Sample Paper

Mathematics

Section - A

In this section, attempt any 16 questions out of Questions 1 – 20.

Each Question is of 1 mark weightage.

1. What is the value of $\tan^{-1}\sqrt{3} - \cot^{-1}(-\sqrt{3})$.

(a) $\frac{\pi}{2}$ (b) $-\frac{\pi}{2}$ (c) 0 (d) 1

2. What is the domain of the function $y = \sin^{-1}(-x^2)$.

(a) $-1 \le x \le 1$ (b) $-\infty \le x < \infty$ (c) $x \le 1$ (d) $x \ge 0$

3. If $A = \begin{bmatrix} 2 & 3 \\ 1 & 0 \end{bmatrix} = P + Q$, where P is symmetric matrix and Q is skew-symmetric matrix then find the matrix P.

(a) $\begin{bmatrix} 5 & 5 \\ 1 & 5 \end{bmatrix}$ (b) $\begin{bmatrix} 2 & 2 \\ 2 & 0 \end{bmatrix}$ (c) $\begin{bmatrix} -1 & 0 \\ 1 & 2 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

4. If $A = [a_{ij}]_{2 \times 2}$, where $a_{ij} = \frac{(j+2j)^2}{2}$, then A is equal to :

(a) $\begin{bmatrix} 9 & 25 \\ 8 & 18 \end{bmatrix}$ (b) $\begin{bmatrix} 9/2 & 25/2 \\ 8 & 18 \end{bmatrix}$ (c) $\begin{bmatrix} 9 & 25 \\ 4 & 9 \end{bmatrix}$ (d) $\begin{bmatrix} 9/2 & 15/2 \\ 4 & 9 \end{bmatrix}$

5. If $A = [a_{ij}]$ is a matrix of order 2 × 2, such that $|A| = -15$ and c_{ij} represents the cofactor of a_{ij}, then $a_{21}c_{21} + a_{22}c_{22}$ is equal to :

(a) – 15 (b) 15 (c) 0 (d) 1

6. If $A = \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}$ is such that $A^2 = I$, then the value of $1 - \alpha^2 - \beta\gamma$ is :

(a) – 1 (b) 0

(c) 1 (d) Can't be calculated

7. Evaluate $\frac{dy}{dx}$, if $y = \frac{8^x}{x^8}$.

(a) $\frac{8x^7}{8^x \log 8}$ (b) $\frac{8^x \log 8}{8x^7}$ (c) $\frac{8^x}{x^8}\left[\log 8 - \frac{8}{x}\right]$ (d) None of these

8. What is the slope of the tangent to the curve $y = 4x^3 - 5x$ at $x = 3$.

(a) 109 (b) 71 (c) 103 (d) 98

9. If $R = \{(x, y) : x, y \in I, x^2 + y^2 \le 4\}$ is a relation in I, then domain of R is:

(a) {0, 1, 2} (b) {– 2, – 1, 0}

(c) {– 2, – 1, 0, 1, 2} (d) None of these

10. If the function $f(x) = \begin{cases} kx^2, & if\ x \le 2 \\ 3, & if\ x > 2 \end{cases}$ is continuous at $x = 2$, then find the value of k.

(a) $\frac{3}{4}$ (b) $\frac{2}{3}$ (c) $\frac{1}{2}$ (d) $\frac{1}{3}$

11. If $f(x) = \begin{cases} \frac{1-\cos 4x}{x^2}, & x < 0 \\ a, & x = 0 \\ \frac{\sqrt{x}}{\sqrt{16+x}-4}, & x > 0 \end{cases}$

is continuous at $x = 0$, then $a =$

(a) 4 (b) 6 (c) 8 (d) 5

12. Find the value of $\tan^{-1}\left[2\sin\left(2\cos^{-1}\frac{\sqrt{3}}{2}\right)\right]$.

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{3}$ (c) $\frac{2\pi}{3}$ (d) $\frac{\pi}{2}$

13. The function $f(x) = \tan x - 4x$ is strictly decreasing on:

(a) $\left(-\frac{\pi}{3}, \frac{\pi}{3}\right)$ (b) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ (c) $(-\pi, \pi)$ (d) None of these

14. The relation R = $\{(a, a), (a, b), (a, c), (b, b), (b, c), (c, a), (c, b), (c, c)\}$ on the set A = $\{a, b, c\}$ is :

(a) Reflexive (b) Transitive (c) Symmetric (d) All of these

15. What is the principal value of $\sin^{-1}\left(\sin\frac{2\pi}{3}\right)$?

(a) $\frac{\pi}{6}$ (b) $\frac{2\pi}{3}$ (c) $\frac{\pi}{2}$ (d) $\frac{\pi}{3}$

16. The number of all possible matrices of order 3×3 with each entry 0 or 1 is :

(a) 264 (b) 308 (c) 512 (d) 500

17. Evaluate the determinant $\Delta = \begin{vmatrix} \log_3 512 & \log_4 3 \\ \log_3 8 & \log_4 9 \end{vmatrix}$

(a) $\frac{15}{2}$ (b) 12 (c) $\frac{14}{3}$ (d) 6

18. Find $\frac{dy}{dx}$ if $y = 3x^3 - 4x^2$.

(a) $\frac{3}{4}x^4 - \frac{4}{3}x^3$ (b) $12x^2 - 8x$ (c) $9x^2 - 8x$ (d) $16x^2 - 8x$

19. The function $f(x) = \log(\cos x)$ is :

(a) Increasing in $[0, \pi]$ (b) Decreasing in $\left[0, \frac{\pi}{2}\right]$

(c) Decreasing in $[0, \pi]$ (d) Increasing in $\left[0, \frac{\pi}{2}\right]$

20. Find $\left.\frac{dy}{dx}\right|_{x=\frac{\pi}{2}}$ where $y = e^{\sin x}$.

(a) 1 (b) -1 (c) 2 (d) 0

Section - B

In this section, attempt any 16 questions out of the Questions 21 - 40.

