Sequence and Series

SEQUENCE

A sequence is a set of terms arranged in a definite order with a rule for obtaining the terms.

e.g. 1 , 1/2 , 1/3 , ..., 1/n , ... is a sequence .

A sequence is a function whose domain is the set N of natural numbers. Since the domain for every sequence is the set of N natural numbers, therefore a sequence is represented by its range. $f : N \to R$, then $f(n) = t_n$, $n \in N$ is called a sequence and is denoted by

$\{f(1), f(2), f(3), \ldots\ldots\}$

$= \{t_1, t_2, t_3, \ldots\ldots\} = \{t_n\}$

Real Sequence : A sequence whose all terms are real is called a real sequence.

Examples :

(i) 2, 5, 8, 11, ...

(ii) 4, 1, –2, –5, ...

(iii) 3, –9, 27, –81, ...

Types of Sequence : On the basis of the number of terms there are two types of sequence.

(i) Finite sequence : A sequence is said to be finite if it has finite number of terms.

(ii) Infinite sequence : A sequence is said to be infinite if it has infinite number of terms.

Series : By adding or substracting the terms of a sequence, we get an expression which is called a series. If $a_1, a_2, a_3, \ldots, a_n$ is a sequence, then the expression $a_1 + a_2 + a_3 + \ldots + a_n$ is a series.

Example. (i) 1 + 2 + 3 + 4 + ... + n

(ii) 2 + 4 + 8 + 16 + ...

Progression : It is not necessary that the terms of a sequence always described by some explicit formula of the n^{th} term. Those sequences whose terms follow certain patterns are called progressions.

SOLVED EXAMPLE

Example-1

Write down the sequence whose nth term is

$(-1)^n \left(\frac{3n+2}{5}\right)$

Sol. Let $a_n = (-1)^n \left(\frac{3n+2}{5}\right)$

Putting = 1, 2, 3, 4,...... successively, we get

$a_1 = (-1)^1 \left(\frac{3.1+2}{5}\right) = -1$

$a_2 = (-1)^2 \left(\frac{3.2+2}{5}\right) = 8/5$

$a_3 = (-1)^3 \left(\frac{3.3+2}{5}\right) = -11/5$

$a_4 = (-1)^4 \left(\frac{3.4+2}{5}\right) = 14/5$

....

Hence we obtain the sequence –1, 8/5, –11/5, 14/5,......

Example-2

If sum of n terms of a sequence is given by $S_n = 2n^2 + 3n$, find its 50^{th} term.

Sol. Let t_n is n^{th} term of the sequence so $t_n = S_n - S_{n-1}$

$= 2n^2 + 3n - 2(n-1)^2 - 3(n-1) = 4n + 1$

so $t_{50} = 201$.

ARITHMETIC PROGRESSION (AP)

AP is a sequence whose consecutive terms increase or decrease by a fixed number . This fixed number is called the common difference . If a is the first term & d the common difference, then AP can be written as $a, a+d, a+2d, \ldots\ldots, a+(n-1)d, \ldots\ldots$.

n^{th} term of this AP, $t_n = a + (n-1)d$, where $d = a_n - a_{n-1}$.
The sum of the first n terms of the AP is given by ;

$$S_n = \frac{n}{2}[2a + (n-1)d] = \frac{n}{2}[a + l].$$

where l is the last term.

Remarks :

(i) If each term of an A.P. is increased, decreased, multiplied or divided by the same non zero number, then the resulting sequence is also an AP .

(ii) Three numbers in AP can be taken as $a-d$, a, $a+d$; four numbers in AP can be taken as $a-3d, a-d, a+d, a+3d$; five numbers in AP are $a-2d, a-d, a, a+d, a+2d$ & six terms in AP are $a-5d, a-3d, a-d, a+d, a+3d, a+5d$ etc.

(iii) The common difference can be zero, positive or negative .

(iv) The sum of the terms in an AP which are equidistant from the beginning & end is constant and equal to the sum of first & last terms .

(v) Any term of an AP (except the first) is equal to half the sum of terms which are equidistant from it . $a_n = 1/2\,(a_{n-k}+a_{n+k})$, $k<n$.
For $k=1$, $a_n = (1/2)(a_{n-1}+a_{n+1})$;
For $k=2$, $a_n = (1/2)(a_{n-2}+a_{n+2})$ and so on .

(vi) $t_r = S_r - S_{r-1}$

(vii) If a, b, c are in AP $\Rightarrow 2b = a+c$.

SOLVED EXAMPLE

Example-3

If t_{54} of an A.P. is – 61 and $t_4 = 64$, find t_{10}.

Sol. Let a be the first term and d be the common difference

so $t_{54} = a + 53d = -61$ ....(i)

and $t_4 = a + 3d = 64$ ....(ii)

equation (i) – (ii) we get

$\Rightarrow 50d = -125$

$$d = -\frac{5}{2}$$

$$\Rightarrow a = \frac{143}{2} \text{ so } t_{10} = \frac{143}{2} + 9\left(-\frac{5}{2}\right) = 49$$

Example-4

Find the number of terms in the sequence 4, 12, 20, ...,108.

Sol. $a = 4, d = 8$ so $108 = 4 + (n-1)8$

$\Rightarrow n = 14$

Example-5

Find the sum of all the three digit natural numbers which on division by 7 leaves remainder 3.

Sol. All these numbers are 101, 108, 115,....... 997, to find n.

$997 = 101 + (n-1)\,7$

$$\Rightarrow n = 129 \text{ so } S = \frac{129}{2}[101 + 997]$$

$= 70821.$

Example-6

The sum of n terms of two A.Ps. are in ratio $\dfrac{7n+1}{4n+27}$. Find the ratio of their 11th terms.

Sol. Let a_1 and a_2 be the first terms and d_1 and d_2 be the common differences of two A.P.s respectively, then

$$\frac{\frac{n}{2}[2a_1+(n-1)d_1]}{\frac{n}{2}[2a_2+(n-1)d_2]} = \frac{7n+1}{4n+27}$$

$$\Rightarrow \frac{a_1+\left(\frac{n-1}{2}\right)d_1}{a_2+\left(\frac{n-1}{2}\right)d_2} = \frac{7n+1}{4n+27}$$

For ratio of 11th terms

Put $\dfrac{n-1}{2} = 10 \Rightarrow n = 21$ on both sides

$$\text{so, } \frac{a_1+10d_1}{a_2+10d_2} = \frac{7(21)+1}{4(21)+27} = \frac{148}{111} = \frac{4}{3}$$

Example-7

The sum of first three terms of an A.P. is 27 and the sum of their squares is 293 . Find the sum to 'n' terms of the A.P.

Sol. Let $a-d$, a, $a+d$ be the numbers $\Rightarrow a = 9$

Also $(a-d)^2 + a^2 + (a+d)^2 = 293$.

$\Rightarrow 3a^2 + 2d^2 = 293$

$\Rightarrow d^2 = 25 \Rightarrow d = \pm 5$

therefore numbers are 14, 9, 4.

Hence the A.P. is 4, 9, 14, ... or 14, 9, 4, ...

$$\Rightarrow s_n = \frac{n}{2}[5n+3] \text{ or } s_n = \frac{n}{2}[33-5n]$$

Example-8

If $\dfrac{1}{b+c}, \dfrac{1}{c+a}, \dfrac{1}{a+b}$ are in A.P., prove that a^2, b^2, c^2 are also in A.P.

Sol. $\because \dfrac{1}{b+c}, \dfrac{1}{c+a}, \dfrac{1}{a+b}$ are in A.P.

$$\Rightarrow \frac{1}{c+a} - \frac{1}{b+c} = \frac{1}{a+b} - \frac{1}{c+a}$$

$$\Rightarrow \frac{b+c-c-a}{(c+a)(b+c)} = \frac{c+a-a-b}{(a+b)(c+a)}$$

$$\Rightarrow \frac{b-a}{b+c} = \frac{c-b}{a+b} \Rightarrow b^2 - a^2 = c^2 - b^2$$

$\Rightarrow a^2, b^2, c^2$ are in A.P.

GEOMETRIC PROGRESSION (GP)

GP is a sequence of numbers whose first term is non zero & each of the succeeding terms is equal to the proceeding terms multiplied by same constant . Thus in a GP the ratio of successive terms is constant. This constant factor is called the **COMMON RATIO** of the G.P. Therefore a, ar, ar^2, ar^3, ar^4, ... is a GP with a as the first term & r as common ratio .

(i) n^{th} term of a G.P. $= a r^{n-1}$

(ii) Sum of the I^{st} n terms i.e. $S_n = \dfrac{a\left(r^n - 1\right)}{r-1}$, if $r \neq 1$.

(iii) Sum of an infinite GP when $|r| < 1$ when $n \to \infty$

$r^n \to 0$ if $|r| < 1$ therefore, $S_\infty = \dfrac{a}{1-r}\ \left(|r| < 1\right)$.

(iv) If each term of a GP be multiplied or divided by the same non-zero quantity, the resulting sequence is also a GP .

(v) Any 3 consecutive terms of a GP can be taken as a/r, a, ar ; any 4 consecutive terms of a GP can be taken as a/r^3, a/r, ar, ar^3 & so on.

(vi) If a, b, c are in GP $\Rightarrow b^2 = ac$.

Sum to *n* terms of a G.P. is given by

Let a is first term and r is common ratio of G.P. then sum is given by

$$S_n = \frac{a(1-r^n)}{1-r} \text{ provided } r \neq 1$$

SOLVED EXAMPLE

Example-9

If the first term of G.P. is 7, its n^{th} term is 448 and sum of first n terms is 889, then find the fifth term of G.P.

Sol. Given a = 7 the first term, $t_n = ar^{n-1} = 7(r)^{n-1} = 448$

$\Rightarrow 7r^n = 448\,r$

Also $S_n = \dfrac{a(r^n-1)}{r-1} = \dfrac{7(r^n-1)}{r-1}$

$\Rightarrow 889 = \dfrac{448r - 7}{r-1}$

$\Rightarrow r = 2.$

Hence $T_5 = ar^4 = 7(2)^4 = 112$

Example-10

Find three numbers in G.P. having sum 19 and product 216.

Sol. Let the three numbers be $\dfrac{a}{r}$, a, ar

so $a\left[\dfrac{1}{r} + 1 + r\right] = 19$ ....(i)

and $a^3 = 216 \Rightarrow a = 6$

so from (i) $6r^2 - 13r + 6 = 0. \Rightarrow r = \dfrac{3}{2}, \dfrac{2}{3}$

Hence the three numbers are 4, 6, 9.

Example-11

Evaluate 7 + 77 + 777 + ... upto n terms.

Sol. Let S = 7 + 77 + 777 + ...upto n terms.

$= \dfrac{7}{9}\,[9 + 99 + 999 + ...]$

$= \dfrac{7}{9}\,[(10-1) + (10^2-1) + (10^3-1) + ... + \text{upto n terms}]$

$= \dfrac{7}{9}\,[10 + 10^2 + 10^3 + ...+ 10^n - n]$

$= \dfrac{7}{9}\left(\dfrac{10(10^n-1)}{9} - n\right) = \dfrac{7}{81}\,[10^{n+1} - 9n - 10]$

Example-12

If the third and fourth terms of an arithmetic sequence are increased by 3 and 8 respectively, then the first four terms form a geometric sequence. Find

(i) the sum of the first four terms of A.P.

(ii) second term of the G.P.

Sol. a, (a + d), (a + 2d), (a + 3d) in A.P.

a, a + d, (a + 2d + 3), (a + 3d + 8) are in G.P.

hence a + d = ar

also $r = \dfrac{a+d}{a} = \dfrac{a+2d+3}{a+d} = \dfrac{a+3d+8}{a+2d+3}$

$\therefore\ \dfrac{d+3}{d} = \dfrac{d+5}{d+3}$

$\Rightarrow d^2 + 6d + 9 = d^2 + 5d$

$\Rightarrow d = -9$

$\therefore\ \dfrac{a-9}{a} = \dfrac{a-15}{a-9}$

$\Rightarrow a^2 - 18a + 81 = a^2 - 15a$

$\Rightarrow 3a = 81 \Rightarrow a = 27$

hence A.P. is 27, 18, 9, 0, G.P. is 27, 18, 12, 8

(i) sum of the first four terms of A.P. = 54

(ii) 2^{nd} term of G.P. = 18

ARITHMETICO-GEOMETRIC SERIES

A series each term of which is formed by multiplying the corresponding term of an AP & GP is called the Arithmetico-Geometric Series . e.g. $1 + 3x + 5x^2 + 7x^3 + ...$

Here 1, 3, 5, ... are in AP & $1, x, x^2, x^3$ ..... are in GP .

Sum of n terms of an Arithmetico-Geometric Series

Let $S_n = a+(a+d)r+(a+2d)r^2+.....+[a+(n-1)d]\,r^{n-1}$

then $S_n = \dfrac{a}{1-r} + \dfrac{dr(1-r^{n-1})}{(1-r)^2} - \dfrac{[a+(n-1)d]r^n}{1-r}, r \neq 1$.

Sum To Infinity : If $|r| < 1$ & $n \to \infty$ then $\lim\limits_{n\to\infty} r^n = 0$. $S_\infty = \dfrac{a}{1-r} + \dfrac{dr}{(1-r)^2}$.

SOLVED EXAMPLE

Example-13

Find the sum to n terms of the series $\frac{1}{2}+\frac{2}{4}+\frac{3}{8}+\frac{4}{16}+\frac{5}{32}+........$ Also find the sum if it exist if $n \to \infty$.

Sol. $S = \frac{1}{2}+\frac{2}{4}+\frac{3}{8}+\frac{4}{16}+\frac{5}{32}+........+\frac{n}{2^n}$ ....(1)

$\frac{S}{2} = \frac{1}{4}+\frac{2}{8}+\frac{3}{16}+...............+\frac{n-1}{2^n}+\frac{n}{2^{n+1}}$ ....(2)

By (1) – (2)

$$\frac{S}{2} = \frac{1}{2}+\frac{1}{2^2}+\frac{1}{2^3}+\frac{1}{2^4}+........+\frac{1}{2^n}-\frac{n}{2^{n+1}}$$

$$= \frac{\frac{1}{2}\left(1-\frac{1}{2^n}\right)}{1-\frac{1}{2}} - \frac{n}{2^{n+1}} = 1-\frac{1}{2^n}-\frac{n}{2^{n+1}}$$

$$S_n = 2\left[\frac{2^{n+1}-2-n}{2^{n+1}}\right] = \frac{2^{n+1}-2-n}{2^n}.$$

If $n \to \infty$ then $S_\infty = \lim\limits_{n\to\infty}\left[2-\frac{1}{2^{n-1}}-\frac{n}{2^n}\right] = 2$

HARMONIC PROGRESSION (HP)

A sequence is said to HP if the reciprocals of its terms are in AP .

If the sequence $a_1, a_2, a_3, ... , a_n$ is an HP then $1/a_1, 1/a_2, ... , 1/a_n$ is an AP & converse. Here we do not have the formula for the sum of the n terms of an HP . For HP whose first term is a & second term is b, the n^{th} term is

$t_n = \dfrac{ab}{b+(n-1)(a-b)}$. If a, b, c are in HP

$\Rightarrow b = \dfrac{2ac}{a+c}$ or $\dfrac{a}{c} = \dfrac{a-b}{b-c}$.

Example-14

If $\frac{1}{a}+\frac{1}{c}+\frac{1}{a-b}+\frac{1}{c-b} = 0$ prove that a, b, c are in HP unless b = a + c.

Sol. $\Rightarrow \dfrac{a+c}{ac}+\dfrac{c-b+a-b}{(a-b)(c-b)} = 0$

$\Rightarrow \dfrac{a+c}{ac}+\dfrac{(a+c)-2b}{ac-b(a+c)+b^2} = 0$

Let $a + c = \lambda$ $\therefore \dfrac{\lambda}{ac}+\dfrac{\lambda-2b}{ac-b\lambda+b^2} = 0$

$\Rightarrow \dfrac{ac\lambda - b\lambda^2 + b^2\lambda + ac\lambda - 2abc}{ac(ac-b\lambda+b^2)} = 0$

$\Rightarrow ac\lambda - b\lambda^2 + b^2\lambda + ac\lambda - 2abc = 0$

$\Rightarrow 2ac(\lambda - b) - b\lambda(\lambda - b) = 0$

$\Rightarrow (2ac - b\lambda)(\lambda - b) = 0 \Rightarrow \lambda = b$ or $\lambda = \dfrac{2ac}{b}$

$\Rightarrow a + c = b$ or $a + c = \dfrac{2ac}{b}$ $(\because a + c = \lambda)$

$\Rightarrow a + c = b$ or $b = \dfrac{2ac}{a+c}$

$\Rightarrow$ a, b, c are in H.P. or a + c = b.

Example-15

If p^{th}, q^{th}, r^{th} terms of an H.P. be a, b, c respectively, prove that

$(q-r)bc + (r-p)ac + (p-q)ab = 0$

Sol. Let 'x' be the first term and 'd' be the common difference of the corresponding A.P..

so $\dfrac{1}{a} = x + (p-1)d$ ....(i)

$\dfrac{1}{b} = x + (q-1)d$ ....(ii)

$\dfrac{1}{c} = x + (r-1)d$ ....(iii)

(i) – (ii) $\Rightarrow ab(p-q)d = b - a$ ....(iv)

(ii) – (iii) $\Rightarrow bc(q-r)d = c - b$ ....(v)

(iii) – (i) $\Rightarrow ac(r-p)d = a - c$ ....(vi)

(iv) + (v) + (vi) gives

$bc(q-r) + ac(r-p) + ab(p-q) = 0$.

ARITHMETIC, GEOMETRIC & HARMONIC MEANS (AM, GM & HM)

Arithmetic Mean : If three terms are in AP then the middle term is called the AM between the other two, so if a, b, c are in AP, b is AM of a & c.

So, A.M. of two numbers a and b is $\frac{a+b}{2}$

AM for any n positive number $a_1, a_2, \dots, a_n$ is ;

$$A = \frac{a_1 + a_2 + a_3 + \dots + a_n}{n}.$$

n-Arithmetic Means Between Two Numbers : If a, b are any two given numbers & $a, A_1, A_2, \dots, A_n$, b are in AP then $A_1, A_2, \dots A_n$ are the n AM's between a & b.

$$A_1 = a + d = a + \frac{b-a}{n+1},$$

$$A_2 = a + 2d = a + \frac{2(b-a)}{n+1}$$

, ... ,

$$A_n = a + nd = a + \frac{n(b-a)}{n+1}$$

Note : Sum of n AM's inserted between a & b is equal to n times the single AM between a & b i.e. $\sum_{r=1}^{n} A_r = nA$ where A is the single AM between a & b.

Geometric Means : If a, b, c are in GP, b is the GM between a & c.

$b^2 = ac$, therefore $b = \sqrt{ac}$; $a > 0$, $c > 0$.

Geometric means of n positive numbers $a_1, a_2, \dots a_n$ is $(a_1.a_2 \dots a_n)^{1/n}$

n-Geometric Means Between a , b : If a, b are two given numbers & $a, G_1, G_2, \dots, G_n$, b are in GP. Then $G_1, G_2, G_3, \dots, G_n$ are n GMs between a & b.

$G_1 = ar = a(b/a)^{1/n+1}$, $G_2 = ar^2 = a(b/a)^{2/n+1}$, ... , $G_n = ar^n = a(b/a)^{n/n+1}$

Note : The product of n GMs between a & b is equal to the nth power of the single GM between a & b i.e. $\prod_{r=1}^{n} G_r = (G)^n$ where G is the single GM between a & b.

Harmonic Mean : If a, b, c are in HP, b is the HM between a & c, then $b = 2ac/[a+c]$.

If a, b are two given number such that

$a, H_1, H_2, H_3 \dots H_n$, b are in H.P.

then $H_1, H_2, H_3 \dots H_n$, are n harmonic means between a and b.

$\because a, H_1, H_2, H_3 \dots H_n, b \rightarrow$ H.P.

$$\therefore \frac{1}{a}, \frac{1}{H_1}, \frac{1}{H_2}, \frac{1}{H_3} \dots \frac{1}{H_n}, \frac{1}{b} \rightarrow \text{A.P.}$$

Common difference $(d) = \frac{\frac{1}{b} - \frac{1}{a}}{n+1}$

So, $\frac{1}{H_1} = \frac{1}{a} + d$, $\frac{1}{H_2} = \frac{1}{a} + 2d$ ... $\frac{1}{H_n} = \frac{1}{a} + nd$

INEQUALITIES

(i) A.M. ≥ G.M. ≥ H.M.

Let $a_1, a_2, \dots\dots, a_n$ be n positive real numbers, then we define their arithmetic mean (A), geometric mean (G) and harmonic mean (H) as

$$A = \frac{a_1 + a_2 + \dots\dots + a_n}{n},$$

$$G = (a_1 a_2 \dots\dots a_n)^{1/n} \text{ and } H = \frac{n}{\left(\frac{1}{a_1} + \frac{1}{a_2} + \frac{1}{a_3} + \dots\dots \frac{1}{a_n}\right)}$$

It can be shown that $A \geq G \geq H$. Moreover equality holds at either place if and only if

$a_1 = a_2 = \dots\dots\dots = a_n$

(ii) Weighted Means

Let $a_1, a_2, \dots\dots, a_n$ be n positive real numbers and $w_1, w_2, \dots\dots, w_n$ be n positive rational numbers. Then we define weighted Arithmetic mean (A*), weighted Geometric mean (G*) and weighted harmonic mean (H*) as

$$A^* = \frac{a_1 w_1 + a_2 w_2 + \dots + a_n w_n}{w_1 + w_2 + \dots + w_n},$$

$$G^* = (a_1^{w_1}.a_2^{w_2} \dots a_n^{w_n})^{\frac{1}{w_1 + w_2 + \dots w_n}}$$

$$\text{and } H^* = \frac{w_1 + w_2 + \dots + w_n}{\frac{w_1}{a_1} + \frac{w_2}{a_2} + \dots + \frac{w_n}{a_n}}.$$

$A^* \geq G^* \geq H^*$ More over equality holds at either place if & only if $a_1 = a_2 i \dots\dots = .a_n$

(iii) Cauchy's Schwartz Inequality:

If $a_1, a_2, \dots\dots a_n$ and $b_1, b_2, \dots\dots, b_n$ are 2n real numbers, then

$(a_1 b_1 + a_2 b_2 + \dots\dots + a_n b_n)^2 \leq (a_1^2 + a_2^2 + \dots\dots + a_n^2)(b_1^2 + b_2^2 + \dots\dots + b_n^2)$ with the equality holding if and only if

$$\frac{a_1}{b_1} = \frac{a_2}{b_2} = \dots\dots = \frac{a_n}{b_n}.$$

SOLVED EXAMPLE

Example-16

Insert 4 H.M between $\frac{2}{3}$ and $\frac{2}{13}$.

Sol. Let 'd' be the common difference of corresponding A.P..

So $d = \frac{\frac{13}{2} - \frac{3}{2}}{5} = 1.$

$\therefore \quad \frac{1}{H_1} = \frac{3}{2} + 1 = \frac{5}{2} \qquad \text{or } H_1 = \frac{2}{5}$

$\frac{1}{H_2} = \frac{3}{2} + 2 = \frac{7}{2} \qquad \text{or } H_2 = \frac{2}{7}$

$\frac{1}{H_3} = \frac{3}{2} + 3 = \frac{9}{2} \qquad \text{or } H_3 = \frac{2}{9}$

$\frac{1}{H_4} = \frac{3}{2} + 4 = \frac{11}{2} \qquad \text{or } H_4 = \frac{2}{11}.$

Example-17

If a, b, c > 0, prove that $\frac{a}{b} + \frac{b}{c} + \frac{c}{a} \geq 3$

Sol. Using the relation A.M. ≥ G.M. we have

$$\frac{\frac{a}{b}+\frac{b}{c}+\frac{c}{a}}{3} \geq \left(\frac{a}{b}.\frac{b}{c}.\frac{c}{a}\right)^{\frac{1}{3}} \Rightarrow \frac{a}{b}+\frac{b}{c}+\frac{c}{a} \geq 3$$

Example-18

If one AM 'a' and two GMN;s p and q be inserted between any two given numbers then show that $p^3 + q^3 = 2\,apq$.

