SIMPLE HARMONIC MOTION

Contents

JEE (Advance) Syllabus

Simple Harmonic Motion : Linear and angular simple harmonic motions

JEE (Main) Syllabus

Simple Harmonic Motion : Periodic motion – period, frequency, displacement as a function of time. Periodic functions. Simple harmonic motion (SHM) and its equation; phase; oscillations of a spring – restoring force and force constant; energy in SHM – kinetic and potential energies; simple pendulum – derivation of expression for its time period; free, forced and damped oscillations (qualitative ideas only), resonance.

Note: ✍ Marked Questions can be used for Revision.

SIMPLE HARMONIC MOTION

1. PERIODIC MOTION

When a body or a moving particle repeats its motion along a definite path after regular interval of time, its motion is said to be **Periodic Motion** and interval of time is called **time period** or harmonic motion period (T). The path of periodic motion may be linear, circular, elliptical or any other curve. For example, rotation of earth about the sun.

2. OSCILLATORY MOTION

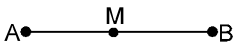
To and Fro' type of motion is called an **Oscillatory Motion**. It need not be periodic and need not have fixed extreme positions. For example, motion of pendulum of a wall clock.
The oscillatory motions in which energy is conserved are also periodic.
The force / torque (directed towards equilibrium point) acting in oscillatory motion is called restoring force / torque.
Damped oscillations are those in which energy is consumed due to some resistive forces and hence total mechanical energy decreases.

3. SIMPLE HARMONIC MOTION

If the restoring force/ torque acting on the body in oscillatory motion is directly proportional to the displacement of body/particle and is always directed towards equilibrium position then the motion is called simple Harmonic Motion (SHM). It is the simplest (easy to analyze) form of oscillatory motion.

TYPES OF SHM

(a) Linear SHM : When a particle undergoes to and fro motion about an equilibrium position, along a straight line. A and B are extreme positions. M is mean position. AM = MB = Amplitude



(b) Angular SHM : When a body/particle is free to rotate oscillate about a given axis on a curved path.

EQUATION OF SIMPLE HARMONIC MOTION (SHM) :

The necessary and sufficient condition for SHM is

$$F = -kx$$

where k = positive constant for a SHM = Force constant
x = displacement from mean position.

or $$m\frac{d^2x}{dt^2} = -kx$$

$\Rightarrow$ $$\frac{d^2x}{dt^2} + \frac{k}{m}x = 0 \quad \text{[differential equation of SHM]}$$

$\Rightarrow$ $$\frac{d^2x}{dt^2} + \omega^2 x = 0 \qquad \text{where } \omega = \sqrt{\frac{k}{m}}$$

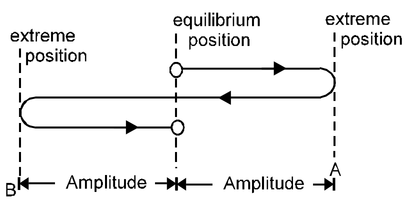
It's solution is $x = A\sin(\omega t + \phi)$

CHARACTERISTICS OF SHM

Note : In the figure shown, path of the particle is on a straight line.

(a) Displacement - It is defined as the distance of the particle from the mean position at that instant. Displacement in SHM at time t is given by $x = A\sin(\omega t + \phi)$

(b) Amplitude - It is the maximum value of displacement of the particle from its equilibrium position.



Amplitude = $\frac{1}{2}$ [distance between extreme points or positions]

It depends on energy of the system.

(c) Angular Frequency (ω) : $\omega = \frac{2\pi}{T} = 2\pi f$ and its unit is rad/sec.

(d) Frequency (f) : Number of oscillations completed in unit time interval is called frequency of oscillations, $f = \frac{1}{T} = \frac{\omega}{2\pi}$, its units is sec^{-1} or Hz.

(e) Time period (T) : Smallest time interval after which the oscillatory motion gets repeated is called time period, $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$

SOLVED EXAMPLE

Example 1. For a particle performing SHM, equation of motion is given as $\frac{d^2x}{dt^2} + 4x = 0$. Find the time period.

Solution : $\frac{d^2x}{dt^2} = -4x$ $\qquad \omega^2 = 4$ $\qquad \omega = 2$

Time period; $T = \frac{2\pi}{\omega} = \pi$

(f) Phase : The physical quantity which represents the state of motion of particle (eg. its position and direction of motion at any instant).
The argument $(\omega t + \phi)$ of sinusoidal function is called instantaneous phase of the motion.

(g) Phase constant (ϕ) : Constant ϕ in equation of SHM is called phase constant or initial phase.
It depends on initial position and direction of velocity.

(h) Velocity(v) : Velocity at an instant is the rate of change of particle's position w.r.t time at that instant.
Let the displacement from mean position is given by

$$x = A\sin(\omega t + \phi)$$

Velocity, $\quad v = \frac{dx}{dt} = \frac{d}{dt}[A\sin(\omega t + \varphi)]$

$$v = A\omega\cos(\omega t + \phi) \qquad \text{or,} \qquad v = \omega\sqrt{A^2 - x^2}$$

At mean position (x = 0), velocity is maximum.

$$v_{max} = \omega A$$

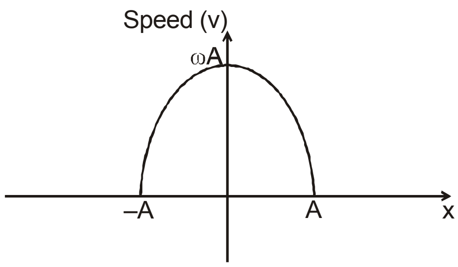
At extreme position (x = A), velocity is minimum.

$$v_{min} = \text{zero}$$

GRAPH OF SPEED (v) VS DISPLACEMENT (x):

$$v = \omega\sqrt{A^2 - x^2} \qquad v^2 = \omega^2(A^2 - x^2)$$

$$v^2 + \omega^2x^2 = \omega^2A^2 \qquad \frac{v^2}{\omega^2A^2} + \frac{x^2}{A^2} = 1$$



GRAPH WOULD BE AN ELLIPSE

(i) Acceleration : Acceleration at an instant is the rate of change of particle's velocity w.r.t. time at that instant.

Acceleration, $a = \frac{dv}{dt} = \frac{d}{dt}[A\omega \cos(\omega t + \phi)]$

$a = -\omega^2 A \sin(\omega t + \phi)$

$a = -\omega^2 x$

Note • Negative sign shows that acceleration is always directed towards the mean position.

At mean position (x = 0), acceleration is minimum.

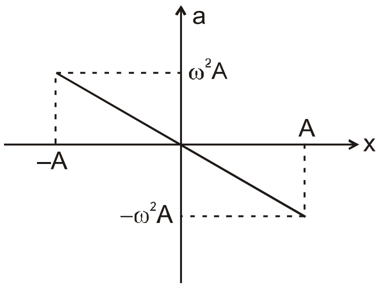
a_{min} = zero

At extreme position (x = A), acceleration is maximum.

$a_{max} = \omega^2 A$

GRAPH OF ACCELERATION (A) VS DISPLACEMENT (x)

$a = -\omega^2 x$



SOLVED EXAMPLE

Example 2. The equation of particle executing simple harmonic motion is x = (5 m) sin $\left[(\pi s^{-1})t + \frac{\pi}{3}\right]$. Write down the amplitude, time period and maximum speed. Also find the velocity at t = 1 s.

Solution : Comparing with equation $x = A \sin(\omega t + \delta)$,

we see that the amplitude = 5 m,

and time period $= \frac{2\pi}{\omega} = \frac{2\pi}{\pi\, s^{-1}}$ = 2s.

The maximum speed $= A\omega = 5\text{ m} \times \pi\, s^{-1} = 5\pi$ m/s.

The velocity at time $t = \frac{dx}{dt} = A\omega \cos(\omega t + \delta)$

At t = 1 s,

$v = (5\text{ m})(\pi\, s^{-1}) \cos\left(\pi + \frac{\pi}{3}\right) = -\frac{5\pi}{2}$ m/s.

Example 3. A particle executing simple harmonic motion has angular frequency 6.28 s^{-1} and amplitude 10 cm. Find (a) the time period, (b) the maximum speed, (c) the maximum acceleration, (d) the speed when the displacement is 6 cm from the mean position, (e) the speed at t = 1/6 s assuming that the motion starts from rest at t = 0.

Solution :

(a) Time period $= \frac{2\pi}{\omega} = \frac{2\pi}{6.28}$ s = 1 s.

(b) Maximum speed $= A\omega = (0.1\text{ m})(6.28\, s^{-1})$
$= 0.628$ m/s.

(c) Maximum acceleration $= A\omega^2$
$= (0.1\text{ m})(6.28\, s^{-1})^2$
$= 4\text{ m/s}^2$.

(d) $v = \omega\sqrt{A^2 - x^2} = (6.28\, s^{-1})\sqrt{(10\text{ cm})^2 - (6\text{ cm})^2} = 50.2$ cm/s.

(e) At t = 0, the velocity is zero i.e., the particle is at an extreme. The equation for displacement may be written as

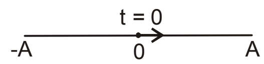
$x = A \cos\omega t$.

The velocity is $v = -A\,\omega \sin \omega t$.

At $t = \frac{1}{6}$ s, $v = -(0.1\text{ m})(6.28\text{ s}^{-1}) \sin\left(\frac{6.28}{6}\right)$

$= (-0.628\text{ m/s}) \sin\frac{\pi}{3} = 54.4$ cm/s.

Example 4. A particle starts from mean position and moves towards positive extreme as shown. Find the equation of the SHM. Amplitude of SHM is A.



Solution : General equation of SHM can be written as $x = A \sin(\omega t + \phi)$

At $t = 0$, $x = 0$

$\therefore$ $0 = A \sin\phi$

$\therefore$ $\phi = 0, \pi$ $\phi \in [0, 2\pi)$

Also; at $t = 0$, $v = +$ve

$\therefore$ $A\omega \cos\phi = +$ve

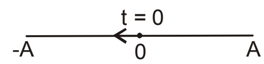
or, $\phi = 0$

Hence, if the particle is at mean position at t = 0 and is moving towards +ve extreme, then the equation of SHM is given by $x = A \sin\omega t$

Similarly

for

$\phi = \pi$



$\therefore$ equation of SHM is $x = A \sin(\omega t + \pi)$

or, $x = -A \sin\omega t$

Note: • **If mean position is not at the origin, then we can replace x by $x - x_0$ and the eqn. becomes $x - x_0 = A \sin(\omega t + \phi)$, where x_0 is the position co-ordinate of the mean position.**

SOLVED EXAMPLE

Example 5. A particle is performing SHM of amplitude "A" and time period "T". Find the time taken by the particle to go from 0 to A/2.

Solution : Let equation of SHM be $x = A \sin \omega t$

when $x = 0$, $t = 0$

when $x = A/2$; $A/2 = A \sin \omega t$

or $\sin \omega t = 1/2$ $\omega t = \pi/6$

$\frac{2\pi}{T} t = \pi/6$ $t = T/12$

Hence, time taken is T/12, where T is time period of SHM.

Example 6. A particle of mass 2 kg is moving on a straight line under the action of force $F = (8 - 2x)$ N. It is released at rest from x = 6 m.

(a) Is the particle moving simple harmonically.

(b) Find the equilibrium position of the particle.

(c) Write the equation of motion of the particle.

(d) Find the time period of SHM.

Solution : $F = 8 - 2x$

or $F = -2(x - 4)$

at equilibrium position $F = 0$

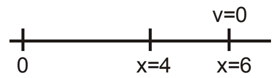
$\Rightarrow$ $x = 4$ is equilibrium position

Hence the motion of particle is SHM with force constant 2 and equilibrium position x = 4.

(a) Yes, motion is SHM.

(b) Equilibrium position is x = 4

(c) At x = 6 m, particle is at rest i.e. it is one of the extreme position



Hence amplitude is A = 2 m and initially particle is at the extreme position.

$\therefore$ Equation of SHM can be written as

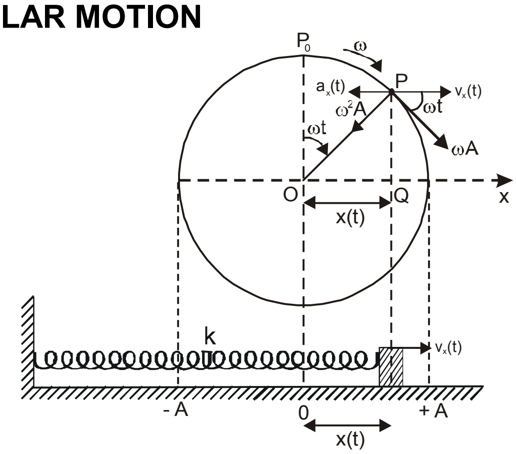
$$x - 4 = 2\cos\omega t\text{ , where } \omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{2}{2}} = 1$$

i.e. $x = 4 + 2\cos t$

(d) Time period, $T = \frac{2\pi}{\omega} = 2\pi$ sec.

4. SHM AS A PROJECTION OF UNIFORM CIRCULAR MOTION

Consider a particle moving on a circle of radius A with a constant angular speed ω as shown in figure.



Suppose the particle is on the top of the circle (Y-axis) at t = 0. The radius OP makes an angle $\theta = \omega t$ with the Y-axis at time t. Drop a perpendicular PQ on X-axis. The components of position vector, velocity vector and acceleration vector at time t on the X-axis are

$x(t) = A\sin\omega t$

$v_x(t) = A\omega\cos\omega t$

$a_x(t) = -\omega^2 A\sin\omega t$

Above equations show that the foot of perpendicular Q executes a simple harmonic motion on the X-axis. The amplitude is A and angular frequency is ω. Similarly the foot of perpendicular on Y-axis will also execute SHM of amplitude A and angular frequency ω [$y(t) = A\cos\omega t$]. The phases of the two simple harmonic motions differ by $\pi/2$.

5. GRAPHICAL REPRESENTATION OF DISPLACEMENT, VELOCITY & ACCELERATION IN SHM

Displacement, $x = A\sin\omega t$

Velocity, $v = A\omega\cos\omega t = A\omega\sin(\omega t + \frac{\pi}{2})$ or $v = \omega\sqrt{A^2 - x^2}$

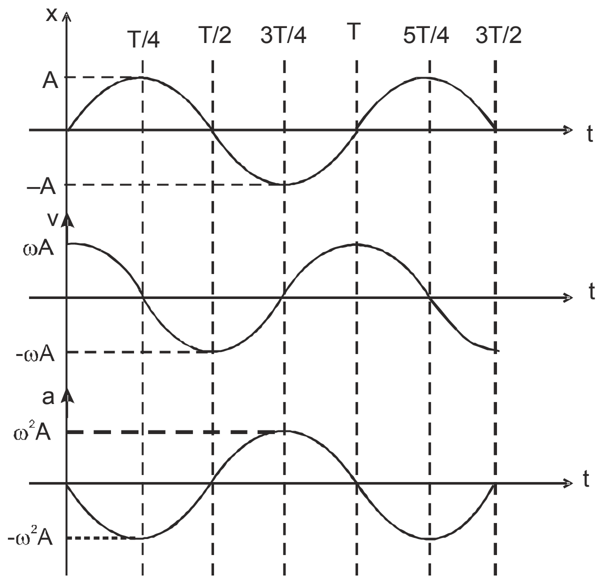
Acceleration, $a = -\omega^2 A\sin\omega t = \omega^2 A\sin(\omega t + \pi)$ or $a = -\omega^2 x$

Note : • $v = \omega\sqrt{A^2 - x^2}$

$a = -\omega^2 x$

These relations are true for any equation of x.

time, t	**0**	**T/4**	**T/2**	**3T/4**	**T**
displacement, x	0	A	0	– A	0
Velocity, v	$A\omega$	0	$-A\omega$	0	$A\omega$
acceleration, a	0	$-\omega^2 A$	0	$\omega^2 A$	0



1. All the three quantities displacement, velocity and acceleration vary harmonically with time, having same period.
2. The velocity amplitude is ω times the displacement amplitude ($v_{max} = \omega A$).
3. The acceleration amplitude is ω^2 times the displacement amplitude ($a_{max} = \omega^2 A$).
4. In SHM, the velocity is ahead of displacement by a phase angle of $\frac{\pi}{2}$.
5. In SHM, the acceleration is ahead of velocity by a phase angle of $\frac{\pi}{2}$.

6. ENERGY OF SHM

Kinetic Energy (KE)

$$\frac{1}{2} mv^2 = \frac{1}{2} m\omega^2 (A^2 - x^2) = \frac{1}{2} k (A^2 - x^2) \text{ (as a function of x)}$$

$$= \frac{1}{2} m A^2\omega^2 \cos^2 (\omega t + \theta) = \frac{1}{2} KA^2 \cos^2 (\omega t + \theta) \text{ (as a function of t)}$$

$$KE_{max} = \frac{1}{2}kA^2 \quad ; \quad \langle KE \rangle_{0-T} = \frac{1}{4}kA^2 \quad ; \quad \langle KE \rangle_{0-A} = \frac{1}{3}kA^2$$

Frequency of KE = 2 × (frequency of SHM)

Potential Energy (PE)

$$\frac{1}{2} Kx^2 \text{ (as a function of x)} = \frac{1}{2} kA^2 \sin^2 (\omega t + \theta) \text{ (as a function of time)}$$

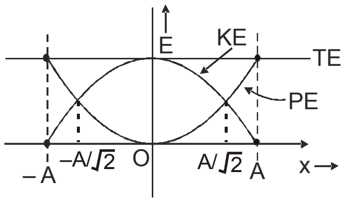
Total Mechanical Energy (TME)

Total mechanical energy = Kinetic energy + Potential energy

$$= \frac{1}{2} k (A^2 - x^2) + \frac{1}{2} Kx^2 = \frac{1}{2} KA^2$$

Hence total mechanical energy is constant in SHM.

Graphical Variation of energy of particle in SHM.



SOLVED EXAMPLE

Example 7. A particle of mass 0.50 kg executes a simple harmonic motion under a force F = – (50 N/m)x. If it crosses the centre of oscillation with a speed of 10 m/s, find the amplitude of the motion.

Solution : The kinetic energy of the particle when it is at the centre of oscillation is

$$E = \frac{1}{2} mv^2 \qquad = \frac{1}{2}(0.50 \text{ kg})(10 \text{ m/s})^2 \qquad = 25 \text{ J}.$$

The potential energy is zero here. At the maximum displacement x = A, the speed is zero and hence the kinetic energy is zero. The potential energy here is $\frac{1}{2} kA^2$. As there is no loss of energy,

$$\frac{1}{2} kA^2 = 25 \text{ J} \qquad \text{.............(i)}$$

The force on the particle is given by

F = – (50 N/m)x.

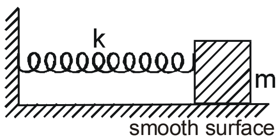
Thus, the spring constant is k = 50 N/m.

Equation (i) gives

$$\frac{1}{2}(50 \text{ N/m}) A^2 = 25 \text{ J} \qquad \text{or,} \qquad A = 1 \text{ m}.$$

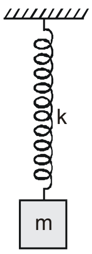
7. SPRING-MASS SYSTEM

(1)



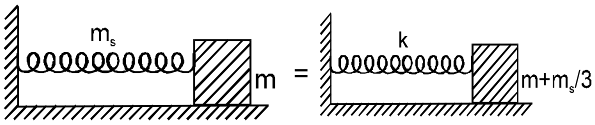
$$T = 2\pi\sqrt{\frac{m}{k}}$$

(2)



$$T = 2\pi\sqrt{\frac{m}{k}}$$

(3) If spring has mass m_s then



$$T = 2\pi\sqrt{\frac{m+\frac{m_s}{3}}{k}}$$

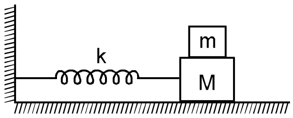
SOLVED EXAMPLE

Example 8. A particle of mass 200 g executes a simple harmonic motion. The restoring force is provided by a spring of spring constant 80 N/m. Find the time period.