Each Question is of 1 mark weightage.

21. If $x = a \sin^3 t$ and $y = a \cos^3 t$, then what is the value of $\frac{dy}{dx}$?

(a) $\tan t$ (b) $-\cot t$ (c) $-\tan t$ (d) $\cot t$

22. The domain of R is:

(a) {2, 4, 8} (b) {2, 4, 6, 8} (c) {2, 4, 6} (d) {1, 2, 3, 4}

23. Let R be a relation on the set N be defined by $\{(x, y) : x, y \in N, 2x + y = 41\}$. Then, R is:

(a) Reflexive (b) Symmetric (c) Transitive (d) None of these

24. Range of $f(x) = \sin^{-1} x + \tan^{-1} x + \sec^{-1} x$ is:

(a) $\left(\frac{\pi}{4}, \frac{3\pi}{4}\right)$ (b) $\left[\frac{\pi}{4}, \frac{3\pi}{4}\right]$ (c) $\left\{\frac{\pi}{4}, \frac{3\pi}{4}\right\}$ (d) None of these

25. The value of $\sin^2\left(\cos^{-1}\frac{1}{2}\right)+\cos^2\left(\sin^{-1}\frac{1}{3}\right)$ is:

(a) $\frac{17}{36}$ (b) $\frac{59}{36}$ (c) $\frac{36}{59}$ (d) None of these

26. If $A = \begin{bmatrix} 0 & -1 & 2 \\ 1 & 0 & 3 \\ -2 & -3 & 0 \end{bmatrix}$, then $A + 2A^T$ equals:

(a) A (b) $-A^T$ (c) A^T (d) $2A^T$

27. If $x^y \cdot y^x = 16$, then $\frac{dy}{dx}$ at (2, 2) is:

(a) 1 (b) – 1 (c) 0 (d) None of these

28. The value of k which makes the function defined by $f(x) = \begin{cases} \sin\frac{1}{x}, & \text{if } x \neq 0 \\ k, & \text{if } x = 0 \end{cases}$, continous at $x = 0$ is:

(a) 8 (b) 1 (c) – 1 (d) None of these

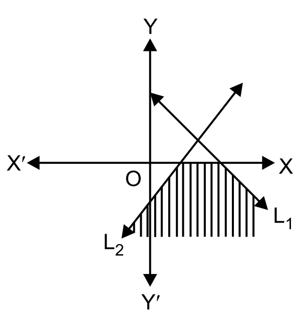
29. The area of a triangle with vertices (– 3, 0), (3, 0) and (0, k) is 9 sq. units. The value of k will be:

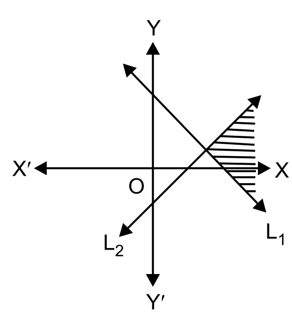
(a) 9 (b) 3 (c) – 9 (d) 6

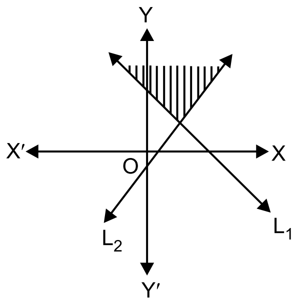
30. A corner point of a feasible region is a point in the region, which is the .... of two boundary lines.

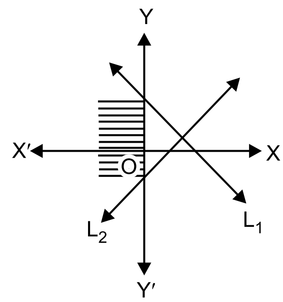
(a) Union (b) Difference (c) Intersection (d) None of these

31. The graphical solution of linear inequalities $x + y \geq 5$ and $x - y \leq 3$, where $L_1 \equiv x + y = 5$ and $L_2 \equiv x - y = 3$, is:

(a) 

(b) 

(c) 

(d) 

32. If A = $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$ and $f(x) = (1 + x)(1 - x)$, then $f(\text{A})$ is:

(a) $-4\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ (b) $-8\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ (c) $4\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ (d) None of these

33. If $\begin{vmatrix} 6i & -3i & 1 \\ 4 & 3i & -1 \\ 40 & 3 & i \end{vmatrix} = x + iy$, then:

(a) $x = 3, y = 1$ (b) $x = 1, y = 3$ (c) $x = 0, y = 3$ (d) $x = 0, y = 0$

34. The tangent to the curve given by $x = e^t . \cos t$, $y = e^t \cdot \sin t$ at $t = \frac{\pi}{4}$ makes with X-axis an angle:

(a) 0 (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

35. If A = $\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, find A^{-1}.

(a) $\frac{1}{7}\begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$ (b) $\frac{1}{7}\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ (c) $\frac{1}{7}\begin{bmatrix} -2 & 1 \\ -1 & -3 \end{bmatrix}$ (d) $\frac{1}{7}\begin{bmatrix} 2 & 1 \\ 1 & -3 \end{bmatrix}$

