POLYNOMIALS

(MATHS)

DPP - 03 CLASS -10th TOPIC - RELATIONSHIP BETWEEN ZEROS & COEFF.

Q.1 Find the zeros of each of the following quadratic polynomials and verify the relationship between the zeros and their coefficients:

(i)
$$f(x) = x^2 - 2x - 8$$

- (ii) $g(s) = 4s^2 4s + 1$
- (iii) $h(t)=t^2 15$
- (iv) $f(x) = 6x^2 3 7x$
- (v) $p(x) = x^2 + 2\sqrt{2x} 6$
- (vi) $q(x)=\sqrt{3x^2+10x+7\sqrt{3}}$
- (vii) $f(x) = x^2 (\sqrt{3} + 1)x + \sqrt{3}$
- (viii) $g(x)=a(x^2+1)-x(a^2+1)$
- (ix) $h(s) = 2s^2 (1 + 2\sqrt{2})s + \sqrt{2}$
- (x) $f(v) = v^2 + 4\sqrt{3}v 15$
- (xi) $p(y) = y^2 + (3\sqrt{5}/2)y 5$
- (xii) $q(y) = 7y^2 (11/3)y 2/3$
- **Q.2** For each of the following, find a quadratic polynomial whose sum and product respectively of the zeros are as given. Also, find the zeros of these polynomials by factorization.
 - (i) -8/3, 4/3
 - (ii) 21/8, 5/16
 - (iii) -2√3, -9
 - (iv) $-3/2\sqrt{5}, -1/2$
- **Q.3** If α and β are the zeros of the quadratic polynomial $f(x) = x^2 5x + 4$, find the value of $1/\alpha + 1/\beta 2\alpha\beta$.
- **Q.4** If α and β are the zeros of the quadratic polynomial $p(y) = 5y^2 7y + 1$, find the value of $1/\alpha + 1/\beta$.
- **Q.5** If α and β are the zeros of the quadratic polynomial $f(x)=x^2 x 4$, find the value of $1/\alpha+1/\beta-\alpha\beta$.
- **Q.6** If α and β are the zeroes of the quadratic polynomial $f(x) = x^2 + x 2$, find the value of $1/\alpha 1/\beta$.
- **Q.7** If one of the zero of the quadratic polynomial $f(x) = 4x^2 8kx 9$ is negative of the other, then find the value of k.
- **Q.8** If the sum of the zeroes of the quadratic polynomial f(t)=kt² + 2t + 3k is equal to their product, then find the value of k.
- **Q.9** If α and β are the zeros of the quadratic polynomial $p(x) = 4x^2 5x 1$, find the value of $\alpha^2\beta+\alpha\beta^2$.
- **Q.10** If α and β are the zeros of the quadratic polynomial $f(t)=t^2-4t+3$, find the value of $\alpha^4\beta^3+\alpha^3\beta^4$.

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SOL.1 (i) f(x) = x^2 - 2x - 8

Given,

f(x) = x^2 - 2x - 8

To find the zeros, we put f(x) = 0

\Rightarrow x^2 - 2x - 8 = 0

\Rightarrow x^2 - 4x + 2x - 8 = 0

\Rightarrow x(x - 4) + 2(x - 4) = 0

\Rightarrow (x - 4)(x + 2) = 0

This gives us 2 zeros, for

x = 4 and x = -2

Hence, the zeros of the quadratic equation are 4 and -2.

Now, for verification

Sum of zeros = - coefficient of x / coefficient of x^2

4 + (-2) = -(-2) / 1

2 = 2
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Product of roots = constant / coefficient of x^2

-8 = -8

Therefore, the relationship between zeros and their coefficients is verified.

