4 Work, Energy and Power

The term *work* was introduced in 1826 by the French mathematician Gaspard-Gustave de Coriolis as "weight lifted through a height", which is based on the use of early steam engines to lift buckets of water out of flooded ore mines. In physics, a force is said to do work if, when acting on a body, there is a displacement of the point of application in the direction of the force. The work is the product of force an displacement.

Note

What does the word work make you think of? You probably think of tasks that you would rather not do. For example, you might think it is work to take leaves in your front yard or run laps around a school track. Your idea of work might be quite different from another person's idea. If you are a member of a sports team, you might like to run. In everyday life, the word work means different things to different people.

But in science, work has a specific meaning. In science work relates to force, motion, and energy.

Taking into account the kinetic and potential energies of a system, we have the law of conservation of mechanical energy, which provides a way of understanding mechanical problems that is based on Newton's laws but often provides new or different insights.

Let us consider some cases where work is not done:

- Work is zero if applied force is zero ($W = 0$ if $F = 0$): If a block is moving on a smooth horizontal surface (frictionless), no work will be done. Note that the block may have large displacement but no work gets done.
- Work is zero if $\cos\theta$ is zero or $\theta = \frac{\pi}{2}$. This explains why no work is done by the porter in carrying the load.

As the porter carries the load by lifting it upwards and the moving forward it is obvious the angle between the force applied by the porter and the displacement is $90°$.

Now that we have seen how to evaluate the work done by a force on an object, let us explore the power of this approach. Solving problems using Newton's second law can be difficult if the forces involved are complicated.

An alternative approach to such problems is to relate the speed of an object to the work done on the object by some net external force.

If the work done by the net force on the object can be calculated for a given displacement, the change in the object's speeds is easy to evaluate.

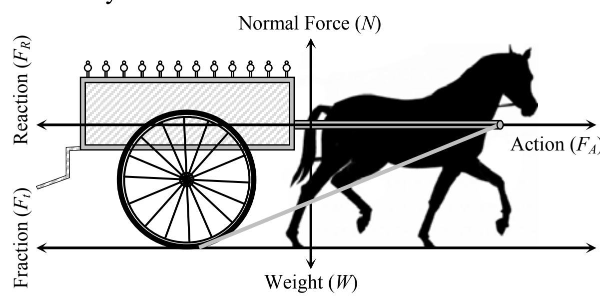


Figure: 4.1

- **Work** $(W) = \vec{F}\cdot\vec{s} = \vec{F}s\cos\theta$

  Where, θ=angle between force F and displacements s.Unit of work in M.K.S. or S.I. system is joule.
- Work done by a force is positive of $\theta < 90°$
- Work done by a force is zero if $\theta = 90°$
- Work done by a force is negative if $180° > \theta > 90°$
- **Power** $P = \frac{\text{Work}}{\text{Time}} = \frac{W}{t} = \vec{F}\cdot\vec{\upsilon} = F\upsilon\cos\theta$

  Unit of power in S.I. system is watt.
- **Kinetic Energy** $(T) = \frac{1}{2}m\upsilon^2$ Kinetic energy is never negative.
- **Potential energy** $U = -\int_{r_0}^{r}\vec{F}\cdot\vec{dr}$

  Where, r_0 is reference position for zero potential energy. Referred to zero potential energy at earth's surface $U = mgh$. Referred to zero potential energy at ∞, $U = \frac{-GM_e m}{r}$ where M_e = mass of earth and r= distance of body of mass m from earth's centre. Potential energy may be positive or negative.

- Elastic potential energy $U = \frac{1}{2}kx^2$
- Mechanical energy $E = T + U$
- Under conservation force $E = K + U =$ constant
- Under non-conservative forces, total energy of universe remains constant.

Work Energy Theorem

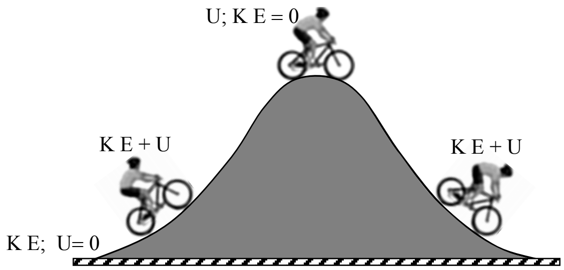


Figure: 4.2

Work = gain in kinetic energy $= \frac{1}{2}m\upsilon_2^2 - \frac{1}{2}m\upsilon_1^2$