36. If $x = a \cos^3 \theta$ and $y = a \sin^3 \theta$, then $1 + \left(\frac{dy}{dx}\right)^2$ is equal to:

(a) $\tan \theta$ (b) $\tan^2 \theta$ (c) 1 (d) $\sec^2 \theta$

37. If given constraints are $5x + 4y \geq 2$, $x \leq 6$ and $y \leq 7$, then the maximum value of the function $Z = x + 2y$, is:

(a) 13 (b) 14 (c) 15 (d) 20

38. If A = $\begin{bmatrix} a & b \\ b & a \end{bmatrix}$ and $\text{A}^2 = \begin{bmatrix} \alpha & \beta \\ \beta & \alpha \end{bmatrix}$, then:

(a) $\alpha = a^2 + b^2, \beta = ab$ (b) $\alpha = a^2 + b^2, \beta = 2ab$ (c) $\alpha = a^2 + b^2, \beta = a^2 - b^2$ (d) $\alpha = 2ab, \beta = a^2 + b^2$

39. Derivative of $\cos x^3 \sin^2 (x^5)$ with respect to x is:

(a) $10x^4 (\sin x^5)(\cos x^5)(\cos x^3) - 3x^2 \sin x^3 \sin^2 x^5$ (b) $5x^3 (\sin x^5)(\cos x^5)(\cos x^3) - 3x^2 \sin x^3 \sin^2 x^5$

(c) $5x^3 (\sin x^5)(\cos x^5)(\cos x^3) - 6x^2 \sin x^3 \sin^2 x^5$ (d) None of the above

40. In a LPP, the maxium value of the objective function $z = ax + by$ is always:

(a) infinite (b) finite (c) 0 (d) None of these

Section - C

In this section, attempt any 8 questions. Each question is of 1-mark weightage.

Questions 46-50 are based on a Case-Study.

41. The corner points of the feasible region determined by the following system of linear inequalities: $2x + y \leq 10$, $x + 3y \leq 15$, $x, y \geq 0$ are (0, 0), (5, 0), (3, 4) and (0, 5). Let $Z = px + qy$, where $p, q > 0$. Condition on p and q so that the maximum of Z occurs at both (3, 4) and (0, 5) is:

(a) $p = q$ (b) $p = 2q$ (c) $p = 3q$ (d) $q = 3p$

42. If $y = (1 + x^{1/4})(1 + x^{1/2})(1 - x^{1/4})$, then $\frac{dy}{dx}$ is equal to :

(a) 1 (b) – 1 (c) x (d) $\sqrt{x}$

43. If the points (3, – 2), (x, 2), (8, 8) are collinear, then find the value of x:

(a) 2 (b) 3 (c) 4 (d) 5

44. If $A = \begin{bmatrix} 0 & 2 & -3 \\ -2 & 0 & -1 \\ 3 & 1 & 0 \end{bmatrix}$, then A is a:

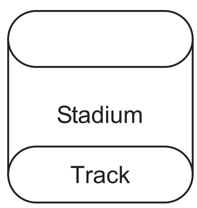
(a) Symmetric matrix (b) Skew-symmetric matrix

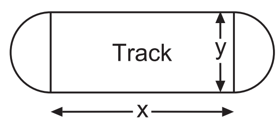
(c) Diagonal matrix (d) None of these

45. If $f(x) = \begin{cases} ax^2 + b, & 0 \le x < 1 \\ 4, & x = 1 \\ x + 3, & 1 < x \le 2 \end{cases}$, then the value of ($a$, b) for which $f(x)$ cannot be continuos at a = 1, is:

(a) (2, 2) (b) (3, 1) (c) (4, 0) (d) (5, 2)

In a stadium a running track of 440 m is to be laid out enclosing a football field. The shape of track is a rectangle with a semi-circle at each end. They want to keep maximum area of rectangular portion.





Give answers of the following questions :

46. Show the relationship of y in terms of x if $x + y$ represents the length and breadth of rectangular field :

(a) $\frac{440 - 2x}{\pi}$ (b) $\frac{2x - 440}{\pi}$ (c) $\frac{440 - x}{\pi}$ (d) $\frac{x - 440}{\pi}$

47. If A shows area of field (rectangular portion), the function of A in terms of x will be :

(a) $x\left(\frac{440 - x}{\pi}\right)$ (b) $\frac{1}{\pi}(440 - x^2)$ (c) $\frac{220 - 2x^2}{\pi}$ (d) $\frac{1}{\pi}(440x - 2x^2)$

48. At what value of x area will be maximum ?

(a) 120 (b) 110 (c) 100 (d) 220

49. What is the maximum area of rectangular field ?

(a) 7700π m^2 (b) 24200 m^2 (c) 7700 m^2 (d) 24200 π m^2

50. The value of y will be :

(a) 70 m (b) 70 π (c) $\frac{70}{\pi}$ (d) 110

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Answers

Sample Paper

Section - A

1. (b) $-\frac{\pi}{2}$

Explanation: We have, $\tan^{-1}\sqrt{3} - \cot^{-1}(-\sqrt{3}) = \tan^{-1}\left(\tan\frac{\pi}{3}\right) - \cot^{-1}\left(\cot\frac{5\pi}{6}\right)$

$$= \frac{\pi}{3} - \frac{5\pi}{6} = \frac{2\pi - 5\pi}{6}$$

$$= \frac{-3\pi}{6} = -\frac{\pi}{2}$$

2. (a) $-1 \le x \le 1$

Explanation: $y = \sin^{-1}(-x^2) \Rightarrow \sin y = -x^2$

Now, $-1 \le -x^2 \le 1$ (since $-1 \le \sin y \le 1$)

