CHAPTER 05

Engineering Economy

5.1 Introduction

Engineering economy is the science of applying economics to select the best from a group of various engineering design. An options if implemented will produce a time pattern of consequences which must be evaluated, compared and predicted in relation to their costs. Thus the essential prerequisite of successful engineering application is economic feasibility.

The concept of economy study may be summarized with the following points:

(a) All alternatives physically capable of achieving the engineering objective should be clearly defined.

(b) The physical consequences of each alternative should be identified and evaluated in terms of money units.

(c) The difference between alternatives should be the basis for comparison and they should be compared on a uniform basis as interest rate etc.

5.2 Interest and Interest Formulae

Interest: The term interest is used to designate a rental for the use of money.

Interest Rate: An interest rate is the ratio of the gain received from an investment over a period of time, usually one year and the investment is usually taken as Rs. 100. It is expressed in the percentage form and designated as 'i'.

The Time Value of Money: Time value of money means that two equal amounts at different points of time do not have equal value if the interest rate is greater than zero. It is the relationship between interest and time that leads to the concept of 'the time value of money'.

Simple Interest: When the total interest earned or charged is directly proportional to the principal involved, the interest rate and the number of interest periods for which the principal is computed, the interest is called simple

$$I = (P)\ (n)\ (i)$$

where I = total interest, P = principal amount lent or borrowed

n = number of years (interest periods), i = interest rate per year (per interest period)

5.2.1 Compound Interest

Whenever the interest charge for any interest period (a year, for example) is based on the remaining principal amount plus any accumulated interest charges upto the beginning of that period, the interest is said to be compound.

5.2.2 Notations and Cash Flow

i = Annual interest rate

n = Number of annual interest periods

P = Present sum of money i.e. present worth at zero time

F = Future sum of money i.e. future worth equal to the compound amount at the end of 'n' years

A = A single payment, in a series of 'n' equal payments, made at the end of each annual interest period

G = Uniform period-by-period increase or decrease in amount (the arithmetic gradient)

5.2.3 Cash Flow Diagram

The graphic presentation of each value plotted at appropriate time is called a cash flow diagram. The normal conventions for cash flow diagrams are as follows:

(a) The horizontal axis is the time scale with progression of time moving from left to right. The value indicated on time scale (viz., 0, 1, 2 ,... n) indicates the end of the respective period.

(b) The arrows signify cash flow, normally downward arrows represent disbursement or costs and upward arrows represent receipts or benefits

(A) Interest Formulae for Single Payment Series

- Figure below shows a cash flow diagram involving a present single sum (P), a future single sum (F), separated by 'n' years with interest rate 'i' per year;

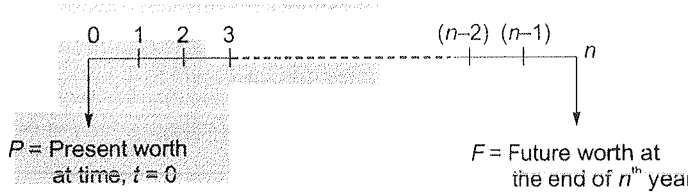


Fig. 5.1 Cash Flow Diagram for Single Payment

Case-I.

Finding F when P is given.

At the end of n years,

$$F = P(1 + i)^n$$

$$\frac{F}{P} = (1 + i)^n$$

The quantity $(1 + i)^n$ is commonly called **"single payment compound amount factor"** indicated by the functional notations as $\left(\frac{F}{P}, i, n\right)$

So, $$F = P\left(\frac{F}{P}, i, n\right)$$

$$\therefore \quad \left(\frac{F}{P}, i, n\right) = \text{Single payment compound amount factor} = (1 + i)^n.$$

Case-II.

Finding P when F is given.

From equation $F = P(1 + i)^n$, solving this for P gives the relation

$$P = F\left[\frac{1}{(1+i)^n}\right]$$

The quantity $\frac{1}{(1+i)^n}$ is called **"single payment present worth factor"** indicated by the functional symbol as $\left(\frac{P}{F}, i, n\right)$

$$\therefore \quad \left(\frac{P}{F}, i, n\right) = \text{Single payment present worth factor} = \frac{1}{(1+i)^n}$$

(B) **Interest Formulae for Equal Payment Series**

- Figure shows a general cash flow diagram involving a series of uniform (equal) payments, each of amount A, occurring at the end of each year with interest rate 'i' per year.

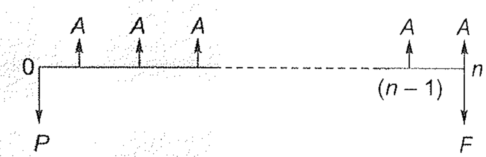


Fig. 5.2 Cash Flow Diagram for Equal Payments

Case-III.

Finding P when A is given.

