BINOMIAL THEOREM

1. BINOMIAL THEOREM

If $a, b \in R$ and $n \in N$, then

$(a + b)^n = {}^nC_0\, a^n b^0 + {}^nC_1\, a^{n-1} b^1 + {}^nC_2\, a^{n-2} b^2 + ... + {}^nC_n\, a^0 b^n$

REMARKS :

1. If the index of the binomial is n then the expansion contains n + 1 terms.
2. In each term, the sum of indices of a and b is always n.
3. Coefficients of the terms in binomial expansion equidistant from both the ends are equal.
4. $(a-b)^n = {}^nC_0 a^n b^0 - {}^nC_1 a^{n-1} b^1 + {}^nC_2 a^{n-2} b^{2-} ... + (-1)^n\, {}^nC_0 a^0 b^n$.

2. GENERAL TERM AND MIDDLE TERMS IN EXPANSION OF $(a + b)^n$

$t_{r+1} = {}^nC_r\, a^{n-r} b^r$

t_{r+1} is called a general term for all $r \in W$ and $0 \le r \le n$. Using this formula we can find any term of the expansion.

MIDDLE TERM (S) :

1. In $(a + b)^n$ if n is even then the number of terms in the expansion is odd. Therefore there is only one middle term and it is $\left(\frac{n+2}{2}\right)^{th}$ term.
2. In $(a + b)^n$, if n is odd then the number of terms in the expansion is even. Therefore there are two middle terms and those are $\left(\frac{n+1}{2}\right)^{th}$ and $\left(\frac{n+3}{2}\right)^{th}$ terms.

3. NUMERICALLY GREATEST TERM

The term with greatest numerical/absolute value in the expansion of $(a + b)^n$ can be found in following way

(i) Find the value of $\dfrac{n+1}{1+\left|\frac{a}{b}\right|}$

(ii) If it equals an integer, say m, then t_m and t_{m+1} are numerically greatest terms.

(iii) If it is not an integer, then t_{m+1} is numerically greatest term (where m is the integral part of $\dfrac{n+1}{1+\left|\frac{a}{b}\right|}$).

Also middle terms in binomial expansions have the greatest binomial coefficients. (${}^nC_0, {}^nC_1, {}^nC_2 .... {}^nC_n$ are called Binomial Coefficients).

4. BINOMIAL COEFFICENTS

The coefficients ${}^nC_0, {}^nC_1, {}^nC_2,..., {}^nC_n$ in the expansion of $(a+b)^n$ are called the binomial coefficients and denoted by $C_0, C_1, C_2, ..., C_n$ respectively

Now

$(1 + x)^n = {}^nC_0 x^0 + {}^nC_1 x^1 + {}^nC_2 x^2 + ... + {}^nC_n x^n$ ..... (i)

Put x = 1.

$(1 + 1)^n = {}^nC_0 + {}^nC_1 + {}^nC_2 + ... + {}^nC_n$

$\therefore$ $2^n = {}^nC_0 + {}^nC_1 + {}^nC_2 + ... + {}^nC_n$

$\therefore$ ${}^nC_0 + {}^nC_1 + {}^nC_2 + ... + {}^nC_n = 2^n$

$\therefore$ $C_0 + C_1 + C_2 + ... + C_n = 2^n$

$\therefore$ The sum of all binomial coefficients is 2^n.

Put x = –1, in equation (i),

$(1-1)^n = {}^nC_0 - {}^nC_1 + {}^nC_2 - ... + (-1)^n\, {}^nC_n$

$\therefore$ $0 = {}^nC_0 - {}^nC_1 + {}^nC_2 - ... + (-1)^n\, {}^nC_n$

$\therefore$ ${}^nC_0 - {}^nC_1 + {}^nC_2 - {}^nC_3 + ... + (-1)^n\, {}^nC_n = 0$

$\therefore$ ${}^nC_0 + {}^nC_2 + {}^nC_4 + ... = {}^nC_1 + {}^nC_3 + {}^nC_5 + ...$

$\therefore$ $C_0 + C_2 + C_4 + ... = C_1 + C_3 + C_5 + ...$

$C_0, C_2, C_4, ...$ are called as even coefficients

$C_1, C_3, C_5 ...$ are called as odd coefficients

Let $C_0 + C_2 + C_4 + ... = C_1 + C_3 + C_5 + ... = k$

Now $C_0 + C_1 + C_2 + C_3 + ... + C_n = 2^n$

$\therefore$ $(C_0 + C_2 + C_4 + ...) + (C_1 + C_3 + C_5 ...) = 2^n$

$\therefore$ $k + k = 2^n$

$2k = 2^n$

$\therefore$ $k = \frac{2^n}{2}$

$\therefore$ $k = 2^{n-1}$

$\therefore$ $C_0 + C_2 + C_4 + ... = C_1 + C_3 + C_5 + ... = 2^{n-1}$

$\therefore$ The sum of even coefficients = The sum of odd coefficients $= 2^{n-1}$

Properties of Binomial Coefficient

(i) $C_0 + C_1 + C_2 + ... + C_n = 2^n$

(ii) $C_0 - C_1 + C_2 - .... + (-1)^n C_n = 0$

(iii) $C_0 + C_2 + C_4 + ... = C_1 + C_3 + C_5 + ... = 2^{n-1}$.

(iv) ${}^nC_{r_1} = {}^nC_{r_2} \Rightarrow r_1 = r_2$ or $r_1 + r_2 = n$

(v) ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$

(vi) $r\,{}^nC_r = n\,{}^{n-1}C_{r-1}$

Some Important Results

(i) Differentiating $(1 + x)^n = C_0 + C_1x + C_2x^2 + ... + C_nx^n$, on both sides we have,

$n(1 + x)^{n-1} = C_1 + 2C_2x + 3C_3x^2 + ... + nC_nx^{n-1}$ ....(1)

Put $x = 1$

$\Rightarrow$ $n2^{n-1} = C_1 + 2C_2 + 3C_3 + ... + nC_n$

Put $x = -1$

$\Rightarrow$ $0 = C_1 - 2C_2 + ... + (-1)^{n-1} nC_n$.

Differentiating (1) again and again we will have different results.

(ii) Integrating $(1 + x)^n$, we have,

$$\frac{(1+x)^{n+1}}{n+1} + C = C_0x + \frac{C_1x^2}{2} + \frac{C_2x^3}{3} + ... + \frac{C_nx^{n+1}}{n+1}$$

(where C is a constant)

Put $x = 0$, we get $C = -\frac{1}{(n+1)}$

Therefore

$$\frac{(1+x)^{n+1} - 1}{n+1} = C_0x + \frac{C_1x^2}{2} + \frac{C_2x^3}{3} + ... + \frac{C_nx^{n+1}}{n+1} \quad ... (2)$$

Put $x = 1$ in (2) we get

$$\frac{2^{n+1} - 1}{n+1} = C_0 + \frac{C_1}{2} + ... + \frac{C_n}{n+1}$$

Put $x = -1$ in (2) we get,

$$\frac{1}{n+1} = C_0 - \frac{C_1}{2} + \frac{C_2}{3} - ....$$

5. BINOMIAL THEOREM FOR ANY INDEX

If n is any real number and $|x| < 1$ then

$$(1 + x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + ...$$

Here there are infinite number of terms in the expansion, The general term is given by

$$t_{r+1} = \frac{n(n-1)(n-2)...(n-r+1)x^r}{r!}, r \geq 0$$

NOTES :

(i) Expansion is valid only when $-1 < x < 1$

(ii) ${}^nC_r$ can not be used because it is defined only for natural number, so ${}^nC_r$ will be written as

$$\frac{n(n-1).........(n-r+1)}{r!}$$

(iii) As the series never terminates, the number of terms in the series is infinite.

(iv) If first term is not 1, then make it unity in the following way. $(a + x)^n = a^n(1 + x/a)^n$ if $\left|\frac{x}{a}\right| < 1$

NOTES :

While expanding $(a + b)^n$ where n is a negative integer or a fraction, reduce the binomial to the form in which the first term is unity and the second term is numerically less than unity.

Particular expansion of the binomials for negative index, $|x|<1$

1. $\frac{1}{1+x}=(1+x)^{-1}$

$=1-x+x^2-x^3+x^4-x^5+.....$

2. $\frac{1}{1-x}=(1-x)^{-1}$

$=1+x+x^2+x^3+x^4+x^5+.....$

3. $\frac{1}{(1+x)^2}=(1+x)^{-2}$

$=1-2x+3x^2-4x^3+.....$

4. $\frac{1}{(1-x)^2}=(1-x)^{-2}$

$=1+2x+3x^2+4x^3+.....$

6. MULTINOMIAL EXPANSION

In the expansion of $(x_1 + x_2 +........ +x_n)^m$ where $m, n \in N$ and $x_1, x_2, ..., x_n$ are independent variables, we have

(i) Total number of terms $= {}^{m+n-1}C_{n-1}$

(ii) Coefficient of $x_1^{r_1} x_2^{r_2} x_3^{r_3} ....x_n^{r_n}$ (where $r_1 + r_2 + ... + r_n = m, r_i \in N \cup \{0\}$ is $\frac{m!}{r_1!r_2!.......r_n!}$

(iii) Sum of all the coefficients is obtained by putting all the variables x_i equal to 1.

SOLVED EXAMPLES

Example – 1

Expand

(i) $(2x^2+3)^4$ (ii) $\left(2x^2-\frac{1}{x}\right)^6$

Sol. **(i)** $(2x^2+3)^4=$

$={}^4C_0(2x^2)^4(3)^0+{}^4C_1(2x^2)^3(3)^1+{}^4C_2(2x^2)^2$

$(3)^2+{}^4C_3(2x^2)^1(3)^3+{}^4C_4(2x^2)^0(3)^4$

$=(1)\,16x^8(1)+4\,(8x^6)\,(3)+6\,(4x^4)\,(9)+4\,(2x^2)\,27+(1)\,(1)\,81$

$$\because \left\{\begin{array}{l}{}^4C_0={}^4C_4=1,\ {}^4C_1={}^4C_3=4\\ {}^4C_2=\frac{4!}{2!\,2!}=\frac{4\times3\times2!}{2!\times2}=6\end{array}\right\}$$

$=16x^8+96x^6+216x^4+216x^2+81$

(ii) $\left(2x^2-\frac{1}{x}\right)^6={}^6C_0\,(2x^2)^6\left(\frac{1}{x}\right)^0-$

${}^6C_1(2x^2)^5\left(\frac{1}{x}\right)^1+{}^6C_2\,(2x^2)^4\left(\frac{1}{x}\right)^2-$

${}^6C_3\,(2x^2)^3\left(\frac{1}{x}\right)^3+{}^6C_4\,(2x^2)^2$

$\left(\frac{1}{x}\right)^4-{}^6C_5\,(2x^2)\left(\frac{1}{x}\right)^5+{}^6C_6\,(2x^2)^0\left(\frac{1}{x}\right)^6$

$=(1)\,64x^{12}(1)-(6)\,(32)\,x^{10}\times\frac{1}{x}+15\,(16)\,x^8\times\frac{1}{x^2}$

$-20\times 8x^6\times\frac{1}{x^3}+15(4)\,x^4\times\frac{1}{x^4}$

$-6(2x^2)\times\frac{1}{x^5}+(1)\,(1)\frac{1}{x^6}$

$$\because \left\{\begin{array}{l}{}^6C_0={}^6C_6=1,\ {}^6C_1={}^6C_5=6\\ {}^6C_2=\frac{6!}{2!4!}=\frac{6\times5\times4!}{2\times4!}=15\\ {}^6C_3=\frac{6!}{3!3!}=\frac{6\times5\times4\times3!}{3\times2\times3!}=20\end{array}\right\}$$

$=64x^{12}-192x^9+240x^6-160x^3+60-\frac{12}{x^3}+\frac{1}{x^6}$

Example – 2

Expand $(1+x+x^2)^3$.

