i), we get

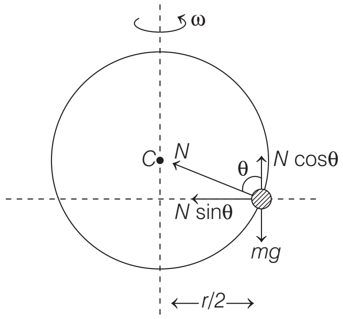
$$\frac{1}{\cos\theta} = \frac{l\omega^2}{g}$$

or $$\omega^2 = \frac{g}{l \cos\theta}$$



$\therefore$ Time period, $$t = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{l \cos\theta}{g}}$$

13. Let N = normal reaction of wire loop acting towards centre.



Then, component $N \cos\theta$ balances weight of bead,

$$\Rightarrow \quad N \cos\theta = mg \qquad \ldots\text{(i)}$$

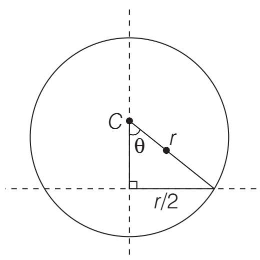
and component $N \sin\theta$ provides necessary centripetal pull on the bead,

$$\Rightarrow \quad N \sin\theta = m\left(\frac{r}{2}\right)\omega^2 \qquad \ldots\text{(ii)}$$

From Eqs. (i) and (ii), we have

$$\tan\theta = \frac{r\omega^2}{2g} \qquad \ldots\text{(iii)}$$

Now, from geometry of figure,



$$\tan\theta = \frac{\frac{r}{2}}{\sqrt{r^2 - \left(\frac{r}{2}\right)^2}} = \frac{r}{2\left(\frac{\sqrt{3}}{2}\right)r} = \frac{1}{\sqrt{3}} \qquad \ldots\text{(iv)}$$

Put this value in Eq. (iii), we get

$$\omega^2 = \frac{2g}{\sqrt{3}\,r}$$

14. Here, $r = 1.5$ m, $m = 0.4$ kg; $g = 9.8\ \text{ms}^{-2}$

The minimum speed at the lowest point of the vertical circle is

$$v_L = \sqrt{5\,gr} = \sqrt{5 \times 9.8 \times 1.5}$$

$$v_L = 8.6\ \text{ms}^{-1}$$

15. Maximum tension that string can bear

$$= 3.7\ \text{kg-wt} = 37\ \text{N}$$

Tension at lowest point of vertical loop $= mg + m\omega^2 r$

$$= 5 + 2\,\omega^2$$

$\therefore$ $$37 = 5 + 2\,\omega^2$$

or $$\omega = 4\ \text{rad/s}$$

16. Here, $n = 60\ \text{rpm} = \dfrac{60}{60} = 1\ \text{rps}$

$$m = 18\text{g} = 18 \times 10^{-3}\ \text{kg}$$

$$r = 8\ \text{cm} = 8 \times 10^{-2}\ \text{m}$$

Centrifugal force, $F = mr\omega^2 = mr\,(2\pi\nu)^2$

$$= 4\pi^2 mr\,n^2$$

$$= 4 \times \frac{22}{7} \times \frac{22}{7} \times (18 \times 10^{-3}) \times (8 \times 10^{-2}) \times 1^2$$

$$F = 5.689 \times 10^{-2}\ \text{N}$$

17. Here, mass of stone, $m = 2$ kg

Length of string, $r = 0.5$ m

Breaking tension, $T = 900$ N

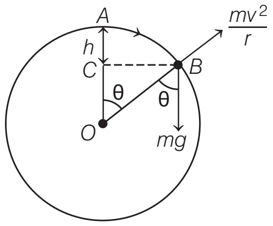
At $$T = mr\omega^2$$

or $$\omega^2 = \frac{T}{mr} = \frac{900}{2 \times 0.5} = 900$$

$\Rightarrow$ $$\omega = \sqrt{900} = 30\ \text{rad/s}$$

18. If v is velocity acquired at B, then

$$v^2 = 2\,gh$$



The particle will leave the sphere at B, when

$$\frac{mv^2}{r} \geq mg\cos\theta$$

$$\frac{2\,gh}{r} = \frac{g\,(r-h)}{r}$$

which gives, $$h = \frac{r}{3}$$

$\therefore$ $$n = 3$$

19. $\alpha = \dfrac{\tau}{I} = \dfrac{FR}{mR^2/2} = \dfrac{2F}{mR}$

$$\alpha = \frac{2 \times 20}{20 \times (0.2)} = 10\ \text{rad/s}^2$$

$$\omega^2 = \omega_0^2 + 2\alpha\Delta\theta$$

$$(50)^2 = 0^2 + 2(10)\,\Delta\theta$$

$\Rightarrow$ $$\Delta\theta = \frac{2500}{20}$$

$$= 125\ \text{rad}$$

Number of revolutions $n = \dfrac{125}{2\pi} \approx 20$

20. We know, $\theta = \left(\dfrac{\omega_1 + \omega_2}{2}\right)t$

Let number of revolutions be N.

$\therefore$ $$2\pi N = 2\pi\left(\frac{900 + 2460}{60 \times 2}\right) \times 26$$

$$N = 728$$