Chapter 7 – Mensuration

Practice Set 7.1

1. **Find the volume of a cone if the radius of its base is 1.5 cm and its perpendicular height is 5 cm.**

Solution:
Given radius of cone ,r = 1.5cm
Height of cone ,h = 5cm
$\therefore$Volume of the cone , V = $(1/3)\pi r^2 h$
$\therefore V = (1/3)\times(22/7)\times 1.52\times 5$
$\therefore V = 11.785\ cm^3$
$= 11.79\ cm^3$
Hence, the volume of the cone is $11.79\ cm^3$.

2. **Find the volume of a sphere of diameter 6 cm.**

Solution:
Given diameter of sphere , d = 6cm
$\therefore$Radius r = d/2 = 3
Volume of a sphere, $V = (4/3)\pi r^3$
$\therefore V = (4/3)\times(22/7)\times 3^3$
$\therefore V = 113.14cm^3$
Hence, the volume of the sphere is $113.14cm^3$.

3. **Find the total surface area of a cylinder if the radius of its base is 5 cm and height is 40 cm.**

Solution:
Given radius of cylinder, r = 5cm
Height of cylinder, h = 40cm
Total surface area of cylinder = $2\pi r(r+h)$
$= 2\times(22/7)\times 5\times(5+40)$
$= 2\times(22/7)\times 5\times 45$
$= 1414.28cm^2$
Hence, Total surface area of cylinder is $1414.28cm^2$.

4. **Find the surface area of a sphere of radius 7 cm.**

Solution:
Given radius of sphere , r = 7cm
Surface area, $A = 4\pi r^2$
$\therefore A = 4\times(22/7)\times 7^2$
$= 616\ cm^2$
Hence, surface area of sphere is $616cm^2$.

5. The dimensions of a cuboid are 44 cm, 21 cm, 12 cm. It is melted and a cone of height 24 cm is made. Find the radius of its base.

Solution:

Given length of cuboid , l = 44 cm
Breadth of cuboid ,b = 21 cm
Height of cuboid , h = 12cm
Volume of the cuboid , V = $l$bh
∴V = 44×21×12 = 11088cm^3
Given Height of cone ,h = 24cm
Since cuboid is melted and a cone is made, the volume will be same.
∴Volume of cone = Volume of cuboid.
$(1/3)\pi r^2h = 11088$
$(1/3)\times(22/7)\times r^2\times 24 =$
$11088\ r^2 =$
$11088/8\times(22/7)\ r^2 = 441$
Taking square root
r = 21
Hence, radius of the cone is 21 cm.

6. Observe the measures of pots in figure 7.8 and 7.9. How many jugs of water can the cylindrical pot hold?

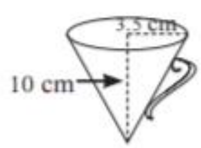


Fig 7.8
conical water jug

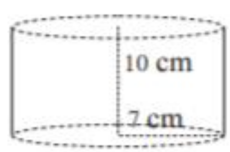


Fig 7.9
cylindrical water pot

Solution:

Given radius of jug , r = 3.5 cm
Height of jug , h = 10 cm
Volume of conical jug, $V = (1/3)\pi r^2h$
$= (1/3)\times\pi\times 3.5^2\times 10 = (122.5/3)\times\pi$
Given Radius of pot ,R = 7cm
Height of pot, H = 10 cm
Volume of pot = πR^2H
$= \pi\times 7^2\times 10 = 490\pi$
Number of jugs of water the cylindrical pot can hold = Volume of pot/Volume of conical jug
$= 490\pi \div (122.5/3)\times\pi$
$= 490\pi\times(3/122.5\pi)$
= 12
Hence, the pot can hold 12 jugs of water.

7. A cylinder and a cone have equal bases. The height of the cylinder is 3 cm and the area of its base is 100 cm^2 .The cone is placed upon the cylinder. Volume of the solid figure so formed is 500 cm^3. Find the total height of the figure.

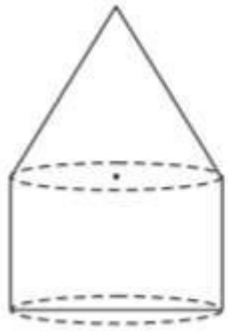


Fig 7.10

Solution:

Given height of cylinder , h = 3cm

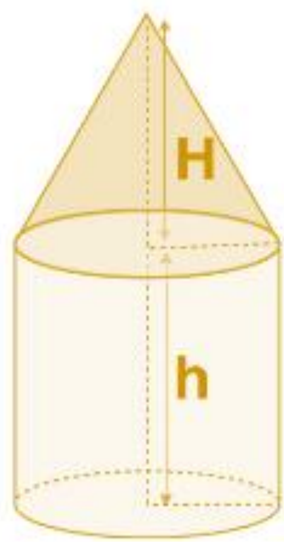
Base area of cylinder , $\pi r^2 = 100cm^2$ …………..(i)

Volume of solid = $500cm^3$

Given cylinder and cone have equal bases.

∴ the radii will be equal.

∴ Radius of cone = radius of cylinder = r



Let height of cone be H

Volume of solid = volume of cylinder + volume of cone ∴500 = $\pi r^2 h + (1/3) \pi r^2 H$

∴500 = 100×3+(1/3)×100×H [∵ πr^2 = 100, base area of cylinder and cone are equal]

∴500 = 300+(100/3)H

⇒ (100/3)H = 200

⇒ H = 3×200/100

= 6

Hence, height of cone is 6cm.

Total height of figure = h+H

= 3+6 = 9cm

Hence, the total height of the figure is 9cm.

8. In figure 7.11, a toy made from a hemisphere, a cylinder and a cone is shown. Find the total area of the toy.

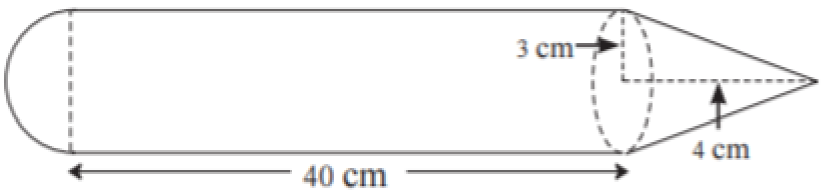


Fig. 7.11

Solution:

Given: For the conical Part,
height ,h = 4 cm, radius , r =
3 cm For the cylindrical part,
height ,H = 40 cm, radius ,r =
3 cm For the hemispherical
part, radius ,r = 3 cm

Slant height of cone (l) = $\sqrt{(h^2+r^2)}$

= $\sqrt{(4^2+3^2)}$ = $\sqrt{(16+9)}$

= $\sqrt{25}$ = 5 cm

Curved surface area of hemisphere = $2\pi r^2$

= $2\times\pi\times3^2$

= 18π cm^2

Curved surface area of cylinder = $2\pi rH$

= $2\times\pi\times3\times40$

= 240π cm^2

Curved surface area of cone = πrl

= $\pi\times3\times5$

= 15π cm^2

Total area of the toy

= Curved surface area of cone + Curved surface area of cylinder + Curved surface area of hemisphere

= $15\pi + 240\pi + 18\pi = 273\pi$ cm^2

Hence, total area of the toy is 273π cm^2.