Sol. Let $y=x+x^2$. Then,

$(1+x+x^2)^3=(1+y)^3={}^3C_0+{}^3C_1\,y+{}^3C_2y^2+{}^3C_3y^3$

$=1+3y+3y^2+y^3=1+3\,(x+x^2)+3\,(x+x^2)^2+(x+x^2)^3$

$=1+3\,(x+x^2)+3(x^2+2x^3+x^4)+\{{}^3C_0x^3\,(x^2)^0+{}^3C_1\,x^{3-1}\,(x^2)^1$

$+\,{}^3C_2x^{3-2}\,(x^2)^2+{}^3C_3\,x^0\,(x^2)^3\}$

$=1+3\,(x+x^2)+3(x^2+2x^3+x^4)+(x^3+3x^4+3x^5+x^6)$

$=x^6+3x^5+6x^4+7x^3+6x^2+3x+1$

Example – 3

Prove that $(\sqrt{5}+1)^5-(\sqrt{5}-1)^5=352$

Sol. $(\sqrt{5}+1)^5-(\sqrt{5}-1)^5=$

$$=\begin{bmatrix} {}^5C_0(\sqrt{5})^5+{}^5C_1(\sqrt{5})^4(1)+{}^5C_2(\sqrt{5})^3(1)^2 \\ +{}^5C_3(\sqrt{5})^2(1)^3+{}^5C_4(\sqrt{5})(1)^4 \\ +{}^5C_5(\sqrt{5})^0(1)^5 \end{bmatrix}$$

$$-\begin{bmatrix} {}^5C_0(\sqrt{5})^5-{}^5C_1(\sqrt{5})^4(1)+{}^5C_2(\sqrt{5})^3(1)^2 \\ -{}^5C_3(\sqrt{5})^2(1)^3+{}^5C_4(\sqrt{5})(1)^4 \\ -{}^5C_5(\sqrt{5})^0(1)^5 \end{bmatrix}$$

$$=2\left[{}^5C_1 5^2+{}^5C_3.5+{}^5C_5\right]$$

$$=2\left[5\times25+10\times5+1\right]$$

$$\left\{\begin{matrix} {}^5C_0={}^5C_5=1;\ {}^5C_4=5; \\ {}^5C_2={}^5C_3=\frac{5.4}{2.1}=10;\ {}^5C_1=5 \end{matrix}\right\}$$

$=2[125+51]$

$=352$

Example – 4

Using binomial theorem compute $(99)^5$.

Sol. $(99)^5=(100-1)^5={}^5C_0(100)^5-{}^5C_1(100)^4+{}^5C_2(100)^3$

$-{}^5C_3(100)^2+{}^5C_4(100)^1-{}^5C_5(100)^0$

$=(100)^5-5\times(100)^4+10\times(100)^3-10\times(100)^2+5\times100-1$

$=10^{10}-5\times10^8+10^7-10^5+5\times10^2-1$

$=(10^{10}+10^7+5\times10^2)-(5\times10^8+10^5+1)$

$=10010000500-500100001=9509900499$

Example – 5

Use the binomial theorem to find the exact value of $(10.1)^5$.

Sol. $(10.1)^5=(10+0.1)^5$

$=10^5+{}^5C_1 10^4(.1)+{}^5C_2 10^3(.1)^2+{}^5C_3 10^2(.1)^3$

$+{}^5C_4 10(.1)^4+{}^5C_5(.1)^5$

$=100000+5\times10^4(.1)+10\times(10^3)(.01)+10\times10^2(.001)$

$+5\times10(.0001)+0.00001$

$=100000+5000+100+1+0.005+0.00001=105101.00501$

Example – 6

Prove that $\sum_{r=1}^{5}{}^5C_r=31$

Sol. $\sum_{r=1}^{5}{}^5C_r={}^5C_1+{}^5C_2+{}^5C_3+{}^5C_4+{}^5C_5$

$$=\frac{5!}{1!4!}+\frac{5!}{2!3!}+\frac{5!}{3!2!}+\frac{5!}{4!1!}+\frac{5!}{5!0!}$$

$$=\frac{5}{1}+\frac{5.4}{2}+\frac{5.4}{2}+\frac{5}{1}+1$$

$=5+10+10+5+1=31$

Example – 7

Find n, if ${}^nC_6:{}^{n-3}C_3=33:4$.

Sol. Given, ${}^nC_6:{}^{n-3}C_3=33:4$.

$$\therefore\ \frac{n!}{6!(n-6)!}\times\frac{3!(n-3-3)!}{(n-3)!}=\frac{33}{4}$$

$$\text{or}\ \frac{n!}{(n-3)!}.\frac{3!}{6!}=\frac{33}{4}\ \text{or}\ \frac{n(n-1)(n-2)}{6.5.4}=\frac{33}{4}$$

or $n(n-1)(n-2)=6.5.33=11.3.3.2.5$

or $n(n-1)(n-2)=11.(3.3).(2.5)=11.10.9\quad\therefore\ n=11$

Example – 8

If ${}^nC_8={}^nC_6$ determine n and hence ${}^nC_2$.

Sol. Given, ${}^nC_8={}^nC_6$

We know that ${}^nC_x={}^nC_y$ then $x=y$ or $x+y=n$

$\Rightarrow n=8+6$

$\Rightarrow\ n=14$

Now ${}^nC_2={}^{14}C_2=\dfrac{14\times13}{2!}=91$

Example – 9

If ${}^{15}C_{3r} = {}^{15}C_{r+3}$, find r.

Sol. We know that if ${}^{n}C_{x} = {}^{n}C_{y}$, then x = y or x + y = n

$${}^{15}C_{3r} = {}^{15}C_{r+3}$$

$\therefore$ Either $3r = r + 3 \Rightarrow r = \frac{3}{2}$,

which is not possible, since r is an integer.

or $3r + r + 3 = 15 \Rightarrow r = 3$.

Hence r = 3.

Example – 10

Find the third term in the expansion of $\left(2x^2 + \frac{3}{2x}\right)^8$

Sol. Let $a = 2x^2,\ b = \frac{3}{2x}, n = 8$

For third term, r = 2

$t_{r+1} = {}^{n}C_{r}\, a^{n-r}\, b^{r}$

$$= {}^{8}C_{2}\,(2x^2)^{8-2}\left(\frac{3}{2x}\right)^2$$

$$= \frac{8.7.6!}{2!6!}(2x^2)^6 \times \frac{9}{4x^2}\left[\because {}^{8}C_{2} = \frac{8!}{2!6!}\right]$$

$$= \frac{8.7}{2} \times 2^6 \times x^{12} \times \frac{9}{4x^2}$$

$$= 63 \times 64x^{10} = 4032x^{10}$$

Example – 11

Find the fifth term in the expansion of $\left(x^2 - \frac{4}{x^3}\right)^{11}$

Sol. Let, $a = x^2,\ b = \frac{-4}{x^3}, n = 11$

For fifth term, r = 4

$\therefore\ t_{r+1} = {}^{n}C_{r}\, a^{n-r}.\ b^{r}$

$$\therefore\ t_5 = {}^{11}C_{4}\,(x^2)^{11-4}\left(\frac{-4}{x^3}\right)^4$$

$$\therefore\ t_5 = \frac{11!}{4!\,7!}(x^2)^7 \times \frac{4^4}{x^{12}}$$

$$\therefore\ t_5 = \frac{11 \times 10 \times 9 \times 8 \times 7!}{4 \times 3 \times 2 \times 7!} x^{14} \times \frac{256}{x^{12}}$$

$$\therefore\ t_5 = 330 \times 256x^2 \quad \Rightarrow t_5 = 84480x^2$$

Example – 12

Given that the 4th term in the expansion of $\left(px + \frac{1}{x}\right)^n$ is $\frac{5}{2}$, find n and p.

Sol. Given expansion is $\left(px + \frac{1}{x}\right)^n$

Given, $T_4 = \frac{5}{2}$

$$\therefore\ {}^{n}C_{3}\,(px)^{n-3}\left(\frac{1}{x}\right)^3 = \frac{5}{2}$$

$$\Rightarrow {}^{n}C_{3}\, p^{n-3}\, x^{n-3}.\frac{1}{x^3} = \frac{5}{2}$$

$$\Rightarrow \frac{n!}{3!(n-3)!}.p^{n-3}\, x^{n-6} = \frac{5}{2} \quad ...(1)$$

Since R.H.S. of (1) is independent of x,

therefore n – 6 = 0 $\quad \therefore$ n = 6.

From (1), $\frac{6!}{3!\,3!}.p^3 = \frac{5}{2}$

$$\Rightarrow 20p^3 = \frac{5}{2}$$

$$\Rightarrow p^3 = \frac{1}{8} = \left(\frac{1}{2}\right)^3 \quad \therefore p = \frac{1}{2}.$$

Hence n = 6 and $p = \frac{1}{2}$.

Example – 13

Given positive integers $r > 1$, $n > 2$ and the coefficient of $(3r)^{th}$ and $(r+2)^{th}$ terms in the binomial expansion of $(1+x)^{2n}$ are equal . Then :

(a) $n = 2r$ (b) $n = 2r + 1$

(c) $n = 3r$ (d) none of these

Ans. (a)

Sol. In the expansion $(1+x)^{2n}$, $t_{3r} = {}^{2n}C_{3r-1}\,(x)^{3r-1}$

and $t_{r+2} = {}^{2n}C_{r+1}\,(x)^{r+1}$

Since, binomial coefficients of t_{3r} and t_{r+2} are equal.

$\therefore\ {}^{2n}C_{3r-1} = {}^{2n}C_{r+1}$

$\Rightarrow 3r - 1 = r + 1$ or $2n = (3r - 1) + (r + 1)$

$\Rightarrow 2r = 2$ or $2n = 4r$

$\Rightarrow r = 1$ or $n = 2r$

But $r > 1$

$\therefore$ We take, $n = 2r$

Example – 14

If the coefficients of three consecutive terms in the expansion of $(1 + a)^n$ are in the ratio 1 : 7 : 42, find n.

Sol. Let the three consecutive terms in the expansion of $(1+a)^n$ be rth, (r + 1)th and (r + 2)th terms respectively.

In the expansion of $(1 + a)^n$,

coefficient of rth term $= {}^nC_{r-1}$,

coefficient of (r + 1)th term $= {}^nC_r$.

coefficient or (r + 2)th term $= {}^nC_{r+1}$

Given, ${}^nC_{r-1} : {}^nC_r : {}^nC_{r+1} = 1 : 7 : 42$.

$$\therefore\ \frac{{}^nC_{r-1}}{{}^nC_r} = \frac{1}{7}$$

$$\Rightarrow \frac{n!}{(r-1)!(n-r+1)!}\cdot\frac{r!(n-r)!}{n!} = \frac{1}{7}$$

$$\Rightarrow \frac{r}{n-r+1} = \frac{1}{7}$$

$\Rightarrow 7r = n - r + 1$

$\Rightarrow n - 8r = -1$ .... (1)

$$\text{And } \frac{{}^nC_r}{{}^nC_{r+1}} = \frac{7}{42}$$

$$\Rightarrow \frac{n!}{r!(n-r)!}\cdot\frac{(r+1)!(n-r-1)!}{n!} = \frac{1}{6}$$

$$\Rightarrow \frac{r+1}{n-r} = \frac{1}{6}$$

$\Rightarrow 6r + 6 = n - r \quad \therefore\ n - 7r = 6$ .... (2)

Now, (2) – (1) $\Rightarrow r = 7$

From (1), n = 55.

Example – 15

If in the expansion of $(1 + x)^n$, the coefficients of 14th, 15th and 16th terms in A.P. find n.

Sol. The coefficients of 14th, 15th and 16th tems in the expansion of $(1 + x)^n$ will be ${}^nC_{13}$, ${}^nC_{14}$ and ${}^nC_{15}$ respectively.

Given, ${}^nC_{13}$, ${}^nC_{14}$ and ${}^nC_{15}$ are in A.P.

$\therefore\ {}^nC_{14} - {}^nC_{13} = {}^nC_{15} - {}^nC_{14}$

or $2\,.\,{}^nC_{14} = {}^nC_{13} + {}^nC_{15}$

$$\text{or}\quad 2.\frac{n!}{(14)!(n-14)!} = \frac{n!}{(13)!(n-13)!} + \frac{n!}{(15)!(n-15)!}$$

Multiplying both sides by 15! (n – 13)!, we get

$$2.\frac{15!(n-13)!}{14!(n-14)!} = \frac{15!(n-13)!}{13!(n-13)!} + \frac{15!(n-13)!}{15!(n-15)!}$$

or $2.15\,(n-13) = 15.14 + (n-13)\,(n-14)$

or $30n - 390 = 210 + n^2 - 27n + 182$

or $n^2 - 57n + 782 = 0$

or $(n-34)\,(n-23) = 0$

Hence n = 23 or 34.

