CHAPTER – 7
COORDINATE GEOMETRY

DISTANCE FORMULA

The distance between any two points $A(x_1, y_1)$ and $B(x_2, y_2)$ is given by

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$or \quad AB = \sqrt{(\text{difference of abscissae})^2 + (\text{difference of ordinates})^2}$$

Distance of a point from origin

The distance of a point P(x, y) from origin O is given by OP = $\sqrt{x^2 + y^2}$

Problems based on geometrical figure

To show that a given figure is a

- ☞ Parallelogram – prove that the opposite sides are equal
- ☞ Rectangle – prove that the opposite sides are equal and the diagonals are equal.
- ☞ Parallelogram but not rectangle – prove that the opposite sides are equal and the diagonals are not equal.
- ☞ Rhombus – prove that the four sides are equal
- ☞ Square – prove that the four sides are equal and the diagonals are equal.
- ☞ Rhombus but not square – prove that the four sides are equal and the diagonals are not equal.
- ☞ Isosceles triangle – prove any two sides are equal.
- ☞ Equilateral triangle – prove that all three sides are equal.
- ☞ Right triangle – prove that sides of triangle satisfies Pythagoras theorem.

IMPORTANT QUESTIONS

Show that the points (1, 7), (4, 2), (–1, –1) and (– 4, 4) are the vertices of a square.

Solution : Let A(1, 7), B(4, 2), C(–1, –1) and D(– 4, 4) be the given points.

$$AB = \sqrt{(1-4)^2 + (7-2)^2} = \sqrt{9+25} = \sqrt{34}$$

$$BC = \sqrt{(4+1)^2 + (2+1)^2} = \sqrt{25+9} = \sqrt{34}$$

$$CD = \sqrt{(-1+4)^2 + (-1-4)^2} = \sqrt{9+25} = \sqrt{34}$$

$$DA = \sqrt{(1+4)^2 + (7-4)^2} = \sqrt{25+9} = \sqrt{34}$$

$$AC = \sqrt{(1+1)^2 + (7+1)^2} = \sqrt{4+64} = \sqrt{68}$$

$$BD = \sqrt{(4+4)^2 + (2-4)^2} = \sqrt{64+4} = \sqrt{68}$$

Since, AB = BC = CD = DA and AC = BD, all the four sides of the quadrilateral ABCD are equal and its diagonals AC and BD are also equal. Therefore, ABCD is a square.

Find a point on the y-axis which is equidistant from the points A(6, 5) and B(– 4, 3).

Solution : We know that a point on the y-axis is of the form (0, y). So, let the point P(0, y) be equidistant from A and B. Then $AP^2 = BP^2$

$\Rightarrow (6 - 0)^2 + (5 - y)^2 = (-4 - 0)^2 + (3 - y)^2$

$\Rightarrow 36 + 25 + y^2 - 10y = 16 + 9 + y^2 - 6y \Rightarrow 4y = 36 \Rightarrow y = 9$

So, the required point is (0, 9).

Questions for practice

1. Show that the points A(1, 2), B(5, 4), C(3, 8) and D(–1, 6) are vertices of a square.
2. Show that the points A(5, 6), B(1, 5), C(2, 1) and D(6, 2) are vertices of a square.
3. Show that the points A(1, –3), B(13, 9), C(10, 12) and D(–2, 0) are vertices of a rectangle.
4. Show that the points A(1, 0), B(5, 3), C(2, 7) and D(–2, 4) are vertices of a rhombus.
5. Prove that the points A(–2, –1), B(1, 0), C(4, 3) and D(1, 2) are vertices of a parallelogram.
6. Find the point on x-axis which is equidistant from (7, 6) and (–3, 4).
7. Find the point on the x-axis which is equidistant from (2, –5) and (–2, 9).
8. Find a point on the y-axis which is equidistant from the points A(5, 2) and B(– 4, 3).
9. Find a point on the y-axis which is equidistant from the points A(5, – 2) and B(– 3, 2).
10. Find the values of y for which the distance between the points P(2, – 3) and Q(10, y) is 10 units.
11. Find the value of a , if the distance between the points A (–3, –14) and B (a, –5) is 9 units.
12. If the point A (2, – 4) is equidistant from P (3, 8) and Q (–10, y), find the values of y. Also find distance PQ.