Sol. Let two numbers are A and B.

if one AM 'a' is inserted between A & B then

A, a, B...........are in A.P.

therefore $\alpha = \frac{A+B}{2}$ or $\boxed{A+B=2a}$ ..(1)

If two GM's p & q are inserted between A & B then

A, p, q, B.......are in G.P.

therefore $p^2 = Aq$ & $q^2 = BP$

or $\boxed{A = \frac{p^2}{q}}$ ...(2)

or $\boxed{B = \frac{q^2}{p}}$ ...(3)

Adding equation (2) & (3)

$A + B = \frac{p^2}{q} + \frac{q^2}{p} \qquad \text{or } 2a = \frac{p^3+q^3}{pq}$

(using equation (1))

or $\boxed{p^3 + q^3 = 2apq}$

ARITHMETIC MEAN OF m^th^ POWER

Let a_1, a_2 ... , a_n be n positive real numbers and let m be a real number, then

$$\frac{a_1^m + a_2^m + \ldots + a_n^m}{n} \geq \left(\frac{a_1 + a_2 + \ldots + a_n}{n}\right)^m$$

, if $m \in R-[0, 1]$

However if $m \in (0, 1)$, then

$$\frac{a_1^m + a_2^m + \ldots + a_n^m}{n} \leq \left(\frac{a_1 + a_2 + \ldots + a_n}{n}\right)^m$$

Obviously if $m \in \{0, 1\}$, then

$$\frac{a_1^m + a_2^m + \ldots + a_n^m}{n} = \left(\frac{a_1 + a_2 + \ldots + a_n}{n}\right)^m$$

Example-19

Prove that $a^4 + b^4 + c^4 \geq abc\,(a + b + c)$, [a, b, c > 0]

Sol. Using mth power inequality, we get

$$\frac{a^4+b^4+c^4}{3} \geq \left(\frac{a+b+c}{3}\right)^4$$

$$= \left(\frac{a+b+c}{3}\right)\left(\frac{a+b+c}{3}\right)^3$$

$$\geq \left(\frac{a+b+c}{3}\right)[(abc)^{1/3}]^3 \quad (\because \text{ A.M} \geq \text{G.M})$$

or $$\frac{a^4+b^4+c^4}{3} \geq \left(\frac{a+b+c}{3}\right)abc$$

$\therefore \quad a^4 + b^4 + c^4 \geq abc\,(a + b + c).$

Example-20

Prove that $\frac{s}{s-a} + \frac{s}{s-b} + \frac{s}{s-c} \geq \frac{9}{2}$, if s = a + b + c, [a, b, c > 0]

Sol. A.M. ≥ H.M.

$$\Rightarrow \frac{(a+b)+(b+c)+(c+a)}{3} \geq \frac{3}{\frac{1}{a+b}+\frac{1}{b+c}+\frac{1}{c+a}}$$

$$\Rightarrow \frac{1}{a+b}+\frac{1}{b+c}+\frac{1}{c+a} \geq \frac{9}{2(a+b+c)}$$

SIGMA NOTATIONS:

Sigma (Σ) notation

Σ indicates sum i.e., $\sum_{i=1}^{n} i = \sum n = 1+2+3+......+n$

(i) $\sum_{i=1}^{n} \frac{i+1}{i+2} = \frac{1+1}{1+2} + \frac{2+1}{2+2} + \frac{3+1}{3+2} + ... + \frac{n+1}{n+2}$

(ii) $\sum_{i=1}^{m} a = a + a + ... + a$ m times

$= am$ where a is constant

(iii) $\sum_{i=1}^{m} ai = a\sum_{i=1}^{m} i = a(1+2+....+m)$

(iv) $\sum_{i=1}^{m} (i^3 - 2i^2 + i) = \sum_{i=1}^{m} i^3 - 2\sum_{i=1}^{m} i^2 + \sum_{i=1}^{m} i$

Therorems :

(a) $\sum_{r=1}^{n} (a_r \pm b_r) = \sum_{r=1}^{n} a_r \pm \sum_{r=1}^{n} b_r$

(b) $\sum_{r=1}^{n} ka_r = \sum_{r=1}^{n} a_r$

(c) $\sum_{r=1}^{n} k =$ nk : where k is a constant.

Important Results

(i) Sum of first n natural numbers

$$\Sigma n = 1+2+....+n$$

$$= \frac{n(n+1)}{2}$$

(ii) Sum of squares of first n natural numbers

$$\Sigma n^2 = 1^2 + 2^2 + ... + n^2$$

$$= \frac{n(n+1)(2n+1)}{6}$$

(iii) Sum of cubes of first n natural numbers

$$\Sigma n^3 = 1^3 + 2^3 + 3^3 + ... + n^3$$

$$= \left(\frac{n(n+1)}{2}\right)^2 = (\Sigma n)^2$$

(iv) Sum of n terms of a sequence $T_n = an^3 + bn^2 + cn + d$

$$\boxed{S_n = a\Sigma n^3 + b\Sigma n^2 + c\Sigma n + dn}$$

Example-21

Find the sum of the series

$3.5 + 6.8 + 9.11 + ... +$ upto n terms

Sol. **n^{th} *term of 3, 6, 9, ... is 3*n**

n^{th} term of 5, 8, 11, ... is $(3n + 2)$

$\therefore$ $T_n = 3n(3n+2) = 9n^2 + 6n$

$\therefore$ $S_n = 9\Sigma n^2 + 6\Sigma n$

$$= \frac{9n(n+1)(2n+1)}{6} + \frac{6n(n+1)}{2}$$

$$= \frac{3}{2}n(n+1)[2n+1+2]$$

$$= \frac{3n(n+1)(2n+3)}{2}$$

METHOD OF DIFFERENCES

Let u_1, u_2, u_3...... be a sequence, such that $u_2 - u_1, u_3 - u_2$,............ is either an A.P. or a G.P. then nth term u_n of this sequence is obtained as follows

$S = u_1 + u_2 + u_3 + ... + u_n$ ...(i)

$S = \quad u_1 + u_2 + ...+ u_{n-1} + u_n$ ...(ii)

(i) – (ii)

$\Rightarrow$ $u_n = u_1 + (u_2 - u_1) + (u_3 - u_2) + ... + (u_n - u_{n-1})$

Where the series $(u_2 - u_1) + (u_3 - u_2) + ... + (u_n - u_{n-1})$ is either in A.P. or in G.P. then we can find u_n and hence sum of this series as $S = \sum_{r=1}^{k} u_r$

Remark : It is not always necessary that the series of first order of differences i.e. $u_2 - u_1, u_3 - u_2 ... u_n - u_{n-1}$ is always either in A.P. or in G.P. in such case let $u_1 = T_1$, $u_2 - u_1 = T_2, u_3 - u_2 = T_3$ ....$,u_n - u_{n-1} = T_n$.

So $u_n = T_1 + T_2 + ... + T_n$ .... (i)

$u_n = \quad T_1 + T_2 + ...+ T_{n-1} + T_n$ .... (ii)

(i) – (ii)

$\Rightarrow$ $T_n = T_1 + (T_2 - T_1) + (T_3 - T_2) + ... + (T_n + T_{n-1})$

Now, the series $(T_2 - T_1) + (T_3 - T_2) + ... + (T_n - T_{n-1})$ is series of second order of differences and when it is either in A.P. or in G.P., then $u_n = u_1 + \Sigma T_r$

Otherwise in the similar way we find series of higher order of differences and the n^{th} term of the series.

If possible express r^{th} term as difference of two terms as $t_r = f(r) - f(r \pm 1)$. This can be explained with help of examples given below.

SOLVED EXAMPLE

Example-22

Find the sum to n terms of the series,

$0.7 + 7.7 + 0.77 + 77.7 + 0.777 + 777.7 + 0.7777 + \ldots\ldots$

where n is even .

Sol. $n = 2m$

$s = (0.7 + 0.77 + 0.777 + \ldots\ldots \text{ m term}) + (7.7 + 77.7 + 777.7 + \ldots\ldots \text{ m terms})$

$$= \frac{7}{9}(0.9 + 0.99 + 0.999 + \ldots\ldots \text{ m terms})$$

$$+ \frac{7}{90}((10^2 - 1) + (10^3 - 1) + \ldots\ldots + (10^{m+1} - 1))$$

$$= \frac{7}{9}\left\{\frac{n}{2} - \frac{1}{9}\left(1 - \frac{1}{10^{n/2}}\right)\right\}$$

$$+ \frac{7}{90}\left\{\frac{100}{9}\left(10^{n/2} - 1\right) - \frac{n}{2}\right\}$$

Example-23

Find the sum to n-terms $3 + 7 + 13 + 21 + \ldots\ldots$

Sol. Let $S = 3 + 7 + 13 + 21 + \ldots\ldots\ldots\ldots + T_n \ldots\ldots\ldots\ldots\ldots\ldots$(i)

$S = \quad 3 + 7 + 13 + \ldots\ldots\ldots\ldots\ldots\ldots + T_{n-1} + T_n \ldots\ldots\ldots\ldots\ldots$(ii)

(i) – (ii) $\Rightarrow T_n = 3 + 4 + 6 + 8 + \ldots\ldots\ldots\ldots\ldots\ldots\ldots + (T_n - T_{n-1})$

$$= 3 + \frac{n-1}{2}[8 + (n-2)2]$$

$= 3 + (n-1)(n+2) = n^2 + n + 1$

Hence $S = \Sigma(n^2 + n + 1) = \Sigma n^2 + \Sigma n + \Sigma 1$

$$= \frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2} + n = \frac{n}{3}(n^2 + 3n + 5)$$

Example-24

Find the nth term and the sum of n term of the series 2, 12, 36, 80, 150, 252

Sol. $S = 2 + 12 + 36 + 80 + 150 + 252 + \ldots\ldots + T_n$ .... (i)

$S = \quad 2 + 12 + 36 + 80 + 150 + \ldots\ldots\ldots + T_{n-1} + T_n$.. (ii)

(i) – (ii) $\Rightarrow T_n = 2 + 10 + 24 + 44 + 70 + 102 + \ldots\ldots + (T_n - T_{n-1})$ .... (iii)

$T_n = 2 + 10 + 24 + 44 + 70 + 102 + \ldots\ldots\ldots + (T_{n-1} - T_{n-2}) + (T_n - T_{n-1})$ ..... (iv)

(iii) – (iv)

$$\Rightarrow T_n - T_{n-1} = 2 + 8 + 14 + 20 + 26 + \ldots. = \frac{n}{2}[4 + (n-1)6]$$

$= n[3n-1] \Rightarrow T_n - T_{n-1} = 3n^2 - n$

$\therefore$ general term of given series is

$\Sigma\, T_n - T_{n-1} = \Sigma\, 3n^2 - n = n^3 + n^2.$

Hence sum of this sereis is

$$S = \Sigma\, n^3 + \Sigma\, n^2 = \frac{n^2(n+1)^2}{4} + \frac{n(n+1)(2n+1)}{6}$$

$$= \frac{n(n+1)}{12}(3n^2 + 7n + 2) = \frac{1}{12}n(n+1)(n+2)(3n+1)$$

Example-25

Find the sum of the series to n terms whose general term is $2n + 1$.

Sol. $S_n = \Sigma T_n = \Sigma(2n + 1)$

$$= 2\Sigma n + \Sigma 1 = \frac{2(n+1)n}{2} + n = n^2 + 2n$$

Example-26

(a) Sum the following series to infinity

$$\frac{1}{1\cdot4\cdot7} + \frac{1}{4\cdot7\cdot10} + \frac{1}{7\cdot10\cdot13} + \ldots\ldots\ldots.$$

(b) Sum the following series upto n-terms

$1\cdot2\cdot3\cdot4 + 2\cdot3\cdot4\cdot5 + 3\cdot4\cdot5\cdot6 + \ldots\ldots\ldots.$

Sol. **(a)** $T_n = \dfrac{1}{[1 + (n-1)3][1 + 3n][4 + 3n]}$

$$= \frac{1}{(3n-2)(2n+1)(3n+4)}$$

$$= \frac{1}{6}\left[\frac{1}{(3n-2)(3n+1)} - \frac{1}{(3n+1)(3n+4)}\right]$$

$$T_1 = \frac{1}{6}\left[\frac{1}{1\cdot4} - \frac{1}{4\cdot7}\right]$$

$$T_2 = \frac{1}{6}\left[\frac{1}{4\cdot7} - \frac{1}{7\cdot10}\right]$$

$$\vdots \qquad \vdots \qquad \vdots$$

$$T_n = \frac{1}{6}\left[\frac{1}{(3n-2)(3n+1)} - \frac{1}{(3n+1)(3n+4)}\right]$$

$$\therefore \quad S_n = \sum T_n = \frac{1}{6}\left[\frac{1}{1\cdot4} - \frac{1}{(3n+1)(3n+4)}\right]$$

$$= \left[\frac{1}{24} - \frac{1}{6(3n+1)(3n+4)}\right] \quad \text{as } n \to \infty$$

$$\therefore \; S_\infty = \frac{1}{24}$$

(b) $1\cdot 2\cdot 3\cdot 4+2\cdot 3\cdot 4\cdot 5+3\cdot 4\cdot 5\cdot 6+............$

$$T_n=\frac{1}{5}n(n+1)(n+2)(n+3)\,[(n+4)-(n-1)]$$

$$\therefore\quad T_1=\frac{1}{5}[1\cdot 2\cdot 3\cdot 4\cdot 5-0]$$

$$T_2=\frac{1}{5}[2\cdot 3\cdot 4\cdot 5\cdot 6-1\cdot 2\cdot 3\cdot 4\cdot 5]$$

$$\vdots \qquad \vdots \qquad \vdots$$

$$T_n=\frac{1}{5}[n(n+1)(n+2)(n+3)(n+4)$$
$$-(n-1)n(n+1)(n+2)(n+3)]$$

$$\therefore\quad S_n=\sum T_n$$

$$=\frac{1}{5}[n(n+1)(n+2)(n+3)(n+4)]$$

$$=\frac{1}{5}n(n+1)(n+2)(n+3)(n+4)$$

Example-27

Sun to n terms of the series

$$\frac{4}{1.2.3}+\frac{5}{2.3.4}+\frac{6}{3.4.5}+........$$

Sol. Let

$$T_r=\frac{r+3}{r(r+1)(r+2)}=\frac{1}{(r+1)(r+2)}+\frac{3}{r(r+1)(r+2)}$$

$$=\left[\frac{1}{r+1}-\frac{1}{r+2}\right]+\frac{3}{2}\left[\frac{1}{r(r+1)}-\frac{1}{(r+1)(r+2)}\right]$$

$$\therefore\quad S=\left[\frac{1}{2}-\frac{1}{n+2}\right]+\frac{3}{2}\left[\frac{1}{2}-\frac{1}{(n+1)(n+2)}\right]$$

$$=\frac{5}{4}-\frac{1}{n+2}\left[1+\frac{3}{2(n+1)}\right]=\frac{5}{4}-\frac{1}{2(n+1)(n+2)}[2n+5]$$

Example-28

Find the sum of the infinite series

$$\frac{3}{1^2.2^2}+\frac{5}{2^2.3^2}+\frac{7}{3^2.4^2}+.....$$

Sol. Here $u_n=\dfrac{2n+1}{n^2(n+1)^2}=\dfrac{(n+1)^2-n^2}{n^2(n+1)^2}=\dfrac{1}{n^2}-\dfrac{1}{(n+1)^2}$

$$\therefore\quad u_1=\frac{1}{1^2}-\frac{1}{2^2},$$

$$u_2=\frac{1}{2^2}-\frac{1}{3^2}\ ....\ u_n=\frac{1}{n^2}-\frac{1}{(n+1)^2}$$

By addition, $S_n=1-\dfrac{1}{(n+1)^2}$

If $n\to\infty$, then $S_n\to 1$. Hence the sum of the infinite series is 1.

Example-29

If $1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+....=\log_e 2$, then sum

$$\frac{1}{1.3}+\frac{1}{2.5}+\frac{1}{3.7}+\frac{1}{4.9}+..................$$

Sol. If S denote the sum of the given series, then

$$S=\frac{2}{2.3}+\frac{2}{4.5}+\frac{2}{6.7}+\frac{2}{8.9}+....$$

$$=2\left[\left(\frac{1}{2}-\frac{1}{3}\right)+\left(\frac{1}{4}-\frac{1}{5}\right)+\left(\frac{1}{6}-\frac{1}{7}\right)+\left(\frac{1}{8}-\frac{1}{9}\right)+.....\right]$$

$$=2\left[1-\left(1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\frac{1}{5}-...\right)\right]$$

$$=2\,[1-\log_e 2]=2-2\log_e 2.$$

Example-30

If a, b, c, d be four distinct positive quantities in G.P., then show that

(a) $a+d>b+c$

(b) $\dfrac{1}{cd}+\dfrac{1}{ab}>2\left(\dfrac{1}{bd}+\dfrac{1}{ac}-\dfrac{1}{ad}\right)$.

Sol. Since a, b, c d are in G.P., therefore

(a) using A.M. > G.M., we have for the first three terms

$$\frac{a+c}{2}>b\quad \text{i.e. } a+c>2b \quad ....(1)$$

$$\frac{b+d}{2}>c\quad \text{i.e. } b+d>2c \quad ....(2)$$

From results (1) and (2), we have $a+c+b+d>2b+2c$ i.e. $a+d>b+c$

which is the desired result.

(b) using G.M. > H.M., we have for the first three terms

$$b>\frac{2ac}{a+c}\quad \text{i.e. } ab+bc>2ac\ ...(3)$$

and for the last three terms $c>\dfrac{2bd}{b+d}$

i.e. $bc+cd>2bd$ ....(4)

From results (3) and (4), we have

$ab+cd+2bc>2ac+2bd$ i.e. $ab+cd>2(ac+bd-bc)$

i.e. $\dfrac{1}{cd}+\dfrac{1}{ab}>2\left(\dfrac{1}{bd}+\dfrac{1}{ac}-\dfrac{1}{ad}\right)$

[dividing both sides by abcd] which is the desired result.

EXERCISE-I

Arithmetic Progression and its properties

Q.1 The sum of 24 terms of the following series $\sqrt{2}+\sqrt{8}+\sqrt{18}+\sqrt{32}+.........$ is

(1) 300 (2) $300\sqrt{2}$
(3) $200\sqrt{2}$ (4) $100\sqrt{2}$

Q.2 If the p^{th}, q^{th} and r^{th} term of an arithmetic sequence are a, b and c respectively, then the value of $[a(q-r)+b(r-p)+c(p-q)]=$

(1) 1 (2) –1
(3) 0 (4) 1/2

Q.3 Let T_r be the r^{th} term of an A.P. for $r = 1, 2, 3,....$. If for some positive integers m, n we have $T_m=\frac{1}{n}$ and $T_n=\frac{1}{m}$, then T_{mn} equals

(1) $\frac{1}{mn}$ (2) $\frac{1}{m}+\frac{1}{n}$ (3) 1 (4) 0

Q.4 If $1, \log_9(3^{1-x}+2), \log_3(4.3^x-1)$ are in A.P. then x equals

(1) $\log_3 4$ (2) $1-\log_3 4$
(3) $1-\log_4 3$ (4) $\log_4 3$

Q.5 If the ratio of the sum of n terms of two A.P.'s be $(7n+1):(4n+27)$, then the ratio of their 11^{th} terms will be

(1) $2:3$ (2) $3:4$
(3) $4:3$ (4) $5:6$

Q.6 The ratio of the sums of first n even numbers and odd numbers will be

(1) $1:n$ (2) $(n+1):1$
(3) $(n+1):n$ (4) $(n-1):1$

Q.7 If S_n denotes the sum of n terms of an arithmetic progression, then the value of $(S_{2n}-S_n)$ is equal to

(1) $2S_n$ (2) S_{3n} (3) $\frac{1}{3}S_{3n}$ (4) $\frac{1}{2}S_n$

Q.8 The sum of the integers from 1 to 100 which are not divisible by 3 or 5 is

(1) 2489 (2) 4735
(3) 2317 (4) 2632

Q.9 Let S_n denotes the sum of n terms of an A.P. If $S_{2n}=3S_n$, then ratio $\frac{S_{3n}}{S_n}=$

(1) 4 (2) 6 (3) 8 (4) 10

Arithmetic Mean

Q.10 A number is the reciprocal of the other. If the arithmetic mean of the two numbers be $\frac{13}{12}$, then the numbers are

(1) $\frac{1}{4},\frac{4}{1}$ (2) $\frac{3}{4},\frac{4}{3}$ (3) $\frac{2}{5},\frac{5}{2}$ (4) $\frac{3}{2},\frac{2}{3}$

Geometric Progression and its properties

Q.11 If $x, 2x+2, 3x+3$, are in G.P., then the fourth term is

(1) 27 (2) –27
(3) 13.5 (4) –13.5

Q.12 The 6^{th} term of a G.P. is 32 and its 8^{th} term is 128, then the common ratio of the G.P. is

(1) – 1 (2) 2 (3) 4 (4) – 4

Q.13 The sum of 100 terms of the series $.9+.09+.009.........$ will be

(1) $1-\left(\frac{1}{10}\right)^{100}$ (2) $1+\left(\frac{1}{10}\right)^{106}$
(3) $1-\left(\frac{1}{10}\right)^{106}$ (4) $1+\left(\frac{1}{10}\right)^{100}$

Q.14 The sum of the series $6+66+666+..........$ upto n terms is

(1) $(10^{n-1}-9n+10)/81$
(2) $2(10^{n+1}-9n-10)/27$
(3) $2(10^{n}-9n-10)/27$
(4) $(10^{n+1}-9n-10)/27$

Q.15 If a, b, c are in G.P., then

(1) a^2, b^2, c^2 are in G.P.

(2) $a^2(b+c), c^2(a+b), b^2(a+c)$ are in G.P.

(3) $\frac{a}{b+c}, \frac{b}{c+a}, \frac{c}{a+b}$ are in G.P.

(4) a^2, b^2, c^2 are in A.P.

Q.16 Consider an infinite G.P. with first term a and common ratio r, its sum is 4 and the second term is 3/4, then

(1) $a = \frac{7}{4}, r = \frac{3}{7}$ (2) $a = \frac{3}{2}, r = \frac{1}{2}$

(3) $a = 2, r = \frac{3}{8}$ (4) $a = 3, r = \frac{1}{4}$

[Geometric Mean]

Q.17 If G be the geometric mean of x and y, then

$\frac{1}{G^2 - x^2} + \frac{1}{G^2 - y^2} =$

(1) G^2 (2) $\frac{1}{G^2}$ (3) $\frac{2}{G^2}$ (4) $3G^2$

A.G.P.

Q.18 $1 + \frac{3}{2} + \frac{5}{2^2} + \frac{7}{2^3} + ...\infty$ is equal to

(1) 3 (2) 6 (3) 9 (4) 12

Q.19 The sum of the first n terms of the series $\frac{1}{2} + \frac{3}{4} + \frac{7}{8} + \frac{15}{16} + ...$ is

(1) $2^n - n - 1$ (2) $1 - 2^{-n}$

(3) $n + 2^{-n} - 1$ (4) $2^n - 1$

Harmonic Progression and Harmonic mean

Q.20 Which number should be added to the numbers 13, 15, 19 so that the resulting numbers be the consecutive terms of a H.P.

(1) 7 (2) 6 (3) –6 (4) –7

Q.21 If 5th term of a H.P. is $\frac{1}{45}$ and 11^{th} term is $\frac{1}{69}$, then its 16^{th} term will be

(1) 1/89 (2) 1/85 (3) 1/80 (4) 1/79

Q.22 If the 7^{th} term of a H.P. is $\frac{1}{10}$ and the 12^{th} term is $\frac{1}{25}$, then the 20^{th} term is

(1) $\frac{1}{37}$ (2) $\frac{1}{41}$ (3) $\frac{1}{45}$ (4) $\frac{1}{49}$

Q.23 If H is the harmonic mean between p and q, then the value of $\frac{H}{p} + \frac{H}{q}$ is

(1) 2 (2) $\frac{pq}{p+q}$ (3) $\frac{p+q}{pq}$ (4) $\frac{2pq}{p+q}$

Relation between A.M., G.M., H.M.