If A exists at end of each year for n years with i rate of interest, the present worth P is obtained by summing the present worth of each payment of amount A

$$P = \frac{A}{(1+i)} + \frac{A}{(1+i)^2} + \frac{A}{(1+i)^3} + \ldots + \frac{A}{(1+i)^n}$$

$$= A\left[\frac{1}{(1+i)} + \frac{1}{(1+i)^2} + \frac{1}{(1+i)^3} + \ldots + \frac{1}{(1+i)^n}\right]$$

The series in the bracket is in the geometric progression whose first term (a) is $\frac{1}{(1+i)}$ and geometric ratio (r) is $\frac{1}{(1+i)}$

Sum of G.P. is given by a $\left(\frac{1-r^n}{1-r}\right)$

$$\therefore \quad P = A.\frac{1}{(1+i)}\left[\frac{1-\left(\frac{1}{1+i}\right)^n}{1-\left(\frac{1}{1+i}\right)}\right] = A\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right]$$

The quantity $\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right]$ is called **"uniform series present worth factor"** indicated by the function notation as $\left[\frac{P}{A}, i, n\right]$

$$\left[\frac{P}{A}, i, n\right] = \text{uniform (equal) series present worth factor} = \frac{(1+i)^n - 1}{i(1+i)^n}$$

Case-IV.

Finding A when P is given.

$$A = P\left[\frac{i(1+i)^n}{(1+i)^n - 1}\right]$$

The quantity in the bracket is called **"capital recovery factor"** using the functional symbol as $\left(\frac{A}{P}, i, n\right)$.

Case-V.

Finding F when A is given

$$P = \frac{F}{(1+i)^n} = \left[\frac{(1+i)^n - 1}{i(1+i)^n}\right]$$

$$\therefore \qquad F = A\left[\frac{(1+i)^n - 1}{i}\right]$$

The factor in the bracket is called **"equal payment series compound amount factor"** given by fundamental notation as $\left(\frac{F}{A}, i, n\right)$

Case-VI.

Finding A when F is given

$$A = F\left[\frac{i}{(1+i)^n - 1}\right]$$

The quantity in the bracket $\left[\frac{i}{(1+i)^n - 1}\right]$ is called **"sinking fund factor"** using functional notation as $\left(\frac{A}{F}, i, n\right)$

Hence,
$$A = F\left(\frac{F}{A}, i, n\right).$$

Summary of various Compound Interest Factors:

Compound Interest Factors					
	To find	Given	Factor by which given is multiplied	Factor Name	Functional notation
Single payment	F	P	$(1+i)^n$	Single payment compound amount factor	$\left(\frac{F}{P}, i, n\right)$
Single payment	P	F	$\frac{1}{(1+i)^n}$	Single payment present worth factor	$\left(\frac{P}{F}, i, n\right)$
Uniform Series Payment	P	A	$\frac{(1+i)^n - 1}{i(1+i)^n}$	Uniform series present worth factor	$\left(\frac{P}{A}, i, n\right)$
Uniform Series Payment	A	P	$\frac{i(1+i)^n}{(1+i)^n - 1}$	Capital recovery factor	$\left(\frac{A}{P}, i, n\right)$
Uniform Series Payment	F	A	$\frac{(1+i)^n - 1}{i}$	Uniform series compound amount factor	$\left(\frac{F}{A}, i, n\right)$
Uniform Series Payment	A	F	$\frac{i}{(1+i)^n - 1}$	Sinking fund factor	$\left(\frac{A}{F}, i, n\right)$

(C) Interest Formulae for Uniform Gradient Payment Series

- Some economic analysis problems involve receipts or disbursements that are projected to increase or decrease by a uniform amount each period, thus contributing an arithmetic series. In general, a uniformly increasing series of payment for n interest periods may be expressed as A_1, A_1+g, A_1+2g, A_1+3g.....$A_1+(n-1)g$ as shown in figure, where A_1 denotes the first year end payment in the series and 'g' the annual change in magnitude called gradient amounts.

  A_1 = Payment at the end of the first year

  g = Annual change in gradient

  n = Number of years

  A = Equivalent annual payment of the series

  A_2 = Equivalent annual payment of the gradient series $\{0, g, 2g, \ldots (n-1)g\}$ at the end of successive years.

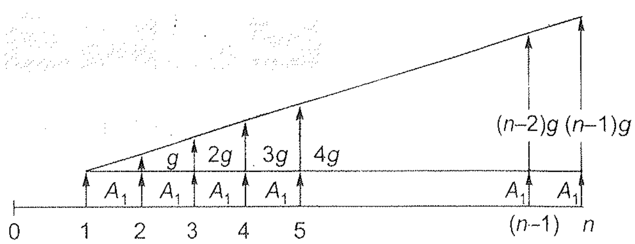
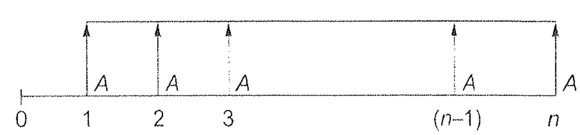