Solution : The time period is

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{200\times10^{-3}\,kg}{80\ N/m}} = 2\pi \times 0.05\ s = 0.31\ s.$$

Example 9. The friction coefficient between the two blocks shown in figure is μ and the horizontal plane is smooth. (a) If the system is slightly displaced and released, find the time period. (b) Find the magnitude of the frictional force between the blocks when the displacement from the mean position is x. (c) What can be the maximum amplitude if the upper block does not slip relative to the lower block ?



Solution : (a) For small amplitude, the two blocks oscillate together. The angular frequency is

$\omega = \sqrt{\frac{k}{M+m}}$ and so the time period $T = 2\pi\sqrt{\frac{M+m}{k}}$.

(b) The acceleration of the blocks at displacement x from the mean position is

$$a = -\omega^2 x = \left(\frac{-kx}{M+m}\right)$$

The resultant force on the upper block is, therefore, $ma = \left(\frac{-mkx}{M+m}\right)$

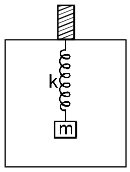
This force is provided by the friction of the lower block. Hence, the magnitude of the frictional force is $\left(\frac{mk\,|x|}{M+m}\right)$

(c) Maximum force of friction required for simple harmonic motion of the upper block is $\frac{mkA}{M+m}$ at the extreme positions. But the maximum frictional force can only be μ mg. Hence

$$\frac{mkA}{M+m} = \mu\, mg \qquad \text{or,} \qquad A = \frac{\mu(M+m)g}{k}$$

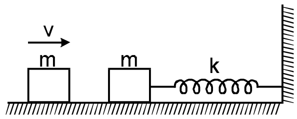
Example 10. A block of mass m is suspended from the ceiling of a stationary elevator through a spring of spring constant k and suddenly, the cable breaks and the elevator starts falling freely. Show that block now executes a simple harmonic motion of amplitude mg/k in the elevator.

Solution : When the elevator is stationary, the spring is stretched to support the block. If the extension is x, the tension is kx which should balance the weight of the block.



Thus, x = mg/k. As the cable breaks, the elevator starts falling with acceleration 'g'. We shall work in the frame of reference of the elevator. Then we have to use a pseudo force mg upward on the block. This force will 'balance' the weight. Thus, the block is subjected to a net force kx by the spring when it is at a distance x from the position of unstretched spring. Hence, its motion in the elevator is simple harmonic with its mean position corresponding to the unstretched spring. Initially, the spring is stretched by x = mg/k, where the velocity of the block (with respect to the elevator) is zero. Thus, the amplitude of the resulting simple harmonic motion is mg/k.

Example 11. The left block in figure collides inelastically with the right block and sticks to it. Find the amplitude of the resulting simple harmonic motion.

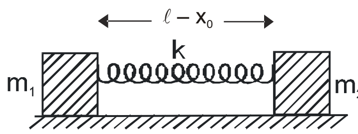


Solution : Assuming the collision to last for a small interval only, we can apply the principle of conservation of momentum. The common velocity after the collision is $\frac{v}{2}$. The kinetic energy = $\frac{1}{2}$ (2m) $\left(\frac{v}{2}\right)^2$

= $\frac{1}{4}$ mv^2. This is also the total energy of vibration as the spring is unstretched at this moment. If the amplitude is A, the total energy can also be written as $\frac{1}{2}$ kA^2. Thus,

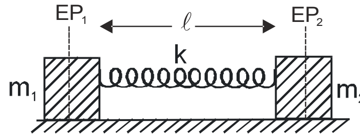
$\frac{1}{2}$kA^2 = $\frac{1}{4}$ mv^2, giving A = $\sqrt{\frac{m}{2k}}$ v.

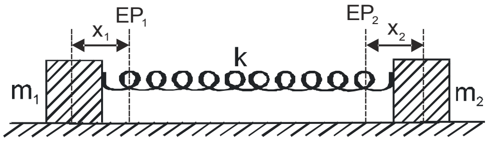
Example 12. Two blocks of mass m_1 and m_2 are connected with a spring of natural length ℓ and spring constant k. The system is lying on a smooth horizontal surface. Initially spring is compressed by x_0 as shown in figure.



Show that the two blocks will perform SHM about their equilibrium position. Also (a) find the time period, (b) find amplitude of each block and (c) length of spring as a function of time.

Solution : (a) Here both the blocks will be in equilibrium at the same time when spring is in its natural length. Let EP_1 and EP_2 be equilibrium positions of block A and B as shown in figure.





Let at any time during oscillations, blocks are at a distance of x_1 and x_2 from their equilibrium positions.

As no external force is acting on the spring block system

$\therefore \quad (m_1 + m_2)\Delta x_{cm} = m_1x_1 - m_2x_2 = 0$ or $\quad m_1x_1 = m_2x_2$

For 1st particle, force equation can be written as

$$k(x_1 + x_2) = -m_1\frac{d^2x_1}{dt^2} \qquad \text{or,} \qquad k\left(x_1 + \frac{m_1}{m_2}x_1\right) = -m_1a_1$$

$$\text{or,} \quad a_1 = -\frac{k(m_1+m_2)}{m_1m_2}x_1 \qquad \therefore \quad \omega^2 = \frac{k(m_1+m_2)}{m_1m_2}$$

Hence, $T = 2\pi\sqrt{\dfrac{m_1m_2}{k(m_1+m_2)}} = 2\pi\sqrt{\dfrac{\mu}{K}}$ where $\mu = \dfrac{m_1m_2}{(m_1+m_2)}$ which is known as reduced mass

Ans (a)

Similarly time period of 2nd particle can be found. Both will be having the same time period.

(b) Let the amplitude of blocks be A_1 and A_2.

$$m_1A_1 = m_2A_2$$

By energy conservation;

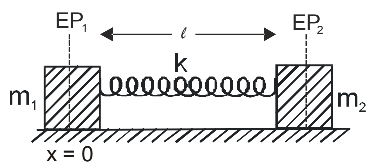
$$\frac{1}{2}k(A_1 + A_2)^2 = \frac{1}{2}k\,x_0^2 \qquad \text{or,} \qquad A_1 + A_2 = x_0$$

$$\text{or,} \quad A_1 + A_2 = x_0 \qquad \text{or,} \qquad A_1 + \frac{m_1}{m_2}A_1 = x_0$$

$$\text{or,} \quad A_1 = \frac{m_2x_0}{m_1+m_2} \qquad \text{Similarly,} \qquad A_2 = \frac{m_1x_0}{m_1+m_2}$$

(c) Consider equilibrium position of 1st particle as origin, i.e. x = 0 .

x co-ordinate of particles can be written as



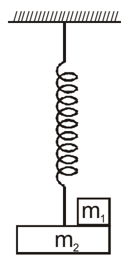
$x_1 = A_1\cos\omega t \quad$ and $\quad x_2 = \ell - A_2\cos\omega t$

Hence, length of spring at time t can be written as;

length $= x_2 - x_1$

$\qquad = \ell - (A_1 + A_2)\cos\omega t$

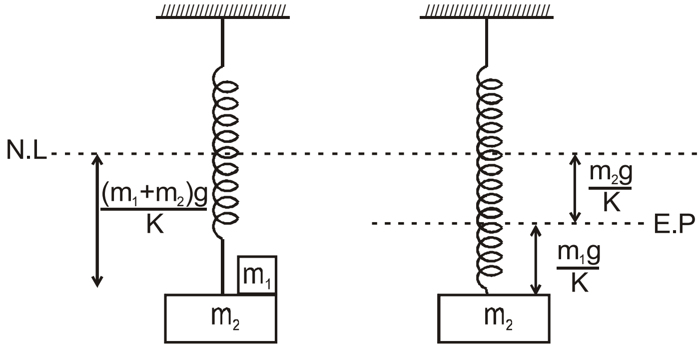
Example 13. The system is in equilibrium and at rest. Now mass m_1 is removed from m_2. Find the time period and amplitude of resultant motion. Spring constant is K.



Solution : Initial extension in the spring

$$x = \frac{(m_1+m_2)g}{K}$$

Now, if we remove m_1, equilibrium position(E.P.) of m_2 will be $\dfrac{m_2\,g}{K}$ below natural length of spring.



At the initial position, since velocity is zero i.e. it is the extreme position.

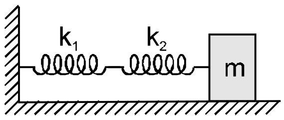
Hence Amplitude $= \dfrac{m_1 g}{K}$

Time period $= 2\pi\sqrt{\dfrac{m_2}{K}}$

Since only block of mass m_2 is oscillating

8. COMBINATION OF SPRINGS

Series Combination :



Total displacement $x = x_1 + x_2$

Tension in both springs $= k_1 x_1 = k_2 x_2$

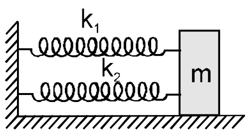
$\therefore$ Equivalent spring constant in series combination K_{eq} is given by :

$$1/k_{eq} = 1/k_1 + 1/k_2 \quad \Rightarrow \quad T = 2\pi\sqrt{\frac{m}{k_{eq}}}$$

Note :

- In series combination, tension is same in all the springs & extension will be different. (If k is same then deformation is also same)
- In series combination , extension of springs will be reciprocal of its spring constant.
- Spring constant of spring is reciprocal of its natural length
  $\because\ k \propto 1/\ell$
  $\therefore\ k_1 \ell_1 = k_2 \ell_2 = k_3 \ell_3$
- If a spring is cut in 'n' pieces then spring constant of one piece will be nk.

Parallel combination :



Extension is same for both springs but force acting will be different.

Force acting on the system = F

$\therefore \quad F = -(k_1 x + k_2 x) \quad \Rightarrow \quad F = -(k_1 + k_2)\, x \quad \Rightarrow \quad F = -k_{eq} x$

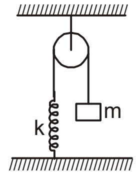
$\therefore \quad k_{eq} = k_1 + k_2 \quad \Rightarrow \quad T = 2\pi\sqrt{\dfrac{m}{k_{eq}}}$

9. METHOD'S TO DETERMINE TIME PERIOD, ANGULAR FREQUENCY IN S.H.M.

(a) Force / torque method

(b) Energy method

SOLVED EXAMPLE



Example 14. The string, the spring and the pulley shown in figure are light. Find the time period of the mass m.

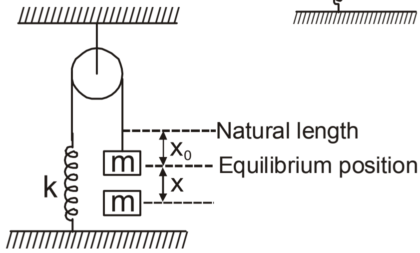
Solution (a) **Force Method**

Let in equilibrium position of the block, extension in spring is x_0.

$\therefore \quad kx_0 = mg \qquad$ -- (1)

Now if we displace the block by x in the downward direction, net force on the block towards mean position is

$F = k(x + x_0) - mg \quad = kx \quad$ using (1)



Hence the net force is acting towards mean position and is also proportional to x.So, the particle will perform S.H.M. and its time period would be

$$T = 2\pi\sqrt{\frac{m}{k}}$$

(b) **Energy Method**

Let gravitational potential energy is to be zero at the level of the block when spring is in its natural length.

Now at a distance x below that level, let speed of the block be v.

Since total mechanical energy is conserved in S.H.M.

$$\therefore \quad -mgx + \frac{1}{2}kx^2 + \frac{1}{2}mv^2 = \text{constant}$$

Differentiating w.r.t. time, we get

$-mgv + kxv + mva = 0$

where a is acceleration.

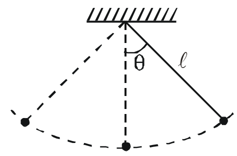
$$\therefore \quad F = ma = -kx + mg \qquad \text{or} \qquad F = -k\left(x - \frac{mg}{k}\right)$$

This shows that for the motion, force constant is k and equilibrium position is $x = \frac{mg}{k}$.

So, the particle will perform S.H.M. and its time period would be $T = 2\pi\sqrt{\frac{m}{k}}$

10. SIMPLE PENDULUM

If a heavy point mass is suspended by a weightless, inextensible and perfectly flexible string from a rigid support, then this arrangement is called a simple pendulum



Time period of a simple pendulum $T = 2\pi\sqrt{\frac{\ell}{g}}$

(some times we can take $g = \pi^2$ for making calculation simple)

Note :

- If angular amplitude of simple pendulum is more, then time period

$$T = 2\pi\sqrt{\frac{\ell}{g}}\left(1 + \frac{\theta_0^2}{16}\right)$$ **(Not in JEE, For other exams)**

where θ_0 is in radians.

- General formula for time period of simple pendulum when ℓ is comparable to radius of Earth R.

$$T = 2\pi\sqrt{\frac{1}{g\left(\frac{1}{R}+\frac{1}{\ell}\right)}}$$ where, R = Radius of the earth

- Time period of simple pendulum of infinite length is maximum and is given by: $T = 2\pi\sqrt{\frac{R}{g}}$ = 84.6 min (Where R is radius of earth)
- Time period of seconds pendulum is 2 sec and ℓ = 0.993 m.
- Simple pendulum performs angular S.H.M. but due to small angular displacement, it is considered as linear S.H.M.
- If time period of clock based on simple pendulum increases then clock will be slow but if time period decrease then clock will be fast.
- If g remains constant & $\Delta\ell$ is change in length, then $\frac{\Delta T}{T}\times 100 = \frac{1}{2}\frac{\Delta\ell}{\ell}\times 100$
- If ℓ remain constant & Δg is change in acceleration then, $\frac{\Delta T}{T}\times 100 = -\frac{1}{2}\frac{\Delta g}{g}\times 100$
- If $\Delta\ell$ is change in length & Δg is change in acceleration due to gravity then,

$$\frac{\Delta T}{T}\times 100 = \left[\frac{1}{2}\frac{\Delta\ell}{\ell} - \frac{1}{2}\frac{\Delta g}{g}\right]\times 100$$

SOLVED EXAMPLE

Example 15 A simple pendulum of length 40 cm oscillates with an angular amplitude of 0.04 rad. Find (a) the time period, (b) the linear amplitude of the bob, (c) the speed of the bob when the string makes 0.02 rad with the vertical and (d) the angular acceleration when the bob is in momentary rest. Take g = 10 m/s^2.

Solution : (a) The angular frequency is $\omega = \sqrt{g/\ell} = \sqrt{\frac{10\,m/s^2}{0.4m}} = 5\ s^{-1}$

the time period is $\frac{2\pi}{\omega} = \frac{2\pi}{5s^{-1}}$ = 1.26 s.

(b) Linear amplitude = 40 cm × 0.04 = 1.6 cm

(c) Angular speed at displacement 0.02 rad is

$\Omega = (5\ s^{-1})\ \sqrt{(0.04)^2 - (0.02)^2}$ rad = 0.17 rad/s.

where speed of the bob at this instant

= (40 cm) × 0.175^{-1} = 6.8 cm/s.

(d) At momentary rest, the bob is in extreme position.

Thus, the angular acceleration

$\alpha = (0.04\ rad)\ (25\ s^{-2}) = 1\ rad/s^2$.

Time Period of Simple Pendulum in accelerating Reference Frame :

$$T = 2\pi\sqrt{\frac{\ell}{g_{eff.}}} \text{ where}$$

$g_{eff.}$ = Effective acceleration in accelerating reference system = $|\vec{g} - \vec{a}|$, at mean position

$\vec{a}$ = acceleration of the point of suspension w.r.t. ground.

Condition for applying this formula: $|\vec{g} - \vec{a}|$ = constant

Also $g_{eff} = \dfrac{\text{Net tension in string}}{\text{mass of bob}}$ at mean position

SOLVED EXAMPLE

Example 16. A simple pendulum is suspended from the ceiling of a car accelerating uniformly on a horizontal road. If the acceleration is a_0 and the length of the pendulum is ℓ, find the time period of small oscillations about the mean position.

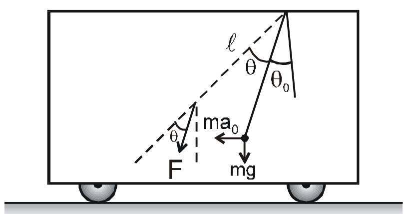
Solution : We shall work in the car frame. As it is accelerated with respect to the road, we shall have to apply a pseudo force ma_0 on the bob of mass m.

For mean position, the acceleration of the bob with respect to the car should be zero. If θ_0 be the angle made by the string with the vertical, the tension, weight and the pseudo force will add to zero in this position.

Hence, resultant of mg and ma_0 (say F = $m\sqrt{g^2 + a_0^2}$) has to be along the string.

$$\therefore \quad \tan\theta_0 = \frac{ma_0}{mg} = \frac{a_0}{g}$$

Now, suppose the string is further deflected by an angle θ as shown in figure.



Now, restoring torque can be given by

$$(F \sin\theta)\, \ell = -m\ell^2\alpha$$

Substituting F and using $\sin\theta \simeq \theta$, for small θ.

$$(m\sqrt{g^2 + a_0^2})\,\ell\theta = -m\ell^2\alpha$$

or, $\alpha = -\dfrac{\sqrt{g^2 + a_0^2}}{\ell}\theta$ so; $\omega^2 = \dfrac{\sqrt{g^2 + a_0^2}}{\ell}$

This is an equation of simple harmonic motion with time period

$$T = \frac{2\pi}{\omega} = 2\pi\frac{\sqrt{\ell}}{(g^2 + a_0^2)^{1/4}}$$

If forces other then $m\vec{g}$ acts then :

$$T = 2\pi\sqrt{\frac{\ell}{g_{eff.}}} \text{ where } \quad g_{eff.} = \left|\vec{g} + \frac{\vec{F}}{m}\right|$$

$\vec{F}$ = constant force acting on 'm'.

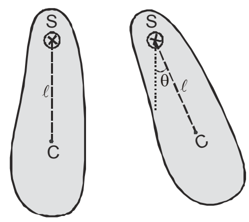
SOLVED EXAMPLE

Example 17. A simple pendulum of length 'ℓ' and having bob of mass 'm' is doing angular SHM inside water. A constant buoyant force equal to half the weight of the bob is acting on the ball. Find the time period of oscillations?

Solution : Here $g_{eff.} = g - \frac{mg/2}{m} = g/2$.

Hence $T = 2\pi\sqrt{\frac{2\ell}{g}}$

11. COMPOUND PENDULUM / PHYSICAL PENDULUM



When a rigid body is suspended from an axis and made to oscillate about that then it is called compound pendulum.

C = Position of center of mass

S = Point of suspension

ℓ = Distance between point of suspension and center of mass (it remains constant during motion)

For small angular displacement "θ" from mean position

The restoring torque is given by

$\tau = -mg\ell\sin\theta$

$\tau = -mg\ell\theta$ $\quad\because$ for small θ, $\sin\theta \simeq \theta$

or, $I\alpha = -mg\ell\theta$ $\quad$ where, I = Moment of inertia about point of suspension.

or, $\alpha = -\frac{mg\ell}{I}\theta$ $\quad$ or, $\omega^2 = \frac{mg\ell}{I}$

Time period, $T = 2\pi\sqrt{\frac{I}{mg\ell}}$ $\quad$ $I = I_{CM} + m\ell^2$

Where I_{CM} = moment of inertia relative to the axis which passes from the center of mass & parallel to the axis of oscillation.

$$T = 2\pi\sqrt{\frac{I_{CM} + m\ell^2}{mg\ell}}$$

where $I_{CM} = mk^2$

k = gyration radius (about axis passing from centre of mass)

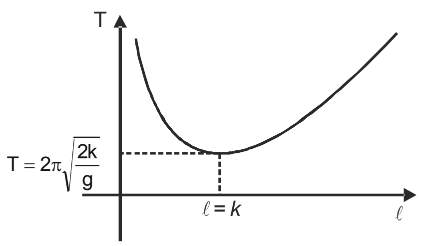
$$T = 2\pi\sqrt{\frac{mk^2 + m\ell^2}{mg\ell}} \qquad T = 2\pi\sqrt{\frac{k^2 + \ell^2}{\ell g}} = 2\pi\sqrt{\frac{L_{eq}}{g}}$$

$L_{eq} = \frac{k^2}{\ell} + \ell$ = equivalent length of simple pendulum ;

T is minimum when ℓ = k.