Example – 16

The term independent of x in $\left[\sqrt{\left(\frac{x}{3}\right)}+\sqrt{\left(\frac{3}{2x^2}\right)}\right]^{10}$ is:

(a) 1 (b) $^{10}C_1$

(c) 5/12 (d) none of these

Ans. (d)

Sol. $T_{r+1} = {}^{10}C_r\left(\sqrt{\frac{x}{3}}\right)^{10-r}\left(\sqrt{\frac{3}{2x^2}}\right)^{r}$

Equating x power to zero

$$\frac{10-r}{2}-r=0$$

$$10-3r=0$$

$$\Rightarrow r=\frac{10}{3}$$

Independent of 'x' term is not possible

Example – 17

Find the constant term (term independent of x) in the expansion of

(i) $\left(2x+\frac{1}{3x^2}\right)^9$ (ii) $\left(x-\frac{2}{x^2}\right)^{15}$

Sol. Let $a=2x,\ b=\frac{1}{3x^2}, n=9$

$$t_{r+1} = {}^nC_r\, a^{n-r}\,.\, b^r$$

$$t_{r+1} = {}^9C_r(2x)^{9-r}\left(\frac{1}{3x^2}\right)^r$$

$$t_{r+1} = {}^9C_r(2)^{9-r}\left(\frac{1}{3}\right)^r x^{-2r}.x^{9-r}$$

$$t_{r+1} = {}^9C_r(2)^{9-r}\left(\frac{1}{3}\right)^r x^{9-3r}$$

To get term independent of x, must have

$x^{9-3r}=x^0$

$9-3r=0 \quad \Rightarrow -3r=-9 \quad \Rightarrow r=3$

$$\therefore\ {}^9C_3(2)^{9-3}\left(\frac{1}{3}\right)^3$$

$$\frac{9!}{3!6!}\times 2^6\times\frac{1}{27}=\frac{9\times8\times7\times6!}{3\times2\times6!}\times64\times\frac{1}{27}$$

$$=\frac{28\times64}{9}=\frac{1792}{9}$$

Constant term independent of $x=\frac{1792}{9}$

(ii) Let $a=x,\ b=\frac{-2}{x^2}, n=15$

$$t_{r+1} = {}^nC_r\, a^{n-r}\,.\, b^r$$

$$t_{r+1} = {}^{15}C_r(x)^{15-r}\left(\frac{-2}{x^2}\right)^r$$

$$t_{r+1} = {}^{15}C_r(x)^{15-r}(-2)^r x^{-2r}$$

$$t_{r+1} = {}^{15}C_r(-2)^r(x)^{15-3r}$$

To get constant term independent of x,

$x^{15-3r}=x^0$

$15-3r=0 \quad \Rightarrow -3r=-15 \quad \Rightarrow r=5$

$$\therefore\ {}^{15}C_5(-2)^5=\frac{15!}{5!10!}(-32)$$

$$=\frac{15\times14\times13\times12\times11\times10!}{5\times4\times3\times2\times10!}\times-32$$

$$=-77\times39\times32=-96096$$

Constant term independent of $x=-96096$

Example – 18

Find the middle term (s) in the expansion of

(i) $\left(\frac{x}{y}+\frac{y}{x}\right)^{12}$ (ii) $\left(x^4-\frac{1}{x^3}\right)^{11}$

Sol. **(i)** Let $a=\frac{x}{y}, b=\frac{y}{x}, n=12$

n is even.

$\therefore \left(\frac{n+2}{2}\right)=\left(\frac{12+2}{2}\right)=\frac{14}{2}=7$

7^{th} term is middle term,

$t_{r+1} = {}^nC_r a^{n-r}.\, b^r$

For 7th term, r = 6

$$t_7 = {}^{12}C_6\left(\frac{x}{y}\right)^{12-6}\left(\frac{y}{x}\right)^6$$

$$t_7 = \frac{12!}{6!6!}\times\left(\frac{x}{y}\right)^6\times\left(\frac{y}{x}\right)^6$$

$$t_7 = \frac{12\times11\times10\times9\times8\times7\times6!}{6\times5\times4\times3\times2\times6!}$$

$t_7 = 77\times12 = 924$

$\therefore$ Middle term = 924

(ii) Let $a = x^4$, $b=\frac{-1}{x^3}, n=11$

n is odd. $\left(\frac{n+1}{2}\right)=\left(\frac{11+1}{2}\right)=6,$

$\left(\frac{n+3}{2}\right)=\left(\frac{11+3}{2}\right)=7$

6th and 7th term are middle term,

$t_{r+1} = {}^nC_r a^{n-r}.\, b^r$

For t_6, r = 5

$$t_6 = {}^{11}C_5\,(x^4)^{11-5}\left(\frac{-1}{x^3}\right)^5$$

$$t_6 = \frac{11!}{5!\,6!}x^{24}\left(\frac{-1}{x^{15}}\right)$$

$$t_6 = \frac{11\times10\times9\times8\times7\times6!}{5\times4\times3\times2\times1\times6!}(-x^9)$$

$= -462\,x^9$

For t_7, r = 6

$$t_7 = {}^{11}C_6\,(x^4)^{11-6}\left(\frac{-1}{x^3}\right)^6$$

$$t_7 = \frac{11!}{6!5!}x^{20}\times\frac{1}{x^{18}}$$

$$t_7 = \frac{11\times10\times9\times8\times7\times6!}{6!\times5\times4\times3\times2}x^2$$

$t_7 = 11\times3\times2\times7 = 462x^2$

Example – 19

The middle term in expansion of $\left(x^2+\frac{1}{x^2}+2\right)^n$ is :

(a) $\frac{n!}{[(n/2)!]^2}$ (b) $\frac{2n!}{[(n/2)!]^2}$

(c) $\frac{1.3.5........(2n+1)}{n!}2^n$ (d) $\frac{(2n)!}{(n!)^2}$

Ans. (d)

Sol. Middle term in expansion of

$$\left(x^2+\frac{1}{x^2}+2\right)^n=\left(x+\frac{1}{x}\right)^{2n}$$

So, (n + 1)th term

$$\Rightarrow {}^{2n}C_n=\frac{(2n)!}{n!n!}$$

Example – 20

Find the middle terms(s) in the expansion of $\left(x^2+\frac{1}{x}\right)^7$

Sol. Let $a = x^2$, $b = \frac{1}{x}$, $n = 7$

n is odd.

$$\left(\frac{n+1}{2}\right)=\left(\frac{7+1}{2}\right)=4 \text{ and } \left(\frac{n+3}{2}\right)=\left(\frac{7+3}{2}\right)=5$$

4th and 5th terms are middle terms,

for t_4, $r = 3$

$t_{r+1} = {}^nC_r a^{n-r} . b^r$

$$t_4 = {}^7C_3 (x^2)^{7-3}\left(\frac{1}{x}\right)^3$$

$$t_4 = \frac{7!}{4!3!}\times(x^2)^4\times\frac{1}{x^3}$$

$$t_4 = \frac{7\times6\times5\times4!}{4!\times3\times2}\times x^8\times\frac{1}{x^3}$$

$t_4 = 35x^5$

For t_5, $r = 4$

$$t_5 = {}^7C_4\left(x^2\right)^{7-4}\left(\frac{1}{x}\right)^4$$

$$t_5 = \frac{7!}{4!3!}\times x^6\times\frac{1}{x^4}$$

$$t_5 = \frac{7\times6\times5\times4!}{4!\times3\times2}\times x^2 = 35x^2$$

Example – 21

Find the coefficient of x^9 in $(1 + 3x + 3x^2 + x^3)^{15}$.

Sol. $(1 + 3x + 3x^2 + x^3)^{15} = [(1 + x)^3]^{15} = (1 + x)^{45}$

$\therefore$ Coefficient of x^9 in $(1 + 3x + 3x^2 + x^3)^{15}$

$=$ coefficient of x^9 in $(1 + x)^{45}$

$= {}^{45}C_9$ [Since in the expansion of $(1 + x)^n$, coefficient of $x^r = {}^nC_r$]

$$= \frac{45!}{9!36!}$$

Example – 22

Find the value of the expression ${}^{47}C_4 + \sum_{j=1}^{5} {}^{52-j}C_3$.

Sol. Given expression $= {}^{47}C_4 + \sum_{j=1}^{5} {}^{52-j}C_3$.

$= {}^{47}C_4 + ({}^{51}C_3 + {}^{50}C_3 + {}^{49}C_3 + {}^{48}C_3 + {}^{47}C_3)$

$= {}^{47}C_4 + ({}^{47}C_3 + {}^{48}C_3 + {}^{49}C_3 + {}^{50}C_3 + {}^{51}C_3)$

$= ({}^{47}C_4 + {}^{47}C_3) + ({}^{48}C_3 + {}^{49}C_3 + {}^{50}C_3 + {}^{51}C_3)$

$= {}^{48}C_4 + {}^{48}C_3 + {}^{49}C_3 + {}^{50}C_3 + {}^{51}C_3$ $[\because {}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r]$

$= ({}^{48}C_4 + {}^{48}C_3) + ({}^{49}C_3 + {}^{50}C_3 + {}^{51}C_3)$

$= ({}^{49}C_4 + {}^{49}C_3) + ({}^{50}C_3 + {}^{51}C_3)$

$= ({}^{50}C_4 + {}^{50}C_3) + {}^{51}C_3 = {}^{51}C_4 + {}^{51}C_3 = {}^{52}C_4$.

Example – 23

The sum of coefficient in $(1 + x – 3x^2)^{2134}$ is

(a) –1 (b) 1

(c) 0 (d) 2^{2134}

Ans. (b)

Sol. Sum of co-efficients can be obtained by substituting x = 1

$\therefore$ Sum of co-efficients $= (1 + 1 – 3)^{2134} = (–1)^{2134} = 1$

Example – 24

The sum of the coefficients in the expansion of $(6a – 5b)^n$, where n is a positive integer, is

(a) 1 (b) –1

(c) 2^n (d) $2^{n–1}$

Ans. (a)

Sol. Sum of coefficients we get when a = b = 1

$(6 – 5)^n = 1$

Example – 25

Prove that $\sum_{r=0}^{n} 3^r \cdot {}^nC_r = 4^n$.

Sol. $(1+x)^n = {}^nC_0 x^0 + {}^nC_1 x + {}^nC_2 x^2 + ... + {}^nC_n x^n$

$\therefore \quad {}^nC_0 x^0 + {}^nC_1 x + {}^nC_2 x^2 + ... + {}^nC_n x^n = (1+x)^n \qquad ....(1)$

Now $\sum_{r=0}^{n} 3^r \cdot {}^nC_r$

$= {}^nC_0 3^0 + {}^nC_1 3^1 + {}^nC_2 3^2 + ... + {}^nC_n 3^n$

$= (1+3)^n$

$= 4^n$.

Example – 26

Numerically greatest term, in the expansion of $(8 - 5x)^{18}$, (where $x = 2/5$) is :

(a) 1623×2^{24} (b) 1623×2^{22}

(c) 1623×2^{23} (d) none of these

Ans. (d)

Sol. $(8-5x)^{18} = 8^{18}\left(1-\frac{5x}{8}\right)^{18}$, $a = 1, b = -\frac{5}{8}\times\frac{2}{5} = -\frac{1}{4}$

$$\frac{(n+1)|b|}{|b|+|a|} = \frac{19\times\frac{1}{4}}{\frac{5}{4}} = 3.8$$

T_4 is greatest term

$T_4 = {}^{18}C_3\, 8^{15}\, (-2)^3$, so it is negative

Example – 27

Show that $2^{4n} - 2^n(7n+1)$ is some multiple of the square of 14, where n is a postive integer.

Sol. $2^{4n} - 2^n(7n+1) = (16)^n - 2^n(7n+1)$

$= (2+14)^n - 2^n \cdot 7n - 2^n$

$= (2^n + {}^nC_1 2^{n-1} \cdot 14 + {}^nC_2 2^{n-2} \cdot 14^2 + ... + 14^n) - 2^n \cdot 7n - 2^n$

$= 14^2 ({}^nC_2 2^{n-2} + {}^nC_3 2^{n-3} 14 + ... + 14^{n-2})$

$+ (2^n + {}^nC_1 \cdot 2^{n-1} \cdot 14 - 2^n \cdot 7n - 2^n)$

$= 14^2 ({}^nC_2 2^{n-2} + {}^nC_3 2^{n-3} \cdot 14 + ... + 14^{n-2})$

$+ (2^n + n2^{n-1} \cdot 2 \cdot 7 - 2^n \cdot 7n - 2^n)$

$= 14^2 ({}^nC_2 \cdot 2^{n-2} + {}^nC_3 \cdot 2^{n-3} \cdot 14 + ... + 14^{n-2})$

This is divisible by 14^2 i.e. by 196 for all positive integral value of n.