Fig. 5.3 Uniform Gradient Payment Series



$$A = A_1 + A_2$$

where $$A_2 = F\left(\frac{A}{F}, i, n\right) = \left[\frac{i}{(1+i)^n - 1}\right]$$

and F is the future amount equivalent to the gradient series. The future amount equivalent to the gradient series can be derived from the table as follows:

Table 5.1 : Gradient Series and an equivalent Set of Series

End of year	Gradient Series	Set of series equivalent to the Gradient series
1	0	0
2	g	g
3	2g	g+g
4	3g	g + g + g
.	.	.
.	.	.
.	.	.
n – 1	(n – 2)g	g + g + g + ... +g
n	(n – 1)g	g + g + g + ... g + g

$$F = g\left[\frac{F}{A},i(n-1)\right] + g\left[\frac{F}{A},i,(n-2)\right] + \ldots + g\left[\frac{F}{A},i,2\right] + g\left[\frac{F}{A},i,1\right]$$

$$= g\left[\frac{(1+i)^{n-1}-1}{i}\right] + g\left[\frac{(1+i)^{n-2}-1}{i}\right] + \ldots + g\left[\frac{(1+i)^{2}-1}{i}\right] + g\left[\frac{(1+i)^{1}-1}{i}\right]$$

$$= \frac{g}{i}\left[(1+i)^{n-1} + (1+i)^{n-2} + \ldots + (1+i)^{2} + (1+i)^{1} - (n-1)1\right]$$

$$= \frac{g}{i}\left[(1+i)^{n-1} + (1+i)^{n-2} + \ldots + (1+i)^{2} + (1+i)^{1} + 1(1+i)^{0}\right] - \frac{ng}{i}$$

The terms in the brackets consist n terms, 1st term being $(1 + i)^0$ and ratio being $(1 + i)$ of geometric progression.

$$= \frac{g}{i}\left[(1+i)^{0}\frac{1-(1+i)^{n}}{1-(1+i)}\right] - \frac{ng}{i} = \frac{g}{i}\left[\frac{(1+i)^{n}-1}{i}\right] - \frac{ng}{i}$$

$$A_2 = F\left[\frac{i}{(1+i)^{n}-1}\right] = \frac{g}{i}\left[\frac{(1+i)^{n}-1}{i}\right]\left[\frac{i}{(1+i)^{n}-1}\right] - \frac{n}{g}\left[\frac{i}{(1+i)^{n}-1}\right]$$

$$A_2 = \frac{g}{i} - \frac{ng}{i}\left[\frac{i}{(1+i)^{n}-1}\right] = g\left[\frac{1}{i} - \frac{n}{i}\left(\frac{A}{F},i,n\right)\right]$$

The resulting factor $\left[\frac{1}{i} - \frac{n}{i}\left(\frac{A}{F},i,n\right)\right]$ is called **gradient factor for annual compounding interest**

and will be designated $\left(\frac{A}{G},i,n\right)$

∴ A_2 = Equivalent annual cost of set of gradient series

$$= g\left(\frac{A}{G},i,n\right)$$

Example:

If a man receives an annual salary of Rs. 8,000 increasing at the rate of Rs. 500 a year, what is his equivalent uniform salary for a period of 10 years, present worth and future worth at an interest rate of 10%?

$$A = A_1 + g\left[\frac{A}{G}, i, n\right]$$

$$= 8000 + 500\left[\frac{1}{0.10} - \frac{10}{0.10}\left(\frac{0.10}{1.10^{10} - 1}\right)\right]$$

$$= 8000 + 500 \times 3.7255 = \text{Rs. } 9862.75$$

$$P = A\left[\frac{P}{A}, 10\%, 10\right] = \text{Rs. } 9862.75 \times 6.0104 = \text{Rs. } 59{,}279$$

$$F = A\left[\frac{F}{A}, 10\%, 10\right] = \text{Rs. } 9862.75 \times 16.338 = \text{Rs. } 161{,}137$$

Nominal and Effective Interest Rates

Very often compounding of interest is done twice, four times or twelve times a year as per agreement. The interest rates associated with this more frequent compounding are normally quoted on an annual basis. Where the actual or effective rate of interest is 3%, interest compounded each six month period the annual or nominal, interest rate is quoted as '6% per year compounded semiannually'. Thus the nominal rate of interest is expressed on an annual basis and it is determined by multiplying the actual or 'effective rate per interest period times the number of interest periods per year. An expression for the effective annual interest rate may be derived from effective interest rate $= \left(1 + \frac{r}{c}\right)^c - 1$ where r is nominal interest rate and c is no. of interest periods per year. If the compounding is annual then $r = i$.

Do You Know?

All annuities (uniform series-end of year payment) start from the end of first year are called ordinary annuities. If the payment does not begin until some later date, the annuity is known as "deferred annuity".

5.3 Capitalised Cost

Capitalised worth is defined as the present worth of perpetual service. In other words, it is the present worth of a uniform series of annuity of infinite period. If there exists a present or principal amount P and this earns interest at the rate of i period, then the end of period perpetual payment A can be made from this principal as $P = \frac{A}{i}$. This can be obtained from the figure.