$T_{min} = 2\pi\sqrt{\frac{2k}{g}}$

Graph of T vs ℓ



SOLVED EXAMPLE

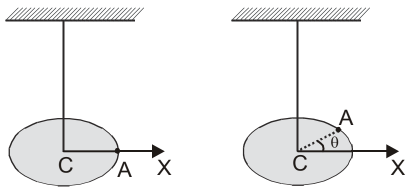
Example 18. A uniform rod of length 1.00 m is suspended through an end and is set into oscillation with small amplitude under gravity. Find the time period of oscillation. (g = 10 m/s²)

Solution : For small amplitude the angular motion is nearly simple harmonic and the time period is given by

$$T = 2\pi\sqrt{\frac{I}{mg(\ell/2)}} = 2\pi\sqrt{\frac{(m\ell^2/3)}{mg(\ell/2)}} = 2\pi\sqrt{\frac{2\ell}{3g}} = 2\pi\sqrt{\frac{2\times1.00\,m}{3\times10\,m/s^2}} = \frac{2\pi}{\sqrt{15}}\ s.$$

12. TORSIONAL PENDULUM

In torsional pendulum, an extended object is suspended at the centre by a light torsion wire. A torsion wire is essentially inextensible, but is free to twist about its axis. When the lower end of the wire is rotated by a slight amount, the wire applies a restoring torque causing the body to oscillate (rotate) about vertical wire, when released. The restoring torque produced is given by



$\tau = -C\theta$ where, C = Torsional constant

or, $I\alpha = -C\theta$ where, I = Moment of inertia about the vertical axis.

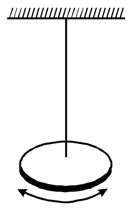
or, $\alpha = -\frac{C}{I}\theta$

$\therefore$ Time Period, $T = 2\pi\sqrt{\frac{I}{C}}$

SOLVED EXAMPLE

Example 19. A uniform disc of radius 5.0 cm and mass 200 g is fixed at its centre to a metal wire, the other end of which is fixed to a ceiling. The hanging disc is rotated about the wire through an angle and is released. If the disc makes torsional oscillations with time period 0.20 s, find the torsional constant of the wire.

Solution : The situation is shown in figure. The moment of inertia of the disc about the wire is



$$I = \frac{mr^2}{2} = \frac{(0.200\,kg)(5.0\times10^{-2}m)^2}{2} = 2.5\times10^{-4}\ kg - m^2.$$

The time period is given by

$$T = 2\pi\sqrt{\frac{I}{C}} \quad \text{or,} \quad C = \frac{4\pi^2 I}{T^2} = \frac{4\pi^2(2.5\times10^{-4}kg - m^2)}{(0.20\,s)^2} = 0.25\ \frac{kg - m^2}{s^2}.$$

13. SUPERPOSITION OF TWO SHM'S

In same direction and of same frequency.

$x_1 = A_1 \sin \omega t$

$x_2 = A_2 \sin(\omega t + \theta)$, then resultant displacement

$x = x_1 + x_2 = A_1 \sin \omega t + A_2 \sin(\omega t + \theta) = A \sin(\omega t + \phi)$

where $A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\theta}$ & $\phi = \tan^{-1}\left[\frac{A_2\sin\theta}{A_1 + A_2\cos\theta}\right]$

If $\theta = 0$, both SHM's are in phase and $A = A_1 + A_2$

If $\theta = \pi$, both SHM's are out of phase and $A = |A_1 - A_2|$

The resultant amplitude due to superposition of two or more than two SHM's of this case can also be found by phasor diagram also.

In same direction but are of different frequencies.

$x_1 = A_1 \sin \omega_1 t$

$x_2 = A_2 \sin \omega_2 t$

then resultant displacement $x = x_1 + x_2 = A_1 \sin \omega_1 t + A_2 \sin \omega_2 t$ This resultant motion is not SHM.

In two perpendicular directions.

$x = A \sin \omega t$

$y = B \sin (\omega t + \theta)$

Case (i) : If $\theta = 0$ or π then $y = \pm (B/A)\, x$. So path will be straight line & resultant displacement will be $r = (x^2 + y^2)^{1/2} = (A^2 + B^2)^{1/2} \sin\omega t$

which is equation of SHM having amplitude $\sqrt{A^2 + B^2}$

Case (ii) : If $\theta = \frac{\pi}{2}$ then $x = A \sin \omega t$

$y = B \sin (\omega t + \pi/2) = B \cos \omega t$

so, resultant will be $\frac{x^2}{A^2} + \frac{y^2}{B^2} = 1$. i.e. equation of an ellipse and if A = B, then superposition will be an equation of circle. This resultant motion is not SHM.

Superposition of SHM's along the same direction (using phasor diagram)

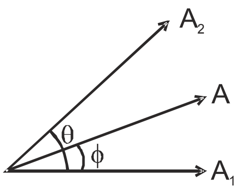
If two or more SHM's are along the same line, their resultant can be obtained by vector addition by making phasor diagram.

1. Amplitude of SHM is taken as length(magnitude) of vector.

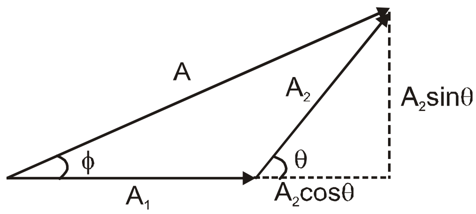
2. Phase difference between the vectors is taken as the angle between these vectors. The magnitude of resultant vector gives resultant amplitude of SHM and angle of resultant vector gives phase constant of resultant SHM.

For example; $x_1 = A_1 \sin \omega t$

$x_2 = A_2 \sin (\omega t + \theta)$



Phasor Diagram



Phasor diagram

If equation of resultant SHM is taken as $x = A \sin (\omega t + \phi)$

$$A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\theta} \quad \Rightarrow \quad \tan\phi = \frac{A_2 \sin\theta}{A_1 + A_2\cos\theta}$$

SOLVED EXAMPLE

Example 20. Find the amplitude of the simple harmonic motion obtained by combining the motions

$x_1 = (2.0 \text{ cm}) \sin\omega t$

and $x_2 = (2.0 \text{ cm}) \sin (\omega t + \pi/3)$.

Solution : The two equations given represent simple harmonic motions along X-axis with amplitudes A_1 = 2.0 cm and A_2 = 2.0 cm. The phase difference between the two simple harmonic motions is $\pi/3$. The resultant simple harmonic motion will have an amplitude A given by

$$A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\delta} \qquad = \sqrt{(2.0\text{cm})^2 + (2.0\text{ cm})^2 + 2(2.0\text{cm})^2\cos\frac{\pi}{3}} = 3.5 \text{ cm}$$

Example 21. $x_1 = 3\sin\omega t$; $x_2 = 4\cos\omega t$

Find (i) amplitude of resultant SHM. (ii) equation of the resultant SHM.

Solution : First write all SHM's in terms of sine functions with positive amplitude. Keep "ωt" with positive sign.

$\therefore$ $x_1 = 3\sin\omega t$

$x_2 = 4\sin(\omega t + \pi/2)$

$$A = \sqrt{3^2 + 4^2 + 2\times3\times4\cos\frac{\pi}{2}} = \sqrt{9+16} = \sqrt{25} = 5$$

$$\tan\phi = \frac{4\sin\frac{\pi}{2}}{3+4\cos\frac{\pi}{2}} = \frac{4}{3} \qquad \phi = 53^\circ \qquad \text{equation } x = 5\sin(\omega t + 53^\circ)$$

Example 22 $x_1 = 5\sin(\omega t + 30^\circ)$

$x_2 = 10\cos(\omega t)$

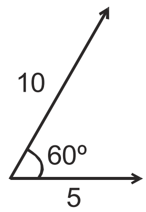
Find amplitude of resultant SHM.

Solution : $x_1 = 5\sin(\omega t + 30^\circ)$

$x_2 = 10\sin(\omega t + \frac{\pi}{2})$

$$A = \sqrt{5^2 + 10^2 + 2\times5\times10\cos 60^\circ}$$

$$= \sqrt{25+100+50} \qquad = \sqrt{175} \qquad = 5\sqrt{7}$$



Phasor Diagram

Example 23 A particle is subjected to two simple harmonic motions

$x_1 = A_1\sin\omega t$

and $x_2 = A_2\sin(\omega t + \pi/3)$.

Find (a) the displacement at t = 0, (b) the maximum speed of the particle and (c) the maximum acceleration of the particle.

Solution : (a) At $t = 0$, $x_1 = A_1\sin\omega t = 0$

and $x_2 = A_2\sin(\omega t + \pi/3)$

$$= A_2\sin(\pi/3) = \frac{A_2\sqrt{3}}{2}.$$

Thus, the resultant displacement at t = 0 is

$$x = x_1 + x_2 = A_2\frac{\sqrt{3}}{2}$$

(b) The resultant of the two motions is a simple harmonic motion of the same angular frequency ω. The amplitude of the resultant motion is

$$A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos(\pi/3)} \qquad = \sqrt{A_1^2 + A_2^2 + A_1A_2}.$$

The maximum speed is

$$u_{max} = A\,\omega = \omega\sqrt{A_1^2 + A_2^2 + A_1A_2}$$

(c) The maximum acceleration is

$$a_{max} = A\,\omega^2 = \omega^2\sqrt{A_1^2 + A_2^2 + A_1A_2}$$

Example 24. A particle is subjected to two simple harmonic motions in the same direction having equal amplitudes and equal frequency. If the resultant amplitude is equal to the amplitude of the individual motions, find the phase difference between the individual motions.

Solution : Let the amplitudes of the individual motions be A each. The resultant amplitude is also A. If the phase difference between the two motions is δ,

$$A = \sqrt{A^2 + A^2 + 2A.\,A.\cos\delta}$$

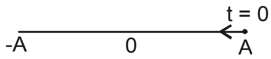
$$A = A\sqrt{2(1+\cos\delta)} = 2A\cos\frac{\delta}{2}$$

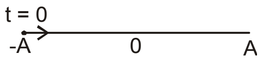
$$\cos\frac{\delta}{2} = \frac{1}{2}$$

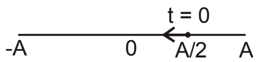
$$\delta = 2\pi/3.$$

MISCELLANEOUS SOLVED EXAMPLE

Problem 1. Write the equation of SHM for the situations shown below:

(a) 

(b) 

(c) 

Solution : **(a)** At t = 0 , x = +A

$x = A\sin(\omega t + \phi)$

$A = A\sin(\phi)$

$\phi = \pi/2$

$$x = A\sin\left(\omega t + \frac{\pi}{2}\right) = A\cos(\omega t)$$

(b) At t = 0 , x = – A

$x = A\sin(\omega t + \phi)$

$-A = A\sin\phi$

$$\phi = \frac{3\pi}{2}$$

$$x = A\sin\left(\omega t + \frac{3\pi}{2}\right)$$

$x = -A\cos(\omega t)$

(c) At t = 0 , $x = \frac{A}{2}$

$x = A\sin(\omega t + \phi)$

$$\frac{A}{2} = A\sin(\omega t + \phi)$$

$$\frac{1}{2} = \sin\phi \Rightarrow \phi = 30^\circ\ ,\ 150$$

Particle is moving towards the mean position and in negative direction.

velocity $v = A\omega \cos(\omega t + \phi)$

At $t = 0$, $v = -ve$

$v = A\omega\cos\phi$

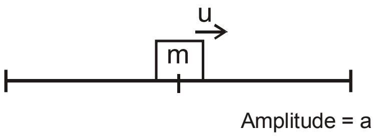
hence $\phi = 150°$

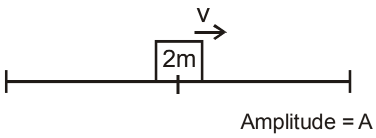
$x = A \sin(\omega t + 150°)$

Ans. (**a**) $x = A \cos\omega t$; (**b**) $x = -A \cos\omega t$; (**c**) $x = A \sin(\omega t + 150°)$

Problem 2. Block A of mass m is performing SHM of amplitude a. Another block B of mass m is gently placed on A when it passes through mean position and B sticks to A. Find the time period and amplitude of new SHM.

Solution :





Time period of mass m = $2\pi\sqrt{\dfrac{m}{K}}$

Time period of mass 2m = $2\pi\sqrt{\dfrac{2m}{K}}$

At mean position, Kinetic energy = Total Energy

For mass m : $\frac{1}{2}mu^2 = \frac{1}{2}m\omega^2a^2$ .... (1)

For mass 2m : $\frac{1}{2}2mv^2 = \frac{1}{2}2m\left(\frac{\omega}{\sqrt{2}}\right)^2 A^2$ .... (2)

By Conservation of momentum

$mu = 2mv$

$v = \dfrac{u}{2}$

$\therefore \ \frac{1}{2}2m\left(\frac{u}{2}\right)^2 = \frac{1}{2}2m\left(\frac{\omega}{\sqrt{2}}\right)^2 A^2$ .... (3)

Divide equation (1) & (3)

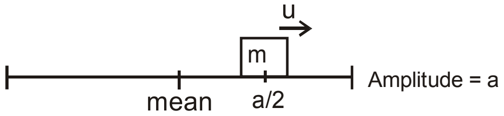
$4 = \dfrac{2a^2}{A^2}$

New Amplitude $A = \dfrac{a}{\sqrt{2}}$

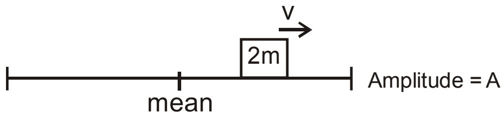
Ans. $T = 2\pi\sqrt{\dfrac{2m}{K}}$ Amplitude = $\dfrac{a}{\sqrt{2}}$

Problem 3. Repeat the above problem assuming B is placed on A at a distance $\frac{a}{2}$ from mean position.

Solution : $T = 2\pi\sqrt{\frac{m}{K}}$



$T = 2\pi\sqrt{\frac{2m}{K}}$



By conservation of momentum

$mu = 2mv \Rightarrow v = \frac{u}{2}$

Kinetic Energies at $\frac{a}{2}$

For mass m : $\frac{1}{2}mu^2 = \frac{1}{2}m\omega^2\left[a^2 - \left(\frac{a}{2}\right)^2\right]$ .... (1)

For mass 2m : $\frac{1}{2}2mv^2 = \frac{1}{2}2m\left(\frac{\omega}{\sqrt{2}}\right)^2\left[A^2 - \left(\frac{a}{2}\right)^2\right]$ .... (2)

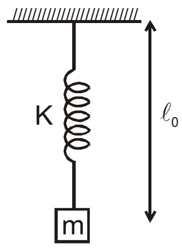
$$\frac{1}{2}2m\frac{u^2}{4} = \frac{1}{2}2m\frac{\omega^2}{2}\left[A^2 - \frac{a^2}{4}\right] \text{ .... (3)}$$

Dividing equation (1) & (3)

$$2 = \frac{a^2 - \frac{a^2}{4}}{A^2 - \frac{a^2}{4}}$$

New Amplitude $A = \sqrt{\frac{5}{8}}\,a$ **Ans.** $T = 2\pi\sqrt{\frac{2m}{K}}$, Amplitude = $a\sqrt{\frac{5}{8}}$

Problem 4. The block is allowed to fall, slowly from the position where spring is in its natural length. Find, maximum extension in the string.



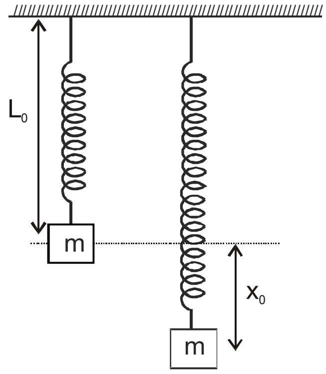
Solution : Since the block falls slowly from rest the maximum extension occurs when

$mg = Kx_0$

$x_0 = \frac{mg}{K}$ is maximum extension **Ans.** $\frac{mg}{K}$

Problem 5. In the above problem if block is released from there, what would be maximum extension.

Solution : Let x_0 = maximum extension



applying conservation of energy

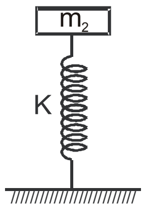
$$mgx_o = \frac{1}{2}Kx_0^2$$

The velocity at the point of maximum extension is zero.

$$x_0 = \frac{2mg}{K} \text{ is maximum extension}$$

Ans. $\frac{2mg}{K}$

Problem 6. Block of mass m_2 is in equilibrium as shown in figure. Another block of mass m_1 is kept gently on m_2. Find the time period of oscillation and amplitude.

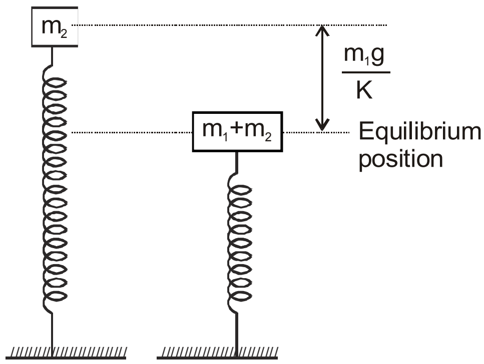


Solution : Time period $T = 2\pi\sqrt{\frac{m_1+m_2}{K}}$

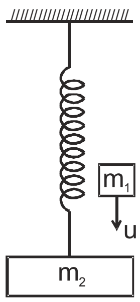
At initial position since velocity is zero it is the extreme position.

Amplitude $A = \frac{m_1 g}{K}$

Ans. $T = 2\pi\sqrt{\frac{m_1+m_2}{K}}$ Amplitude $= \frac{m_1 g}{K}$



Problem 7. Block of mass m_2 is in equilibrium and at rest. The mass m_1 moving with velocity u vertically downwards collides with m_2 and sticks to it. Find the energy of oscillation.

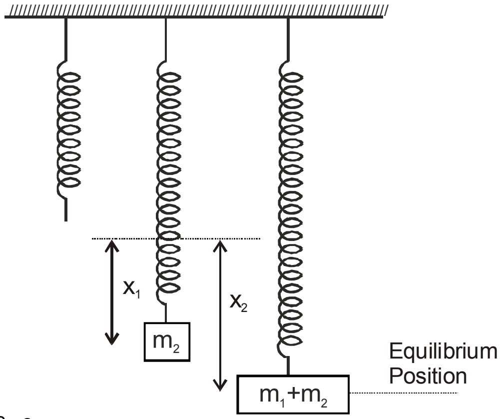


Solution : Let the velocity of m_1 & m_2 be v after collision

By conservation of momentum

$$m_1u = (m_1 + m_2)v$$

$$v = \frac{m_1u}{m_1+m_2}$$



Hence, $KE = \frac{1}{2}(m_1+m_2)v^2 = \frac{1}{2}\frac{m_1^2}{m_1+m_2}u^2$

$m_2g = Kx_1$

$(m_1 + m_2)g = Kx_2$

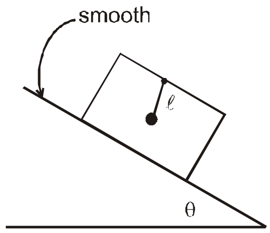
$PE = \frac{1}{2}K(x_2 - x_1)^2 = \frac{1}{2}K\left(\frac{m_1g}{K}\right)^2 \Rightarrow PE = \frac{1}{2}\frac{m_1^2g^2}{K}$

Therefore energy of oscillation is -

E = KE + PE

$$E = \frac{1}{2}\frac{m^2u^2}{m_1+m_2} + \frac{m_1^2g^2}{2K} \Rightarrow = \frac{1}{2}\left[\frac{m_1^2u^2}{m_1+m_2} + \frac{m_1^2g^2}{K}\right]$$ **Ans.**

Problem 8. A box is placed on a smooth inclined plane and it is free to move. A simple pendulum is attached in the block. Find its time period.