Note : If $n = 1$, ${}^nC_2 = 0$, ${}^nC_3 = 0$ etc.

$\therefore$ Given expression $= 14^2 \times 0 = 0$, which is divisible by 196.

Example – 28

Using binomial theorem, prove that $6^n - 5n$ always leaves the remainder 1 when divided by 25 for all positive integers n.

Sol. $6^n - 5n = (1+5)^n - 5n$

$= (1 + {}^nC_1 5 + {}^nC_2 5^2 + ... + {}^nC_n 5^n) - 5n$

$= (1 + n \cdot 5 + {}^nC_2 \cdot 5^2 + ... + {}^nC_n 5^n) - 5n$

$= 1 + {}^nC_2 5^2 + {}^nC_3 5^3 + ... + {}^nC_n 5^n$

$= 1 + 25({}^nC_2 + {}^nC_3 \cdot 5 + ... + {}^nC_n 5^{n-2})$

$= 1 + 25.k$ where k is a positive integer.

$\therefore$ When $6^n - 5n$ is divided by 25, remainder will be 1 for all positive integer n.

Example – 29

Which number is larger, $(1.2)^{4000}$ or 800 ?

Sol. $(1.2)^{4000} = (1 + 0.2)^{4000}$

$= {}^{4000}C_0 + {}^{4000}C_1 (0.2)$ + sum of positive terms

$= 1 + 4000(0.2)$ + a positive number

$= 1 + 800$ + a positive number

> 800

Hence $(1.2)^{4000} > 800$.

Example – 30

The coefficient of x^n in expansion of $(1+x)(1-x)^n$ is

(a) $(n-1)$ (b) $(-1)^n(1-n)$

(c) $(-1)^{n-1}(n-1)^2$ (d) $(-1)^{n-1} n$

Ans. (b)

Sol. $(1+x)(1-x)^n = (1-x)^n + x(1-x)^n$

$\therefore$ Coefficient of x^n is $= (-1)^n + (-1)^{n-1}\, {}^nC_1$

$= (-1)^n [1 - n]$

Example – 31

The coefficient of x^7 in the expansion of $(1 - x - x^2 + x^3)^6$ is

(a) –132 (b) –144

(c) 132 (d) 144

Ans. (b)

Sol. $(1 - x - x^2 + x^3)^6 = ((1 - x)(1 - x^2))^6 = (1 - x)^6 (1 - x^2)^6$

$= (1 - {}^6C_1 x + {}^6C_2 x^2 - {}^6C_3 x^3 + {}^6C_4 x^4 - {}^6C_5 x^5 + {}^6C_6 x^6)$

$(1 - {}^6C_1 x^2 + {}^6C_2 x^4 - {}^6C_3 x^6 + {}^6C_4 x^8 - {}^6C_5 x^{10} + {}^6C_6 x^{12})$

Coeff. of $x^7 = (-{}^6C_1)(-{}^6C_3) + (-{}^6C_3)({}^6C_2) + (-{}^6C_5)(-{}^6C_1)$

$= 6.20 - 20 . 15 + 6 . 6 = 120 - 300 + 36 = -144$

Example – 32

If the coefficients of x^3 and x^4 in the expansion of $(1 + ax + bx^2)(1 - 2x)^{18}$ in powers of x are both zero, then (a, b) is equal to :

(a) $\left(16, \frac{272}{3}\right)$ (b) $\left(16, \frac{251}{3}\right)$

(c) $\left(14, \frac{251}{3}\right)$ (d) $\left(14, \frac{272}{3}\right)$

Ans. (a)

Sol. On expanding the given expression we get,

$\Rightarrow (1 + ax + bx^2)(1 - {}^{18}C_1 (2x) + {}^{18}C_2 (2x)^2 - {}^{18}C_3 (2x)^3 + ... + {}^{18}C_{18} (2x)^{18})$

Coefficient of x^3,

$\Rightarrow -{}^{18}C_1 (2b) + {}^{18}C_2 (4a) - 8.\ {}^{18}C_3 = 0$

$\Rightarrow 51a = 3b + 544$ ..(1)

Similarly coefficient of x^4,

$\Rightarrow {}^{18}C_4 (2)^4 - {}^{18}C_3 . 8a + {}^{18}C_2 4b = 0$

$\Rightarrow 32a = 3b + 240$ ..(2)

On solving (1) and (2) we get,

$\Rightarrow a = 16$ and $b = \frac{272}{3}$

Example – 33

Find the term independent of x in $(1 + x)^m \left(1 + \frac{1}{x}\right)^n$

Sol. Given expression $= (1+x)^m \left(1+\frac{1}{x}\right)^n = (1+x)^m \left(\frac{x+1}{x}\right)^n$

$= \frac{(1+x)^{m+n}}{x^n} = x^{-n}(1+x)^{m+n}$

$\therefore$ Required term independent of x

$=$ coefficient of x^0 in $x^{-n}(1 + x)^{m+n}$

$=$ coefficient of x^n in $(1 + x)^{m+n}$

$= {}^{m+n}C_n = \frac{(m+n)!}{n!m!}$

[Since in the expansion of $(1 + x)^n$, co-efficient of $x^r = {}^nC_r$]

Example – 34

Find the coefficient of x^5 in the expansion of the product $(1 + 2x)^6 (1 - x)^7$.

Sol. $(1 + 2x)^6 = [1 + {}^6C_1 (2x) + {}^6C_2 (2x)^2 + {}^6C_3 (2x)^3 + {}^6C_4 (2x)^4 + {}^6C_5 (2x)^5 + {}^6C_6 (2x)^6]$ .....(1)

Again, $(1 - x)^7 = 1 - {}^7C_1 x + {}^7C_2 x^2 - {}^7C_3 x^3 + {}^7C_4 x^4 - {}^7C_5 x^5 + {}^7C_6 x^6 - {}^7C_7 x^7$

$= 1 - 7x + 21x^2 - 35x^3 + 35x^4 - 21x^5 + 7x^6 - x^7$ .....(2)

Now $(1 + 2x)^6 (1 - x)^7$

$= (1 + 12x + 60x^2 + 160x^3 + 240x^4 + 192x^5 + ...)$
$\times (1 - 7x + 21x^2 - 35x^3 + 35x^4 - 21x^5 + ...)$

$\therefore$ Required coefficient of x^5 in the product

$= 1 \times (-21) + 12 \times 35 + 60 \times (-35) + 160 \times 21 + 240 \times (-7) + 192 \times 1$

$= -21 + 420 - 2100 + 3360 - 1680 + 192 = 171$

Example – 35

Simplify first three terms in the expansion of the following

(i) $(1+2x)^{-4}$ (ii) $(5+4x)^{-1/2}$

Sol. **(i)** $(1+2x)^{-4}=$

$$1+(-4)(2x)+\frac{(-4)(-4-1)}{2!}(2x)^2+.................$$

$$=1-8x+\frac{(-4)(-5)}{2}(4x^2)+.................$$

$=1–8x+40x^2+.....$

(ii) $$(5+4x)^{\frac{-1}{2}}=5^{\frac{-1}{2}}\left[1+\frac{4x}{5}\right]^{\frac{-1}{2}}$$

$$=5^{\frac{-1}{2}}\left[1+\left(\frac{-1}{2}\right)\left(\frac{4x}{5}\right)+\frac{\left(\frac{-1}{2}\right)\left(\frac{-1}{2}-1\right)}{2!}\left(\frac{4x}{5}\right)^2+...\right]$$

$$=5^{\frac{-1}{2}}\left[1-\frac{2x}{5}+\frac{\left(\frac{-1}{2}\right)\left(\frac{-3}{2}\right)}{2}\times\frac{16x^2}{25}+...\right]$$

$$=5^{\frac{-1}{2}}\left[1-\frac{2x}{5}+\frac{6x^2}{25}+....\right]$$

Example – 36

Find the coefficient of x^4 in the expansion of $\frac{1+x}{1-x}$ if $|x|<1$

Sol. $$\frac{1+x}{1-x}=(1+x)(1-x)^{-1}$$

$$=(1+x)\left[1+\frac{(-1)}{1!}(-x)\frac{(-1)(-1-1)}{2!}(-x)^2\right.$$

$$\left.+\frac{(-1)(-1-1)(-1-2)}{3!}(-x)^3.....\text{to}\ \infty\right]$$

$=(1+x)(1+x+x^2+x^3+x^4+......\text{ to }\infty)$

$=[1+x+x^2+x^3+x^4+.......\text{ to }\infty]+[x+x^2+x^3+x^4+.........\text{ to }\infty]$

$=1+2x+2x^2+2x^3+2x^4+2x^5+......\text{ to }\infty$

Hence coefficient of $x^4=2$

Example – 37

Find the square root of 99 correct to 4 places of deicmal.

Sol. $$(99)^{1/2}=(100-1)^{1/2}=\left[100\left(1-\frac{1}{100}\right)\right]^{\frac{1}{2}}$$

$$=\left[100\left(1-\frac{1}{100}\right)\right]^{\frac{1}{2}}$$

$=(100)^{1/2}[1–0.01]^{1/2}$

$$=10\left[1+\frac{\frac{1}{2}}{1!}(-0.01)+\frac{\frac{1}{2}\left(\frac{1}{2}-1\right)}{2!}(-0.01)^2+........to\ \infty\right]$$

$=10\,[1–0.005-0.0000125+.........\text{ to }\infty]$

$=10\,(.9949875)=9.94987=9.9499$

Example – 38

The number of terms in the expansion of $(2x+3y-4z)^n$, is

(a) $n+1$ (b) $n+3$

(c) $\frac{(n+1)(n+2)}{2}$ (d) none of these

Ans. (c)

Sol. Number of terms $= {}^{n+r-1}C_{r-1}$

$= {}^{n+3-1}C_{3-1}$

$$=\frac{(n+2)(n+1)}{2}$$

Example – 39

The coefficient of $(a^3 b^6 c^8 d^9 e f)$ in the expansion of $(a + b + c - d - e - f)^{31}$ is :

(a) 123210 (b) 23110

(c) 3110 (d) none of these

Ans. (d)

Sol. The coefficient of $a^3 b^6 c^8 d^9 e f$ in expansion of

$(a + b + c - d - e - f)^{31}$ is zero as that term is not possible in expansion as sum of powers in not 31.

Example – 40

Find the total number of terms in the expansion of $(1 + a + b)^{10}$ and coefficient of a^2b^3.

