PERMUTATION AND COMBINATION

1. FUNDAMENTAL PRINCIPLE OF COUNTING (counting without actual counting):

Let an event A can occur in 'm' different ways and another event B can occur in 'n' different ways.

(a) **Multiplication Rule :** The total number of different ways of simultaneous occurrence of both events in a definite order is m× n. This can be extended to any number of events.

(b) **Addition Rule :** The total number of different ways of happening exactly one of the events is m + n.

Example : There are 23 IITs in India and let each IIT offers 15 different courses, then the IITJEE topper can select the IIT and course in 23 × 15 = 345 number of ways.

Example : There are 23 IITs & 31 NITs in India, then a student who cleared both IITJEE Advance & JEE Main exams can select an institute in (23 + 31) = 54 number of ways.

SOLVED EXAMPLE

Example 1 : There are 4 buses running from Kota to Jaipur and 5 buses running from Jaipur to Delhi. In how many ways a person can travel from Kota to Delhi via Jaipur by bus?

Solution : Let E_1 be the event of travelling from Kota to Jaipur & E_2 be the event of travelling from Jaipur to Delhi by the person.

E_1 can happen in 4 ways and E_2 can happen in 5 ways.

Since both the events E_1 and E_2 are to be happened in order, simultaneously,

the number of ways = 4 × 5 = 20.

Example 2 : A college offers 6 courses in the morning and 4 in the evening. The number of ways a student can select exactly one course, either in the morning or in the evening-

(A) 6 (B) 4 (C) 10 (D) 24

Solution : The student has 6 choices from the morning courses out of which he can select one course in 6 ways.

For the evening course, he has 4 choices out of which he can select one in 4 ways.

Hence the total number of ways 6 + 4 = 10. **Ans. (C)**

2. PERMUTATION & COMBINATION :

(a) **Permutation :** Each of the arrangements in a definite order which can be made by taking some or all of the things at a time is called a PERMUTATION. In permutation, order of appearance of things is taken into account; when the order is changed, a different permutation is obtained.

${}^nP_r$ denotes the number of permutations of n **different** things, taken r at a time ($n \in N, r \in W, r \leq n$)

$${}^nP_r = n(n-1)(n-2)\ldots\ldots\ldots\ldots(n-r+1) = \frac{n!}{(n-r)!}$$

Note : **(i)** ${}^nP_n = n!$, ${}^nP_0 = 1$, ${}^nP_1 = n$

(ii) Number of arrangements of n **distinct** things taken all at a time = n!

(b) **Combination :** Each of the groups or selections which can be made by taking some or all of the things without considering the order of the things in each group is called a COMBINATION.

^nC_r denotes the number of combinations of n different things taken r at a time ($n \in N$, $r \in W$, $r \leq n$)

$$^nC_r = \frac{n!}{r!(n-r)!}$$

Note : $^nP_r = {^nC_r}.\, r!$

SOLVED EXAMPLE

Example 3 : If a denotes the number of permutations of (x + 2) things taken all at a time, b the number of permutations of x things taken 11 at a time and c the number of permutations of (x – 11) things taken all at a time such that a = 182 bc, then the value of x is

(A) 15 (B) 12 (C) 10 (D) 18

Solution: $^{x+2}P_{x+2} = a \Rightarrow a = (x+2)! \Rightarrow$

$$^{x}P_{11} = b \Rightarrow b = \frac{x!}{(x-11)!}$$

and $^{x-11}P_{x-11} = c \Rightarrow c = (x-11)!$

$$\because a = 182bc \Rightarrow (x+2)! = 182\frac{x!}{(x-11)!}(x-11)! \Rightarrow (x+2)(x+1) = 182 = 14 \times 13$$

$\therefore x+1 = 13 \quad \Rightarrow x = 12$ **Ans. (B)**

Example 4 : A box contains 5 different red and 6 different white balls. In how many ways can 6 balls be drawn so that there are atleast two red balls ?

Solution : The selections of 6 balls, consisting of atleast two red balls from 5 red and 6 white balls, can be made in the following ways

Red balls(5)	White balls(6)	Number of ways
2	4	$^5C_2 \times {^6C_4} = 150$
3	3	$^5C_3 \times {^6C_3} = 200$
4	2	$^5C_4 \times {^6C_2} = 75$
5	1	$^5C_5 \times {^6C_1} = 6$

Therefore total number of ways = 431 **Ans.**

Example 5 : If all the letters of the word 'TOUGH' are arranged in all possible manner as they are in a dictionary, then find the rank of the word 'TOUGH'.

Solution: First of all, arrange all letters of given word alphabetically : 'GHOTU'

Total number of words starting with G _ _ _ _ = 4! = 24

Total number of words starting with H _ _ _ _ = 4! = 24

Total number of words starting with O _ _ _ _ = 4! = 24

Total number of words starting with TG _ _ _ = 3! = 6

Total number of words starting with TH_ _ _ = 3! = 6

Total number of words starting with TOG_ _ = 2! = 2

Total number of words starting with TOH_ _ = 2! = 2

Total number of words starting with TOUGH = 1

$\therefore$ Rank of the word **TOUGH** = 24 + 24 + 24 + 6 + 6 + 2 + 2 + 1 = 89 **Ans.**

Example 6 : How many numbers between 10 and 10,000 can be formed by using the digits 1, 2, 3, 4, 5 if If (i) No digit is repeated in any number. (ii) Digits can be repeated.

Solution: Number of two digit numbers = 5 × 4 = 20

Number of three digit numbers = 5 × 4 × 3 = 60

Number of four digit numbers = 5 × 4 × 3 × 2 = 120

Total = 200

Example 7 : How many three digit can be formed using the digits 1, 2, 3, 4, 5, without repetition of digits? How many of these are even?

Solution: Three places are to be filled with 5 different objects.

$\therefore$ Number of ways = ${}^5C_3 \cdot 3! = 60$

For the 2nd part, unit digit can be filled in two ways & the remaining two digits can be filled in ${}^4P_2$ ways.

$\therefore$ Number of even numbers = $2 \times {}^4P_2 = 24$.

Example 8 : A delegation of four students is to be selected from a total of 12 students. In how many ways can the delegation be selected, if-

(a) all the students are equally willing ?

(b) two particular students have to be included in the delegation ?

(c) two particular students do not wish to be together in the delegation ?

(d) two particular students wish to be included together only ?

(e) two particular students refuse to be together and two other particular students wish to be together only in the delegation ?

Solution: (a) Formation of delegation means selection of 4 out of 12. Hence the number of ways = ${}^{12}C_4 = 495$.

(b) If two particular students are already selected. Here we need to select only 2 out of the remaining 10. Hence the number of ways = ${}^{10}C_2 = 45$.

(c) The number of ways in which both are selected = 45. Hence the number of ways in which the two are not included together = 495 – 45 = 450

(d) There are two possible cases

(i) Either both are selected. In this case, the number of ways in which the selection can be made = 45.

(ii) Or both are not selected. In this case all the four students are selected from the remaining ten students. This can be done in ${}^{10}C_4 = 210$ ways.

Hence the total number of ways of selection = 45 + 210 = 255

(e) We assume that students A and B wish to be selected together and students C and D do not wish to be together. Now there are following 6 cases.

(i) (A, B, C) selected, (D) not selected

(ii) (A, B, D) selected, (C) not selected

(iii) (A, B) selected, (C, D) not selected

(iv) (C) selected, (A, B, D) not selected

(v) (D) selected, (A, B, C) not selected

(vi) A, B, C, D not selected

For (i) the number of ways of selection = $^8C_1 = 8$

For (ii) the number of ways of selection = $^8C_1 = 8$

For (iii) the number of ways of selection = $^8C_2 = 28$

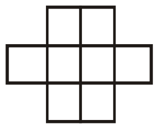
For (iv) the number of ways of selection = $^8C_3 = 56$

For (v) the number of ways of selection = $^8C_3 = 56$

For (vi) the number of ways of selection = $^8C_4 = 70$

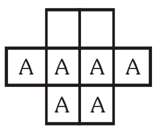
Hence total number of ways = 8 + 8 + 28 + 56 + 56 + 70 = 226. **Ans.**

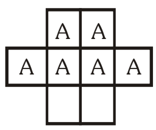
Example 9 : In the given figure of squares, 6 A's should be written in such a manner that every row contains at least one 'A'. In how many number of ways is it possible ?



(A) 24 (B) 25 (C) 26 (D) 27

Solution: There are 8 squares and 6 'A' in given figure. First we can put 6 'A' in these 8 squares by $^8C_6$ number of ways.

(I) 

(II) 

According to question, atleast one 'A' should be included in each row. So after subtracting these two cases, number of ways are = $(^8C_6 - 2) = 28 - 2 = 26$. **Ans. (C)**

Example 10 : There are four coplanar parallel lines. If any 5 points are taken on each of the lines, the maximum number of triangles with vertices at these points is :

(A) 1140 (B) 1130 (C) 1100 (D) 1010

Solution: Total no. of triangles

= No. of combination of three points – No. of combination of 3 collinear points

= $^{20}C_3 - {^5C_3} \times 4 = 1100$ **Ans. (C)**

Problems for Self Practise - 01 :

(1) Find the number of natural numbers from 1 to 1000 having none of their digits repeated.

(2) The number of signals that can be made with 3 flags each of different colour by hoisting 1 or 2 or 3 above the other, is:

(3) Find the number of ways of selecting 5 members from a committee of 5 men & 2 women such that all women are always included.

(4) Out of first 20 natural numbers, 3 numbers are selected such that there is exactly one even number. How many different selections can be made ?

(5) How many four letter words can be made from the letters of the word 'PROBLEM'. How many of these start as well as end with a vowel ?

Answers : (1) 738 (2) 15 (3) 10 (4) 450 (5) 840, 40

3. PERMUTATIONS OF ALIKE OBJECTS :

Case-I : Taken all at a time -

The number of permutations of n things taken all at a time : when p of them are similar of one type, q of them are similar of second type, r of them are similar of third type and the remaining n – (p + q+ r) are all different is: $\frac{n!}{p!\ q!\ r!}$.

SOLVED EXAMPLE

Example 11 : In how many ways the letters of the word "ARRANGE" can be arranged without altering the relative position of vowels & consonants.

Solution: The consonants in their positions can be arranged in $\frac{4!}{2!}$ = 12 ways.

The vowels in their positions can be arranged in $\frac{3!}{2!}$ = 3 ways

$\therefore$ Total number of arrangements = 12 × 3 = 36 **Ans.**

Example 12 : How many numbers can be formed with the digits 1, 2, 3, 4, 3, 2, 1 so that the odd digits always occupy the odd places?

(A) 17 (B) 18 (C) 19 (D) 20

Solution: There are 4 odd digits (1, 1, 3, 3) and 4 odd places (first, third, fifth and seventh). At these places the odd digits can be arranged in $\frac{4!}{2!2!} = 6$ ways

Then at the remaining 3 places, the remaining three digits (2, 2, 4) can be arranged in $\frac{3!}{2!} = 3$ ways

$\therefore$ The required number of numbers = 6 × 3 = 18. **Ans. (B)**

Example 13 : (a) How many permutations can be made by using all the letters of the word HINDUSTAN ?

(b) How many of these permutations begin and end with a vowel ?

(c) In how many of these permutations, all the vowels come together ?

(d) In how many of these permutations, none of the vowels come together ?

(e) In how many of these permutations, do the vowels and the consonants occupy the same relative positions as in HINDUSTAN ?

Solution: (a) The total number of permutations = Arrangements of nine letters taken all at a time $= \frac{9!}{2!} = 181440$.

(b) We have 3 vowels and 6 consonants, in which 2 consonants are alike. The first place can be filled in 3 ways and the last in 2 ways. The rest of the places can be filled in $\frac{7!}{2!}$ ways.

Hence the total number of permutations = $3 \times 2 \times \frac{7!}{2!} = 15120$.

(c) Assume the vowels (I, U, A) as a single letter. The letters (IUA), H, D, S, T, N, N can be arranged in $\frac{7!}{2!}$ ways. Also IUA can be arranged among themselves in 3! = 6 ways.

Hence the total number of permutations = $\frac{7!}{2!} \times 6 = 15120$.

(d) Let us divide the task into two parts. In the first, we arrange the 6 consonants as shown below in $\frac{6!}{2!}$ ways.

× C × C × C × C × C × C × (Here C stands for a consonant and × stands for a gap between two consonants)

Now 3 vowels can be placed in 7 places (gaps between the consonants) in ${}^7C_3.3! = 210$ ways.

Hence the total number of permutations = $\frac{6!}{2!} \times 210 = 75600$.

(e) In this case, the vowels can be arranged among themselves in 3! = 6 ways.

Also, the consonants can be arranged among themselves in $\frac{6!}{2!}$ ways.

Hence the total number of permutations = $\frac{6!}{2!} \times 6 = 2160$. **Ans.**

Example 14 : If all the letters of the word 'PROPER' are arranged in all possible manner as they are in a dictionary, then find the rank of the word 'PROPER' .

Solution: First of all, arrange all letters of given word alphabetically : EOPPRR

Total number of words starting with-

E _ _ _ _ _ $= \dfrac{5!}{2!2!} = 30$ O _ _ _ _ _ $= \dfrac{5!}{2!2!} = 30$

PE _ _ _ _ $= \dfrac{4!}{2!} = 12$ PO _ _ _ _ $= \dfrac{4!}{2!} = 12$

PP _ _ _ _ $= \dfrac{4!}{2!} = 12$ PRE _ _ _ = 3! = 6

PROE _ _ = 2! = 2 PROPER = 1= 1

Rank of the word PROPER = 105 **Ans.**

Example 15 : Find the total number of 4 letter words formed using four letters from the word "PARALLELOPIPED'.

Solution: Given letters are PPP, LLL, AA, EE, R, O, I, D.