Solution : Let F_p = Pseudo force

For equilibrium position

$T\sin\alpha + F_P = mg\sin\theta$

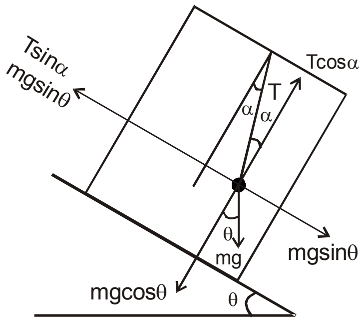
$T\sin\alpha + ma = mg\sin\theta$

since $a = g\sin\theta$

$T\sin\alpha + mg\sin\theta = mg\sin\theta$

$T\sin\alpha = 0$

$\therefore \alpha = 0$



Hence in equilibrium position the string is perpendicular to incline plane.

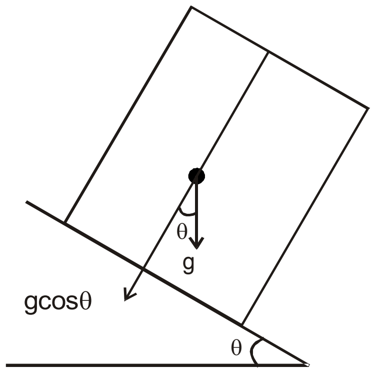
Therefore effective 'g' is

$g_{eff} = g\cos\theta$

Time period

$$T = 2\pi\sqrt{\frac{\ell}{g_{eff}}}$$

$$T = 2\pi\sqrt{\frac{\ell}{g\cos\theta}}$$ **Ans.**



Problem 9. $x_1 = 5 \sin \omega t$

$x_2 = 5 \sin (\omega t + 53^\circ)$

$x_3 = -10 \cos \omega t$

Find amplitude of resultant SHM

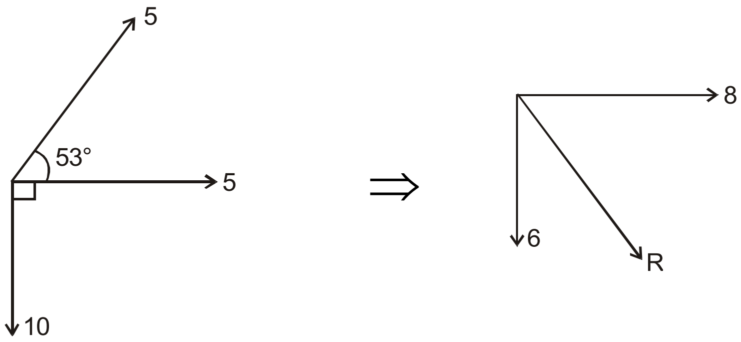
Solution : $x_1 = 5\sin\omega t$

$x_2 = 5 \sin (\omega t + 53^\circ)$

$x_3 = -10 \cos \omega t$

we can write $x_3 = 10 \sin(\omega t + 270^\circ)$

Finding the resultant amplitude by vector notation.



Resultant Amplitude $|R| = \sqrt{8^2 + 6^2} = 10$ **Ans.**

Exercise # 1

PART - I : SUBJECTIVE QUESTIONS

SECTION (A) : EQUATION OF SHM

A-1. The equation of a particle executing SHM is $x = (5m)\sin\left[(\pi s^{-1})t + \frac{\pi}{6}\right]$. Write down the amplitude, initial phase constant, time period and maximum speed.

A-2. A particle having mass 10 g oscillates according to the equation x = (2.0 cm) sin [(100 s^{-1}) t+ π/6]. Find (a) the amplitude, the time period and the force constant (b) the position, the velocity and the acceleration at t = 0.

A-3. The equation of motion of a particle which started at t = 0 is given by x = 5 sin (20 t + π/3) where x is in centimetre and t in second. When does the particle

(a) first come to rest?

(b) first have zero acceleration?

(c) first have maximum speed ?

A-4. A particle is executing SHM with amplitude A and has maximum velocity v_0. Find its speed when it is located at distance of $\frac{A}{2}$ from mean position.

A-5. At an instant a particle in S.H.M. located at distance 2 cm from mean position, have magnitudes of velocity and acceleration 1m/s and 10 m/s^2 respectively. Find the amplitude and the time period of the motion.

A-6. A particle is executing SHM. Find the positions of the particle where its speed is 8 cm/s, If maximum magnitudes of its velocity and acceleration are 10 cm/s and 50 cm/s^2 respectively.

SECTION (B) : ENERGY

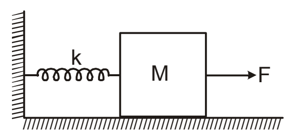
B-1. A particle performing SHM with amplitude 10cm. At What distance from mean position the kinetic energy of the particle is thrice of its potential energy?

B-2. An object of mass 0.2 kg executes simple harmonic oscillations along the x–axis with a frequency of (25 / π) Hz. At the position x = 0.04, the object has kinetic energy of 0.5 J and potential energy 0.4 J. Find the amplitude of oscillations **[1994 ; 2M]**

SECTION (C) : SPRING MASS SYSTEM

C-1. A vertical spring-mass system with lower end of spring is fixed, made to undergo small oscillations. If the spring is stretched by 25cm, energy stored in the spring is 5J .Find the mass of the block if it makes 5 oscillations each second.

C-2. A spring mass system is shown in figure .spring is initially unstretched. A man starts pulling the block with constant force F. Find

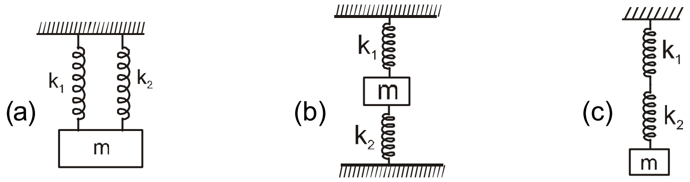


(a) The amplitude and the time period of motion of the block

(b) The K.E. of the block at mean position

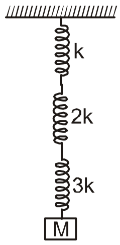
(c) The energy stored in the spring when the block passes through the mean position

C-3. Three spring mass systems are shown in figure. Assuming gravity free space, find the time period of oscillations in each case. What should be the answer if space is not gravity free ?



C-4. A spring of force constant 'k' is cut into two parts whose lengths are in the ratio 1 : 2. The two parts are now connected in parallel and a block of mass 'm' is suspended at the end of the combined spring. Find the period of oscillation performed by the block.

C-5. Spring mass system is shown in figure. Find the elastic potential energy stored in each spring when block is at its mean position. Also find the time period of vertical oscillations. The system is in vertical plane.



SECTION (D) : SIMPLE PENDULUM

D-1. Find the length of seconds pendulum at a place where $g = \pi^2$ m/s^2 .

D-2. Instantaneous angle (in radian) between string of a simple pendulum and vertical is given by $\theta = \frac{\pi}{180}\sin 2\pi t$.

Find the length of the pendulum if $g = \pi^2$ m/s^2

D-3. A pendulum clock is accurate at a place where g = 9.8 m/s^2. Find the value of g at another place where clock becomes slow by 24 seconds in a day (24 hrs).

D-4. A pendulum is suspended in a lift and its period of oscillation is T_0 when the lift is stationary.

(i) What will be the period T of oscillation of pendulum, if the lift begins to accelerate downwards with an acceleration equal to $\frac{3g}{4}$?

(ii) What must be the acceleration of the lift for the period of oscillation of the pendulum to be $\frac{T_0}{2}$?

D-5. A simple pendulum of length ℓ is suspended through the ceiling of a lift. Find its time period if lift is (a) at rest (b) moving with uniform velocity (c) going up with acceleration a_0 (d) Going down with acceleration a_0 [where $a_0 < g$]

D-6. A 0.1kg ball is attached to a string 1.2m long and suspended as a simple pendulum. At a point 0.2 m below the point of suspension a peg is placed, which the string hits when the pendulum comes down. If the mass is pulled a small distance to one side and released what will be the time period of the motion.

(g = 10 m/sec^2)

SECTION (E) : COMPOUND PENDULUM & TORSIONAL PENDULUM

E-1. Compound pendulums are made of :

(a) A rod of length ℓ suspended through a point located at distance $\ell/4$ from centre of rod.

(b) A ring of mass m and radius r suspended through a point on its periphery.

(c) A uniform square plate of edge a suspended through a corner.

(d) A uniform disc of mass m and radius r suspended through a point r/2 away from the centre.

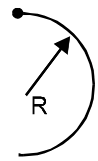
Find the time period in each case.

E-2. Two compound pendulums are made of :

(a) A disc of radius r and

(b) A uniform rod of length L. Find the minimum possible time period and distance between centre and point of suspension in each case.

E-3. A half ring of mass m, radius R is hanged at its one end in vertical plane and is free to oscillate in its plane. Find frequency for small oscillations about mean position of the half ring.



SECTION (F) : SUPERPOSITION OF SHM

F-1. A particle is subjected to two SHM's simultaneously

$X_1 = a_1 \text{Sin}\omega t$ and $X_2 = a_2 \text{Sin}(\omega t + \phi)$

Where $a_1 = 3.0$ cm , $a_2 = 4.0$ cm.

Find Resultant amplitude if the phase difference ϕ has values (a) 0^0 (b) 60^0 (c) 90^0

F-2. A particle is subjected to three SHM's in same direction simultaneously each having equal amplitude a and equal time period. The phase of the second motion is 30^0 ahead of the first and the phase of the third motion is 30^0 ahead of the second. Find the amplitude of the resultant motion.

F-3. A particle simultaneously participates in two mutually perpendicular oscillations $x = \sin \pi t$ & $y = 2\cos 2\pi t$. Write the equation of trajectory of the particle.

SECTION (G) : FOR JEE-MAIN

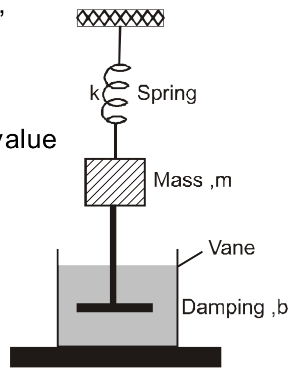
G-1. In forced oscillation of a particle, the amplitude is maximum for a frequency ω_1 of the force, while the energy is maximum for a frequency ω_2 of the force. What is the relation between ω_1 and ω_2 ?

G-2. For the damped oscillator shown in Fig , the mass of the block is 200 g, k = 80 N m^{-1} and the damping constant b is 40 g s^{-1}. Calculate

(a) The period of oscillation,

(b) Time taken for its amplitude of virbrations to drop to half of its initial value

(c) The time for the mechanical energy to drop to half initial value.



PART - II : OBJECTIVE QUESTIONS

*** Marked Questions may have one or more than one correct option.**

SECTION (A) : EQUATION OF SHM

A-1. According to a scientists, he applied a force $F = -cx^{1/3}$ on a particle and the particle is performing SHM. No other force acted on the particle. He refuses to tell whether c is a constant or not. Assume that he had worked only with positive x then :

(A) as x increases c also increases
(B) as x increases c decreases
(C) as x increases c remains constant
(D) the motion cannot be SHM

A-2. A particle performing SHM takes time equal to T (time period of SHM) in consecutive appearances at a perticular point. This point is :

(A) An extreme position
(B) The mean position
(C) Between positive extreme and mean position
(D) Between negative extreme and mean position

A-3. A particle executing linear SHM. Its time period is equal to the smallest time interval in which particle acquires a particular velocity $\vec{v}$, the magnitude of $\vec{v}$ may be :

(A) Zero (B) V_{max} (C) $\frac{V_{max}}{2}$ (D) $\frac{V_{max}}{\sqrt{2}}$

A-4. Two SHM's are represented by $y = a \sin(\omega t - kx)$ and $y = b \cos(\omega t - kx)$. The phase difference between the two is :

(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{6}$ (D) $\frac{3\pi}{4}$

A-5. How long after the beginning of motion is the displacement of a harmonically oscillating particle equal to one half its amplitude if the period is 24s and particle starts from rest.

(A) 12s (B) 2s (C) 4s (D) 6s

A-6. A particle is made to under go simple harmonic motion. Find its average acceleration in one time period.

(A) $\omega^2 A$ (B) $\frac{\omega^2 A}{2}$ (C) $\frac{\omega^2 A}{\sqrt{2}}$ (D) zero

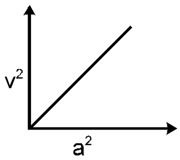
A-7. A particle performing SHM on the y axis according to equation $y = A + B \sin\omega t$. Its amplitude is :

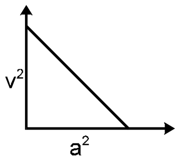
(A) A (B) B (C) A + B (D) $\sqrt{A^2 + B^2}$

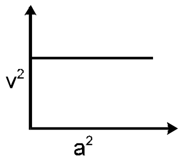
A-8. Two particles execute S.H.M. of same amplitude and frequency along the same straight line from same mean position. They cross one another without collision, when going in opposite directions, each time their displacement is half of their amplitude. The phase-difference between them is

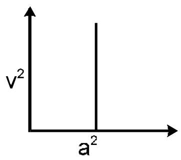
(A) 0° (B) 120° (C) 180° (D) 135°

A-9. A mass M is performing linear simple harmonic motion, then correct graph for acceleration a and corresponding linear velocity v is

(A) 

(B) 

(C) 

(D) 

SECTION (B) : ENERGY

B-1. A body executing SHM passes through its equilibrium. At this instant, it has

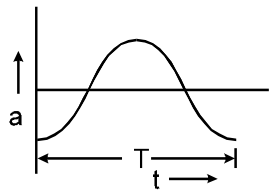
(A) maximum potential energy (B) maximum kinetic energy

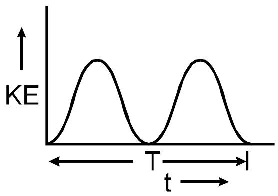
(C) minimum kinetic energy (D) maximum acceleration

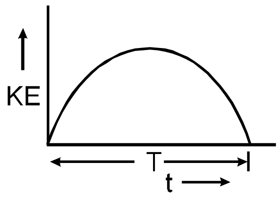
B-2. The K.E. and P.E of a particle executing SHM with amplitude A will be equal when its displacement is

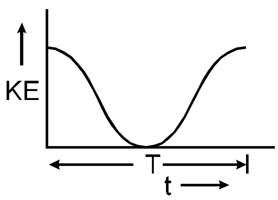
(A) $\sqrt{2}A$ (B) $\frac{A}{2}$ (C) $\frac{A}{\sqrt{2}}$ (D) $\sqrt{\frac{2}{3}}A$

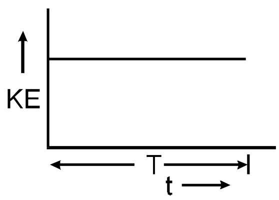
B-3. Acceleration a versus time t graph of a body in SHM is given by a curve shown below. T is the time period. Then corresponding graph between kinetic energy KE and time t is correctly represented by



(A) 

(B) 

(C) 

(D) 

B-4. A particle of mass m oscillating with amplitude A and angular frequency ω. its average energy in one time period is ?

(A) $\frac{1}{2}m\omega^2A^2$ (B) $\frac{1}{4}m\omega^2A^2$ (C) $m\omega^2A^2$ (D) zero

B-5. In SHM particle oscillates with frequency υ then find the frequency of oscillation of its kinetic energy.

(A) υ (B) $\upsilon/2$ (C) 2υ (D) zero

B-6. A body is executing simple harmonic motion. At a displacement x, its potential energy is E_1 and at a displacement y, its potential energy is E_2. The potential energy E at a displacement (x + y) is

(A) $E_1 + E_2$ (B) $\sqrt{E_1^2 + E_2^2}$ (C) $E_1 + E_2 + 2\sqrt{E_1 E_2}$ (D) $\sqrt{E_1 E_2}$

B-7. A particle performs S.H.M. of amplitude A along a straight line. When it is at a distance $\frac{\sqrt{3}}{2}$ A from mean position, its kinetic energy gets increased by an amount $\frac{1}{2}$ m ω^2 A^2 due to an impulsive force. Then its new amplitude becomes:

(A) $\frac{\sqrt{5}}{2}$ A (B) $\frac{\sqrt{3}}{2}$ A (C) $\sqrt{2}$ A (D) $\sqrt{5}$ A

SECTION (C) : SPRING MASS SYSTEM

C-1. Two spring mass systems having equal mass and spring constant k_1 and k_2. If the maximum velocities in two systems are equal then ratio of amplitude of 1st to that of 2nd is :

(A) $\sqrt{k_1/k_2}$ (B) k_1/k_2 (C) k_2/k_1 (D) $\sqrt{k_2/k_1}$

C-2. A toy car of mass m is having two similar rubber ribbons attached to it as shown in the figure. The force constant of each rubber ribbon is k and surface is frictionless. The car is displaced from mean position by x cm and released. At the mean position the ribbons are undeformed. Vibration period is

(A) $2\pi\sqrt{\frac{m(2k)}{k^2}}$ (B) $\frac{1}{2\pi}\sqrt{\frac{m(2k)}{k^2}}$ (C) $2\pi\sqrt{\frac{m}{k}}$ (D) $2\pi\sqrt{\frac{m}{k+k}}$

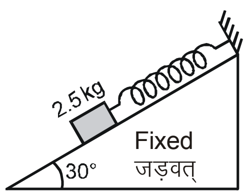
C-3. A mass of 1 kg attached to the bottom of a spring has a certain frequency of vibration. The following mass has to be added to it in order to reduce the frequency by half : **[REE – 1988]**

(A) 1 kg (B) 2 kg (C) 3 kg (D) 4 kg

C-4. Frequency of a block in spring mass system is υ. If it is taken in a lift moving with constant acceleration upward, then frequency will :

(A) decrease (B) increase (C) remain constant (D) none

C-5. A smooth inclined plane having angle of inclination 30° with horizontal has a mass 2.5 kg held by a spring which is fixed at the upper end as shown in figure. If the mass is taken 2.5 cm up along the surface of the inclined plane, the tension in the spring reduces to zero. If the mass is then released, the angular frequency of oscillation in radian per second is

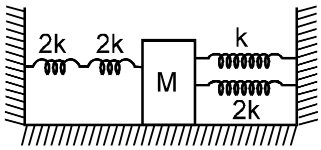


(A) 0.707 (B) 7.07 (C) 1.414 (D) 14.14

C-6. A particle executes simple harmonic motion under the restoring force provided by a spring. The time period is T. If the spring is divided in two equal parts and one part is used to continue the simple harmonic motion, the time period will

(A) remain T (B) become 2T (C) become T/2 (D) become $T/\sqrt{2}$

C-7. Four massless springs whose force constants are 2k, 2k, k and 2k respectively are attached to a mass M kept on a frictionless plane (as shown in figure). If the mass M is displaced in the horizontal direction, then the frequency of the system. **[JEE 1990]**



(A) $\frac{1}{2\pi}\sqrt{\frac{k}{4M}}$ (B) $\frac{1}{2\pi}\sqrt{\frac{4k}{M}}$ (C) $\frac{1}{2\pi}\sqrt{\frac{k}{7M}}$ (D) $\frac{1}{2\pi}\sqrt{\frac{7k}{M}}$

C-8. The total mechanical energy of a particle of mass m executing SHM with the help of a spring is $E = (1/2)m\omega^2A^2$. If the particle is replaced by another particle of mass m/2 while the amplitude A remains same. New mechanical energy will be :

(A) $\sqrt{2}$ E (B) 2E (C) E/2 (D) E

SECTION (D) : SIMPLE PENDULUM

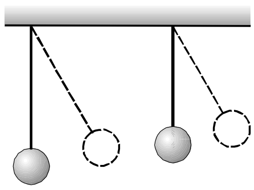
D-1. A pendulum clock that keeps correct time on the earth is taken to the moon. It will run

(A) at correct rate (B) 6 times faster (C) $\sqrt{6}$ times faster (D) $\sqrt{6}$ times slower

D-2. Two pendulums begin to swing simultaneously. The first pendulum makes 9 full oscillations when the other makes 7. Find the ratio of length of the two pendulums.