Sol. Total number of terms $= {}^{10+3-1}C_{3-1} = {}^{12}C_2 = 66$

Coefficient of $a^2b^3 = \dfrac{10!}{2!\times 3!\times 5!} = 2520$

EXERCISE - 1 : BASIC OBJECTIVE QUESTIONS

Binomial theorem for positive integral index

1. If $\frac{1}{\underline{|8}}+\frac{1}{\underline{|9}}=\frac{x}{\underline{|10}}$, then x is equal to

(a) 100 (b) 90

(c) 170 (d) none of these

2. The expansion $(x+a)^n={}^nC_0x^n+{}^nC_1x^{n-1}a^1+.....+{}^nC_na^n$ is valid when n is

(a) an integer (b) a natural number

(c) a rational number (d) none of these

3. $(x+\sqrt{x^3-1})^5+(x-\sqrt{x^3-1})^5$ is a polynomial of degree

(a) 5 (b) 6

(c) 7 (d) 8

4. $(1.003)^4$ is nearly equal to

(a) 1.012 (b) 1.0012

(c) 0.988 (d) 1.003

5. The number of non-zero terms in the expansion of

$$\left[\left(1+3\sqrt{2}\,x\right)^9+\left(1-3\sqrt{2}\,x\right)^9\right] \text{ is}$$

(a) 9 (b) 10

(c) 5 (d) None of these

General term of binomial expansion

6. The term void of x in the expanion of $\left(x-\frac{3}{x^2}\right)^{18}$ is

(a) ${}^{18}C_6$ (b) ${}^{18}C_6 3^6$

(c) ${}^{18}C_{12}$ (d) ${}^{18}C_6 3^{12}$

7. If n $\in$ N and the coefficients of x^{-7} and x^{-8} to the expansion of $\left(2+\frac{1}{3x}\right)^n$ are equal then n =

(a) 56 (b) 15

(c) 45 (d) 55

8. If n,p $\in$ N and in the expansion of $(1+x)^n$ the coefficient of p^{th} and $(p+1)^{th}$ terms are respectively p and q. The p + q =

(a) ${}^{n+3}C_p$ (b) ${}^{n+1}C_1$

(c) ${}^{n+2}C_1$ (d) ${}^nC_p$

9. 5th term from the end in the expansion of $\left(\frac{x^3}{2}-\frac{2}{x^2}\right)^{12}$ is

(a) $-7920\,x^{-4}$ (b) $7920\,x^4$

(c) $7920\,x^{-4}$ (d) $-7920\,x^4$

10. If the coefficients of (r + 4)th term and (2r + 1)th term in the expansion of $(1+x)^{18}$ are equal, then r =

(a) 3 (b) 5

(c) 3 or 5 (d) none of these

11. The term independent of x in the expansion of $\left(2x-\frac{3}{x^2}\right)^9$ is

(a) $3^3 . {}^9C_3$ (b) $2^6 . 3^3\,{}^9C_3$

(c) $-3^3 . {}^9C_3$ (d) $-2^6 . 3^3 . {}^9C_3$

12. In the expansion of $\left(\frac{1}{2}x^{\frac{1}{3}}+x^{-\frac{1}{5}}\right)^8$, the term independent of x is

(a) T_5 (b) T_7

(c) T_6 (d) T_8

13. The term independent of x in the expansion of

$$\left[\left(t^{-1}-1\right)x+\left(t^{-1}+1\right)^{-1}x^{-1}\right]^8 \text{ is :}$$

(a) $56\left(\frac{1-t}{1+t}\right)^3$ (b) $56\left(\frac{1+t}{1-t}\right)^3$

(c) $70\left(\frac{1-t}{1+t}\right)^4$ (d) $70\left(\frac{1+t}{1-t}\right)^4$

14. The greatest value of the term independent of x, as α varies over R, in the expansion of $\left(x\cos\alpha+\frac{\sin\alpha}{x}\right)^{20}$ is :

(a) ${}^{20}C_{10}$ (b) ${}^{20}C_{19}$

(c) ${}^{20}C_6$ (d) ${}^{20}C_{10}\left(\frac{1}{2}\right)^{10}$

15. The coefficient of x^8y^{10} in the expansion of $(x+y)^{18}$ is

(a) ${}^{18}C_8$ (b) ${}^{18}P_{10}$

(c) 2^{18} (d) None of these

16. If the term independent of x in the expansion of $\left(\sqrt{x}-\frac{\lambda}{x^2}\right)^{10}$ is 405, then λ equals

(a) –3 (b) 3

(c) 3 or –3 (d) None of these

17. If $(1+ax)^m = 1+8x+24x^2+...$, then the value of a and m are respectively.

(a) 4,2 (b) 2,4

(c) 1,8 (d) None of these

Application of binomial theorem

18. If $n \in N$ then $49^n + 16n - 1$ is divisible by

(a) 3 (b) 19

(c) 64 (d) 29

19. Remainder when 7^{100} is divided by 25 is

(a) 1 (b) 24

(c) 18 (d) none of these

20. The co-efficient of x^4 in the expansion of $(1+x+x^2+x^3)^n$ is

(a) ${}^nC_4$ (b) ${}^nC_4 + {}^nC_2$

(c) ${}^nC_4 + {}^nC_1 + {}^nC_4 . {}^nC_2$ (d) ${}^nC_4 + {}^nC_2 + {}^nC_1 . {}^nC_2$

21. Co-efficient of x^5 in the expansion of $(x^2-x-2)^5$ is :

(a) –83 (b) –82

(c) –81 (d) 0

22. The number of integral terms in the expansion of $\left(\sqrt{3}+\sqrt{2}\right)^{500}$ is :

(a) 128 (b) 129

(c) 251 (d) 512

23. The coefficient of x^{99} in $(x+1)(x+3)(x+5)\ .....\ (x+199)$ is

(a) $1+2+3+....+99$ (b) $1+3+5+....+199$

(c) 1.3.5. .... 199 (d) None of these

24. The coefficient of x^{17} in the expansion of $(x-1)(x-2)\ ....\ (x-18)$ is

(a) 342 (b) –171

(c) $\frac{171}{2}$ (d) 684

25. The coefficient of x^{50} in the binomial expansion of $(1+x)^{1000} + x(1+x)^{999} + x^2(1+x)^{998} + ... + x^{1000}$ is:

(a) $\frac{(1000)!}{(50)!(950)!}$ (b) $\frac{(1000)!}{(49)!(951)!}$

(c) $\frac{(1001)!}{(51)!(950)!}$ (d) $\frac{(1001)!}{(50)!(951)!}$

Binomial coefficient problems

26. If ${}^nC_3 = {}^nC_2$, then n is equal to

(a) 2 (b) 3

(c) 5 (d) none of these

27. If ${}^{n+1}C_4 = 9\ {}^nC_2$, then n =

(a) 10 (b) 9

(c) 12 (d) 11

28. If $n \in N$ and $(1+x)^n = 1 + a_1x + a_2x^2 + ... + a_nx^n$. If a_1, a_2 and a_3 are in A.P., then the value of *n* is

(a) 4 (b) 5

(c) 6 (d) 7

29. The sum ${}^rC_r + {}^{r+1}C_r + {}^{r+2}C_r + ... + {}^nC_r\ (n \geq r)$ equals

(a) ${}^nC_{r+1}$ (b) ${}^{n+1}C_{r+1}$

(c) ${}^{n+1}C_{r-1}$ (d) ${}^{n+1}C_r$

30. The sum of coefficient in the expansion of $(1+x-3x^2)^{3148}$ is

(a) 8 (b) 7

(c) 1 (d) –1

31. If the sum of the coefficients in the expansion of $(a^2x^2 - 6ax + 11)^{10}$, where a is constant, is 1024, then the value of a is :

(a) 5 (b) 1

(c) 2 (d) 3

32. If $n \in N$ and $(1-x+x^2)^n = a_0 + a_1x + a_2x^2 + ... + a_{2n}x^{2n}$, then $a_0 + a_2 + a_4 + ... + a_{2n}$ is equal to :

(a) $\frac{3^n+1}{2}$ (b) $\frac{3^n-1}{2}$

(c) $3^n - \frac{1}{2}$ (d) $3^n + \frac{1}{2}$

33. The sum of the numerical coefficients in the expansion of $\left(1+\frac{x}{3}+\frac{2y}{3}\right)^{12}$, is

(a) 1 (b) 2

(c) 2^{12} (d) none of these

34. In the expansion of $(1 + x)^{50}$, the sum of the coefficient of odd powers of x is

(a) 0 (b) 2^{49}

(c) 2^{50} (d) 2^{51}

35. Sum of the last 30 coefficients in the expansion of $(1 + x)^{59}$, when expanded in ascending powers of x is

(a) 2^{59} (b) 2^{58}

(c) 2^{30} (d) 2^{29}

36. ${}^{10}C_1+{}^{10}C_2+{}^{10}C_3+............+{}^{10}C_{10}=$

(a) 512 (b) 511

(c) 1024 (d) none of these

37. The value of ${}^nC_0-{}^nC_1+{}^nC_2-....+(-1)^n\,{}^nC_n$ is

(a) 1 (b) n

(c) 2^n (d) 0

38. $\sum_{r=0}^{n} {}^{2n+1}C_{2r+1}$ is equal to

(a) $2^{2n}-1$ (b) 2^{2n}

(c) $2^{2n+1}-1$ (d) 2^{2n+1}

39. Let $S_1=\sum_{j=1}^{10} j(j-1)\,{}^{10}C_j$, $S_2=\sum_{j=1}^{10} j\,{}^{10}C_j$ and

$S_3=\sum_{j=1}^{10} j^2\,{}^{10}C_j$

Statement I : $S_3=55\times2^9$.

Statement II : $S_1=90\times2^8$ and $S_2=10\times2^8$.

(a) Statement I is false, Statement II is true

(b) Statement I is true, Statement II is true; Statement II is a correct explanation for Statement I

(c) Statement I is true, Statement II is true; Statement II is not a correct explanation for Statement I

(d) Statement I is true, Statement II is false

40. **Statement I :** $\sum_{r=0}^{n}(r+1)\,.\,{}^nC_r=(n+2)\,2^{n-1}$

Statement II : $\sum_{r=0}^{n}(r+1)\,{}^nC_r.x^r=(1+x)^n+nx\,(1+x)^{n-1}$

(a) Statement I is false, Statement II is true

(b) Statement I is true, Statement II is true; Statement II is a correct explanation for Statement I

(c) Statement I is true, Statement II is true; Statement II is not a correct explanation for Statement I

(d) Statement I is true, Statement II is false

41. a, b, c, d are any four consecutive co-efficients of any binomial expansion, then $\frac{a+b}{a}, \frac{b+c}{b}, \frac{c+d}{c}$ are in :

(a) A.P.

(b) G.P.

(c) H.P.

(d) arithmetico geometric progression

42. The value of

$\left({}^7C_0+{}^7C_1\right)+\left({}^7C_1+{}^7C_2\right)+...+\left({}^7C_6+{}^7C_7\right)$ is

(a) 2^7-1 (b) 2^8-2

(c) 2^8-1 (d) 2^8

Numerical Value Type Questions

43. If $\lfloor n+2 = 210 \lfloor n-1$, then the value of n is equal to

44. The total number of terms in the expansion of $(x+a)^{100}+(x-a)^{100}$ after simplification is

45. If ${}^{2n}C_3 : {}^nC_2 :: 44 : 1$, then the value of n is

46. If ${}^{32}C_{2n-1}={}^{32}C_{n-3}$, then n =

47. The coefficient of x^3 in $\left(\sqrt{x^5}+\frac{3}{\sqrt{x^3}}\right)^6$ is :

48. If the last term in the binomial expansion of $\left(2^{1/3}-\frac{1}{\sqrt{2}}\right)^n$ is $\left(\frac{1}{3^{5/3}}\right)^{\log_3 8}$, then the 5th term form the beginning is :

49. If rth term in the expansion of $\left(x^2+\frac{1}{x}\right)^{12}$ is independent of x, then r =

50. The middle term in the expansion of $\left(\frac{2x^2}{3}+\frac{3}{2x^2}\right)^{10}$ is

51. If T_2/T_3 in the expansion of $(a + b)^n$ and T_3/T_4 in the $(a + b)^{n+3}$ are equal, then n =

52. If $n \in N$ and second, third and fourth terms in the expansion of $(x + a)^n$ are 240, 720 and 1080 respectively, then the value of n is

53. If the sum of binomial coefficients in the expansion $\left(2x+\frac{1}{x}\right)^n$ is 256, then term independent of x is

54. Coefficient of x^5 in the expansion of $(1 + x^2)^5 (1 + x)^4$ is

55. The number of irrational terms in the expansion of $\left(4^{1/5}+7^{1/10}\right)^{45}$ is

EXERCISE - 2 : PREVIOUS YEAR JEE MAIN QUESTIONS

1. If the number of terms in the expansion of $\left(1-\frac{2}{x}+\frac{4}{y}\right)^n$, $x \neq 0$, is 28, then the sum of the coefficients of all the terms in this expansion, is : **(2016)**

(a) 2187 (b) 243

(c) 729 (d) 64

2. For $x \in R$, $x \neq -1$, if

$(1+x)^{2016}+x(1+x)^{2015}+x^2(1+x)^{2014}+......+x^{2016}=\sum_{i=0}^{2016} a_i x^i$,

then a_{17} is equal to : **(2016/Online Set–1)**

(a) $\frac{2017!}{17!\,2000!}$ (b) $\frac{2016!}{17!\,1999!}$

(c) $\frac{2017!}{2000!}$ (d) $\frac{2016!}{16!}$

3. If the coefficients of x^{-2} and x^{-4} in the expansion of $\left(x^{\frac{1}{3}}+\frac{1}{2x^{\frac{1}{3}}}\right)^{18}$, $(x>0)$, are m and n respectively, then $\frac{m}{n}$ is equal to : **(2016/Online Set–2)**

(a) 182 (b) $\frac{4}{5}$

(c) $\frac{5}{4}$ (d) 27

4. The value of $\left({}^{21}C_1-{}^{10}C_1\right)+\left({}^{21}C_2-{}^{10}C_2\right)+\left({}^{21}C_3-{}^{10}C_3\right)+\left({}^{21}C_4-{}^{10}C_4\right)+.....+\left({}^{21}C_{10}-{}^{10}C_{10}\right)$ is: **(2017)**