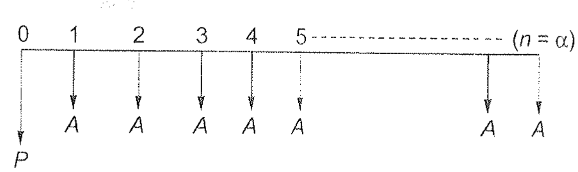


Fig. 5.4 Capitalised Cost

$$P = A\left[\frac{P}{A}, i.\alpha\right] = A\left[\frac{(1+i)^\alpha - 1}{i(1+i)^\alpha}\right] = A\left[\frac{1 - \frac{1}{(1+i)^\alpha}}{i}\right] = \frac{A}{i}$$

The P is often spoken as the capitalized value of A. In providing for perpetual care of some structure or the maintenance, we often encounter a special type of perpetuity. A certain amount X may be needed every K periods to provide a fund for replacement or maintenance. The most common method is to convert X amount occurring at end of K period into an equivalent cash flow of equal annual payment A and then let this A be assumed to extend infinity.

Another approach may be as follows; what principal amount P, when compounded at i per period for K periods will at the end of K^{th} period equal to X needed plus P to be available for accumulating interest to provide the next X payment. This can be stated as

$$P\left(\frac{F}{P}, i, K\right) = X + P$$

$$\therefore \qquad P = \frac{X}{\left[\left(\frac{F}{P}, i, K\right) - 1\right]} = \frac{X}{\left[(1+i)^K - 1\right]}$$

5.4 Comparison of Alternatives

In engineering economy, the prospective respective receipts and disbursements of two or more alternatives proposals must be placed on an equivalent basis for comparison. This may be accomplished by the proper use of interest formulae developed in the previous article. The most common bases for comparison are

A. Present worth amount
B. Annual equivalent amount
C. Capitalized amount
D. Rate of return method

5.4.1 The Present Worth amount

The method is described as follows,

1. Indicate all expenses and benefits at appropriate point of time in the form of cash flow separately for each alternative. Represent downward arrows for expenses (costs) and upward arrows for benefits or savings. In the algebraic expression + sign is for expenses and –ve sign is for benefits.
2. Shift the amount at zero time by using proper interest formulae and find total present worth amount of each alternative and compare. There may be two cases.

Case-I :

When service life of each alternative is same.

Case-II :

When service life of alternatives are different.

5.4.2 Annual Equivalent Amount

The annual equivalent amount is another basis for comparison that has characteristics similar to the present worth amount. The cash flow is converted in a series of equal annual amount by first calculating the present worth amount for the original series and then multiplying it with capital recovery factor $\left(\frac{A}{P}, i, n\right)$.

The comparison is made by seeing the annual equivalent amount of each alternative.

5.4.3 The Capitalized Amount

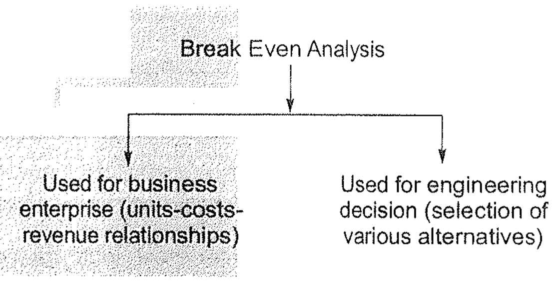
The annual equivalent amount obtained is assumed to extend for infinite period and the capitalized amount is obtained by using $\frac{A}{i}$. The equivalent annual amount is calculated for replacement and maintenance cost and added to the initial cost of the alternative.

5.4.4 Rate of Return Method

It is defined as the interest rate that reduces the present worth amount of a series of receipts and disbursements to zero for each alternative. In economic terms, the fundamental concept of rate of return is that it is the rate of interest earned on the unrecovered balance of an investment so that unrecovered balance is zero at end of investment's life.

The computation of rate of return generally requires a trial and error solution until the i^* can be interpolated. The common convention used is '+' sign for cash inflows and '–' signs for cash outflows.

5.5 Break Even Analysis



5.5.1 Break Even Analysis for Business Enterprise

Break even analysis implies that at some point in the business operations, total revenue equals total cost. Basically, Break Even Analysis is concerned with finding the point at which revenue and cost agree exactly, hence the term Breakeven (Point). The Break Even Point is therefore the volume of output at which neither a profit nor a loss is incurred. If the production rate is greater than that of the Break Even Point (BEP), a profit will result. When, the production rate is less than that of the (BEP), a loss will be incurred.

5.5.2 Break Even Chart

A break even chart is a graphical representation of the relationship between costs and revenue for all possible volumes of output. The main objective of the graphic device is to determine the break even point and profit potential under varying conditions of output and costs. In brief it may be called a continuous income (profit and loss) statements.