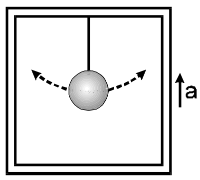
(A) $\frac{49}{81}$ (B) $\frac{7}{9}$ (C) $\frac{50}{81}$ (D) $\frac{1}{2}$

D-3. Two pendulums at rest start swinging together. Their lengths are respectively 1.44 m and 1 m. They will again start swinging in same phase together after (of longer pendulum) :



(A) 1 vibration (B) 3 vibrations (C) 4 vibrations (D) 5 vibrations

D-4. A scientist measures the time period of a simple pendulum as T in a lift at rest. If the lift moves up with acceleration as one fourth of the acceleration of gravity, the new time period is



(A) $\frac{T}{4}$ (B) 4T (C) $\frac{2}{\sqrt{5}}T$ (D) $\frac{\sqrt{5}}{2}T$

D-5. A simple pendulum has some time period T. What will be the percentage change in its time period if its amplitude is decreased by 5%?

(A) 6 % (B) 3 % (C) 1.5 % (D) 0 %

SECTION (E) : COMPOUND PENDULUM & TORSIONAL PENDULUM

E-1. A 25 kg uniform solid sphere with a 20 cm radius is suspended by a vertical wire such that the point of suspension is vertically above the centre of the sphere. A torque of 0.10 N-m is required to rotate the sphere through an angle of 1.0 rad and then the orientation is maintained. If the sphere is then released, its time period of the oscillation will be :

(A) π second (B) $\sqrt{2}\pi$ second (C) 2π second (D) 4π second

E-2. A metre stick swinging about its one end oscillates with frequency f_0. If the bottom half of the stick was cut off, then its new oscillation frequency will be :

(A) f_0 (B) $\sqrt{2}\, f_0$ (C) $2f_0$ (D) $2\sqrt{2}\, f_0$

SECTION (F) : SUPERPOSITION OF SHM

F-1. When two mutually perpendicular simple harmonic motions of same frequency, amplitude and phase are superimposed

(A) the resulting motion is uniform circular motion.

(B) the resulting motion is a linear simple harmonic motion along a straight line inclined equally to the straight lines of motion of component ones.

(C) the resulting motion is an elliptical motion, symmetrical about the lines of motion of the components.

(D) the two S.H.M. will cancel each other.

F-2. The position of a particle in motion is given by $y = C\sin\omega t + D\cos\omega t$ w.r.t. origin. Then motion of the particle is:

(A) SHM with amplitude C+D (B) SHM with amplitude $\sqrt{C^2 + D^2}$

(C) SHM with amplitude $\dfrac{(C+D)}{2}$ (D) not SHM

F-3. A simple harmonic motion is given by $y = 5\,(\sin 3\pi t + \sqrt{3}\cos 3\pi t)$. What is the amplitude of motion if y is in m ?

(A) 100 cm (B) 5 m (C) 200 cm (D) 1000 cm

F-4. The position vector of a particle moving in x-y plane is given by

$\vec{r} = (A\sin\omega t)\,\hat{i} + (A\cos\omega t)\,\hat{j}$ then motion of the particle is :

(A) SHM (B) on a circle (C) on a straight line (D) with constant acceleration

F-5. The displacement of a particle executing periodic motion is given by $y = 4\cos^2(0.5t)\sin(1000\,t)$. The given expression is composed by minimum :

(A) four SHMs (B) three SHMs (C) one SHM (D) None of these

SECTION (G) : FOR JEE MAIN

G-1. When an oscillator completes 100 oscillation its ampliutde reduced to $\frac{1}{3}$ of initial value. What will be its amplitude, when it completes 200 oscillation :

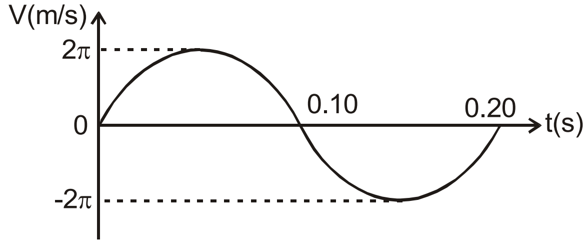
(1) $\frac{1}{8}$ (2) $\frac{2}{3}$ (3) $\frac{1}{6}$ (4) $\frac{1}{9}$

G-2. The damping force on an oscillator is directly proportional to the velocity. The units of the constant of proportionality are :
[AIPMT_Pre_2012]

(1) $kgms^{-1}$ (2) $kgms^{-2}$ (3) kgs^{-1} (4) kgs

PART - III : MATCH THE COLUMN

1. A simple harmonic oscillator consists of a block attached to a spring with k = 200 N/m. The block slides on a frictionless horizontal surface, with equilibrium point x = 0. A graph of the block's velocity v as a function of time t is shown. Correctly match the required information in the left column with the values given in the right column. (use $\pi^2 = 10$)



	Left Column		**Right Column**
(A)	The block's mass in kg	(p)	– 0.20
(B)	The block's displacement at t = 0 in metres	(q)	– 200
(C)	The block's acceleration at t = 0.10 s in m/s^2	(r)	0.20
(D)	The block's maximum kinetic energy in Joule	(s)	4.0

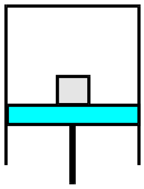
2. In the column-I, a system is described in each option and corresponding time period is given in the column-II. Suitably match them.

Column-I	**Column-II**
(A) A simple pendulum of length 'ℓ' oscillating with small amplitude in a lift moving down with retardation g/2.	(p) $T = 2\pi\sqrt{\frac{2\ell}{3g}}$
(B) A block attached to an end of a vertical spring, whose other end is fixed to the ceiling of a stationary lift, stretches the spring by length 'ℓ' in equilibrium. It's time period when lift moves up with an acceleration g/2 is	(q) $T = 2\pi\sqrt{\frac{\ell}{g}}$
(C) The time period of small oscillation of a uniform rod of length 'ℓ' smoothly hinged at one end. The rod oscillates in vertical plane.	(r) $T = 2\pi\sqrt{\frac{2\ell}{g}}$
(D) A cubical block of edge 'ℓ' and specific gravity 1/2 is in equilibrium with some volume inside water filled in a large fixed container. Neglect viscous forces and surface tension. The time period of small oscillations of the block in vertical direction is	(s) $T = 2\pi\sqrt{\frac{\ell}{2g}}$

Exercise # 2

PART - I : ONLY ONE OPTION CORRECT TYPE

1. A block of mass m is resting on a piston as shown in figure which is moving vertically with a SHM of period 1 s. The minimum amplitude of motion at which the block and piston separate is :

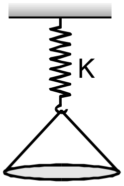


(A) 0.25 m (B) 0.52 m (C) 2.5 m (D) 0.15 m

2. The potential energy of a particle of mass 'm' situated in a unidimensional potential field varies as $U(x) = U_0 [1 - \cos ax]$, where U_0 and a are constants. The time period of small oscillations of the particle about the mean position–

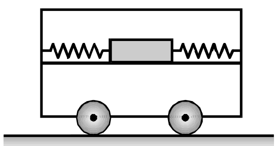
(A) $2\pi\sqrt{\frac{m}{aU_0}}$ (B) $2\pi\sqrt{\frac{am}{U_0}}$ (C) $2\pi\sqrt{\frac{m}{a^2U_0}}$ (D) $2\pi\sqrt{\frac{a^2m}{U_0}}$

3. A solid ball of mass m is made to fall from a height H on a pan suspended through a spring of spring constant K as shown in figure. If the ball does not rebound and the pan is massless, then amplitude of oscillation is



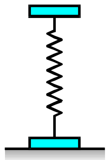
(A) $\frac{mg}{K}$ (B) $\frac{mg}{k}\left(1+\frac{2HK}{mg}\right)^{1/2}$ (C) $\frac{mg}{K}+\left(\frac{2HK}{mg}\right)^{1/2}$ (D) $\frac{mg}{K}\left[1+\left(1+\frac{2HK}{mg}\right)^{1/2}\right]$

4. Two springs, each of spring constant k, are attached to a block of mass m as shown in the figure. The block can slide smoothly along a horizontal platform clamped to the opposite walls of the trolley of mass M. If the block is displaced by x cm and released, the period of oscillation is :



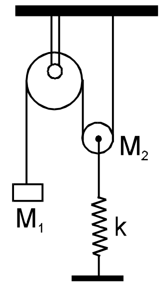
(A) $T = 2\pi\sqrt{\frac{Mm}{2k}}$ (B) $T = 2\pi\sqrt{\frac{(M+m)}{kmM}}$ (C) $T = 2\pi\sqrt{\frac{mM}{2k(M+m)}}$ (D) $T = 2\pi\frac{(M+m)^2}{k}$

5. Two plates of same mass are attached rigidly to the two ends of a spring as shown in figure. One of the plates rests on a horizontal surface and the other results a compression y of the spring when it is in equilibrium state. The further minimum compression required, so that when the force causing compression is removed the lower plate is lifted off the surface, will be :



(A) 0.5 y (B) 3y (C) 2y (D) y

6. What would be the period of the free oscillations of the system shown here if mass M_1 is pulled down a little force constant of the spring is k, mass of fixed pulley is negligible and movable pulley is smooth



(A) $T = 2\pi\sqrt{\frac{M_1 + M_2}{k}}$ (B) $T = 2\pi\sqrt{\frac{M_1 + 4M_2}{k}}$

(C) $T = 2\pi\sqrt{\frac{M_2 + 4M_1}{k}}$ (D) $T = 2\pi\sqrt{\frac{M_2 + 3M_1}{k}}$

7. The bob in a simple pendulum of length ℓ is released at t = 0 from the position of small angular displacement θ_0. Linear displacement of the bob at any time t from the mean position is given by

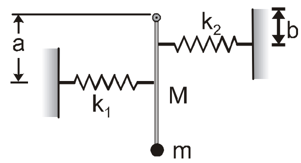
(A) $\ell\theta_0 \cos\sqrt{\frac{g}{\ell}}\, t$ (B) $\ell\sqrt{\frac{g}{\ell}}\, t\cos\theta_0$ (C) $\ell g \sin\theta_0$ (D) $\ell\theta_0 \sin\sqrt{\frac{g}{\ell}}\, t$

8. The period of small oscillations of a simple pendulum of length ℓ if its point of suspension O moves a with a constant acceleration $\alpha = \alpha_1\hat{i} - \alpha_2\hat{j}$ with respect to earth is ($\hat{i}$ and $\hat{j}$ are unit vectors in horizontal and vertically upward directions respectively)

(A) $T = 2\pi\sqrt{\frac{\ell}{\{(g-\alpha_2)^2 + \alpha_1^2\}^{1/2}}}$ (B) $T = 2\pi\sqrt{\frac{\ell}{\{(g-\alpha_1)^2 + \alpha_2^2\}^{1/2}}}$

(C) $T = 2\pi\sqrt{\frac{\ell}{g}}$ (D) $T = 2\pi\sqrt{\frac{\ell}{(g^2 + \alpha_1^2)^{1/2}}}$

9. A rod of mass M and length L is hinged at its one end and carries a particle of mass m at its lower end. A spring of force constant k_1 is installed at distance a from the hinge and another of force constant k_2 at a distance b as shown in the figure. If the whole arrangement rests on a smooth horizontal table top, the frequency of vibration is



(A) $\frac{1}{2\pi}\sqrt{\frac{k_1a^2 + k_2b^2}{L^2(m + \frac{M}{3})}}$ (B) $\frac{1}{2\pi}\sqrt{\frac{k_2 + k_1}{M + m}}$

(C) $\frac{1}{2\pi}\sqrt{\frac{k_2 + k_1\frac{a^2}{b^2}}{4\frac{M}{3} + m}}$ (D) $\frac{1}{2\pi}\sqrt{\frac{k_1 + \frac{k_2b^2}{a^2}}{\frac{4}{3}m + M}}$

10. A simple pendulum ; a physical pendulum; a torsional pendulum and a spring–mass system, each of same frequency are taken to the Moon. If frequencies are measured on the moon, which system or systems will have it unchanged ?

(A) spring–mass system and torsional pendulum.

(B) only spring–mass system.

(C) spring–mass system and physical pendulum.

(D) None of these

11. A particle moves along the X-axis according to the equation $x = 10 \sin^3 (\pi t)$. The amplitudes and frequencies of component SHMs are

(A) amplitude 30/4, 10/4 ; frequencies 3/2, 1/2

(B) amplitude 30/4, 10/4 ; frequencies 1/2, 3/2

(C) amplitude 10, 10 ; frequencies 1/2, 1/2

(D) amplitude 30/4, 10 ; frequencies 3/2, 2

PART - II : INTEGER TYPE QUESTIONS

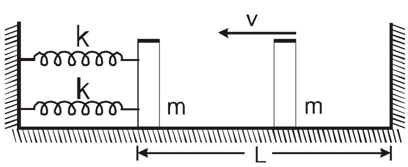
1. Two particles A and B are performing SHM along x and y-axis respectively with equal amplitude and frequency of 2 cm and 1 Hz respectively. Equilibrium positions of the particles A and B are at the co-ordinates (3, 0) and (0, 4) respectively. At t = 0, B is at its equilibrium position and moving towards the origin, while A is nearest to the origin and moving away from the origin. If the maximum and minimum distances between A and B is S_1 and S_2. Find $S_1 + S_2$ (in cm).

2. Two particles P and Q describe S.H.M. of same amplitude a, same frequency f along the same straight line from the same mean position. The maximum distance between the two particles is $a\sqrt{2}$. If the initial phase difference between the particles is $\frac{\pi}{N}$ then find N:

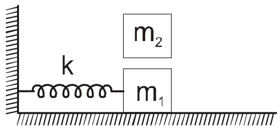
3. A street car moves rectilinearly from station A (here car stops) to the next station B (here also car stops) with an acceleration varying according to the law f = a - bx, where a and b are positive constants and x is the distance from station A. If the maximum distance between the two stations is $x = \frac{Na}{b}$. Then find N.

4. A particle is oscillating in a straight line about a centre O, with a force directed towards O. When at a distance 'x' from O, the force is mn^2x where 'm' is the mass and 'n' is a constant. The amplitude is a = 15 cm. When at a distance $\sqrt{3}\,\frac{a}{2}$ from O the particle receives a blow in the direction of motion which generates an extra velocity na. If the velocity is away from O at the time of blow and the new amplitude becomes $K\sqrt{3}$ cm. The find K.

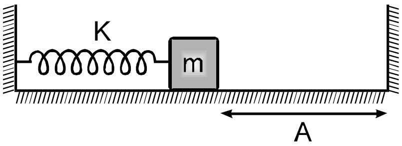
5. The right block in figure moves at a speed V towards the left block placed in equilibrium. All the surfaces are smooth and all the collisions are elastic. If the time period of periodic motion is $\left(\pi\sqrt{\frac{m}{xk}} + \frac{yL}{v}\right)$. Then find x + y. (Neglect the width of the blocks).



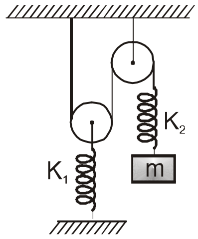
6. A block of mass 4kg attached with spring of spring constant 100 N/m executing SHM of amplitude 0.1m on smooth horizontal surface as shown in figure. If another block of mass 5 kg is gently placed on it, at the instant it passes through the mean position and new amplitude of motion is n^{-1} meter. Then find n. (assuming that two blocks always move together)



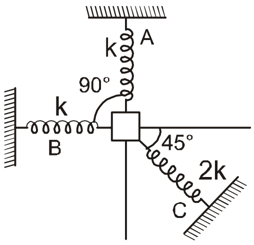
7. In the figure shown the spring is relaxed and mass m is attached to the spring. The spring is compressed by 2 A and released at t=0. Mass m collides with the wall and loses two third of its kinetic energy and returns. Starting from t=0,find time taken (in sec.) by it to come back to rest again (instant at which spring is again under maximum compression). Take $\sqrt{\frac{m}{k}} = \frac{12}{\pi}$



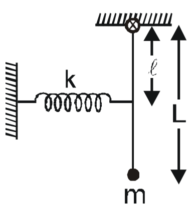
8. For given spring mass system, if the time period of small oscillations of block about its mean position is $\pi\sqrt{\frac{nm}{k}}$. Then find n. Assume ideal conditions. The system is in vertical plane and take $K_1 = 2K$, $K_2 = K$.



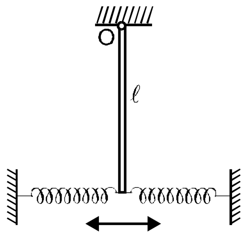
9. A block of mass m is attached to three springs A,B and C having force constants k, k and 2k respectively as shown in figure. If the block is slightly pushed against spring C. If the frequency of oscillations $\sqrt{\frac{Nk}{m}}$. Then find N. The system is placed on horizontal smooth surface.



10. A particle of mass m is suspended at the lower end of a thin rod of negligible mass. The upper end of the rod is free to rotate in the plane of the page about a horizontal axis passing through the point O. The spring is undeformed when the rod is vertical as shown in fig. If it is displaced from its mean position then the period of oscillation is $T = 2\pi\sqrt{\left\{\frac{xmL^2}{(k\ell^2 + ymgL)}\right\}}$ .Find x+ y.



11. If the frequency of small oscillations of a thin uniform vertical rod of mass m and length ℓ hinged at the point O (Fig.) is $\sqrt{\frac{n}{l}}$, then find n. The force constant for each spring is K/2 and take K= $\frac{2mg}{\ell}$. The springs are of negligible mass. (g = 10 m/s^2)

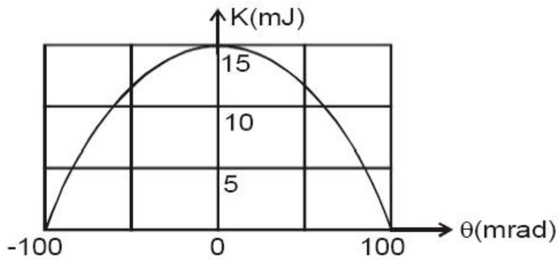


12. A thin uniform plate shaped as an equilateral triangle with a side ℓ performs small oscillations about the horizontal axis coinciding with one of its sides. If the oscillation period is T = $\pi\sqrt{\frac{\ell\sqrt{n}}{g}}$. Find n

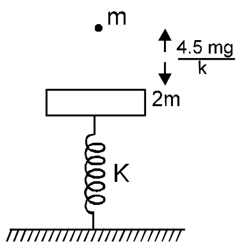
13. The period of oscillation of a simple pendulum of length L suspended from the roof of a vehicle which moves without friction down on inclined plane of inclination $\alpha = 60°$ is given by $\pi\sqrt{\frac{XL}{g}}$ then find X.