$\Rightarrow$ $1 \ge x^2 \ge -1$

$\Rightarrow$ $-1 \le x^2 \le 1$

$\Rightarrow$ $|x| \le 1$

$\Rightarrow$ $-1 \le x \le 1$

3. (b) $\begin{bmatrix} 2 & 2 \\ 2 & 0 \end{bmatrix}$

Explanation: We have, $A = \begin{bmatrix} 2 & 3 \\ 1 & 0 \end{bmatrix}$

$\therefore$ $A' = \begin{bmatrix} 2 & 1 \\ 3 & 0 \end{bmatrix}$

Since, P is symmetric matrix

$\therefore$ $P = \frac{1}{2}(A + A')$

$\Rightarrow$ $P = \frac{1}{2}\begin{bmatrix} 2+2 & 3+1 \\ 1+3 & 0+0 \end{bmatrix}$

$\Rightarrow$ $P = \frac{1}{2}\begin{bmatrix} 4 & 4 \\ 4 & 0 \end{bmatrix}$

$\Rightarrow$ $P = \begin{bmatrix} 2 & 2 \\ 2 & 0 \end{bmatrix}$

4. (b) $\begin{bmatrix} 9/2 & 25/2 \\ 8 & 18 \end{bmatrix}$

Explanation: Here,

$$a_{11} = \frac{(1+2\times1)^2}{2} = \frac{9}{2}$$

$$a_{12} = \frac{(1+2\times2)^2}{2} = \frac{25}{2}$$

$$a_{21} = \frac{(2+2\times1)^2}{2} = 8$$

and

$$a_{22} = \frac{(2+2\times2)^2}{2} = 18$$

So, the required matrix $A = \begin{bmatrix} 9/2 & 25/2 \\ 8 & 18 \end{bmatrix}$

5. (a) – 15

Explanation: Given : $|A| = -15$

Let $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$

So, $|A| = a_{11}a_{22} - a_{12}a_{21}$

$\Rightarrow$ $-15 = a_{11}a_{22} - a_{12}a_{21}$ ...(i)

Let c_{ij} represent the cofactor of a_{ij}.

Then $c_{21} = (-1)^{2+1} \times a_{12} = -a_{12}$

$c_{22} = (-1)^{2+2} \times a_{11} = a_{11}$

Now, $a_{21}c_{21} + a_{22}c_{22} = a_{21}(-a_{12}) + a_{22}(a_{11})$

$= -a_{21}a_{12} + a_{22}a_{11}$

$= a_{22}a_{11} - a_{21}a_{12} = -15$ [Using (i)]

6. (b) 0

Explanation: $A^2 = I$ (Given)

$$\begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}\begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$\Rightarrow$ $$\begin{bmatrix} \alpha^2+\beta\gamma & \alpha\beta-\alpha\beta \\ \alpha\gamma-\alpha\gamma & \beta\gamma+\alpha^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

On comparing the corresponding elements

$$\alpha^2 + \beta\gamma = 1$$

$$1 - \alpha^2 - \beta\gamma = 0$$

7. (c) $\frac{8^x}{x^8}\left[\log 8 - \frac{8}{x}\right]$

Explanation:

$$y = \frac{8^x}{x^8}$$

Differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{x^8(8^x.\log 8) - 8^x(8.x^7)}{x^{16}} \qquad \left[\because \frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v.u' - uv'}{v^2}\right]$$

$$= 8^x.\ x^{-8}\log 8 - 8.8^x.\ x^{-9} = \frac{8^x \log 8}{x^8} - \frac{8.8^x}{x^9}$$

$\Rightarrow$ $$\frac{dy}{dx} = \frac{8^x}{x^8}\left[\log 8 - \frac{8}{x}\right]$$

8. (c) 103

Explanation: Given, $y = 4x^3 - 5x$

Differentiating w.r.t. x, $\frac{dy}{dx} = 12x^2 - 5$

$\therefore$ $$\left(\frac{dy}{dx}\right)_{(\text{at } x=3)} = 12(3)^2 - 5$$

$$= 108 - 5 = 103.$$

9. (c) $\{-2, -1, 0, 1, 2\}$

Explanation: Given, $R = \{(x, y) : x, y \in I, x^2 + y^2 \leq 4\}$

$= \{(0, 0), (0, -1), (0, 1), (0, -2) \ldots (-2, 0)\}$

$\therefore$ Domain of $R = \{x : (x, y) \in R = \{-2, -1, 0, 1, 2\}$.