(a) $2^{21}-2^{11}$ (b) $2^{21}-2^{10}$

(c) $2^{20}-2^{9}$ (d) $2^{20}-2^{10}$

5. The coefficient of x^{-5} in the binomial expansion of $\left(\frac{x+1}{x^{\frac{2}{3}}-x^{\frac{1}{3}}+1}-\frac{x-1}{x-x^{\frac{1}{2}}}\right)^{10}$, where $x \neq 0, 1$, is : **(2017/Online Set–2)**

(a) 1 (b) 4

(c) –4 (d) –1

6. The sum of the co-efficients of all odd degree terms in the expansion of

$\left(x+\sqrt{x^3-1}\right)^5+\left(x-\sqrt{x^3-1}\right)^5, (x>1)$ is : **(2018)**

(a) 2 (b) -1

(c) 0 (d) 1

7. If n is the degree of the polynomial

$\left[\frac{2}{\sqrt{5x^3+1}-\sqrt{5x^3-1}}\right]^8+\left[\frac{2}{\sqrt{5x^3+1}+\sqrt{5x^3-1}}\right]^8$ and m is the coefficient of x^n in it, then the ordered pair (n, m) is equal to : **(2018/Online Set–1)**

(a) $(24, (10)^8)$ (b) $(8, 5(10)^4)$

(c) $(12, (20)^4)$ (d) $(12, 8(10)^4)$

8. The coefficient of x^{10} in the expansion of $(1+x)^2(1+x^2)^3(1+x^3)^4$ is equal to : **(2018/Online Set–2)**

(a) 52 (b) 56

(c) 50 (d) 44

9. The coefficient of x^2 in the expansion of the product $(2-x^2).((1+2x+3x^2)^6+(1-4x^2)^6)$ is : **(2018/Online Set–3)**

(a) 107 (b) 106

(c) 108 (d) 155

10. The sum of the series

$2\cdot{}^{20}C_0+5\cdot{}^{20}C_1+8\cdot{}^{20}C_2+11\cdot{}^{20}C_3+...+62\cdot{}^{20}C_{20}$

is equal to: **(8-04-2019/Shift-1)**

(a) 2^{26} (b) 2^{25}

(c) 2^{23} (d) 2^{24}

11. The sum of the co-efficient of all even degree terms in x in the expansion of

$\left(x+\sqrt{x^3-1}\right)^6+\left(x-\sqrt{x^3-1}\right)^6, (x>1)$ is equal to :

(8-04-2019/Shift-1)

12. If the fourth term in the binomial expansion of

$\left(\sqrt{x^{\frac{1}{1+\log_{10}x}}}+x^{\frac{1}{12}}\right)^6$ is equal to 200, and $x>1$, then the value of x is: **(8-04-2019/Shift-2)**

(a) 100 (b) 10

(c) 10^3 (d) 10^4

13. If the fourth term in the Binomial expansion of

$\left(\frac{2}{x}+x^{\log_8 x}\right)^6$ $(x>0)$ is 20×8^7, then a value of x is :

(9-04-2019/Shift-1)

(a) 8^3 (b) 8^{-2}

(c) 8 (d) 8^2

14. If some three consecutive coefficients in the binomial expansion of $(x+1)^n$ in powers of x are in the ratio 2:15:70, then the average of these three coefficients is:

(9-04-2019/Shift-2)

(a) 964 (b) 232

(c) 227 (d) 625

15. If the coefficients of x^2 and x^3 are both zero, in the expansion of the expression $\left(1+ax+bx^2\right)(1-3x)^{15}$ in powers of x, then the ordered pair (a, b) is equal to :

(10-04-2019/Shift-1)

(a) (28, 861) (b) (–54, 315)

(c) (28, 315) (d) (–21, 714)

16. The smallest natural number n, such that the coefficient of x in the expansion of $\left(x^2+\frac{1}{x^3}\right)^n$ is ${}^nC_{23}$, is:

(10-4-2019/Shift-2)

(a) 38 (b) 58

(c) 23 (d) 35

17. The coefficient of x^{18} in the product $(1+x)(1-x)^{10}(1+x+x^2)^9$ is **(12-04-2019/Shift-1)**

18. If ${}^{20}C_1+\left(2^2\right){}^{20}C_2+\left(3^2\right){}^{20}C_3+...+\left(20^2\right){}^{20}C_{20}=A\left(2^\beta\right)$, then the ordered pair (A, β) is equal to _____.

(12-04-2019/Shift-2)

(a) (420, 19) (b) (420, 18)

(c) (380, 18) (d) (380, 19)

19. The term independent of x in the expansion of

$\left(\frac{1}{60}-\frac{x^8}{81}\right).\left(2x^2-\frac{3}{x^2}\right)^6$ is equal to ______.

(12-04-2019/Shift-2)

(a) -72 (b) 36

(c) -36 (d) -108

20. If the fractional part of the number $\frac{2^{403}}{15}$ is $\frac{k}{15}$, then k is equal to: **(9-01-2019/Shift-1)**

21. The coefficient of t^4 in the expansion of $\left(\frac{1-t^6}{1-t}\right)^3$

(9-01-2019/Shift-2)

(a) 14 (b) 15

(c) 10 (d) 12

22. If the third term in the binomial expansion of $\left(1+x^{\log_2 x}\right)^5$ equals 2560, then a possible value of x is:

(10-1-2019/Shift-1)

(a) $\frac{1}{4}$ (b) $4\sqrt{2}$

(c) $\frac{1}{8}$ (d) $2\sqrt{2}$

23. If $\sum_{i=1}^{20}\left(\frac{{}^{20}C_{i-1}}{{}^{20}C_i+{}^{20}C_{i-1}}\right)^3=\frac{k}{21}$, then k equals:

(10-1-2019/Shift-1)

24. The positive value of λ for which the co-efficient of x^2 in the expression $x^2\left(\sqrt{x}+\frac{\lambda}{x^2}\right)^{10}$ is 720, is:

(10-01-2019/Shift-2)

(a) 4 (b) $2\sqrt{2}$

(c) $\sqrt{5}$ (d) 3

25. If $\sum_{r=0}^{25}\left\{{}^{50}C_r\cdot{}^{50-r}C_{25-r}\right\}=K\left({}^{50}C_{25}\right)$, then K is equal to:

(10-01-2019/Shift-2)

(a) $(25)^2$ (b) $2^{25}-1$

(c) 2^{24} (d) 2^{25}

26. The sum of the real values of x for which the middle term in the binomial expansion of $\left(\frac{x^3}{3}+\frac{3}{x}\right)^8$ equals 5670 is,

(11-01-2019/Shift-1)

27. Let $(x+10)^{50}+(x-10)^{50}=a_0+a_1x+a_2x^2+\ldots..+a_{50}x^{50}$, for all $x\in R$, then $\frac{a_2}{a_0}$ is equal to :

(11-01-2019/Shift-2)

(a) 12.50 (b) 12.00

(c) 12.25 (d) 12.75

28. Let $S_n=1+q+q^2+\ldots\ldots.+q^n$ and

$T_n=1+\left(\frac{q+1}{2}\right)+\left(\frac{q+1}{2}\right)^2+\ldots\ldots\ldots+\left(\frac{q+1}{2}\right)^n$ where q is a real number and $q\neq 1$.

If ${}^{101}C_1+{}^{101}C_2.S_1+\ldots\ldots.+{}^{101}C_{101}.S_{100}=\alpha T_{100}$,

then α is equal to : **(11-01-2019/Shift-2)**

(a) 2^{99} (b) 202

(c) 200 (d) 2^{100}

29. A ratio of the 5^{th} term from the beginning to the 5th term from the end in the binomial expansion of $\left(2^{\frac{1}{3}}+\frac{1}{2(3)^{\frac{1}{3}}}\right)^{10}$ is **(12-01-2019/Shift-1)**

(a) $1:2(6)^{\frac{1}{3}}$ (b) $1:4(16)^{\frac{1}{3}}$

(c) $4(36)^{\frac{1}{3}}:1$ (d) $2(36)^{\frac{1}{3}}:1$

30. The total number of irrational terms in the binomial expansion of $\left(7^{\frac{1}{5}}-3^{\frac{1}{10}}\right)^{60}$ is **(12-01-2019/Shift-2)**

(a) 55 (b) 49

(c) 48 (d) 54

31. If ${}^nC_4$, ${}^nC_5$, and ${}^nC_6$ are in A.P., then n can be:

(12-01-2019/Shift-2)

(a) 9 (b) 14

(c) 11 (d) 12

32. Let $\alpha>0,\beta>0$ be such that $\alpha^3+\beta^2=4$. If the maximum value of the term independent of x in the binomial expansion of $\left(\alpha x^{\frac{1}{9}}+\beta x^{-\frac{1}{6}}\right)^{10}$ is $10k$, then k is equal to :

(2-9-2020/Shift-1)

(a) 176 (b) 336

(c) 352 (d) 84

33. For a positive integer n, $\left(1+\frac{1}{x}\right)^n$ is expanded in increasing powers of x. If three consecutive coefficients in this expansion are in the ratio, 2:5:12, then n is equal to ....

(2-09-2020/Shift-2)

34. If the number of integral terms in the expansion of $\left(3^{\frac{1}{2}}+5^{\frac{1}{8}}\right)^n$ is exactly 33, then the least value of n is :

(3-09-2020/Shift-1)

(a) 128 (b) 248

(c) 256 (d) 264

35. If the term independent of x in the expansion of $\left(\frac{3}{2}x^2-\frac{1}{3x}\right)^9$ is k, then 18 k is equal to :

(3-09-2020/Shift-2)

(a) 5 (b) 9

(c) 7 (d) 11

36. Let $\left(2x^2+3x+4\right)^{10}=\sum_{r=0}^{20}a_r x^r$. Then $\frac{a_7}{a_{13}}$ is equal to ......

(4-09-2020/Shift-1)

37. If for some positive integer n, the coefficients of three consecutive terms in the binomial expansion of $(1+x)^{n+5}$ are in the ratio 5:10:14, then the largest coefficient in this expansion is : **(4-9-2020/Shift-2)**

(a) 792 (b) 252

(c) 462 (d) 330

38. The natural number m, for which the coefficient of x in the binomial expansion of $\left(x^m+\frac{1}{x^2}\right)^{22}$ is 1540, is ……………. **(5-09-2020/Shift-1)**

39. The coefficient of x^4 in the expansion of $\left(1+x+x^2+x^3\right)^6$ in powers of x, is___ **(5-09-2020/Shift-2)**

40. If $\{p\}$ denotes the fractional part of the number p, then $\left\{\frac{3^{200}}{8}\right\}$, is equal to: **(6-09-2020/Shift-1)**

(a) $\frac{5}{8}$ (b) $\frac{1}{8}$

(c) $\frac{7}{8}$ (d) $\frac{3}{8}$

41. If the constant term in the binomial expansion of $\left(\sqrt{x}-\frac{k}{x^2}\right)^{10}$ is 405, then $|k|$ equals:

(6-09-2020/Shift-2)

(a) 1 (b) 9

(c) 2 (d) 3

42. If the sum of the coefficients of all even powers of x in the product $\left(1+x+x^2+x^3.....+x^{2n}\right)$ $\left(1-x+x^2-x^3.....+x^{2n}\right)$ is 61, then n is equal to

(7-01-2020/Shift-1)

43. The coefficient of x^7 in the expression $(1+x)^{10}+x(1+x)^9+x^2(1+x)^8+.......+x^{10}$ is :

(7-01-2020/Shift-2)

(a) 420 (b) 330

(c) 210 (d) 120

44. If α and β be the coefficients of x^4 and x^2 respectively in the expansion of $\left(x+\sqrt{x^2-1}\right)^6+\left(x-\sqrt{x^2-1}\right)^6$, then

(8-01-2020/Shift-2)

(a) $\alpha+\beta=-30$ (b) $\alpha-\beta=-132$

(c) $\alpha+\beta=60$ (d) $\alpha-\beta=60$

45. The coefficient of x^4 in the expansion of $\left(1+x+x^2\right)^{10}$ is

(9-01-2020/Shift-1)