Cases	No.of ways of selection	No.of ways of arrangements	Total
All distinct	${}^8C_4$	${}^8C_4 \times 4!$	1680
2 alike, 2 distinct	${}^4C_1 \times {}^7C_2$	${}^4C_1 \times {}^7C_2 \times \dfrac{4!}{2!}$	1008
2 alike, 2 other alike	${}^4C_2$	${}^4C_2 \times \dfrac{4!}{2!2!}$	36
3 alike, 1 distinct	${}^2C_1 \times {}^7C_1$	${}^2C_1 \times {}^7C_1 \times \dfrac{4!}{3!}$	56
		Total	2780

Ans.

Problems for Self Practise - 02 :

In how many ways we can select 4 letters from the letters of the word MISSISSIPPI?

Answers : 21

4. FORMATION OF GROUPS :

(i) Number of ways in which (m + n + p) **different things** can be divided into three different groups containing m, n & p things respectively is $\dfrac{(m+n+p)!}{m!\,n!\,p!}$, $(m \neq n \neq p)$

(ii) Number of ways in which (3m + 2n + p) **different things** can be divided into five different groups containing m, m, m, n, n & p things respectively is $\dfrac{(3m+2n+p)!}{(m!)^3(n!)^2 p!.3!.2!}$.

SOLVED EXAMPLE

Example 16 : In how many ways 8 different toys can be distributed between 3 brothers such that no one gets same number of toys and yonger brother receives more toys?

Solution: Number of ways = $\left(\frac{8!}{1!2!.5!}+\frac{8!}{1!.3!.4!}\right)\times 2!=896$

Example 17 : In how many ways 10 persons can be divided into 5 pairs?

Solution: We have each group having 2 persons and the qualitative characteristic are same (Since there is no purpose mentioned or names for each pair).

Thus the number of ways = $\frac{10!}{(2!)^5\,5!}$ = 945.

Example 18 : In how many ways can 15 students be divided into 3 groups of 5 students each such that 2 particular students are always together ? Also find the number of ways if these groups are to be sent to three different colleges.

Solution: Here first we separate those two particular students and make 3 groups of 5,5 and 3 of the remaining 13 so that these two particular students always go with the group of 3 students.

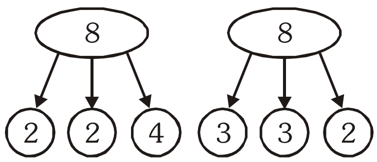
$\therefore$ Number of ways = $\frac{13!}{5!5!3!}.\frac{1}{2!}$.

Now if these groups are to be sent to three different colleges, total number of

ways = $\frac{13!}{5!5!3!}.\frac{1}{2!}.3!$ **Ans.**

Example 19 : In how many ways can 8 different books be distributed among 3 students if each receives at least 2 books ?

Solution: If each receives at least two books, then the division trees would be as shown below :



(i) (ii)

The number of ways of division for tree in figure (i) is $\left[\frac{8!}{(2!)^2 4!2!}\right]$.

The number of ways of division for tree in figure (ii) is $\left[\frac{8!}{(3!)^2 2!2!}\right]$.

The total number of ways of distribution of these groups among 3 students

is $\left[\frac{8!}{(2!)^2 4!2!}+\frac{8!}{(3!)^2 2!2!}\right]\times 3!$ = 2940 **Ans.**

Problems for Self Practise - 03 :

(1) 9 persons enter a lift from ground floor of a building which stops in 10 floors (excluding ground floor), if it is known that persons will leave the lift in groups of 2, 3, & 4 in different floors. In how many ways this can happen?

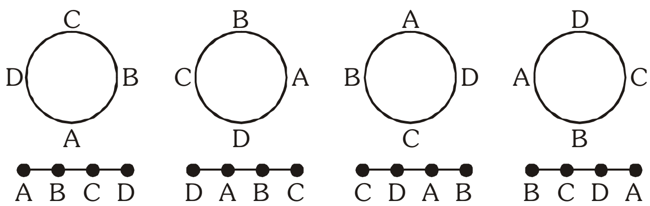
(2) In how many ways one can make four equal heaps using a pack of 52 playing cards?

(3) Find the number of ways in which 16 constables can be assigned to patrol 8 villages, 2 for each.

(4) In how many ways can 6 different books be distributed among 3 students such that none gets equal number of books and each gets atleast one book ?

Answers : (1) 907200 **(2)** $\frac{52!}{(13!)^4\,4!}$ **(3)** $\frac{16!}{(2!)^8 8!}\times 8!$ **(4)** 360

5. CIRCULAR PERMUTATION :



(a) (b) (c) (d)

Let us consider that persons A,B,C,D are sitting around a round table. If all of them (A,B,C,D) are shifted by one place in anticlockwise order, then we will get Fig.(b) from Fig.(a). Now, if we shift A,B,C,D in anticlockwise order, we will get Fig.(c). Again, if we shift them, we will get Fig.(d) and in the next time, Fig.(a).

Thus, we see that if 4 persons are sitting at a round table, they can be shifted four times and the four different arrangements, thus obtained will be the same, because anticlockwise order of A,B,C,D does not change.

But if A,B,C,D are sitting in a row and they are shifted in such an order that the last occupies the place of first, then the four arrangements will be different.

Thus, if there are 4 things, then for each circular arrangement number of linear arrangements is 4.

Similarly, if n different things are arranged along a circle, for each circular arrangement number of linear arrangements is n.

Therefore, the number of linear arrangements of n different things is n × (number of circular arrangements of n different things). Hence, the number of circular arrangements of n different things is -

1/n × (number of linear arrangements of n different things) = $\frac{n!}{n}$ = (n–1)!

Therefore note that :

(i) The number of circular permutations of n different things taken all at a time is : (n – 1)!.

If clockwise & anti-clockwise circular permutations are considered to be same, then it is : $\frac{(n-1)!}{2}$.

(ii) The number of circular permutations of n different things taking r at a time distinguishing clockwise & anticlockwise arrangements is : $\frac{{}^nP_r}{r}$

Example 20 : In how many ways can 5 boys and 5 girls be seated at a round table so that no two girls are together?

(A) $5! \times 5!$ (B) $5! \times 4!$ (C) $\frac{1}{2}(5!)^2$ (D) $\frac{1}{2}(5! \times 4!)$

Solution: Leaving one seat vacant between two boys, 5 boys may be seated in 4! ways. Then at remaining 5 seats, 5 girls sit in 5! ways. Hence the required number of ways = $4! \times 5!$

Ans. (B)

Example 21 : The number of ways in which 7 girls can stand in a circle so that they do not have same neighbours in any two arrangements ?

(A) 720 (B) 380 (C) 360 (D) none of these

Solution: Seven girls can stand in a circle by $\frac{(7-1)!}{2!}$ number of ways, because there is no difference in anticlockwise and clockwise order of their standing in a circle.

$\therefore$ $\frac{(7-1)!}{2!} = 360$ **Ans. (C)**

Example 22 : The number of ways in which 20 different pearls of two colours can be set alternately on a necklace, there being 10 pearls of each colour, is

(A) $9! \times 10!$ (B) $5(9!)^2$ (C) $(9!)^2$ (D) none of these

Solution: Ten pearls of one colour can be arranged in $\frac{1}{2}.(10-1)!$ ways. The number of arrangements of 10 pearls of the other colour in 10 places between the pearls of the first colour = 10!

$\therefore$ The required number of ways $= \frac{1}{2} \times 9! \times 10! = 5\,(9!)^2$ **Ans. (B)**

<u>Problems for Self Practise</u> - 04 :

(1) In how many ways can 3 men and 3 women be seated around a round table such that all men are always together ?

(2) Find the number of ways in which 6 persons out of 5 men & 5 women can be seated at a round table such that 2 men are never together.

(3) In how many ways can 8 persons be seated on two round tables of capacity 5 & 3.

Answers : (1) 36 **(2)** 5400 **(3)** 2688

6. SELECTION OF ONE OR MORE OBJECTS

(a) Number of ways in which atleast one object may be selected out of 'n' distinct objects, is

$$^nC_1 + {^nC_2} + {^nC_3} + \ldots\ldots\ldots\ldots\ldots + {^nC_n} = 2^n - 1$$

(b) Number of ways in which atleast one object may be selected out of 'p' alike objects of one type, 'q' alike objects of second type and 'r' alike objects of third type, is

$$(p + 1)\,(q + 1)\,(r + 1) - 1$$

(c) Number of ways in which atleast one object may be selected from 'n' objects where 'p' alike of one type, 'q' alike of second type and 'r' alike of third type and rest $n - (p + q + r)$ are different, is $(p + 1)\,(q + 1)\,(r + 1)\,2^{n-(p+q+r)} - 1$

SOLVED EXAMPLE

Example 23 : There are 10 different flowers in a basket. In how many ways we can select atleast one of them?

Solution: We may select 1 flower, 2 flowers,........, 10 flowers.

$\therefore$ The number of ways = ${}^{10}C_1 + {}^{10}C_2 + ... + {}^{10}C_{10} = 2^{10} - 1. = 1023$

Example 24 : There are 12 fruits in a basket of which 5 are apples, 4 mangoes and 3 bananas (fruits of same species are identical). How many ways are there to select atleast one fruit?

Solution: Let x be the number of apples being selected

y be the number of mangoes being selected and

z be the number of bananas being selected.

Then $x = 0, 1, 2, 3, 4, 5$ $y = 0, 1, 2, 3, 4$ $z = 0, 1, 2, 3$

Total number of triplets (x, y, z) is $6 \times 5 \times 4 = 120$

Exclude (0, 0, 0)

$\therefore$ Number of combinations = 120 – 1 = 119.

Example 25 : There are 3 books of mathematics, 4 of science and 5 of english. How many different collections can be made such that each collection consists of-

(i) one book of each subject ?

(ii) at least one book of each subject ?

(iii) at least one book of english ?

Solution: (i) ${}^3C_1 \times {}^4C_1 \times {}^5C_1 = 60$

(ii) $(2^3-1)(2^4-1)(2^5-1) = 7 \times 15 \times 31 = 3255$

(iii) $(2^5-1)(2^3)(2^4) = 31 \times 128 = 3968$ **Ans.**

Note: Let $N = p^a.\ q^b.\ r^c$ .... where p, q, r........ are distinct primes & a, b, c....... are natural numbers then :

(a) The total numbers of divisors of N including 1 & N is = $(a + 1)(b + 1)(c + 1)$.......

(b) The sum of these divisors is

$$= (p^0 + p^1 + p^2 + ...+ p^a)(q^0 + q^1 + q^2 + ...+ q^b)(r^0 + r^1 + r^2 + ...+ r^c)...$$

(c) Number of ways in which N can be resolved as a product of two factor is =

$$\frac{1}{2}\ (a+1)\ (b+1)\ (c+1)......\ \text{if N is not a perfect square}$$

$$\frac{1}{2}\ \left[(a+1)\ (b+1)\ (c+1)......+1\right]\ \text{if N is a perfect square}$$

(d) Number of ways in which a composite number N can be resolved into two factors which are relatively prime (or coprime) to each other is equal to 2^{n-1} where n is the number of different prime factors in N.

(e) All divisors except the number itself are called proper divisors.

Example 26 : Find the number of proper divisors of the number 38808. Also find the sum of these divisors.

Solution: **(i)** The number $38808 = 2^3 . 3^2 . 7^2 . 11$

Hence the total number of proper divisors (excluding itself i.e.38808)

$= (3 + 1)(2 + 1)(2 + 1)(1 + 1) - 1 = 71$

(ii) The sum of these divisors

$= (2^0 + 2^1 + 2^2 + 2^3)(3^0 + 3^1 + 3^2)(7^0 + 7^1 + 7^2)(11^0 + 11^1) - 38808$

$= (15)(13)(57)(12) - 38808 = 133380 - 38808 = 94572.$ **Ans.**

Example 27 : In how many ways the number 18900 can be split in two factors which are relative prime (or coprime)?

Solution: Here N = 18900 = $2^2 . 3^3 . 5^2 . 7^1$

Number of different prime factors in 18900 = n = 4

Hence number of ways in which 18900 can be resolved into two factors which are relative prime (or coprime) = $2^{4-1} = 2^3 = 8$. **Ans.**

Example 28 : Find the total number of proper factors of the number 35700. Also find

(i) sum of all these factors,

(ii) sum of the odd proper divisors,

(iii) the number of proper divisors divisible by 10 and the sum of these divisors.

Solution: $35700 = 5^2 \times 2^2 \times 3^1 \times 7^1 \times 17^1$

The total number of factors is equal to the total number of selections from (5,5), (2,2), (3), (7) and (17), which is given by 3 × 3 × 2 × 2 × 2 = 72.

These include 35700. Therefore, the number of proper divisors (excluding 35700) is 72 – 1 = 71

(i) Sum of all these factors (proper) is :

$(5^0 + 5^1 + 5^2)(2^0 + 2^1 + 2^2)(3^0 + 3^1)(7^0 + 7^1)(17^0 + 17^1) - 35700$

$= 31 \times 7 \times 4 \times 8 \times 18 - 35700 = 89292$

(ii) The sum of odd proper divisors is :

$(5^0 + 5^1 + 5^2)(3^0 + 3^1)(7^0 + 7^1)(17^0 + 17^1)$

$= 31 \times 4 \times 8 \times 18 = 17856$

(iii) The number of proper divisors divisible by 10 is equal to number of selections from (5,5), (2,2), (3), (7), (17) consisting of at least one 5 and at least one 2 and 35700 is to be excluded and is given by 2 × 2 × 2 × 2 × 2 – 1= 31.

Sum of these divisors is :

$(5^1 + 5^2)(2^1 + 2^2)(3^0 + 3^1)(7^0 + 7^1)(17^0 + 17^1) - 35700$

$= 30 \times 6 \times 4 \times 8 \times 18 - 35700 = 67980$ **Ans.**

Problems for Self Practise - 05 :

(1) From 5 apples, 4 mangoes & 3 bananas, in how many ways we can select atleast two fruits of each variety if (i) fruits of same species are identical? (ii) fruits of same species are different?

(2) Find the number of ways in which the number 94864 can be resolved as a product of two factors.

(3) Find the number of different sets of solution of xy = 1440.