(ii)
$$g(s) = 4s^2 - 4s + 1$$

Given,
 $g(s) = 4s^2 - 4s + 1$
To find the zeros, we put $g(s) = 0$
 $\Rightarrow 4s^2 - 4s + 1 = 0$
 $\Rightarrow 4s^2 - 2s - 2s + 1 = 0$
 $\Rightarrow 2s(2s - 1) - (2s - 1) = 0$
 $\Rightarrow (2s - 1)(2s - 1) = 0$
This gives us 2 zeros, for
 $s = 1/2$ and $s = 1/2$
Hence, the zeros of the quadratic equation are $1/2$ and $1/2$.
Now, for verification

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Sum of zeros = - coefficient of s / coefficient of s^2
1/2 + 1/2 = -(-4)/4
1 = 1
Product of roots = constant / coefficient of s^2
1/2 \ge 1/2 = 1/4
1/4 = 1/4
Therefore, the relationship between zeros and their coefficients is verified.
(iii) h(t)=t^2 - 15
Given,
h(t) = t^2 - 15 = t^2 + (0)t - 15
To find the zeros, we put h(t) = 0
\Rightarrow t<sup>2</sup> - 15 = 0
\Rightarrow (t + \sqrt{15})(t - \sqrt{15}) = 0
This gives us 2 zeros, for
t = \sqrt{15} and t = -\sqrt{15}
Hence, the zeros of the quadratic equation are \sqrt{15} and \sqrt{15}.
Now, for verification
Sum of zeros = - coefficient of t / coefficient of t^2
\sqrt{15} + (-\sqrt{15}) = -(0) / 1
0 = 0
Product of roots = constant / coefficient of t^2
\sqrt{15} \text{ x} (-\sqrt{15}) = -15/1
-15 = -15
Therefore, the relationship between zeros and their coefficients is verified.
(iv) f(x) = 6x^2 - 3 - 7x
Given,
f(x) = 6x^2 - 3 - 7x
To find the zeros, we put f(x) = 0
\Rightarrow 6x^2 - 3 - 7x = 0
\Rightarrow 6x^2 - 9x + 2x - 3 = 0
\Rightarrow 3x(2x - 3) + 1(2x - 3) = 0
\Rightarrow (2x-3)(3x+1) = 0
This gives us 2 zeros, for
x = 3/2 and x = -1/3
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Hence, the zeros of the quadratic equation are 3/2 and -1/3. Now, for verification Sum of zeros = - coefficient of x / coefficient of x^2 3/2 + (-1/3) = -(-7)/67/6 = 7/6Product of roots = constant / coefficient of x^2 $3/2 \times (-1/3) = (-3)/6$ -1/2 = -1/2Therefore, the relationship between zeros and their coefficients is verified. (v) $p(x) = x^2 + 2\sqrt{2x} - 6$ Given, $p(x) = x^2 + 2\sqrt{2x} - 6$ To find the zeros, we put p(x) = 0 $\Rightarrow x^2 + 2\sqrt{2x} - 6 = 0$ \Rightarrow x² + 3 $\sqrt{2x}$ - $\sqrt{2x}$ - 6 = 0 \Rightarrow x(x + 3 $\sqrt{2}$) - $\sqrt{2}$ (x + 3 $\sqrt{2}$) = 0 \Rightarrow (x - $\sqrt{2}$)(x + 3 $\sqrt{2}$) = 0 This gives us 2 zeros, for $x = \sqrt{2}$ and $x = -3\sqrt{2}$ Hence, the zeros of the quadratic equation are $\sqrt{2}$ and $-3\sqrt{2}$. Now, for verification Sum of zeros = – coefficient of x / coefficient of x^2 $\sqrt{2} + (-3\sqrt{2}) = -(2\sqrt{2})/1$ $-2\sqrt{2} = -2\sqrt{2}$ Product of roots = constant / coefficient of x^2 $\sqrt{2} \times (-3\sqrt{2}) = (-6) / 2\sqrt{2}$ $-3 \ge 2 = -6/1$ -6 = -6 Therefore, the relationship between zeros and their coefficients is verified. (vi) $q(x) = \sqrt{3x^2 + 10x + 7\sqrt{3}}$ Given.