10. (a) $\frac{3}{4}$

Explanation:

$$(\text{L.H.L at } x = 2) = \lim_{x \to 2^-} f(x)$$

$$= \lim_{x \to 2^-} kx^2$$

and $f(2) = k(2)^2 = 4k$

$$(\text{R.H.L at } x = 2) = \lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} 3 = 3$$

$\because f(x)$ is continuous at $x = 2$

$$\lim_{x \to 2^-} f(x) = f(2) = \lim_{x \to 2^+} f(x)$$

$\Rightarrow$ $4k = 3$

$\Rightarrow$ $k = \frac{3}{4}$

11. (c) 8

Explanation: Since $f(x)$ is continuous at $x = 0$

$\therefore$ LHL at $x = 0 = f(0) =$ RHL at $x = 0$

$$\lim_{x \to 0} \frac{1-\cos 4x}{x^2} = a = \lim_{x \to 0} \frac{\sqrt{x}}{\sqrt{16+\sqrt{x}}-4}$$

$$\lim_{x \to 0} \frac{4 \sin 4x}{2x} = a = \lim_{x \to 0} \sqrt{16+\sqrt{x}}+4$$

$$\lim_{x \to 0} \frac{4\sin 4x}{4x} \times \frac{4x}{2x} = a = \lim_{x \to 0} \sqrt{16+\sqrt{x}}+4$$

$\therefore$ $a = 8.$

12. (b) $\frac{\pi}{3}$

Explanation: $\tan^{-1}\left[2\sin\left(2\cos^{-1}\frac{\sqrt{3}}{2}\right)\right] = \tan^{-1}\left(2\sin\left(2\times\frac{\pi}{6}\right)\right)$ $\left[\because \cos^{-1}\frac{\sqrt{3}}{2} = \frac{\pi}{6}\right]$

$$= \tan^{-1}\left(2 \sin \frac{\pi}{3}\right)$$

$$= \tan^{-1}\left(2\times\frac{\sqrt{3}}{2}\right)$$

$$= \tan^{-1}(\sqrt{3}) = \frac{\pi}{3}$$

13. (a) $\left(-\frac{\pi}{3}, \frac{\pi}{3}\right)$

Explanation: $f(x) = \tan x - 4x$

$\Rightarrow$ $f'(x) = \sec^2 x - 4$

When $\frac{-\pi}{3} < x < \frac{\pi}{3}$, $1 < \sec x < 2$

Therefoer, $1 < \sec^2 x < 4$

$\Rightarrow$ $-3 < (\sec^2 x - 4) < 0$

Thus for $\frac{-\pi}{3} < x < \frac{\pi}{3}$, $f'(x) < 0$

Hence, f is strictly decreasing on $\left(-\frac{\pi}{3}, \frac{\pi}{3}\right)$

14. (a) Reflexive

Explanation: R is reflexive, since $(a, a), (b, b), (c, c) \in$ R

R is not symmetric, since $(a, b) \in$ R but $(b, a) \notin$ R

R is not transitive, since $(b, c) \in$ R and $(c, a) \in$ R but $(b, a) \notin$ R.

Thus, R is only reflexive.

15. (d) $\frac{\pi}{3}$

Explanation:

$$\sin^{-1}\left(\sin\frac{2\pi}{3}\right) = \sin^{-1}\left[\sin\left(\pi-\frac{\pi}{3}\right)\right]$$

$$= \sin^{-1}\left(\sin\frac{\pi}{3}\right) = \frac{\pi}{3} \qquad \left[\because \frac{\pi}{3} \in \left[-\frac{\pi}{2},\frac{\pi}{2}\right]\right]$$

16. (c) 512

Explanation: Number of elements of 3×3 matrix $= 3 \times 3 = 9$

Number of ways to write 0 or 1 at one place = 2

Number of ways to write 0 or 1 at nine places

$$= 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$$

$$= 2^9 = 512$$

$\therefore$ There are 512 possible matrices.

17. (a) $\frac{15}{2}$

Explanation: We have,

$$\Delta = \begin{vmatrix} \log_3 512 & \log_4 3 \\ \log_3 8 & \log_4 9 \end{vmatrix}$$

$$\Rightarrow \quad \Delta = \begin{vmatrix} \log_3 2^9 & \log_{2^2} 3 \\ \log_3 2^3 & \log_{2^2} 3^2 \end{vmatrix}$$

$$\Rightarrow \quad \Delta = \begin{vmatrix} 9\log_3 2 & \frac{1}{2}\log_2 3 \\ 3\ \log_3 2 & \frac{2}{2}\log_2 3 \end{vmatrix} \qquad \left[\because \log_{a^p} m^n = \frac{n}{p}\log_a m\right]$$

$$\Rightarrow \quad \Delta = (9\log_3 2) \times (\log_2 3) - \left(\frac{1}{2}\log_2 3\right)(3\log_3 2)$$

$$\Rightarrow \quad \Delta = 9(\log_3 2 \times \log_2 3) - \frac{3}{2}(\log_2 3 \times \log_3 2)$$

$$\Rightarrow \quad \Delta = 9 - \frac{3}{2}$$

$$\Rightarrow \quad \Delta = \frac{15}{2} \qquad [\because \log_b a \times \log_a b = 1]$$

18. (c) $9x^2 - 8x$

Explanation: $y = 3x^3 - 4x^2$

Differentiating w.r.t. x, we get

$$\frac{dy}{dx} = 9x^2 - 8x$$

19. (b) decreasing in $\left[0, \frac{\pi}{2}\right]$

Explanation: Given, $f(x) = \log(\cos x)$

Differentiate $f(x)$ w.r.t x, we get

$$f'(x) = \frac{1}{\cos x}(-\sin x) = -\tan x$$

But we know that $\tan x > 0$ for $\left[0, \frac{\pi}{2}\right]$

$\Rightarrow$ $f'(x) < 0$ for $x \in \left[0, \frac{\pi}{2}\right]$

$\Rightarrow f(x)$ is decreasing in $\left[0, \frac{\pi}{2}\right]$.