Section formula

The coordinates of the point P(x, y) which divides the line segment joining the points A(x_1, y_1) and B(x_2, y_2), internally, in the ratio $m_1 : m_2$ are

$$\left(\frac{m_1x_2+m_2x_1}{m_1+m_2},\frac{m_1y_2+m_2y_1}{m_1+m_2}\right)$$

Mid-point formula

The coordinates of the point P(x, y) which is the midpoint of the line segment joining the points A(x_1, y_1) and B(x_2, y_2), are $\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$

IMPORTANT QUESTIONS

Find the coordinates of the point which divides the line segment joining the points (4, – 3) and (8, 5) in the ratio 3 : 1 internally.

Solution : Let P(x, y) be the required point.

Using the section formula, $x=\frac{m_2x_1+m_1x_2}{m_1+m_2}, y=\frac{m_2y_1+m_1y_2}{m_1+m_2}$ we get

$$x=\frac{3(8)+1(4)}{3+1}=7, y=\frac{3(5)+1(-3)}{3+1}=3$$

Therefore, (7, 3) is the required point.

In what ratio does the point (– 4, 6) divide the line segment joining the points A(– 6, 10) and B(3, – 8)?

Solution : Let (– 4, 6) divide AB internally in the ratio k : 1.

Using the section formula, $x=\frac{m_2x_1+m_1x_2}{m_1+m_2}, y=\frac{m_2y_1+m_1y_2}{m_1+m_2}$ we get

$$y=\frac{k(-8)+1(10)}{k+1}=6$$

$$\Rightarrow -8k+10=6k+6 \Rightarrow -8k-6k=6-10$$

$$\Rightarrow -14k=-4 \Rightarrow k=\frac{4}{14}=\frac{2}{7}$$

Therefore, the point (– 4, 6) divides the line segment joining the points A(– 6, 10) and B(3, – 8) in the ratio 2 : 7.

Questions for practice

16. Find the coordinates of the point which divides the join of (–1, 7) and (4, –3) in the ratio 2 : 3.

17. Find the coordinates of the points of trisection of the line segment joining (4, –1) and (–2, –3).

18. Find the coordinates of the points of trisection (i.e., points dividing in three equal parts) of the line segment joining the points A(2, – 2) and B(– 7, 4).

19. Find the ratio in which the y-axis divides the line segment joining the points (5, – 6) and (–1, – 4). Also find the point of intersection.

20. Find the ratio in which the line segment joining the points (– 3, 10) and (6, – 8) is divided by (– 1, 6).

21. Find the ratio in which the line segment joining A(1, – 5) and B(– 4, 5) is divided by the x-axis. Also find the coordinates of the point of division.

22. Find the coordinates of the points which divide the line segment joining A(– 2, 2) and B(2, 8) into four equal parts.

23. If the points A(6, 1), B(8, 2), C(9, 4) and D(p, 3) are the vertices of a parallelogram, taken in order, find the value of p.

24. If (1, 2), (4, y), (x, 6) and (3, 5) are the vertices of a parallelogram taken in order, find x and y.

25. In what ratio does the x–axis divide the line segment joining the points (– 4, – 6) and (–1, 7)? Find the coordinates of the point of division.

26. If P (9a – 2, –b) divides line segment joining A (3a + 1, –3) and B (8a, 5) in the ratio 3 : 1, find the values of a and b.

27. If (a, b) is the mid-point of the line segment joining the points A (10, –6) and B (k, 4) and a – 2b = 18, find the value of k and the distance AB.

28. The centre of a circle is (2a, a – 7). Find the values of a if the circle passes through the point (11, –9) and has diameter $10\sqrt{2}$ units.

29. The line segment joining the points A (3, 2) and B (5,1) is divided at the point P in the ratio 1:2 and it lies on the line 3x – 18y + k = 0. Find the value of k.

30. Find the coordinates of the point R on the line segment joining the points P (–1, 3) and Q (2, 5) such that PR = $\frac{3}{5}$PQ.

31. Find the values of k if the points A (k + 1, 2k), B (3k, 2k + 3) and C (5k – 1, 5k) are collinear.

32. Find the ratio in which the line 2x + 3y – 5 = 0 divides the line segment joining the points (8, –9) and (2, 1). Also find the coordinates of the point of division.

33. The mid-points D, E, F of the sides of a triangle ABC are (3, 4), (8, 9) and (6, 7). Find the coordinates of the vertices of the triangle.