 $q(x) = \sqrt{3x^2 + 10x + 7\sqrt{3}}$ To find the zeros, we put q(x) = 0 $\Rightarrow \sqrt{3x^2 + 10x + 7\sqrt{3}} = 0$ $\Rightarrow \sqrt{3}x^{2} + 3x + 7x + 7\sqrt{3}x = 0$ $\Rightarrow \sqrt{3}x(x + \sqrt{3}) + 7(x + \sqrt{3}) = 0$ $\Rightarrow (x + \sqrt{3})(\sqrt{3}x + 7) = 0$ This gives us 2 zeros, for $x = -\sqrt{3} \text{ and } x = -7/\sqrt{3}$ Hence, the zeros of the quadratic equation are $-\sqrt{3}$ and $-7/\sqrt{3}$. Now, for verification Sum of zeros = - coefficient of x / coefficient of x^{2} $-\sqrt{3} + (-7/\sqrt{3}) = -(10)/\sqrt{3}$ $(-3-7)/\sqrt{3} = -10/\sqrt{3}$ $-10/\sqrt{3} = -10/\sqrt{3}$ Product of roots = constant / coefficient of x^{2} $(-\sqrt{3}) x (-7/\sqrt{3}) = (7\sqrt{3})/\sqrt{3}$ 7 = 7Therefore, the relationship between zeros and their coefficients is verified.

Given,

$$f(x) = x^{2} - (\sqrt{3} + 1)x + \sqrt{3}$$
To find the zeros, we put $f(x) = 0$

$$\Rightarrow x^{2} - (\sqrt{3} + 1)x + \sqrt{3} = 0$$

$$\Rightarrow x^{2} - \sqrt{3}x - x + \sqrt{3} = 0$$

$$\Rightarrow x(x - \sqrt{3}) - 1 (x - \sqrt{3}) = 0$$

$$\Rightarrow (x - \sqrt{3})(x - 1) = 0$$
This gives us 2 zeros, for
 $x = \sqrt{3}$ and $x = 1$
Hence, the zeros of the quadratic equation are $\sqrt{3}$ and 1.
Now, for verification

(vii) $f(x) = x^2 - (\sqrt{3} + 1)x + \sqrt{3}$

Sum of zeros = – coefficient of x / coefficient of x^2

$$\sqrt{3} + 1 = -(-(\sqrt{3} + 1)) / 1$$

 $\sqrt{3} + 1 = \sqrt{3} + 1$

Product of roots = constant / coefficient of x^2

$$1 \ge \sqrt{3} = \sqrt{3} / 1$$
$$\sqrt{3} = \sqrt{3}$$

Therefore, the relationship between zeros and their coefficients is verified.

(viii) $g(x)=a(x^2+1)-x(a^2+1)$ Given, $g(x) = a(x^2+1) - x(a^2+1)$ To find the zeros, we put g(x) = 0 \Rightarrow a(x²+1)-x(a²+1) = 0 \Rightarrow ax² + a - a²x - x = 0 \Rightarrow ax² - a²x - x + a = 0 \Rightarrow ax(x - a) - 1(x - a) = 0 \Rightarrow (x - a)(ax - 1) = 0 This gives us 2 zeros, for x = a and x = 1/aHence, the zeros of the quadratic equation are a and 1/a. Now, for verification Sum of zeros = - coefficient of x / coefficient of x^2 $a + 1/a = -(-(a^2 + 1))/a$ $(a^2 + 1)/a = (a^2 + 1)/a$ Product of roots = constant / coefficient of x^2 $a \ge 1/a = a / a$

1 = 1

Therefore, the relationship between zeros and their coefficients is verified.

(ix) h(s) =
$$2s^2 - (1 + 2\sqrt{2})s + \sqrt{2}$$

Given,

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h(s) = 2s<sup>2</sup> - (1 + 2√2)s + √2

To find the zeros, we put h(s) = 0

⇒ 2s<sup>2</sup> - (1 + 2√2)s + √2 = 0

⇒ 2s<sup>2</sup> - 2√2s - s + √2 = 0

⇒ 2s(s - √2) -1(s - √2) = 0

⇒ (2s - 1)(s - √2) = 0

This gives us 2 zeros, for

x = √2 and x = 1/2

Hence, the zeros of the quadratic equation are √3 and 1.