The variable cost is the ordinate between FF' and FV. Line OS is the revenue line and a linear relationship is utilised to describe the total sales. The point at which total cost line meets the revenue line is known as BEP (Break Even Point). The BEP can be described in the forms of output units or turnover of that output units. The horizontal line (or the distance) between BEP and output being produced is called **margin or safety, and it is expressed as a percentage of the total turnover.** If this distance is large, it indicates that the profits will be there even if there is a drop in production because of

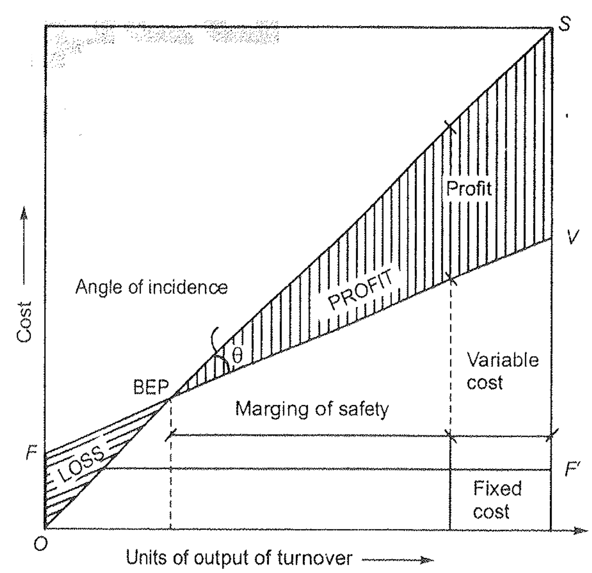


Fig. 5.5 Break Even Chart

some unavoidable reasons. The angle between sale line and total cost line at BEP is called **angle of incidence (θ). If this angle is large, rate of profit will be high.**

For the most favourable position in business the angle of incidence and margin of safety should be as high as possible.

5.5.3 Functions of Break Even Chart

The various functions of Break Even Chart are as follows:

1. It is an important tool of business management and it gives crystal clear view of the position of a business.
2. BEP of the chart marks clearly no profit no loss situation.
3. It is useful in understanding the effects of variations of fixed cost, variable cost and revenue on the profitability of a business. So it is used to portray the effects of proposed changes in operational policy.
4. Angle of incidence and margin of safety indicates the business position—whether favourable or unfavourable.
5. It indicates likely profits or losses at various outputs levels.
6. It is a decision making tool in the hands of management.

Limitations of Break Even Chart:

1. When the market conditions may not remain constant over the range of projected capacity it becomes difficult to classify the costs as fixed or variable, the BEP may not portray the true picture.
2. The break even chart is a tool for a short run analysis.
3. The total cost line (sum of variable cost and fixed cost) need not be straight line. In actual practice, costs do not vary in direct proportion.
4. The revenue line may also not be a straight line.
5. It represents a static picture whereas business operations are very much dynamic.
6. Analysis of break even chart presents additional difficulty when a company produces a variety of products.

5.6 Depreciation

Depreciation is defined as the loss in value of an asset with the passage of time. A reserve fund should be created to realize the cost of asset at the end of its life and it must not be confused with the profit. The main purpose of the depreciation is to provide for the recovery of capital that has been involved in the possession of the physical property.

Types of Depreciation:

A common classification of depreciation is as follows:

(a) Physical depreciation (b) Functional depreciation (c) Contingent depreciation

(a) Physical Depreciation

Depreciation resulting in physical impairment of an asset is known as physical depreciation. This results in lowering the ability of the asset to render its intended service. The primary cause of physical depreciation is wear and tear because of its constant use such as abrasion, shocks, vibration, impact etc. and the deterioration due to action of elements such as corrosion of pipe, chemical decomposition.

(b) Functional Depreciation

Functional depreciation often called OBSOLESCENCE and is defined as the loss in the value of the property due to change in fashion, design or structure or due to inadequacy to meet the growing demand, necessity of replacement due to new invention being more economical and more efficient etc.

(c) Contingent Depreciation

It is due to:

1. accidents (due to negligence)
2. disease (pollution of water, parasites)
3. diminution of supply (natural gas, electricity water etc.

Table 5.2 Difference between Depreciation and Obsolescence

Depreciation	Obsolescence
1. This is the physical loss in the value of property due to wear and tear and decay etc.	1. This is the functional loss in the value of the property due to change in design, structure, fashion, utility, demand etc.
2. It depends upon its original condition, quality of maintenance and mode of use.	2. It depends upon the technological advancement, art etc.
3. It varies with age.	3. Not dependent on age.
4. There are several methods for calculating the amount of depreciation.	4. No method is available for calculation of obsolescence.

Salvage Value (or Resale Value)

It is the value of the property at the end of its utility period without being dismantled. Salvage value implies that the property has further utility. A structure that has been well maintained and carefully handled will fetch more salvage value.

Scrap Value

The value of a property realized when it become absolutely useless except for sale as junk is its scrap value. The utility of the article is assumed to be zero.

Book Value

It is defined as the value of the property shown in the account books in that particular year i.e. the original cost less total depreciation till that year.

