Chapter 14. Factorisations

Question 1

Find the common factors of the given terms: $(i)4x,16x$ $(ii)3x^2y^3,27x^3y^2,51x^2y^2z$.

Solution:

$(i)4x,16x$

$4x = 1\times 4\times x$

$16x = 1\times 4\times 4\times x$

common factor $= 4x$

$(ii)3x^2y^3,27x^3y^2,51x^2y^2z$

$3x^2y^3 = 1\times 3\times x\times x\times y\times y\times y$

$27x^3y^2 = 1\times 3\times 3\times 3\times x\times x\times x\times y\times y$

$51x^2y^2z = 1\times 3\times 3\times 3\times 3\times x\times x\times y\times y\times z$

common factor $= 3x^2y^2$

Question 2

Factorise the following expressions:

$(i)\,5xy - 25x^3y^2$

$(ii)\,x(3x-y) - 5y(3x-y) - z(3x-y)$

$(iii)\,(a+b)(x+y) + (2a+3)(x+y) - (3a+4b)(x+y)$.

Solution:

$(i)5xy - 25x^3y^2$

$5xy - 25x^3y^2 = 5xy - 5\times 5xy\times x^2y$

$= 5xy(1-5x^2y)$

$(ii)\,x(3x-y) - 5y(3x-y) - z(3x-y)$

$= (3x-y)(x-5y-z)$

$(iii)\,(a+b)(x+y) + (2a+3)(x+y) - (3a+4b)(x+y)$

$= (x+y)[(a+b) + (2a+3b) - (3a+4b)] = (x+y)[3a+4b-3a-4b] = 0$

Question 3

Factorise: $(i) ap^2 + bp^2 + aq^2 + bq^2$ $(ii) 1 - c^2 + b^2 - c^2b^2$ $(iii) ab\left(x^2 + 1\right) + x\left(a^2 + b^2\right)$

Solution:

$$(i) ap^2 + bp^2 + aq^2 + bq^2$$
$$= p^2(a+b) + q^2(a+b) = (a+b)\left(p^2 + q^2\right)$$
$$(ii) 1 - c^2 + b^2 - c^2b^2$$
$$= \left(1 - c^2\right) + b^2\left(1 - c^2\right)$$
$$= \left(1 - c^2\right)\left(1 + b^2\right)$$
$$(iii) ab\left(x^2 + 1\right) + x\left(a^2 + b^2\right)$$
$$= abx^2 + ab + a^2x + b^2x$$
$$= abx^2 + a^2x + b^2x + ab$$
$$= ax(bx + a) + b(bx + a)$$
$$= (ax + b)(bx + a)$$

Question 4

Factorise: $(i)16l^2+24lm+9m^2$ $(ii)x^2+2\sqrt{3}x+3$ $(iii)16(x+y)^2-40(x+y)(x-y)+2y(x-y)^2$ $(iv)x^2+\frac{1}{x^2}+3-2x-\frac{2}{x}$.

Solution:

$(i)16l^2+24lm+9m^2$

$=(4l)^2+2\times(4l)\times(3m)+(3m)^2$

$=(4l+3m)^2$

$(ii)x^2+2\sqrt{3}x+3$

$=(x)^2+2\times x\times\sqrt{3}+\left(\sqrt{3}\right)^2$

$=\left(x+\sqrt{3}\right)^2$

$(iii)16(x+y)^2-40(x+y)(x-y)+2y(x-y)^2$

Let $(x+y)=p,\ (x-y)=q$

$16p^2-40pq+25q^2=(4p)^2-2\times4p\times5q+(5q)^2$

$=(4p-5q)^2$

$(4p-5q)^2=\{4(x+y)-5(x-y)\}^2$

$=\{4x+4y-5x+5y\}^2$

$=(-x+9y)^2$

$=(x-9y)^2$

$(iv)x^2+\frac{1}{x^2}+3-2x-\frac{2}{x}$

$=x^2+2\times x\times\frac{1}{x}+\frac{1}{x^2}-2\left(x+\frac{1}{x}\right)+1$

$=\left(x+\frac{1}{x}\right)^2-2\left(x+\frac{1}{x}\right)+1$

$=\left(x+\frac{1}{x}-1\right)^2$

Question 5

Factorize: $(i)\,64x^2y^2-81z^2$ $(ii)\,128ax^2-242ay^2$ $(iii)16p^3-4p$.

Solution:

$(i)\,64x^2y^2-81z^2$

$=(8xy)^2-(9z)^2$

$=(8xy+9z)(8xy-9z)$

$(ii)\,128ax^2-242ay^2$

$=2a\left(64x^2-121y^2\right)$

$=2a\left\{(8x)^2-(11y)^2\right\}$

$=2a(8x+11y)(8x-11y)$

$(iii)16p^3-4p$

$=4p\left(4p^2-1\right)$

$=4p\left((2p)^2-1^2\right)$

$=4p(2p-1)(2p+1)$

Question 6

Factorise: $(i)48(5a-2b)^2-75(2a-5b)^2$ $(ii)x^4+x^2y^2+y^4$ $(iii)x^8-y^8$.