46. In the expansion of $\left(\frac{x}{\cos\theta}+\frac{1}{x\sin\theta}\right)^{16}$, if l_1 is the least value of the term independent of x when $\frac{\pi}{8}\le\theta\le\frac{\pi}{4}$ and l_2 is the least value of the term independent of x when $\frac{\pi}{16}\le\theta\le\frac{\pi}{8}$, then the ratio $l_2 : l_1$ is equal to :

(9-1-2020/Shift-2)

(a) 16 : 1 (b) 8 : 1

(c) 1 : 8 (d) 1 : 16

47. If $C_r = {}^{25}C_r$ and $C_0 + 5.\,C_1 + 9.\,C_2 + ... + 101.\,C_{25} = 2^{25}.\,k$ then k is equal to **(9-1-2020/Shift-2)**

48. The coefficient of x^{256} in the expansion of $(1-x)^{101}(x^2+x+1)^{100}$ is: **(20-07-2021/Shift-1)**

(a) $-{}^{100}C_{16}$ (b) ${}^{100}C_{16}$

(c) ${}^{100}C_{15}$ (d) $-{}^{100}C_{15}$

49. The number of rational terms in the binomial expansion of $\left(4^{\frac{1}{4}}+5^{\frac{1}{6}}\right)^{120}$ is ______. **(20-07-2021/Shift-1)**

50. For the natural numbers m, n, if $(1-y)^m(1+y)^n = 1 + a_1y + a_2y^2 + ... + a_{m+n}y^{m+n}$ and $a_1 = a_2 = 10$, then the value of $(m+n)$ is equal to **(20-07-2021/Shift-2)**

(a) 88 (b) 64

(c) 100 (d) 80

51. If b is very small as compared to the value of a, so that the cube and other higher powers of $\frac{b}{a}$ can be neglected in the identity

$$\frac{1}{a-b}+\frac{1}{a-2b}+\frac{1}{a-3b}.........+\frac{1}{a-nb} = \alpha n + \beta n^2 + \gamma n^3,$$

then the value of γ is is ? **(25-07-2021/Shift-1)**

(a) $\frac{b^2}{3a^3}$ (b) $\frac{a+b}{3a^2}$

(c) $\frac{a^2+b}{3a^3}$ (d) $\frac{a+b^2}{3a^3}$

52. The term independent of 'x' in the expansion of $\left(\frac{x+1}{x^{2/3}-x^{1/3}+1}-\frac{x-1}{x-x^{1/2}}\right)^{10}$, where $x \neq 0, 1$ is equal to ________ ? **(25-07-2021/Shift-1)**

53. The ratio of the coefficient of the middle term in the expansion of $(1+x)^{20}$ and the sum of the coefficients of two middle terms in expansion of $(1+x)^{19}$ is ? **(25-07-2021/Shift-1)**

54. The probability that a randomly selected 2 digit number belongs to the set $(n \in N : (2^n - 2)$ is a multiple of $3)$ is equal to : **(27-07-2021/Shift-1)**

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$

(c) $\frac{2}{3}$ (d) $\frac{1}{6}$

55. If the coefficients of x^7 in $\left(x^2+\frac{1}{bx}\right)^{11}$ and x^{-7} in $\left(x-\frac{1}{bx^2}\right)^{11}$, $b \neq 0$, are equal, then the value of b is equal to: **(27-07-2021/Shift-1)**

(a) –1 (b) 2

(c) –2 (d) 1

56. A possible value of 'x', for which the ninth term in the expansion of $\left\{3^{\log_3\sqrt{25^{x-1}+7}}+3^{\left(-\frac{1}{8}\right)\log_3\left(5^{x-1}+1\right)}\right\}^{10}$ in the increasing powers of $3^{\left(\frac{1}{8}\right)\log_3\left(5^{x-1}+1\right)}$ is equal to 180, is: **(27-07-2021/Shift-2)**

(a) 2 (b) 1

(c) 0 (d) –1

57. The number of elements in the set $\{n \in \{1, 2, 3, ..., 100\} \mid (11)^n > (10)^n + (9)^n\}$ is ______. is **(22-07-2021/Shift-2)**

58. If the constant term, in binomial expansion of $\left(2x^r+\frac{1}{x^2}\right)^{10}$ is 180, then r is equal to _____. **(22-07-2021/Shift-2)**

59. The sum of all those terms which are rational numbers in the expansion of $\left(2^{\frac{1}{3}}+3^{\frac{1}{4}}\right)^{12}$ is: **(25-07-2021/Shift-2)**

(a) 27 (b) 89

(c) 35 (d) 43

60. If the greatest value of the term independent of 'x' in the expansion of $\left(x\sin\alpha + a\frac{\cos\alpha}{x}\right)^{10}$ is $\frac{10!}{(5!)^2}$, then the value of 'a' is equal to: **(25-07-2021/Shift-2)**

(a) 2 (b) –1

(c) 1 (d) –2

61. The lowest integer which is greater than $\left(1+\frac{1}{10^{100}}\right)^{10^{100}}$ is ______. **(25-07-2021/Shift-2)**

(a) 3 (b) 4

(c) 2 (d) 1

62. If the coefficients of x^7 and x^8 in the expansion of $\left(2+\frac{x}{3}\right)^n$ are equal, then the value of n is equal to___. **(25-07-2021/Shift-2)**

63. Let $n \in N$ and $[x]$ denote the greatest integer less than or equal to x. If the sum of $(n+1)$ terms of ${}^nC_0, 3.{}^nC_1, 5.{}^nC_2, 7.{}^nC_3, \ldots$..is equal to $2^{100}.101$, then $2\left[\frac{n-1}{2}\right]$ is equal to________. **(25-07-2021/Shift-2)**

64. If the sum of the coefficients in the expansion of $(x+y)^n$ is 4096, than greatest coefficient in the expansion is_______ ? **(01-09-2021/Shift-2)**

65. Let $\binom{n}{k}$ denote ${}^nC_k$ and $\left[\begin{matrix} n \\ k \end{matrix}\right] = \begin{cases} \binom{n}{k}, & \text{if } 0 \le k \le n \\ 0, & \text{otherwise} \end{cases}$

If $A_k = \sum_{i=0}^{9}\binom{9}{i}\left[\begin{matrix} 12 \\ 12-k+i \end{matrix}\right] + \sum_{i=0}^{8}\binom{8}{i}\left[\begin{matrix} 13 \\ 13-k+i \end{matrix}\right]$ and $A_4 - A_3 = 190p$, then p is equal to _______. **(26-08-2021/Shift-2)**

66. $\sum_{k=0}^{20}\left({}^{20}C_k\right)^2$ is equal to: **(27-08-2021/Shift-1)**

(a) ${}^{41}C_{20}$ (b) ${}^{40}C_{20}$

(c) ${}^{40}C_{21}$ (d) ${}^{40}C_{19}$

67. If ${}^{20}C_r$ is the co-efficient of x^r in the expansion of $(1+x)^{20}$, then the value of $\sum_{r=0}^{20} r^2\ {}^{20}C_r$ is equal to : **(26-08-2021/Shift-1)**

(a) 420×2^{19} (b) 420×2^{18}

(c) 380×2^{18} (d) 380×2^{19}

68. $3\times 7^{22} + 2\times 10^{22} - 44$ when divided by 18 leaves the remainder ___________. **(27-08-2021/Shift-2)**

69. If $\left(\frac{3^6}{4^4}\right)k$ is the term independent of x in the binomial expansion of $\left(\frac{x}{4}-\frac{12}{x^2}\right)^{12}$, then k is equal to __________ ? **(31-08-2021/Shift-1)**

70. If the coefficient of a^7b^8 in the expansion of $(a+2b+4ab)^{10}$ is $K.2^{16}$, then K is equal to______. **(31-08-2021/Shift-2)**

71. Let n be a positive integer. Let

$$A = \sum_{k=0}^{n}(-1)^k\ {}^nC_k\left[\left(\frac{1}{2}\right)^k + \left(\frac{3}{4}\right)^k + \left(\frac{7}{8}\right)^k + \left(\frac{15}{16}\right)^k + \left(\frac{31}{32}\right)^k\right]$$

If $63A = 1 - \frac{1}{2^{30}}$, then n is equal to ____. **(16-03-2021/Shift-2)**

72. If n is the number of irrational terms in the expansion of $\left(3^{\frac{1}{4}} + 5^{\frac{1}{8}}\right)^{60}$, then $(n-1)$ is divisible by **(16-03-2021/Shift-1)**

(a) 8 (b) 26

(c) 7 (d) 30

73. The value of $\sum_{r=0}^{6}\left({}^6C_r \cdot {}^6C_{6-r}\right)$ is equal to : **(17-03-2021/Shift-2)**

(a) 1024 (b) 1124

(c) 1324 (d) 924

74. Let the coefficients of third, fourth and fifth terms in the expansion of $\left(x+\frac{a}{x^2}\right)^n$, $x \neq 0$, be in the ratio 12 : 8 : 3. Then the term independent of x in the expansion, is equal to ....

(Round off the answer to nearest integer)

(17-03-2021/Shift-2)

75. If $(2021)^{3762}$ is divided by 17, then the remainder is ……………..

(17-03-2021/Shift-1)

76. The term independent of x in the expansion of

$$\left[\frac{x+1}{x^{2/3}-x^{1/3}+1}-\frac{x-1}{x-x^{1/2}}\right]^{10}, \ x \neq 1, \text{ is equal to}$$

_______.

(18-03-2021/Shift-2)

77. Let ${}^nC_r$ denote the binomial coefficient of x^r in the expansion of $(1+x)^n$. If

$$\sum_{k=0}^{10}(2^2+3k)\,{}^nC_k=\alpha.3^{10}+\beta.2^{10}, \ \alpha,\beta\in R, \text{ then } \alpha+\beta$$

is equal to _______.

(18-03-2021/Shift-2)

78. Let $(1+x+2x^2)^{20}=a_0+a_1x+a_2x^2+......+a_{40}x^{40}$.

Then, $a_1+a_3+a_5+.....+a_{37}$ is equal to :

(18-03-2021/Shift-1)

(a) $2^{20}(2^{20}-21)$ (b) $2^{19}(2^{20}+21)$

(c) $2^{19}(2^{20}-21)$ (d) $2^{20}(2^{20}+21)$

79. If $n \geq 2$ is a positive integer, then the sum of the series

$${}^{n+1}C_2+2\left({}^2C_2+{}^3C_2+{}^4C_2+....+{}^nC_2\right) \text{ is}$$

(24-02-2021/Shift-2)

(a) $\frac{n(n+1)^2(n+2)}{12}$ (b) $\frac{n(n+1)(2n+1)}{6}$

(c) $\frac{n(n-1)(2n+1)}{6}$ (d) $\frac{n(2n+1)(3n+1)}{6}$

80. For integers n and r, let $\binom{n}{r}=\begin{cases}{}^nC_r, & \text{if } n\geq r\geq 0\\ 0, & \text{otherwise}\end{cases}$

The maximum value of k for which the sum

$$\sum_{i=0}^{k}\binom{10}{i}\binom{15}{k-i}+\sum_{i=0}^{k+1}\binom{12}{i}\binom{13}{k+1-i} \text{ is maximum, is}$$

equal to ____.