Stopping distance of a vehicle on a rough surface. $s = \frac{\upsilon^2}{2\mu g}$

Conservation of Momentum

- **Recoiling of a gun**

Figure: 4.3

By the law of conservation of linear momentum:

$m_G\vec{v}_G + m_B\vec{v}_B = 0$

So, recoil velocity $\vec{v}_G = -\frac{m_B}{m_G}\vec{v}_B$

- If 'n' bullets each of mass m are fired per unit time from a machine gun, then the force required to hold the gun $= v\left(\frac{dm}{dt}\right) = v(mn) = mnv$

Power

From a practical view point, it is interesting to know not only the amount of energy transferred to or from a system but also the rate at which the transfer occurred. This is a particularly interesting issue for living creatures because the maximum work per second, or power output, of an animal varies greatly with output duration. To handle such problems in a systematic way, **power is defined as the time rate of energy transfer.**

If an external force is applied to an object and if the work done by this force is W in the time interval Δt, then the **average power** during this interval is defined as the ratio of the work done to the time interval: $\bar{P} \equiv \frac{Q}{\Delta t}$

Example 1. A porter lifts a luggage of 15 kg from the ground and put it on his head, 1.5 m above the ground. Calculate the work done by him on the luggage. (take g = 10 m/s²)

Solution: Mass of luggage, $m = 15\ kg$

Displacement, $d = 1.5\ m$

Acceleration due to gravity, $g = 10\ m/s^2$

Work done, $W = Fd = mgd$

W = 15 × 10 × 1.5 = 225 J

Example 2. A force of 10 N displaces a body by 5 m, the angle between force and displacement is 60º, then find the work done.

Solution: Force, $F = 10\ N$,

Displacement, $d = 5m$,

Angle between force and displacement, $\theta = 60°$,

Work done, $W = Fd\ \cos\theta = 10 \times 5 \times \cos 60°$

$\therefore \quad \cos 60° = \frac{1}{2}$

Then, $W = 10 \times 5 \times \frac{1}{2}$

$\Rightarrow$ W = 25 J

Example 3. What is the work to be done to increase the velocity of a car from 30 km/h to 60 km/h. If mass of the car is 1500 kg.

Solution: Mass of car, m = 1500 kg.

Initial velocity, u = 30 km/h = 8.33 m/s.

Final velocity, v = 60 km/h = 16.67 m/s.

Work done, $W = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$

$W = \frac{1}{2} \times 1500[(16.67)^2 - (8.33)^2]$

= 750(277.9 – 69.4)

W = 750 × 208.5 = 156375 J.

$W = 1.56 \times 10^5 J$.

Example 4. A body of mass 10 kg is kept at a height 10 m from the ground, when it is released after sometime its kinetic energy becomes 450 J. What will be the potential energy of the body at the instant ?

Solution: At a height of 10 m. The mechanical energy of the body,

E = Kinetic energy + potential energy

$E = m\,(o)^2$ + mgh ($\because$ initial velocity of the body is zero)

$E = 10 \times 10 \times 10 = 1000$ J.

After sometime the kinetic energy is 450 J. Suppose at that instant potential energy is U, then by the law of conservation of mechanical energy.

$E = 450 + U$

$1000 = 450 + U$ or $U = 1000 - 450 \Rightarrow U = 550\,J.$

Example 5. A boy of mass 50 kg runs up a staircase of 45 steps in 9 s. If the height of each step is 15 cm. Find his power. (g = 10 m/s²)

Solution: Mass of man, m = 50 kg.

Height covered,

$h = 45 \times 15 = 675$ cm = 6.75 m

Power $P = \dfrac{W}{t} = \dfrac{mgh}{t} = \dfrac{50 \times 10 \times 6.75}{9}$

$\Rightarrow P = 375$ watt

Multiple Choice Questions

1. Work done upon a body is:
a. a vector quantity **b.** a scalar quantity
c. **a.** and **b.** both are correct **d.** none of these

2. Work done:
a. is always positive
b. is always negative
c. can be positive, negative or zero
d. none of these

3. No work is done when :
a. a nail is plugged in a wooden board
b. a box is pushed along a horizontal floor
c. there is no component of force parallel to the direction of motion
d. there is no component of force perpendicular to the direction of motion

4. A body at rest can have:
a. speed **b.** velocity
c. momentum **d.** energy

5. Types of mechanical energies are:
a. kinetic energy only
b. potential energy only
c. kinetic energy and potential energy both
d. neither kinetic energy nor potential energy

6. Work means:
a. effort **b.** interview
c. achievement **d.** get-together

7. Work is done on a body when:
a. force acts on the body but the body is not displaced
b. force does not act on the body but it is displaced
c. force acts on the body in a direction perpendicular to the direction of the displacement of the body
d. force acts on the body and they body is either displaced in the direction of force or opposite to the direction of force.