Q.24 If the arithmetic mean of two numbers be A and geometric mean be G, then the numbers will be

(1) $A \pm (A^2 - G^2)$

(2) $\sqrt{A} \pm \sqrt{A^2 - G^2}$

(3) $A \pm \sqrt{(A+G)(A-G)}$

(4) $\frac{A \pm \sqrt{(A+G)(A-G)}}{2}$

Q.25 If the arithmetic, geometric and harmonic means between two positive real numbers be A, G and H, then

(1) $A^2 = GH$ (2) $H^2 = AG$

(3) $G = AH$ (4) $G^2 = AH$

Q.26 If a^2, b^2, c^2 be in A.P., then $\frac{a}{b+c}, \frac{b}{c+a}, \frac{c}{a+b}$ will be in

(1) A.P. (2) G.P. (3) H.P. (4) A.G.P.

Miscelleneous Series

Q.27 The sum of the series $1 + (1+2) + (1+2+3) + ...$ upto n terms, will be

(1) $n^2 - 2n + 6$ (2) $\frac{n(n+1)(2n-1)}{6}$

(3) $n^2 + 2n + 6$ (4) $\frac{n(n+1)(n+2)}{6}$

Q.28 $\dfrac{1^3+2^3+3^3+4^3+........12^3}{1^2+2^2+3^2+4^2+.........12^2}=$

(1) $\dfrac{234}{25}$ (2) $\dfrac{243}{35}$ (3) $\dfrac{263}{27}$ (4) $\dfrac{236}{35}$

Q.29 If the n^{th} term of a series be $3+n(n-1)$, then the sum of n terms of the series is

(1) $\dfrac{n^2+n}{3}$ (2) $\dfrac{n^3+8n}{3}$

(3) $\dfrac{n^2+8n}{5}$ (4) $\dfrac{n^2-8n}{3}$

Q.30 If the sum of $1+\dfrac{1+2}{2}+\dfrac{1+2+3}{3}+.....$ to n terms is S, then S is equal to

(1) $\dfrac{n(n+3)}{4}$ (2) $\dfrac{n(n+2)}{4}$

(3) $\dfrac{n(n+1)(n+2)}{6}$ (4) n^2

EXERCISE-II

Q.1 The first term of an A.P. of consecutive integer is p^2+1. The sum of (2p + 1) terms of this series can be expressed as

(1) $(p+1)^2$ (2) $(2p+1)(p+1)^2$

(3) $(p+1)^3$ (4) $p^3+(p+1)^3$

Q.2 If $a_1, a_2, a_3,........$ are in A.P. such that $a_1+a_5+a_{10}+a_{15}+a_{20}+a_{24}=225$, then $a_1+a_2+a_3+.....+a_{23}+a_{24}$ is equal to

(1) 909 (2) 75

(3) 750 (4) 900

Q.3 The sum of integers from 1 to 100 that are divisible by 2 or 5 is

(1) 2550 (2) 1050

(3) 3050 (4) none of these

Q.4 The interior angles of a polygon are in A.P. If the smallest angle is 120º & the common difference is 5º, then the number of sides in the polygon is:

(1) 7 (2) 9

(3) 16 (4) none of these

Q.5 Consider an A.P. with first term 'a' and the common difference 'd'. Let S_k denote the sum of its first K terms.

If $\dfrac{S_{kx}}{S_x}$ is independent of x, then

(1) a = d/2 (2) a = d

(3) a = 2d (4) none of these

Q.6 If $x \in R$, the numbers $5^{1+x}+5^{1-x}$, a/2, 25^x+25^{-x} form an A.P. then 'a' must lie in the interval:

(1) [1, 5] (2) [2, 5]

(3) [5, 12] (4) $[12, \infty)$

Q.7 The sum of the series $\dfrac{1}{\log_2 4}+\dfrac{1}{\log_4 4}+\dfrac{1}{\log_8 4}+$

(1) $\dfrac{1}{2}n(n+1)$ (2) $\dfrac{1}{12}n(n+1)(2n+1)$

(3) $\dfrac{1}{n(n+1)}$ (4) $\dfrac{1}{4}n(n+1)$

Q.8 Sum of the series $S = 1^2-2^2+3^2-4^2+.....-2002^2+2003^2$ is

(1) 2007006 (2) 1005004

(3) 2000506 (4) None of these

Q.9 If a and b are p^{th} and q^{th} terms of an AP, then the sum of its (p + q) terms is

(1) $\dfrac{p+q}{2}\left[a-b+\dfrac{a+b}{p-q}\right]$

(2) $\dfrac{p+q}{2}\left[a+b+\dfrac{a-b}{p-q}\right]$

(3) $\dfrac{p-q}{2}\left[a+b+\dfrac{a+b}{p+q}\right]$

(4) None of these

Q.10 There are n A.M's between 3 and 54, such that the 8th mean: $(n-2)^{th}$ mean:: 3: 5. The value of n is.

(1) 12 (2) 16 (3) 18 (4) 20

Q.11 The third term of a G.P. is 4. The product of the first five terms is
(1) 4^3 (2) 4^5
(3) 4^4 (4) None of these

Q.12 If S is the sum of infinity of a G.P. whose first term is 'a', then the sum of the first n terms is
(1) $S\left(1-\frac{a}{S}\right)^n$ (2) $S\left[1-\left(1-\frac{a}{S}\right)^n\right]$
(3) $a\left[1-\left(1-\frac{a}{S}\right)^n\right]$ (4) None of these

Q.13 For a sequence $\{a_n\}$, $a_1 = 2$ and $\frac{a_{n+1}}{a_n} = \frac{1}{3}$. Then $\sum_{r=1}^{20} a_r$ is
(1) $\frac{20}{2}[4+19\times 3]$ (2) $3\left(1-\frac{1}{3^{20}}\right)$
(3) $2(1-3^{20})$ (4) None of these

Q.14 α, β be the roots of the equation $x^2 - 3x + a = 0$ and γ, δ the roots of $x^2 - 12x + b = 0$ and numbers $\alpha, \beta, \gamma, \delta$ (in this order) form an increasing G.P., then
(1) $a = 3, b = 12$ (2) $a = 12, b = 3$
(3) $a = 2, b = 32$ (4) $a = 4, b = 16$

Q.15 In a G.P. of positive terms, any term is equal to the sum of the next two terms. The common ratio of the G.P. is
(1) $2 \cos 18^\circ$ (2) $\sin 18^\circ$
(3) $\cos 18^\circ$ (4) $2 \sin 18^\circ$

Q.16 The rational number, which equals the number $2.\overline{357}$ with recurring decimal is
(1) $\frac{2355}{1001}$ (2) $\frac{2379}{997}$
(3) $\frac{2355}{999}$ (4) none of these

Q.17 If sum of the infinite G.P., $p, 1, \frac{1}{p}, \frac{1}{p^2}, \frac{1}{p^3}, \ldots$ is $\frac{9}{2}$, then value of p is
(1) $3, \frac{3}{2}$ (2) $\frac{2}{3}, 2$ (3) $\frac{3}{2}, 1$ (4) $\frac{1}{3}, 2$

Q.18 The sum to 10 terms of the series $\sqrt{2}+\sqrt{6}+\sqrt{18}+\sqrt{54}+\ldots$ is
(1) $121(\sqrt{6}+\sqrt{2})$ (2) $\frac{121}{2}(\sqrt{3}+1)$
(3) $243(\sqrt{3}+1)$ (4) $243(\sqrt{3}-1)$

Q.19 If p is positive, then the sum to infinity of the series, $\frac{1}{1+p} - \frac{1-p}{(1+p)^2} + \frac{(1-p)^2}{(1+p)^3} - \ldots\ldots$ is
(1) 1/2 (2) 3/4 (3) 1 (4) None of these

Q.20 If $x > 0$, and $\log_2 x + \log_2(\sqrt{x}) + \log_2(\sqrt[4]{x}) + \log_2(\sqrt[8]{x}) + \log_2(\sqrt[16]{x}) + \ldots = 4$, then x equals
(1) 2 (2) 3 (3) 4 (4) 5

Q.21 If $3 + \frac{1}{4}(3+d) + \frac{1}{4^2}(3+2d) + \ldots + \text{upto } \infty = 8$, then the value of d is
(1) 9 (2) 5 (3) 1 (4) None of these

Q.22 If the sum of the roots of the quadratic equation, $ax^2 + bx + c = 0$ is equal to sum of the squares of their reciprocals, then $\frac{a}{c}, \frac{b}{a}, \frac{c}{b}$ are in
(1) A.P. (2) G.P. (3) H.P. (4) none of these

Q.23 If $x = \sum_{n=0}^{\infty} a^n$, $y = \sum_{n=0}^{\infty} b^n$, $z = \sum_{n=0}^{\infty} c^n$ where a, b, c are in AP and $|a| < 1, |b| < 1, |c| < 1$, then x, y, z are in
(1) HP
(2) Arithmetic-Geometric Progression
(3) AP
(4) GP

Q.24 If a, b, c are in A.P. p, q, r are in H.P. and ap, bq, cr are in G.P., then $\frac{p}{r}+\frac{r}{p}$ is equal to
(1) $\frac{a}{c}+\frac{c}{a}$ (2) $\frac{a}{c}-\frac{c}{a}$
(3) $\frac{b}{q}+\frac{q}{b}$ (4) $\frac{b}{q}-\frac{a}{p}$

Q. 25 If $a^x = b^y = c^z = d^t$ and a, b, c, d are in G.P., then x, y, z, t are in
(1) A.P. (2) G.P. (3) H.P. (4) none of these

Q.26 If A, G & H are respectively the A.M., G.M. & H.M. of three positive numbers a, b, & c, then the equation whose roots are a, b, & c is given by:
(1) $x^3 - 3Ax^2 + 3G^3x - G^3 = 0$
(2) $x^3 - 3Ax^2 + 3(G^3/H)x - G^3 = 0$
(3) $x^3 + 3Ax^2 + 3(G^3/H)x - G^3 = 0$
(4) $x^3 - 3Ax^2 - 3(G^3/H)x + G^3 = 0$

Q.27 The sum $\sum_{r=2}^{\infty} \frac{1}{r^2-1}$ is equal to

(1) 1 (2) 3/4
(3) 4/3 (4) None of these

Q.28 If $\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+ \ldots$ upto $\infty = \frac{\pi^2}{6}$, then $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\ldots =$

(1) $\pi^2/12$ (2) $\pi^2/24$
(3) $\pi^2/8$ (4) None of these

Q.29 The sum of the first n-terms of the series $1^2 + 2.2^2 + 3^2 + 2.4^2 + 5^2 + 2.6^2 + \ldots\ldots$ is $\frac{n(n+1)^2}{2}$, when n is even. When n is odd, the sum is

(1) $\frac{n(n+1)^2}{4}$ (2) $\frac{n^2(n+2)}{4}$
(3) $\frac{n^2(n+1)}{2}$ (4) $\frac{n(n+2)^2}{4}$

Q.30 $1^2 + 2^2 + \ldots\ldots + n^2 = 1015$, then value of n is
(1) 15 (2) 14
(3) 13 (4) None of these

EXERCISE-III

Q.1 If $a_1, a_2, \ldots\ldots, a_n$ are distinct terms of an A.P., then
(A) $a_1 + 2a_2 + a_3 = 0$
(B) $a_1 - 2a_2 + a_3 = 0$
(C) $a_1 + 3a_2 - 3a_3 - a_4 = 0$
(D) $a_1 - 4a_2 + 6a_3 - 4a_4 + a_5 = 0$

Q.2 If x, |x + 1|, |x – 1| are three terms of an A.P., then its sum upto 20 terms is –
(A) 180 (B) 350
(C) 90 (D) 720

Q.3 For the A.P. given by $a_1, a_2, \ldots\ldots a_n, \ldots\ldots$, the equations satisfied are
(A) $a_1 + 2a_2 + a_3 = 0$
(B) $a_1 - 2a_2 + a_3 = 0$
(C) $a_1 + 3a_2 - 3a_2 - a_4 = 0$
(D) $a_1 - 4a_2 + 6a_3 - 4a_4 + a_5 = 0$

Q.4 The sides of a right triangle form a G.P. The tangent of the smallest angle is

(A) $\sqrt{\frac{\sqrt{5}+1}{2}}$ (B) $\sqrt{\frac{\sqrt{5}-1}{2}}$
(C) $\sqrt{\frac{2}{\sqrt{5}+1}}$ (D) $\sqrt{\frac{2}{\sqrt{5}-1}}$

Q.5 If b_1, b_2, b_3 $(b_i > 0)$ are three successive terms of a G.P. with common ratio r, the value of r for which the inequality $b_3 > 4b_2 - 3b_1$ holds is given by
(A) $r > 3$ (B) $0 < r < 1$
(C) $r = 3.5$ (D) $r = 5.2$

Q.6 If positive numbers a, b, c are in A.P. and a^2, b^2, c^2 are in H.P., then
(A) $a = b = c$ (B) $2b = a + c$
(C) $b^2 = \sqrt{\frac{ac}{8}}$ (D) None of these

Q.7 If the arithmetic mean of two positive numbers a & b $(a > b)$ is twice their geometric mean, then a: b is:
(A) $2+\sqrt{3} : 2-\sqrt{3}$ (B) $7+4\sqrt{3} : 1$
(C) $1 : 7-4\sqrt{3}$ (D) $2 : \sqrt{3}$

Q.8 Let a, x, b be in A.P; a, y, b be in G.P. and a, z, b be in H.P. If $x = y + 2$ and $a = 5z$ then
(A) $y^2 = xz$ (B) $x > y > z$
(C) $a = 9, b = 1$ (D) $a = 1/4, b = 9/4$

Q.9 If $\sum_{r=1}^{n} r(r+1)(2r+3) = an^4 + bn^3 + cn^2 + dn + e$, then
(A) $a + c = b + d$
(B) $e = 0$
(C) $a, b - 2/3, c - 1$ are in A.P.
(D) c/a is an integer

Q.10 The value of $\sum_{r=1}^{n} \frac{1}{\sqrt{a+rx}+\sqrt{a+(r-1)x}}$ is

(A) $\frac{n}{\sqrt{a}+\sqrt{a+nx}}$ (B) $\frac{n}{\sqrt{a}-\sqrt{a+nx}}$
(C) $\frac{\sqrt{a+nx}-\sqrt{a}}{x}$ (D) $\frac{\sqrt{a}+\sqrt{a+nx}}{x}$

Comprehension # 01 (Q. No. 11 to 13)

We know that $1 + 2 + 3 + ... = \frac{n(n+1)}{2} = f(n)$,

$1^2 + 2^2 + 3^2 + ... + n^2 = \frac{n(n+1)(2n+1)}{6} = g(n)$,

$1^3 + 2^3 + 3^3 + ... + n^3 = \left(\frac{n(n+1)}{2}\right)^2 = h(n)$

Q.11 g(n) – g(n – 1) must be equal to

(A) n^2 (B) $(n-1)^2$
(C) $n - 1$ (D) n^3

Q.12 Greatest even natural number which divides g(n) – f(n), for every $n \geq 2$, is

(A) 2 (B) 4
(C) 6 (D) none of these

Q.13 f(n) + 3 g(n) + h(n) is divisible by 1 + 2 + 3 + ... + n

(A) only if n = 1 (B) only if n is odd
(C) only if n is even (D) for all $n \in N$

Comprehension # 02 (Q. No. 14 to 16)

The sum of squares of three distinct real numbers which form an increasing GP is S^2 (common ratio is r). If sum of numbers is αS, then

Q.14 If r = 3 then α^2 cannot lie in

(A) $\left(\frac{1}{3}, 1\right)$ (B) $(1, 2)$
(C) $\left(\frac{1}{3}, 3\right)$ (D) $(1, 3)$

Q.15 If $\alpha^2 = 2$, then the value of common ratio r is greater than

(A) 9 (B) 4 (C) 2 (D) 3

Q.16 If r = 2, then the value of α^2 is $\frac{a}{b}$ (where a and b are coprime) then a + b is

(A) 3 (B) 5 (C) 8 (D) 10

Q.17

Column – I	**Column – II**
(A) If $\log_5 2$, $\log_5(2^x - 5)$ and $\log_5(2^x - 7/2)$ are in A.P., then value of 2x is equal to	(p) 6
(B) Let S_n denote sum of first n terms of an A.P. If $S_{2n} = 3S_n$, then $\frac{S_{3n}}{S_n}$ is	(q) 9
(C) Sum of infinite series $4 + \frac{8}{3} + \frac{12}{3^2} + \frac{16}{3^3} + - - -$ is	(r) 3
(D) The length,breadth, height of a rectangular box are in G.P. The volume is 27, the total surface area is 78. Then the length is (s) 1	(s) 1

Q.18

Column – I	**Column – II**
(A) If $\log_x y$, $\log_z x$, $\log_y z$ are in G.P., xyz = 64 and x^3, y^3, z^3 are in A.P., then $\frac{3x}{y}$ is equal to	(p) 2
(B) The value of $2^{\frac{1}{4}} . 4^{\frac{1}{8}} . 8^{\frac{1}{16}} - - - \infty$ is equal to	(q) 1
(C) If x, y, z are in A.P., then $(x + 2y - z)(2y + z - x)(z + x - y) = kxyz$, where $k \in N$, then k is equal to	(r) 3
(D) There are m A.M. between 1 and 31. If the ratio of the 7^{th} and $(m-1)^{th}$ means is 5 : 9, then $\frac{m}{7}$ is equal to	(s) 4

NUMERICAL BASED QUESTIONS

Q.19 The value of $2^{1/4} \times 4^{1/8} \times 8^{1/6}$ ..... ∞ is

Q.20 An infinite G.P. is selected from $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}$ to converge to 1/7. If $1/2^a$ is the first term of such a G.P., find a.

$1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}$

Q.21 First term of an A.P. of non-constant terms is 3 and its second, tenth and thirty-fourth terms form a G.P., find the common difference.

Q.22 If $\log_x y, \log_z x, \log_y z$ are in G.P. , xyz = 64 and x^3, y^3, z^3 are in A.P., find x + y + z.

Q.23 If a_1, a_2, a_3,........ are in A.P. such that $a_1 + a_5 + a_{10} + a_{15} + a_{20} + a_{24} = 225$, then
$a_1 + a_2 + a_3 + ... + a_{23} + a_{24}$ is equal to

Q.24 If x > 0 and $\log_2 x + \log_2\left(\sqrt{x}\right) + \log_2\left(\sqrt[4]{x}\right) + \log_2\left(\sqrt[8]{x}\right) + \log_2\left(\sqrt[16]{x}\right) + ... = 4$ then x equals

Q.25 If x, y, z be respectively the pth, qth and rth terms of G.P., then

$(q-r) \log x + (r-p) \log y + (p-q) \log z =$

Q.26 The sum $\sum_{r=2}^{\infty} \frac{1}{r^2-1}$ is equal to

Q.27 Consider an infinitely decreasing geometric progression $b_1, b_2, b_3, ..., b_n$,..... whose sum is 3 and sum of its cubes is $\frac{108}{13}$, find the value of b_2.

Q.28 $\frac{1}{2.4} + \frac{1.3}{2.4.6} + \frac{1.3.5}{2.4.6.8} + \frac{1.3.5.7}{2.4.6.8.10} +$ ....∞ is equal to

EXERCISE-IV

JEE-MAIN
PREVIOUS YEAR'S

Q.1 If the 2nd, 5th and 9th terms of a non-constant A.P. are in G.P., then the common ratio of this G.P. is: **[JEE Main-2016]**

(1) 8/5 (2) 4/3 (3) 1 (4) 7/4

Q.2 If the sum of the first ten terms of the series

$$\left(1\frac{3}{5}\right)^2 + \left(2\frac{2}{5}\right)^2 + \left(3\frac{1}{5}\right)^2 + 4^2 + \left(4\frac{4}{5}\right)^2 + ...,$$

is $\frac{16}{5}m$, then m is equal to: **[JEE Main-2016]**

(1) 102 (2) 101
(3) 100 (4) 99

Q.3 For any three positive real numbers a, b and c, then **[JEE Main- 2017]**

(1) a, b and c are in G.P. (2) b, c and a are in G.P.
(3) b, c and a are in A.P. (4) a, b and c are in A.P.

Q.4 Let a, b, c $\in$ R. If $f(x) = ax^2 + bx + c$ is such that a + b + c = 3 and f(x + y) = f(x) + f(y) + xy, $\forall$ x, y $\in$ R, then $\sum_{n=1}^{10} f(n)$ is equal to : **[JEE Main-2017]**

(1) 255 (2) 330
(3) 165 (4) 190

Q.5 Let a1, $a_2, a_3, ..., a_{49}$ be in A.P. such that $\sum_{k=0}^{12} a_{4k+1} = 416$ and $a_9 + a_{43} = 66$. If $a_1^2 + a_2^2 + ...a_{17}^2 = 140m$, then m is equal to - **[JEE Main-2018]**

(1) 68 (2) 34
(3) 33 (4) 66

Q.6 Let A be the sum of the first 20 terms and B be the sum of the first 40 terms of the series **[JEE Main-2018]**
$1^2 + 2.2^2 + 3^2 + 2.4^2 + 5^2 + 2.6^2 + ...$ If $B - 2A = 100\lambda$, then λ is equal to :

(1) 248 (2) 464
(3) 496 (4) 232

Q.7 If a, b and c be three distinct numbers in G.P. and a+b+c = xb then x can not be **[JEE Main - 2019 (January)]**

(1) –2 (2) –3
(3) 4 (4) 2

Q.8 Let $a_1, a_2, ...a_{30}$ be an A.P., $S = \sum_{i=1}^{30} a_i$ and $T = \sum_{i=1}^{15} a_{2i-1}$. If $a_5 = 27$ and S – 2T=75, then a_{10} is equal to **[JEE Main - 2019 (January)]**

(1) 52 (2) 57
(3) 47 (4) 42

Q.9 The sum of the following series $1+6+\frac{9(1^2+2^2+3^2)}{7}+\frac{12(1^2+2^2+3^2+4^2)}{9}+\frac{15(1^2+2^2+....+5^2)}{11}+....$ upto 15 terms, is : **[JEE Main - 2019 (January)]**

(1) 7820 (2) 7830
(3) 7520 (4) 7510

Q.10 Let a, b and c be the 7th, 11th and 13th terms respectively of a non-constant A.P. If these are also the three consecutive terms of a G.P. then $\frac{a}{c}$ is equal to : **[JEE Main - 2019 (January)]**

(1) 1/2 (2) 4
(3) 2 (4) 7/13

Q.11 The sum of all two digit positive numbers which when divided by 7 yield 2 or 5 as remainder is: **[JEE Main - 2019 (January)]**

(1) 1256 (2) 1465
(3) 1365 (4) 1356

Q.12 If 19th terms of non – zero A. P is zero, then its (49th term) : (29th term) is : **[JEE Main - 2019 (January)]**

(1) 4 : 1 (2) 1 : 3
(3) 3 : 1 (4) 2 : 1

Q.13 The sum of an infinite geometric series with positive terms is 3 and the sum of the cubes of its terms is $\frac{27}{19}$. Then the common ratio of this series is :- **[JEE Main - 2019 (January)]**

(1) $\frac{1}{3}$ (2) $\frac{2}{3}$
(3) $\frac{2}{9}$ (4) $\frac{4}{9}$

Q.14 Let $a_1, a_2, ...a_{10}$ be a G. P. If $\frac{a_3}{a_1} = 25$, then $\frac{a_9}{a_5}$ equals :- **[JEE Main - 2019 (January)]**

(1) 5^4 (2) $4(5^2)$
(3) 5^3 (4) $2(5^2)$

Q.15 The product of three consecutive terms of a G.P. is 512. If 4 is added to each of the first and the second of these terms, the three terms now form an A.P. Then the sum of the original three terms of the given G.P. is: **[JEE Main - 2019 (January)]**