Answers : (1) (i) 24 (ii) $2^{12} - 4$ **(2)** 23 **(3)** 36

7. MULTINOMIAL THEOREM :

Number of ways of selecting exactly r objects from (m + n + p) objects, where p are alike of one kind, m alike of second kind & n alike of third kind, is given by coefficient of x^r in the expansion of

$(1 + x + x^2 + ... + x^p)(1 + x + x^2 + ... + x^m)(1 + x + x^2 + ... + x^n)$.

SOLVED EXAMPLE

Example 29 : In how many ways exactly 5 fruits can be selected from 6 mangoes, 5 apples & 4 bananas ?

Solution : Number of Mangoes = x, Number of apples = y, Number of bananas = z

Now $x + y + z = 5$

Total number of ways = coefficient of x^5 in $(x^0 + x^1 + x^2 + ...+ x^5)(x^0 + x^1 + ...+ x^5)(x^0 + x^1 +....+ x^4)$

$\Rightarrow$ coefficient of x^5 in $\frac{1-x^6}{1-x}\cdot\frac{1-x^6}{1-x}\cdot\frac{1-x^5}{1-x}$

$\Rightarrow$ coefficient of x^5 in $(1 - x^5)(1 - x)^{-3}$

$\Rightarrow$ ${}^{3+5-1}C_5 - {}^{3+2-1}C_{2-1}$

$\Rightarrow$ ${}^8C_5 - {}^4C_1 = 52$

8. DISTRIBUTION OF OBJECT :

(a) Distribution of distinct objects : Number of ways in which n distinct things can be distributed to p persons if there is no restriction to the number of things received by them is given by : p^n

(b) Distribution of alike objects : Number of ways to distribute n alike things among p persons so that each may get none, one or more thing(s) is given by ${}^{n+p-1}C_{p-1}$.

SOLVED EXAMPLE

Example 30 : In how many ways can 5 different mangoes, 4 different oranges & 3 different apples be distributed among 3 children such that each gets alteast one mango ?

Solution : 5 different mangoes can be distributed by following ways among 3 children such that each gets atleast 1 :

3 1 1
2 2 1

Total number of ways : $\left(\frac{5!}{3!1!1!2!}+\frac{5!}{2!2!2!}\right)\times 3!$

Now, the number of ways of distributing remaining fruits (i.e. 4 oranges + 3 apples) among 3 children = 3^7 (as each fruit has 3 options).

$\therefore$ Total number of ways $=\left(\frac{5!}{3!2!}+\frac{5!}{(2!)^3}\right)\times 3! \times 3^7$

Example 31 : A is a set containing n elements. A subset P of A is chosen. The set A is reconstructed by replacing the elements of P. A subset Q of A is again chosen. The number of ways of choosing P and Q so that $P \cap Q = \phi$ is :-

Solution : Let $A = \{a_1, a_2, a_3, ... a_n\}$. For $a_i \in A$, we have the following choices :

(i) $a_i \in P$ and $a_i \in Q$ (ii) $a_i \in P$ and $a_i \notin Q$

(iii) $a_i \notin P$ and $a_i \in Q$ (iv) $a_i \notin P$ and $a_i \notin Q$

Out of these only (ii), (iii) and (iv) imply $a_i \notin P \cap Q$. Therefore, the number of ways in which none of $a_1, a_2, ...a_n$ belong to $P \cap Q$ is 3^n. **Ans.**

Example 32 : In how many ways can 12 identical apples be distributed among four children if each gets atleast 1 apple and not more than 4 apples.

Solution : Let x,y,z & w be the number of apples given to the children.

$\Rightarrow x + y + z + w = 12$

Coeff. of x^{12} in $(x^1 + x^2 + x^3 + x^4)^4$

coeff. of x^8 in$(1 - x^4)^4.(1 - x)^{-4}$

coeff. of x^8 in $({}^4C_0 - {}^4C_1.x^4 + {}^4C_2.x^8)(1 - x)^{-4}$

$= {}^{11}C_8 - {}^4C_1.{}^7C_4 + {}^4C_2.{}^3C_0 = 31$

Example 33 : Find the number of non negative integral solutions of the inequation $x + y + z \le 20$.

Solution : Let w be any number ($0 \le w \le 20$), then we can write the equation as :

$x + y + z + w = 20$ (here $x, y, z, w \ge 0$)

Total ways = ${}^{23}C_3$

Example 34 : Find the number of integral solutions of $x + y + z + w < 25$, where $x > -2$, $y > 1$, $z \ge 2$, $w \ge 0$.

Solution : Given $x + y + z + w < 25$

$x + y + z + w + v = 25$ ....(i)

Let $x = -1 + t_1$, $y = 2 + t_2$, $z = 2 + t_3$, $w = t_4$, $v = 1 + t_5$ where $(t_1, t_2, t_3, t_4 \ge 0)$

Putting value of x, y, z, w, v in equation (i)

$\Rightarrow t_1 + t_2 + t_3 + t_4 + t_5 = 21.$

Number of solutions = ${}^{25}C_4$

Example 35 : Find the number of positive integral solutions of the inequation $x + y + z \ge 150$, where $0 < x \le 60$, $0 < y \le 60$, $0 < z \le 60$.

Solution : Let $x = 60 - t_1$, $y = 60 - t_2$, $z = 60 - t_3$ (where $0 \le t_1 \le 59$, $0 \le t_2 \le 59$, $0 \le t_3 \le 59$)

Given $x + y + z \ge 150$ or $x + y + z - w = 150$ (where $0 \le w \le 30$) ....(i)

Putting values of x, y, z in equation (i)

$60 - t_1 + 60 - t_2 + 60 - t_3 - w = 150$

$30 = t_1 + t_2 + t_3 + w$

Total solutions = ${}^{33}C_3$

Example 36 : Find the number of positive integral solutions of $xy = 12$

Solution : $xy = 2^2 \times 3^1$

(i) 3 has 2 ways either 3 can go to x or y

(ii) 2^2 can be distributed between x & y as distributing 2 identical things between 2 persons

(where each person can get 0, 1 or 2 things). Let two person be ℓ_1 & ℓ_2

$\Rightarrow \ell_1 + \ell_2 = 2$ $\Rightarrow {}^{2+1}C_1 = {}^3C_1 = 3$

So total ways = 2 × 3 = 6.

Alternatively : $xy = 12 = 2^2 \times 3^1$

$x = 2^{a_1}3^{a_2}$ $0 \le a_1 \le 2$, $0 \le a_2 \le 1$

$y = 2^{b_1}3^{b_2}$ $0 \le b_1 \le 2$, $0 \le b_2 \le 1$

$2^{a_1+b_1}3^{a_2+b_2} = 2^2 3^1$

$\Rightarrow a_1 + b_1 = 2 \rightarrow {}^3C_1$ ways & $a_2 + b_2 = 1 \rightarrow {}^2C_1$ ways

Number of solutions = ${}^3C_1 \times {}^2C_1 = 3 \times 2 = 6$

Example 37 : Find the number of solutions of the equation xyz = 360 when **(i)** $x,y,z \in N$ **(ii)** $x,y,z \in I$

Solution : **(i)** $xyz = 360 = 2^3 \times 3^2 \times 5$ $(x,y,z \in N)$

$x = 2^{a_1}3^{a_2}5^{a_3}$ (where $0 \le a_1 \le 3$, $0 \le a_2 \le 2$, $0 \le a_3 \le 1$)

$y = 2^{b_1}3^{b_2}5^{b_3}$ (where $0 \le b_1 \le 3$, $0 \le b_2 \le 2$, $0 \le b_3 \le 1$)

$z = 2^{c_1}3^{c_2}5^{c_3}$ (where $0 \le c_1 \le 3$, $0 \le c_2 \le 2$, $0 \le c_3 \le 1$)

$\Rightarrow \quad 2^{a_1}3^{a_2}5^{a_3}.2^{b_1}3^{b_2}5^{b_3}.2^{c_1}3^{c_2}5^{c_3} = 2^3 \times 3^2 \times 5^1$

$\Rightarrow \quad 2^{a_1+b_1+c_1}.3^{a_2+b_2+c_2}.5^{a_3+b_3+c_3} = 2^3 \times 3^3 \times 5^1$

$\Rightarrow \quad a_1 + b_1 + c_1 = 3 \rightarrow {}^5C_2 = 10$

$a_2 + b_2 + c_2 = 2 \rightarrow {}^4C_2 = 6$

$a_3 + b_3 + c_3 = 1 \rightarrow {}^3C_2 = 3$

Total solutions = 10 × 6 × 3 = 180.

(ii) If $x,y,z \in I$ then, (a) all positive (b) 1 positive and 2 negative.

Total number of ways = $180 + {}^3C_2 \times 180 = 720$

<u>Problems for Self Practise</u> - 06 :

(1) In how many ways can 12 identical apples be distributed among 4 boys. (a) If each boy receives any number of apples. (b) If each boy receives atleast 2 apples.

(2) Find the number of integral solutions of x + y + z = 20, if $x \ge -4$, $y \ge 1$, $z \ge 2$

Answers : (1) 455, 35 (2) 253

9. DEARRANGEMENT :

There are n letters and n corresponding envelopes. The number of ways in which letters can be placed in the envelopes (one letter in each envelope) so that no letter is placed in correct envelope is

$$n!\left[1-\frac{1}{1!}+\frac{1}{2!}+.....+\frac{(-1)^n}{n!}\right]$$

SOLVED EXAMPLE

Example 38 : A person writes letters to six friends and addresses the corresponding envelopes. In how many ways can the letters be placed in the envelopes so that

(i) all the letters are in the wrong envelopes.

(ii) at least two of them are in the wrong envelopes.

Solution : (i) The number of ways is which all letters be placed in wrong envelopes

$$= 6!\left(1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}-\frac{1}{5!}+\frac{1}{6!}\right) = 720\left(\frac{1}{2}-\frac{1}{6}+\frac{1}{24}-\frac{1}{120}+\frac{1}{720}\right)$$

= 360 – 120 + 30 – 6 + 1 = 265.

(i) The number of ways in which at least two of them in the wrong envelopes

$$= {}^6C_4 . 2!\left(1-\frac{1}{1!}+\frac{1}{2!}\right) + {}^6C_3 . 3!\left(1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}\right) + {}^6C_2 . 4!\left(1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}\right)$$

$$+ {}^6C_1 . 5!\left(1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}-\frac{1}{5!}\right) + {}^6C_0\, 6!\left(1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}-\frac{1}{5!}+\frac{1}{6!}\right)$$

= 15 + 40 + 135 + 264 + 265 = 719. **Ans.**

Problems for Self Practise - 07 :

(1) There are four balls of different colours and four boxes of colours same as those of the balls. Find the number of ways in which the balls, one in each box, could be placed in such a way that a ball does not go to box of its own colour.

(2) In a match the column question, Column I contain 10 questions and Column II contain 10 answers written in some arbitrary order. In how many ways a student can answer this question so that exactly 6 of his matchings are correct?

Answers : (1) 9 **(2)** 1890

10. EXPONENT OF PRIME NUMBER IN n!

Prime factorisation of n! : Let p be a prime number and n be a positive integer, then exponent of p in n! is denoted by E_p (n!) and is given by

$$E_p(n!) = \left[\frac{n}{p}\right]+\left[\frac{n}{p^2}\right]+\left[\frac{n}{p^3}\right]+.....+\left[\frac{n}{p^k}\right]$$

where, $p^k \le n < p^{k+1}$ and [x] denotes the integral part of x.

If we isolate the power of each prime contained in any number n, then n can be written as

n = $2^{\alpha_1}.3^{\alpha_2}.5^{\alpha_3}.7^{\alpha_4}....$, where α_i are whole numbers.

SOLVED EXAMPLE

Example 39 : Find the exponent of 6 in 50!

Solution : $E_2(50!) = \left[\frac{50}{2}\right]+\left[\frac{50}{4}\right]+\left[\frac{50}{8}\right]+\left[\frac{50}{16}\right]+\left[\frac{50}{32}\right]+\left[\frac{50}{64}\right]$ (where [ ] denotes integral part)

$E_2(50!)$ = 25 + 12 + 6 + 3 + 1 + 0 = 47

$$E_3(50!) = \left[\frac{50}{3}\right]+\left[\frac{50}{9}\right]+\left[\frac{50}{27}\right]+\left[\frac{50}{81}\right]$$

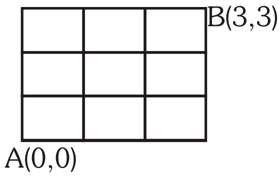
$E_3(50!)$ = 16 + 5 + 1 + 0 = 22

$\Rightarrow$ 50! can be written as 50! = $2^{47}.\,3^{22}$.........

Therefore exponent of 6 in 50! = 22

MISCELLANEOUS EXAMPLES:

Example 40 : In how many ways can a person go from point A to point B if he can travel only to the right or upward along the lines (Grid Problem) ?



Solution : To reach the point B from point A, a person has to travel along 3 horizontal and 3 vertical strips.

Therefore, we have to arrange 3H and 3V in a row. Total number of ways = $\frac{6!}{3!3!} = 20$ ways **Ans.**

Example 41 : Find sum of all numbers formed using the digits 2,4,6,8 taken all at a time and no digit being repeated.

Solution : All possible numbers = 4! = 24

If 2 occupies the unit's place then total numbers = 6

Hence, 2 comes at unit's place 6 times.

Sum of all the digits occuring at unit's place

= 6 × (2 + 4 + 6 + 8)

Same summation will occur for ten's, hundred's & thousand's place. Hence required sum

= 6 × (2 + 4 + 6 + 8) × (1 + 10 + 100 + 1000) = 133320

Example 42 : Find the sum of all the numbers greater than 1000 using the digits 0,1,2,2.

Solution : (i) When 1 is at thousand's place, total numbers formed will be = $\frac{3!}{2!} = 3$

(ii) When 2 is at thousand's place, total numbers formed will be = 3! = 6

(iii) When 1 is at hundred's, ten's or unit's place then total numbers formed will be-

Thousand's place is fixed i.e. only the digit 2 will come here, remaining two places can be filled in 2! ways.

So total numbers = 2!