20. (d) 0

Explanation: We have, $y = e^{\sin x} \Rightarrow \frac{dy}{dx} = e^{\sin x}.\cos x$

$$\therefore \quad \left.\frac{dy}{dx}\right|_{x=\frac{\pi}{2}} = = e^{1}.0 = 0$$

Section - B

21. (b) $-\cot t$

Explanation: Given, $x = a\sin^3 t$

$$\therefore \quad \frac{dx}{dt} = 3a\sin^2 t.\cos t$$

Also, $y = a\cos^3 t$

$$\therefore \quad \frac{dx}{dt} = 3a\cos^2 t(-\sin t)$$

$$\therefore \quad \frac{dy}{dx} = \frac{-3a\cos^2 t.\sin t}{3a\sin^2 t.\cos t} \qquad \left[\because \frac{dy}{dx} = \frac{dy/dt}{dx/dt}\right]$$

$$\Rightarrow \quad \frac{dy}{dx} = -\cot t$$

22. (c) {2, 4, 6}

Explanation: Given, $R = \{(x, y) : x + 2y = 8, x, y \in N\}$

$\therefore$ $R = \{(2, 3), (4, 2), (6, 1)\}$

$\therefore$ Domain of $R = \{x : (x, y) \in R\} = \{2, 4, 6\}$.

23. (d) None of these

Explanation: $R = \{(x, y) : x, y \in N, 2x + y = 41\}$

Reflexive : $(1, 1) \notin R$ as $2.1 + 1 = 3 \neq 41$. R is not reflexive.

Symetric : $(1, 39) \in R$ but $(39, 1) \notin R$. So R is not symmetric.

Transitive : $(20, 1) \in R$ and $(1, 39) \in R$. But $(20, 39) \notin R$, so R is not transitive.

24. (c) $\left\{\frac{\pi}{4}, \frac{3\pi}{4}\right\}$

Explanation: Given

$$f(x) = \sin^{-1} x + \tan^{-1} x + \sec^{-1} x$$

$$\text{Domain of } \sin^{-1} x = [-1, 1]$$

$$\text{Domain of } \tan^{-1} x = (-\infty, \infty)$$

$$\text{Domain of } \sec^{-1} x = (-\infty, \infty) - (-1, 1)$$

$$\text{Domain of } f(x) = [-1, 1] \cap [(-\infty, \infty) \cap [(-\infty, \infty)] - (-1, 1)] = \{-1, 1\}$$

Now,

$$f(-1) = \sin^{-1}(-1) + \tan^{-1}(-1) + \sec^{-1}(-1) = -\frac{\pi}{2} - \frac{\pi}{4} + \pi = \frac{\pi}{4}$$

and

$$f(1) = \sin^{-1}(1) + \tan^{-1}(1) + \sec^{-1}(1) = \frac{\pi}{2} + \frac{\pi}{4} + 0 = \frac{3\pi}{4}$$

$$\text{Range of } f(x) = \left\{\frac{\pi}{4}, \frac{3\pi}{4}\right\}$$

25. (b) $\frac{59}{36}$

Explanation: $\sin^2\left(\cos^{-1}\frac{1}{2}\right) + \cos^2\left(\sin^{-1}\frac{1}{3}\right)$

$$= 1 - \cos^2\left(\cos^{-1}\frac{1}{2}\right) + 1 - \sin^2\left(\sin^{-1}\frac{1}{3}\right) = 1 - \left(\frac{1}{2}\right)^2 + 1 - \left(\frac{1}{3}\right)^2 = 2 - \frac{1}{4} - \frac{1}{9} = \frac{59}{36}$$

26. (c) A^T

Explanation:

$$A^T = \begin{bmatrix} 0 & 1 & -2 \\ -1 & 0 & -3 \\ 2 & 3 & 0 \end{bmatrix} = -\begin{bmatrix} 0 & -1 & 2 \\ 1 & 0 & 3 \\ -2 & -3 & 0 \end{bmatrix} = -A$$

So, $A^T = -A$

$\Rightarrow$ $A + A^T = 0$

Hence, $A + 2A^T = A^T$

27. (b) -1

Explanation: We have, $x^y \cdot y^x = 16$

Taking log on both sides, $\log x^y + \log y^x = \log 16$

$$y \log x + x \log y = \log 16$$

Differentiating both sides, w.r.t. x, we get

$$\frac{y}{x} + \log x \frac{dy}{dx} + \frac{x}{y}\frac{dy}{dx} + \log y = 0$$

At $x = 2$, $y = 2$

$$1 + \log 2\left(\frac{dy}{dx}\right)_{(2,\,2)} + 1\left(\frac{dy}{dx}\right)_{(2,\,2)} + \log 2 = 0$$

$$\Rightarrow \quad \left(\frac{dy}{dx}\right)_{(2,\,2)} = \frac{-(\log 2 + 1)}{\log 2 + 1} = -1$$

28. (d) None of these

Explanation: $\lim_{x \to 0} \sin\frac{1}{x}$ does not exist. Hence, there is no value of k for which the function is continuos at $x = 0$.