(1) 36 (2) 32
(3) 24 (4) 28

Q.16 Let $S_k = \frac{1+2+3+......+k}{k}$. If $S_1^2 + S_2^2 + ... + S_{10}^2 = \frac{5}{12}A$, then A equal to : **[JEE Main - 2019 (January)]**

(1) 283 (2) 301
(3) 303 (4) 156

Q.17 If the sum of the first 15 terms of the series $\left(\frac{3}{4}\right)^3 + \left(1\frac{1}{2}\right)^3 + \left(2\frac{1}{4}\right)^3 + 3^3 + \left(3\frac{3}{4}\right)^3 + ...$ is equal to 225 k, then k is equal to : **[JEE Main - 2019 (January)]**

(1) 108 (2) 27
(3) 54 (4) 9

Q.18 The sum of all natural numbers 'h' such that 100 < n < 200 and H.C.F. (91, n) > 1 is : **[JEE Main-2019(April)]**

(1) 3221
(2) 3121
(3) 3203
(4) 3303

Q.19 The sum $\sum_{k=1}^{20} k\frac{1}{2^k}$ is equal to - **[JEE Main-2019(April)]**

(1) $2-\frac{3}{2^{17}}$ (2) $2-\frac{11}{2^{19}}$
(3) $1-\frac{11}{2^{20}}$ (4) $2-\frac{21}{2^{20}}$

Q.20 Let the sum of the first n terms of a non-constant A.P., $a_1, a_2, a_3, ...$ be $50n + \frac{n(n-7)}{2}A$, where A is a constant. If d is the common difference of this A.P., then the ordered pair (d, a_{50}) is equal to **[JEE Main-2019(April)]**

(1) (A, 50 + 46A)
(2) (A, 50 + 45A)
(3) (50, 50 + 46A)
(4) (50, 50 + 45A)

Q.21 If the sum and product of the first three term in an A.P. are 33 and 1155, rtespectively, then a value of its 11^{th} term is :- **[JEE Main-2019(April)]**

(1) – 25 (2) 25
(3) – 36 (4) – 35

Q.22 The sum of the series $1 + 2 \times 3 + 3 \times 5 + 4 \times 7 + ...$ upto 11^{th} term is :- **[JEE Main-2019(April)]**

(1) 915 (2) 946
(3) 945 (4) 916

Q.23 The sum $1+\frac{1^3+2^3}{1+2}+\frac{1^3+2^3+3^3}{1+2+3}+....$ **[JEE Main-2019(April)]**

$+\frac{1^3+2^3+3^3+....+15^3}{1+2+3+....+15}-\frac{1}{2}(1+2+3+.....+15)]$

(1) 1240 (2) 1860
(3) 660 (4) 620

Q.24 Let a, b and c be in G.P. with common ratio r, where a ≠ and $0 < r \le \frac{1}{2}$. If 3a, 7b and 15c are the first three terms of an A.P. then the 4^{th} term of this A.P. is : **[JEE Main-2019(April)]**

(1) $\frac{7}{3}$ (2) a (3) $\frac{2}{3}a$ (4) 5a

Q.25 The sum $\frac{3\times1^3}{1^2}+\frac{5\times(1^3+2^3)}{1^2+2^2}+\frac{7\times(1^3+2^3+3^3)}{1^2+2^2+3^2}+$ .... up to 10 terms **[JEE Main-2019(April)] 2019]**

(1) 660 (2) 620 (3) 680 (4) 600

Q.26 If $a_1, a_2, a_3, ...a_n$ are in A.P. and $a_1 + a_4 + a_7 +.....+ a_{16} =$ 114, then $a_1 + a_6 + a_{11} + a_{16}$ is equal to : **[JEE Main-2019(April)] 2019]**

(1) 38 (2) 98 (3) 76 (4) 64

Q.27 Let S_n denote the sum of the first n terms of an A.P. If $S_4 = 16$ and $S_6 = –48$, then S_{10} is equal to : **[JEE Main-2019(April)]**

(1) –320 (2) –260
(3) –380 (4) –410

Q.28 For $x \in R$, let [x] denote the greatest integer ≤ x, then the sum of the series

$\left[-\frac{1}{3}\right]+\left[-\frac{1}{3}-\frac{1}{100}\right]+\left[-\frac{1}{3}-\frac{2}{100}\right]+.....+\left[-\frac{1}{3}-\frac{99}{100}\right]$ is **[JEE Main-2019(April)]**

(1) –153 (2) –133
(3) –131 (4) –135

Q.29 If $a_1, a_2, a_3,....$ are in A.P. such that $a_1 + a_7 + a_{16} = 40$, then the sum of the first 15 terms of this A.P. is **[JEE Main-2019(April)]**

(1) 200 (2) 280
(3) 120 (4) 150

Q.30 The greatest positive integer k, for which $49^k + 1$ is a factor of the sum $49^{125} + 49^{124} + ... + 49^2 + 49 + 1$, is **[JEE Main-2020 (January)]**

(1) 32 (2) 63
(3) 65 (4) 60

Q.31 Five numbers are in A.P., whose sum is 25 and product is 2520. If one of these five numbers is $\frac{-1}{2}$, then the greatest number amongst them is **[JEE Main-2020 (January)]**

(1) $\frac{21}{2}$ (2) 16
(3) 27 (4) 7

Q.32 If the sum of the first 40 terms of the series, $3+4+8+9+13+14+18+19+........$is (102)m, then m is equal to: **[JEE Main-2020 (January)]**

(1) 25 (2) 20
(3) 10 (4) 5

Q.33 Let $a_1, a_2, a_3,.......$ be a G.P. such that $a_1 < 0$, $a_1 + a_2 = 4$ and $a_3 + a_4 = 16$. If $\sum_{i=1}^{9} a_i = 4\lambda$, then λ is equal to: **[JEE Main-2020 (January)]**

(1) 171 (2) –513
(3) $\frac{511}{3}$ (4) –171

Q.34 Let $f : R \to R$ be such that for all $x \in R$ $(2^{1+x} + 2^{1-x})$, f(x) and $(3^x + 3^{-x})$ are in A.P., then the minimum value of f(x) is **[JEE Main-2020 (January)]**

(1) 0 (2) 2
(3) 4 (4) 3

Q.35 The sum $\sum_{k=1}^{20}(1+2+3+.......+k)$ is **[JEE Main-2020 (January)]**

Q.36 If the 10th term of an A.P. is $\frac{1}{20}$ and its 20th term is $\frac{1}{10}$, then the sum of its first 200 term is

[JEE Main-2020 (January)]

(1) 100 (2) $50\frac{1}{4}$ (3) $100\frac{1}{2}$ (4) 50

Q.37 The sum, $\sum_{n=1}^{7}\frac{n(n+1)(2n+1)}{4}$ is equal to

[JEE Main-2020 (January)]

Q.38 The product $2^{\frac{1}{4}}, 4^{\frac{1}{16}}, 8^{\frac{1}{48}}, 16^{\frac{1}{128}}$ .... to ∞ is equal to

[JEE Main-2020 (January)]

(1) $2^{\frac{1}{4}}$ (2) $2^{\frac{1}{2}}$ (3) 1 (4) 2

Q.39 Let a_n be the n^{th} term of G.P. of positive terms. If $\sum_{n=1}^{100} a_{2n+1} = 200$ and $\sum_{n=1}^{100} a_{2n} = 100$, then $\sum_{n=1}^{200} a_n$ is equal to : **[JEE Main-2020 (January)]**

(1) 175 (2) 150

(3) 300 (4) 225

Q.40 The number of terms common the two A.P. s 3, 7, 11......, 407 and 2, 9, 16, ..., 709 is______.

[JEE Main-2020 (January)]

Q.41 Let S be the sum of the first 9 terms of the series : $\{x+ka\} + \{x^2+(k+2)a\} + \{x^3+(k+4)a\} + \{x^4+(k+6)a\} + \ldots$ where $a \neq 0$ and $x \neq 1$.

If $S = \frac{x^{10} - x + 45a(x-1)}{x-1}$, then k is equal to

[JEE Main-2020 (September)]:

(1) –3 (2) 1

(3) –5 (4) 3

Q.42 If the sum of first 11 terms of an A.P., $a_1, a_2, a_3 \ldots$ is 0 ($a_1 \neq 0$), then the sum of the A.P., , $a_1, a_3, a_5, \ldots, a_{23}$ is ka_1, where k is equal to : **[JEE Main-2020 (September)]**

(1) $-\frac{121}{10}$ (2) $-\frac{72}{5}$

(3) $\frac{72}{5}$ (4) $\frac{121}{10}$

Q.43 The sum of the first three terms of a G.P. is S and their product is 27. Then all such S lie in:

[JEE Main-2020 (September)]

(1) $(-\infty,-9]\cup[3,\infty)$ (2) $[-3,\infty)$

(3) $(-\infty,-3]\cup[9,\infty)$ (4) $(-\infty,9]$

Q.44 If $|x| < 1$, $|y| < 1$ and $x \neq y$, then the sum to infinity of the following series $(x + y) + (x^2 + xy + y^2) + (x^3 + x^2y + xy^2 + y^3) + \ldots..$ is :

[JEE Main-2020 (September)]

(1) $\frac{x+y+xy}{(1+x)(1+y)}$ (2) $\frac{x+y-xy}{(1-x)(1-y)}$

(3) $\frac{x+y-xy}{(1+x)(1+y)}$ (4) $\frac{x+y+xy}{(1-x)(1-y)}$

Q.45 If m arithmetic means (A. Ms) and three geometric means (G.Ms) are inserted between 3 and 243 such that 4^{th} A.M. is equal to 2^{nd} G.M., then m is equal to _________ . **[JEE Main-2020 (September)]**

Q.46 If the sum of the series

$20 + 19\frac{3}{5} + 19\frac{1}{5} + 18\frac{4}{5} + \ldots$ upto n^{th} term is 488 and the n^{th} term is negative, then **[JEE Main-2020 (September)]**

(1) n = 41 (2) n^{th} term is $-4\frac{2}{5}$

(3) n = 60 (4) n^{th} term is – 4

Q.47 The value of $(0.16)^{\log_{2.5}\left(\frac{1}{3}+\frac{1}{3^2}+\frac{1}{3^3}+\ldots.\infty\right)}$ equal to _____.

[JEE Main-2020 (September)]

Q.48 If the first term of an A.P. is 3 and the sum of its first 25 terms is equal to the sum of its next 15 terms, then the common difference of this A.P. is :

[JEE Main-2020 (September)]

(1) $\frac{1}{6}$ (2) $\frac{1}{4}$

(3) $\frac{1}{7}$ (4) $\frac{1}{5}$

Q.49 Let $a_1, a_2, ...a_n$ be a given A.P. whose common difference is an integer and $S_n = a_1 + a_2 + ...+ a_n$. If $a_1 = 1$, $a_n = 300$ and $15 \le n \le 50$, then the ordered pair (S_{n-4}, a_{n-4}) is equal to **[JEE Main-2020 (September)]**

(1) (2490, 249) (2) (2480, 249)
(3) (2490, 248) (4) (2480, 248)

Q.50 If $1 + (1 - 2^2 \cdot 1) + (1 - 4^2 \cdot 3) + (1 - 6^2 \cdot 5) + ... + (1 - 20^2 \cdot 19) = \alpha - 220\beta$, then an ordered pair (α, β) is equal to : **[JEE Main-2020 (September)]**

(1) (10, 103) (2) (10, 97)
(3) (11, 97) (4) (11, 103)

Q.51 If the sum of the first 20 terms of the series $\log_{(7^{1/2})} x + \log_{(7^{1/3})} x + \log_{(7^{1/4})} x + ...$ is 460, then x is equal to **[JEE Main-2020 (September)]**

(1) 7^2 (2) e^2 (3) $7^{1/2}$ (4) $7^{46/21}$

Q.52 If the sum of the second, third and fourth terms of a positive term G.P. is 3 and the sum of its sixth, seventh and eighth terms is 243, then the sum of the first 50 terms of this G.P. is : **[JEE Main-2020 (September)]**

(1) $\frac{2}{13}(3^{50} - 1)$ (2) $\frac{1}{13}(3^{50} - 1)$

(3) $\frac{1}{26}(3^{49} - 1)$ (4) $\frac{1}{26}(3^{50} - 1)$

Q.53 If $2^{10} + 2^9 \cdot 3^1 + 2^8 \cdot 3^2 + ... + 2 \cdot 3^9 + 3^{10} = S - 2^{11}$, then S is equal to : **[JEE Main-2020 (September)]**

(1) $2 \cdot 3^{11}$ (2) $3^{11} - 2^{12}$ (3) $\frac{3^{11}}{2} + 2^{10}$ (4) 3^{11}

Q.54 If $3^{2\sin 2\alpha - 1}$, 14 and $3^{4 - 2\sin 2\alpha}$ are the first three terms of an A.P. for some α, then the sixth term of this A.P is :
[JEE Main-2020 (September)]

(1) 65 (2) 78 (3) 81 (4) 66

Q.55 Suppose that a function $f : R \to R$ satisfies $f(x + y) = f(x)f(y)$ for all $x, y \in R$ and $f(1) = 3$. If $\sum_{i=1}^{n} f(i) = 363$, then n is equal to ________ . **[JEE Main-2020 (September)]**

Q.56 The common difference of the A.P. $b_1, b_2, ..., b_m$ is 2 more than the common difference of A.P. $a_1, a_2, ... , a_n$. If $a_{40} = -159$, $a_{100} = -399$ and $b_{100} = a_{70}$, then b_1 is equal to:
[JEE Main-2020 (September)]

(1) –127 (2) –81 (3) 127 (4) 81

Q.57 Let a, b, c, d and p be any non zero distinct real numbers such that $(a^2 + b^2 + c^2) p^2 - 2 (ab + bc + cd)p + (b^2 + c^2 + d^2) = 0$. Then : **[JEE Main-2020 (September)]**

(1) a, c, p are in G. P.
(2) a, b, c, d are in A. P.
(3) a, c, p are in A.P.
(4) a, b, c, d are in G.P.

JEE-ADVANCED PREVIOUS YEAR'S

Q.1 Let $a_1, a_2, a_3, ..., a_{11}$ be real numbers satisfying $a_1 = 15$, $27 - 2a_2 > 0$ and $a_k = 2a_{k-1} - a_{k-2}$ for k = 3, 4, ..., 11. If $\frac{a_1^2 + a_2^2 + ... + a_{11}^2}{11} = 90$, then the value of $\frac{a_1 + a_2 + ... + a_{11}}{11}$ is equal to **[IIT JEE–2010]**

Q.2 Let $a_1, a_2, a_3, ..., a_{100}$ be an arithmetic progression with $a_1 = 3$ and $S_p = \sum_{i=1}^{p} a_i$, $1 \le p \le 100$.

For any integer n with $1 \le n \le 20$, let m = 5n. If $\frac{S_m}{S_n}$ does not depend on n, then a_2 is **[IIT JEE-2011]**

Q.3 The minimum value of the sum of real numbers a^{-5}, a^{-4}, $3a^{-3}$, 1, a^8 and a^{10} where $a > 0$ is **[IIT JEE-2011]**

Q.4 Let $a_1, a_2, a_3, ...$ be in harmonic progression with $a_1 = 5$ and $a_{20} = 25$. The least positive integer n for which $a_n < 0$ is **[IIT JEE-2012]**

(A) 22 (B) 23
(C) 24 (D) 25

Q.5 Let $S_n = \sum_{k=1}^{4n} (-1)^{\frac{k(k+1)}{2}} k^2$. Then S_n can take value(s)

[JEE Advanced-2013]

(A) 1056 (B) 1088
(C) 1120 (D) 1332

Q.6 A pack contains n card numbered from 1 to n. Two consecutive numbered card are removed from the pack and the sum of the numbers on the remaining cards is 1224. If the smaller of the numbers on the removed cards is k, then k – 20 = **[JEE Advanced-2013]**

Q.7 Let a,b,c be positive integers such that $\frac{b}{a}$ is an integer. If a,b,c are in geometric progression and the arithmetic mean of a,b,c is b + 2, then the value of $\frac{a^2+a-14}{a+1}$ is **[JEE Advanced-2014]**

Q.8 Suppose that all the terms of an arithmetic progression (A.P.) are natural numbers. If the ratio of the sum of the first seven terms to the sum of the first eleven terms is 6 : 11 and the seventh term lies in between 130 and 140, then the common difference of this A.P. is **[JEE Advanced-2015]**

Q.9 Let $b_i > 1$ for i = 1, 2, ..., 101. Suppose $\log_e b_1, \log_e b_2, ... \log_e b_{101}$ are in Arithmetic Progression (A.P.) with the common difference $\log_e 2$. Suppose $a_1, a_2, ..., a_{101}$ are in A.P. such that $a_1 = b_1$ and $a_{51} = b_{51}$. If $t = b_1 + b_2 + ... + b_{51}$ and $s = a_1 + a_2 + ... + a_{51}$, *then* **[JEE Advanced-2016]**

(A) $s > t$ and $a_{101} > b_{101}$ (B) $s > t$ and $a_{101} < b_{101}$
(C) $s < t$ and $a_{101} > b_{101}$ (D) $s < t$ and $a_{101} < b_{101}$

Q.10 The sides of the right angled triangle are in arithmetic progression. If the triangle has area 24, then what is the length of its smallest side? **[JEE Advanced-2017]**

Q.11 Let AP(a;d) denote the set of all the terms of an infinite arithmetic progression with first term a and common difference d > 0. If AP(1; 3) $\cap$ AP(2;5) $\cap$ AP(3;7) = AP(a ; d) then a + d equals ____ **[JEE Advanced -2019]**

Q.12 Let m be the minimum possible value of $\log_3\left(3^{y_1}+3^{y_2}+3^{y_3}\right)$, where y_1, y_2, y_3 are real numbers for which $y_1 + y_2 + y_3 = 9$. Let M be the maximum possible value of $(\log_3 x_1 + \log_3 x_2 + \log_3 x_3)$, where x_1, x_2, x_3 are positive real numbers for which $x_1 + x_2 + x_3 = 9$. Then the value of $\log_2 (m^3) + \log_3(M^2)$ is ______.

[JEE(Advanced) - 2020]

Q.13 Let a_1, a_2, a_3, ….. be a sequence of positive integers in arithmetic progression with common difference 2. Also, let b_1, b_2, b_3, ….. be a sequence of positive integers in geometric progression with common ratio 2. If $a_1 = b_1 = c$, then the number of all possible values of c, for which the equality

$2(a_1 + a_2 + ...+a_n) = b_1 + b_2 + ... + b_n$

holds for some positive integer n, is ______

[JEE(Advanced) - 2020]

Answer Key

EXERCISE-I

Q.1 (2)	**Q.2** (3)	**Q.3** (3)	**Q.4** (2)	**Q.5** (3)	**Q.6** (3)	**Q.7** (3)	**Q.8** (4)	**Q.9** (2)	**Q.10** (4)
Q.11 (4)	**Q.12** (2)	**Q.13** (1)	**Q.14** (2)	**Q.15** (1)	**Q.16** (4)	**Q.17** (2)	**Q.18** (4)	**Q.19** (3)	**Q.20** (4)
Q.21 (1)	**Q.22** (4)	**Q.23** (1)	**Q.24** (3)	**Q.25** (4)	**Q.26** (1)	**Q.27** (4)	**Q.28** (1)	**Q.29** (2)	**Q.30** (1)

EXERCISE-II

Q.1 (4)	**Q.2** (4)	**Q.3** (3)	**Q.4** (2)	**Q.5** (1)	**Q.6** (4)	**Q.7** (4)	**Q.8** (1)	**Q.9** (2)	**Q.10** (2)
Q.11 (2)	**Q.12** (2)	**Q.13** (2)	**Q.14** (3)	**Q.15** (4)	**Q.16** (3)	**Q.17** (1)	**Q.18** (1)	**Q.19** (1)	**Q.20** (3)
Q.21 (1)	**Q.22** (3)	**Q.23** (1)	**Q.24** (1)	**Q.25** (3)	**Q.26** (2)	**Q.27** (2)	**Q.28** (3)	**Q.29** (3)	**Q.30** (2)

EXERCISE-III

Q.1 (B,D) **Q.2** (A) **Q.3** (B,D) **Q.4** (B,C) **Q.5** (A,B,C,D) **Q.6** (A,B) **Q.7** **(A,B,C)** **Q.8** (A,B,C) **Q.9** (A, B, C, D) **Q.10** (A,C)
Q.11 (A) **Q.12** (A) **Q.13** (D) **Q.14** (A) **Q.15** (C) **Q.16** (D) **Q.17** (A) $\rightarrow$ (p), (B) $\rightarrow$ (p), (C) $\rightarrow$ (q), (D) $\rightarrow$ (q)
Q.18 (A) $\rightarrow$ (r), (B) $\rightarrow$ (p), (C) $\rightarrow$ (s), (D) $\rightarrow$ (p) **Q.19** [0002] **Q.20** [0003] **Q.21** [0001] **Q.22** [0012] **Q.23** [900] **Q.24** [4]
Q.25 [0] **Q.26** [0.75] **Q.27** [0.67] **Q.28** [0.5]

EXERCISE-IV

JEE-MAIN

PREVIOUS YEAR'S

Q.1 (2)	**Q.2** (2)	**Q.3** (3)	**Q.4** (2)	**Q.5** (2)	**Q.6** (1)	**Q.7** (4)	**Q.8** (1)	**Q.9** (1)	**Q.10** (2)
Q.11 (4)	**Q.12** (3)	**Q.13** (2)	**Q.14** (1)	**Q.15** (4)	**Q.16** (3)	**Q.17** (2)	**Q.18** (2)	**Q.19** (2)	**Q.20** (1)
Q.21 (1)	**Q.22** (2)	**Q.23** (4)	**Q.24** (2)	**Q.25** (1)	**Q.26** (3)	**Q.27** (1)	**Q.28** (2)	**Q.29** **(1)**	**Q.30** (2)
Q.31 (2)	**Q.32** (2)	**Q.33** (4)	**Q.34** (4)	**Q.35** [1540]	**Q.36** (3)	**Q.37** [504]	**Q.38** (2)	**Q.39** (2)	**Q.40** [14]
Q.41 (1)	**Q.42** (2)	**Q.43** (3)	**Q.44** (2)	**Q.45** [39]	**Q.46** (4)	**Q.47** [4]	**Q.48** (1)	**Q.49** (3)	**Q.50** (4)
Q.51 (1)	**Q.52** (4)	**Q.53** (4)	**Q.54** (4)	**Q.55** [05.00]	**Q.56** (2)	**Q.57** (4)			

JEE-ADVANCED

PREVIOUS YEAR'S

Q.1 [0]	**Q.2** 3 or 9	**Q.3** [8]	**Q.4** (D)	**Q.5** (A,D)	**Q.6** [5]	**Q.7** [4]	**Q.8** [9]	**Q.9** (B)	**Q.10** [6]
Q.11 [157.00]		**Q.12** [8.00]	**Q.13** [1.00]						

EXERCISE (Solution)

EXERCISE-I

Q.1 (2)

We have $\sqrt{2}+\sqrt{8}+\sqrt{18}+\sqrt{32}+......$

$=1\sqrt{2}+2\sqrt{2}+3\sqrt{2}+4\sqrt{2}+........$

$=\sqrt{2}[1+2+3+4+..........\text{upto 24 terms}]$

$=\sqrt{2}\times\frac{24\times25}{2}=300\sqrt{2}$

Q.2 (3)

Suppose that first term and common difference of A.P.'s are A and D respectively.

Now, p^{th} term $=A+(p-1)D=a$ .....(i)

q^{th} term $=A+(q-1)D=b$ ......(ii)

and r^{th} term $=A+(r-1)D=c$ .....(iii)

So, $a(q-r)+b(r-p)+c(p-q)$

$=a\left\{\frac{b-c}{D}\right\}+b\left\{\frac{c-a}{D}\right\}+c\left\{\frac{a-b}{D}\right\}$

$=\frac{1}{D}(ab-ac+bc-ab+ca-bc)=0$.

Q.3 (3)

$T_m=a+(m-1)d=\frac{1}{n}$

and $T_n=a+(n-1)d=\frac{1}{m}$

On solving $a=\frac{1}{mn}$ and $d=\frac{1}{mn}$

$\therefore T_{mn}=a+(mn-1)d=\frac{1}{mn}+(mn-1)\frac{1}{mn}=1$

Q.4 (2)

The given number are in A.P.