(iv) When 2 is at hundred's, ten's or unit's place then total numbers formed will be-

Thousand's place has 2 options and other two places can be filled in 2 ways.

So total numbers = 2 × 2 = 4

Sum = 10^3 (1 × 3 + 2 × 6) + 10^2 (1 × 2 + 2 × 4) + 10^1(1 × 2 + 2 × 4) + (1 × 2 + 2 × 4)

= 15 × 10^3 + 10^3 + 10^2 + 10

= 16110 **Ans.**

Example 43 : Find the number of positive integral solutions of x + y + z = 20, if $x \neq y \neq z$.

Solution : $x \geq 1$

$y = x + t_1 \qquad t_1 \geq 1$

$z = y + t_2 \qquad t_2 \geq 1$

$x + x + t_1 + x + t_1 + t_2 = 20$

$3x + 2t_1 + t_2 = 20$

(i) $x = 1$ $2t_1 + t_2 = 17$

$t_1 = 1,2 \ldots\ldots\ldots 8 \Rightarrow 8$ ways

(ii) $x = 2$ $2t_1 + t_2 = 14$

$t_1 = 1,2 \ldots\ldots\ldots 6 \Rightarrow 6$ ways

(iii) $x = 3$ $2t_1 + t_2 = 11$

$t_1 = 1,2 \ldots\ldots\ldots 5 \Rightarrow 5$ ways

(vi) $x = 4$ $2t_1 + t_2 = 8$

$t_1 = 1,2,3 \Rightarrow 3$ ways

(v) $x = 5 2t_1 + t_2 = 5$

$t_1 = 1, 2 \Rightarrow 2$ ways

Total = 8 + 6 + 5 + 3 + 2 = 24

But each solution can be arranged by 3! ways.

So total solutions = 24 × 3! = 144.

Example 44 : 10 persons are sitting in a row. In how many ways we can select three of them if adjacent persons are not selected?

Solution : Let $P_1, P_2, P_3, P_4, P_5, P_6, P_7, P_8, P_9, P_{10}$ be the persons sitting in this order.

If three are selected (non consecutive) then 7 are left out.

Let P,P,P,P,P,P,P be the left out & q, q, q be the selected. The number of ways in which these 3 q's can be placed into the 8 positions between the P's (including extremes) is the number ways of required selection.

Thus number of ways = ${}^8C_3 = 56$.

Example 45 : A regular polygon has 20 sides. How many triangles can be drawn by using the vertices, but not using the sides?

Solution : The first vertex can be selected in 20 ways. The remaining two are to be selected from 17 vertices so that they are not consecutive. This can be done in ${}^{17}C_2 - 16$ ways.

$\therefore$ The total number of ways = $20 \times ({}^{17}C_2 - 16)$

But in this method, each selection is repeated thrice.

$\therefore$ Number of triangles = $\dfrac{20 \times ({}^{17}C_2 - 16)}{3}$ = 800.

Exercise # 1

PART-I : SUBJECTIVE QUESTIONS

Section(A) : Fundamental Principle of counting, Problems based on selection and arrangements

A-1. There are nine students (5 boys & 4 girls) in the class. In how many ways

(a) Three distinct books can be distributed between two boys and one girl.

(b) Three distinct books can be distributed between three students in which atleast one is boy.

(c) One book of Maths, two books of Physics and three books of Chemistry can be distributed

(No student can get more than one book in same subject & books are distinct)

A-2. How many of the four digit numbers are there whose atleast one digit is even?

A-3. Mr. Rahul goes to an ATM to withdraw cash but he forgets his four digits ATM PIN. Find his maximum number of unsuccessful attempts to withdraw cash?

A-4. In how many ways we can select a committee of 6 persons from 6 boys and 3 girls, if atleast two boys & atleast two girls must be there in the committee?

A-5. In how many ways five different red balls and five different black balls can be arranged in a row such that.

(i) there is no restriction.

(ii) all red balls are not together.

(iii) no two red balls are together.

(iv) All red balls are together and all black balls are together

(v) Red and black ball are alternatively.

A-6. Let 'm' be the total number of words formed by arranging the letters of the word "SPECIAL" in all possible manner and 'n' be the number of words in which vowels occur alphabetically, then find $\frac{m}{n}$.

A-7. In a question paper there are two parts part A and part B each consisting of 5 questions. In how many ways a student can answer 6 questions, by selecting atleast two from each part?

A-8. Find the total number of three digit of natural number whose atleast one digit is 3.

A-9. Find the number of 6 digit numbers whose last two digits are even. (Repetition of digits are not allowed)

A-10. Consider the five points comprising of the vertices of a square and the intersection point of its diagonals. How many triangles can be formed using these points?

A-11. How many 4 digit numbers are there, without repetition of digits, if each number is divisible by 5?

A-12 Out of seven consonants and four vowels, how many words of six letters can be formed by taking atleast four consonants and two vowels (Assume that each ordered group of letter is a word).

A-13. There are 3 white, 4 blue and 1 red flowers. All of them are taken out one by one and arranged in a row in the order. How many different arrangements are possible (flowers of same colours are similar)?

A-14. Three ladies have brought one child each for admission to a school. The principal wants to interview the six persons one by one subject to the condition that no mother is interviewed before her child. Then find the number of ways in which interviews can be arranged.

A-15. Find the number of words which can be formed by using all letters of the word 'RELIABLE'. If
(i) First & last letters are vowels
(ii) Starts with R and end with E.
(iii) Vowels always occur together
(iv) All vowels are not together
(v) No two vowels are together
(vi) Vowels & consonants are alternate
(vii) Vowels always occupy even places only
(viii) Order of vowels remains same.
(ix) Relative order of vowels & consonants remain same
(x) Number of words formed by selecting 2 vowels and 3 consonants.

A-16. Find the total number of ways of choosing a committee of 2 women & 3 men from 5 women & 6 men, if Mr. John refuses to serve on the committee if Mr. Jony is a member & Mr. Jony can only serve, if Miss Julie is the member of the committee.

A-17. Find the number of ways in which 3 distinct numbers can be selected from the set $\{3^1, 3^2, 3^3, ... 3^{100}, 3^{101}\}$ so that they form a G.P.

A-18. How many four digit natural numbers not exceeding the number 4321 can be formed using the digits 1, 2, 3, 4, if repetition is allowed?

A-19. If $A = \{1, 2, 3, 4 .....n\}$ and $B \subseteq A$; $C \subseteq A$, then the find number of ways of selecting
(i) Sets B and C
(ii) Unordered pairs of B and C
(iii) Order pair of B and C such that $B \cap C = \phi$
(iv) Unordered pair of B and C such that $B \cap C = \phi$
(v) Ordered pair of B and C such that $B \cup C = A$ and $B \cap C = \phi$
(vi) Unordered pair of B and C such that $B \cup C = A$, $B \cap C = \phi$
(vii) Ordered pair of B and C such that $B \cap C$ is singleton

A-20. There are 2 women participating in a chess tournament. Every participant played 2 games with the other participants. The number of games that the men played between themselves exceeded by 66 as compared to the number of games that the men played with the women. Find the number of participants & the total numbers of games played in the tournament.

A-21. How many arithmetic progressions with 10 terms are there, whose first term is in the set {1, 2, 3, 4} and whose common difference is in the set {3, 4, 5, 6, 7} ?

A-22. Find the number of all five digit numbers whose product of digit is divisible by 5.

Section (B): Grouping and Circular Permutation

B-1. In how many ways 5 boys and 4 girls can sit in straight line if 2 girls are together and the other 2 girls are also together but separated from the first 2.

B-2. In how many ways 20 different books can be distributed between 7 students such that 3 students receive 4 books each and remaining students 2 books each.

B-3. In how many ways Rohit can score a century in 20 consecutive ball if he hits either six or four or play a dot ball ?

B-4. (a) In how many ways five people can be distributed in three different rooms if no room must be empty?
(b) In how many ways can five people be arranged in three different rooms if no room must be empty and each room has 5 seats in a single row.

B-5. Prove that $\dfrac{n^2!}{(n!)^n}$ is an integer for all $n \in N$

B-6. In how many ways 5 persons can sit at a round table if two particular persons do not sit together.

B-7. Eight persons including A and B are seated on a circular table. How many arrangements are possible if exactly two persons sit between A and B.

B-8. In how many ways 4 boys and 4 girls can sit around a round table if no two girls are together.

Section (C): Selection of one or more objects

C-1. Find the number of ways of selection of one or more letters from the letters of the word 'INDEPENDENCE' if
- (i) There is no restriction.
- (ii) The letters D & E are selected atleast once.
- (iii) Only one letter is selected.
- (iv) Atleast two letters are selected.

C-2. In how many ways atleast one vowel and atleast one consonant can be selected from the letters of the word 'BETWEEN'.

C-3. Find the number of divisors of the number 162000.
- (i) How many of these are divisible by 4 but not by 16.
- (ii) How many of these are divisible by 15 but not by 25. Also find their sum.

C-4. Let N = 200200, then
- (i) In how many ways N can be expressed as the product of two factors.
- (ii) In how many ways N can be expressed as the product of two relatively prime factors.

C-5. Find the number of divisors of the number $5^5.7^4.11^3$ which are of the form $6\lambda+1$, where $\lambda \in N$.

Section(D): Multinomial theorem and Dearrangement

D-1. Find the number of negative integral solutions of the equation $x + y + z = -20$.

D-2. In how many ways it is possible to divide six identical green, six identical red and six identical blue balls among Ram and Shyam such that each gets equal number of balls.

D-3. If x_1, x_2, x_3 are the whole numbers and gives remainders 0,1 and 2 when divided by 3, Then find the total number of solutions of the equation $x_1+x_2+x_3 = 33$.

D-4. Find the number of solutions of the equation $x + y + z + w = 20$ under the following restrictions:
- (i) x,y,z,w are whole numbers.
- (ii) x,y,z,w are natural numbers.
- (iii) $x,y,z,w \in \{1,2,3,\ldots,10\}$
- (iv) x and y are odd natural numbers while z & w are even natural numbers.

D-5. A person throws a fair dice four times. In how many ways he can score a total of 16.

D-6. Five balls are to be placed in three boxes. In how many different ways these balls can be placed so that no box remains empty if
- (i) balls and boxes are different
- (ii) balls are identical and boxes are different
- (iii) balls are different and boxes are identical
- (iv) balls as well as boxes are identical

D-7. A person writes letters to five friends and addresses on the corresponding envelopes. In how many ways the letters be placed in the envelopes so that
- (i) all letters are in the wrong envelopes.
- (ii) atleast three of them are in the wrong envelopes.

Section(E): Miscellaneous

E-1. For each positive integer k, let S_k denote the increasing arithmetic sequence of integers whose first term is 1 and whose common difference is k. Find the number of values of k for which S_k contains the term 361.

E-2. (i) Find the exponent of 3 in 50!.

(ii) Find the exponent of 12 in 15!.

E-3. Find the total number of ways of selecting 3 numbers from the set of first 50 natural numbers such that their sum is divisible by 3.

E-4. How many four digit numbers can be formed in which sum of first two digits is equal to sum of last two digits. (Assume 0000,0111,0001,…all are four digit numbers).

E-5. Let each side of smallest square of chess board is one unit in length.

(i) Find the total number of rectangles on the chess board with one side even and one side odd.

(ii) In how many ways a pair of smallest square can be selected which has exactly one corner common.

E-6. Determine the number of paths from the origin to the point (5,6) in the cartesian plane which never pass through the points (2,1) and (3,3) in paths consisting only of steps going 1 unit North and 1 unit East.

PART-II : OBJECTIVE QUESTIONS

Section(A): Fundamental Principle of counting, Problems based on selection and arrangements

A-1. Using 6 different flags, how many different signals can be made by using atleast 3 flags, arranging one above the other?

(A) 1890 (B) 1980 (C) 1920 (D) 1800

A-2. 8 chairs are numbered from 1 to 8. Two women & 3 men wish to occupy one chair each. First the women choose the chairs from amongst the chairs marked 1 to 4 and then men select from the remaining chairs. The number of possible arrangements is

(A) ${}^6C_3 . {}^4C_4$ (B) ${}^4P_2 . {}^4P_3$ (C) ${}^4C_3 . {}^4P_3$ (D) ${}^4P_2 . {}^6P_3$

A-3. Number of words that can be formed with the letters of the word 'STRANGE' which neither starts with S nor ends with E, is

(A) 1320 (B) 1440 (C) 3720 (D) 3600

A-4. A rack has 5 different pair of shoes. The number of ways in which 4 shoes can be selected from it so that there will be no complete pair is:

(A) 1920 (B) 200 (C) 110 (D) 80

A-5. Number of permutations of 1,2,3,4,5,6,7,8 and 9 taken all at a time such that the digit 1 appearing somewhere to the left of 2, 3 appearing somewhere to the left of 4 and 5 somewhere to the left of 6, is

(A) 9.7! (B) 8! (C) 5!.4! (D) 8!.4!

A-6. A word has 4 identical letters and some different letters. If the total number of words that can be formed with the letters of the word is 210, then the number of different letters in the word is

(A) 3 (B) 4 (C) 5 (D) 7

A-7. The number of different six digit numbers that can be formed by using all the digits 0,1,1,2,2,2 is

(A) 400 (B) 50 (C) 100 (D) 120

A-8. Number of ways in which 5 A's and 6 B's can be arranged in a row such that the arrangement is a palindrome, is

(A) 6 (B) 10 (C) 8 (D) 12

A-9. There are 6 periods in a working day of a school. Number of ways in which 5 subjects can be arranged if each subject is allotted atleast one period and no period remains vacant, is

(A) 1800 (B) 210 (C) 3600 (D) 360

A-10. There are 10 white balls of different shades and 9 black balls of identical shades. Then the number of arrangements of these balls in a row such that no two black balls are together is

(A) $10!.\ ^{11}P_9$ (B) $10!.\ ^{11}C_9$ (C) 10! (D) 10!. 9!

A-11. There are 2 identical white balls, 3 identical red balls and 4 green balls of different shades. The number of ways in which these balls can be arranged in a row so that atleast one ball is separated from the balls of same colour, is

(A) 6(7! - 4!) (B) 7(6! - 4!) (C) 8! - 5! (D) none of these

A-12. How many different words beginning and ending with a consonant can be formed from the letters of the word 'EQUATION'?