29. (b) 3

Explanation: Area of triangle $= \frac{1}{2}\begin{vmatrix} -3 & 0 & 1 \\ 3 & 0 & 1 \\ 0 & k & 1 \end{vmatrix} = \pm 9$

$\Rightarrow \quad -k(-3-3) = \pm 18$

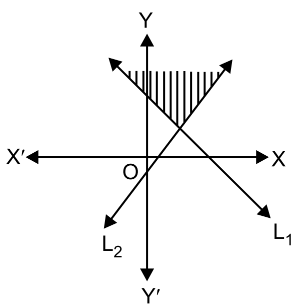
$\Rightarrow \quad 6k = \pm 18$

$\Rightarrow \quad k = \pm 3$

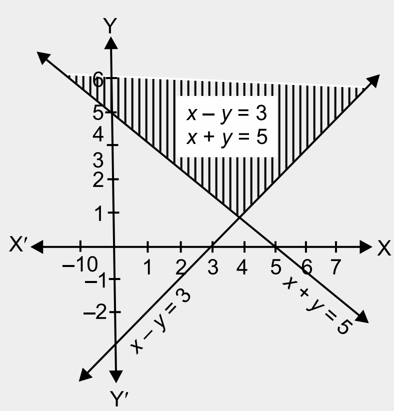
30. (c) Intersection

Explanation: A corner point of a feasible region is a point, in the region, which is the intersection of two boundary lines.

31. (c)



Explanation: The graph of linear equation $x + y = 5$ is drawn in figure. We note that solution of inequality (i) is represented by the shaded region above the line $x + y = 5$, including the points on the line



On the same set of axes, we draw the graph of the equation $x - y = 3$ as shown in figure. Then, we note that inequality (ii) represnts the shaded region above the line $x - y = 3$, including the points on the line.

Clearly, the double shaded region, common to the above two shaded region is the required solution region of the given system of inequalities.

32. (a) $-4\begin{bmatrix}1 & 1\\1 & 1\end{bmatrix}$

Explanation: Given,

$$f(x) = (1 + x)(1 - x)$$
$$= 1 - x^2$$

$\Rightarrow$ $$f(\text{A}) = \text{I} - \text{A}^2 \qquad \text{(Put } x = \text{A)}$$

$\Rightarrow$ $$f(\text{A}) = \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix} - \left\{\begin{bmatrix}1 & 2\\2 & 1\end{bmatrix}\begin{bmatrix}1 & 2\\2 & 1\end{bmatrix}\right\}$$

$$= \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix} - \begin{bmatrix}5 & 4\\4 & 5\end{bmatrix} = \begin{bmatrix}-4 & -4\\-4 & -4\end{bmatrix}$$

$$= -4\begin{bmatrix}1 & 1\\1 & 1\end{bmatrix}$$

33. (d) $x = 0, y = 0$

Explanation: $$\begin{vmatrix}6i & -3i & 1\\4 & 3i & -1\\40 & 3 & i\end{vmatrix} = 1(12 - 120i) + 1(18i + 120i) + i(18i^2 + 12i) \qquad (\because i^2 = -1)$$

$$= 12 - 120i + 18i + 120i + i(-18 + 12i)$$
$$= 12 + 18i - 18i - 12 = 0 = 0 + i0$$
$$= x + iy$$

On comparing we get

$$x = y = 0$$

34. (d) $\frac{\pi}{2}$

Explanation: $$\frac{dx}{dt} = -e^t \cdot \sin t + e^t \cos t$$

$$\frac{dy}{dt} = e^t \cos t + e^t \sin t$$

Therefore $$\left(\frac{dy}{dx}\right)_{t=\frac{\pi}{4}} = \frac{\cos t + \sin t}{\cos t - \sin t} = \frac{1}{-1} = -1$$

35. (a) $\frac{1}{7}\begin{bmatrix}2 & -1\\1 & 3\end{bmatrix}$

Explanation: $$|\text{A}| = 6 + 1 = 7 \neq 0,$$

$\therefore$ A^{-1} exists.

$$(\text{adj A}) = \begin{bmatrix}2 & -1\\1 & 3\end{bmatrix}$$

$$\text{A}^{-1} = \frac{1}{7}\begin{bmatrix}2 & -1\\1 & 3\end{bmatrix}$$

36. (d) $\sec^2 \theta$

Explanation: $\frac{dx}{d\theta} = 3a \cos^2 \theta (- \sin \theta)$

and $\frac{dy}{d\theta} = 3a \sin^2 \theta (\cos \theta)$

Now, $\frac{dy}{dx} = \frac{3a \sin^2 \theta \cos \theta}{-3a \cos^2 \theta \sin \theta} = - \tan \theta$

$\therefore$ $1 + \left(\frac{dy}{dx}\right)^2 = 1 + (- \tan \theta)^2 = 1 + \tan^2 \theta = \sec^2 \theta.$

37. (d) 20

Explanation: Feasible region is ABCDEA and Z = $x + 2y$

38. (b) $\alpha = a^2 + b^2, \beta = 2ab$

Explanation: Given, $A = \begin{bmatrix} a & b \\ b & a \end{bmatrix}, A^2 = \begin{bmatrix} \alpha & \beta \\ \beta & \alpha \end{bmatrix}$

Now, $A^2 = A \cdot A = \begin{bmatrix} a & b \\ b & a \end{bmatrix} \begin{bmatrix} a & b \\ b & a \end{bmatrix}$

$\Rightarrow$ $\begin{bmatrix} \alpha & \beta \\ \beta & \alpha \end{bmatrix} = \begin{bmatrix} a^2 + b^2 & 2ab \\ 2ab & a^2 + b^2 \end{bmatrix}$