[I.I.T. (Scr.) 2000, 1/35]

14. Figure shows the kinetic energy K of a simple pendulum versus its angle θ from the vertical. The pendulum bob has mass 0.2 kg. If the length of the pendulum is equal to $\frac{n}{g}$ meter, then find n (g = 10 m/s^2).



15. In the figure shown a plate of mass 2m is at rest and in equilibrium. A particle of mass m is released from height $\frac{4.5mg}{k}$ from plate. The particle sticks to the plate. Neglecting the duration of collision. Starting from the time when the particle sticks to plate to the time when the spring is in maximum compression for the the first time is $2\pi\sqrt{\frac{m}{ak}}$, then find a.



PART - III : ONE OR MORE THAN ONE CORRECT OPTIONS

1.* A particle moves on the X-axis according to the equation $x = x_0 \sin^2 \omega t$. The motion is simple harmonic

(A) with amplitude $x_0/2$ (B) with amplitude $2x_0$ (C) with time period $\frac{2\pi}{\omega}$ (D) with time period $\frac{\pi}{\omega}$

2.* If y, v and a represent displacement, velocity and acceleration at any instant for a particle executing SHM, which of the following statements are true?

(A) v and y may have same direction

(B) v and a may have same direction

(C) a and y may have same direction

(D) a and v may have opposite direction

3.* Which of the following functions represent SHM?

(A) $\sin 2\omega t$ (B) $\sin^2 \omega t$ (C) $\sin \omega t + 2 \cos \omega t$ (D) $\sin \omega t + \cos 2\omega t$

4.* The time period of a particle in simple harmonic motion is T. A time T/6 after it passes its mean position, its:

(A) velocity will be one half its maximum velocity

(B) displacement will be one half of its amplitude

(C) acceleration will be nearly 86% of its maximum acceleration

(D) KE = PE

5.* For a body executing SHM with amplitude A, time period T, maximum velocity v_{max} and phase constant zero, which of the following statements are correct for $0 \le t \le \frac{T}{4}$ (y is displacement from mean position) ?

(A) At $y = (A/2)$, $v > (v_{max}/2)$ (B) for $v = (v_{max}/2)$, $y > (A/2)$

(C) For $t = (T/8)$, $y > (A/2)$ (D) For $y = (A/2)$, $t < (T/8)$

6.* The speed v of a particle moving along a straight line, when it is at a distance (x) from a fixed point of the line is given by

$$v^2 = 108 - 9x^2 \quad \text{(assuming mean position to have zero phase constant)}$$

(all quantities are in cgs units) :

(A) the motion is uniformly accelerated along the straight line

(B) the magnitude of the acceleration at a distance 3cm from the fixed point is 27 cm/s^2

(C) the motion is simple harmonic about the given fixed point.

(D) the maximum displacement from the fixed point is 4 cm.

7.* The potential energy of a particle of mass 0.1 kg, moving along the x-axis, is given by $U = 5x (x - 4)$ J, where x is in meters. It can be concluded that

(A) the particle is acted upon by a constant force

(B) the speed of the particle is maximum at x = 2 m

(C) the particle executes SHM

(D) the period of oscillation of the particle is $(\pi/5)$ sec

8.* A horizontal plank has a rectangular block placed on it. The plank starts oscillating vertically and simple harmonically with an amplitude of 40 cm. The block just loses contact with the plank when the latter is at momentary rest. Then :

(A) the period of oscillation is $\left(\frac{2\pi}{5}\right)$

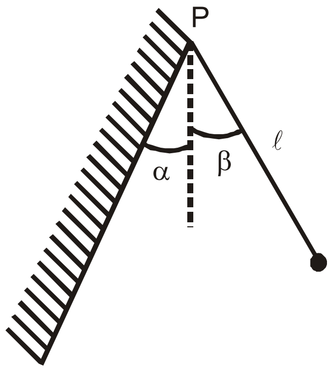
(B) the block weighs double its weight, when the plank is at one of the positions of momentary rest.

(C) the block weighs 0.5 times its weight on the plank halfway up

(D) the block weighs 1.5 times its weight on the plank halfway down

(E) the block weights its true weight on the plank when the latter moves fastest

9.*✍ A ball is hung vertically by a thread of length 'ℓ' from a point 'P' of an inclined wall that makes an angle 'α' with the vertical. The thread with the ball is then deviated through a small angle 'β' ($\beta > \alpha$) and set free. Assuming the wall to be perfectly elastic, the period of such pendulum is/are



(A) $2\sqrt{\frac{\ell}{g}}\left[\sin^{-1}\left(\frac{\alpha}{\beta}\right)\right]$

(B) $2\sqrt{\frac{\ell}{g}}\left[\frac{\pi}{2}+\sin^{-1}\left(\frac{\alpha}{\beta}\right)\right]$

(C) $2\sqrt{\frac{\ell}{g}}\left[\cos^{-1}\left(\frac{\alpha}{\beta}\right)\right]$

(D) $2\sqrt{\frac{\ell}{g}}\left[\cos^{-1}\left(-\frac{\alpha}{\beta}\right)\right]$

10.* The position of a particle at time t moving in x-y plane is given by $\vec{r} = (\hat{i} + 2\hat{j})A\cos\omega t$. Then, the motion of the particle is :

(A) on a straight line (B) on an ellipse (C) periodic (D) SHM

PART - IV : COMPREHENSION

COMPREHENSION - 1

A 2kg block hangs without vibrating at the bottom end of a spring with a force constant of 400 N/m. The top end of the spring is attached to the ceiling of an elevator car. The car is rising with an upward acceleration of 5 m/s^2 when the acceleration suddenly ceases at time t = 0 and the car moves upward with constant speed. (g = 10 m/s^2)

1. What is the angular frequency of oscillation of the block after the acceleration ceases?

(A) $10\sqrt{2}$ rad/s (B) 20 rad/s (C) $20\sqrt{2}$ rad/s (D) 32 rad/s

2. The amplitude of the oscillations is

(A) 7.5 cm (B) 5 cm (C) 2.5 cm (D) 1 cm

3. The initial phase angle observed by an observer in the elevator, taking upward direction to be positive and positive extreme position to have $\pi/2$ phase, is equal to

(A) $-\pi/4$ rad (B) $\pi/2$ rad (C) π rad (D) $3\pi/2$ rad

COMPREHENSION - 2

A particle of mass 'm' moves on a horizontal smooth line AB of length 'a' such that when particle is at any general point P on the line two forces act on it. A force $\frac{mg(AP)}{a}$ towards A and another force $\frac{2mg(BP)}{a}$ towards B.

4. Find its time period when released from rest from mid-point of line AB.

(A) $T = 2\pi\sqrt{\frac{3a}{g}}$ (B) $T = 2\pi\sqrt{\frac{a}{2g}}$ (C) $T = 2\pi\sqrt{\frac{a}{g}}$ (D) $T = 2\pi\sqrt{\frac{a}{3g}}$

5. Find the minimum distance of the particle from B during the motion.

(A) $\frac{a}{6}$ (B) $\frac{a}{4}$ (C) $\frac{a}{3}$ (D) $\frac{a}{8}$

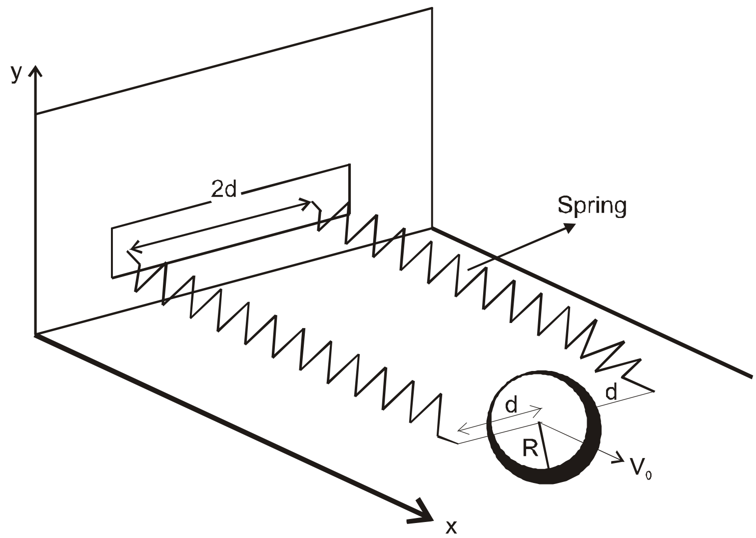
6. If the force acting towards A stops acting when the particle is nearest to B then find the velocity with which it crosses point B.

(A) $\frac{\sqrt{2ga}}{3}$ (B) $\frac{\sqrt{2ga}}{6}$ (C) $\frac{\sqrt{2ga}}{5}$ (D) $\frac{\sqrt{ga}}{3}$

COMPREHENSION - 3

A uniform thin cylindrical disk of mass M and radius R is attached to two identical massless springs of spring constant k which are fixed to the wall as shown in the figure. The springs are attached to the axle of the disk symmetrically on either side at a distance d from its centre. The axle is massless and both the springs and the axle are in a horizontal plane. The unstretched length of each spring is L. The disk is initially at its equilibrium position with its centre of mass (CM) at a distance L from the wall. The disk rolls without slipping with velocity $\vec{V}_0 = V_0\hat{i}$. The coefficient of friction is μ. **[JEE-2008, 3×4/163]**

Figure :



7. The net external force acting on the disk when its centre of mass is at displacement x with respect to its equilibrium position is

(A) $-kx$ (B) $-2kx$ (C) $-\frac{2kx}{3}$ (D) $-\frac{4kx}{3}$

8. The centre of mass of the disk undergoes simple harmonic motion with angular frequency ω equal to

(A) $\sqrt{\frac{k}{M}}$ (B) $\sqrt{\frac{2k}{M}}$ (C) $\sqrt{\frac{2k}{3M}}$ (D) $\sqrt{\frac{4k}{3M}}$

9. The maximum value of V_0 for which the disk will roll without slipping is

(A) $\mu g\sqrt{\frac{M}{k}}$ (B) $\mu g\sqrt{\frac{M}{2k}}$ (C) $\mu g\sqrt{\frac{3M}{k}}$ (D) $\mu g\sqrt{\frac{5M}{2k}}$

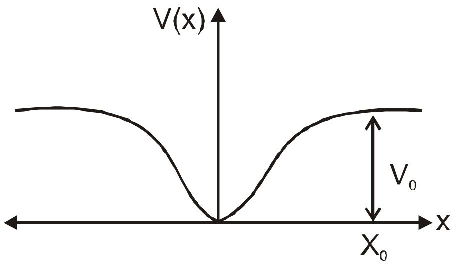
Exercise # 3

PART - I : JEE (ADVANCED) / IIT-JEE PROBLEMS (PREVIOUS YEARS)

*** Marked Questions may have more than one correct option.**

PARAGRAPH FOR QUESTION NOS. 1 TO 3

When a particle of mass m moves on the x-axis in a potential of the form $V(x) = kx^2$, it performs simple harmonic motion. The corresponding time period is proportional to $\sqrt{\frac{m}{k}}$, as can be seen easily using dimensional analysis. However, the motion of a particle can be periodic even when its potential energy increases on both sides of x = 0 in a way different from kx^2 and its total energy is such that the particle does not escape to infinity. Consider a particle of mass m moving on the x-axis. Its potential energy is $V(x) = \alpha x^4$ ($\alpha > 0$) for $|x|$ near the origin and becomes a constant equal to V_0 for $|x| \geq X_0$ (see figure) **[JEE 2010; 9/160, –1]**



1. If the total energy of the particle is E, it will perform periodic motion only if :

(A) $E < 0$ (B) $E > 0$ (C) $V_0 > E > 0$ (D) $E > V_0$

2. For periodic motion of small amplitude A, the time period T of this particle is proportional to :

(A) $A\sqrt{\frac{m}{\alpha}}$ (B) $\frac{1}{A}\sqrt{\frac{m}{\alpha}}$ (C) $A\sqrt{\frac{\alpha}{m}}$ (D) $\frac{1}{A}\sqrt{\frac{\alpha}{m}}$

3. The acceleration of this particle for $|x| > X_0$ is :

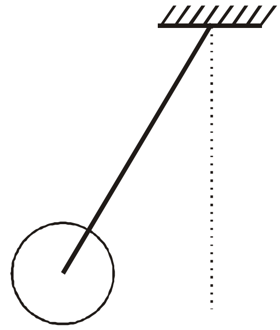
(A) proportional to V_0

(B) proportional to $\frac{V_0}{mX_0}$

(C) proportional to $\sqrt{\frac{V_0}{mX_0}}$

(D) zero

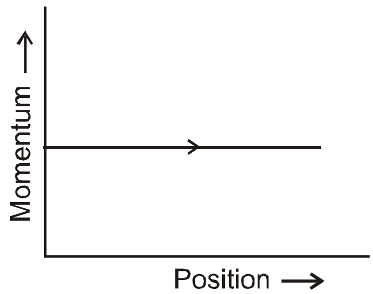
4*. A metal rod of length 'L' and mass 'm' is pivoted at one end. A thin disk of mass 'M' and radius 'R' (<L) is attached at its center to the free end of the rod. Consider two ways the disc is attached: (case A). The disc is not free to rotate about its center and (case B) the disc is free to rotate about its center. The rod-disc system performs SHM in vertical plane after being released from the same displaced position. Which of the following statement(s) is (are) true? **[JEE 2011, 4/160]**



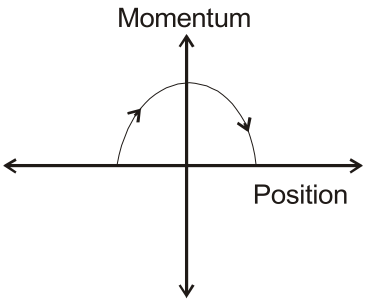
(A) Restoring torque in case A = Restoring torque in case B
(B) Restoring torque in case A **<** Restoring torque in case B
(C) Angular frequency for case A **>** Angular frequency for case B.
(D) Angular frequency for case A < Angular frequency for case B.

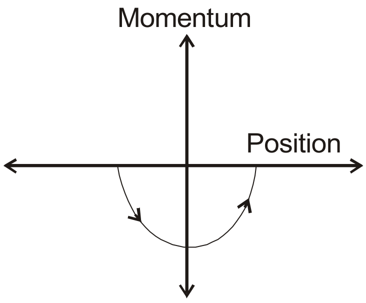
PARAGRAPH FOR QUESTION NOS. 5 TO 7

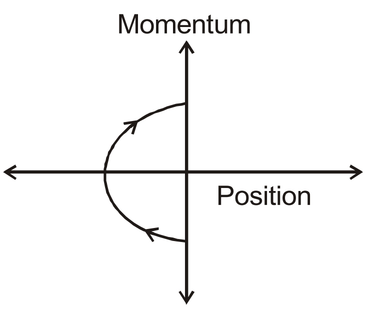
Phase space diagrams are useful tools in analyzing all kinds of dynamical problems. They are especially useful in studying the changes in motion as initial position and momentum are changed. Here we consider some simple dynamical systems in one-dimension. For such systems, phase space is a plane in which position is plotted along horizontal axis and momentum is plotted along vertical axis. The phase space diagram is x(t) vs. p(t) curve in this plane. The arrow on the curve indicates the time flow. For example, the phase space diagram for a particle moving with constant velocity is a straight line as shown in the figure. We use the sign convention in which position or momentum. upwards (or to right) is positive and downwards (or to left) is negative. **[JEE 2011, 3×3/160, –1]**

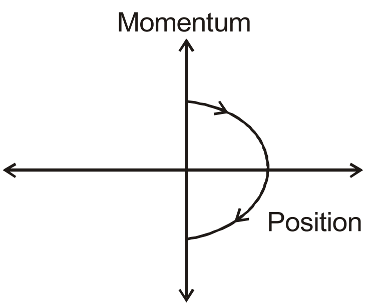


5. The phase space diagram for a ball thrown vertically up from ground is :

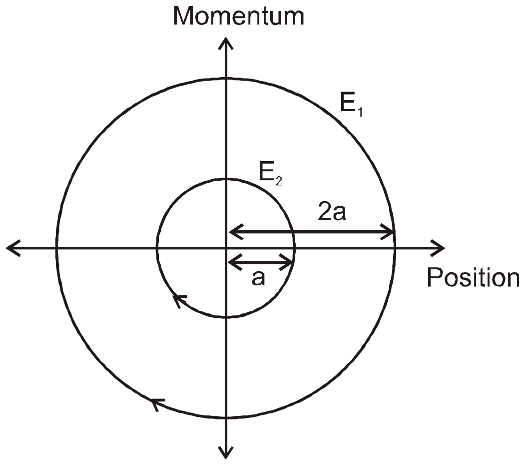
(A)


(B)


(C)


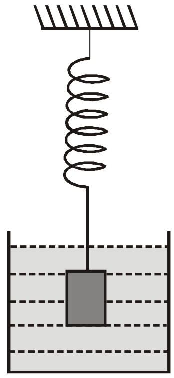
(D)


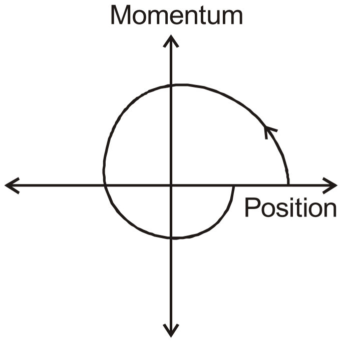
6. The phase space diagram for simple harmonic motion is a circle centered at the origin. In the figure, the two circles represent the same oscillator but for different initial conditions, and E_1 and E_2 are the total mechanical energies respectively. Then

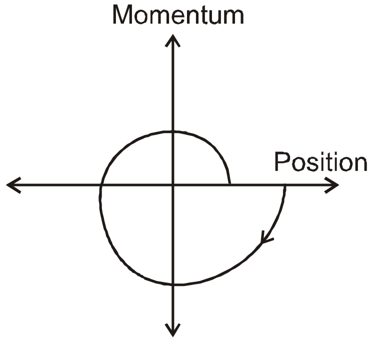


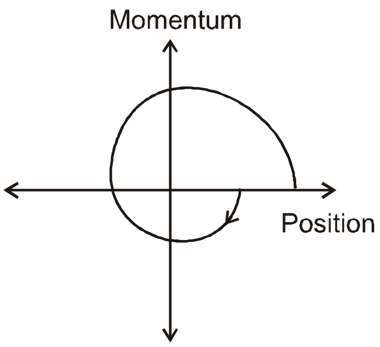
(A) $E_1 = \sqrt{2}\,E_2$ (B) $E_1 = 2\,E_2$ (C) $E_1 = 4\,E_2$ (D) $E_1 = 16\,E_2$

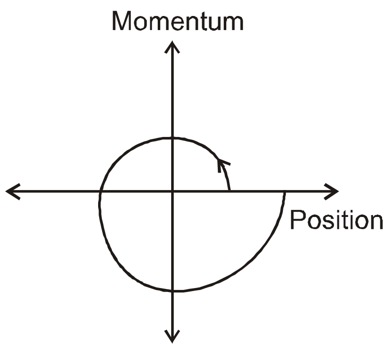
7. Consider the spring-mass system, with the mass submerged in water, as shown in the figure. The phase space diagram for one cycle of this system is **:**



(A)


(B)


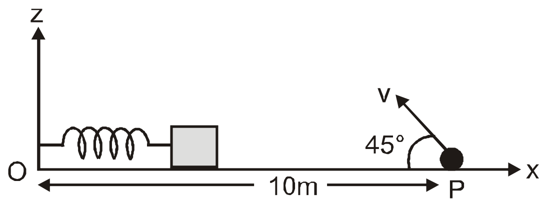
(C)


(D)


8. A point mass is subjected to two simultaneous sinusoidal displacements in x-direction, x_1 (t) = A sin ωt and x_2 (t) = A sin $\left(\omega t + \frac{2\pi}{3}\right)$. Adding a third sinusoidal displacement x_3 (t) = B sin (ωt + ϕ) brings the mass to a complete rest. The values of B and ϕ are **[JEE 2011, 3/160, –1]**

(A) $\sqrt{2}A, \frac{3\pi}{4}$ (B) $A, \frac{4\pi}{3}$ (C) $\sqrt{3}A, \frac{5\pi}{6}$ (D) $A, \frac{\pi}{3}$

9. A small block is connected to one end of a massless spring of un-stretched length 4.9 m. The other end of the spring (see the figure) is fixed. The system lies on a horizontal frictionless surface. The block is stretched by 0.2 m and released from rest at t = 0. It then executes simple harmonic motion with angular frequency $\omega = \frac{\pi}{3} rad/s$. Simultaneously at t = 0, a small pebble is projected with speed v from point P at an angle of 45º as shown in the figure. Point P is at a horizontal distance of 10 cm from O. If the pebble hits the block at t = 1s, the value of v is (take g = 10 m/s²) **[IIT-JEE-2012, Paper-1; 3/70, –1]**



(A) $\sqrt{50}$ m/s (B) $\sqrt{51}$ m/s (C) $\sqrt{52}$ m/s (D) $\sqrt{53}$ m/s

10.* A particle of mass m is attached to one end of a mass–less spring of force constant k, lying on a frictionless horizontal plane. The other end of the spring is fixed. The particle starts moving horizontally from its equilibrium position at time t = 0 with an initial velocity u_0. When the speed of the particle is 0.5 u_0, it collies elastically with a rigid wall. After this collision : **[JEE ADVANCED 2013]**

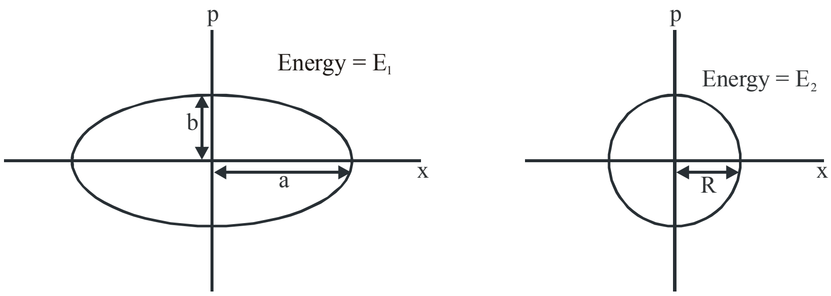
(A) the speed of the particle when it returns to its equilibrium position is u_0 .