Solution:

$$(i)48(5a-2b)^2-75(2a-5b)^2$$
$$=3\left\{16(5a-2b)^2-25(2a-5b^2)\right\}$$
$$=3\left\{4^2(5a-2b)^2-5^2(2a-5b)\right\}$$
$$=3\left\{(20a-8b)^2-(10a-25b)^2\right\}$$
$$=3\left\{(20a-8b)+(10a-25b)\right\}\left\{(20a-8b)-(10a-25b)\right\}$$
$$=3\{30a-33b\}\{10a+17b\}$$
$$=9(10a-11b)(10a+17b)$$

$$(ii)x^4+x^2y^2+y^4$$
$$=x^4+2x^2y^2+y^4-x^2y^2$$
$$=\left(x^2\right)^2+2x^2y^2+\left(y^2\right)^2-(xy)^2$$
$$=\left(x^2+y^2\right)^2-(xy)^2$$
$$=\left\{\left(x^2+y^2\right)+xy\right\}\left\{\left(x^2+y^2\right)-xy\right\}$$
$$=\left(x^2+xy+y^2\right)\left(x^2-xy+y^2\right)$$

$$(iii)x^8-y^8$$
$$=\left(x^4\right)^2-\left(y^4\right)^2$$
$$=\left(x^4+y^4\right)\left(x^4-y^4\right)$$
$$=\left(x^4+y^4\right)\left\{\left(x^2\right)^2-\left(y^2\right)^2\right\}$$
$$=\left(x^4+y^4\right)\left(x^2+y^2\right)\left(x^2-y^2\right)$$

Question 7

Carryout the following divisions: $(i) 48x^5 \div 8x$ $(ii) 16l^2m^2 \div (-64lm)$.

Solution:

$(i) 48x^5 \div 8x$

$= \frac{48x^5}{8x} = 6x^4$

$(ii) 16l^2m^2 \div (-64lm)$

$= \frac{16l^2m^2}{64lm} = \frac{lm}{4}$

$(iii)(xy - ab + bx - ay) \div (x - a)$

$xy - ab + bx - ay = xy - ay + bx - ab$

$= y(x - a) + b(x - a)$

$\frac{xy - ab + bx - ay}{x - a} = \frac{y(x - a) + b(x - a)}{x - a} = y + b$

Question 8

Divide: $(i)\left(6ab-b^2+12ac-2bc\right)\div(b+2c)$ $(ii)\left(x^2+x^3+x+1\right)\div(x+1)$ $(iii)\left(a^3+a-3a^2-3\right)\div(a-3)$ $(iv)\left(1-x^2+2xy-y^2\right)\div(x-1)$.

Solution:

$(i)\left(6ab-b^2+12ac-2bc\right)\div(b+2c)$

$$6ab-b^2+12ac-2bc=6ab+12ac-b^2-2bc$$

$$=6a(b+2c)-b(b+2c)=(b+2c)(6a-b)$$

$$\frac{\left(6ab-b^2+12ac-2bc\right)}{b+2c}=\frac{(b+2c)(6a-b)}{(b+2c)}=6a-b$$

$(ii)\left(x^2+x^3+x+1\right)\div(x+1)$

$$x^3+x^2+x+1=x^2(x+1)+1(x+1)=\left(x^2+1\right)(x+1)$$

$$\frac{x^3+x^2+x+1}{x+1}=\frac{\left(x^2+1\right)(x+1)}{(x+1)}=x^2+1$$

$(iii)\left(a^3+a-3a^2-3\right)\div(a-3)$

$$a^3+a-3a^2-3=a\left(a^2+1\right)-3\left(a^2+1\right)$$

$$=(a-3)\left(a^2+1\right)$$

$$\frac{\left(a^3+a-3a^2-3\right)}{a-3}=\frac{(a-3)\left(a^2+1\right)}{a-3}=a^2+1$$

$(iv)\left(1-x^2+2xy-y^2\right)\div(x-1)$

$$1-x^2+2xy-y^2=1-\left(x^2-2xy+y^2\right)$$

$$=(1)^2-(x-y)^2$$

$$=(1+x-y)\left[1-(x-y)\right]=(1+x-y)(1-x+y)$$

$$\frac{1-x^2+2xy-y^2}{x-1}=\frac{(1+x-y)(1-x+y)}{(x-1)}$$

Question 9

Find & correct the following statements: $(i) 3l + 5m = 8lm$ $(ii) \frac{75x}{x+5} = 5x$ $(iii)(a+5)(a-2) = a^2 + 5$ $(iv)(l-9)^2 = l^2 - 81$.

Solution:

$(i) 3l + 5m = 8lm$

$3l + 5m = 3l + 5m$

$(ii) \frac{75x}{x+5} = 5x$

$\frac{75x}{x+5} = \frac{75x}{x+5}$

$(iii)(a+5)(a-2) = a^2 + 5$

$(a+5)(a-2) = a^2 + 3a - 10$

$(iv)(l-9)^2 = l^2 - 81$

$(l-9)^2 = l^2 - 18l + 81$