Now, $\alpha = a^2 + b^2; \beta = 2ab$

39. (a) $10x^4 (\sin x^5) (\cos x^5) (\cos x^3) - 3x^2 \sin x^3 \sin^2 x^5$

Explanation: Let $y = \cos x^3 \sin^2 (x^5)$

On differentiating both sides w.r.t. x, we get

$$\frac{dy}{dx} = \frac{d}{dx}\{\cos x^3 \sin^2 (x^5)\}$$

$$= \cos x^3 \frac{d}{dx} \sin^2 (x^5) + \sin^2 (x^5) \frac{d}{dx} (\cos x^3)$$

$$\left[\because \frac{d}{dx}(uv) = u \frac{d}{dx} v + v \frac{d}{dx} u\right]$$

$$= (\cos x^3)\ (2 \sin x^5)\ \frac{d}{dx}(\sin x^5) + \sin^2 (x^5)\ (- \sin x^3)\ \frac{d}{dx}(x^3)$$

$$\left[\because \frac{d}{dx} f\{g(x)\} = f'\{g(x)\} \frac{d}{dx} g(x)\right]$$

$$= (\cos x^3) (2 \sin x^5) (\cos x^5) \frac{d}{dx} (x^5) + \sin^2 (x^5) (- \sin x^3) (3x)^2$$

[using chain rule]

$$= (\cos x^3) (2 \sin x^5) (\cos x^5) (5x^4) - \sin^2 (x^5) (\sin x^3) (3x^2)$$

$$= 10x^4 (\sin x^5) (\cos x^5) (\cos x^3) - 3x^2 \sin x^3 \sin^2 x^5.$$

40. (b) Finite

Explanation: In a LPP, the maximum value of the objective function $z = ax + by$ is always finite.

Section - C

41. (d) $q = 3p$

Explanation: The maximum value of Z is unique.

It is given that maximum value of Z occurs at two points (3, 4) and (0, 5). Value of Z at (3, 4) = Value of Z at (0, 5).

$$\Rightarrow \quad p(3) + q(4) = p(0) + q(5)$$
$$3p + 4q = 5q \Rightarrow 3p = q$$

42. (b) – 1

Explanation: We have, $y = (1 + x^{1/4})(1 + x^{1/2})(1 - x^{1/4})$
$$= (1 - x^{1/2})(1 + x^{1/2}) = 1 - x$$
$$\therefore \quad \frac{dy}{dx} = -1.$$

43. (d) 5

Explanation: Since, the given points are collinear.

$$\therefore \quad \frac{1}{2}\begin{vmatrix} 3 & -2 & 1 \\ x & 2 & 1 \\ 8 & 8 & 1 \end{vmatrix} = 0$$
$$\Rightarrow \quad 3(2-8) + 2(x-8) + 1(8x-16) = 0$$
$$\Rightarrow \quad -18 + 2x - 16 + 8x - 16 = 0$$
$$\Rightarrow \quad x = 5$$

44. (b) skew-symmetric matrix

Explanation:
$$A^T = \begin{bmatrix} 0 & -2 & 3 \\ 2 & 0 & 1 \\ -3 & -1 & 0 \end{bmatrix} = -\begin{bmatrix} 0 & 2 & -3 \\ -2 & 0 & -1 \\ 3 & 1 & 0 \end{bmatrix} = -A$$

Since $A^T = -A$, therefore, A is a skew-symmetric matrix.

45. (d) (5, 2)

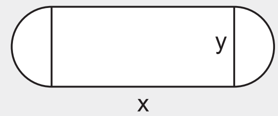
Explanation:
$$\lim_{x \to 1^-} f(x) = \lim_{h \to 0} f(1-h)^2 + b = a + b$$

Function $f(x)$ is discontinuous at $x = 1$.
$$\lim_{x \to 1^-} f(x) \neq f(1)$$
$$\Rightarrow \quad a + b \neq 4$$

Checking options , (5, 2) is the correct answer.

46. (a) $\frac{440-2x}{\pi}$

Explanation: Perimeter of field is 440 m.



$$x = \text{length},\ y = \text{breadth}$$

$$\text{Radius of semicircle} = \frac{y}{2}$$

$$\therefore \quad P = 2x + 2\pi.\left(\frac{y}{2}\right)$$

$$440 = 2x + \pi y$$

$$\frac{440-2x}{\pi} = y$$

47. (d) $\frac{1}{\pi}(440x - 2x^2)$

Explanation: A be the area of rectangular field.

$$A = xy$$

$$A = x\left(\frac{440-2x}{\pi}\right) = \frac{1}{\pi}(440x - 2x^2)$$

48. (b) 110 m

Explanation: Differentiate A w.r. to x,

$$\frac{dA}{dx} = \frac{1}{\pi}(440 - 4x)$$

Put $$\frac{dA}{dx} = 0$$

$$\Rightarrow \quad 440 - 4x = 0$$

$$440 = 4x$$

$$x = 110 \text{ m}$$

49. (c) 7700 m^2

Explanation: Now, $\frac{d^2A}{dx^2} = -4$, –ve *i.e.*, area is maximum at $x = 110$

$$\text{Maximum area, } A = xy = 110 \times \frac{220}{\pi} = \frac{24200}{\pi} = \frac{24200}{22} \times 7$$

$$= 7700 \text{ m}^2$$

50. (a) 70 m

Explanation:

$$y = \frac{440 - 2\times 110}{\pi} = \frac{440-220}{\pi} = \frac{220}{\pi}$$

$$= \frac{220}{22}\times 7 = 70 \text{ m}$$