(B) the time at which the particle passes through the equilibrium position for the first time is $t = \pi\sqrt{\frac{m}{k}}$.

(C) the time at which the maximum compression of the spring occurs is $t = \frac{4\pi}{3}\sqrt{\frac{m}{k}}$.

(D) the time at which the particle passes througout the equilibrium position for the second time is $t = \frac{5\pi}{3}\sqrt{\frac{m}{k}}$.

11. Two independent harmonic oscillators of equal mass are oscillating about the origin with angular frequencies ω_1 and ω_2 and have total energies E_1 and E_2, respectively. The variations of their momenta p with positions x are shown in the figures. If $\frac{a}{b} = n^2$ and $\frac{a}{R} = n$, then the correct equation (s) is (are) **[JEE-Advanced-2015]**



(A) $E_1\omega_1 = E_2\omega_2$ (B) $\frac{\omega_2}{\omega_1} = n^2$ (C) $\omega_1\omega_2 = n^2$ (D) $\frac{E_1}{\omega_1} = \frac{E_2}{\omega_2}$

12. A particle of unit mass is moving along the x-axis under the influence of a force and its total enegy is conserved. Four possible forms of the potential energy of the particle are given in column I (a and U_0 are constants). Match the potential energies in column I to the corresponding statement(s) in column-**II**

[JEE-Advanced-2015]

Column-I	**Column-II**
(A) $U_1(x) = \frac{U_0}{2}\left[1-\left(\frac{x}{a}\right)^2\right]^2$	(P) The force acting on the particle is zero at x = a.
(B) $U_2(x) = \frac{U_0}{2}\left(\frac{x}{a}\right)^2$	(Q) The force acting on the particle is zero at x = 0.
(C) $U_3(x) = \frac{U_0}{2}\left(\frac{x}{a}\right)^2 \exp\left[-\left(\frac{x}{a}\right)^2\right]$	(R) The force acting on the particle is zero at x =–a.
(D) $U_4(x) = \frac{U_0}{2}\left[\frac{x}{a} - \frac{1}{3}\left(\frac{x}{a}\right)^3\right]$	(S) The particle experiences an attractive force towards x = 0 in the region |x| < a.
	(T) The particle with total enegy $\frac{U_0}{4}$ can oscillate about the point x = –a.

13. A block with mass M is connected by a massless spring with stiffness constant k to a rigid wall and moves without friction on a horizontal surface. The block oscillates with small amplitude A about an equilibrium position x_0. Consider two cases : (i) when the block is at x_0; and (ii) when the block is at $x = x_0 + A$. In both the cases, a particle with mass m (<M) is softly placed on the block after which they stick to each other. Which of the following statement(s) is(are) true about the motion after the mass m is placed on the mass M?

[JEE-Advanced-2016]

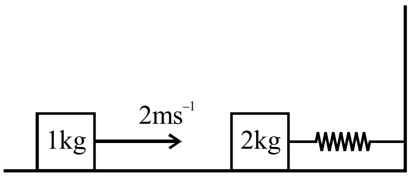
(A) The amplitude of oscillation in the first case changes by a factor of $\sqrt{\frac{M}{m+M}}$, whereas in the second case it remains unchanged

(B) The final time period of oscillation in both the cases is same

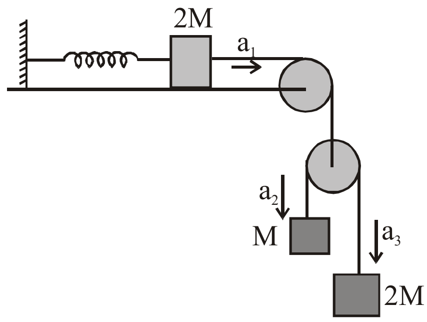
(C) The total energy decreases in both the cases

(D) The instantaneous speed at x_0 of the combined masses decreases in both the cases.

14. A spring-block system is resting on a frictionless floor as shown in the figure. The spring constant is 2.0 N m^{-1} and the mass of the block is 2.0 kg. Ignore the mass of the spring. Initially the spring is in an unstretched condition. Another block of mass 1.0 kg moving with a speed of 2.0 m s^{-1} collides elastically with the first block. The collision is such that the 2.0 kg block does not hit the wall. The distance, in metres, between the two blocks when the spring returns to its unstretched position for the first time after the collision is _____. **[JEE-Advanced-2018]**



15. A block of mass 2M is attached to a massless spring with spring-constant k. This block is connected to two other blocks of masses M and 2M using two massless pulleys and strings. The accelerations of the blocks are a_1, a_2 and a_3 as shown in figure. The system is released from rest with the spring in its unstretched state. The maximum extension of the spring is x_0. Which of the following option(s) is/are correct ? [g is the acceleration due to gravity. Neglect friction] **[JEE-Advanced-2019, P-1]**



(A) $x_0 = \frac{4Mg}{k}$

(B) When spring achieves an extension of $\frac{x_0}{2}$ for the first time, the speed of the block connected to the spring is $3g\sqrt{\frac{M}{5k}}$

(C) $a_2 - a_1 = a_1 - a_3$

(D) At an extension of $\frac{x_0}{4}$ of the spring, the magnitude of acceleration of the block connected to the spring is $\frac{3g}{10}$

PART - II : JEE(MAIN) / AIEEE PROBLEMS (PREVIOUS YEARS)

1. A mass M, attached to a horizontal spring, executes SHM with a amplitude A_1. When the mass M passes through its mean position then a smaller mass m is placed over it and both of them move together with amplitude A_2. The ratio of $\left(\frac{A_1}{A_2}\right)$ is : **[AIEEE - 2011; 4/120, –1]**

(1) $\frac{M}{M+m}$ (2) $\frac{M+m}{M}$ (3) $\left(\frac{M}{M+m}\right)^{1/2}$ (4) $\left(\frac{M+m}{M}\right)^{1/2}$

2. Two particles are executing simple harmonic motion of the same amplitude A and frequency ω along the x-axis. Their mean position is separated by distance X_0 ($X_0 > A$). If the maximum separation between them is (X_0 + A), the phase difference between their motion is : **[AIEEE - 2011, 4/120, –1]**

(1) $\frac{\pi}{2}$ (2) $\frac{\pi}{3}$ (3) $\frac{\pi}{4}$ (4) $\frac{\pi}{6}$

3. If a simple pendulum has significant amplitude (up to a factor of 1/e of original) only in the period between t = 0s to t = τs, then τ may be called the average life of the pendulum. When the spherical bob of the pendulum suffers a retardation (due to viscous drag) proportional to its velocity, with 'b' as the constant of proportionality, the average life time of the pendulum is (assuming damping is small) in seconds :

[AIEEE 2012 ; 4/120, –1]

(1) $\frac{0.693}{b}$ (2) b (3) $\frac{1}{b}$ (4) $\frac{2}{b}$

4. The amplitude of a damped oscillator decreases to 0.9 times its original magnitude is 5s. In another 10s it will decrease to α times its original magnitude, where α equals. **[JEE-Main 2013; 4/120, –1]**

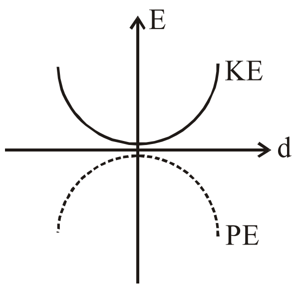
(1) 0.7 (2) 0.81 (3) 0.729 (4) 0.6

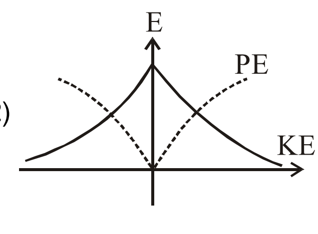
5. A particle moves with simple harmonic motion in a straight line. In first τ s, after starting from rest it travels a distance a, and in next τ s it travels 2a, in same direction, then : **[JEE-Main 2013; 4/120, –1]**

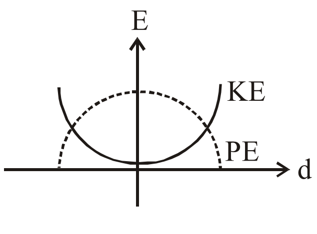
(1) amplitude of motion is 3a (2) time period of oscillations is 8τ

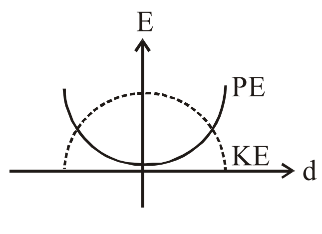
(3) amplitude of motion is 4a (4) time period of oscillations is 6τ

6. For a simple pendulum, a graph is plotted between its kinetic energy (KE) and potential energy (PE) against its displacement d. Which one of the following represents these correctly ? (graphs are schematic and not drawn to scale) **[JEE Main-2015; 4/120, –1]**

(1) 

(2) 

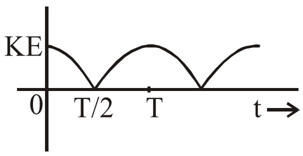
(3) 

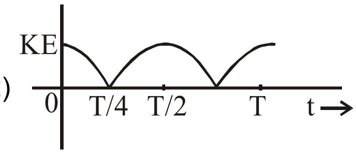
(4) 

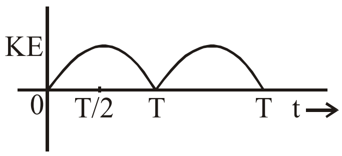
7. A particle performs simple harmonic motion with amplitude A. Its speed is tripled at the instant that it is at a distance $\frac{2A}{3}$ from equilibrium position. The new amplitude of the motion is: **[JEE Main-2016; 4/120, –1]**

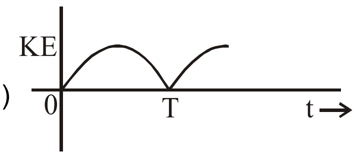
(1) $\frac{7A}{3}$ (2) $\frac{A}{3}\sqrt{41}$ (3) 3A (4) $A\sqrt{3}$

8. A particle is executing simple harmonic motion with a time period T. AT time t = 0, it is at its position of equilibrium. The kinetic energy-time graph of the particle will look like **[JEE Main-2017; 4/120, –1]**

(1) 

(2) 

(3) 

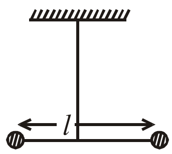
(4) 

9. A silver atom in a solid oscillates in simple harmonic motion in some direction with a frequency of 10^{12}/sec. What is the force constant of the bonds connecting one atom with the other ? (Mole wt. of silver =108 and Avagadro number = 6.02×10^{23} gm mole^{-1}) **[JEE Main 2018; 4/120, –1]**

(1) 7.1 N/m (2) 2.2 N/m (3) 5.5 N/m (4) 6.4 N/m

10. Two masses m and $\frac{m}{2}$ are connected at the two ends of a massless rigid rod of length l. The rod is suspended by a thin wire of torsional constant k at the centre of mass of the rod-mass system(see figure). Because of torsional constant k, the restoring torque is $\tau = k\theta$ for angular displacement 0. If the rod is rotated by θ_0 and released, the tension in it when it passes through its mean position will be:

[JEE Main-2019, Jan; 4/120, –1]

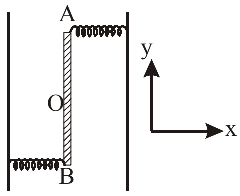


(1) $\frac{3k\theta_0^2}{l}$ (2) $\frac{k\theta_0^2}{2l}$ (3) $\frac{2k\theta_0^2}{l}$ (4) $\frac{k\theta_0^2}{l}$

11. A simple pendulum of length 1 m is oscillating with an angular frequency 10 rad/s. The support of the pendulum starts oscillating up and down with a small angular frequency of 1 rad/s and an amplitude of 10^{-2} m. The relative change in the angular frequency of the pendulum is best given by : **[JEE Main-2019, Jan; 4/120, –1]**

(1) 10^{-3} rad/s (2) 10^{-1} rad/s (3) 1 rad/s (4) 10^{-5} rad/s

12. Two light identical springs of spring constant k are attached horizontally at the two ends of a uniform horizontal rod AB of length ℓ and mass m. The rod is pivoted at its centre 'O' and can rotate freely in horizontal plane. The other ends of the two springs are fixed to rigid supports as shown in figure. The rod is gently pushed through a small angle and released. The frequency of resulting oscillation is: **[JEE Main-2019, Jan; 4/120, –1]**



(1) $\frac{1}{2\pi}\sqrt{\frac{6k}{m}}$ (2) $\frac{1}{2\pi}\sqrt{\frac{2k}{m}}$ (3) $\frac{1}{2\pi}\sqrt{\frac{k}{m}}$ (4) $\frac{1}{2\pi}\sqrt{\frac{3k}{m}}$

13. The displacement of a damped harmonic oscillator is given by $x(t) = e^{-0.1t}\cos(10\pi t + \phi)$. Here t is in seconds. The time taken for its amplitude of vibration to drop to half of its initial value is close to :

[JEE Main-2019, April; 4/120, –1]

(1) 13 s (2) 7 s (3) 27 s (4) 4 s

Answers

Exercise : 1

PART - I

SECTION (A)

A-1. Amplitude = 5 m, Phase constant = $\frac{\pi}{6}$,

Time period = 2 s, Maximum speed = 5π m/s

A-2. (a) 2.0 cm, $\frac{\pi}{50}$ s = 0.063 s, 100 N/m

(b) 1.0 cm, $\sqrt{3}$ m/s, 100 m/s^2

A-3. (a) $\frac{\pi}{120}$ s (b) $\frac{\pi}{30}$ s (c) $\frac{\pi}{30}$ s

A-4. $\frac{\sqrt{3}v_0}{2}$

A-5. $\sqrt{24}$ cm, $\frac{2\pi}{10\sqrt{5}}$ s = 0.28 s

A-6 $\pm\frac{6}{5}$ cm = ± 1.2 cm from the mean position

SECTION (B) :

B-1. ± 5 cm **B-2.** 0.06 m

SECTION (C) :

C-1. $\frac{16}{10\pi^2}$ = 0.16 Kg

C-2. (a) $\frac{F}{k}$, $2\pi\sqrt{\frac{M}{k}}$, (b) $\frac{F^2}{2k}$ (c) $\frac{F^2}{2k}$

C-3. (a) $2\pi\sqrt{\frac{m}{k_1+k_2}}$, $k_{eq.} = k_1 + k_2$;

(b) $2\pi\sqrt{\frac{m}{k_1+k_2}}$, $k_{eq.} = k_1 + k_2$;

(c) $2\pi\sqrt{\frac{m(k_1+k_2)}{k_1k_2}}$, $k_{eq.} = \frac{k_1k_2}{k_1+k_2}$

Answers will remian same

C-4. $T = 2\pi\sqrt{\left(\frac{2m}{9k}\right)}$

C-5. $\frac{M^2g^2}{2k}$, $\frac{M^2g^2}{4k}$ and $\frac{M^2g^2}{6k}$ from above,

time period = $2\pi\sqrt{\frac{11M}{6k}}$

SECTION (D) :

D-1. 1 m **D-2.** 0.25 m

D-3. $\left(\frac{3600}{3601}\right)^2 g = 9.794\ m/s^2$

D-4. (i) $2T_0$ (ii) 3g upwards

D-5. (a) $2\pi\sqrt{\frac{\ell}{g}}$ (b) $2\pi\sqrt{\frac{\ell}{g}}$

(c) $2\pi\sqrt{\frac{\ell}{g+a_0}}$ (d) $2\pi\sqrt{\frac{\ell}{g-a_0}}$

D-6. $\frac{\pi}{\sqrt{g}}(\sqrt{1.2}+1)$ = 2.1 sec.

SECTION (E) :

E-1. (a) $T = 2\pi\sqrt{\frac{7\ell}{12g}}$ (b) $2\pi\sqrt{\frac{2r}{g}}$

(c) $2\pi\sqrt{\frac{\sqrt{8}\,a}{3g}}$ (d) $2\pi\sqrt{\frac{3r}{2g}}$

E-2. (a) $2\pi\sqrt{\frac{r\sqrt{2}}{g}}$, $r/\sqrt{2}$ (b) $2\pi\sqrt{\frac{L}{\sqrt{3}g}}$, $\frac{L}{2\sqrt{3}}$.