Now, for verification

Sum of zeros = - coefficient of s / coefficient of s<sup>2</sup>

\sqrt{2} + 1/2 = -(-(1 + 2\sqrt{2})) / 2
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 $(2\sqrt{2}+1)/2 = (2\sqrt{2}+1)/2$ Product of roots = constant / coefficient of s^2 $1/2 \ge \sqrt{2} = \sqrt{2}/2$ $\sqrt{2} / 2 = \sqrt{2} / 2$ Therefore, the relationship between zeros and their coefficients is verified. (x) $f(v) = v^2 + 4\sqrt{3}v - 15$ Given, $f(v) = v^2 + 4\sqrt{3}v - 15$ To find the zeros, we put f(v) = 0 \Rightarrow v² + 4 $\sqrt{3}$ v - 15 = 0 \Rightarrow v² + 5 $\sqrt{3}$ v - $\sqrt{3}$ v - 15 = 0 \Rightarrow v(v + 5 $\sqrt{3}$) – $\sqrt{3}$ (v + 5 $\sqrt{3}$) = 0 \Rightarrow (v - $\sqrt{3}$)(v + 5 $\sqrt{3}$) = 0 This gives us 2 zeros, for $v = \sqrt{3}$ and $v = -5\sqrt{3}$ Hence, the zeros of the quadratic equation are $\sqrt{3}$ and $-5\sqrt{3}$. Now, for verification Sum of zeros = – coefficient of v / coefficient of v^2 $\sqrt{3} + (-5\sqrt{3}) = -(4\sqrt{3}) / 1$ $-4\sqrt{3} = -4\sqrt{3}$ Product of roots = constant / coefficient of v^2 $\sqrt{3} \times (-5\sqrt{3}) = (-15) / 1$ $-5 \times 3 = -15$ -15 = -15Therefore, the relationship between zeros and their coefficients is verified. (xi) $p(y) = y^2 + (3\sqrt{5}/2)y - 5$ Given. $p(y) = y^2 + (3\sqrt{5}/2)y - 5$

To find the zeros, we put f(v) = 0 $\Rightarrow y^2 + (3\sqrt{5}/2)y - 5 = 0$ $\Rightarrow y^2 - \sqrt{5}/2 y + 2\sqrt{5}y - 5 = 0$ $\Rightarrow y(y - \sqrt{5}/2) + 2\sqrt{5} (y - \sqrt{5}/2) = 0$ $\Rightarrow (y + 2\sqrt{5})(y - \sqrt{5}/2) = 0$ This gives us 2 zeros, for

 $y = \sqrt{5}/2$ and $y = -2\sqrt{5}$

Hence, the zeros of the quadratic equation are $\sqrt{5}/2$ and $-2\sqrt{5}$.

Now, for verification

Sum of zeros = – coefficient of y / coefficient of y^2

 $\sqrt{5/2} + (-2\sqrt{5}) = -(3\sqrt{5/2}) / 1$

 $-3\sqrt{5}/2 = -3\sqrt{5}/2$

Product of roots = constant / coefficient of y^2

$$\sqrt{5/2} \times (-2\sqrt{5}) = (-5) / 1$$

- $(\sqrt{5})^2 = -5$
-5 = -5

Therefore, the relationship between zeros and their coefficients is verified.

(xii)
$$q(y) = 7y^2 - (11/3)y - 2/3$$

Given,

$$q(y) = 7y^2 - (11/3)y - 2/3$$

To find the zeros, we put q(y) = 0

$$\Rightarrow 7y^{2} - (11/3)y - 2/3 = 0$$
$$\Rightarrow (21y^{2} - 11y - 2)/3 = 0$$

$$\Rightarrow 21y^2 - 11y - 2 = 0$$

$$\Rightarrow 21y^2 - 14y + 3y - 2 = 0$$

$$\Rightarrow 7y(3y-2) - 1(3y+2) = 0$$

$$\Rightarrow (3y-2)(7y+1) = 0$$

This gives us 2 zeros, for

$$y = 2/3$$
 and $y = -1/7$

Hence, the zeros of the quadratic equation are 2/3 and -1/7.

Now, for verification

Sum of zeros = – coefficient of y / coefficient of y^2

$$2/3 + (-1/7) = -(-11/3)/7$$

-11/21 = -11/21

Product of roots = constant / coefficient of y^2

$$2/3 \times (-1/7) = (-2/3) / 7$$

Therefore, the relationship between zeros and their coefficients is verified.