9. In the figure 7.12, a cylindrical wrapper of flat tablets is shown. The radius of a tablet is 7 mm and its thickness is 5 mm. How many such tablets are wrapped in the wrapper?

Fig. 7.12

Solution:

Given radius of tablet, r = 7mm

Thickness of tablet , h = 5mm

∴Volume of tablet =

Radius of cylindrical wrapper , R = 14/2 = 7mm

Height of cylindrical wrapper , H = 10cm = 10×0 = 100mm

Let n be the number of tablets that can be wrapped.

∴n = volume of cylindrical wrapper/volume of tablet

$= \pi R^2 H/\pi r^2 h$

$= \pi \times 7^2 \times 100/\pi \times 7^2 \times 5$

= 20

Hence,, number of tablets that can be wrapped in wrapper is 20.

10. Figure 7.13 shows a toy. Its lower part is a hemisphere and the upper part is a cone. Find the volume and the surface area of the toy from the measures shown in the figure.(π=3.14)

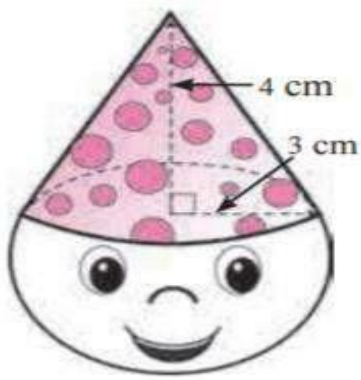


Fig. 7.13

Solution:

Given radius of cone ,r = 3 cm

Height of cone, h = 4 cm

Radius of hemisphere, r = 3cm

Slant height of cone , l =

$\sqrt{(h^2+r^2)}$ ∴$l = \sqrt{(4^2+3^2)}$ =

$\sqrt{(16+9)} = \sqrt{25} = 5$

Curved surface area of cone = $\pi r l$

$= \pi \times 3 \times 5 = 15\pi cm^2$

Curved surface area of hemisphere = $(2/3)\pi r^3$

$= (2/3)\pi \times 3^3 = 18\pi cm^2$

∴Surface area of toy = Curved surface area of cone+ Curved surface area of hemisphere

$= 15\pi + 18\pi = 33\pi = 33 \times 3.14 = 103.62 cm^2$

Volume of cone = $(1/3)\pi r^2 h$

$= (1/3)\pi \times 3^2 \times 4 = 12\pi cm^3$

Volume of hemisphere = $(2/3)\pi r^3$

$= (2/3)\pi\times3^3 = 18\pi cm^3$

∴Volume of toy = Volume of cone+ Volume of hemisphere

$= 12\pi+18\pi = 30\pi = 30\times3.14 = 94.2cm^3$

Hence, the surface area and volume of the toy are $103.62cm^2$ and $94.2cm^3$ respectively.

11. Find the surface area and the volume of a beach ball shown in the figure.

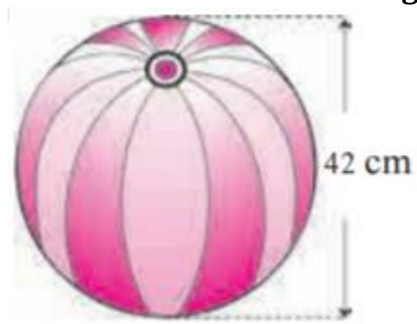


Fig. 7.14

Solution:

Given diameter of ball, d = 42 cm

∴radius of ball, r = 42/2 = 21cm

Surface area of sphere = $4\pi r^2$

$= 4\times\pi\times21^2 = 4\times3.14\times441 = 5538.96\ cm^2$

Volume of sphere = $(4/3)\pi r^3$

$= (4/3)\times\pi\times21^3 = (4/3)\times3.14\times9261 = 38772.72\ cm^3$

Hence, the surface area and volume of ball are $5538.96\ cm^2$ and $38772.72\ cm^3$ respectively.

12. As shown in the figure, a cylindrical glass contains water. A metal sphere of diameter 2 cm is immersed in it. Find the volume of the water.

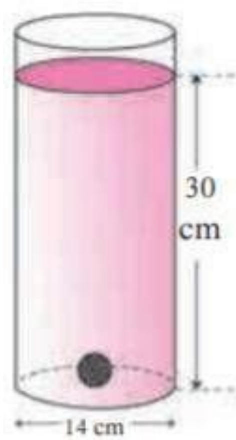


Fig. 7.15

Solution:

Given diameter of cylindrical glass, d = 14cm

∴Radius, R = 14/2 = 7cm

Height of glass in cylindrical glass, H = 30cm

Given diameter of metal sphere = 2cm

∴radius of metal sphere, r = 2/2 = 1cm

Volume of sphere = $(4/3)\pi r^3$

$= (4/3)\times\pi\times 1^3$

$= (4/3)\pi$

$= (4/3)\times(22/7)$

$= 4.19cm^3$

Volume of water with sphere in it = $\pi R^2 H$

$= \pi\times 7^2\times 30$

$= 1470\pi$

$= 4620cm^3$

Volume of water in glass = Volume of water with sphere in it - Volume of sphere

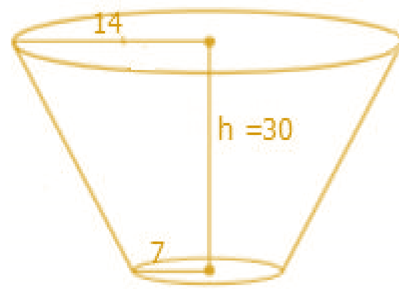
$= 4620-4.19 = 4615.81cm^3$

Hence, the volume of water in glass is $4615.81cm^3$.

Practice Set 7.2

1. The radii of two circular ends of frustum shape bucket are 14 cm and 7 cm. Height of the bucket is 30 cm. How many liters of water it can hold? (1 litre = 1000 cm^3)

Solution:



Given height of bucket, h = 30cm
r_1 = 14cm r_2= 7cm
Volume of a frustum = $(1/3)\pi h(r_1^2+r_2^2+r_1\times r_2)$
$\therefore$Volume of bucket = $(1/3)\pi\times30(14^2+7^2+14\times7)$
= $10\pi\times(196+49+98)$
= 3430π
= 10780cm^3
= 10.78 litres [$\because$ 1 litre = 1000 cm^3]
Hence, the bucket can hold 10.78 litres of water.

2. The radii of ends of a frustum are 14 cm and 6 cm respectively and its height is 6 cm. Find its
i) curved surface area ii) total surface area. iii) volume (π = 3.14)

Solution: Given r_1 = 14cm
r_2= 6cm
Height, h = 6cm
Slant height of frustum $l = \sqrt{[h^2+(r_1-r_2)^2]}$
$= \sqrt{[6^2+(14-6)^2]}$
$= \sqrt{[36+(8)^2]}$
$= \sqrt{[36+64]}$
$= \sqrt{100}$
= 10

(i) Curved surface area of frustum = $\pi l(r_1+r_2)$
= $\pi\times10(14+6)$
= $\pi\times10\times20$
= 3.14×200
= 628cm^2
Hence, curved surface area of frustum is 628cm^2.