(A) 3. 6! (B) 6. 6! (C) 2. 6! (D) 6!

A-13. The number of permutations that can be formed by all the letters of the word 'DISTINCTION' in which all 3 I's appear before 'O', is

(A) $\frac{11!}{4!}$ (B) $\frac{11!}{4!2!}$ (C) $\frac{11!}{2!2!}$ (D) $\frac{11!}{4!2!2!}$

A-14. The number of words formed using the letters of the word 'RELIABILITY' such that vowels and consonants occur alternatively, is

(A) $\frac{5!}{3!}.\frac{6!}{2!}$ (B) $\frac{5!}{3}.\frac{6!}{2!}$ (C) $^2C_1.\frac{5!}{3}.\frac{6!}{2!}$ (D) none of these

A-15. If all the letters of the word 'COVID' are arranged in alphabetical order, then the rank of the word COVID is

(A) 15 (B) 16 (C) 17 (D) 18

A-16. The total number of words formed using the letters of the word 'CHANDRASWAMI' which contains the word 'SCAM', is

(A) 9! (B) $\frac{9!}{2!}$. 4! (C) $\frac{9!}{2!}$ (D) 2 . 9!

A-17. In a G-20 summit, If Mr. Trumph wants to speak before Mr. Putin & Mr. Putin wants to speak after Mr. Modi and remaining speakers can speak at any number, then the number of ways that all the 20 speakers can give their speeches, is

(A) $^{20}C_3$ (B) $^{20}P_8$ (C) $^{20}P_3$ (D) $\frac{20!}{3}$

A-18. Passengers are to travel by a double decker bus which can accommodate 13 in the upper deck and 7 in the lower deck. The number of ways that they can travel if 5 refuse to sit in the upper deck and 8 refuse to sit in the lower deck, is

(A) 25 (B) 18 (C) 21 (D) 15

A-19. How many nine digit numbers can be formed using the digits 2,2,3,3,5,5,6,6,6 so that the odd digits occupy even places?

(A) 7560 (B) 60 (C) 180 (D) 16

A-20. Eight cards bearing number 1,2,3,4,5,6,7,8 are well shuffled. Then in how many cases the top 2 cards will form a pair of twin prime numbers?

(A) 720 (B) 1440 (C) 2880 (D) 2160

A-21. The sum of all four digit numbers which can be formed using the digits 6,7,8,9 is
(A) 2133120 (B) 2133140 (C) 2133150 (D) 2133122

A-22. Out of 16 players of a cricket team , 4 are bowlers and 2 are wicket keepers. A team of 11 players is to be chosen so as to contain at least 3 bowlers and at least 1 wicket keeper. The number of ways in which the team can be selected, is
(A) 2400 (B) 2472 (C) 2500 (D) 960

Section(B): Grouping and Circular Permutation

B-1. A gentleman a party of m + n ($m \neq n$) friends to a dinner and places m at one round table T_1 and n at another round table T_2. If not all people shall have the same neighbour in any two arrangements, then the number of ways in which he can arrange the guests, is
(A) $\frac{(m+n)!}{4mn}$ (B) $\frac{(m+n)!}{2mn}$ (C) $2\frac{(m+n)!}{mn}$ (D) none of these

B-2. Number of ways in which 13 different toys can be distributed between 3 brothers in such a way that distribution among the 2 elder brothers is even and the youngest one receives one toy more, is
(A) $\frac{13!}{5!}$ (B) $\frac{13!}{5!(4!)^2}$ (C) $\frac{13!}{5!(4!)^2\,2!}.3!$ (D) $\frac{13!}{5!(4!)^2}.3!$

B-3. In an eleven storeyed building (Ground floor + ten floor), 9 people enter in a lift cabin from ground floor. It is known that they will leave the lift in groups of 2, 3 and 4 at different floors. The number of ways in which they can get down is
(A) $\frac{10!}{4}$ (B) $\frac{2.10!}{9}$ (C) $\frac{8.9!}{4}$ (D) $\frac{9.9!}{4}$

B-4. The number of ways in 8 different beads can be threaded on to a ring such that 4 particular beads are always together, is
(A) 144 (B) 576 (C) 1680 (D) 288

B-5. In a SAARC summit, 2 Indians, 3 Sri Lankan, 3 Nepalis and 4 Pakistanis are to be seated for a round table conference. Total number of ways in which these delegates can take their seats if delegates of same nationality sit together, is
(A) $2(4!)^2(3!)^2$ (B) $2.\ 3!\ (4!)^3$ (C) $2.\ (3!)^3.\ 4!$ (D) $2\ (4!)^3(3!)^2$

B-6. The number of ways in which 5 flowers from 8 different flowers can be strung to form a garland, is
(A) 1344 (B) 336 (C) 3360 (D) 672

B-7. The number of ways in which 6 boys and 3 girls can stand in a circle so that all girls are together, is
(A) 2160 (B) 4320 (C) 1440 (D) 30240

Section(C) : Selection of one or more objects

C-1. Let m denote the number of ways in which 4 different books can be distributed among 10 persons, each receiving atmost one and let n denote the number of ways of distribution if the books are all alike, Then
(A) m = 4n (B) n = 4m (C) m = 24n (D) none of these

C-2. The number of proper divisors of the number 9680 is
(A) 28 (B) 29 (C) 30 (D) 26

C-3. How many divisors of 21600 are divisible by 4 and 15 but not by 16?
(A) 10 (B) 18 (C) 30 (D) 12

C-4. Product of all even divisors of 1000 is
(A) $32.\ 10^2$ (B) $64.\ 2^{14}$ (C) $64.\ 10^{18}$ (D) $128.\ 10^6$

C-5. There are 420 rooms in a row, whose numbers are 1, 2, 3, ….., 420. Initially all the doors are closed. A person takes 420 rounds of the row, numbers as 1^{st} round, 2^{nd} round, ….420^{th} round. In each round, he interchanges the position of those door numbers whose number is multiple of round number. After 420^{th} round, how many doors will be open?

(A) 20 (B) 419 (C) 21 (D) 18

C-6. The number of words of 4 letters that can be formed using the letters of the word "PARALLELOPIPED" is

(A) 2770 (B) 2780 (C) 2472 (D) 2800

C-7. If fruits of same species are alike, then in how many ways atleast 2 fruits can be selected from 5 Apples, 4 Mangoes, 3 Bananas and 3 different fruits?

(A) 956 (B) 959 (C) 960 (D) 953

Section(D) : Multinomial theorem and Dearrangement

D-1. The number of ways in which 10 identical apples and 8 identical mangoes can be distributed among 6 children such that each child receives atleast one fruit of each species, is

(A) 126 (B) 2646 (C) 2400 (D) 2640

D-2. Six cards are drawn one by one from a set of unlimited number of cards , each card is marked with numbers –1, 0, or 1. Number of different ways in which they can be drawn if the sum of the numbers shown by them vanishes, is

(A) 462 (B) 126 (C) 141 (D) 115

D-3. In a bakery 8 different brands of biscuits are available. In how many ways 6 biscuits can be selected if biscuits of same brand are identical.

(A) ${}^{13}C_6$ (B) ${}^{13}C_8$ (C) 8^6 (D) 6^8

D-4. Number of integral solutions of the equation XYZ = 216, is

(A) 100 (B) 120 (C) 480 (D) 400

D-5. Number of positive integral solutions satisfying the equation $(x_1+x_2+x_3)(y_1 + y_2) = 77$ is

(A) 150 (B)270 (C)420 (D)1024

D-6. A person writes letters to his 5 friends and addresses the corresponding envelopes. Number of ways in which the letters can be placed in the envelope so that atleast two of them are in the wrong envelopes, is

(A)1 (B)2 (C)118 (D)119

D-7. There are six letters $L_1, L_2, L_3, L_4, L_5, L_6$ and their corresponding six envelopes $E_1, E_2, E_3, E_4, E_5, E_6$. Letters having odd value can be put into odd value envelopes and even value letters can be put into even value envelopes so that no letter goes into the right envelope, then number of arrangements is equal to

(A) 4 (B) 44 (C) 9 (D) 6

D-8. Seven cards and seven envelopes are numbered 1, 2, 3, 4, 5, 6, 7 and cards are to be placed in envelopes so that each envelope contains exactly one card and no card is placed in the envelope bearing the same number and moreover the card number 1 is always placed in envelope number 2 and 2 is always placed in envelope numbered 3, then the number of ways it can be done is

(A) 44 (B) 53 (C) 62 (D) 9

Section(E): Miscellaneous

E-1. Number of cyphers at the end of ${}^{2002}C_{1001}$ is

(A) 0 (B) 1 (C) 2 (D) 200

E-2. There are 100 different books in a shelf. Number of ways in which 3 books can be selected so that no two which are neighbours is

(A) ${}^{100}C_3 - 98$ (B) ${}^{97}C_3$ (C) ${}^{96}C_3$ (D) ${}^{98}C_3$

E-3. The number of ways of choosing triplets (x, y, x) such that $z \geq \max\{x, y\}$ and $x, y, z \in \{1, 2, 3, ..., n\}$ is

(A) $\sum_{t=1}^{n} t^2$ (B) ${}^{n+1}C_3 - {}^{n+2}C_3$ (C) $2({}^{n+2}C_3) + {}^{n+1}C_2$ (D) $\left(\frac{n(n+1)^2}{2}\right)$

E-4. Number of ways of selecting pair of black squares in chessboard such that they have exactly one common corner is equal to

(A) 64 (B) 49 (C) 50 (D) 56

PART-III : MATCH THE COLUMN

1.

	Column-I		Column-II
(A)	Number of increasing permutations of m numbers taken from the set of n distinct numbers (n > m), is	(P)	n^m
(B)	There are m men and n monkeys. Number of ways in which every monkey has a master, if a man can have any number of monkeys	(Q)	${}^mC_n$
(C)	Number of ways in n red balls and (m - 1) green balls Can be arranged in a line, so that no two red balls are Together, is	(R)	${}^nC_m$
(D)	Number of ways in which m different toys can be distributed in n children if every child may receive any number of toys, is	(S)	m^n

2.

	Column-I		Column-II
(A)	In an examination, 5 students were found to have their mobiles in their pocket. The invigilator took their mobiles in his possession. After the end of examination, invigilator randomly returned their mobiles. The number of ways in which at most two students get their own mobiles, is	(P)	36
(B)	The number of 4 digit natural numbers such that the product of their digits is 12, is	(Q)	109
(C)	If 7 points out of 12 are in the same straight line and other than these 7 points no three points are collinear, then the number of triangles formed by given points, is	(R)	1260
(D)	The maximum number of points of intersection of 8 unequal Circles and 4 straight lines, is	(S)	185

Exercise # 2

PART-I : OBJECTIVE QUESTIONS

1. The number of positive integers less than 2018 that are divisible by 6 but are not divisible by at least one of the numbers 4 or 9 is .....

(A) 280 (B) 250 (C) 300 (D) 320

2. Consider all 6-digit numbers of the form **abccba** where b is odd. Determine the number of all such 6-digit numbers that are divisible by 7.

(A) 50 (B) 60 (C) 70 (D) 80

3. Number of times is the digit 5 written when listing all numbers from 1 to 10^5?

(A) 45000 (B) 50000 (C) 52500 (D) 47500

4. There are m points on a straight-line AB and n points on the line AC none of them being the point A. Triangles are formed with these points as vertices, when (i) A is excluded (ii) A is included. The ratio of number of triangles in the two cases is :

(A) $\frac{m+n-2}{m+n}$ (B) $\frac{m+n-2}{m+n-1}$

(C) $\frac{m+n-2}{m+n+2}$ (D) $\frac{m(n-1)}{(m+1)(n+1)}$

5. Two classrooms A and B having capacity of 25 and (n – 25) seats respectively. A_n denotes the number of possible seating arrangements of room 'A', when 'n' students are to be seated in these rooms, starting from room 'A' which is to be filled up full to its capacity. If $A_n - A_{n+1} = 25!(^{49}C_{25})$, then 'n' is equal to

(A) 50 (B) 48 (C) 49 (D) 51

6. Number of 3 digit numbers in which the digit at 100^{th} place is greater than the other two digits is

(A) 285 (B) 281 (C) 240 (D) 204

7. Number of words that can be formed using the letters of the word "ENGINEERING" and starts as well as end with a vowel, is

(A) 5040 (B) 4050 (C) 50400 (D) 45360

8. If all words formed using the letters of the word "CORONA" are arranged alphabetically, then 348^{th} word of the sequence is

(A) ROCOAN (B) ROCONA

(C) ROOCNA (D) RCONOA

9. The total number of words that can be formed using the letters of the word "BOOKKEEPING" such that identical letters never occur together, is

(A) 500. 8! (B) 550. 8! (C) 522. 7! (D) 550. 7!

10. A family consists of a grandfather, m sons and daughters and 2n grandchildren. They are to be seated in a row for dinner. The grandchildren wish to occupy the n seats at each end and the grandfather refuses to have a grandchild on either side of him. In how many ways can the family be seated for dinner

(A) (2n – 1)! m! (m – 1) (B) (2n)! (m – 1)! (m – 1) (C) (2n)! m! (m – 1) (D) (2n)! m! m

11. A set contains 11 distinct elements, then the total number of subsets of the set having atleast three elements is

(A) 1981 (B) 1972 (C) 1952 (D) 1947

12. If $\alpha = x_1x_2x_3$ and $\beta = y_1y_2y_3$ are two three digit numbers, then the number of pairs of α and β that can be formed so that α can be subtracted from β without borrowing, is

(A) 55. (45)2 (B) 45. (55)2 (C) 553 (D) 36. (45)2

13. How many 'n' digits positive integers can be formed using the digits 1, 2 and 3 such that each digit is used at least once?

(A) $3(n-1)$ (B) $3^n - 2.2^n + 3$ (C) $3^n - 2.2^n - 3$ (D) $3^n - 3.2^n + 3$

14. Let X = $\{1,2,3,4,....,2020\}$ and A and B are two proper subsets of set X. Then number of ways of selecting unordered pair of sets A and B such $A \cup B$ is also a proper subset of X.