Methods for Calculating Depreciation

There are several methods of calculating depreciation. Some of them are simple and used regularly whereas some are relatively uncommon. The following notations have been used:

C_i = Initial cost of an asset at zero time (This will include the cost of asset +transportation charge + installation and other charges spent initially)

C_S = Salvage value (or scrape value) to be estimated at the end of utility period.

n = Life of the asset.

B_m = Book value at the end of period 'm'

5.6.1 Straight Line Method

In this method, the property is assumed to lose value by a constant amount every year. At the end of the life, the salvage value (or scrap value) left.

$$D_m = \frac{C_i - C_S}{n}$$

$$D_m = D_1 = D_2 = D_3 = D_n$$

$$B_m = C_1 - m\left(\frac{C_i - C_s}{n}\right)$$

Do You Know?

This method is recommended for all the equipments/assets with constant demand and which do not face any obsolescence during their useful life. It is widely used in the case of all civil engineering appliances and applications.

5.6.2 Declining Balance Method (or Constant Percentage Method)

In this method, the property is assumed to lose value annually at a constant percentage of its book value.

Let FDB be the fixed percentage

D_1 = Depreciation for 1st year = $C_i \times \text{FDB}$

∴ B_1 = Book value at end of 1st year

$= C_i - C_i \times \text{FDB} = C_i(1 - \text{FDB})$

D_2 = Depreciation for 2nd year = Book value at beginning of 2nd year × FDB

$= C_i(1 - \text{FDB})\,\text{FDB}$

(Book value at beginning of 2nd = Book value at the end of 1st year) depreciation for second year.

∴ $B_2 = C_i(1 - \text{FDB}) - C_i(1 - \text{FDB})\text{FDB}$

$= C_i(1 - \text{FDB})^2$

Similarly, $B_3 = C_i(1 - \text{FDB})^3$

....

....

∴ Book value at the end of n years = $C_i(1 - \text{FDB})^n$

∴ $C_S = C_i(1 - \text{FDB})^n$

∴ $$\text{FDB} = 1 - \left(\frac{C_S}{C_i}\right)^{\frac{1}{n}}$$

Do You Know?

In some cases, the cost of maintenance increases towards the end of the life, it may be desirable to charge large amount of depreciation in the early period and less in the fag period, so that the combined effect of depreciation and maintenance costs roughly remains the same. This method is most suitable when the asset fears to be obsolete before its life such as electronic equipments etc., but there are two difficulties in using this method. The first difficulty is that this formula can never depreciate to its salvage value. But this is not a serious difficulty. In actual practice, the depreciation for the last year is taken as book value at the beginning of the last year minus salvage value. The other serious difficulty is that this method can not be used when the salvage value of the asset is zero.

5.6.3 Double Declining Balance Method

In this method also, the property is assumed to lose value annually by fixed factor of the book value.

FDDB = Fixed factor for double declining balance method.

FDDB is taken as double the straight line

$$\text{Rate. i.e. FDDB} = \frac{2}{n}$$

The process of calculation of depreciation and book value of each year is the same as adopted in the Declining Balance Method.

Do You Know?

The applicability of the method is also similar i.e. when it is intended to charge larger portion of depreciation in the early life of the property, particularly the asset/equipment faces the obsolescence before the estimated life.

5.6.4 Sum of the Years Digit Method

This method also falls in the category of accelerated type depreciation method like Declining Balance Method and Double Declining Balance Method in which depreciation models assume that the value of an asset decreases at a decreasing rate.

In this method, the digits corresponding to the number of each years of life are listed in reverse order. The sum of these digits is then determined. The depreciation factor for any year is the reverse digit for the year divided by the sum of the digits, and depreciation for a year is the product of depreciation factor of that year and the amount ($C_i - C_S$). The general expression for the annual depreciation for any year (m) when the life is n years is expressed as

$$D_m = (C_i - C_S)\left[\frac{(n-m+1)}{\frac{n(n+1)}{2}}\right]$$

Do You Know?

This method also provides rapid depreciation during the early years of life like the previous two methods but this method has one advantage over them that it enables properties to be depreciated to zero value and is easier to use in practice.

5.6.5 Sinking Fund Method

Sinking fund depreciation model assumes that the value of an asset decreases at an increasing rate. Equal amount (D) is assumed to be deposited into a sinking fund at the end of each year of the assets life. The sinking fund is ordinarily compounded annually, at the end of the estimated life, the amount accumulated equals the total depreciation of the asset (C_i- C_S). It is to be clearly understood that the depreciation amount is the sum of two components. The first component is the amount deposited into sinking fund and the second component is the amount of interest earned on the accumulated value of the sinking fund at the beginning of the particular year in question.