(ii) Total surface area of frustum = $\pi l(r_1+r_2)+\pi r_1^2+\pi r_2^2$
= $\pi\times10(14+6)+\pi\times14^2+\pi\times6^2$
= $\pi\times10\times20+\pi\times196+\pi\times36$

$= 200\pi+196\pi+36\pi$

$= 432\pi$

$= 432\times3.14$

$= 1356.48cm^2$

Hence, Total surface area of frustum is $1356.48cm^2$.

(iii)Volume of frustum $= (1/3)\pi h(r_1^2+r_2^2+r_1\times r_2)$

$= (1/3)\pi\times6(14^2+6^2+14\times6)$

$= 2\pi\times(196+36+84)$

$= 2\times3.14\times316$

$= 1984.48cm^3$

Hence, volume of frustum is $1984.48cm^3$

3. The circumferences of circular faces of a frustum are 132 cm and 88 cm and its height is 24 cm. To find the curved surface area of the frustum complete the following activity. (π = 22/7).

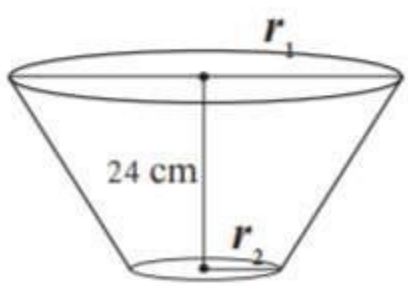


Fig. 7.23

$Circumference_1 = 2\pi r_1 = 132$

$r_1 = 132/2\pi =$ _____

$Circumference_2 = 2\pi r_2 = 88$

$r_2 = 88/2\pi =$ _____

Slant height of frustum $l = \sqrt{[h^2+(r_1-r_2)^2]}$

$= \sqrt{[\_\_^2+(\_\_\_)^2]}$

= _____cm

Curved surface area of frustum $= \pi(r_1+r_2)l$

$= \pi\times$___$\times$___

= ___sq.cm.

Solution:

Circumference$_1$ = $2\pi r_1 = 132$

$r_1 = 132/2\pi = 21cm$ [$\pi=22/7$]

Circumference$_2$ = $2\pi r_2 = 88$

$r_2 = 88/2\pi = 14cm$

Slant height of frustum $l = \sqrt{[h^2+(r_1-r_2)^2]}$

$= \sqrt{[24^2+(21-14)^2]}$ [given h =24]

$= \sqrt{[576+(7)^2]}$

$= \sqrt{[576+49]}$

$= \sqrt{625}$

$= 25cm$

Curved surface area of frustum = $\pi(r_1+r_2)l$

$= \pi\times(21+14)\times 25$

$= \pi\times(35)\times 25$

$= 2750 sq.cm$

Practice set 7.3

1. **Radius of a circle is 10 cm. Measure of an arc of the circle is 54°. Find the area of the sector associated with the arc. ($\pi = 3.14$)**

Solution:
Given radius of circle , $r = 10cm$
Measure of an arc of the circle ,$\theta = 54^\circ$
$\pi = 3.14$
Area of sector , $A = (\theta/360)\pi r^2$
$\therefore A = (54/360)\times3.14\times10^2$
$= (9/60)\times3.14\times100$
$= 47.1cm^2$
Hence, area of the sector is $47.1cm^2$

2. **Measure of an arc of a circle is 80° and its radius is 18 cm. Find the length of the arc. ($\pi = 3.14$)**

Solution:
Given measure of arc, $\theta = 80^\circ$
Radius , $r = 18cm$
$\pi = 3.14$
Length of arc , $l =$
$(\theta/360)2\pi r$ $l =$
$(80/360)\times2\times3.14\times18$
$= 25.12cm$
Hence, the length of the arc is 25.12cm.

3. **Radius of a sector of a circle is 3.5 cm and length of its arc is 2.2 cm. Find the area of the sector.**

Solution:
Given radius of sector of a circle , $r = 3.5cm$
Length of the arc, $l = 2.2cm$
Area of sector ,$A = lr/2$
$\therefore A = 2.2\times3.5/2 = 3.85cm^2$
Hence, the area of the sector is $3.85cm^2$

4. Radius of a circle is 10 cm. Area of a sector of the sector is 100 cm^2 . Find the area of its corresponding major sector. (π = 3.14)

Solution:
Given radius of circle , r = 10cm
Area of minor sector , A = 100cm^2
Area of circle = $\pi r^2 = 3.14\times10^2 = 3.14\times100 = 314cm^2$
Area of major sector = Area of circle - Area of minor sector
= 314-100
= 214cm^2
Hence, the area of the major sector is 214cm^2

5. Area of a sector of a circle of radius 15 cm is 30 cm^2 . Find the length of the arc of the sector.

Solution:
Given radius of circle , r = 15cm
Area of the sector of circle , A = 30cm^2
Area of sector = length of arc ×radius/2 = lr/2
∴Length of arc , l = 2A/r = 2×30/15 =
4cm Hence, the length of the arc is 4cm.

6. In the figure 7.31, radius of the circle is 7 cm and m(arc MBN) = 60°, find
(1) Area of the circle .
(2) A(O - MBN) .
(3) A(O - MCN) .

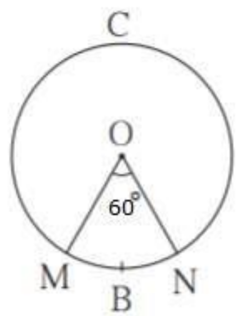


Fig. 7.31

Solution:
Given radius of circle , r = 7cm
(1) Area of the circle , A = πr^2
= $(22/7)\times7^2$
=(22/7)×49
= 154cm^2
Hence, the area of the circle is 154cm^2

(2) Given m(arc MBN),$\theta = 60°$

Area of sector = $(\theta/360)\pi r^2$

$\therefore$A(O-MBN) = $(60/360)\times(22/7)\times 7^2$

= $(1/6)\times 22\times 7$

= $25.67 cm^2$

Hence, area of sector O-MBN is $25.67 cm^2$

(3) Area of major sector = Area of circle -Area of minor sector

$\therefore$A(O-MCN) = 154-25.67

= $128.33 cm^2$

Hence, area of sector O-MCN is $128.33 cm^2$

7. **In figure 7.32, radius of circle is 3.4 cm and perimeter of sector P-ABC is 12.8 cm. Find A(P-ABC).**

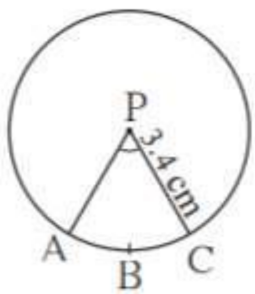


Fig. 7.32

Solution:

Given radius , r = 3.4cm

Perimeter of sector P-ABC = PA+arc ABC +PC

12.8 = 3.4+ length of arc ABC +3.4

$\therefore$length of arc ABC , l = 12.8-3.4-3.4 = 6cm

Area of sector P-ABC = $lr/2$

= $6\times 3.4/2$

= 3×3.4

= $10.2 cm^2$

Hence, A(P-ABC) is $10.2 cm^2$.