8. Force F acts on a body such that force F makes an angle θ with the horizontal direction and the body is also displaced through a distance S in the horizontal direction, then the work done by the force is:
a. FS **b.** $FS\cos\theta$
c. $FS\sin\theta$ **d.** Zero

9. In tug of war work done by winning team is:
a. zero **b.** positive
c. negative **d.** None of these

10. In tug of war work done by losing team is:
a. zero **b.** positive
c. negative **d.** None of these

11. Work done by the force of gravity, when a body is lifted to height h above the ground is:
a. zero **b.** positive
c. negative **d.** None of these

12. When work is done on a body:
a. it gains energy
b. it looses energy
c. its energy remains constant
d. none of these

13. Choose correct relation :
a. $1\,J = 10^5\,erg$ **b.** $1\,J = 10^7\,erg$
c. $1\,J = 10^3\,erg$ **d.** None of these

14. The kinetic energy of an object is K. If its velocity is doubled than its kinetic energy will be:
a. K **b.** $2K$ **c.** $\dfrac{K}{2}$ **d.** $4K$

15. Two bodies of mass 1 kg and 4 kg possess equal momentum. The ratio of their KE:
a. 4 : 1 **b.** 1 : 4
c. 2 : 1 **d.** 1 : 2

16. Which of is not the unit of energy?
a. kilocalorie **b.** kWh
c. erg **d.** watt

17. 1 kg mass has K.E. of 1 J when its speed is:
a. 0.45 ms^{-1} **b.** 1 ms^{-1}
c. 1.4 ms^{-1} **d.** 4.4 ms^{-1}

18. When you compress a spring you do work on it. The elastic potential energy of the spring:
a. increases **b.** decreases
c. disappears **d.** remains constant

19. When a ball is thrown upward, its total energy?
a. increases **b.** decreases
c. remains same **d.** None of these

20. If a stone of mass *'m'* falls a vertical distance *'d'* the decrease in gravitational potential energy is:
a. $\frac{Mg}{d}$ **b.** $\frac{Mg^2}{2}$ **c.** mgd **d.** $\frac{Mg}{d^2}$

21. An object of mass 10 kg falls from height 10 m. Kinetic energy gained by the body will be approximately equal to:
a. 1000 J **b.** 500 J
c. 100 J **d.** None of these

22. A spring is stretched. The potential energy in stretching the spring:
a. remains the same **b.** increases
c. decreases **d.** becomes zero

23. The potential energy of a boy is maximum when he is:
a. standing
b. sleeping on the ground
c. sitting on the ground
d. sitting on chair

24. The potential energy of a freely falling object decreases continuously. What happens to the loss of potential energy?
a. It is continuously converted into sound energy
b. It is continuously converted into kinetic energy
c. It is continuously destroyed
d. None of these

25. A device which converts mechanical energy into electrical energy is known as:
a. electric motor **b.** lever
c. generator **d.** microphone

26. The value of g on moon 1/6th of the value of g on the earth. A man can jump 1.5 m high on the earth. On moon he can jump up to a height of:
a. 9 m **b.** 7.5 m **c.** 6 m **d.** 4.5 m

27. A raised hummer possess :
a. kinetic energy only
b. gravitational potential energy
c. electrical energy
d. sound energy

28. An object of mass 1 kg has a P.E. of 1 J relative to the ground when it is at a height of : ($g = 9.8\ m/s^2$)
a. 0.10 m **b.** 10 m **c.** 9.8 m **d.** 32 m

29. To lift a 5 kg mass to a certain height, amount of energy spent is 245 J. The mass was raised to a height of:
a. 15 m **b.** 10 m **c.** 7.5 m **d.** 5 m

30. Chlorophyll in the plants convert the light energy into:
a. heart energy **b.** chemical energy
c. mechanical energy **d.** electrical energy