SOL.2	(i) -8/3 , 4/3
	A quadratic polynomial formed for the given sum and product of zeros is given by:
	$f(x) = x^2 + -(sum of zeros) x + (product of roots)$
	Here, the sum of zeros is = $-8/3$ and product of zero= $4/3$
	Thus,
	The required polynomial f(x) is,
	$\Rightarrow x^2 - (-8/3)x + (4/3)$
	$\Rightarrow x^2 + 8/3x + (4/3)$
	So, to find the zeros, we put $f(x) = 0$
	$\Rightarrow x^2 + 8/3x + (4/3) = 0$
	$\Rightarrow 3x^2 + 8x + 4 = 0$
	$\Rightarrow 3x^2 + 6x + 2x + 4 = 0$
	$\Rightarrow 3x(x+2) + 2(x+2) = 0$
	$\Rightarrow (x+2) (3x+2) = 0$
	\Rightarrow (x + 2) = 0 and, or (3x + 2) = 0
	Therefore, the two zeros are -2 and $-2/3$.
	(ii) 21/8, 5/16
	A quadratic polynomial formed for the given sum and product of zeros is given by:
	$f(x) = x^2 + -(sum of zeros) x + (product of roots)$
	Here, the sum of zeros is = $21/8$ and product of zero = $5/16$
	Thus,
	The required polynomial f(x) is,
	$\Rightarrow x^2 - (21/8)x + (5/16)$
	$\Rightarrow x^2 - 21/8x + 5/16$
	So, to find the zeros, we put $f(x) = 0$
	$\Rightarrow x^2 - 21/8x + 5/16 = 0$
	$\Rightarrow 16x^2 - 42x + 5 = 0$
	$\Rightarrow 16x^2 - 40x - 2x + 5 = 0$
	$\Rightarrow 8x(2x-5) - 1(2x-5) = 0$
	$\Rightarrow (2x-5) (8x-1) = 0$
	\Rightarrow (2x - 5) = 0 and, or (8x - 1) = 0
	Therefore, the two zeros are $5/2$ and $1/8$.
	(iii) -2√3, -9
	A quadratic polynomial formed for the given sum and product of zeros is given by:

SOL.3

 $f(x) = x^2 + -(sum of zeros) x + (product of roots)$ Here, the sum of zeros is = $-2\sqrt{3}$ and product of zero = -9Thus, The required polynomial f(x) is, \Rightarrow x² - (-2 $\sqrt{3}$)x + (-9) \Rightarrow x² + 2 $\sqrt{3x}$ - 9 So, to find the zeros we put f(x) = 0 $\Rightarrow x^2 + 2\sqrt{3x} - 9 = 0$ \Rightarrow x² + 3 $\sqrt{3}$ x - $\sqrt{3}$ x - 9 = 0 \Rightarrow x(x + 3 $\sqrt{3}$) - $\sqrt{3}$ (x + 3 $\sqrt{3}$) = 0 \Rightarrow (x + 3 $\sqrt{3}$) (x - $\sqrt{3}$) = 0 \Rightarrow (x + 3 $\sqrt{3}$) = 0 and, or (x - $\sqrt{3}$) = 0 Therefore, the two zeros are $-3\sqrt{3}$ and $\sqrt{3}$. (iv) $-3/2\sqrt{5}$, -1/2A quadratic polynomial formed for the given sum and product of zeros is given by: $f(x) = x^2 + -(sum of zeros) x + (product of roots)$ Here, the sum of zeros is = $-3/2\sqrt{5}$ and product of zero = -1/2Thus. The required polynomial f(x) is, \Rightarrow x² - (-3/2 $\sqrt{5}$)x + (-1/2) \Rightarrow x² + 3/2 $\sqrt{5x}$ - 1/2 So, to find the zeros, we put f(x) = 0 \Rightarrow x² + 3/2 $\sqrt{5x}$ - 1/2 = 0 $\Rightarrow 2\sqrt{5x^2 + 3x} - \sqrt{5} = 0$ $\Rightarrow 2\sqrt{5x^2 + 5x - 2x} - \sqrt{5} = 0$ $\Rightarrow \sqrt{5x(2x + \sqrt{5})} - 1(2x + \sqrt{5}) = 0$ \Rightarrow (2x + $\sqrt{5}$) ($\sqrt{5x}$ - 1) = 0 \Rightarrow (2x + $\sqrt{5}$) = 0 and, or ($\sqrt{5x} - 1$) = 0 Therefore, the two zeros are $-\sqrt{5}/2$ and $1/\sqrt{5}$. From the question, it's given that: α and β are the roots of the quadratic polynomial f(x) where a = 1, b = -5 and c = 4 So, we can find