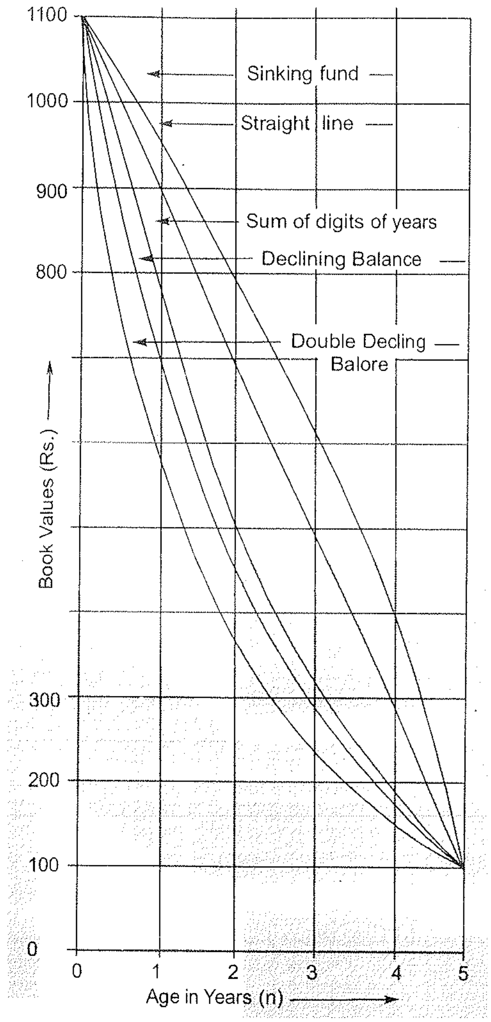


Fig. 5.6 Sinking Fund Method

Let 'i' be the rate of interest. The first component of depreciation which is to be deposited equally at the end of each year

$$= (C_i - C_S)\left[\frac{A}{F}, i, n\right]$$

$$D = [C_i - C_S]\left[\frac{i}{(1+i)^n - 1}\right]$$

Combining both factors into account, the depreciation for m^{th} year by this method is enumerated as

$$D_m = D[1+i]^{m-1}$$

$$m = 1, 2, 3.....$$

$$D_1 = D$$

$$D_2 = D(1+i)$$

$$D_3 = D(1+i)^2$$

$$D_m = ...$$

5.7 Depletion

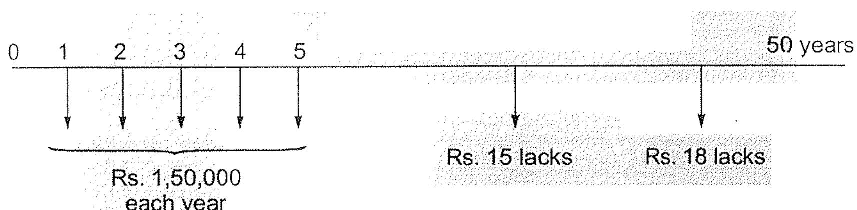
What natural resources are being consumed in producing products or services, the term 'Depletion' is used to indicate the decrease in value of the resource. The term is commonly used in connection with removal of coal from a mine, timber from a forest, stone from a quarry, oil from reservoir etc. When these natural resources are sold or mined, the reserve decreases and value of the property diminishes.

However there is a basic difference in the manner in which the amounts received through depletion and depreciation must be handled. In case of depreciation, the property involved usually is replaced with similar property when it has been fully depreciated. In the case of depletion of mineral or other natural resources, such replacement is not possible. Once the gold has been removed from the mine or the oil from the well, it cannot be replaced. Therefore in the actual operation of many natural resource business, the depletion fund created may be used to acquire new properties such as new mines to give continuity to the enterprise.

$$\text{Depletion for a year} = \left(\frac{\text{Cost of property}}{\text{No. of units in the property}}\right) \times \text{units sold during the year}$$

Example 5.1 The maintenance cost of an office building with design life of 50 years is estimated to be Rs. 1,50,000 for the first five years. An expenditure of Rs. 15 lacks is expected in the 20th years and Rs. 18 lacks in the 35th years. Assuming rate of interest as 10% per annum, what is the equivalent uniform annual cost over the entire design life of 50 years?

Solution:



Let, P = Present worth of various maintenance costs

$$= 150000\left(\frac{P}{A}, 10\%, 5\right) + 1500000\left(\frac{P}{F}, 10\%, 20\right) + 1800000\left(\frac{P}{F}, 10\%, 35\right)$$

$$= 150000\left[\frac{(1+i)^{n_1-1}}{i(1+i)^{n_1}}\right] + 1500000\left[\frac{1}{(1+i)^{n_2}}\right] + 1800000\left[\frac{1}{(1+i)^{n_3}}\right]$$

$$= 150000\left[\frac{(1+0.1)^5 - 1}{0.1(1+0.1)^5}\right] + 1500000\left[\frac{1}{(1+0.1)^{20}}\right] + 1800000\left[\frac{1}{(1+0.1)^{35}}\right]$$

$$= 150000\,(3.7908) + 1500000\,(0.1486) + 1800000\,(0.0356)$$

$$= 568620 + 222900 + 64080$$

$$= 855600$$

Let, A = Equivalent uniform annual cost over the entire design life of 50 years

$$\therefore \quad A = P\left(\frac{A}{P}, 10\%, 50\right) = 855600\frac{i + (1+i)^n}{\left[(1+i)^n - 1\right]} = 855600\frac{(0.1)(1+0.1)^{50}}{\left[(1+0.1)^{50} - 1\right]}$$

$$= 855600\,(0.10086) = \text{Rs. } 86295.82$$

Objective Brain Teasers

Q.1 Break even point is the volume of output at which
(a) Profit without loss is incurred
(b) no profit no loss is incurred
(c) only loss is incurred
(d) none of these