E-3. $\frac{1}{2\pi}\sqrt{\frac{g}{2r}}\sqrt{1+\frac{4}{\pi^2}}$

SECTION (F) :

F-1. (a) 7 cm (b) $\sqrt{37}$ cm = 6.1 cm (c) 5 cm

F-2. $a\sqrt{4+2\sqrt{3}}$ **F-3.** $2x^2 + \frac{y}{2} = 1$

SECTION (G) :

G-1. Both amplitude and energy of the particle can be maximum only in the case of resonance. For resonance to occur, $\omega_1 = \omega_2$

G-2. (a) 0.3 s; (b) 6.93 s; (c) 3.4 s

PART - II

SECTION (A) :

A-1.	(A)	**A-2.**	(A)	**A-3.**	(B)
A-4.	(A)	**A-5.**	(C)	**A-6.**	(D)
A-7.	(B)	**A-8.**	(B)	**A-9.**	(B)

SECTION (B) :

B-1.	(B)	**B-2.**	(C)	**B-3.**	(A)
B-4.	(A)	**B-5.**	(C)	**B-6.**	(C)
B-7.	(C)				

SECTION (C) :

C-1.	(D)	**C-2.**	(C)	**C-3.**	(C)
C-4.	(C)	**C-5.**	(D)	**C-6.**	(D)
C-7.	(B)	**C-8.**	(D)		

SECTION (D) :

D-1.	(D)	**D-2.**	(A)	**D-3.**	(D)
D-4.	(C)	**D-5.**	(D)		

SECTION (E) :

E-1.	(D)	**E-2.**	(B)

SECTION (F) :

F-1.	(B)	**F-2.**	(B)	**F-3.**	(D)
F-4.	(B)	**F-5.**	(B)		

SECTION (G) :

G-1.	(4)	**G-2.**	(3)

PART - III

1. (A) $\rightarrow$ r , (B) $\rightarrow$ p , (C) $\rightarrow$ q , (D) $\rightarrow$ s

2. (A) $\rightarrow$ p ; (B) $\rightarrow$ q ; (C) $\rightarrow$ p ; (D) $\rightarrow$ s

Exercise : 2

PART - I

1.	(A)	**2.**	(C)	**3.**	(B)
4.	(C)	**5.**	(C)	**6.**	(C)
7.	(A)	**8.**	(A)	**9.**	(A)
10.	(A)	**11.**	(B)		

PART - II

1.	10	**2.**	2	**3.**	2
4.	15	**5.**	4	**6.**	15
7.	17	**8.**	12	**9.**	3
10.	2	**11.**	75	**12.**	3
13.	8	**14.**	15	**15.**	3

PART - III

1.	(A)(D)	**2.**	(A)(B)(D)
3.	(A)(B)(C)	**4.**	(A)(C)
5.	(A)(B)(C)(D)	**6.**	(B)(C)
7.	(B)(C)(D)	**8.**	(A)(B)(C)(D)(E)
9.	(B)(D)	**10.**	(A)(C)(D)

PART - IV

1.	(A)	**2.**	(C)	**3.**	(D)
4.	(D)	**5.**	(A)	**6.**	(B)
7.	(D)	**8.**	(D)	**9.**	(C)

Exercise : 3

PART - I

1.	(C)	**2.**	(B)	**3.**	(D)
4.	(A)(D)	**5.**	(D)	**6.**	(C)
7.	(B)	**8.**	(B)	**9.**	(A)
10.	(A,D)	**11.**	(B,D)		

12. (A)-P,Q,R,T; (B)-Q,S; (C)-P,Q,R,S; (D)-P,R,T

13.	(A,B,D)	**14.**	2.09 m	**15.**	(C)

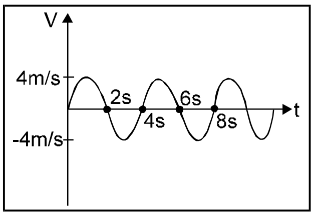
PART - II

1.	(4)	**2.**	(2)	**3.**	(4)
4.	(3)	**5.**	(4)	**6.**	(4)
7.	(1)	**8.**	(2)	**9.**	(1)
10.	(4)	**11.**	(1)	**12.**	(1)
13.	(2)				

RANKER PROBLEMS

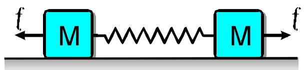
SUBJECTIVE QUESTIONS

1. If velocity of a particle moving along a straight line changes sinusoidally with time as shown in the given graph. Find the average speed over time interval t = 0 to t = 2 (2n – 1) seconds, n being any positive integer.

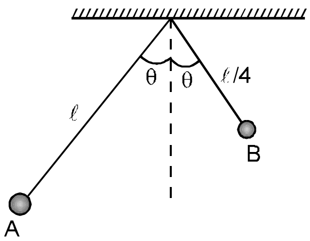


2. Two particles P_1 and P_2 are performing SHM along the same line about the same mean position. Initially they are at their positive extreme positions. If the time period of each particle is 12 sec and the difference of their amplitudes is 12 cm then find the minimum time after which the separation between the particles become 6 cm.

3. A spring of force constant α has two blocks of same mass M connected to each end of the spring as shown in figure. Same force f extends each end of the spring. If the masses are released, then period of vibration is:



4. Two simple pendulums A and B having lengths ℓ and $\ell/4$ respectively are released from the position as shown in figure. Calculate the time after which the release of the two strings become parallel for the first time. Angle θ is very small.



5. A circular ring hung on a nail in a wall, performs oscillations of angular amplitude 4^0 and time period 4 seconds. Then find the :

(a) radius of the circular ring

(b) speed and acceleration of the particle farthest away from the point of suspension as it passes through mean position

(c) acceleration of this particle when it is at an extreme position

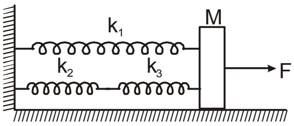
(take g = 10 m/s^2 and $\pi^2 = 10$)

6. A car is moving on a horizontal circular road of radius r with constant speed v. A simple pendulum of length ℓ is is suspended from the ceiling of the car. Find :

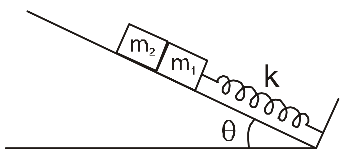
(a) the tension in string when pendulum is always stationary w.r.t the car. Mass of the bob is M.

(b) Find frequency of small oscillations

7. The springs shown in the figure are all unstretched in the beginning when a man starts pulling the block. The man exerts a constant force F on the block. Find the amplitude and the frequency of the motion of the block.



8. Spring of spring constant k is attached with a block of mass m_1 as shown in figure. Another block of mass m_2 placed against m_1



(a) at equilibrium find the compression in the spring

(b) the blocks are pushed a further distance $(2/k(m_1+m_2)g\sin\theta)$ against the spring and released. Find the position where the two blocks separate.

(c) find the common speed of block when they separate.

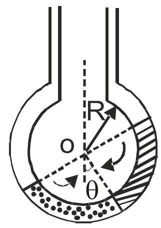
9. A particle of mass 'm' is moving in the x-y plane such that its x and y coordinate vary according to the law $x = a \sin \omega t$ and $y = a \cos \omega t$ where 'a' and 'ω' are positive constants and 't' is time. Find

(a) equation of the path. Name the trajectory (path)

(b) whether the particle moves in clockwise or anticlockwise direction

(c) magnitude of the force on the particle at any time t.

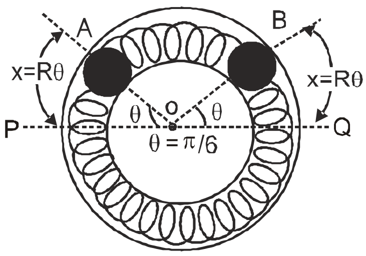
10. Two non–viscous, incompressible and immiscible liquids of densities ρ and 1.5 ρ are poured into the two limbs of a circular tube of radius R and small cross–section kept fixed in a vertical plane as shown in fig. Each liquid occupies one–fourth the circumference of the tube. **(1991 ; 4 + 4 M)**



(a) Find the angle θ that the radius to the interface makes with the vertical in equilibrium position.

(b) If the whole liquid column is given a small displacement from its equilibrium position, show that the resulting oscillations are simple harmonic. Find the time period of these oscillations.

11. Two identical balls A and B, each of mass 0.1 kg, are attached to two identical mass less springs. The spring–mass system is constrained to move inside a rigid smooth pipe bent in the form of a circle as shown in the figure. The pipe is fixed in a horizontal plane. The centres of the balls can move in a circle of radius 0.06 m. Each spring has a natural length of 0.06π metre and spring constant 0.1 N/m. Initially, both the balls are displaced by an angle $\theta = \pi/6$ radian with respect to the diameter PQ of the circle (as shown in fig.) and released from rest. **[1993 ; 6M]**



(i) Calculate the frequency of oscillation of ball B.

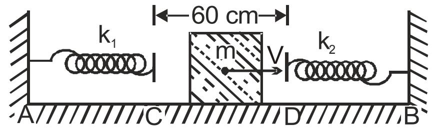
(ii) Find the speed of ball A when A and B are at the two ends of the diameter PQ.

(iii) What is the total energy of the system ?

12. Two light springs of force constant k_1 and k_2 and a block of mass m are in one line AB on a smooth horizontal table such that one end of each spring is fixed to rigid supports and the other end is free as shown in the figure. The distance CD between the free ends of the spring is 60 cm. If the block moves along AB with a velocity 120 cm/s in between the springs, calculate the period of oscillation of the block.

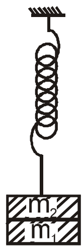
(k_1 = 1.8 N/m, k_2 = 3.2 N/m, m = 200 g)

(1985 ; 6M)

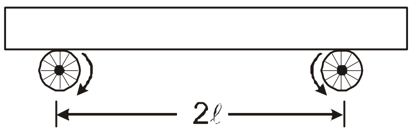


13. Two masses m_1 and m_2 are suspended together by a massless spring of spring constant k (Fig). When the masses are in equilibrium, m_1 is removed without disturbing the system. Find the angular frequency and amplitude of oscillation of m_2 .

(1981 ; 3M)



14. Two wheels which are rotated by some external source with constant angular velocity in opposite directions as shown in figure. A uniform plank of mass M is placed on it symmetrically. The friction co-efficient between each wheel and the plank is μ. Find the frequency of oscillations, when plank is slightly displaced along its length and released.



Answers

1. $\langle v \rangle = \frac{8}{\pi}$ m/s **2.** t = 2s **3.** $2\pi\sqrt{\frac{M}{2\alpha}}$

4. $\frac{\pi}{3}\sqrt{\frac{\ell}{g}}$

5. (a) 2 m ; (b) 4/9 m/s, 4/81 m/s^2 towards the point of suspension; (c) $2\pi/9$ m/s^2 towards mean position,

6. (a) ma ; (b) $\frac{1}{2\pi}\sqrt{a/\ell}$ where a = $\left[g^2 + \frac{v^4}{r^2}\right]^{1/2}$

7. $\frac{F(k_2+k_3)}{k_1 k_2 + k_2 k_3 + k_3 k_1}$, $\frac{1}{2\pi}\sqrt{\frac{k_1 k_2 + k_2 k_3 + k_3 k_1}{M(k_2+k_3)}}$

8. (a) $\frac{(m_1+m_2)\, g \sin\theta}{k}$ (b) when the spring acquires its natural length (c) $\sqrt{\frac{3}{k}}(m_1+m_2)\, g\sin\theta$

9. (a) $x^2 + y^2 = a^2$, circle

(b) The particle moves in clock wise sense.

(c)The magnitude of force = m $\sqrt{a_x^2 + a_y^2}$ = $m\omega^2 a$

10. (a) $\tan^{-1}\left(\frac{1}{5}\right)$ (b) $2\pi\sqrt{\frac{R}{6.11}}$

11. (i) $f = \frac{1}{2\pi}\sqrt{\frac{4\times 0.1}{0.1}} = \frac{1}{\pi}$ Hz (ii) V = 0.0628 (iii) 3.9×10^{-4} J

12. 2.82 s **13.** (i) $\omega = \sqrt{\frac{k}{m_2}}$, (ii) Amplitude = $\frac{m_1 g}{k}$ **14.** $2\pi\sqrt{\frac{\ell}{\mu g}}$

SELF ASSESSMENT PAPER

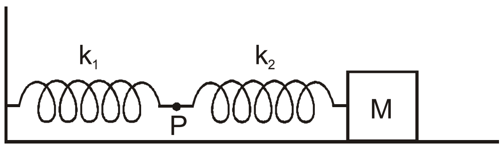
JEE (ADVANCED) PAPER-1

SECTION-1 : ONE OPTION CORRECT TYPE (Maximum Marks - 12)

1. A simple pendulum has time period T_1. The point of suspension is now moved upward according to the relation $y = kt^2$ ($k = 1\ m/s^2$) where y is the vertical displacement, the time period now becomes T_2. The ratio of $\left(\frac{T_1}{T_2}\right)^2$ is : ($g = 10\ m/s^2$)

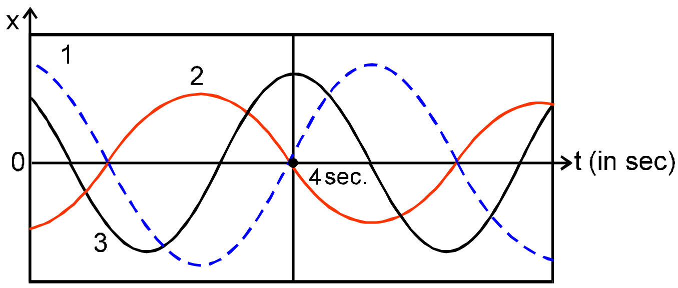
(A) $\frac{5}{6}$ (B) $\frac{6}{5}$ (C) 1 (D) $\frac{4}{5}$

2. The mass M shown in the figure oscillates in simple harmonic motion with amplitude A. The amplitude of the point P is



(A) $\frac{k_1 A}{k_2}$ (B) $\frac{k_2 A}{k_1}$ (C) $\frac{k_1 A}{k_1 + k_2}$ (D) $\frac{k_2 A}{k_1 + k_2}$

3. Graph shows the x(t) curves for three experiments involving a particular spring- block system oscillating in SHM. The kinetic energy of the system is largest at t = 4 sec. for the situation :



(A) 1 (B) 2 (C) 3 (D) Same in all

4. The displacement of an object attached to a spring and executing simple harmonic motion is given by $x = 2 \times 10^{-2} \cos\pi t$ metres. The time at which the maximum speed first occurs is :

(A) 0.5 s (B) 0.75 s (C) 0.125 s (D) 0.25 s

SECTION-2 : ONE OR MORE THAN ONE CORRECT TYPE (Maximum Marks - 32)

5. Three simple harmonic motions in the same direction having the same amplitude a and same period are superposed. If each differs in phase from the next by 45^0, then,

(A) the resultant amplitude is $(1+\sqrt{2})a$

(B) the phase of the resultant motion relative to the first is 90^0.

(C) the energy associated with the resulting motion is $(3+2\sqrt{2})$ times the energy associated with any single motion.

(D) the resulting motion is not simple harmonic.

6. A coin is placed on a horizontal platform which undergoes vertical simple harmonic motion of angular frequency ω.The amplitude of oscillation is gradually increased. The coin will leave contact with the platform for the first time :

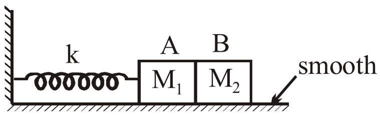
(A) at the highest position of the platform

(B) at the mean position of the platform

(C) for an amplitude of $\frac{g}{\omega^2}$

(D) for an amplitude of $\frac{g^2}{\omega^2}$

7. A Block A of mass M_1 is attached to a spring of spring constant 'k'. Blocks A and B are in contact while the spring is compressed by x_0. The system is released and then the blocks start moving. After some time the contact between the blocks breaks, then choose the correct statement(s).



(A) Blocks will separate at natural length of spring.

(B) After seperation of block B, the block A perform SHM of amplitude $x_0\sqrt{\frac{m_1}{m_1+m_2}}$.

(C) After seperation of block B, the maximum velocity of block A is $x_0\sqrt{\frac{k}{m_1+m_2}}$.

(D) After separation block A will perform SHM of amplitude x_0.

8. A particle is performing SHM with its position given as $x = 2 + 5\sin\left(\pi t+\frac{\pi}{6}\right)$ where x (in m) & t (in sec). Which of the following is/are correct :

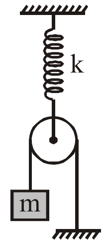
(A) Equilibrium position is at x = 2m

(B) Maximum speed of particle is 5π m/s

(C) At t = 0 particle is 2.5 m away from mean position moving in negative direction

(D) At t = 0, x = 4.5 m; acceleration of particle is $-\pi^2(4.5)$ m/s^2

9. Block m is released from rest when spring is in its natural length (assume pulley is ideal and block does not strike on ground during it's motion in vertical plane) then :



(A) maximum elongation in spring is 4 mg/k

(B) maximum elongation in spring is 2 mg/k

(C) maximum speed of block is $2g\sqrt{\frac{m}{k}}$

(D) maximum speed of block is $g\sqrt{\frac{m}{k}}$

10. A 20 gm particle is subjected to two simple harmonic motions $x_1 = 2 \sin 10t$, $x_2 = 4 \sin\left(10t + \frac{\pi}{3}\right)$ along same straight line. Where x_1 and x_2 are in metre and t is in sec.

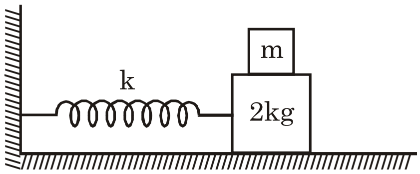
(A) the displacement of the particle at t = 0 will be $2\sqrt{3}$ m

(B) maximum speed of the particle will be $20\sqrt{7}$ m/s

(C) magnitude of maximum acceleration of the particle will be $200\sqrt{7}$ m/s^2.

(D) Energy of the resultant motion will be 28 J.

11. When displaced and released, the 2 kg mass in the figure oscillates on the frictionless horizontal surface with time period $\pi/6$ seconds. A small mass m is placed on the 2 kg block and the coefficient of static friction between the small mass and the 2 kg block is 0.1. (Assume the period is unaffected by adding the small mass).



(A) The maximum amplitude of oscillation before the small mass slips is $\frac{1}{144}$ m

(B) The maximum amplitude of oscillation before the small mass slips is $\frac{1}{72}$ m

(C) maximum velocity of blocks assuming no slipping is $\frac{1}{12}$ m/s

(D) maximum velocity of blocks assuming no slipping is $\frac{1}{24}$ m/s

12. x-t equation of a particle moving along x-axis is given as :

$x = A + A(1 - \cos \omega t)$

(A) Particle oscillates simple harmonically between points x = 2A and x = A.

(B) Velocity of particle is maximum at x = 2A.

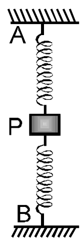
(C) Time taken by the particle in travelling from x = A to x = 3A is π/ω.

(D) Time taken by particle in travelling from x = A to x = 2A is $\pi/2\omega$.

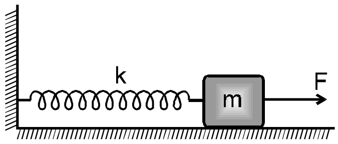
SECTION-3 : NUMERICAL VALUE TYPE (Maximum Marks - 18)

13. A block of mass 1 kg hanging from a vertical spring executes SHM of amplitude 0.2m and time period $\pi/10$sec. Find the maximum force exerted by the spring on the block.

14. Two identical springs are attached to a small block P. The other ends of the springs are fixed at A and B. When P is in equilibrium the extension of top spring is 20 cm and extension of bottom spring is 10 cm. If the period of small vertical oscillations of P about its equilibrium position is $\frac{\pi}{N}$ sec. Find N. (use g =9.8m/s^2).



15. A constant force produces maximum velocity V on the block connected to the spring of force constant K as shown in the fig. When the force constant of spring becomes 4K, if the maximum velocity of the block is V/N. Find N :



16. If the magnitude of average acceleration in half time period from equilibrium position in a simple harmonic motion is $\frac{NA\omega^2}{\pi}$. Find N.

17. Equation of SHM is $x = 10 \sin 10\pi t$. If the distance between the two points where speed is 50π cm/sec. is $10\sqrt{m}$ cm. x is in cm and t is in seconds. Find m

18. A body of mass 2 kg suspended through a vertical spring executes simple harmonic motion of period 4s. If the oscillations are stopped and the body hangs in equilibrium, find the potential energy (in J) stored in the spring.

Answers

1.	(B)	**2.**	(D)	**3.**	(A)	**4.**	(A)	**5.**	(A)(C)
6.	(A)(C)	7.	(A)(B)(C)	**8.**	(A,B)	**9.**	(A,C)	**10.**	(A,B,C,D)
11.	(A,C)	**12.**	(B, C, D)	**13.**	90 N	**14.**	7	**15.**	2
16.	2	**17.**	3	**18.**	40				