31. Kilowatt is the unit of:
a. energy **b.** power
c. force **d.** momentum

32. Work is product of time and:
a. energy **b.** power
c. force **d.** distance

33. A young son work quickly for 2 hours and prepares 16 items in a day. His old father works slowly for either hours and prepare 24 items a day:
a. son has more power **b.** son has more energy
c. both have equal power **d.** both have equal energy

34. One horse power is:
a. 746 W **b.** 550 W **c.** 980 W **d.** 32 W

35. Power of a moving body is stored in the form of:
a. work and distance **b.** force and distance
c. force and velocity **d.** force and time

36. A weight lifter lifts 240 kg from the ground to a height of 2.5 m in 3 seconds his average power is:
a. 1960 W **b.** 19.6 W **c.** 1.96 W **d.** 196 W

37. Which of the following is not the unit of power?
a. J/s **b.** Watt **c.** kJ/h **d.** kWh

38. A body of mass *m* is moving in a circle of radius *r* with a constant speed *v*. The force on the body is $\frac{mv^2}{r}$ and is directed towards the centre. What is the work done by this force in moving the body over half the circumference of the circle?
a. $\frac{mv^2}{\pi r^2}$ **b.** Zero **c.** $\frac{mv^2}{r^2}$ **d.** $\frac{\pi r^2}{mv^2}$

39. If the unit of force and length each be increased by four times, then the unit of energy is increased by:

a. 16 times **b.** 8 times **c.** 2 times **d.** 4 times

40. A man pushes a wall and fails to displace it. He does

a. negative work

b. positive but not maximum work

c. no work at all

d. maximum work

41. The same retarding force is applied to stop a train. The train stops after 80 *m*. If the speed is doubled, then the distance will be:

a. same **b.** doubled

c. halved **d.** four times

42. A body of mass 10 *kg* is dropped to the ground from a height of 10 *metres*. The work done by the gravitational force is: $(g = 9.8\,m/\sec^2)$

a. – 490 Joules **b.** + 490 Joules

c. – 980 Joules **d.** + 980 Joules

43. Which of the following is a scalar quantity?

a. Displacement **b.** Electric field

c. Acceleration **d.** Work

44. The work done in pulling up a block of wood weighing 2 kN for a length of 10 *m* on a smooth plane inclined at an angle of 15° with the horizontal is:

a. 4.36 kJ **b.** 5.17 kJ **c.** 8.91 kJ **d.** 9.82 kJ

45. A force of 5 *N* acts on a 15 *kg* body initially at rest. The work done by the force during the first second of motion of the body is:

a. 5 *J* **b.** $\frac{5}{6}J$ **c.** 6 *J* **d.** 75*J*

46. Which of the following is a unit of energy?

a. Unit **b.** Watt

c. Horse Power **d.** None

47. If force and displacement of particle in direction of force are doubled. Work would be:

a. Double **b.** 4 times **c.** Half **d.** $\frac{1}{4}$ times

48. Two bodies of masses 1 *kg* and 5 *kg* are dropped gently from the top of a tower. At a point 20 *cm* from the ground, both the bodies will have the same:

a. Momentum **b.** Kinetic energy

c. Velocity **d.** Total energy

49. A ball is released from the top of a tower. The ratio of work done by force of gravity in first, second and third second of the motion of the ball is:

a. 1 : 2 : 3 **b.** 1 : 4 : 9

c. 1 : 3 : 5 **d.** 1 : 5 : 3

50. A spring of force constant 800 *N/m* has an extension of 5 *cm*. The work done in extending it from 5 *cm* to 15 *cm* is:

a. 16 *J* **b.** 8 *J* **c.** 32 *J* **d.** 24 *J*

51. When a spring is stretched by 2 *cm*, it stores 100 *J* of energy. If it is stretched further by 2 *cm*, the stored energy will be increased by:

a. 100 *J* **b.** 200 *J* **c.** 300 *J* **d.** 400 *J*

52. A spring when stretched by 2 mm its potential energy becomes 4 J. If it is stretched by 10 mm, its potential energy is equal to:

a. 4 *J* **b.** 54 *J* **c.** 415 *J* **d.** None

53. A spring of spring constant 5×10^3 *N/m* is stretched initially by 5 *cm* from the unscratched position. Then the work required to stretch it further by another $5\,cm$ is:

a. 6.25 *N-m* **b.** 12.50 *N-m*

c. 18.75 *N-m* **d.** 25.00 *N-m*

54. Two bodies of masses m_1 and m_2 have equal kinetic energies. If p_1 and p_2 are their respective momentum, then ratio $p_1 : p_2$ is equal to:

a. $m_1 : m_2$ **b.** $m_2 : m_1$

c. $\sqrt{m_1} : \sqrt{m_2}$ **d.** $m_1^2 : m_2^2$

55. Work done in raising a box depends on:

a. How fast it is raised

b. The strength of the man

c. The height by which it is raised

d. None of the above

56. A light and a heavy body have equal momenta. Which one has greater KE?

a. The light body **b.** The heavy body

c. The KE are equal **d.** Data is incomplete

57. A body at rest may have

a. Energy **b.** Momentum

c. Speed **d.** Velocity

58. The kinetic energy possessed by a body of mass *m* moving with a velocity *v* is equal to $\frac{1}{2}mv^2$, provided

a. The body moves with velocities comparable to that of light

b. The body moves with velocities negligible compared to the speed of light
c. The body moves with velocities greater than that of light
d. None of the above statement is correct

59. When work is done on a body by an external force, its
a. kinetic energy increases
b. potential energy increases
c. Both kinetic and potential energies may increase
d. Sum of kinetic and potential energies remains constant

60. A light and a heavy body have equal kinetic energy. Which one has a greater momentum?
a. The light body
b. The heavy body
c. Both have equal momentum
d. It is not possible to say anything without additional information

61. If the linear momentum is increased by 50%, the kinetic energy will increase by
a. 50% **b.** 100% **c.** 125% **d.** 25%

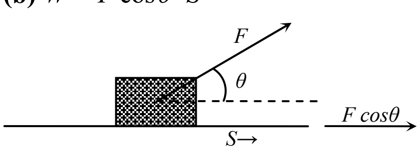
62. If the stone is thrown up vertically and return to ground, its potential energy is maximum.
a. during the upward journey
b. at the maximum height
c. during the return journey
d. at the bottom

ANSWERS

1.	**2.**	**3.**	**4.**	**5.**	**6.**	**7.**	**8.**	**9.**	**10.**
b	c	c	d	c	c	d	b	b	c
11.	**12.**	**13.**	**14.**	**15.**	**16.**	**17.**	**18.**	**19.**	**20.**
c	a	b	d	a	d	c	a	c	c
21.	**22.**	**23.**	**24.**	**25.**	**26.**	**27.**	**28.**	**29.**	**30.**
a	b	a	b	c	a	b	a	d	b
31.	**32.**	**33.**	**34.**	**35.**	**36.**	**37.**	**38.**	**39.**	**40.**
b	b	a	a	c	a	d	b	a	c
41.	**42.**	**43.**	**44.**	**45.**	**46.**	**47.**	**48.**	**49.**	**50.**
d	d	d	b	b	a	d	c	c	b
51.	**52.**	**53.**	**54.**	**55.**	**56.**	**57.**	**58.**	**59.**	**60.**
c	d	c	c	c	a	a	b	c	b
61.	**62.**								
c	b								

SOLUTIONS

8. **(b)** $W = F\cos\theta \cdot S$



14. **(d)** $K \cdot E = \frac{1}{2}mv^2$

$\Rightarrow$ $(K \cdot E)_{New} = 4(K \cdot E) \Rightarrow v_{New} = 2v$

15. **(a)** $E = \frac{P^2}{2m}$

$\Rightarrow$ $\frac{E_1}{E_2} = \frac{m_2}{m_1} = \frac{4}{1}$

As P – constant

21. **(a)** $v^2 = u^2 + 2gh$

$\Rightarrow$ $u = 0$

$\Rightarrow$ $v^2 = 2 \times 10 \times 10 = 200$

$\therefore$ $E = \frac{1}{2}mv^2 = \frac{1}{2}10 \times 200 = 1000J$

26. **(a)** $v^2 = u^2 + 2gh$

$\Rightarrow$ $\frac{h_m}{h_e} = \frac{g_e}{g_m}$

$\Rightarrow$ $h_m = h_e \times \frac{g}{\frac{1}{6}g}$ 1.5 6 $= 9m$

28. **(a)** $h = \frac{U}{mgh}$

29. **(d)** $h = \frac{U}{mgh}$

36. **(a)** $P = \frac{U}{t} \Rightarrow P = \frac{mgh}{t}$

38. **(b)** Work done by centripetal force is always zero, because force and instantaneous displacement are always perpendicular.