(A) $\frac{4^{2020} - 3^{2020}}{2}$ (B) $\frac{4^{2020} - 3^{2020} + 2^{2020} - 1}{2}$

(C) $\frac{4^{2020} - 3^{2020} + 2^{2020}}{2}$ (D) None of these

15. How many ways are there to invite one of three friends for dinner on 6 successive nights such that no friend is invited more than three times?

(A) $\frac{6\times 6!}{2!3!} + \frac{6!}{3!3!} + \frac{6!}{2!2!2!}$ (B) $\frac{6\times 6!}{2!3!} + 6\times\frac{6!}{3!3!} + \frac{6!}{2!2!2!}$

(C) $\frac{6\times 6!}{2!3!} + 3\times\frac{6!}{3!3!} + \frac{6!}{2!2!2!}$ (D) $\frac{3\times 6!}{2!3!} + 3\times\frac{6!}{3!3!} + \frac{6!}{2!2!2!}$

16. If 10 identical dice are rolled, then total number of possible outcomes are

(A) 6^{10} (B) $\frac{6^{10}}{10!}$ (C) ${}^{15}C_5$ (D) None of these

17. Number of ways in which a pack of 52 playing cards can be distributed equally among 4 players such that each player have face cards of the same suit, is

(A) $\frac{36!.4!}{(9!)^4}$ (B) $\frac{36!}{(9!)^4}$ (C) $\frac{40!}{(10!)^4}$ (D) $\frac{40!.4!}{(10!)^4}$

18. There are six line segments of length 2, 3, 4, 5, 6, 7 units, the number of triangles that can be formed by these line segments is

(A) ${}^6C_3 - 7$ (B) ${}^6C_3 - 4$ (C) ${}^6C_3 - 5$ (D) ${}^6C_3 - 6$

19. There are m apples and n oranges to be placed in a line such that the two extreme fruits being both oranges. Let P denotes the number of arrangements if the fruits of same species are different and Q is the corresponding figure when the fruits of same species are alike, then the value of P/Q is equal to

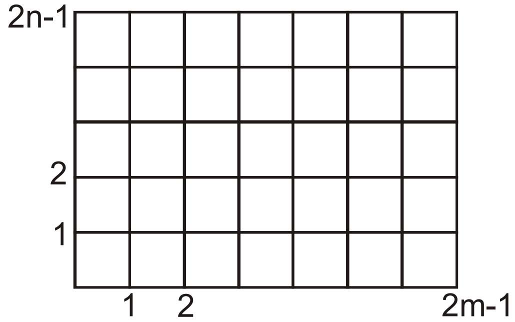
(A) ${}^mP_2 . {}^nP_n . (n-2)!$ (B) ${}^nP_2 . {}^mP_m . (n-2)!$

(C) ${}^mP_2 . {}^nP_n . (m-2)!$ (D) ${}^nP_2 . {}^mP_m . (n-2)!$

20. The number of intersection points of diagonals of 2021 sides regular polygon, which lie inside the polygon.

(A) ${}^{2021}C_4$ (B) ${}^{2020}C_2$ (C) ${}^{2020}C_4$ (D) ${}^{2021}C_2$

21. A rectangle with sides 2m - 1 and 2n - 1 is divided into squares of unit length by drawing parallel lines as shown in the diagram, then the number of rectangles possible with odd side lengths is



(A) $(m + n - 1)^2$ (B) 4^{m+n-1} (C) m^2n^2 (D) $m(m + 1)n(n + 1)$

22. Total number of positive integers less than 1155 which are relatively prime with 1155 are

(A) 240 (B) 480 (C) 504 (D) 380

23. If r, s, t are prime numbers and p, q are the positive integers such that the LCM of p, q is $r^2t^4s^2$, then the number of ordered pair (p, q) is

(A) 252 (B) 254 (C) 225 (D) 224

24. The number of 7 digit integers, with sum of digits equal to 10 and formed by using the digits 1, 2 and 3 only, is

(A) 55 (B) 66 (C) 77 (D) 88

PART-II : NUMERICAL QUESTIONS

1. If repetition of digit is not allowed, then how many five digit numbers which are divisible by 3 can be formed using the digits?

2. An old man while dialling a 7-digit telephone number remembers that the first four digits consists of one 1's, one 2's and two 3's. He also remembers that the number is even while has no memorising of the 4^{th} and 5^{th} digits but he remembers that the sum of 4^{th} and 5^{th} digits 9 and the number has exactly one zero. Find the maximum number of distinct trials he has to try to make sure that he dials the correct telephone number.

3. Number of ways in which five vowels and ten decimal digits can be placed in a row such that between any two vowels odd number of digits are placed and both end places are occupied by vowels is $n(10!)(5!)$, then find the value of n.

4. Find the number of ways in which 8 different coins can be distributed among 3 brothers such that each one receives at least one and at most 4 coins.

5. Let E denote the set of all natural numbers n such that 3 < n < 100 and the set {1, 2, 3, ... , n) can be partitioned in to 3 subsets with equal sums. Find the number of elements of E.

6. Find the number of 3-digit numbers. (including all numbers) which have any one digit is the average of the other two digits.

7. Six persons A, B, C, D, E, and F are to be seated at a circular table. In how many ways this can be done if A must have either B or C on his right and B must have either C or D on his right?

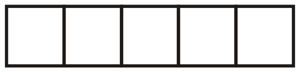
8. In how many ways a team of 6 horses can be selected from a stud of 16 so that there shall be 3 out of A B C A' B' C' but never A A' , B B' or C C' together.

9. There are 10 straight lines in a plane, such that no 3 of them are concurrent and no two of them are parallel to each other. If points of intersection of above lines are joined, maximum number of lines (excluding old lines) thus formed are 'n'. Find the sum of reciprocal of divisors of 'n'.

10. Six married couples are sitting in a room. Find the total number of ways in which 4 people can be selected so that they form at least one couple.

11. Find the total number of permutations which can be formed out of the letters of the word "RELIABLE" taking four letters together.

12. Find the total number of words each having 3 consonants and 3 vowels that can be formed from the letters of the word 'CIRCUMFERENCE'. In how many of these words all three C's occur together?



13. In a hockey series between team X and Y, they decide to play till a team wins 10 matches. The number of ways in which team X wins the series is $\frac{1}{2}\,{}^{20}C_m$, then find the value of m.

14. In a shooting competition a man can score 0, 2, 4 or 6 points in each shot. Then find the total number of ways in which he can score total 24 points in 7 shots.

15. Find the total number of natural numbers less than 10^6 whose sum of digits is equal to 10.

16. A box contains 6 balls which may be all of different colours or three each of two colours or two each of three colours. If balls of same colour are identical, then find the total number of ways of selecting 3 balls from the box.

17. There are balls available in 3 different colours (at least four of each colour). Balls are all alike except of the colour. If 'm' denotes the number of arrangements of four balls if no arrangement consists of all balls of same colour and 'n' denotes the corresponding figure when every arrangement consists of balls of each colour, then find the value of $\frac{m}{n}$.

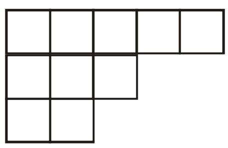
18. If sum of all the six digit numbers formed using the digits 2, 3, 3, 4, 4, 4 is N, then find the value of $\frac{N}{1111110}$.

19. Lines y = x + i and y = – x + j are drawn in x-y plane such that $i \in \{1,2,3,4\}$ and $j \in \{1,2,3,4,5,6\}$.If m represents the total number of rectangles which are not squares formed by these lines and n represents the total number of triangles formed by the given lines & X-aixs, then find the value of m/n.

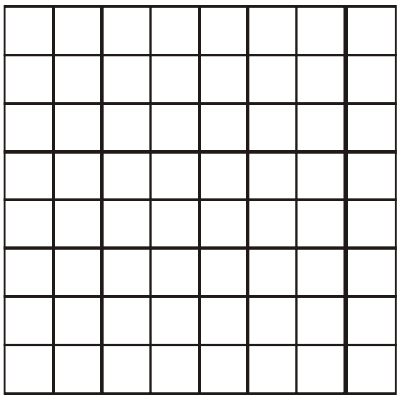
20. Three vertices of a convex polygon are selected. If the number of triangles that can be constructed using the vertices of polygon such that none of the sides of the triangle is also the side of the polygon is 30, then find the number of sides if the polygon.

21. Let P_n denotes the number of ways in which three people can be selected out of 'n' people sitting on a round table while no two of them are consecutive. If $P_n - P_{(n+1)} = 27$, then find the value of 'n'.

22. Find the total number of ways in which 5 X's can be placed in the square of given figure so that no row remains empty.



23. On a chess board as shown in figure, A and B are two insects which starts moving towards each other with same constant speed. Insect A can move only to the right or upward along the lines while insect B can move only to the left or downward along the lines of the chess board. Find the total number of ways the two insects can meet at some point during the trip.



24. Consider all 4 element subsets of the set A = {1, 2, 3, ... 8}. The arithmetic mean of the greatest elements of these 4 element subsets is .....

25. We have four sets S_1, S_2, S_3, S_4 each containing a number of parallel lines. The set S_i contains i + 1 parallel lines i = 1, 2, 3, 4. A line in S_i is not parallel to lines in S_j when $i \neq j$. In how many points do these lines intersect ?

26. A_1, A_2, A_3 .... A_{15} is a 15 sided regular polygon. The number of distinct equilateral triangles in the plane of the polygon, with exactly two of their vertices from the set $\{A_1, A_2, A_3 \text{ .... } A_{15}\}$ is .....

PART - III : ONE OR MORE THAN ONE CORRECT QUESTION

1. A question paper consists of two sections. Section A has 7 questions and section B has 8 questions. A student has to answer 10 questions out of these 15 questions. The number of ways in which he can answer if he must answer at least 3 of the section A and at least 4 of the section B is

(A) ${}^7C_3 . {}^8C_1 + {}^7C_4 . {}^8C_2 + {}^7C_5 . {}^8C_3 + {}^7C_6 . {}^8C_4$ (B) 2926
(C) 3003 (D) ${}^{15}C_{10} - {}^7C_2 - {}^8C_3$

2. In an examination, there are 4 compulsory and 3 optional subjects. A student has to appear in all 7 papers and to pass the examination he must pass all the 4 compulsory subjects and at least two optional subjects. The number of ways in which he can fail is

(A) $({}^4C_1 + {}^4C_2 + {}^4C_3 + {}^4C_4)\,({}^3C_2 + {}^3C_3)$ (B) $({}^4C_1 + {}^4C_2 + {}^4C_3 + {}^4C_4) + ({}^3C_2 + {}^3C_3)$
(C) 19 (D) 60

3. A kindergarten teacher has 25 kids in her class. She takes 5 of them at a time, to a museum as often as she can, without taking the same 5 kids more than once. Then the number of visits, the teacher makes to the museum exceeds that of a kid by

(A) ${}^{25}C_5 - {}^{24}C_5$ (B) ${}^{24}C_5$ (C) ${}^{24}C_4$ (D) ${}^{25}C_5 - {}^{24}C_4$

4. Number of ways in which 3 different numbers in an A. P. can be selected from the first n natural numbers is

(A) $\dfrac{(n-2)(n-4)}{4}$ if n is even (B) $\dfrac{n^2-4n+5}{2}$ if n is odd

(C) $\dfrac{(n-1)^2}{4}$ if n is odd (D) $\dfrac{n(n-2)}{4}$ if n is even

5. Total number of words formed using the letters of the word " APRAJITA" are

(A) 1680 if all three A's appear before I

(B) 192 if vowels and consonants are alternate

(C) 96 if relative position of vowels and consonants remain same

(D) 3600 if two A's are together but separated from third A

6. If all the letters of the word "TEJAS" are arranged in all possible ways and put in dictionary order, then

(A) The 83^{rd} word is SEJAT (B) The 82^{nd} word is SEJTA

(C) The 50^{th} word is JAETS (D) The 79^{th} word is SEAJT

7. The number of ways in which 8 balls of different colours can be arranged in a row so that the two balls of particular colours (say red and white) may never come together is

(A) $8! - 2.\,7!$ (B) $6.\,7!$ (C) $2.\,6!\,.7C2$ (D) none of these

8. The combinatorial coefficient C(n,r) is equal to

(A) Number of possible subsets of r members from a set of n distinct members.

(B) Number of possible binary messages of length n with exactly r 1's.

(C) Number of non-decreasing 2-D paths from the lattice point (0,0) to (r,n).

(D) Number of ways of selecting r things from n different things when a particular thing is always included plus the number of ways of selection r thing from n things, when a particular thing is always excluded.

9. There are 10 questions, each question is either True or False. Number of different sequences of incorrect answers is equal to

(A) Number of ways in which a normal coin tossed 10 times would fall in a definite order if both heads and tails are present.

(B) Number of ways in which a multiple-choice question containing 10 alternatives with one or more than one correct alternative, can be answered.

(C) Number of ways in which it is possible to draw a sum of money with 10 coins of different denominations taken some or all at a time.

(D) Number of different selections of 10 indistinguishable things taken some or all at a time.

10. The maximum number of permutations of 2n letters in which there are only two letters a and b, taken all at a time is given by:

(A) ${}^{2n}C_n$ (B) $\dfrac{2.6.10.....(4n-6).(4n-2)}{1.2.3.....(n-1).n}$

(C) $\dfrac{(n+1).(n+2).(n+3).....(2n-1).2n}{1.2.3.....(n-1).n}$ (D) $2^n.\dfrac{1.3.5.....(2n-3).(2n-1)}{n!}$

11. Number of ways in which the letters of the word 'BULBUL' can be arranged in a line in a definite order is also equal to the

(A) number of ways in which 2 alike Apples and 4 alike Mangoes can be distributed in 3 children so that each child receives any number of fruits.

(B) number of ways in which 6 different books can be tied up into 3 bundles, if each bundle has equal number of books.

