Structure of Geometry

Exercise 6:

Solution 1:

In direct proof, we deduce a statement from the data by means of logical arguments. From this statement we then deduce another statement and through such chain of statements, we derive the statement to be proved by means of logical arguments.

Solution 2:

Given: x is an even number.
To prove: $x + 1$ is an odd number.
Proof: $x = 2k$ $\quad [\because x \text{ is an even number and } k \in N]$
$\therefore\ x + 1 = 2k + 1$
$\therefore\ x + 1 = 2\left(k + \frac{1}{2}\right)$
As $k \in N$, $\left(k + \frac{1}{2}\right) \notin N$.
$\therefore$ $x + 1$ is not a multiple of 2. So $x + 1$ is not an even number.
$\therefore\ x + 1$ is an odd number.

Solution 3:

1. Jayendra can eat five cups of ice cream or less than five cups of ice cream.
   But he cannot eat more than five cups of ice cream.
2. Every youth should contribute 10 hours per month for social services.
   But less than 10 hours is not permissible.
3. $m + 7 = 10$ has just one solution, not more than one solution.
4. A line can intersect a circle in two points or it can intersect a circle in less than two points, but it can never intersect a circle in more than two points.

Solution 4:

1. Data : $X \subset Y$ and $Y \subset X$
   To Prove : $X = Y$

2. Data : Triangle.
   To Prove : The sum of measures of all the three angles of a triangle is $180°$.

3. Data: Set B is not an empty set.
   To prove: Set B has atleast two subsets.

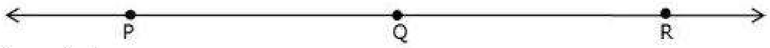
4. Data: Today is Sunday.
   To prove: The school has a holiday.

Solution 5:

1. Implication is a conditional statement of the type 'if p, then q,' Here, p is called the sufficient condition for q and q is called the necessary condition for p
2. '$x + 5 = 7$' is the sufficient condition and $x = 2$ is the necessary condition.
3. There are three parts of a theorem:
   1. Hypothesis or Data
   2. Conclusion or To prove
   3. Proof
4. Proofs in geometry are divided into two types:
   1. Direct proof
   2. Indirect proof
5. Two types of indirect proof are:
   1. Methods of exhausting alternatives
   2. Methods of reduction ad absurdum
6. Main parts of the structure of modern geometry are:
   1. Defined terms
   2. Undefined terms
   3. Postulates and
   4. Theorems

Exercise 6.1:

Solution 1:



Given that,

$PQ = QR \quad ...(1)$

By Euclid's 2nd axiom, if equals are added to equals, the wholes are equal.

So, add PQ to both sides of equation (1).

$\Rightarrow PQ + PQ = PQ + QR.$

By Euclid's 4th axiom, things which coincide with one another are equal to one another.

Here PR coincides with PQ + QR, so we get,

$2PQ = PR$

By Euclid's 7th axiom, things which are halves of the same thing are equal to one another.

Multiplying both sides by $\frac{1}{2}$

$\Rightarrow \frac{1}{2}(2PQ) = \frac{1}{2}(PR)$

$\Rightarrow PQ = \frac{1}{2}PR$

Solution 2:

Given, PR = QS

∴ PR coincides with PQ + QR, QS coincides with

QR + RS and by Euclid's 4^{th} axiom, we get,

PQ + QR = QR + RS

Then, by Euclid's 3^{rd} axiom

PQ + QR – QR = QR + RS – QR

∴ PQ + RS

Solution 3:

False.

Infinitely many lines can pass through a single point.

Solution 4(1):

a. Solid – Surface – Line – Point

A solid has three dimensions, a surface has two dimensions, a line has one and a point has no dimension.

Solution 4(2):

c.0

A point has no dimension.

Solution 4(3):

d. 2

A surface has 2 dimensions.

Solution 4(4):

b. 13 Chapters

Euclid divided his famous treatise 'the elements' into 13 chapters.

Solution 4(5):

b. Thales

Pythagoras was a student of Thales.

Solution 4(6):

d. Theorem

A theorem needs proof.

Solution 4(7):

c. a postulate

Euclid stated that all right angles are equal to each other in the form of a postulate.

Solution 4(8):

c. a postulate

'Lines are parallel to each other if they do not intersect' is stated in the form of a postulate.