$\therefore 2\log_9(3^{1-x}+2)=\log_3(4.3^x-1)+1$

$\Rightarrow 2\log_{3^2}(3^{1-x}+2)=\log_3(4.3^x-1)+\log_3 3$

$\Rightarrow \frac{2}{2}\log_3(3^{1-x}+2)=\log_3[3(4.3^x-1)]$

$\Rightarrow 3^{1-x}+2=3(4.3^x-1)$

$\Rightarrow \frac{3}{y}+2=12y-3,$ where $y=3^x$

$\Rightarrow 12y^2-5y-3=0$

$y=\frac{-1}{3}$ or $\frac{3}{4}\Rightarrow 3^x=\frac{-1}{3}$ or $3^x=\frac{3}{4}$

$x=\log_3(3/4)\Rightarrow x=1-\log_3 4$.

Q.5 (3)

Let S_n and S'_n be the sums of n terms of two A.P.'s and T_{11} and T'_{11} be the respective 11^{th} terms, then

$$\frac{S_n}{S'_n}=\frac{\frac{n}{2}[2a+(n-1)d]}{\frac{n}{2}[2a'+(n-1)d']}=\frac{7n+1}{4n+27}$$

$$\Rightarrow \frac{a+\frac{(n-1)}{2}d}{a'+\frac{(n-1)}{2}d'}=\frac{7n+1}{4n+27}$$

Now put $n=21$,

we get $\frac{a+10d}{a'+10d'}=\frac{T_{11}}{T'_{11}}=\frac{148}{111}=\frac{4}{3}$.

Note : If ratio of sum of n terms of two A.P.'s are given in terms of and ratio of their terms are to be found then put . Here we put .

Q.6 (3)

Let $S_{Even}=2+4+6+8+........\infty$ .....(i)

and $S_{Odd}=1+3+5+7+9+.......\infty$ .....(ii)

Sum $S_E\frac{n}{2}\left[4+(n-1)2\right]=\frac{n}{2}[2n+2]=\frac{n}{2}2(n+1)$

and $S_O=\frac{n}{2}[2+(n-1)2]=\frac{n}{2}(2n)$

Now $\frac{S_E}{S_O}=\frac{(n+1)}{n}$ or $S_E:S_O=(n+1):n$

Q.7 (3)

$$S_{2n}-S_n=\frac{2n}{2}\{2a+(2n-1)d\}-\frac{n}{2}\{2a+(n-1)d\}$$

$= \frac{n}{2}\{4a + 4nd - 2d - 2a - nd + d\} = \frac{n}{2}\{2a + (3n-1)d\}$

$= \frac{1}{3}.\frac{3n}{2}\{2a + (3n-1)d\} = \frac{1}{3}S_{3n}$

Q.8 (4)

Let $S = 1 + 2 + 3 + ... + 100$

$= \frac{100}{2}(1+100) = 50(101) = 5050$

Let $S_1 = 3 + 6 + 9 + 12 + ... + 99$

$= 3(1 + 2 + 3 + 4 + ... + 33)$

$= 3.\frac{33}{2}(1+33) = 99 \times 17 = 1683$

Let $S_2 = 5 + 10 + 15 + ... + 100$

$= 5(1 + 2 + 3 + ... + 20)$

$= 5.\frac{20}{2}(1+20) = 50 \times 21 = 1050$

Let $S_3 = 15 + 30 + 45 + ... + 90$

$= 15(1 + 2 + 3 + ... + 6)$

$= 15.\frac{6}{2}(1+6) = 45 \times 7 = 315$

$\therefore$ Required sum $= S - S_1 - S_2 + S_3$

$= 5050 - 1683 - 1050 + 315 = 2632.$

Q.9 (2)

$S_{2n} = 3S_n$

$\Rightarrow \frac{2n}{2}\{2a + (2n-1)d\} = 3.\frac{n}{2}\{2a + (n-1)d\}$

$2a = (n+1)d$

Put $2a = (n+1)d$ in $\frac{S_{3n}}{S_n}$, we get its value 6.

Q.10 (4)

Suppose that required numbers a and b. Therefore according to the conditions $a = \frac{1}{b}$

and $\frac{a+b}{2} = \frac{13}{12} \Rightarrow a + b = \frac{13}{6}$

$\Rightarrow a + \frac{1}{a} = \frac{13}{6} \Rightarrow 6a^2 - 13a + 6 = 0$

$\Rightarrow \left(a - \frac{3}{2}\right)\left(a - \frac{2}{3}\right) = 0 \Rightarrow a = \frac{3}{2}$ and $b = \frac{2}{3}$

$\Rightarrow a = \frac{2}{3}$ and $b = \frac{3}{2}$

Trick : Find the A.M. of option (1), (2), (3), (4) one by one.

Q.11 (4)

Given that x, 2x + 2, 3x + 3 are in G.P.

Therefore, $(2x + 2)^2 = x(2x + 2)^2 = x(3x + 3)$

$\Rightarrow x^2 + 5x + 4 = 0$

$\Rightarrow (x+4)(x+1) = 0 \Rightarrow x = -1, -4$

Now first term a = x

Second term $ar = 2(x+1) \Rightarrow r = \frac{2(x+1)}{x}$

then 4[th] term $= ar^3 = x\left[\frac{2(x+1)}{x}\right]^3 = \frac{8}{x^2}(x+1)^3$

Putting x = –4

We get $T_4 = \frac{8}{16}(-3)^3 = -\frac{27}{2} = -13.5$

Q.12 (2)

$T_6 = 32$ and $T_8 = 128 \Rightarrow ar^5 = 32$ .....(i)

and $ar^7 = 128$ .....(ii)

Dividing (ii) by (i), $r^2 = 4 \Rightarrow r = 2$

Q.13 (1)

Series is a G.P. with $a = 0.9 = \frac{9}{10}$ and $r = \frac{1}{10} = 0.1$

$\therefore S_{100} = a\left(\frac{1-r^{100}}{1-r}\right) = \frac{9}{10}\left(\frac{1-\frac{1}{10^{100}}}{1-\frac{1}{10}}\right) = 1 - \frac{1}{10^{100}}.$

Q.14 (2)

Given series $6 + 66 + 666 + ... +$ upto n terms

$= \frac{6}{9}(9 + 99 + 999 + ...\text{ upto terms})$

$= \frac{2}{3}(10 + 10^2 + 10^3 + ... + \text{ upto terms})$

$= \frac{2}{3}\left(\frac{10(10^n - 1)}{10-1} - n\right) = \frac{1}{27}[20(10^n - 1) - 18n]$

$= \frac{2(10^{n+1} - 9n - 10)}{27}$

Q.15 (1)

$\because$ *a*, *b*, *c* are in G.P.

$\therefore \frac{b}{a} = \frac{c}{b} = r$

$\Rightarrow \frac{b^2}{a^2} = \frac{c^2}{b^2} = r^2$ $a^2,\ b^2, c^2$ are in G.P.

Q.16 (4)

Here $\frac{a}{1-r} = 4$ and $ar = \frac{3}{4}$. Dividing these,

$r(1-r) = \frac{3}{16}$ or $16r^2 - 16r + 3 = 0$

or $(4r-3)(4r-1) = 0$

$r = \frac{1}{4}, \frac{3}{4}$ and $a = 3, 1$ so $(a, r) = \left(3, \frac{1}{4}\right), \left(1, \frac{3}{4}\right)$.

Q.17 (2)

As given $G = \sqrt{xy}$

$$\therefore \frac{1}{G^2 - x^2} + \frac{1}{G^2 - y^2} = \frac{1}{xy - x^2} + \frac{1}{xy - y^2}$$

$$= \frac{1}{x-y}\left\{-\frac{1}{x} + \frac{1}{y}\right\} = \frac{1}{xy} = \frac{1}{G^2}.$$

Q.18 (4)

It is an arithmetico-geometric series

$$S_\infty = \frac{a}{1-r} + \frac{dr}{(1-r)^2} = \frac{1}{1-\frac{1}{2}} + \frac{2}{\left(1-\frac{1}{2}\right)^2} = \frac{2}{\frac{1}{2}} + \frac{2}{\frac{1}{4}}$$

$$= 4 + 8 = 12$$

Q.19 (3)

The sum of the first n terms is

$$S_n = \left(1-\frac{1}{2}\right) + \left(1-\frac{1}{2^2}\right) + \left(1-\frac{1}{2^3}\right) + \left(1-\frac{1}{2^4}\right)$$

$$+ \ldots\ldots + \left(1-\frac{1}{2^n}\right) = n - \left\{\frac{1}{2} + \frac{1}{2^2} + \ldots\ldots + \frac{1}{2^n}\right\}$$

$$= n - \frac{1}{2}\left(\frac{1-\frac{1}{2^n}}{1-\frac{1}{2}}\right) = n - \left(1-\frac{1}{2^n}\right) = n - 1 + 2^{-n}.$$

Trick : Check for $n = 1, 2$ i.e. $S_1 = \frac{1}{2}, S_2 = \frac{5}{4}$ and

(3) $\Rightarrow S_1 = \frac{1}{2}$ and $S_2 = 2 + 2^{-2} - 1 = \frac{5}{4}$.

Q.20 (4)

Suppose that x to be added then numbers 13, 15, 19 so that new numbers $x+13, 15+x, 19+x$ will be in H.P.

$$\Rightarrow (15+x) = \frac{2(x+13)(19+x)}{x+13+x+19}$$

$\Rightarrow x^2 + 31x + 240 = x^2 + 32x + 247 \Rightarrow x = -7$

Trick : Such type of questions should be checked with the options.

Q.21 (1)

Here 5^{th} term of the corresponding

A.P. $= a + 4d = 45$ …..(i)

and 11^{th} term of the corresponding

A.P. $= a + 10d = 69$ …..(ii)

From (i) and (ii), we get $a = 29, d = 4$

Therefore 16^{th} term of the corresponding A.P. $= a + 15d = 29 + 15 \times 4 = 89$.

Hence 16^{th} term of the H.P. is $\frac{1}{89}$.

Q.22 (4)

Considering corresponding A.P.

$a + 6d = 10$ and $a + 11d = 25 \Rightarrow d = 3, a = -8$

Hence term of the corresponding H.P. is $\frac{1}{49}$

Q.23 (1)

As given $H = \frac{2pq}{p+q}$

$$\therefore \frac{H}{p} + \frac{H}{q} = \frac{2q}{p+q} + \frac{2p}{p+q} = \frac{2(p+q)}{p+q} = 2.$$

Q.24 (3)

A.M. $= \frac{a+b}{2} = A$ and G.M. $= \sqrt{ab} = G$

On solving a and b are given by the values

$A \pm \sqrt{(A+G)(A-G)}$.

Trick : Let the numbers be 1, 9. Then A=5 and G=3. Now put these values in options.

Here (3) $\Rightarrow 5 \pm \sqrt{8 \times 2}$ i.e. 9 and 1.

Q.25 (4)

Let $A = \frac{a+b}{2}, G = \sqrt{ab}$ and $H = \frac{2ab}{a+b}$.

Then $G^2 = ab$ .....(i)

and $AH = \left(\frac{a+b}{2}\right) \cdot \frac{2ab}{a+b} = ab$ .....(ii)

From (i) and (ii), we have $G^2 = AH$

Q.26 (1)

Since a^2, b^2, c^2 be in A.P. Then

$b^2 - a^2 = c^2 - b^2$

$\Rightarrow (b-a)(b+a) = (c-b)(c+b)$

$\Rightarrow \frac{b-a}{c+b} = \frac{c-b}{b+a}$

$\Rightarrow \frac{(b-a)(a+b+c)}{(c+a)(b+c)} = \frac{(c-b)(a+b+c)}{(a+b)(c+a)}$

$\Rightarrow \frac{b^2+bc-ac-a^2}{(c+a)(b+c)} = \frac{c^2+ac-ab-b^2}{(a+b)(c+a)}$

$\Rightarrow \frac{b}{c+a} - \frac{a}{b+c} = \frac{c}{a+b} - \frac{b}{c+a}$

Hence $\frac{a}{b+c}, \frac{b}{c+a}, \frac{c}{a+b}$ be in A.P.

Q.27 (4)

Here $T_n = \frac{n(n+1)}{2}$

Therefore $S_n = \frac{1}{2}\left\{\Sigma n^2 + \Sigma n\right\} = \frac{n(n+1)(n+2)}{6}$

Q.28 (1)

$\frac{1^3+2^3+3^3+4^3+.........+12^3}{1^2+2^2+3^2+4^2+........+12^2}$

$= \frac{\left(\sum_{n=1}^{12} n^3\right)}{\left(\sum_{n=1}^{12} n^2\right)} = \left[\frac{n(n+1)}{2}\right]^2 \times \frac{6}{n(n+1)(2n+1)}$

$= \frac{3}{2} \cdot \frac{n(n+1)}{(2n+1)} = \frac{3}{2} \cdot \frac{12 \cdot 13}{25} = \frac{234}{25}$

[Putting n=12].

Q.29 (2)

Here, $T_n = 3 + n(n-1) = 3 + n^2 - n$

Now sum $S = \Sigma T_n = \Sigma(3 + n^2 - n)$

$= 3n + \frac{1}{6}n(n+1)(2n+1) - \frac{n(n+1)}{2}$

$= \frac{1}{6}n(n+1)[2n+1-3] + 3n = \frac{n^3+8n}{3}$

Q.30 (1)

$T_n = \frac{1+2+3+.....+n}{n} = \frac{n(n+1)}{2n} = \frac{1}{2}(n+1)$

Hence, $S = \frac{1}{2}(\Sigma n + n) = \frac{1}{2}\left\{\frac{n(n+1)}{2} + n\right\} = \frac{n(n+3)}{4}$.

EXERCISE-II

Q.1 (4)

$S = \frac{2p+1}{2}\,[2(p^2+1)+2p]$

$= (2p+1)\,(p^2+1+p)$

$= 2p^3 + 3p^2 + 3p + 1 = p^3 + (p+1)^3$

Q.2 (4)

$a_1 + a_5 + a_{10} + a_{15} + a_{20} + a_{24} = 225$

$\Rightarrow$ $3\,(a_1 + a_{24}) = 225$ (sum of terms equidistant from beginning and end are equal)

$a_1 + a_{24} = 75$

Now $a_1 + a_2 + ... + a_{23} + a_{24}$

$= \frac{24}{2}\,[a_1 + a_{24}] = 12 \times 75 = 900$

Q.3 (3)

Sum of the integer divided by 2

$= 2 + 4 + ...+ 98 + 100$

$= \frac{50}{2}\,[2.2 + (50-1)2]$

$= 50\,[51] = 2550$

Sum of the integer divided by 5

$= 5 + 10 + ...95 + 100$

$= \frac{20}{2}\,[5 + 100] = 1050$

Sum of the integer divided by 10

$\frac{10}{2}[10+100]=550$

Sum of the integers divided by 5 or 10

$=2550+1050-550=3050$

Q.4 (2)

Let a, a + d, a + 2d .... are Interior angles

$\because$ sum of interior angles = $(n-2)\pi$, where n is the number of sides

$\therefore$ $a = 120°, d = 5°$

$\Rightarrow$ $\frac{n}{2}[240° + (n-1)5°] = (n-2)180°$

$\Rightarrow$ $n^2 = 25n - 144 \Rightarrow n = 16, 9$ but $n \neq 16$

because if n = 16, then an interior angle will be 180° which is not possible So n = 9

Q.5 (1)

$$\frac{S_{kx}}{S_x} = \frac{\frac{kx}{2}[2a+(kx-1)d]}{\frac{x}{2}[2a+(x-1)d]} = k\left[\frac{2a+(kx-1)d}{2a+(x-1)d}\right]$$

If $2a - d = 0$, then $\frac{S_{kx}}{S_x}$ is independent of x

So $d = 2a$

Q.6 (4)

$x \in R$

$5^{1+x} + 5^{1-x}$, a/2 , $5^{2x} + 5^{-2x}$ are in A.P

$a = (5^{2x} + 5^{-2x}) + (5^{1+x} + 5^{1-x})$

$a = (5^{2x} + 5^{-2x}) + 5(5^x + 5^{-x})$

$= (5^x - 5^{-x})^2 + 2 + 5(5^{x/2} - 5^{-x/2})^2 + 10$

$a = 12 + (5^x - 5^{-x})^2 + 5(5^{x/2} - 5^{-x/2})^2$

$\Rightarrow a \geq 12$

Q.7 (4)

$$S = \frac{1}{\log_2 4} + \frac{1}{\log_4 4} + \frac{1}{\log_8 4} + ... + \frac{1}{\log_{2^n} 4}$$

$$= \frac{1}{2} + \frac{1}{1} + \frac{1}{2/3} + ... + \frac{1}{2/n}$$

$$= \frac{1}{2} + 1 + \frac{3}{2} + \frac{4}{2} + ... + \frac{n}{2}$$

$$= \frac{n(n+1)}{4}$$ **Ans**

Q.8 (1)

Given that

$S = 1^2 - 2^2 + 3^2 - 4^2 + ... 2002^2 + 2003^2$

$= 1 + (3^2 - 2^2) + (5^2 - 4^2) + ... + (2003^2 - 2002^2)$

$= 1 + 2 + 3 + 4 + 5 + ... + 2002 + 2003$

$= \frac{2003}{2}[1 + 2003] = 2003(1002) = (2000+3)(1000+2)$

$= 2007006$

Q.9 (2)

Let AP be A, A + d, A + 2d, ...

$T_p = a, T_q = b$

$\therefore a = A + (p-1)d$ ...(i)

$b = A + (q-1)d$ ...(ii)

$\Rightarrow$ subtract (i) & (ii) $\frac{a-b}{p-q} = d$

add (i) & (ii) $a + b = 2A + (p + q - 1)d - d$

$\Rightarrow$ $2A + (p + q - 1)d = (a + b) + d$

$$S_{p+q} = \frac{(p+q)}{2}[2A + (p + q - 1)d]$$

$$= \frac{(p+q)}{2}\left[a + b + \frac{a-b}{p-q}\right]$$

Q.10 (2)

$\frac{(54-3)}{n+1} = d$

$d = \frac{51}{n+1} \Rightarrow \frac{A_8}{A_{n-2}} = \frac{3}{5}$

$$\Rightarrow \frac{3 + 8\frac{51}{n+1}}{3 + (n-2)\frac{51}{n+1}} = \frac{3}{5}$$

$$\Rightarrow \frac{3n + 3 + 408}{3n + 3 + 51n - 102} = \frac{3}{5}$$

$\Rightarrow 15n + 2055 = 162n - 297$

$\Rightarrow 147n = 2352$

$n = 16$

Q.11 (2)

Let the GP be a, ar^2, ar^3, ...

Now, $ar^2 = 4$

$\therefore a \cdot ar \cdot ar^2 \cdot ar^3 \cdot ar^4 = a^5 r^{10} = (ar^2)^5 = 4^5$

Q.12 (2)

Let the GP be a, ar^2, ar^3, ...

We know that sum of G.P. is possible $\Rightarrow |r| < 1$

$S = \frac{a}{1-r} \Rightarrow r = \left(1 - \frac{a}{S}\right)$

$$S_n = \frac{a(1-r^n)}{1-r} = \frac{a\left(1-\left(1-\frac{a}{S}\right)^n\right)}{\frac{a}{S}} = S\left[1-\left(1-\frac{a}{S}\right)^n\right]$$

Q.13 (2)

Given,

$$a_1 = 2, \& \ \frac{a_{n+1}}{a_n} = \frac{1}{3}$$

$$= r, \ \sum_{r=1}^{20} a_r = \frac{a_1(1-r^{20})}{1-r} = \frac{2\left(1-\left(\frac{1}{3}\right)^{20}\right)}{\frac{2}{3}} = 3\left(1-\frac{1}{3^{20}}\right)$$

Q.14 (3)

Given that,

$x^2 - 3x + a = 0$ with roots α, β; $x^2 - 12x + b = 0$ with roots γ, δ

Also, $\alpha, \beta, \gamma, \delta$ in increasing G.P.

Let $\alpha, \beta, \gamma, \delta$ be $\alpha, \alpha r, \alpha r^2, \alpha r^3$

$\alpha + \alpha r = 3$ & $\alpha r^2 + \alpha r^3 = 12$

$\alpha(1 + r) = 3$ & $\alpha r^2 (1 + r) = 12$

& $r^2 . 3 = 12$

$\therefore \alpha = \frac{3}{1+r}$ & $r^2 = 4 \Rightarrow \alpha = \frac{3}{3} \Rightarrow \alpha = 1 \qquad r = 2,$

$r = -2$ is rejected since

$\because$ G.P. increasing

$\alpha(\alpha r) = a$ & $(\alpha r^2)(\alpha r^3) = b$

$a = \alpha(\alpha r) = 1\,(1.2) = 2$

$b = (\alpha r^2)(\alpha r^3) = (1.4)(1.8) = 32$

Q.15 (4)

Let GP be $a_1, a_2, ..., a_k, ...$ with first term a & common ratio r,

$a_k = a_{k+1} + a_{k+2} \quad \forall\, a_k > 0$

$\Rightarrow ar^{k-1} = ar^k + ar^{k+1} \ \Rightarrow \ r > 0$

$\Rightarrow 1 = r + r^2 \Rightarrow r^2 + r - 1 = 0$

$\Rightarrow r = \frac{-1 \pm \sqrt{1+4}}{2}$ {r = –ve rejected}

$\Rightarrow r = \frac{\sqrt{5}-1}{2} = 2\left(\frac{\sqrt{5}-1}{4}\right) = 2 \sin 18^\circ$

Q.16 (3)

$y = 2.\overline{357}$

$y = 2.357357357$..................(1)

$1000\,y = 2357.357357$...........(2)

so $999y = 2355$

$y = \frac{2355}{999}$

Q.17 (1)

$S = \frac{p}{1-\frac{1}{p}} = \frac{9}{2} \quad \Rightarrow \frac{p^2}{p-1} = \frac{9}{2}$

$\Rightarrow 2p^2 - 9p + 9 = 0$

$\Rightarrow 2p^2 - 6p - 3p + 9 = 0$

$\Rightarrow (2p - 3)(p - 3) = 0$

$p = 3/2 \ , 3$

Q.18 (1)

Let $S = \sqrt{2} + \sqrt{6} + \sqrt{18} + \sqrt{54} + ... +$ (10 terms)

$= \sqrt{2}\ (1 + \sqrt{3} + \sqrt{9} + \sqrt{27} + ... +$ (10 terms))

$= \sqrt{2}\ (1 + 3^{1/2} + 3^1 + 3^{3/2} + ... +)$

$= \sqrt{2}\ \ 1. \frac{(1-(\sqrt{3})^{10})}{(1-\sqrt{3})} = \frac{\sqrt{2}((\sqrt{3})^{10}-1)}{(\sqrt{3}-1)}$

$= \frac{\sqrt{2}(3^5-1)(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)} = \frac{\sqrt{2}}{2}\ 242\,(\sqrt{3}+1)$

$= \sqrt{2}\ (121)(\sqrt{3}+1) = 121\,(\sqrt{6}+\sqrt{2})$

Q.19 (1)

Let $S = \frac{1}{(1+p)} - \frac{1-p}{(1+p)^2} + \frac{(1-p)^2}{(1+p)^3} \ ... \infty$

$-1 < r = -\left(\frac{1-p}{1+p}\right) < 1 \qquad \because \ p > 0$

It is a GP with first term $= \frac{1}{1+p}$ & common ratio r

$\therefore S = \frac{a}{1-r} = \frac{\left(\frac{1}{1+p}\right)}{1+\frac{1-p}{1+p}} = \frac{1}{1+p+1-p} = \frac{1}{2}$

Q.20 (3)

If $x > 0$

$\log_2 x + \log_2(\sqrt{x}) + \log_2 \sqrt[4]{x} + \log_2 \sqrt[8]{x} + \log_2 \sqrt[16]{x} + \ldots = 4$

$\Rightarrow \log_2 x + \log_2 x^{1/2} + \log_2 x^{1/4} + \ldots = 4$

$\Rightarrow (\log_2 x)\left[1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots\right] = 4$

$\Rightarrow (\log_2 x)\frac{1}{1-\frac{1}{2}} = 4 \Rightarrow \log_2 x = 2 \Rightarrow x = 4$

Q.21 (1)

$3 + \frac{1}{4}(3+d) + \frac{1}{4^2}(3+2d) + \ldots + \infty = 8$

$a = 3, r = \frac{1}{4}$

Sum of AGP upto ∞

$S_\infty = \frac{a}{1-r} + \frac{dr}{(1-r)^2}$

$\Rightarrow 8 = \frac{3}{(3/4)} + \frac{d\left(\frac{1}{4}\right)}{3^2/4^2} \Rightarrow 8 = 4 + \frac{4d}{3^2} \Rightarrow 4 = \frac{4d}{3^2} \Rightarrow d = 3^2 \Rightarrow d = 9$

Q.22 (3)

$\alpha + \beta = \frac{1}{\alpha^2} + \frac{1}{\beta^2} \Rightarrow \alpha + \beta = \frac{\alpha^2+\beta^2}{\alpha^2\beta^2}$

$\Rightarrow \quad -\frac{b}{a} = \frac{b^2 - 2ac}{c^2} \Rightarrow -bc^2 = ab^2 - 2a^2c$

$\Rightarrow \quad ab^2 + bc^2 = 2a^2c \Rightarrow \frac{b}{c} + \frac{c}{a} = \frac{2a}{b}$

So $\frac{c}{a}, \frac{a}{b}, \frac{b}{c}$ are in A.P. $\Rightarrow \frac{a}{c}, \frac{b}{a}, \frac{c}{b}$ are in H.P.