8. **In figure 7.33 O is the centre of the sector. ∠ ROQ = ∠ MON = 60°. OR = 7 cm, and OM = 21 cm. Find the lengths of arc RXQ and arc MYN. ($\pi = 22/7$)**

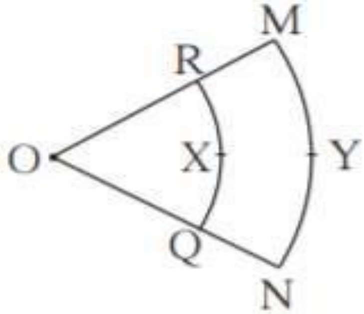


Fig. 7.33

Solution:

Given ∠ ROQ = ∠ MON ,θ= 60°.

Radius OR, r = 7 cm

Radius OM, R = 21 cm

Length of arc RXQ = (θ/360)2πr

= (60/360)×2×(22/7)×7

= (1/6)× 2×22

= 22/3

= 7.33cm

Hence, Length of arc RXQ is 7.33cm.

Length of arc MYN = (θ/360)2πR

= (60/360)×2×(22/7)×21

= (1/6)× 2×22×3

= 22cm

Hence, Length of arc MYN is 22cm.

9. In figure 7.34, if A(P-ABC) = 154 cm^2 radius of the circle is 14 cm, find

(1) ∠ APC.

(2) l(arc ABC) .

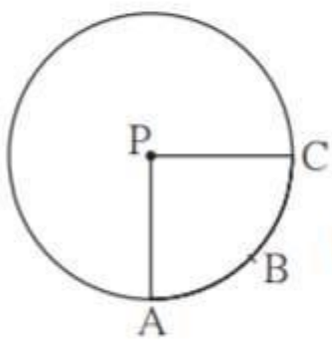


Fig. 7.34

Solution:

Given radius of circle , r = 14cm

Area ,A(P-ABC) = $154cm^2$

(1) Let ∠APC = θ

A(P-ABC) = $(θ/360)πr^2$

∴154 = $(θ/360)×(22/7)×14^2$

∴θ = (154×360×7)/(22×14×14)

∴θ = 90˚

Hence, ∠APC is 90˚.

(2)length of arc, l(arc ABC) = $(\theta/360)2\pi r$

∴ l(arc ABC) =
$(90/360)\times 2\times(22/7)\times 14$ =
$(1/4)\times 2\times 22\times 2 = 22$cm.
Hence, l(arc ABC) is 22cm.

10. Radius of a sector of a circle is 7 cm. If measure of arc of the sector is -
(1) 30°
(2) 210°
(3) three right angles;
find the area of the sector in each case.

Solution:
Given radius of sector, r = 7cm
(1) Measure of arc, $\theta = 30^\circ$
∴Area of sector =
$(\theta/360)\pi r^2$
$= (30/360)\times(22/7)\times 7^2$
$= (1/12)\times(22/7)\times 49$
$= (1/12)\times 22\times 7$
$= 12.83cm^2$
Hence, the area of sector is $12.83cm^2$

(2) Measure of arc, $\theta = 210^\circ$
∴Area of sector = $(\theta/360)\pi r^2$
$= (210/360)\times(22/7)\times 7^2$
$= (7/12)\times(22/7)\times 49$
$= (7/12)\times 22\times 7$
$= 89.83cm^2$
Hence, the area of sector is $89.83cm^2$

(3) Measure of arc, θ = 3 right angles = $3\times 90 = 270^\circ$
∴Area of sector = $(\theta/360)\pi r^2$
$= (270/360)\times(22/7)\times 7^2$
$= (3/4)\times(22/7)\times 49$
$= (3/4)\times 22\times 7$
$= 115.5\ cm^2$
Hence, the area of sector is $115.5\ cm^2$

11. The area of a minor sector of a circle is 3.85 cm^2 and the measure of its central angle is 36°. Find the radius of the circle.

Solution:

Given area of minor sector = $3.85cm^2$

Measure of central angle, $\theta = 36^\circ$

Area of sector =
$(\theta/360)\pi r^2$ ∴3.85 =
$(36/360)\times(22/7)\times r^2$ r^2 =
$(3.85\times360\times7)/(36\times22)$
$r^2 = (3.85\times10\times7)/22$
$r^2 = 12.25$
⇒ r =
3.5cm

Hence, the radius of the circle is 3.5cm

12. In figure 7.35, □ PQRS is a rectangle. If PQ = 14 cm, QR = 21 cm, find the areas of the parts x, y and z .

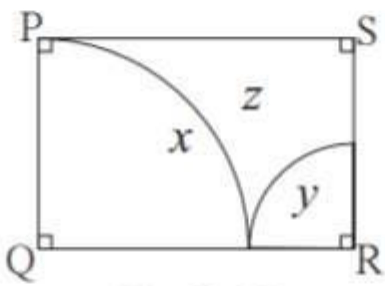
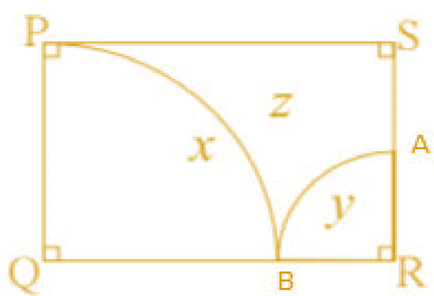


Fig. 7.35

Solution:



Given PQ =
14cm QR =
21cm ∠Q = θ
= 90°

Area of part x = $(\theta/360)\pi r^2$

Area of part x = $(90/360)\times(22/7)\times14^2$
= 11×14
= $154cm^2$

Consider sector (R-BYA)

QR = QB+BR

∴ BR = 21-14 = 7cm

∴AR = 7cm [radius of same circle]

Area of part y = $(\theta/360)\pi r^2$

Area of part y = $(90/360)\times(22/7)\times7^2$

= 11×7/2

= $38.5 cm^2$

Area of rectangle PQRS = length ×breadth

= QR×PQ

= 21×14

= $294 cm^2$

∴Area of part z = Area of rectangle PQRS - [Area of part x+ Area of part y]

∴Area of part z = 294-(154+38.5)

= 294-192.5

= $101.5 cm^2$

Hence, area of part x is $154 cm^2$, area of part y is $38.5 cm^2$ and area of part z is $101.5 cm^2$

13. △ LMN is an equilateral triangle. LM = 14 cm. As shown in figure, three sectors are drawn with vertices as centres and radius7 cm. Find, (1) A (△ LMN)
(2) Area of any one of the sectors.
(3) Total area of all the three sectors.
(4) Area of the shaded region.