(C) coefficient of $x^2y^2z^2$ in the expansion of $(x + y + z)^6$.

(D) number of ways in which 6 different prizes can be distributed equally in 3 children.

12. Total number of positive integral solutions of the equation $10 < x + y + z \leq 25$ is

(A) ${}^{28}C_3 - {}^{13}C_3$ (B) ${}^{25}C_3 - {}^{10}C_3$ (C) 2990 (D) 2180

13. Let $N = 2^3.3^5.5^7$, then which of the following statement is true about N ?

(A) Number of divisors which are divisible by 3 but not by 12 is equal to 80.

(B) Sum of all divisors is equal to $\dfrac{(2^4-1)(3^6-1)(5^8-1)}{8}$.

(C) Sum of reciprocal of all divisors is equal to $\dfrac{(2^4-1)(3^6-1)(5^8-1)}{8N}$.

(D) If x and y are two coprime numbers such that x.y = N, then number of ordered pairs of (x, y) is equal to 4.

14. There are 10 seats in a row of which 4 are to be occupied. The number of ways of arranging 4 persons so that no two persons sit side by side, is

(A) ${}^{7}C_4$ (B) $4.\ {}^{7}P_3$ (C) ${}^{7}C_3 . 4!$ (D) 840

15. ${}^{50}C_{36}$ is divisible by

(A) 19 (B) 25 (C) 361 (D) 125

16. The number of ways in which 200 different things can be divided into groups of 100 pairs, is

(A) $\dfrac{200!}{2^{100}}$ (B) $\dfrac{200!}{2^{100}.100!}$

(C) $\left(\dfrac{101}{2}\right)\left(\dfrac{102}{2}\right)\left(\dfrac{103}{2}\right)...\left(\dfrac{200}{2}\right)$ (D) 1. 3. 5……199

17. If repetition of digits are not allowed, then total number of 7 digit numbers that can be formed using the non-zero digits, is

(A) 12. 7! if product of any 5 consecutive digits is divisible by 5

(B) 11. 7! if number is divisible by 3

(C) 36. 7! if product of all digits of the number is divisible by

(D) 12. 7! if number is divisible by 3

PART - IV : COMPREHENSION

Comprehension # 1

16 players $P_1, P_2, P_3,....,P_{16}$ take part in a tennis tournament. Lower suffix player is better than any higher suffix player. These players are to be divided into 4 groups each comprising of 4 players and the best from each group is selected for semifinals.

1. Number of ways in which they can be divided into 4 equal groups if players P_1, P_2, P_3 and P_4 are in different groups, is

(A) $\dfrac{11!}{36}$ (B) $\dfrac{11!}{72}$ (C) $\dfrac{11!}{108}$ (D) $\dfrac{11!}{216}$

2. Number of ways in which these 16 players can be divided into four equal groups such that when the best player is selected from each group, P_6 is one among them, is

(A) $\dfrac{20.12!}{(4!)^3}$ (B) $\dfrac{12!}{(4!)^3}$ (C) $\dfrac{20.12!}{(4!)^3\,3!}$ (D) $\dfrac{12!}{(4!)^3\,3!}$

Comprehension # 2

Consider the word W = MISSISSIPPI

3. Total number of combinations of 5 letters taken from the letters of the word 'W' is

(A) 22 (B) 25 (C) 27 (D) 19

4. Total number of permutations of all the letters of the word 'W' if all the S's as well as all the P's are separated, is

(A) $\frac{10!}{4!3!}$ (B) $\frac{10!}{4!}$ (C) $\frac{10!}{4.4!}$ (D) $\frac{10!}{4!4!}$

Comprehension # 3

A mega pizza is to be sliced n times and S_n denotes the maximum number of pieces after n slice.

5. Relation between S_n and S_{n-1} is

(A) $S_n = S_{n-1} + n + 3$ (B) $S_n = S_{n-1} + n + 2$ (C) $S_n = S_{n-1} + n + 1$ (D) $S_n = S_{n-1} + n$

6. If the mega pizza is to be distributed among 60 persons, each one of them get at least one piece, then minimum number of ways of slicing the mega pizza is

(A) 10 (B) 9 (C) 11 (D) 8

Exercise # 3

PART - I : JEE (ADVANCED) / IIT-JEE PROBLEMS (PREVIOUS YEARS)

*** Marked Questions may have more than one correct option.**

1. Let S = {1, 2, 3, 4}. The total number of unordered pairs of disjoint subsets of S is equal to **[IIT-JEE-2010, Paper-2, (5, –2), 79]**

(A) 25 (B) 34 (C) 42 (D) 41

2. The total number of ways in which 5 balls of different colours can be distributed among 3 persons so that each person gets at least one ball is **[IIT-JEE 2012, Paper-1, (3, –1), 70]**

(A) 75 (B) 150 (C) 210 (D) 243

Paragraph for Question Nos. 3 and 4

Let a_n denote the number of all n-digit positive integers formed by the digits 0,1 or both such that no consecutive digits in them are 0. Let b_n = the number of such n-digit integers ending with digit 1 and c_n = the number of such n-digit integers ending with digit 0.

3. Which of the following is correct ? **[IIT-JEE 2012, Paper-2, (3, –1), 66]**

(A) $a_{17} = a_{16} + a_{15}$ (B) $c_{17} \neq c_{16} + c_{15}$ (C) $b_{17} \neq b_{16} + c_{16}$ (D) $a_{17} = c_{17} + b_{16}$

4. The value of b_6 is

(A) 7 (B) 8 (C) 9 (D) 11

5. Let $n_1 < n_2 < n_3 < n_4 < n_5$ be positivie integers such that $n_1 + n_2 + n_3 + n_4 + n_5 = 20$. Then the number of such distinct arrangements $(n_1, n_2, n_3, n_4, n_5)$ is **[JEE (Advanced) 2014, Paper-1, (3, 0)/60]**

6. Let $n \geq 2$ be an integer. Take n distinct points on a circle and join each pair of points by a line segment. Colour the line segment joining every pair of adjacent points by blue and the rest by red. If the number of red and blue line segments are equal, then the value of n is **[JEE (Advanced) 2014, Paper-1, (3, 0)/60]**

7. Six cards and six envelopes are numbered 1, 2, 3, 4, 5, 6 and cards are to be placed in envelopes so that each envelope contains exactly one card and no card is placed in the envelope bearing the same number and moreover the card numbered 1 is always placed in envelope numbered 2. Then the number of ways it can be done is **[JEE (Advanced) 2014, Paper-2, (3, –1)/60]**

(A) 264 (B) 265 (C) 53 (D) 67

8. Let n be the number of ways in which 5 boys and 5 girls can stand in a queue in such a way that all the girls stand consecutively in the queue. Let m be the number of ways in which 5 boys and 5 girls can stand in a queue in such a way that exactly four girls stand consecutively in the queue. Then the value of $\frac{m}{n}$ is **[JEE (Advanced) 2015, P-1 (4, 0) /88]**

9. A debate club consists of 6 girls and 4 body. A team of 4 members is to be selected from this club including the selection of a captain (from among these 4 member) for the team. If the team has to include at most one boy, then the number of ways of selecting the team is **[JEE(Advanced)-2016, 3(–1)]**

(A) 380 (B) 320 (C) 260 (D) 95

10. Words of length 10 are formed using the letters A, B, C, D, E, F, G, H, I, J. Let x be the number of such words where no letter is repeated; and let y be the number of such words where exactly one letter is repeated twice and no other letter is repeated. Then $\frac{y}{9x} =$ **[JEE(Advanced)-2017, 3]**

11. Let S = {1, 2, 3,.....,9}. For k = 1,2, ..., 5, let N_k be the number of subsets of S, each containing five elements out of which exactly k are odd. Then $N_1 + N_2 + N_3 + N_4 + N_5$ = **[JEE(Advanced)-2017, 3(–1)]**

(A) 125 (B) 252 (C) 210 (D) 126

12. The number of 5 digit numbers which are divisible by 4, with digits from the set {1, 2, 3, 4, 5} and the repetition of digits is allowed, is ______ **[JEE(Advanced)-2018, 3(0)]**

13. In a high school, a committee has to be formed from a group of 6 boys M_1, M_2, M_3, M_4, M_5, M_6 and 5 girls G_1, G_2, G_3, G_4, G_5.

(i) Let α_1 be the total number of ways in which the committee can be formed such that the committee has 5 members, having exactly 3 boy and 2 girls.

(ii) Let α_2 be the total number of ways in which the committe can be formed such that the committee has at least 2 members, and having an equal number of boys and girls.

(iii) Let α_3 be the total number of ways in which the committe can be formed such that the committee has 5 members, at least 2 of them being girls.

(iv) Let α_4 be the total number of ways in which the committee can be formed such that the commitee has 4 members, having at least 2 girls and such that both M_1 and G_1 are **NOT** in the committee together.

LIST-I	LIST-II
P. The value of α_1 is	**1.** 136
Q. The value of α_2 is	**2.** 189
R. The value of α_3 is	**3.** 192
S. The value of α_4 is	**4.** 200
	5. 381
	6. 461

The correct option is :-

(A) **P → 4; Q → 6, R → 2; S → 1** (B) **P → 1; Q → 4; R → 2; S → 3**

(C) **P → 4; Q → 6, R → 5; S → 2** (D) **P → 4; Q → 2; R → 3; S → 1**

[JEE(Advanced)-2018, 3(–1)]

14. Five person A,B,C,D and E are seated in a ciruclar arrangement. If each of them is given a hat of one of the three colours red, blue and green ,then the number of ways of distributing the hats such that the persons seated in adjacent seats get different coloured hats is **[JEE(Advanced)-2019, 3(0)]**

PART - II : JEE(MAIN) / AIEEE PROBLEMS (PREVIOUS YEARS)

1. **Statement-1 :** The number of ways of distributing 10 identical balls in 4 distinct boxes such that no box is empty is ${}^9C_3$. **[AIEEE 2011, I, (4, –1), 120]**

Statement-2 : The number of ways of choosing any 3 places from 9 different places is ${}^9C_3$.

(1) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.

(2) Statement-1 is true, Statement-2 is true; Statement-2 is **not** a correct explanation for Statement-1.

(3) Statement-1 is true, Statement-2 is false.

(4) Statement-1 is false, Statement-2 is true.

2. There are 10 points in a plane, out of these 6 are collinear. If N is the number of triangles formed by joining these points. then : **[AIEEE 2011, II, (4, –1), 120]**

(1) $N \leq 100$ (2) $100 < N \leq 140$ (3) $140 < N \leq 190$ (4) $N > 190$

3. Assuming the balls to be identical except for difference in colours, the number of ways in which one or more balls can be selected from 10 white, 9 green and 7 black balls is : **[AIEEE-2012, (4, –1)/120]**

(1) 880 (2) 629 (3) 630 (4) 879

4. Let T_n be the number of all possible triangles formed by joining vertices of an n-sided regular polygon. If $T_{n+1} - T_n = 10$, then the value of n is : **[AIEEE - 2013, (4, –¼),360]**

(1) 7 (2) 5 (3) 10 (4) 8

5. The number of integers greater than 6,000 that can be formed, using the digits 3, 5, 6, 7 and 8, without repetition, is : **[JEE(Main) 2015, (4, – ¼), 120]**

(1) 216 (2) 192 (3) 120 (4) 72

6. If all the words (with or without meaning) having five letters, formed using the letters of the word SMALL and arranged as in a dictionary; then the position of the word SMALL is : **[JEE (Main)-2016]**

(1) 58^{th} (2) 46^{th} (3) 59^{th} (4) 52^{nd}

7. A man X has 7 friends, 4 of them are ladies and 3 are men. His wife Y also has 7 friends, 3 of them are ladies and 4 are men. Assume X and Y have no common friends. Then the total number of ways in which X and Y together can throw a party inviting 3 ladies and 3 men, so that 3 friends of each of X and Y are in this party, is : **[JEE (Main)-2017]**

(1) 484 (2) 485 (3) 468 (4) 469

8. From 6 different novels and 3 different dictionaries, 4 novels and 1 dictionary are to be selected and arranged in a row on a shelf so that the dictionary is always in the middle. The number of such arrangements is- **[JEE(Main)-2018]**

(1) less than 500 (2) at least 500 but less than 750

(3) at least 750 but less than 1000 (4) at least 1000

9. Consider a class of 5 girls and 7 boys. The number of different teams consisting of 2 girls and 3 boys that can be formed from this class, if there are two specific boys A and B, who refuse to be the members of the same team, is: **[JEE(Main)-Jan 2019]**

(1) 200 (2) 300 (3) 500 (4) 350

10. The number of natural numbers less than 7,000 which can be formed by using the digits 0,1,3,7,9 (repitition of digits allowed) is equal to : **[JEE(Main)-Jan 2019]**

(1) 250 (2) 374 (3) 372 (4) 375

11. Total number of 6-digit numbers in which only and all the five digits 1, 3, 5, 7 and 9 appear, is: **[JEE(Main)-Jan 2020]**

(1) $\frac{5}{2}(6!)$ (2) 5^6 (3) $\frac{1}{2}(6!)$ (4) 6!