$\Rightarrow$ $W = \vec{F}.\vec{s} = Fs\cos\theta = Fs\cos(90°) = 0$

39. **(a)** Work = Force × Displacement (length). If unit of force and length be increased by four times then the unit of energy will increase by 16 times.

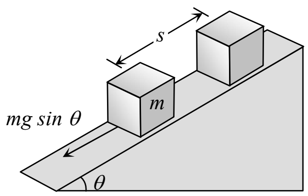
40. **(c)** No displacement is there.

41. **(d)** Stopping distance $S \propto u^2$. If the speed is doubled then the stopping distance will be four times.

42. **(d)** As the body moves in the direction of force therefore work done by gravitational force will be positive.
$W = Fs = mgh = 10 \times 9.8 \times 10 = 980J$

44. **(b)** $W = mg\sin\theta \times s$

$= 2\times10^3 \times \sin15° \times 10 = 5.17\,kJ$



45. **(b)** $W = Fs = F\times\frac{1}{2}at^2 \left[\text{from } s = ut + \frac{1}{2}at^2\right]$

$\Rightarrow W = F\left[\frac{1}{2}\left(\frac{F}{m}\right)t^2\right]$

$= \frac{F^2t^2}{2m} = \frac{25\times(1)^2}{2\times15}$

$= \frac{25}{30} = \frac{5}{6}\,J$

46. **(a)** Both part will have numerically equal momentum and lighter part will have more velocity.

47. **(d)** Watt and Horsepower are the unit of power.

48. **(c)** Velocity of fall is independent of the mass of the falling body.

49. **(c)** When the ball is released from the top of tower then ratio of distances covered by the ball in first, second and third second:

$h_I : h_{II} : h_{III} = 1 : 3 : 5$: [because $h_n \propto (2n-1)$]

$\therefore$ Ratio of work done, $mgh_I : mgh_{II} : mgh_{III}$

$= 1 : 3 : 5$

50. **(b)** $W = \frac{1}{2}k(x_2^2 - x_1^2)$

$= \frac{1}{2}\times800\times\left(15^2 - 5^2\right)\times10^{-4} = 8\,J$

51. **(c)** $100 = \frac{1}{2}kx^2$ (given)

$W = \frac{1}{2}k\left(x_2^2 - x_1^2\right)$

$= \frac{1}{2}k\left[(2x)^2 - x^2\right]$

$= 3\times\left(\frac{1}{2}kx^2\right) = 3\times100 = 300\,J$

52. **(d)** $U = \frac{1}{2}kx^2$ if x becomes 5 times then energy will become 25 times.

i.e. $4\times25 = 100\,J$

53. **(c)** $W = \frac{1}{2}k(x_2^2 - x_1^2)$

$= \frac{1}{2}\times5\times10^3\left(10^2 - 5^2\right)\times10^{-4}$

$= 18.75\,J$

54. **(c)** $P = \sqrt{2mE}$

$\therefore$ $P \propto \sqrt{m}$ (if E = const.)

$\therefore$ $\frac{P_1}{P_2} = \sqrt{\frac{m_1}{m_2}}$

55. **(c)** Work in raising a box = (weight of the box) × (height by which it is raised)

56. **(a)** $E = \frac{P^2}{2m}$ if

P = constant then $E \propto \frac{1}{m}$

57. **(a)** Body at rest may possess potential energy.

58. **(b)** Due to theory of relativity.

60. **(b)** $P = \sqrt{2mE}$ if E are equal then $P \propto \sqrt{m}$

i.e. heavier body will possess greater momentum.

61. **(c)** Let $P_1 = P$, $P_2 = P_1 + 50\%$ of

$P_1 = P_1 + \frac{P_1}{2} = \frac{3P_1}{2}$

$E \propto P^2 \Rightarrow \frac{E_2}{E_1} = \left(\frac{P_2}{P_1}\right)^2$

$= \left(\frac{3P_1/2}{P_1}\right)^2 = \frac{9}{4}$

$\Rightarrow$ $E_2 = 2.25,$

$E = E_1 + 1.25\,E_1$

$\therefore$ $E_2 = E_1 + 125\%$ of E_1

i.e. kinetic energy will increase by 125%.

62. **(b)** Potential energy = mgh

Potential energy is maximum when h is maximum.

□ □ □