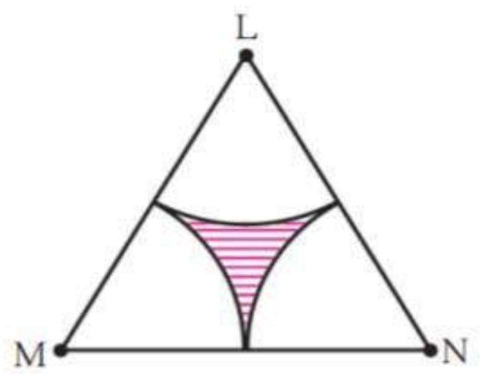


Fig. 7.36

Solution:

(1) Given △LMN is an equilateral triangle . LM = 14 cm

∴Area of △LMN = $(\sqrt{3}/4)a^2$

Here a represents the side of equilateral triangle.

a = 14

∴Area of △LMN = $(\sqrt{3}/4)\times 14^2$

= $49\sqrt{3}$

= $84.87 cm^2$

Hence, area of △LMN is $84.87 cm^2$

(2) Since $\triangle LMN$ is equilateral, $\angle L = \angle M = \angle N = 60^\circ$

$\therefore \theta = 60^\circ$

Given r = 7

Area of sector = $(\theta/360)\pi r^2$

$\therefore$Area of sector = $(60/360)\times 22/7\times 7^2$

$= 11\times 7/3$

$= 25.67cm^2$

Hence, area of one sector = $25.67cm^2$

(3) Total area of 3 sectors = 3×area of one sector

$= 3\times 25.67$

$= 77.01cm^2$

Hence, total area of 3 sectors is $77.01cm^2$.

(4) Area of shaded region = Area of $\triangle LMN$ **-** Area of 3 sectors

$= 84.87 - 77.01$

$= 7.86cm^2$

Hence, area of shaded region is $7.86cm^2$.

Problem Set 7.4

1. In figure 7.43, A is the centre of the circle. ∠ ABC = 45° and AC = 7√2 cm. Find the area of segment BXC.

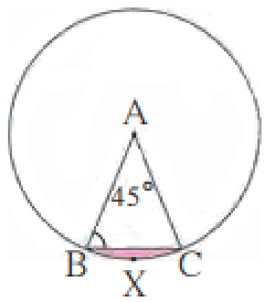


Fig. 7.43

Solution:

Given radius of circle, r = 7√2cm

AB = AC [radii of same circle]

∴∠ ABC = ∠ACB = 45° [isosceles triangle theorem]

In△ABC,

∠A = θ = 90˚ [Angle sum property of triangle]

Area of segment BXC = r^2[(πθ/360) - (sinθ/2)]

= $(7\sqrt{2})^2$[3.14×90/360 - (sin90˚)/2]

= 98×[(3.14/4)-(1/2)]

= 98×[0.785-
0.5] = 27.93cm^2

Hence, area of segment BXC is 27.93cm^2.

2. In the figure 7.44, O is the centre of the circle. m(arc PQR) = 60° OP = 10 cm. Find the area of the shaded region. (π = 3.14, √3 = 1.73)

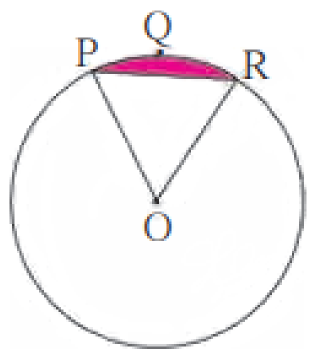


Fig. 7.44

Solution:

Given radius OP , r = 10cm

m(arc PQR) , θ = 60°

Area of segment PQR = r^2[(πθ/360) - (sinθ/2)]

= 10^2[3.14×(60/360)- sin60˚/2]

$= 100[3.14\times(1/6) - \sqrt{3}/4]$
$= 100[(3.14/6) - 1.73/4]$
$= (314/6)-(173/4)$
$= 52.33 - 43.25$
$= 9.08 cm^2$

Hence, area of shaded region is $9.08cm^2$.

3. In the figure 7.45, if A is the centre of the circle. ∠ PAR = 30°, AP = 7.5, find the area of the segment PQR (π = 3.14)

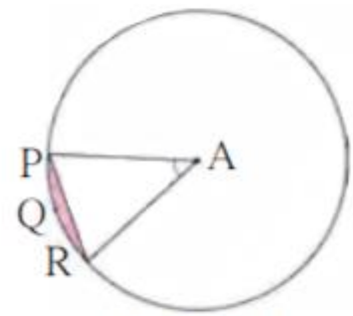


Fig. 7.45

Solution:

Given radius AP , r = 7.5

Central angle ∠PAR = θ = 30˚

Area of segment PQR = $r^2[(\pi\theta/360) - (\sin\theta/2)]$
$= 7.5^2[3.14\times(30/360) - \sin 30^\circ/2]$
$= 56.25[3.14\times(1/12) - 1/4]$
$= 56.25[(3.14/12) - (3/12)]$
$= 56.25\times0.14/12$
$= 0.65625cm^2$

Hence, area of segment PQR is $0.65625cm^2$.

4. In the figure 7.46, if O is the centre of the circle, PQ is a chord. ∠POQ = 90°, area of shaded region is 114 cm^2 , find the radius of the circle. (π = 3.14)

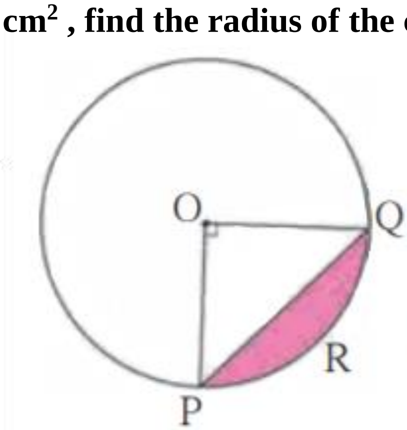


Fig. 7.46

Solution:

Given ∠POQ = θ = 90°, area of shaded region PRQ = $114cm^2$

Area of segment PRQ = $r^2[(\pi\theta/360) - (\sin\theta/2)]$

$\therefore 114 = r^2[3.14\times90/360 - \sin90^\circ/2]$

$\therefore 114 = r^2[3.14\times(1/4) - 1/2]$

$\therefore 114 = r^2[0.785-0.5]$

$\therefore 114 = r^2\times0.285$

$\Rightarrow r^2 = 114/0.285 = 400$

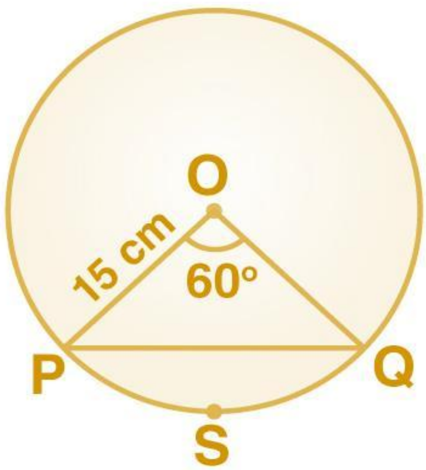
Taking square root on both sides

r = 20cm

Hence, the radius of the circle is 20cm.