Answers

Exercise # 1

PART-I

SECTION-(A)

A-1. (a) 240 (b) 480 (c) 326592
A-2. 8375
A-3. 9999
A-4. 65
A-5. (i) 10! (ii) 10! – 6! × 5! (iii) 86,400 (iv) 28,800 (v) 28,800
A-6. 6
A-7. 200
A-8. 252
A-9. 31080
A-10. 8
A-11. 952
A-12 301 × 6!
A-13. 280
A-14. 90
A-15. (i) 2160 (ii) 360 (iii) 720 (iv) 9360 (v) 1440 (vi) 288 (vii) 144 (viii) 840 (ix) 144 (x) 294
A-16. 124
A-17. 2500
A-18. 229
A-19. (i) 4^n (ii) $2^{n-1}(2^n + 1)$ (iii) 3^n (iv) $\frac{3^n - 1}{2} + 1$ (v) 2^n (vi) 2^{n-1} (vii) ${}^nC_1 . 3^{n-1}$
A-20. 13, 156
A-21. 20
A-22. 57232

SECTION-(B)

B-1. 43200 **B-2.** $\frac{20!}{(4!)^3(2!)^4\,4!.3!}.7!$

B-3. $\frac{20!}{10!10!} + \frac{20!}{7!12!} + \frac{20!}{4!14!2!} + \frac{20!}{16!3!}$

B-4. (a) 150 (b) 270000
B-6. 12
B-7. 1440
B-8. 144

SECTION-(C)

C-1. (i) 479 (ii) 256 (iii) 6 (iv) 473
C-2. 45
C-3. 100, 40, 20
C-4. (i) 48 (ii) 16
C-5. 59

SECTION-(D)

D-1. 171
D-2. 37
D-3. 66
D-4. (i) ${}^{23}C_3$ (ii) ${}^{19}C_3$ (iii) ${}^{19}C_3 - 4.{}^{9}C_3$ (iv) ${}^{10}C_3$
D-5. 95
D-6. (i) 150 (ii) 6 (iii) 25 (iv) 2
D-7. (i) 44 (ii) 109

SECTION-(E)

E-1. 24
E-2. (i) 22 (ii) 5
E-3. ${}^{16}C_3 + 2.\ {}^{17}C_3 + {}^{16}C_1 . {}^{17}C_1 . {}^{17}C_1$
E-4. 670
E-5. (i) 640 (ii) 98
E-6. 184

PART-II

SECTION-(A)

A-1.	(C)	**A-2.**	(D)
A-3.	(C)	**A-4.**	(D)
A-5.	(A)	**A-6.**	(D)
A-7.	(B)	**A-8.**	(B)
A-9.	(A)	**A-10.**	(B)
A-11.	(A)	**A-12.**	(B)
A-13.	(D)	**A-14.**	(A)
A-15.	(D)	**A-16.**	(C)
A-17.	(D)	**A-18.**	(C)
A-19.	(B)	**A-20.**	(C)
A-21.	(A)	**A-22.**	(B)

SECTION-(B)

B-1.	(A)	**B-2.**	(B)
B-3.	(A)	**B-4.**	(D)
B-5.	(C)	**B-6.**	(D)
B-7.	(B)		

SECTION-(C)

C-1.	(C)	**C-2.**	(B)
C-3.	(D)	**C-4.**	(C)
C-5.	(A)	**C-6.**	(B)
C-7.	(D)		

SECTION-(D)

D-1.	(B)	**D-2.**	(C)
D-3.	(A)	**D-4.**	(D)
D-5.	(C)	**D-6.**	(D)
D-7.	(A)	**D-8.**	(B)

SECTION-(E)

E-1.	(B)	**E-2.**	(D)
E-3.	(A)	**E-4.**	(B)

PART-III

1.	**A-R, B-S, C-Q, D-P**
2.	**A-Q, B-P, C-S, D-R**

Exercise # 2

PART-I

1.	(A)	**2.**	(C)
3.	(B)	4.	(A)
5.	(A)	**6.**	(A)
7.	(C)	**8.**	(B)
9.	(D)	**10.**	(C)
11.	(A)	**12.**	(B)
13.	(D)	**14.**	(B)
15.	(C)	**16.**	(C)
17.	(D)	**18.**	(A)
19.	(D)	**20.**	(A)
21.	(C)	**22.**	(B)
23.	(C)	**24.**	(C)

PART-II

1.	744	**2.**	192
3.	20	**4.**	4620
5.	64	**6.**	117
7.	18	**8.**	960
9.	2.97	**10.**	480
11.	606	**12.**	22100, 52
13.	10	**14.**	1918
15.	2997	**16.**	31
17.	2.16 or 2.17	**18.**	20
19.	2.67	**20.**	9
21.	10	**22.**	175
23.	12870	**24.**	7.2
25.	71	**26.**	195

PART - III

1.	(A), (B) (D)	**2.**	(B), (C)
3.	(B), (D)	**4.**	(C), (D)
5.	(A), (B), (C), (D)	**6.**	(B), (C), (D)
7.	(A), (B), (C)	**8.**	(A), (B) (D)
9.	(B), (C)	**10.**	(A), (B),(C), (D)
11.	(A), (C), (D)	**12.**	(D)
13.	(A), (B), (C)	**14.**	(B), (C), (D)
15.	(A), (B)	**16.**	(B), (C), (D)
17.	(A), (B), (C)		

PART - IV

1.	(C)	**2.**	(A)
3.	(B)	**4.**	(D)
5.	(D)	**6.**	(C)

Exercise # 3

PART - I

1.	(A)	**2.**	(B)
3.	(A)	**4.**	(B)
5.	(7)	**6.**	(5)
7.	(C)	**8.**	(5)
9.	(A)	**10.**	5
11.	(D)	**12.**	(625)
13.	(C)	**14.**	(30.0)

PART - II

1.	(1)	**2.**	(1)
3.	(4)	**4.**	(2)
5.	(2)	**6.**	(1)
7.	(2)	**8.**	(4)
9.	(2)	**10.**	(2)
11.	(1)		

SUBJECTIVE QUESTIONS

1. How many positive integers are there such that n is a divisor of one of the numbers 10^{40}, 20^{30}?

2. Six cards are drawn one by one from a set of unlimited number of cards, each card is marked with numbers – 1, 0 or 1. Number of different ways in which they can be drawn if the sum of the numbers shown by them vanishes, is:

3. A five letter word is to be formed such that the letters appearing in the odd numbered positions are taken from the letters which appear without repetition in the word "MATHEMATICS". Further the letters appearing in the even numbered positions are taken from the letters which appear with repetition in the same word "MATHEMATICS". The number of ways in which the five letter word can be formed is:

4. 6 blue, 7 green and 10 white balls are arranged in row such that every blue ball is between and green and a white ball. Moreover, a white ball and a green ball must not be next to each other. The number of such arrangements is ....

5. In how many ways 4 square are can be chosen on a chess-board, such that all the squares lie in a diagonal line.

6. Find the number of functions $f : A \to B$ where n(A) = m , n(B) = t , which are non decreasing,

7. Find the number of ways of selecting 3 vertices from a polygon of sides '2n+1' such that centre of polygon lie inside the triangle.

8. A operation * on a set A is said to be binary, if $x * y \in A$, for all $x, y \in A$, and it is said to be commutative , if $x * y = y * x$ for all $x, y \in A$. Now if $A = \{a_1, a_2, ..., a_n\}$, then find the following -

(i) Total number of binary operations of A

(ii) Total number of binary operation on A such that

$a_i * a_j \neq a_i * a_k$, if $j \neq k$.

(iii) Total number of binary operations on A such that $a_i * a_j < a_i * a_{j+1}$ $\forall$ i, j

9. The integers from 1 to 1000 are written in order around a circle. Starting at 1, every fifteenth number is marked (that is 1, 16, 31, ... etc.). This process in continued untill a number is reached which has already been marked, then find number of unmarked numbers.

10. Find the number of ways in which n '1' and n '2' can be arranged in a row so that upto any point in the row no. of '1' is more than or equal to no. of '2'

11. Find the number of positive integers less than 2310 which are relatively prime with 2310.

12. In maths paper there is a question on "Match the column" in which column A contains 6 entries & each entry of column A corresponds to exactly one of the 6 entries given in column B written randomly. 2 marks are awarded for each correct matching & 1 mark is deducted from each incorrect matching. A student having no subjective knowledge decides to match all the 6 entries randomly. Find the number of ways in which he can answer, to get atleast 25 % marks in this question.

13. Find the number of positive unequal integral solution of the equation x + y + z = 20.

14. If we have 3 identical white flowers and 6m identical red flowers. Find the number of ways in which a garland can be made using all the flowers.

15. The number of combinations of n letters together out of 3n letters of which n are a and n are b and the rest unlike.

16. In a row, there are n rooms, whose door no. are 1,2,.......,n, initially all the door are closed. A person takes 81 round of the row, numbers as 1^{st} round, 2^{nd} round .... n^{th} round. In each round, he interchage the position of those door number, whose number is multiple of the round number. Find out after 81^{st} round,How many doors will be open.

17. Mr. Sibbal walk up 15 steps, going up either 1 or 2 steps with each stride there is explosive material on the 8^{th} step so he cannot step there. Then number of ways in which Mr. Sibbal can go up.

18. A batsman scores exactly a century by hitting fours and sixes in twenty consecutive balls. In how many different ways can he do it if some balls may not yield runs and the order of boundaries and verbounderies are taken into account

19. In how many ways can two distinct subsets of the set A of $k(k \geq 2)$ elements be selected so that they have exactly two common elements.

20. How many 5 digit numbers can be made having exactly two identical digit.

21. In how many ways can$(2n + 1)$ identical balls be placed in 3 distinct boxes so that any two boxes together will contain more balls than the third box.

22. Let f(n) denote the number of different ways in which the positive integer 'n' can be expressed as sum of 1s and 2s.

for example $f(4) = 5$ {2 + 2, 2 + 1 + 1, 1 + 2 + 1, 1 + 1 + 2, 1 + 1 + 1 + 1}. Now that order of 1s and 2s is important. Then determine f(f(6))

23. Prove that $(n!)!$ is divisible by $(n!)^{(n-1)!}$

24. A user of facebook which is two or more days older can send a friend request to some one to join facebook. If initially there is one user on day one then find a recurrence relation for a_n where a_n is number of users after n days.

25. Let X = {1, 2, 3,.....,10}. Find the the number of pairs {A, B} such $A \subseteq X$. $B \subseteq X$. $A \neq B$ and $A \cap B = \{5, 7, 8\}$.

26. Consider a 20-sided convex polygon K, with vertices $A_1, A_2, \ldots, A_{20}$ in that order. Find the number of ways in which three sides of K can be chosen so that every pair among them has at least two sides of K between them. (For example $(A_1A_2, A_4A_5, A_{11}A_{12})$ is an admissible triple while $(A_1A_2, A_4A_5, A_{19}A_{20})$ is not).

27. Thirty volunteers are distributed to three polling booths. Each booth must have at least one and all must have different number of volunteers allotted. Thne the number of ways of allocating volunteers is :

28. Seven points are marked on the circumference of a circle and all pairs of points are joined by straight lines. No three of these lines have a common point and any two intersect at a point inside the circle. Into how many regions s the interior of the circle divided by the these lines ?

29. The number of ways in which 26 identical chocolates be distributed between Amy, Bob, Cathy and Daniel so that each receives at least one chocolate and Amy receives more chocolates than Bob is ....

30. If N is the number of triangles of different shapes (i.e. not similar) whose angles are all integers (in degrees), what is N/100 ?

31. Find the number of 4-digit numbers (in base 10) having non-zero digits and which are divisible by 4 but not by 8.

32. Find the number of all integer-sided isosceles obtuse-angled triangles with perimeter 2008.

33. Let ABC be a triangle. An interior point P of ABC is said to be good if we can find exactly 27 rays emanating from P intersecting the sides of the triangle ABC such that the triangle is divided by these rays into 27 smaller triangles of equal area. Determine the number of good points for a given triangle ABC.

34. Let $\sigma = (a_1, a_2, a_3, ..., a_n)$ be a permutation of (1, 2, 3, ..., n). A pair (a_i,a_j) is said to correspond to an inversion of σ, if i < j but $a_i > a_j$. (Example : In the permutation (2, 4, 5, 3, 1), there are 6 inversions corresponding to the pairs (2, 1), (4, 3), (4, 1), (5, 3) (5, 1), (3, 1) .) How many permutations of (1, 2, 3, ... n), ($n \geq 3$), have exactly **two** inversions.?

35. We will say that a rearrangement of the letters of a word has no fixed letters, if no column has the same letter repeated when the rearrangement is placed directly below the word. For instant, H B R A T A is a rearrangement with no fixed letters of B H A R A T. How many distinguishable rearrangement with no fixed letters does B H A R A T have ? (The two A's are considered identical)

36. what is the number of ways in which one can choose 60 units square from a 11 × 11 chessboard such that no two chosen square have a side in common ?

37. What is the number of ways in which one can colour the square of a 4 × 4 chessboard with colours red and blue such that each row as well as each column has exactly two red squares and blue squares ?

38. A positive integer k is said to be good if there exists a partition of {1, 2, 3, ... , 20} in to disjoint proper subsets such that the sum of the numbers in each subset of the partition is k. How many good numbers are there ?

39. 13 boys are sitting in a row in a theatre. After the intermission, they return and are seated such that either they occupy the same seat or the adjacent seat in such a way that it differs from the original arrangement. The number of ways this is possible is ....

40. Find the number of permutations $x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8$, of the integers –3, –2, –1, 0, 1, 2, 3, 4 that satisfy the chain of inequalities $x_1x_2 \leq x_2x_3 \leq x_3x_4 \leq x_4x_5 \leq x_5x_6 \leq x_6x_7 \leq x_7x_8$.

Answers

1. 2301	**2.** 141	**3.** 540
4. 1980	**5.** 364	**6.** $(t+m-1)c_m$ ways
7. $\frac{2n+1}{3}({}^{2n}C_2 - 3.{}^{n}C_2)$	**8.** (i) n^{n^2} (ii) $(n!)^n$ (iii) 1	**9.** 800
10. $\frac{{}^{2n}C_n}{n+1}$	**11.** 480	**12.** 56 ways
13. 144	**14.** $3m^2 + 3m + 1$	**15.** $(n + 2). 2^{n-1}$
16. 9	**17.** 441	**18.** $\frac{20!}{10!10!} + \frac{20!}{7!12!} + \frac{20!}{4!\,14!\,2!} + \frac{20!}{16!\,3!}$
19. $\frac{k(k-1)}{4}((3)^{k-2}-1)$	**20.** 18480	**21.** $\frac{n(n+1)}{2}$
22. 377	**24.** $a_n = a_{n-1} + a_{n-2}$	**25.** 2186
26. 520	**27.** 366	**28.** 57
29. 1078	**30.** 27	**31.** 729
32. 86	**33.** $^{26}C_2$	**34.** $\frac{(n+1)(n-2)}{2}$
35. 84	**36.** 62	**37.** 90
38. 6	**39.** 376	**40.** 21