Q.23 (1)

Given that,

$x = \sum_{n=0}^{\infty} a^n, y \sum_{n=0}^{\infty} b^n, z = \sum_{n=0}^{\infty} c^n \quad \begin{cases} |a|<1 \\ |b|<1 \\ |c|<1 \end{cases}$

$x = \frac{1}{1-a}, y = \frac{1}{1-b}, z = \frac{1}{1-c}$

$\because$ a, b, c are in A.P.

$\Rightarrow 1-a, 1-b, 1-c$ are also in A.P.

$\frac{1}{1-a}, \frac{1}{1-b}, \frac{1}{1-c}$ are in H.P. $\Rightarrow$ x, y, z in H.P.

Q.24 (1)

a, b, c in A.P. $\Rightarrow 2b = a + c$ ...(i)

p, q, r, in H.P. $\Rightarrow q = \frac{2pr}{p+r}$ ...(ii)

ap, bq, cr in G.P. $\Rightarrow b^2q^2 = acpr$ ...(iii)

From (ii) & (iii), we get

$\Rightarrow \frac{b^2.4(pr)^2}{(p+r)^2} = ac\,pr$

$\Rightarrow \frac{(a+c)^2 pr}{(p+r)^2} = ac$ (from (i))

$\Rightarrow \frac{(p+r)^2}{pr} = \frac{(a+c)^2}{ac} \Rightarrow \frac{p^2+r^2}{pr} + 2 = \frac{a^2+c^2}{ac} + 2$

$\Rightarrow \frac{p}{r} + \frac{r}{p} = \frac{a}{c} + \frac{c}{a}$.

Q. 25 (3)

$a^x = b^y = c^z = d^t = k$ and a, b, c, d are in G.P.

a, b, c are in G.P. $\Rightarrow$ So $b^2 = ac$

$\Rightarrow \quad k^{2/y} = k^{1/x + 1/z} \Rightarrow \frac{2}{y} = \frac{1}{x} + \frac{1}{z}$

$\Rightarrow$ x, y, z are in H.P.

$\because$ b, c, d are in GP

then $\frac{2}{z} = \frac{1}{y} + \frac{1}{t} \Rightarrow$ y, z, t are in HP

So x, y, z, t are in H.P.

Q.26 (2)

$AM = A = \frac{a+b+c}{3}$

$GM = G = (abc)^{1/3}$

$HM = H = \frac{3abc}{ab+bc+ca} = \frac{3G^3}{ab+bc+ca}$

Equation whose roots are a,b,c

$\Rightarrow x^3 - (a+b+c)x^2 + (\Sigma ab)x - abc = 0$

$\Rightarrow x^3 - 3Ax^2 + \frac{3G^3}{H}.x - G^3 = 0$ **Ans**

Q.27 (2)

Let $S = \sum_{r=2}^{\infty} \frac{1}{r^2-1}$

$= \sum_{r=2}^{\infty} \frac{1}{(r-1)(r+1)} = \frac{1}{2}\sum_{r=2}^{\infty}\left(\frac{1}{r-1} - \frac{1}{r+1}\right)$

$$=\frac{1}{2}\left[1-\frac{1}{3}+\frac{1}{2}-\frac{1}{4}+\frac{1}{3}-\frac{1}{5}+....+\frac{1}{n-1}-\frac{1}{n+1}+....\right]$$

when $n\to\infty \Rightarrow \frac{1}{n+1}\to 0$

$$\therefore S=\frac{1}{2}\left[1+\frac{1}{2}\right]=\frac{3}{4}.$$

Q.28 (3)

Let $S=\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+....\infty=\frac{\pi^2}{6}$

Now $S_{even}=\frac{1}{2^2}+\frac{1}{4^2}+\frac{1}{6^2}+....\infty$

$$=\frac{1}{2^2}\left[1+\frac{1}{2^2}+\frac{1}{3^2}+....\infty\right]=\frac{1}{2^2}\ \frac{\pi^2}{6}=\frac{\pi^2}{24}$$

$S_{odd}=\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+....\infty$

$=S-S_{even}$

$$=\frac{\pi^2}{6}-\frac{\pi^2}{24}=\frac{3\pi^2}{24}=\frac{\pi^2}{8}$$

Q.29 (3)

$1^2+2.2^2+3^2+2.4^2+5^2+2.6^2+......$n terms

$=\frac{n(n+1)^2}{2}$, when n is even

$1^2+2\,.\,2^2+3^2+.....\,2\,.\,n^2=n\,\frac{(n+1)^2}{2}$

when n is odd n + 1 is even

$1^2+2\,.\,2^2+3^2+.....\,n^2+2\,.\,(n+1)^2$

$=(n+1)\,\frac{(n+2)^2}{2}$

$1^2+2\,.\,2^2+3^2+.....\,n^2=(n+1)\left[\frac{(n+2)^2}{2}-2(n+1)\right]$

$=\frac{(n+1)\,n^2}{2}$

Q.30 (2)

Given that,

$1^2+2^2+...\,n^2=1015$

$\frac{n(n+1)(2n+1)}{6}=1015$

Put $n=15\Rightarrow \frac{15\times16\times31}{6}=1240\Rightarrow n\neq15$

Put $n=14\Rightarrow \frac{14\times15\times29}{6}=1015\Rightarrow n=14$

EXERCISE-III

Q.1 (B,D)

(D) $a_1+4a_2+6a_3-4a_4+a_5=0$

$a-4(a+d)+6(a+2d)-4(a+3d)+(a+4d)$

$=0-0=0$

Like wise we can check other options

Q.2 (A, B)

since x, $|x+1|$, $|x-1|$ are in A.P.

so $2|x+1|=x+|x-1|$ .... (i)

Case-I If $x<-1$, then (i) becomes

$-2(x+1)=x-(x-1)$

$\Rightarrow x=-\frac{3}{2}$

then series $-\frac{3}{2},\frac{1}{2},\frac{5}{2},......$

$\therefore\quad S_{20}=\frac{20}{2}[-3+(20-1)2]=350$

Case-II If $-1\le x\le 1$, then (i) becomes

$2(x+1)=x-(x-1)\Rightarrow x=-1/2$

then series $-\frac{1}{2},\frac{1}{2},\frac{3}{2}$

So $S_{20}=\frac{20}{2}[-1+(20-1)1]=10\times18=180$

Case-III If $x\ge1$, then (i) becomes

$2(x+1)=x+x-1$

$2=-1$ impossible.

Q.3 (B,D)

$a_1,a_2,....,a_n,....$are in AP

$a_2=\frac{a_1+a_3}{2}\Rightarrow a_1+a_3-2a_2=0$

$\&\ a_1-2a_2+a_3=0$ ...(i)

$\&\ -2(a_2-2a_3+a_4)=0$ ...(ii)

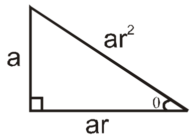
& $a_3 - 2a_4 + a_5 = 0$ .....(iii)

Adding (i) (ii) & (iii), we get

$a_1 - 4a_2 + 6a_3 - 4a_4 + a_5 = 0$

Q.4 (B,C)

Case - I



$r > 1$

$a^2 + a^2r^2 = a^2r^4$

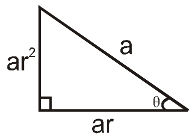
$\Rightarrow r^4 - r^2 - 1 = 0$

$r^2 = \dfrac{\sqrt{5}+1}{2}$

$r = \sqrt{\dfrac{\sqrt{5}+1}{2}}$ tangent of smallest angle = $\tan\theta = \dfrac{1}{r}$

$= \sqrt{\left(\dfrac{2}{\sqrt{5}+1}\right)}$

Case - II



$0 < r < 1$

$a^2 = a^2r^2 + a^2r^4$

$r^4 + r^2 - 1 = 0$

$r^2 = \dfrac{\sqrt{5}-1}{2}$

$r = \sqrt{\dfrac{\sqrt{5}-1}{2}}$

tangent of smallest angle = $\tan\theta = r = \sqrt{\dfrac{\sqrt{5}-1}{2}}$

Q.5 (A,B,C,D)

b_1, b_2, b_3 are in G.P. $\therefore b_3 > 4b_2 - 3b_1 \Rightarrow r^2 > 4r - 3$

$\Rightarrow \quad r^2 - 4r + 3 > 0$

$\Rightarrow \quad (r-1)(r-3) > 0$ So $0 < r < 1$ and $r > 3$

Q.6 (A,B)

$2b = a + c$

$4b^2 = a^2 + c^2 + 2ac$

$\Rightarrow a^2 + c^2 = 4b^2 - 2ac$ & $b^2 = \dfrac{2a^2c^2}{a^2 + c^2}$

$\Rightarrow b^2(4b^2 - 2ac) = 2a^2c^2$

$\Rightarrow b^2(2b^2 - ac) = a^2c^2$

$\Rightarrow 2b^4 - b^2(ac) - (ac)^2 = 0$

$\Rightarrow (b^2 - ac)(2b^2 + ac) = 0$

$\Rightarrow b^2 = ac \Rightarrow$ a, b, c in G.P.

& a, b, c in A.P.

$\Rightarrow a = b = c$

or $2b^2 + ac = 0 \Rightarrow b^2 = -\dfrac{ac}{2}$

$\Rightarrow a, b, \dfrac{-c}{2}$ in G.P.

Q.7 **A,B,C**

$\dfrac{\frac{a+b}{2}}{\sqrt{ab}} = \dfrac{2}{1} \Rightarrow \dfrac{a+b}{2\sqrt{ab}} = \dfrac{2}{1}$

use compendendo and dividendo rule

$\Rightarrow \dfrac{a+b+2\sqrt{ab}}{a+b-2\sqrt{ab}} = \dfrac{3}{1} \Rightarrow \dfrac{\sqrt{a}+\sqrt{b}}{\sqrt{a}-\sqrt{b}} = \dfrac{\sqrt{3}}{1}$

$\Rightarrow \dfrac{2\sqrt{a}}{2\sqrt{b}} = \dfrac{\sqrt{3}+1}{\sqrt{3}-1} \Rightarrow \dfrac{a}{b} = \dfrac{3+1+2\sqrt{3}}{3+1-2\sqrt{3}}$

$\Rightarrow \dfrac{a}{b} = \dfrac{2+\sqrt{3}}{2-\sqrt{3}} = \dfrac{(2+\sqrt{3})(2+\sqrt{3})}{4-3} = 7 + 4\sqrt{3}$ **Ans**

Q.8 A,B,C

Given, $x = y + 2$, $a = 5z$

a, x, b in A.P $\Rightarrow 2x = a + b$

a, y, b in G.P $\Rightarrow y^2 = ab$

a, z, b in H.P $\Rightarrow z = \dfrac{2ab}{a+b} \Rightarrow z = \dfrac{2y^2}{2x} \Rightarrow y^2 = zx$

A.M. > G.M. > H.M.

$x > y > z$

$\dfrac{a}{5} = \dfrac{2ab}{a+b}$

$a^2 + ab = 10ab \Rightarrow a(a - 9b) = 0 \Rightarrow a \neq 0$ or $a = 9b$

$y = x - 2$

$\therefore y = \dfrac{2x-4}{2} \Rightarrow y^2 = \dfrac{(a+b-4)^2}{4}$

$\Rightarrow a^2 + b^2 - 2ab - 8a - 8b + 16 = 0$

$\Rightarrow 4b^2 - 5b + 1 = 0 \Rightarrow (b-1)(4b-1) = 0$

$\Rightarrow b = 1, a = 9$

or $b = 1/4, a = 9/4$

Q.9 (A, B, C, D)

$\sum_{r=1}^{n} r(r+1)(2r+3) = an^4 + bn^3 + cn^2 + dn + e$

$\sum_{r=1}^{n} (r^2+r)(2r+3)$

$= \sum_{r=1}^{n} (2r^3 + 5r^2 + 3r)$

$= 2.\frac{n^2(n+1)^2}{4} + 5\frac{n(n+1)(2n+1)}{6} + 3\frac{n(n+1)}{2}$

$= \frac{n(n+1)}{2}\left[n(n+1) + \frac{5}{3}(2n+1) + 3\right]$

$= \frac{n(n+1)}{2}\left[\frac{6(n^2+n) + 10(2n+1) + 18}{6}\right]$

$= \frac{n(n+1)}{12}[6n^2 + 26n + 28]$

$= \frac{1}{12}[6n^4 + 26n^3 + 28n^2 + 6n^3 + 26n^2 + 28n]$

$= \frac{1}{12}[6n^4 + 32n^3 + 54n^2 + 28n]$

$a = \frac{6}{12}, b = \frac{32}{12}, c = \frac{54}{12}, d = \frac{28}{12} = \frac{7}{3}, e = 0$

so $a + c = b + d$

$b - \frac{2}{3} = \frac{32}{12} - \frac{2}{3} = \frac{24}{12}$

$c - 1 = \frac{42}{12}$

so $a, b-2/3, c-1$ are in A.P

& $\frac{c}{a} = \frac{54}{6} = 9$ is an integer

Q.10 (A,C)

$\sum_{r=1}^{n} \frac{1}{\sqrt{a+rx} + \sqrt{a+(r-1)x}}$

$= \sum_{1}^{n} \frac{\sqrt{a+rx} - \sqrt{a+(r-1)x}}{x}$

$= \frac{1}{x}\sum_{1}^{n}(\sqrt{a+rx} - \sqrt{a+(r-1)x})$

$= \frac{1}{x}[\sqrt{a+nx} - \sqrt{a}]$

Upon rationalizing

$= \frac{1}{x}\frac{a+nx-a}{\sqrt{a+nx}+\sqrt{a}} = \frac{n}{\sqrt{a+nx}+\sqrt{a}}$

Q.11 (A)

$g(n) - g(n-1) = 1^2 + 2^2 + 3^2 + \ldots\ldots\ldots + (n-1)^2 + n^2 - (1^2 + 2^2 + 3^2 + \ldots\ldots\ldots + (n-1)^2) = n^2$

Q.12 (A)

$g(n) - f(n) = \frac{n(n+1)(2n+1)}{6} - \frac{n(n+1)}{2} = \frac{n(n+1)}{2}$

$\left(\frac{2n+1}{3} - 1\right) = \frac{n(n+1)}{2}\ \frac{2n-2}{3}$

$= \frac{n(n+1)(n-1)}{3} = \frac{(n-1)\,n\,(n+1)}{3}$

for $n = 2\ \frac{(n-1)\,n\,(n+1)}{3} = \frac{1.2.3}{3}$ which is divisible by 2 but not by 2^2

$\therefore$ greatest even integer which divides $\frac{(n-1)\,n\,(n+1)}{3}$, for every $n \in N, n \geq 2$, is 2

Q.13 (D)

$f(n) + 3g(n) + h(n) =$

$\frac{n(n+1)}{2} + \frac{n(n+1)(2n+1)}{2} + \left(\frac{n(n+1)}{2}\right)^2$

$= \frac{n(n+1)}{2}\left(1 + 2n + 1 + \frac{n(n+1)}{2}\right) = (1 + 2 + 3 + \ldots. + n)$

$\left(2n + 2 + \frac{n(n+1)}{2}\right)$

$\Rightarrow$ for all $n \in N$

Q.14 (A)

Q.15 (C)

Q.16 (D)

Sol. **14, 15, 16**

Let the 3 numbers in strictly increasing GP are $\frac{a}{r}, a, ar (r > 1)$

According to passage $\frac{a^2}{r^2} + a^2 + a^2r^2 = S^2$

or $a^2\left(\frac{1}{r^2}+1+r^2\right) = S^2$

or $a^2\left(\frac{1}{r}+1+r\right)\left(\frac{1}{r}-1+r\right) = S^2$ ...(i)

and $\frac{a}{r} + a + ar = \alpha S$

or $a\left(\frac{1}{r}+1+r\right) = \alpha S$

$\therefore$ $a^2\left(\frac{1}{r}+1+r\right)^2 = \alpha^2 S^2$ ..(ii)

Dividing Eq. (ii) by (i), then $\left(\dfrac{\frac{1}{r}+1+r}{\frac{1}{r}-1+r}\right) = \alpha^2$

$\Rightarrow (1+r+r^2) = \alpha^2(1-r+r^2)$

$\Rightarrow (\alpha^2-1)r^2-(\alpha^2+1)r+\alpha^2-1=0$ ...(iii)

(1) $r = 3 \therefore r = \frac{11}{7}$

(2) Put $\alpha^2 = 2$ in eq. (iii), then $r^2 - 3r + 1 = 0$

$\therefore$ $r = \frac{3\pm\sqrt{5}}{2} = \frac{3+\sqrt{5}}{2} \therefore r > 2$

(3) Put $r = 2$ in eq. (iii), then

$4(\alpha^2-1)-2(\alpha^2+1)+a^2-1=0$

or $3\alpha^2 - 7 = 0 \therefore \alpha^2 = 7/3 = a/b \Rightarrow a+b = 10$

Q.17 (A) $\rightarrow$ (p), (B) $\rightarrow$ (p), (C) $\rightarrow$ (q), (D) $\rightarrow$ (q)

(A) $2\log_5(2^x-5) = \log_5 2 + \log_5(2^x - 7/2)$

$(2^x-5)^2 = 2\left(2^x - \frac{7}{2}\right)$

$t^2 - 10t + 25 = 2t - 7$ {put $2^x = t$}

$t^2 - 12t + 32 = 0$

$\therefore t = 8, 4$

$\therefore 2^x = 4$ or $2^x = 8$

$\therefore x = 2, 3$ $2^x - 5 > 0$

$\therefore 2x = 6 \therefore 2^x > 5$

so only solution x = 3

(B) $\frac{S_{2n}}{S_n} = \dfrac{\frac{2n}{2}[2a+(2n-1)d]}{\frac{n}{2}[2a+(n-1)d]} = 3$

$= \frac{2a+(2n-1)d}{2a+(n-1)d} = \frac{3}{2}$

$\therefore d = \frac{2a}{n+1}$

Now $\frac{S_{3n}}{S_n} = \dfrac{\frac{3n}{2}[2a+(3n-1)d]}{\frac{n}{2}[2a+(n-1)d]}$

$= \dfrac{3\left[2a+(3n-1)\frac{2a}{n+1}\right]}{\left[2a+(n-1)\frac{2a}{n+1}\right]} = \frac{3[(n+1)+(3n-1)]}{(n+1)+(n-1)}$

$= \frac{3.4n}{2n} = 6$

(C) $S = 4 + \frac{8}{3} + \frac{12}{3^2} + \frac{16}{3^3} + \ldots\ldots\ldots\ldots \frac{4n}{3^{n-1}}$ ...(i)

$\frac{S}{3} = \frac{4}{3} + \frac{8}{3^2} + \frac{12}{3^3} + \ldots\ldots\ldots\ldots + \frac{4m}{3^n}$ ...(ii)

(i) – (ii) we get

$\frac{2}{3}S = 4 + \frac{4}{3} + \frac{4}{3^2} + \frac{4}{3^3} + \ldots\ldots\ldots + \frac{4}{3^{n-1}} + \frac{4n}{3^n}$

$= 4\left(\dfrac{1-\left(\frac{1}{3}\right)^n}{1-\frac{1}{3}}\right) - \frac{4n}{3^n}$

$S = \frac{4\cdot 3}{2}\cdot\frac{3}{2}\left(1-\left(\frac{1}{3}\right)^n\right) - \frac{4n}{3^n}\cdot\frac{3}{2}$

$\therefore S_\infty = 9$

(D) $\ell, \ell r, \ell r^2$

$\therefore \ell^3 r^3 = 27$ (volume)

$\ell r = 3$

surface area

$2(\ell \,.\, \ell r + \ell r \,.\, \ell r^2 + \ell r^2 \,.\, \ell) = 78$

$\ell^2(r + r^2 + r^3) = 39 \Rightarrow \ell^2\left(\frac{3}{\ell} + \frac{3^2}{\ell^2} + \frac{3^3}{\ell^3}\right) = 39$

$3\ell + 3^2 + \frac{3^3}{\ell} = 39 \Rightarrow \ell + 3 + \frac{9}{\ell} = 13$

$\therefore \quad \ell^2 - 10\ell + 9 = 0 \Rightarrow \ell = 1, 9$

$\Rightarrow \quad \ell r = 3$ and $\ell > \ell r$

$\therefore \quad r = \frac{1}{3} \therefore \ell = 9$

Q.18 (A) $\rightarrow$ (r), (B) $\rightarrow$ (p), (C) $\rightarrow$ (s), (D) $\rightarrow$ (p)

(A) $2(\log_z x)^2 = \log_x y . \log_y z$ G.P

$(\log_z x)^2 = \log_x z$

$(\log_z x)^3 - 1 = 0$

$\log_z x = 1$, $(\log_z x)^2 + \log_z x + 1 = 0$ (not possible)

$\therefore x = z$ ....(1)

$2y^3 = x^3 + z^3$ x^3, y^3, z^3 in A.P

$2y^3 = 2x^3$

$x = y$ ....(2)

$\therefore x = y = z$

Given $xyz = 64$, $\therefore x = y = z = 4$

$\therefore \frac{3x}{y} = 3$

(B) $S_\infty = 2^{1/4} . 4^{1/8} . 8^{1/16}$................$\infty$

$S_\infty = 2^{1/4} . 2^{2/8} . 2^{3/16}$................$\infty$

$S_\infty = 2^{1/4 + 2/8 + 3/16 ....\infty} = 2^{S'_\infty}$

Let $S'_\infty = \frac{1}{4} + \frac{2}{8} + \frac{3}{16}$ ....