5. A chord PQ of a circle with radius 15 cm subtends an angle of 60° with the centre of the circle. Find the area of the minor as well as the major segment. (π = 3.14, √3 = 1.73)

Solution:



Given central angle θ = 60°

Radius ,r = 15cm

Let chord PQ subtend ∠POQ = 60° at centre.

∴ θ = 60°

Area of minor segment = $r^2[(\pi\theta/360) - (\sin\theta/2)]$

$= 15^2[3.14\times(60/360) - \sin60^\circ/2]$

$= 225[3.14\times(1/6) - \sqrt{3}/4]$

$= 225[(3.14/6) - 1.73/4]$

$= 225[(6.28- 5.19)/12]$

$= 20.44$

Hence, area of minor segment is $20.44cm^2$.

Area of circle = πr^2

$= 3.14\times15^2$

$= 3.14\times225$

$= 706.5cm^2$

Area of major segment = Area of circle - area of minor segment

$= 706.5 - 20.44$

$= 686.06cm^2$

Hence, area of major segment is $686.06cm^2$.

Problem Set 7

1. Choose the correct alternative answer for each of the following questions.
(1) The ratio of circumference and area of a circle is 2:7. Find its circumference.
(A) 14π (B) 7/π (C) 7π (D) 14/π

Solution:
Circumference of circle = $2\pi r$
Area of circle = πr^2
Given ratio of circumference and area = 2:7
$\therefore 2\pi r/\pi r^2 = 2/7$
$\Rightarrow 2/r = 2/7$
$\Rightarrow r = 7$cm
$\therefore$Circumference = $2\pi r = 2\pi\times7 = 14\pi$.
Hence, option A is the answer.

(2) If measure of an arc of a circle is 160° and its length is 44 cm, find the circumference of the circle. (A) 66 cm (B) 44 cm (C) 160 cm (D) 99 cm

Solution:
Given measure of arc $\theta = 160^\circ$
Length of arc = 44cm
$\therefore$ Length of arc = $(\theta/360)2\pi r$
$44 = (160/360)\times2\pi r$
$\Rightarrow 2\pi r = 44\times360/160 = 99$
Circumference = $2\pi r$
Hence, circumference is 99cm
So option D is the answer.

(3) Find the perimeter of a sector of a circle if its measure is 90 ° and radius is 7 cm. (A) 44 cm (B) 25 cm (C) 36 cm (D) 56 cm

Solution:
Given radius , r = 7cm $\theta = 90^\circ$
Perimeter of a sector = 2r + length of arc
$= 2r + (\theta/360)2\pi r$
$= 2\times7 + (90/360)\times2\times(22/7)\times7$
$= 14+11 = 25$cm
Hence, option B is the answer.

(4) Find the curved surface area of a cone of radius 7 cm and height 24 cm.
(A) 440 cm^2 (B) 550 cm^2 (C) 330 cm^2 (D) 110 cm^2

Solution:
Given height , h = 24cm
Radius , r = 7cm
Slant height $l = \sqrt{(h^2+r^2)}$
$= \sqrt{(24^2+7^2)}$
$= \sqrt{(576+49)}$
$=\sqrt{625}$
= 25cm
Curved surface area of a cone = $\pi r l$
= 22/7)×7×25
= 22×25
=550cm^2
Hence, option B is the answer.

(5) The curved surface area of a cylinder is 440 cm^2 and its radius is 5 cm. Find its height.
(A) 44 /π cm (B) 22π cm (C) 44π cm (D) 22/π cm

Solution:
Given radius , r = 5cm
Curved surface area = 440cm^2
Curved surface area = 2πrh
∴440 = 2πrh
⇒ h = 440/2πr
= 440/(2π×5)
= 440/10π
= 44π
Hence, option C is the answer.

(6) A cone was melted and cast into a cylinder of the same radius as that of the base of the cone. If the height of the cylinder is 5 cm, find the height of the cone.
(A) 15 cm (B) 10 cm (C) 18 cm (D) 5 cm

Solution:
Given radius of both cone and cylinder
are equal. Height of cylinder,H = 5cm
Let height of cone be h.
Since cone is melted and cast into a cylinder, volume of cone is equal to volume of cylinder.
∴$(1/3)\pi r^2 h = \pi r^2 H$
⇒h = 5×3 = 15cm
Hence, option A is the answer.

(7) Find the volume of a cube of side 0.01 cm.
(A) 1 cm^3 (B) 0.001 cm^3 (C) 0.0001 cm^3 (D) 0.000001 cm^3

Solution:
Volume of a cube ,V= a^3
Given side , a = 0.01 ∴V
= 0.01^3 = 0.000001cm^3
Hence, option D is the
answer.

(8) Find the side of a cube of volume 1 m^3 .
(A) 1 cm (B) 10 cm (C) 100 cm (D)1000 cm
Solution:
Given volume , V = 1m^3
Volume of cube of side a = a^3
∴a^3 = 1m^3
⇒ a =1m = 100cm
Hence, option C is the answer.

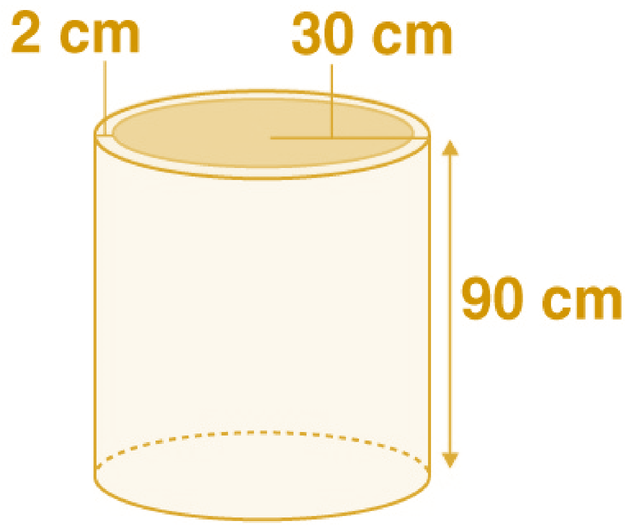
2. A washing tub in the shape of a frustum of a cone has height 21 cm. The radii of the circular top and bottom are 20 cm and 15 cm respectively. What is the capacity of the tub ?(π = 22/7)

Solution:
Given r_1 =
20cm r_2 =
15cm Height ,
h = 21cm
Volume of frustum ,V = $(1/3)\pi h(r_1^2+r_2^2+r_1\times r_2)$
∴V = $(1/3)\times(22/7)\times21\times(20^2+15^2+20\times15)$
∴V = 22×(400+225+300)
∴V = 22×925
∴V = 22×925 = 20350cm^3
= 20.34litres [∵ 1 litre = 1000cm^3]

Hence, the capacity of the tub is 20.34 litres.

3*. Some plastic balls of radius 1 cm were melted and cast into a tube. The thickness, length and outer radius of the tube were 2 cm, 90 cm and 30 cm respectively. How many balls were melted to make the tube?