MCQ QUESTIONS (1 mark)

1. If the distance between the points (2, –2) and (–1, x) is 5, one of the values of x is
(a) –2 (b) 2 (c) –1 (d) 1

2. The mid-point of the line segment joining the points A (–2, 8) and B (– 6, – 4) is
(a) (– 4, – 6) (b) (2, 6) (c) (– 4, 2) (d) (4, 2)

3. The distance of the point P (2, 3) from the x-axis is
(a) 2 (b) 3 (c) 1 (d) 5

4. The distance between the points A (0, 6) and B (0, –2) is
(a) 6 (b) 8 (c) 4 (d) 2

5. The distance of the point P (–6, 8) from the origin is

$\sqrt{\ }$

6. The distance between the points (0, 5) and (–5, 0) is
(a) 5 (b) $5\sqrt{2}$ (c) $2\sqrt{5}$ (d) 10

7. AOBC is a rectangle whose three vertices are vertices A (0, 3), O (0, 0) and B (5, 0). The length of its diagonal is
(a) 5 (b) 3 (c) $\sqrt{34}$ (d) 4

8. The perimeter of a triangle with vertices (0, 4), (0, 0) and (3, 0) is
(a) 5 (b) 12 (c) 11 (d) 7 + $\sqrt{5}$

9. The points (–4, 0), (4, 0), (0, 3) are the vertices of a
(a) right triangle (b) isosceles triangle
(c) equilateral triangle (d) scalene triangle

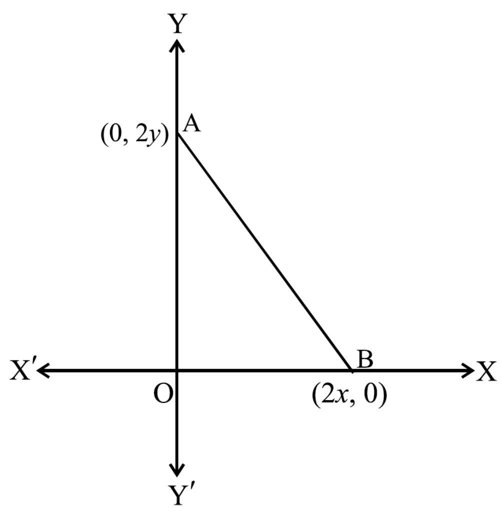
10. The point which divides the line segment joining the points (7, –6) and (3, 4) in ratio 1 : 2 internally lies in the
(a) I quadrant (b) II quadrant
(c) III quadrant (d) IV quadrant

11. The point which lies on the perpendicular bisector of the line segment joining the points A (–2, –5) and B (2, 5) is
(a) (0, 0) (b) (0, 2) (c) (2, 0) (d) (–2, 0)

12. The fourth vertex D of a parallelogram ABCD whose three vertices are A (–2, 3), B (6, 7) and C (8, 3) is
(a) (0, 1) (b) (0, –1) (c) (–1, 0) (d) (1, 0)

13. If the point P (2, 1) lies on the line segment joining points A (4, 2) and B (8, 4), then
(a) AP = $\frac{1}{3}$AB (b) AP = PB (c) PB = $\frac{1}{3}$AB (d) AP = $\frac{1}{2}$AB

14. The coordinates of the point which is equidistant from the three vertices of the Δ AOB as shown in the Fig. 7.1 is
(a) (x, y) (b) (y, x)
(c) $\left(\frac{x}{2}, \frac{y}{2}\right)$ (d) $\left(\frac{y}{2}, \frac{x}{2}\right)$



15. If P$\left(-\frac{a}{3}, 4\right)$ is the mid-point of the line segment joining the points Q (– 6, 5) and R (– 2, 3), then the value of a is
(a) – 4 (b) – 12 (c) 12 (d) – 6

16. The perpendicular bisector of the line segment joining the points A (1, 5) and B (4, 6) cuts the y-axis at
(a) (0, 13) (b) (0, –13)
(c) (0, 12) (d) (13, 0)

17. A circle drawn with origin as the centre passes through (13/2 ,0). The point which does not lie in the interior of the circle is
(a) $\left(\frac{-3}{4}, 1\right)$ (b) $\left(2, \frac{7}{3}\right)$ (c) $\left(5, \frac{-1}{2}\right)$ (d) $\left(-6, \frac{5}{2}\right)$

18. A line intersects the y-axis and x-axis at the points P and Q, respectively. If (2, –5) is the mid-point of PQ, then the coordinates of P and Q are, respectively
(a) (0, – 5) and (2, 0) (b) (0, 10) and (– 4, 0)
(c) (0, 4) and (– 10, 0) (d) (0, – 10) and (4, 0)

19. If the distance between the points (4, p) and (1, 0) is 5, then the value of p is
(a) 4 only (b) ± 4 (c) – 4 only (d) 0

20. If the points A (1, 2), O (0, 0) and C (a, b) are collinear, then
(a) a = b (b) a = 2b (c) 2a = b (d) a = –b