$S'_n = \frac{1}{4} + \frac{2}{8} + \frac{3}{16} .... \frac{n}{2^{n+1}}$ ...(i)

$\frac{S'_n}{2} = \frac{1}{8} + \frac{2}{16} + ... + \frac{n-1}{2^{n+1}} + \frac{n}{2^{n+2}}$ ...(ii)

(i) – (ii) we get

$\frac{S'_n}{2} = \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + ... \frac{1}{2^{n+1}} - \frac{n}{2^{n+2}}$

$\frac{S'_n}{2} = \frac{1}{4}\left(\frac{1-(1/2)^n}{1-1/2}\right) - \frac{n}{2^{n+2}}$

$S'_n = \frac{2.2}{4}\left(1-\left(\frac{1}{2}\right)^n\right) - \frac{2n}{2^{n+2}}$

$S'_\infty = 1$

$\therefore S_\infty = 2^{S'_\infty} = 2$

(C) x,y,z are in A.P

$y = \frac{x+z}{2}$, or $2y = x + z$

$(x + 2y - z)(x + z + z - x)(z + x - y)$

$= (x + (x + z) - z)(x + z + z - x)(2y - y)$

$= 2x .2z .y = 4xyz$

$\therefore k = 4$

(D) $d = \frac{31-1}{m+1} = \frac{30}{m+1}$

$\frac{A_7}{A_{m-1}} = \frac{1 + 7\frac{30}{m+1}}{1 + (m-1)\frac{30}{m+1}} = \frac{5}{9}$

$\Rightarrow \frac{m+211}{31m-29} = \frac{5}{9}$

$\Rightarrow 146m = 2044$

$\Rightarrow m = 14$

$\therefore \frac{m}{7} = 2$

NUMERICAL VALUE BASED

Q.19 [0002]

$2^{1/4} \times 4^{1/8} \times 8^{1/16} ... = 2^{\frac{1}{4}+\frac{2}{8}+\frac{3}{16}+...}$

Now, $S = \frac{1}{4} + \frac{2}{8} + \frac{3}{16} + ...$

$\Rightarrow \frac{1}{2}S = \frac{1}{8} + \frac{2}{16} + ...$

$\therefore S - \frac{1}{2}S = \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + ...$

$\Rightarrow \frac{1}{2}S = \frac{\frac{1}{4}}{1-\frac{1}{2}}$

$\Rightarrow S = 1$

So the given product is 2.

Q.20 [0003]

Let common ratio is $\frac{1}{2^b}$

and $S_\infty = \frac{a}{1-r} = \frac{\frac{1}{2^a}}{1-\frac{1}{2^b}} = \frac{1}{7}$

$\Rightarrow b = 3$ & $a = b$

$\Rightarrow b = 3$ & $a = b$

Hence, a = 3

Q.21 [0001]

$T_2 = 3+d, T_{10} = 3+9d, T_{34} = 3+33d$

since $T_2 . T_{10}, T_{34}$ are in G.P

$T_{10}^{\ 2} = T_2 T_{34}$

$\Rightarrow (3+9d)^2 = (3+d)(3+33d)$

$\Rightarrow d = 0,1$

hence d = 1

Q.22 [0012]

According to question,

$\frac{\log_z x}{\log_x y} = \frac{\log_y z}{\log_z x} \Rightarrow (\log x)^3 = (\log z)^3$

$\Rightarrow x = z$

Since $2y^3 = x^3 + z^3 \Rightarrow x^3 = y^3$ or $x = y$

given $xyz = 64$ & $x = y = z$

$\therefore x = y = z = 4$

$\& x + y + z = 12$

Q.23 [900]

$a_1 + a_5 + a_{10} + a_{15} + a_{20} + a_{24} = 225$

$\Rightarrow$ $3(a_1 + a_{24}) = 225$ (sum of terms equidistant from beginning and end are equal)

$a_1 + a_{24} = 75$

Now $a_1 + a_2 + ... + a_{23} + a_{24} = \frac{24}{2}[a_1 + a_{24}]$

$= 12 \times 75 = 900$

Q.24 **[4]**

We can write the given equation as

$\log_2\left(x^{1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+..........}\right) = 4$

$\Rightarrow \log_2(x^2) = 4 \Rightarrow x^2 = 2^4 \Rightarrow x = 4$

Q.25 **[0]**

$T_p = AR^{p-1} = x$

$\log x = \log A + (p-1)\log R$

Similary write $\log y$, $\log z$

Multiply by $q - r$, $r - p$ and $p - q$ and add we get,

$(q-r)\log x + (r-p)\log y + (p-q)\log z = 0$

Q.26 **[0.75]**

$S = \sum_{r=2}^{\infty} \frac{1}{r^2 - 1} = \sum_{r=2}^{\infty} \frac{1}{(r-1)(r+1)}$

$T_r = \frac{1}{(r-1)(r+1)} = \frac{1}{2}\left[\frac{1}{r-1} - \frac{1}{r+1}\right]$

$\Rightarrow S_\infty = \frac{3}{4}$]

Q.27 **[0.67]**

b_1, b_2, b_3...........

or $a, ar, ar^2, ... \infty$

Given $\frac{a}{1-r} = 3$ and $\frac{a^3}{1-r^3} = \frac{108}{13}$

solving $a = 2$ and $r = \frac{1}{3}$ or $r = 3$ (rejected)

$\therefore$ $ar = \frac{2}{3} = b_2$

Q.28 **[0.5]**

$T_n = \frac{1.3.5.........(2n-1)}{2.4.6............2n(2n+2)}[(2n+2) - (2n+1)]$

$T_n = \frac{1.3.5.........(2n-1)}{2.4.6............2n} -$

$\frac{1.3.5.........(2n-1)(2n+1)}{2.4.6............2n(2n+2)}$

$\therefore S_n = \sum T_n = \frac{1}{2} - \frac{1.3.5.........(2n+1)}{2.4.6............2n(2n+2)}$

Note that $S_\infty = \frac{1}{2}$

EXERCISE-IV

JEE-Main
PREVIOUS YEAR'S

Q.1 (2)

Let A be first term & d be com diff.

$T_2 = A + d$, $T_5 = A + 4d$ & $T_9 = A + 8d$

$T_5^2 = T_2 . T_9$

$(A + 4d)^2 = (A + d)(A + 8d)$

$A^2 + 16d^2 + 8Ad = A^2 + 9Ad + 8d^2$

$8d^2 - Ad = 0$

$\Rightarrow d(8d - A) = 0 \Rightarrow A = 8d$

$T_2 = 9d$, $T_5 = 12d$, $T_9 = 16d$

Com ratio $= \dfrac{12d}{9d} = \dfrac{4}{3}$

Q.2 (2)

$$\left(\frac{8}{5}\right)^2 + \left(\frac{12}{5}\right)^2 + \left(\frac{16}{5}\right)^2 + \left(\frac{20}{5}\right)^2 + \left(\frac{24}{5}\right)^2 + \ldots$$

$$= \frac{8^2 + 12^2 + 16^2 + \ldots}{25} = \frac{4^2\left(2^2 + 3^2 + 4^2 + \ldots + 11^2\right)}{25}$$

$$= \frac{16}{25} \times \left[\frac{11 \times 12 \times 23}{6} - 1\right] = \frac{16}{25} \times 101$$

Q.3 (3)

$(15a)^2 + (3b)^2 + (5c)^2 - (15a)(5c) - (15a)(3b) - (3b)(5c) = 0$

$\frac{1}{2}$ $[(15a - 3b)^2 + (3b - 5c)^2 + (5c - 15a)^2]$

It is possible when $15a = 3b = 5c$

$\therefore\ b = \dfrac{5c}{3}$, $a = \dfrac{c}{3}$

$a + b = 2c$

$\Rightarrow$ b, c, a in A.P.

Q.4 (2)

$f(x) = ax^2 + bx + c$

$f(1) = a + b + c = 3$

Now $f(x + y) = f(x) + f(y) = xy$

put $y = 1$

$f(x + 1) = f(x) + f(1) + x$

$f(x + 1) = f(x) + x + 3$

Now

$f(2) = 7$

$f(3) = 12$

Now

$S_n = 3 + 7 + 12 + t_n$ ....(i)

$S_n = \quad 3 + 7 + ...t_{n-1} + t_n$ ....(ii)

On subtracting (ii) from (i)

$t_n = 3 + 4 + 5 + ...$upto n terms

$$t_n = \frac{(n^2 + 5n)}{2}$$

$$S_n = \sum t_n = \sum \frac{(n^2 + 5n)}{2}$$

$$S_n = \frac{1}{2}\left[\frac{n(n+1)(2n+1)}{6} + \frac{5n(n+1)}{2}\right]$$

$S_{10} = 330$

Q.5 (2)

$$\sum_{k=0}^{12} a_{4k+1} = 416$$

$$\Rightarrow \frac{13}{2}[2a_1 + 48d] = 416$$

$\Rightarrow a_1 + 24d = 32$ .....(i)

$a_9 + a_{43} = 66 \Rightarrow 2a_1 + 50d = 66$ .....(ii)

$2a_1 + 48d = 64$ by .....(i)

d = 1 and $a_1 = 8$

$$\Rightarrow 140m = \sum_{r=1}^{17} a_r^2 = \sum_{r=1}^{17}\left[8 + (r-1).1^2\right]$$

$$\Rightarrow 140m = \sum_{r=1}^{17}(r+7)^2 \qquad \Rightarrow 140m = \sum_{r=1}^{24} r^2 - \sum_{r=1}^{7} r^2$$

$$\Rightarrow 140m = \frac{24.25.49}{6} - \frac{7.8.15}{6}$$

$$\Rightarrow 140m = \frac{7.8.5}{6}[105 - 3]$$

$\Rightarrow 140m = 280.17 \Rightarrow m = 34$

Q.6 (1)

$$B - 2A = \sum_{r=1}^{40} T_r - 2\sum_{r=1}^{20} T_r = \sum_{r=21}^{40} T_r - \sum_{r=1}^{20} T_r$$

$B - 2A = m\,(21^2 + 2.22^2 + 23^2 + 2.24^2 + ... + 40^2)$
$- (1^2 + 2.2^2 + 3^2 + 2.4^2 .... + 20^2)$
$= 20\,[22 + 2.24 + 26 + 2.28 + ... + 60]$

$$= 20\left[\underbrace{(22+24+26...60)}_{20\text{terms}}+\underbrace{(24+28+......+60)}_{10\text{terms}}\right]$$

$$= 20\left[\frac{20}{2}(22+60)+\frac{10}{2}(24+60)\right]=10[20.82+10.84]$$

$$=100[164+84]=100.248$$

Q.7 **(4)**

$a + ar + ar^2 = xar$

since $a \neq 0$ so $\frac{r^2+r+1}{r} = x;\ 1+r+\frac{1}{r} = x$

$\because r+\frac{1}{r} \in (-\infty,-2] \cup [2,\infty) \Rightarrow x \in (-\infty,-1] \cup [3,\infty)$

Q.8 **(1)**

$$S=\sum_{i=1}^{30} a_i,\ T=\sum_{i=1}^{15} a_{2i-1},\quad a_5=27,\quad S-2T=75$$

Let $a_i = a + (i-1)D$

$S = a_1 + a_2 + a_3 + ... + a_{30}$

$T = a_1 + a_3 + a_5 + ...+a_{29}$

$\therefore 2T = 2a_1 + 2a_3 + 2a_5 + ...+ 2a_{29}$

$S - 2T = (a_2 - a_1) + (a_4 - a_3) + (a_6 - a_5) +.......+ (a_{30} - a_{29}) = 75$

$= 15D$

But $S - 2T = 75 \quad \Rightarrow 15D = 75 \quad \Rightarrow D = 5$

Now $a_5 = 27 \Rightarrow a + 4D = 27$

$\therefore a = 27 - 20 \Rightarrow a = 7$

now $a_{10} = a + 9D$

$= 7 + 45 = 52$

Q.9 (1)

$$T_n = \frac{(3+(n-1)\times 3)(1^2+2^2+...+n^2)}{(2n+1)}$$

$$T_n = \frac{3.\frac{n^2(n+1)(2n+1)}{6}}{(2n+1)} - \frac{n^2(n+1)}{2}$$

$$S_{15} = \frac{1}{2}\sum_{n=1}^{15}(n^3+n^2) = \frac{1}{2}$$

$$\left[\left(\frac{15(15+1)}{2}\right)^2 + \frac{15\times 16\times 31}{6}\right] = 7820$$

Q.10 (2)

$a = A + 6d$

$b = A + 10d$

$c = A + 12d$

a, b, c are in G.P.

$\Rightarrow (A + 10d)^2 = (A + 6d)(a + 12d)$

$\Rightarrow \frac{A}{d} = -14$

$$\frac{a}{c} = \frac{A+6d}{A+12d} = \frac{6+\frac{A}{d}}{12+\frac{A}{d}} = \frac{6-14}{12-14} = 4$$

Q.11 (4)

Two digit numbers of the from $7\lambda + 2$ are 16, 23 .... , 93

Two digit numbers of the from $7\lambda + 5$ are 12, 19 .... , 96

Sum of all the above number equals to

$$\frac{12}{2}(16+93)+\frac{13}{2}(12+96) = 654+702 = 1356$$

Q.12 (3)

$a + 18d = 0 \Rightarrow a = -18d$

$$\frac{t_{49}}{t_{29}} = \frac{a+48d}{a+28d} = \frac{-18d+48d}{-18d+28d}$$

$$= \frac{30d}{10d} = 3$$

Q.13 (B)

$$\frac{a}{1-r} = 3$$

Cube both sides

$$\frac{a^3}{(1-r)^3} = 27 \qquad ....(1)$$

and $\frac{a^3}{1-r^3} = \frac{27}{19} \qquad ....(2)$

(1)/(2) gives $\frac{1-r^3}{(1-r)^3} = 19 \quad \Rightarrow \quad r = \frac{2}{3}$

Q.14 (A)

$$\frac{a_3}{a_1} = \frac{a_1 r^2}{a_1} = r^2$$

$$\Rightarrow r^2 = 25$$

Now $\frac{a_9}{a_5} = \frac{a_1 r^8}{a_1 r^4} = r^4 = (25)^2 = 5^4$

Q.15 (4)

Let the numbers be $\frac{a}{r}$, a, ar

Given $a^3 = 512 \Rightarrow a = 8$

Now given $\frac{8}{r} + 4, 12, 8r$ are in A.P.

$\Rightarrow 2r^2 - 5r + 2 = 0$

$\Rightarrow r = \frac{1}{2}$ or 2

Numbers are 4, 8, 16 or 16, 8, 4

Sum of numbers = 4 + 8 + 16 = 28

Q.16 (3)

$S_k = \frac{k+1}{2}$

$\sum_{k=i}^{10}\left(\frac{k+1}{2}\right)^2 = \frac{5}{12}A$

$2^2 + 3^2 + ...11^2 = \frac{5A}{3}$

$\frac{11\times12\times23}{6} - 1 = \frac{5A}{3}$

$505\times\frac{3}{5} = A$

$A = 303$

Q.17 (2)

$S = \left(\frac{3}{4}\right)^3 + \left(\frac{6}{4}\right)^3 + \left(\frac{9}{4}\right)^3 + \left(\frac{12}{4}\right)^3 + ...15\text{ term}$

$= \frac{27}{64}\sum_{r=1}^{15} r^3$

$= \frac{27}{64}\cdot\left[\frac{15(15+1)}{2}\right]^2$

= 225 K (Given in question)

K = 27

Q.18 (2)

S_A = sum of numbers between 100 & 200 which are divisible by 7.

$\Rightarrow S_A = 105 + 112 + ... + 196$

$S_A = \frac{14}{2}[105 + 196] = 2107$

S_B = Sum of numbers between 100 & 200 which are divisible by 13.

$S_B = 104 + 117 + ... + 195 = \frac{8}{2}[104 + 195] = 1196$

S_C = Sum of numbers between 100 & 200 which are divisible by both 7 & 13.

$S_C = 182$

$\Rightarrow$ H.C.F. $(91, n) > 1 = S_A + S_B - S_C = 3121$

Q.19 (2)

$S = \sum_{k=1}^{20}\frac{1}{2^k}$

$S = \frac{1}{2} + \frac{2}{2^2} + \frac{3}{3^2} + ... + \frac{20}{2^{20}}$

$S\times\frac{1}{2} = \frac{1}{2^2} + \frac{2}{2^3} + ... + \frac{19}{2^{20}} + \frac{20}{2^{21}}$

$\Rightarrow \left(1 - \frac{1}{2}\right)S = \frac{1}{2} + \frac{1}{2^2} + ... + \frac{1}{2^{20}} - \frac{20}{2^{21}}$

$\Rightarrow S = 2 - \frac{11}{2^{19}}$

Q.20 (1)

$S_n = 50n + \frac{n(n-7)}{2}A$

$T_n = S_n - S_{n-1}$

$= 50n + \frac{n(n-7)}{2}A - 50(n-1) - \frac{(n-1)(n-8)}{2}A$

$= 50 + \frac{A}{2}\left[n^2 - 7n - n^2 + 9n - 8\right]$

$= 50 + A(n-4)$

$d = T_n - T_{n-1}$

$= 50 + A(n-4) - 50 - A(n-5)$

$= A$

$T_{50} = 50 + 46A$

$(d, A_{50}) = (A, 50 + 46A)$

Q.21 (1)

$a - d + a + a + d = 33 \Rightarrow a = 11$

$a(a^2 - d^2) = 1155$

$121 - d^2 = 105$

$d^2 = 16 \Rightarrow d = \pm 4$

If $d = 4$ then I^{st} term = 7

If $d = -4$ then I^{st} term = 15

$T_{11} = 7 + 40 = 47$

OR $T_{11} = 15 - 40 = -25$

Q.22 (2)

$T_r = r(2r-1)$

$\Sigma = \Sigma 2r^2 - \Sigma r$

$$S = \frac{2.n(n+1)(2n+1)}{6} - \frac{n(n+1)}{2}$$

$$S_{11} = \frac{2}{6}\cdot(11)(12)(23) - \frac{11(12)}{2} = (44)(23) - 66 = 946$$

Q.23 (4)

$$\text{Sum} = \sum_{n=1}^{15}\frac{1^3+2^3+.....n^3}{1+2+.....+n} - \frac{1}{2}.\frac{15.16}{2}$$

$$= \sum_{n=1}^{15}\frac{n(n+1)}{2} - 60$$

$$= \sum_{n=1}^{15}\frac{n(n+1)(n+2-(n-1))}{6} - 60$$

$$= \frac{15.16.17}{6} - 60 = 620$$

Q.24 (2)

$b = ar$

$c = ar^2$

3a, 7b and 15 c are in A.P

$\Rightarrow 14b = 3a + 15c$

$\Rightarrow 14(ar) = 3a + 15\,ar^2$

$\Rightarrow 14r = 3 + 15r^2$

$\Rightarrow 15r^2 - 14r + 3 = 0 \quad \Rightarrow (3r-1)(5r-3) = 0 \quad r = \frac{1}{3}, \frac{3}{5}$

Only acceptable value is $r = \frac{1}{3}$, because

$r \in \left(0, \frac{1}{2}\right]$

$\therefore$ c.d $= 7b - 3a = 7ar - 3a = \frac{7}{3}a - 3a = -\frac{2}{3}a$

$\therefore$ 4th term $= 15\,c - \frac{2}{3}a = \frac{15}{9}a - \frac{2}{3}a = a$

Q.25 (1)

$$T_n = \frac{\left(3+(n-1)\times 2\left(1^3+2^3+...+n^3\right)\right)}{\left(1^2+2^2+...+n^2\right)}$$

$$= \frac{3}{2}n(n+1) = \frac{n(n+1)(n+2)-(n-1)n(n+1)}{2}$$

$$\Rightarrow S_n = \frac{n(n+1)(n+2)}{2}$$

$\Rightarrow S_{10} = 660$

Q.26 (3)

$a_1 + a_4 + a_7 + a_{10} + a_{13} + a_{16} = 114$

$\Rightarrow \frac{6}{2}(a_1 + a_{16}) = 114$

$\Rightarrow a_1 + a_{16} = 38$

So, $a_1 + a_6 + a_{11} + a_{16} = \frac{4}{2}(a_1 + a_{16})$

$= 2 \times 38 = 76$

Q.27 (1)

$2\{2a + 3d\} = 16$

$3(2a + 5d) = -48$

$2a + 3d = 8$

$2a + 5d = -16$

$d = -12$

$S_{10} = 5\,\{44 - 9 \times 12\}$

$= -320$

Q.28 (2)

$$\underbrace{\left[-\frac{1}{3}\right]+\left[-\frac{1}{3}-\frac{1}{100}\right]+...+\left[-\frac{1}{3}-\frac{66}{100}\right]}_{(-1)67}$$

$$+\underbrace{\left[-\frac{1}{3}-\frac{66}{100}\right]+...+\left[-\frac{1}{3}-\frac{99}{100}\right]}_{-2(33)} = -133$$

Q.29 (1)

$a_1 + a_7 + a_{16} = 40$

$a + a + 6d + a + 15d = 40$

$\Rightarrow 3a + 21d = 40 \quad \Rightarrow a + 7d = \frac{40}{3}$

$$S_{15} = \frac{15}{2}(2a + 14d) = 15(a + 7d)$$

$$S_{15} = 15 \times \frac{40}{3} \Rightarrow 200$$

$S_{15} = 200$

Q.30 (2)

$$\frac{(49)^{126}-1}{48} = \frac{\left((49)^{63}+1\right)\left((49)^{63}-1\right)}{48}.$$

Q.31 (2)

Let terms be a – 2d, a – d, a, a + d, a + 2d

Sum = 25 ⇒ 5a = 25 ⇒ a = 5

Product = 2520

$(5-2d)(5-d)\,5\,(5+d)(5+2d) = 2520$

$\Rightarrow (25-4d^2)(25-d^2) = 504$

$\Rightarrow 625 - 100d^2 - 25d^2 + 4d^4 = 504$

$\Rightarrow 4d^4 - 125d^2 + 625 - 504 = 0$

$\Rightarrow 4d^4 - 125d^2 + 121 = 0$

$\Rightarrow 4d^4 - 121d^2 - 4d^2 + 121 = 0$

$\Rightarrow (d^2-1)(4d^2-121) = 0$

$\Rightarrow d = \pm 1;\ d = \pm\frac{11}{2}$

$d = \pm 1$, does not give $\frac{-1}{2}$ as a term

$\therefore d = \frac{11}{2}$

$\therefore$ Largest term = 5 + 2d = 5 + 11 = 16.

Q.32 (2)

$S = \underbrace{3+4}+\underbrace{8+9}+\underbrace{13+14}+\underbrace{18+19}$.........40 terms

$S = 7+17+27+37+47+$........20 terms

$S_{40} = \frac{20}{2}[2\times 7\,(19)\,10] = 10\,[14+190]$

$= 10[2040] = (102)(20) \quad \Rightarrow m = 20$

Q.33 (4)

$a_1 + a_2 = 4 \Rightarrow a_1 + a_1r = 4$ ....(i)

$a_3 + a_4 = 16 \Rightarrow a_1r^2 + a_1r^3 = 16$ ....(ii)

$\frac{1}{r^2} + \frac{1}{4} \Rightarrow r^2 = 4$

$r = \pm 2$

$r = 2,\ a_1(1+2) = 4 \Rightarrow a_1 = \frac{4}{3}$

$r = -2,\ a_1(1-2) = 4 \Rightarrow a_1 = -4$

$$\sum_{i=1}^{a} a_i = \frac{a_1\left(r^q-1\right)}{r-1} - \frac{(-4)\left((-2)^9-1\right)}{-2-1} = \frac{4}{3}(-513) = 4\lambda$$

$\lambda = -171$

Q.34 (4)

$$f(x) = \left(\frac{2^{1-x}+2^{1+x}+3^x+3^{-x}}{2}\right)$$

Using A.M. ≥ G.M.

$f(x) \geq 3$.