Solution:



Given radius of plastic ball, r = 1cm
Thickness of tube = 2cm
Height of tube, h = 90cm
Outer radius, R = 30cm
Inner radius of tube ,r_1 = outer radius - thickness
= 30-2 = 28cm
Volume of plastic needed for tube = volume of outer tube - volume of inner tube
= $\pi R^2h - \pi r_1^2h$
= $\pi h(R^2-r_1^2)$
= $\pi\times 90(30^2-28^2)$
= $\pi\times 90(900-784)$
= $\pi\times 90\times 116$ = $10440\pi cm^3$
Volume of a plastic ball = $(4/3)\pi r^3$
= $(4/3)\pi\times 1^3$
= $(4/3)\pi cm^3$
Number of balls melted to make tube = Volume of plastic needed for tube/ Volume of a plastic ball
= $10440\pi\div(4/3)\pi$
= $10440\pi\times 3/4\pi$
= 7830
Hence, the number of balls needed to make tube is 7830.

4. A metal parallelopiped of measures 16 cm × 11cm × 10cm was melted to make coins. How many coins were made if the thickness and diameter of each coin was 2 mm and 2 cm respectively?

Solution:

Given: For the parallelopiped.,
length (l) = 16 cm, breadth (b) = 11 cm,
height (h) = 10 cm
For the cylindrical coin,
thickness (H) = 2 mm,
diameter (D) 2 cm
To find: Number of coins made.
Volume of parallelopiped = l × b × h
= 16 × 11 × 10
= 1760 cm^3
Thickness of coin (H) = 2 mm
= 0.2 cm …[∵ 1 cm = 10 mm]
Diameter of coin (D) = 2 cm
Radius of coin (R) = $\frac{D}{2} = \frac{2}{2} = 1\text{cm}$
Volume of one coin = $\pi R^2 H$

$= \frac{22}{7} \times 1^2 \times 0.2$

$= \frac{4.4}{7}\ \text{cm}^3$

Number of coins that were made

$= \frac{\text{Volume of parallelopiped}}{\text{Volume of one coin}} = \frac{1760}{\frac{4.4}{7}}$

$= \frac{1760 \times 7}{4.4} = \frac{1760 \times 7 \times 10}{44} = 2800$

∴ 2800 coins were made by melting the parallelopiped.

5. The diameter and length of a roller is 120 cm and 84 cm respectively. To level the ground, 200 rotations of the roller are required. Find the expenditure to level the ground at the rate of ₹ 10 per sq.m.

Solution:

Given: For the cylindrical roller,
diameter (d) =120 cm,
length = height (h) = 84 cm
To find: Expenditure of levelling the ground.
Diameter of roller (d) = 120 cm
Radius of roller (R) = $\frac{D}{2} = \frac{120}{2} = 60\text{cm}$
Curved surface area of roller = 2πrh

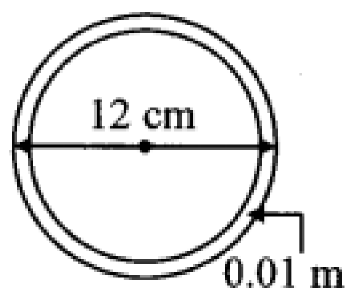
$= 2 \times \frac{22}{7} \times 60 \times 84$

$= 2 \times 22 \times 60 \times 12$
$= 31680\ cm^2$
$= \frac{31680}{100 \times 100}\ m^2$ [∵ 1 m = 100 cm]
$= 3.168\ m^2$

Now, area of ground levelled in one rotation = curved surface area of roller
$= 3.168\ m^2$
∴ Area of ground levelled in 200 rotations
$= 3.168 \times 200 = 633.6\ m^2$
Rate of levelling = ₹ 10 per m^2
∴ Expenditure of levelling the ground
= 633.6 × 10 = ₹ 6336
∴ The expenditure of levelling the ground is ₹ 6336.

6. The diameter and thickness of a hollow metal sphere are 12 cm and 0.01 m respectively. The density of the metal is 8.88 gm per cm3. Find the outer surface area and mass of the sphere, [π = 3.14]

Solution:
Given: For the hollow sphere,
diameter (D) =12 cm, thickness = 0.01 m
density of the metal = 8.88 gm per cm^3
To find:
i. Outer surface area of the sphere
ii. Mass of the sphere.



Diameter of the sphere (D)= 12 cm
∴ Radius of sphere (R) = d2 = 122 = 6 cm
∴ Surface area of sphere = $4\pi R^2$
$= 4 \times 3.14 \times 6^2$
$= 452.16\ cm^2$
Thickness of sphere = 0.01 m
= 0.01 × 100 cm … [∵ 1 m = 100 cm]
= 1 cm
∴ Inner radius of the sphere (r)
= Outer radius – thickness of sphere
= 6 – 1 = 5 cm
∴ Volume of hollow sphere
= Volume of outer sphere – Volume of inner sphere
$= \frac{4}{3}\pi R^3 - \frac{4}{3}\pi r^3$
$= \frac{4}{3}\pi(R^3 - r^3)$
$= \frac{4}{3} \times 3.14 \times (6^3 - 5^3)$

$= \frac{4}{3} \times 3.14 \times (216 - 125)$

$= \frac{4}{3} \times 3.14 \times 91$

$= \frac{1142.96}{3} = 380.99 \text{ cm}^3$

Now, density of metal = $\frac{\text{Mass of sphere}}{\text{Volume of sphere}}$

$\therefore$ 8.88 = $\frac{\text{Mass of sphere}}{380.99}$

$\therefore$ Mass of sphere = 8.88 × 380.99

= 3383.19 gm

$\therefore$ The outer surface area and the mass of the sphere are 452.16 cm^2 and 3383.19 gm respectively.

7. A cylindrical bucket of diameter 28 cm and height 20 cm was full of sand. When the sand in the bucket was poured on the ground, the sand got converted into a shape of a cone. If the height of the cone was 14 cm, what was the base area of the cone?

Solution:

Given: For the cylindrical bucket,

diameter (d) = 28 cm, height (h) = 20 cm

For the conical heap of sand,

height (H) = 14 cm

To find: Base area of the cone (πR^2).

Diameter of the bucket (d) = 28 cm

Radius of bucket (r) = $\frac{d}{2} = \frac{28}{2}$ = 14 cm

Volume of bucket = $\pi^2 h$

$= \frac{22}{7} \times 14^2 \times 20$

= 22 × 14 × 2 × 20

=12320 cm^3

Volume of conical heap = $\frac{1}{3}\pi R^2 H$

$= \frac{1}{3} \times \pi R^2 \times 14$

$= \frac{14}{3}\pi R^2 \text{ cm}^2$

But, volume of bucket = volume of conical heap

$\therefore$ 12320 = $\frac{14}{3}\pi R^2$

$\therefore$ $\pi R^2 = \frac{12320 \times 3}{14}$

= 2640 cm^2

The base area of the cone is 2640 cm^2.