Sum of the roots = α + β = -b/a = - (-5)/1 = 5

Product of the roots = $\alpha\beta$ = c/a = 4/1 = 4

To find, $1/\alpha + 1/\beta - 2\alpha\beta$ $\Rightarrow [(\alpha + \beta)/\alpha\beta] - 2\alpha\beta$ \Rightarrow (5)/4 - 2(4) = 5/4 - 8 = -27/4 From the question, it's given that: SOL.4 α and β are the roots of the quadratic polynomial f(x) where a =5, b = -7 and c = 1 So, we can find Sum of the roots = α + β = -b/a = - (-7)/5 = 7/5 Product of the roots = $\alpha\beta$ = c/a = 1/5 To find, $1/\alpha + 1/\beta$ $\Rightarrow (\alpha + \beta) / \alpha \beta$ $\Rightarrow (7/5)/(1/5) = 7$ From the question, it's given that: SOL.5 α and β are the roots of the quadratic polynomial f(x) where a = 1, b = -1 and c = -4 So, we can find Sum of the roots = α + β = -b/a = - (-1)/1 = 1 Product of the roots = $\alpha\beta$ = c/a = -4 /1 = -4 To find, $1/\alpha + 1/\beta - \alpha\beta$ $\Rightarrow [(\alpha + \beta) / \alpha \beta] - \alpha \beta$ $\Rightarrow [(1)/(-4)] - (-4) = -1/4 + 4 = 15/4$ **SOL.6** From the question, it's given that: α and β are the roots of the quadratic polynomial f(x) where a = 1, b = 1 and c = -2 So, we can find Sum of the roots = α + β = -b/a = -(1)/1 = -1 Product of the roots = $\alpha\beta$ = c/a = -2 /1 = -2 To find, $1/\alpha - 1/\beta$ $\Rightarrow [(\beta - \alpha) / \alpha \beta]$ $\frac{\beta - \alpha}{\alpha \beta} = \frac{\beta - \alpha}{\alpha \beta} \times \frac{\alpha - \beta}{\alpha \beta} = \frac{\sqrt{\alpha + \beta^2 - 4\alpha \beta}}{\alpha \beta} = \frac{\sqrt{1 + 8}}{2} = \frac{\sqrt{9}}{2} = \frac{3}{2} \Rightarrow$ SOL.7 From the question, it's given that: The quadratic polynomial f(x) where a = 4, b = -8k and c = -9And, for roots to be negative of each other, let the roots be α and $-\alpha$. So, we can find Sum of the roots = $\alpha - \alpha = -b/a = -(-8k)/1 = 8k = 0$ [:: $\alpha - \alpha = 0$] \Rightarrow k = 0

SOL.8 Given,

The quadratic polynomial $f(t)=kt^2 + 2t + 3k$, where a = k, b = 2 and c = 3k.

And,

Sum of the roots = Product of the roots

 $\Rightarrow (-b/a) = (c/a)$ $\Rightarrow (-2/k) = (3k/k)$ $\Rightarrow (-2/k) = 3$ $\therefore k = -2/3$

SOL.9 From the question, it's given that:

 α and β are the roots of the quadratic polynomial p(x) where a = 4, b = -5 and c = -1 So, we can find Sum of the roots = $\alpha+\beta$ = -b/a = - (-5)/4 = 5/4 Product of the roots = $\alpha\beta$ = c/a = -1/4 To find, $\alpha^2\beta+\alpha\beta^2$ $\Rightarrow \alpha\beta(\alpha+\beta)$ $\Rightarrow (-1/4)(5/4) = -5/16$

SOL.10 From the question, it's given that:

α and β are the roots of the quadratic polynomial f(t) where a = 1, b = -4 and c = 3 So, we can find Sum of the roots = α+β = -b/a = -(-4)/1 = 4Product of the roots = αβ = c/a = 3/1 = 3To find, $α^4β^3+α^3β^4$ $\Rightarrow α^3β^3 (α+β)$ $\Rightarrow (αβ)^3 (α+β)$ $\Rightarrow (3)^3 (4) = 27 x 4 = 108$