Q.2 Capital recovery factor considers
(a) developing a predetermined sum at a future data at a uniform rate
(b) consuming a predetermined sum till a future data at a uniform rate
(c) developing a predetermined sum at a future date at a uniformly varying rate
(d) consuming a predetermined sum till a future date at uniformly varying rate

Q.3 Which one of the following statements applies to the declining balance method of depreciation accounting?
(a) Uniform write-off of cost throughout the service is aimed
(b) Greater write-off in the early years is aimed
(c) Smaller write-off in early years is aimed
(d) A varying rate of depreciation is applied on the basis of market value of the asset

Q.4 For four levels of investment (whether or not technology changes are also implicit), the following estimates of annualised costs and annualised benefits are made as in the given table:

Plan	Annualised benefits (Rs. in Lakhs)	Annualised Cost (Rs. in Lakhs)
A	5000	2000
B	5250	2300
C	5850	2700
D	6100	3000

Out of these four, the plan of choice
(a) could be either *C* or *D*
(b) could be either *A*, *B* or *D*
(c) would be *D*
(d) would be *C*

Q.5 A machine costing Rs 8500 will have a scrap value of Rs 300. Machines of this class have a working hour average life of 25000 hours. What will be the depreciation charge at the end of the first year, if the machine is operated for a total of 1500 hours? (use straight line method of depreciation)
(a) Rs. 492.00 (b) Rs. 542.00
(c) Rs. 548.50 (d) Rs. 692.00

Q.6 An equipment costs Rs. 25 Lakhs with an estimated salvage value of Rs. 5 Lakhs after 5 years of useful life. What is the approximate equated annual cost for use of the equipment ?
(a) Rs. 4 Lakhs (b) Rs. 10 Lakhs
(c) Rs. 13 Lakhs (d) Rs. 17 Lakhs

Q.7 A machine costs Rs. 16,000. By constant rate of declining balance method of depreciation, its salvage value after an expected life of 3 years is Rs. 2,000. The rate of depreciation is
(a) 0.25 (b) 0.30
(c) 0.40 (d) 0.50

Q.8 The average investment value of an equipment over its 5-year life is Rs. 53000 whereas the annual depreciation is Rs. 11000. Cost of money is 15% pa and taxes are at 5% of investment value. The equipment is used for 1800 hours during each year. The hourly ownership cost for the equipment will be
(a) Rs. 37.77 (b) Rs. 24
(c) Rs. 18 (d) Rs. 12

Q.9 The principal or capitalized amount is sufficient to provide for a replacement costing Rs. 100,000 at the end of each fifth year from now. Continuing for ever, (i = 10% per year) is
(a) 11,000 (b) 50,000
(c) 163,797 (d) 1,50,000

Q.10 A man receives an annual salary of Rs. 8000 increasing at the rate of 500 a year. Than his equivalent salary for a period of 10 years is (given interest rate is 10%).

(a) 8500 (b) 8050

(c) 9000 (d) 9862.75

Answers

1. (b)	2. (b)	3. (b)	4. (d)	5. (a)
6. (a)	7. (d)	8. (b)	9. (c)	10. (d)

Hints & Solution

5. (a)

$$D_m = \frac{8500 - 300}{25000} = 0.328 \text{ Rs/hr}$$

$$\therefore \quad D_1 = 0.328 \times 1800 = \text{Rs. } 492$$

6. (a)

Equated annual cost or annual depreciation

$$= \frac{\text{Initial value} - \text{Salvage value}}{\text{Useful life of equipment in years}}$$

$$= \frac{2500000 - 500000}{5} = 400000$$

7. (d)

Rate of depreciation

$$r_D = 1 - \left(\frac{C_s}{C_i}\right)^{1/n}$$

Initial cost,

$$C_i = 16000$$

Salvage value,

$$C_s = 2000$$

$$n = 3$$

$$\therefore \quad r_D = 1 - \left(\frac{2000}{16000}\right)^{1/3} = 0.50$$

9. (c)

$$P = \frac{x}{\left[(1+i)^k - 1\right]} = \frac{100000}{(1+0.10)^5 - 1}$$

$$= 163{,}797$$

10. (d)

$$A = A_1 + g\left[\frac{A}{G}, i, n\right]$$

$$= 8000 + 500\left[\frac{1}{0.10} - \frac{10}{0.10}\left(\frac{0.10}{1.10^{10} - 1}\right)\right]$$

$$= 800 + 500 \times 3.7255$$

$$= 9862.75$$