8. The radius of a metallic sphere is 9 cm. It was melted to make a wire of diameter 4 mm. Find the length of the wire.

Solution:

Given: For metallic sphere,

radius (R) = 9 cm

For the cylindrical wire,

diameter (d) = 4 mm

To find: Length of wire (h).

Volume of sphere = $\frac{4}{3}\pi R^3$

$= \frac{4}{3} \times \pi \times 9^3$

$= 972\pi\ cm^3$

Diameter of wire (d) = 4 mm

$= \frac{4}{10}$ cm [∵ 1 cm = 10 mm]

= 0.4 cm

Radius of wire (r) = $\frac{d}{2} = \frac{0.4}{2} = 0.2$ cm

Volume of wire = $\pi r^2 h$

$= \pi(0.2)^2 h = 0.04\pi h\ cm^3$

But, volume of wire = volume of sphere

$0.04\ \pi h = 972\ \pi$

$h = \frac{972}{0.04}$

$= \frac{97200}{4}$

= 24300 cm

$= \frac{24300}{100}$ m [1 m = 100 cm]

h = 243 m

∴ The length of the wire is 243 m.

9. The area of a sector of a circle of 6 cm radius is 157t sq.cm. Find the measure of the arc and length of the arc corresponding to the sector.

Solution:

Given: Radius (r) = 6 cm,

area of sector = 15 π cm^2

To find: i. Measure of the arc (θ),

ii. Length of the arc (l)

Area of sector = $\frac{\theta}{360} \times \pi r^2$

$15\pi = \frac{\theta}{360} \times \pi \times 6^2$

$15\pi = \frac{\theta}{360} \times \pi \times 36$

$15 = \frac{\theta}{10}$ $\therefore \theta = 150°$

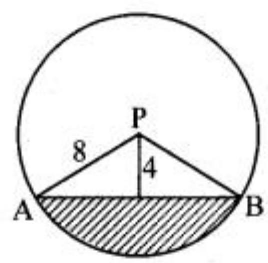
Also, area of sector = $\frac{\text{length of the arc} \times \text{radius}}{2}$

$15\pi = \frac{\text{lenth of the arc} \times 6}{2}$

length of the arc = $\frac{15\pi \times 2}{6} = 5\pi$ cm

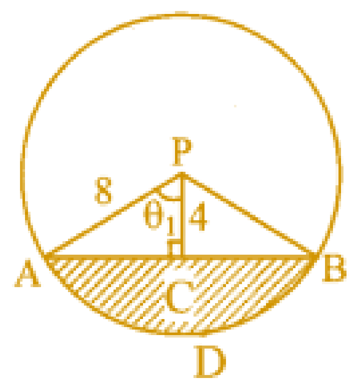
∴ The measure of the arc and the length of the arc are 150° and 5π cm respectively.

10. In the adjoining figure, seg AB is a chord of a circle with centre P. If PA = 8 cm and distance of chord AB from the centre P is 4 cm, find the area of the shaded portion.



(π = 3.14, 3–√ = 1.73)

Solution:



Given: Radius (r) = PA = 8 cm,

PC = 4 cm

To find: Area of shaded region.

Let $\angle APC = \theta_1$

In ΔACP = 90°

$\cos \theta_1 = \frac{PC}{AP} = \frac{4}{8} = \frac{1}{2}$

But, $\cos 60° = \frac{1}{2}$

$\theta_1 = 60°$

Similarly, we can show that, ∠BPC = 60°

∠APB = ∠APC + ∠BPC … [Angle sum property]

∴ θ = 60° + 60° = 120°

∴ $\sin 60° = \frac{AC}{8}$ ∴ $\frac{\sqrt{3}}{2} = \frac{AC}{8}$

∴ $AC = 4\sqrt{3}$ cm

Now, AB = 2 AC

… [Perpendicular drawn from the centre of the circle to the chord bisects the chord]

$= 2 \times 4\sqrt{3}$

$= 8\sqrt{3}$ cm

$\therefore \quad A(\Delta APB) = \frac{1}{2} \times AB \times PC$

$= \frac{1}{2} \times 8\sqrt{3} \times 4$

$= 16\sqrt{3}$.

$= 16 \times 1.73$

$= 27.68 \text{ cm}^2$

Area of shaded region = A(P-ADB) – A(ΔAPB)

= 66.98 – 27.68 = 39.30 cm^2

∴ The area of the shaded region is 39.30 cm^2.

11. In the adjoining figure, square ABCD is inscribed in the sector A-PCQ. The radius of sector C-BXD is 20 cm. Complete the following activity to find the area of shaded region.

Solution : Side of square ABCD = radius of sector C-BXD = 20 cm

Area of square = $(\text{side})^2 = 20^2 = 400$ ..... (I)

Area of shaded region inside the square

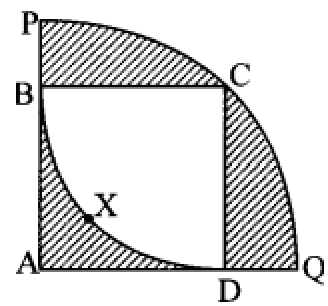
= Area of square ABCD – Area of sector C-BXD

$= 400 - \frac{\theta}{360} \times \pi r^2$

$= 400 - \frac{90}{360} \times \frac{3.14}{1} \times \frac{400}{1}$

$= 400 - 314$

$= 86$



Radius of bigger sector = Length of diagonal of square ABCD

$= 20\sqrt{2}$

Area of the shaded regions outside the square

= Area of sector A-PCQ – Area of square ABCD

= A(A-PCQ) – A(□ABCD)

$= \left(\frac{\theta}{360} \times \pi \times r^2\right) - AB^2$

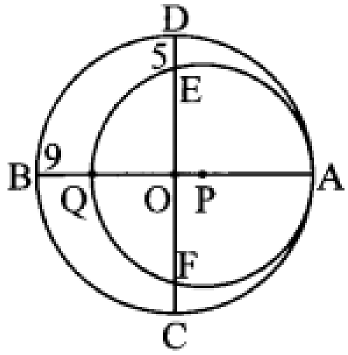
$= \frac{90}{360} \times 3.14\ (20\sqrt{2})^2 - (20)^2$

$= 628 - 400$

$= 228$ cm^2

∴ total area of the shaded region = 86 + 228 = 314 sq.cm.

12. In the adjoining figure, two circles with centres O and P are touching internally at point A. If BQ = 9, DE = 5, complete the following activity to find the radii of the circles.



Solution : Let the radius of the bigger circle be R and that of smaller circle be r.

OA, OB, OC and OD are the radii of the bigger circle

$\therefore$ OA = OB = OC = OD = R

PQ = PA = r

OQ = OB − BQ = [R-9]

OE = OD − DE = [R-5]

As the chords QA and EF of the circle with centre P intersect in the interior of the circle, so by the property of internal division of two chords of a circle,

OQ × OA = OE × OF

[R-9] × R = [R-5] × [R-5] .... ($\because$ OE = OF)

$R^2 - 9R = R^2 - 10R + 25$

R = [25]

AQ = 2r = AB − BQ

2r = 50 − 9 = 41

r = [41/2] = [20.5]