Q.35 [1540]

$$\sum_{k=1}^{20}(1+2+3+\ldots\ldots+k) = \sum_{k=1}^{20}\frac{k(k+1)}{2}$$

$$= \frac{1}{2}\sum_{k=1}^{20}(k^2+k) \quad = \frac{1}{2}\left[\frac{20\,(21)\,(41)}{6} + \frac{20\,(21)}{2}\right]$$

$$= \frac{1}{2}\left[\frac{420\times 41}{6} + \frac{20\times 21}{2}\right] = \frac{1}{2}[2870+210] = 1540.$$

Q.36 (3)

$T_{10} = \frac{1}{20} = a + 9d$ .... (i)

$T_{20} = \frac{1}{10} = a + 19d$ ....(ii)

$\Rightarrow a = \frac{1}{200}, d = \frac{1}{200}$

$\Rightarrow S_{200} = \frac{200}{2}\left[\frac{2}{200} + \frac{109}{200}\right] = \frac{201}{2} = 100\frac{1}{2}$

Q.37 **504**

$$\frac{1}{4}\left[\sum_{n=1}^{7}\left(2n^3+3n^2+n\right)\right]$$

$$\frac{1}{4}\left[2\left(\frac{7.8}{2}\right)^2 + 3\left(\frac{7.8.15}{6}\right) + \frac{7.8}{2}\right]$$

$$\frac{1}{4}[2\times 49\times 16 + 28\times 15 + 28]$$

$$\frac{1}{4}[1568+420+28] = 504$$

Q.38 (2)

$$2^{\frac{1}{4}+\frac{2}{16}+\frac{3}{48}+\ldots\ldots\infty} = 2^{\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\ldots\ldots\infty} = \sqrt{2}$$

Q.39 (2)

Let GP is a, ar, ar^2 ....

$$\sum_{n=1}^{100} a_{2n+1} = a_3 + a_5 + a_{201} = 200 \Rightarrow \frac{ar^2(r^{200}-1)}{r^2-1} = 200 \ \ldots(1)$$

$$\sum_{n=1}^{100} a_{2n} = a_2 + a_4 + a_{200} = 100 = \frac{ar^2(r^{200}-1)}{r^2-1} = 100 \ \ldots(2)$$

Form (1) and (2) r = 2

add both

$\Rightarrow a_2 + a_3 + a_{200} + a_{201} = 300 \Rightarrow r(a_1 + \ldots\ldots\ldots a_{200}) = 300$

$$\sum_{n=1}^{200} a_n = \frac{300}{r} = 150$$

Q.40 [14]

First common = 23

common difference = 7 × 4 = 28

Last term ≤ 407

$\Rightarrow \quad 23 + (n-1) \times 28 \le 407$

$\Rightarrow (n-1) \times 28 \le 384$

$\Rightarrow n \le 13.71 + 1$

$n \le 14.71$

So $\quad n = 14$

Q.41 (1)

Series $(x + ka) + (x^2 + (k+2)a) + ...9$ terms

$\Rightarrow S = (x + x^2 + x^3 + ...9 \text{ terms}) + a[k + (k+2) + (k+4) + ...9 \text{ terms}]$

$$\Rightarrow S = \frac{x(x^9 - 1)}{x-1} + \frac{9}{2}[2ak + 8 \times (+2a)]$$

$$\Rightarrow S = \frac{x^{10} - x}{x-1} + \frac{9ka + 72a}{1} = \frac{x^{10} + 45a(x-1)}{x-1} \text{ (given)}$$

$$\Rightarrow \frac{x^{10} - x + 9a(k+8)(x-1)}{x-1} = \frac{x^{10} - x + 45a(x-1)}{x-1}$$

$\Rightarrow 9a(k+8) = 45a$

$\Rightarrow k + 8 = 5$

$\Rightarrow k = -3$

Q.42 (2)

Let common difference be d.

$\because a_1 + a_2 + a_3 + ... + a_{11} = 0$

$$\therefore \frac{11}{2}\{2a_1 + 10.d\} = 0$$

$\therefore a_1 + 5d = 0$

$$\therefore d = -\frac{a_1}{5} \quad ....(1)$$

Now $a_1 + a_3 + a_5 + ... + a_{23}$

$= a_1 + (a_1 + 2d) + (a_1 + 4d) + ... + (a_1 + 22d)$

$$= 12a_1 + 2d\frac{11 \times 12}{2}$$

$$= 12\left(a_1 + 11.-\frac{a_1}{5}\right)$$

$$= 12 \times \left(-\frac{6}{5}\right)a_1$$

$$= -\frac{72}{5}a_1$$

Q.43 (3)

Let $\frac{a}{r}$, a, ar be terms of G.P.

$$\therefore a\left(\frac{1}{r} + 1 + r\right) = S \quad ....(i)$$

and $a^3 = 27$

$\Rightarrow a = 3 \quad .....(ii)$

$$S = 3 + 3\left(r + \frac{1}{r}\right)$$

As if $f(x) = x + \frac{1}{x}$ then $f(x) \in (-\infty, -2] \cup [2, \infty)$

$\Rightarrow 3f(x) \in (-\infty, -6] \cup [6, \infty)$

$\Rightarrow 3 + 3f(x) \in (-\infty, -3] \cup [9, \infty)$

$\Rightarrow S \in (-\infty, -3] \cup [9, \infty)$

Q.44 (2)

$(x + y) + (x^2 + y^2 + xy) + (x^3 + x^2y + xy^2 + y^3) + ... \infty$

$$= \frac{1}{x-y}\left[(x^2 - y^2) + (x^3 - y^3) + (x^4 - y^4) + ...\infty\right]$$

$$= \frac{1}{x-y}\left[\frac{x^2}{1-x} - \frac{y^2}{1-y}\right] = \frac{(x-y)(x+y-xy)}{(x-y)(1-x)(1-y)}$$

$$= \frac{x+y-xy}{(1-x)(1-y)}$$

Q.45 (39)

$3, A_1, A_2 .... A_m, 243$

As $243 = 3 + (m+1)d$

$$\Rightarrow d = \frac{240}{(m+1)}$$

Also $3, G_1, G_2, G_3, 243$

$\therefore$ As $G_2^2 = 243 \times 3$

$G_2 = \sqrt{243 \times 3} = 27$

$$\text{Now, } A_4 = 3 + 4d = 3 + \frac{960}{(m+1)}$$

As $A_5 = G_2$

$$24 = 3 + \frac{960}{(m+1)} \quad \therefore m = 39$$

Q.46 (4)

$$S_n = 20 + 19\frac{3}{5} + 19\frac{1}{5} + 18\frac{4}{5} + ...$$

$\because S_n = 488$

$$\frac{n}{2}\left\{40 + (n-1).\left(-\frac{2}{5}\right)\right\} = 488$$

$$n\left\{20 - \frac{n-1}{5}\right\} = 488$$

$n^2 - 101n + 2440 = 0$

$\therefore (n-40)(n-61) = 0$

For negative term n = 61

$$\therefore \; n^{th} \text{ term} = T_{61} = 20 + 60.\left(-\frac{2}{5}\right) = -4$$

Q.47 [4]

$$(0.16)^{\log_{2.5}\left(\frac{1}{3}+\frac{1}{3^2}+\frac{1}{3^3}+....\infty\right)}$$

As sum of GP upt o infinity $= \dfrac{a}{1-r}$

$$\therefore \; \frac{1}{3}+\frac{1}{3^2}+\frac{1}{3^2}+....\infty = \frac{\frac{1}{3}}{1-\frac{1}{3}} = \frac{1}{2}$$

$$\therefore \; (0.16)^{\log_{2.5}\left(\frac{1}{2}\right)}$$

$$\text{As } 0.16 = \frac{16}{100} = \left(\frac{4}{10}\right)^2 = \left(\frac{10}{4}\right)^{-2} = (2.5)^{-2}$$

$$\therefore \; (2.5)^{-2\log_{2.5}\left(\frac{1}{2}\right)} = \left(\frac{1}{2}\right)^{-2} = 4$$

Q.48 (1)

Here a = 3 and $S_{25} = S_{15}$

$\Rightarrow 2S_{25} = S_{40}$

$$\Rightarrow 2\times\frac{25}{2}[6+24d] = \frac{40}{2}[6+39d]$$

$\Rightarrow 25[6+32d] = 20[6+39d]$

$\Rightarrow 150 + 600\,d = 120 + 780\,d$

$\Rightarrow 180\,d = 30$

$$\Rightarrow d = \frac{1}{6}$$

Q.49 (3)

$a_1 = 1$ and $a_n = 300$ and $d \in Z$

$300 = 1 + (n-1)d$

$$\Rightarrow d = \frac{299}{(n-1)} = \frac{23\times13}{(n-1)}$$

$\therefore$ n – 1 = 13 or 23 (as d is integer)

$\Rightarrow$ n = 14 or 24 $\quad\Rightarrow$ n = 24 and d = 13

$a_{20} = 1 + 19 . 13 = 248$

$$S_{20} = 20\frac{(248+1)}{2} \quad = 2490$$

Q.50 (4)

$1 + (1 - 2^2 . 1) + (1 - 4^2 . 3) + (1 - 6^2 . 5) + ...(1 - 20^2.19)$

$$S = 1 + \sum_{r=1}^{10}\left[1-(2r)^2(2r-1)\right]$$

$$= 1 + \sum_{r=1}^{10}\left(1-8r^3+4r^2\right) = 1 + 10 - \sum_{r=1}^{10}\left(8r^3-4r^2\right)$$

$$= 11 - 8\left(\frac{10\times11}{2}\right)^2 + 4\times\left(\frac{10\times11\times2}{6}\right)$$

$= 11 - 2\times(110)^2 + 4\times55\times7$

$= 11 - 220\,(110 - 7)$

$= 11 - 220 \times 103 = \alpha - 220\,\beta \Rightarrow \alpha = 11$

$\beta = 103$

$\Rightarrow (11, 103)$

Q.51 (1)

$S = \log_7 x^2 + \log_7 x^3 + \log_7 x^4 + ...$ 20 terms

$\Rightarrow \log_7 x\,(x^2 \cdot x^3 \cdot x^4 \cdot ....x^{21}) = 460$ given

$\Rightarrow \log_7 x^{(2+3+4....21)} = 460$

$\Rightarrow (2 + 3 + 4 + ... + 21)\log_7 x = 460$

$$\Rightarrow \frac{20}{2}\,(2+21)\log_7 x = 460$$

$$\log_7 x = \frac{460}{230} = 2 \qquad \Rightarrow \quad x = 7^2 = 49$$

Q.52 (4)

Let the first term be 'a' and common ratio be 'r'

$\because ar(1 + r + r^2) = 3$ ...(i)

and $ar^5(1 + r + r^2) = 243$ ...(ii)

From (i) and (ii),

$$r^4 = 81 \Rightarrow r = 3 \text{ and } a = \frac{1}{13}$$

$$S_{50} = \frac{a\left(r^{50}-1\right)}{r-1} = \frac{3^{50}-1}{26}$$

Q.53 (4)

LHS is G.P. of common ratio $\frac{3}{2}$

$$\therefore \frac{2^{10}\left(1-\left(\frac{3}{2}\right)^{11}\right)}{\left(1-\frac{3}{2}\right)} = S - 2^{11} \Rightarrow 2^{10}\frac{\left(\frac{3^{11}-2^{11}}{2^{11}}\right)}{\frac{1}{2}} = S - 2^{11}$$

$\Rightarrow S = 3^{11}$

Q.54 (4)

Given $3^{2\sin 2\alpha - 1}$, 14, $3^{4-2\sin 2\alpha}$ are in A.P.

So $3^{2\sin 2\alpha - 1} + 3^{4-2\sin 2\alpha} = 28$

$\Rightarrow \frac{3^{2\sin 2\alpha}}{3} + \frac{81}{3^{2\sin 2\alpha}} = 28$

$\Rightarrow \frac{t}{3} + \frac{81}{t} = 28$ {Put $3^{2\sin 2\alpha} = t$}

$\Rightarrow t^2 - 84t + 243 = 0 \Rightarrow t = 81, t = 3$

$\Rightarrow$ When $t = 81$, when $t = 3$

$\Rightarrow \sin 2\alpha = 2$ (Not possible) $\quad 2\sin 2\alpha = 1$

$\sin 2\alpha = \frac{1}{2}$

$2\alpha = \frac{\pi}{6}$

$\alpha = \frac{\pi}{12}$

So, I term $a = 3^{o} = 1$, $d = 14 - 1 = 13$

Now, $T_6 = a + 5d = 1 + 65 = 66$

Q.55 (05.00)

$\because f(x+y) = f(x).f(y) \quad \forall x, \in R \ f(1) = 3.$

$\Rightarrow f(x) = 3^x \Rightarrow f(i) = 3^i$

$\Rightarrow \sum_{i=1}^{n} f(i) = 363$

$\Rightarrow 3 + 3^2 + 3^3 + ... + 3^n = 363$

$\frac{3(3^n - 1)}{3-1} = 363$

$3^n - 1 = \frac{363 \times 2}{3} = 242$

$3^n = 243 = 3^5$

$n = 5$

Q.56 (2)

Let common difference of series

$a_1, a_2, a_3, ..., a_n$ be d.

$\because a_{40} = a_1 + 39d = -159$ ...(i)

and $a_{100} = a_1 + 99d = -399$ ...(ii)

From eqn. (ii) and (i)

$d = -4$ and $a_1 = -3$

Common difference of $b_1, b_2, b_3, ...$ be (–2).

$\therefore b_{100} = a_{70}$

$\therefore b_1 + 99(-2) = (-3) + 69(-4)$

$\therefore b_1 = 198 - 279$

$\therefore b_1 = -81$

Q.57 (4)

$(a^2p^2 + 2abp + b^2) + (b^2p^2 + 2bcp + c^2) + (c^2p^2 + 2cdp + d^2) = 0$

$\Rightarrow (ap + b)^2 + (bp + c)^2 + (cp + d)^2 = 0$

$\therefore ap + b = bp + c = cp + d = 0$

$\Rightarrow \frac{b}{a} = \frac{c}{b} = \frac{d}{c} \quad \therefore$ a, b, c, d are in G.P.

JEE-ADVANCED

PREVIOUS YEAR'S

Q.1 0

$a_1 = 15$

$\frac{a_k + a_{k-2}}{2} = a_{k-1}$ for $k = 3, 4, ..., 11 \Rightarrow a_1, a_2, ..., a_{11}$

are in AP

$a_1 = a = 15$

$\frac{a_1^2 + a_2^2 + ... + a_n^2}{11} = 90 \Rightarrow$

$\frac{(15)^2 + (15+d)^2 + ... + (15+10d)^2}{11} = 90$

$\Rightarrow 9d^2 + 30d + 27 = 0 \Rightarrow d = -3$ or $-\frac{9}{7}$

Since $27 - 2a_2 > 0 \Rightarrow a_2 < \frac{27}{2}$

$\Rightarrow d = -3$

$\frac{a_1 + a_2 + ... + a_{11}}{11} = \frac{11}{2}\ \frac{[30 + 10(-3)]}{11} = 0$

Q.2 3 or 9, both 3 and 9 (The common difference of the arithmatic progression can be either 0 or 6, and accordingly the second term can be either 3, or 9 ; thus the answers 3, or 9, or both 3 and 9 are acceptable.)

$$\frac{S_m}{S_n} = \frac{S_{5n}}{S_n} = \frac{\frac{5n}{2}[6+(5n-1)d]}{\frac{n}{2}[6+(n-1)d]} = \frac{5[(6-d)+5nd]}{[(6-d)+nd]}$$

; $d = 6$ or $d = 0$

Now if $d = 0$ then $a_2 = 3$ else $a_2 = 9$ for single choice more appropriate choice is 9, but in principal, question seems to have an error.

$\therefore \quad a_2 = 3 + 6 = 9$

Q.3 8

A.M. ≥ G.M.

$$\frac{\frac{1}{a^5}+\frac{1}{a^4}+\frac{1}{a^3}+\frac{1}{a^3}+\frac{1}{a^3}+1+a^8+a^{10}}{8}$$

$$\geq \left(\frac{1}{a^5}.\frac{1}{a^4}.\frac{1}{a^3}.\frac{1}{a^3}.\frac{1}{a^3}.1.a^8.a^{10}\right)^{1/8}$$

$\Rightarrow \quad \frac{1}{a^5}+\frac{1}{a^4}+\frac{3}{a^3}+1+a^8+a^{10} \geq 8(1)^{1/8}$

$\Rightarrow$ minimum value of

$\frac{1}{a^5}+\frac{1}{a^4}+\frac{3}{a^3}+1+a^3+a^{10} = 8$, at $a = 1$

Q.4 (D)

Corresponding A.P.

$\frac{1}{5}$,..................... $\frac{1}{25}$ (20^{th} term)

$$\frac{1}{25} = \frac{1}{5} + 19\,d$$

$$\Rightarrow d = \frac{1}{19}\left(\frac{-4}{25}\right) = -\frac{4}{19\times 25}$$

$a_n < 0$

$$\frac{1}{5} - \frac{4}{19\times 25}\times(n-1) < 0 \;\Rightarrow\; \frac{19\times 5}{4} < n-1$$

$\Rightarrow \quad n > 24.75$

Q.5 (A,D)

$$S_n = \sum_{k=1}^{4n}(-1)^{\frac{k(k+1)}{2}}k^2$$

$= -1^2 - 2^2 + 3^2 + 4^2 - 5^2 - 6^2 + 7^2 + 8^2 + ...$

$= (3^2 - 1) + (4^2 - 2^2) + (7^2 - 5^2) + (8^2 - 6^2)......$

$$= 2[\underbrace{4+6+12+14+20+22+......}_{2n \text{ terms}}]$$

$$= 2[\underbrace{(4+12+20\,......)}_{n \text{ terms}} + \underbrace{(6+14+22\,........)}_{n \text{ terms}}]$$

$$= 2\left[\frac{n}{2}(4\times 2+(n-1)8)\frac{n}{2}(2\times 6+(n+1)8)\right]$$

$= 2[n(4 + 4n - 4) + n(6 + 4n - 4)]$

$= 2(4n^2 + (4n + 2)n) = 2(8n^2 + 2n) = 4n(4n + 1)$

(A) $1056 = 32 \times 33 \quad n = 8$

(B) $1088 = 32 \times 34$

(C) $1120 = 32 \times 35$

(D) $1332 = 36 \times 37 \quad n = 9$

Q.6 5

Numbers removed are k and k + 1.

now $\frac{n(n+1)}{2} - k - (k+1) = 1224$

$\Rightarrow \quad n^2 + n - 4k = 2450 \Rightarrow n^2 + n - 2450 = 4k$

$\Rightarrow \quad (n + 50)(n - 49) = 4k \qquad \Rightarrow n > 49$

<u>Alternative</u>

$\therefore$ To satisfy this equation n should be of the form of (4p + 1) or (4p + 2) taking n = 50

$\Rightarrow \quad 4k = 100 \Rightarrow k = 25 \therefore k - 20 = 5$

Now if we take $n = 53 \quad \Rightarrow k = 103 \quad \Rightarrow n < k$ so not possible. Hence $n \geq 53$ will not be possible.

Q.7 4

Let $b = ar$, $c = ar^2 \Rightarrow r$ is Integers. Also $\frac{a+ar+ar^2}{3}$

$= ar + 2 \Rightarrow a + ar^2 = 2ar + 6$

$\Rightarrow \quad a(r-1)^2 = 6 \Rightarrow r$ must be 2 and a = 6. Thus

$$\frac{a^2+a-14}{a+1} = \frac{36+6-14}{7} = 4 \text{ Ans.}$$

Q.8 9

$$\frac{S_7}{S_{11}} = \frac{6}{11}$$

$$\frac{\frac{7}{2}[2a+6d]}{\frac{11}{2}[2a+10d]} = \frac{6}{11}$$ Given $130 < a + 6d < 140$

$$\frac{7(a+3d)}{11(a+5d)} = \frac{6}{11}$$

$7a + 21d = 6a + 30d$ $\Rightarrow 130 < 15d < 140$

$a = 9d$ Hence $d = 9$

$a = 81$

Hence $d = 9$

Alternative :

Let the AP be a, a + d, a + 2d, ...

where $a, d \in N$

Given $\frac{S_7}{S_{11}} = \frac{6}{11}$ and $130 < a + 6d < 140$..... (2)

$$\Rightarrow \frac{\frac{7}{2}\{2a+6d\}}{\frac{11}{2}\{2a+10d\}} = \frac{6}{11}$$

$$\Rightarrow \frac{14a+42d}{22a+110d} = \frac{6}{11}$$

$\Rightarrow$ $154a + 462d = 132a + 660d$

$\Rightarrow$ $22a = 198d$

$\Rightarrow$ $a = \frac{99d}{11} = 9d$

(2) $\Rightarrow$ $\therefore$ $130 < 9d + 6d < 140$

$\Rightarrow$ $8.6 < d < 9.3$

$\therefore$ $d = 9$

Q.9 (B)

$\log_e b_1, \log_e b_2, ..., \log_e b_{101}$ are in A.P

$\Rightarrow b_1, b_2, b_3 .....b_{101}$ *are in G.P.*

commen deffirence of A.P. is $\log_e 2$,

$\therefore$ common ratio of G.P. is 2

$a_1, a_2 ......a_{101}$ are in A.P

$a_1 = b_1 = A(Let)$

$$Now,\ t = \frac{b_1(2^{51}-1)}{2-1} = b_1(2^{51}-1) = A(2^{51}-1)$$

$$s = \frac{51}{2}(2a_1 + (51-1)d) = \frac{51}{2}(2A + 50d)$$

Now, $a_{51} = b_{51} \Rightarrow A + 50d = A\,2^{50} \Rightarrow 50d = A(2^{50}-1)$

$$\therefore\ s = \frac{51}{2}\left[2A + A(2^{50}-1)\right]$$

$$\Rightarrow s = \frac{51}{2}A\left[2^{50}+1\right]$$

$$s = A\left[51.2^{49} + \frac{51}{2}\right] = A\left[4.2^{49} + 47.2^{49} + \frac{51}{2}\right]$$

$$= A\left[2^{51} - 1 + 47.2^{49} + \frac{51}{2} + 1\right]$$

$$= A(2^{51}-1) + A\left(47.2^{49} + \frac{53}{2}\right)$$

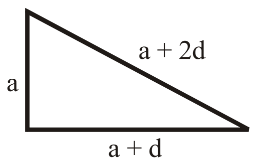
$$\Rightarrow s = t + A\left(47.2^{49} + \frac{53}{2}\right) \Rightarrow s > t$$

$Now, a_{101} = A + 100d = A + 2A(2^{50}-1) = A(2^{51}-1)$

$b_{101} = A.2^{100}$

Clearly $b_{101} > a_{101}$

Q.10 (6)



$\frac{1}{2}a(a+d) = 24$ $\Rightarrow$ $a(a+d) = 48$

............(1)

$a^2 + (a+d)^2 = (a+2d)^2$ $\Rightarrow$ $3d^2 + 2ad - a^2 = 0$

$(3d - a)(a + d) = 0$

$\Rightarrow$ $3d = a$ ($\because a + d \neq 0$)

$\Rightarrow$ $d = 2$

$a = 6$

so smallest side = 6

Q.11 (157.00)

We equate the general terms of three respective A.P.'s as $1 + 3a = 2 + 5b = 3 + 7c$

$\Rightarrow$ 3 divides $1 + 2b$ and 5 divides $1 + 2c$

$\Rightarrow 1 + 2c = 5, 15, 25$ etc.

So, first such terms are possible when $1 + 2c = 15$ i.e. $c = 7$

Hence, first term = a = 52

$d = \text{LCM}(3,5,7) = 105$

$\Rightarrow a + d = 157$

Q.12 (8.00)

$$\frac{3^{y_1}+3^{y_2}+3^{y_3}}{3} \geq \left[3^{(y_1+y_2+y_3)}\right]^{\frac{1}{3}}$$

$$\Rightarrow 3^{y_1}+3^{y_2}+3^{y_3} \geq 3^4$$

$$\Rightarrow \log_3\left(3^{y_1}+3^{y_2}+3^{y_3}\right) \geq 4$$

$$\Rightarrow m = 4$$

Also, $\frac{x_1+x_2+x_3}{3} \geq \sqrt[3]{x_1x_2x_3}$

$\Rightarrow x_1x_2x_3 \leq 27$

$\Rightarrow \log_3 x_1 + \log_3 x_2 + \log_3 x_3 \leq 3$

$\Rightarrow M = 3$

Thus, $\log_2(m^3)+\log_3(M^2) = 6+2$

$= 8$

Q.13 (1.00)

Given $2(a_1+a_2+....a_n) = b_1+b_2+....b_n$

$$\Rightarrow 2\times\frac{n}{2}\left(2c+(n-2)x_2\right) = c\left(\frac{2^n-1}{2-1}\right)$$

$\Rightarrow 2n^2-2n = c(2^n-1-2n)$

$$\Rightarrow c = \frac{2n^2-2n}{2^n-1-2n} \in \mathbb{N}$$

So, $2n^2-2n \geq 2^n-1-2n$

$\Rightarrow 2n^2+1 \geq 2^n \quad \Rightarrow n < 7$

$\Rightarrow$ n can be 1, 2, 3....,

Checking c against these values of n we get c = 12 (when n = 3)

Hence number of such c = 1