(24-02-2021/Shift-2)

81. The value of

$$-{}^{15}C_1+2\,{}^{15}C_2-3\,{}^{15}C_3+\cdots-15\,{}^{15}C_{15}$$
$$+\,{}^{14}C_1+{}^{14}C_3+{}^{14}C_5+\cdots+{}^{14}C_{11}$$

(24-02-2021/Shift-1)

(a) 2^{14} (b) $2^{13}-13$

(c) 2^{16} (d) $2^{13}-14$

82. If the reminder when x is divided by 4 is 3, then the remainder when $(2020+x)^{2022}$ is divided by 8 is_______.

(25-02-2021/Shift-2)

83. The maximum value of the term independent of 't' in the expansion of $\left(tx^{\frac{1}{5}}+\frac{(1-x)^{\frac{1}{10}}}{t}\right)^{10}$ where $x\in(0,1)$ is

(26-02-2021/Shift-1)

(a) $\frac{10!}{3(5!)^2}$ (b) $\frac{2.10!}{3\sqrt{3}(5!)^2}$

(c) $\frac{10!}{\sqrt{3}(5!)^2}$ (d) $\frac{2.10!}{3(5!)^2}$

84. Let $m,n\in N$ and $\gcd(2,n)=1$. If

$$30\binom{30}{0}+29\binom{30}{1}+.....+2\binom{30}{28}+1\binom{30}{29}=n\cdot 2^m, \text{ then}$$

$n+m$ is equal to _____. $\left(\text{Here }\binom{n}{k}={}^nC_k\right)$

(26-02-2021/Shift-1)

85. If the fourth term in the expansion of $(x+x^{\log_2 x})^7$ is 4480, then the value of x where $x\in N$ is equal to :

(17-03-2021/Shift-1)

(a) 4 (b) 2

(c) 3 (d) 1

EXERCISE - 3 : ADVANCED OBJECTIVE QUESTIONS

Objective Questions I [Only one correct option]

1. The ratio of the coefficient of x^{10} in $(1-x^2)^{10}$ and the term independent of x in $\left(x-\frac{2}{x}\right)^{10}$, is

(a) 1 : 16 (b) 1 : 32
(c) 1 : 64 (d) none of these

2. If the fourth term in the expansion of $\left(px+\frac{1}{x}\right)^n$ is independent of x, then the value of term is

(a) $5p^3$ (b) $10p^3$
(c) $20p^3$ (d) none of these

3. The greatest coefficent in the expansion of $(1 + x)^{2n}$ is

(a) ${}^{2n}C_n$ (b) ${}^{2n}C_{n-1}$
(c) ${}^{2n}C_{n-2}$ (d) none of these

4. Which of the following expression is divisible by 1225 ?

(a) $6^{2n} - 35n - 1$ (b) $6^{2n} - 35n + 1$
(c) $6^{2n} - 35n$ (d) $6^{2n} - 35n + 2$

5. The number of distinct terms in the expansion of $(x + y - z)^{16}$ is

(a) 136 (b) 153
(c) 16 (d) 17

6. The total number of terms in the expansion of $(a + b + c + d)^n$, $n \in N$ is

(a) $\frac{n(n+1)(n+2)}{6}$ (b) $\frac{n(n+1)(n+2)(n+3)}{6}$

(c) $\frac{(n+1)(n+2)(n+3)}{6}$ (d) none of these

7. If rth and (r + 1)th term in the expansion of $(1 + x)^n$ are equal, then n =

(a) $\frac{(1+x)r-x}{4x}$ (b) $\frac{(1+x)r-x}{3x}$

(c) $\frac{(1+x)r-x}{x}$ (d) $\frac{(1+x)r-x}{r}$

8. In the binomial expansion of $(a - b)^n$, $n \geq 5$, the sum of the 5^{th} and 6^{th} terms is zero. Then, a/b equals

(a) $\frac{n-5}{6}$ (b) $\frac{n-4}{5}$

(c) $\frac{5}{n-4}$ (d) $\frac{6}{n-5}$

9. The greatest value of the term independent of x in the expansion of $(x \sin\alpha + x^{-1}\cos\alpha)^{10}$, $\alpha \in R$, is

(a) 2^5 (b) $\frac{10!}{(5!)^2}$

(c) $\frac{10!}{2^5 \times (5!)^2}$ (d) none of these

10. If the ratio of 7th term from the beginning to the seventh term from the end in the expansion of $\left(\sqrt[3]{2}+\frac{1}{\sqrt[3]{3}}\right)^x$ is $\frac{1}{6}$ then x, is

(a) 9 (b) 6, 15
(c) 12, 9 (d) none of these

11. The sum of coefficients of the two middle terms in the expansions of $(1 + x)^{2n-1}$ is equal to :

(a) ${}^{(2n-1)}C_n$ (b) ${}^{(2n-1)}C_{n+1}$
(c) ${}^{2n}C_{n-1}$ (d) ${}^{2n}C_n$

12. The greatest term (numerically) in the expansion of $(2 + 3x)^9$, when $x = \frac{3}{2}$, is

(a) $\frac{5 \times 3^{11}}{2}$ (b) $\frac{5 \times 3^{13}}{2}$

(c) $\frac{7 \times 3^{13}}{2}$ (d) none of these

13. The greatest term (numerically) in the expansion of $(3 - 5x)^{11}$, when $x = \frac{1}{5}$ is

(a) 55×3^9 (b) 46×3^9
(c) 55×3^6 (d) none of these

14. If 7^{103} is divided by 25, then the remainder is

(a) 20 (b) 16
(c) 18 (d) 15

15. The last digit of the number $(32)^{32}$ is

(a) 4 (b) 6
(c) 8 (d) none of these

16. $9^7 + 7^9$ is divisible by

(a) 6 (b) 24
(c) 64 (d) 72

17. The number $5^{25} - 3^{25}$ is divisible by :

(a) 2 (b) 3
(c) 5 (d) 7

18. If $0 \le r \le n$, then the coefficient of x^r in the expansion of $P = 1 + (1+x) + (1+x)^2 + ... + (1+x)^n$ is

(a) ${}^nC_r$ (b) ${}^{n+1}C_{r+1}$
(c) ${}^nC_{r+1}$ (d) none of these

19. The coefficient of x^{20} in the expansion of $(1+x^2)^{40} \cdot \left(x^2 + 2 + \frac{1}{x^2}\right)^{-5}$ is

(a) ${}^{30}C_{10}$ (b) ${}^{30}C_{25}$
(c) 1 (d) none of these

20. The integral part of $(\sqrt{2}+1)^6$ is:

(a) 198 (b) 197
(c) 196 (d) 163

21. If $\frac{1}{1-2x+x^2} = 1 + a_1x + a_2x^2 + ...$, then the value of a_r is

(a) 2r (b) r + 1
(c) r (d) r – 1

22. The value of ${}^{50}C_4 + \sum_{r=1}^{6} {}^{56-r}C_3$ is

(a) ${}^{56}C_4$ (b) ${}^{56}C_3$
(c) ${}^{55}C_3$ (d) ${}^{55}C_4$

23. If n is a positive integer greater than 1, then $a - {}^nC_1(a-1) + {}^nC_2(a-2) - .... + (-1)^n(a-n)$ is equal to

(a) n (b) a
(c) 0 (d) none of these

24. If $(1+x)^{15} = C_0 + C_1x + C_2x^2 + ... + C_{15}x^{15}$, then ${}^{15}C_0^2 - {}^{15}C_1^2 + {}^{15}C_2^2 - {}^{15}C_3^2 + ... {}^{15}C_{15}^2$ is equal to

(a) 0 (b) 1
(c) –1 (d) none of these

25. The sum $1.\,{}^{20}C_1 - 2.\,{}^{20}C_2 + 3.\,{}^{20}C_3 - ..... - {}^{20}C_{20}$ is equal to

(a) 2^{19} (b) 0
(c) $2^{20} - 1$ (d) none of these

26. $1.\,{}^nC_1 + 2,\,{}^nC_2 + 3.\,{}^nC_3 + ... + n.\,{}^nC_n$ is equal to

(a) $\frac{n(n+1)}{4}.2^n$ (b) $2^{n+1} - 3$
(c) $n2^{n-1}$ (d) none of these

27. If C_r stands for ${}^nC_r$, then the sum of the series

$$\frac{2\left(\frac{n}{2}\right)!\left(\frac{n}{2}\right)!}{n!}\left[C_0^2 - 2C_1^2 + 3C_2^2 - .... + (-1)^n(n+1)C_n^2\right]$$

where n is an even positive integer, is equal to :

(a) $(-1)^{n/2}(n+2)$ (b) $(-1)^n(n+1)$
(c) $(-1)^{n/2}(n+1)$ (d) none of these

28. If $A = {}^{2n}C_0 .\, {}^{2n}C_1 + {}^{2n}C_1\, {}^{2n-1}C_1 + {}^{2n}C_2\, {}^{2n-2}C_1 + ...$then A is

(a) 0 (b) 2^n
(c) $n2^{2n}$ (d) 1

29. The coefficient of x^{50} in the expansion : $(1+x)^{1000} + 2x(1+x)^{999} + 3x^2(1+x)^{998} + ...+1001$ terms

(a) ${}^{1002}C_{50}$ (b) ${}^{1002}C_{51}$
(c) ${}^{1005}C_{50}$ (d) ${}^{1005}C_{48}$

30. If $s_n = \sum_{r=0}^{n} \frac{1}{{}^nC_r}$ and $t_n = \sum_{r=0}^{n} \frac{r}{{}^nC_r}$ then $\frac{t_n}{s_n}$ is equal to

(a) $\frac{n}{2}$ (b) $\frac{n}{2} - 1$
(c) n – 1 (d) $\frac{2n-1}{2}$

31. If 7 divides $32^{32^{32}}$, the remainder is :

(a) 1 (b) 0
(c) 4 (d) 6

32. The term independent of x in the expansion of $(1+x)^n \left(1 + \frac{1}{x}\right)^n$ is :

(a) $C_0^2 + 2C_1^2 + 3C_2^2 + ... + (n+1)C_n^2$
(b) $(C_0 + C_1 + C_2 + ... + C_n)^2$
(c) $C_0^2 + C_1^2 + ... + C_n^2$
(d) none of these

33. The number of terms in the expansion of $\left(x^2+1+\frac{1}{x^2}\right)^n$, $n \in N$ is.

(a) 2n (b) 3n

(c) 2n + 1 (d) 3n + 1

34. Integral part of $(7+4\sqrt{3})^n$ is $(n \in N)$

(a) an even number

(b) an odd number

(c) an even or an odd number depending upone the of n

(d) none of these

Objective Questions II [One or more than one correct option]

35. In the expansion of $(x + y + z)^{25}$

(a) every term is of the form ${}^{25}C_r \cdot {}^{r}C_k \cdot x^{25-r} \cdot y^{r-5} \cdot z^k$

(b) the coefficient of $x^8y^9z^9$ is 0

(c) the number of term is 325

(d) none of these.

36. Element in set of values of r for which,

${}^{18}C_{r-2} + 2.\,{}^{18}C_{r-1} + {}^{18}C_r \geq {}^{20}C_{13}$ is :

(a) 9 (b) 5

(c) 7 (d) 10

37. The expansion of $(3x + 2)^{-1/2}$ is valid in ascending powers of x, if x lies in the interval

(a) (0, 2/3) (b) (-3/2, 3/2)

(c) (-2/3, 2/3) (d) $(-\infty, -3/2)\ (3/2, \infty)$

Numerical Value Type Questions

38. If ${}^{n}C_{r-1} = 36$, ${}^{n}C_r = 84$ and ${}^{n}C_{r+1} = 126$, then r =

39. Let the co-efficients of x^n in $(1 + x)^{2n}$ & $(1 + x)^{2n-1}$ be P & Q respectively, then $\left(\frac{P+Q}{P}\right)^4 =$

40. Sum of square of all possible values of 'r' satisfying the equation

${}^{39}C_{3r-1} - {}^{39}C_{r^2} = {}^{39}C_{r^2-1} - {}^{39}C_{3r}$ is :

41. If $\frac{1}{1!10!}+\frac{1}{2!9!}+\frac{1}{3!8!}+\ldots\ldots+\frac{1}{10!1!}=\frac{(2^{10}-1)}{k10!}$ then find the value of k.

42. The coefficient of x^{99} in the polynomial $(x - 1)(x - 2) \ldots. (x - 100)$ is .....

Match the Following

Each question has two columns. Four options are given representing matching of elements from Column-I and Column-II. Only one of these four options corresponds to a correct matching.For each question, choose the option corresponding to the correct matching.

43. Match the entries in Column-I representing in n with their values given in Column-II.

Column I	Column II
(A) ${}^{16}C_n + {}^{16}C_{n+1} + {}^{17}C_{n+2} \geq {}^{18}C_{2n-1}$	**(P)** 15
(B) ${}^{16}C_{n+5} \leq {}^{17}C_{n+6}$	**(Q)** 6
(C) $12 \times ({}^{n}C_6)^2 \leq 7 \times ({}^{n+1}C_5) \times ({}^{n+1}C_7)$	**(R)** 7
(D) $2 \times ({}^{n-1}C_4 - {}^{n-1}C_3) < 5 \times ({}^{n-2}C_2)$	**(S)** 12

The correct matching is

(a) A–Q, R ; B–Q, R ; C–P, Q, R, S; D–Q, R

(b) A–Q ; B–Q, R ; C–P, Q, R, S; D–Q, R

(c) A–Q, R ; B–Q ; C–P, Q, R, S; D–Q, R

(d) A–Q, R ; B–Q, R ; C–P, Q, S; D–Q, R

44. Match the following with their no. of terms.

Column-I	Column-II
(A) $(x_1 + x_2 + x_3 \ldots\ldots + x_n)^3$	**(P)** infinite
(B) $(x_1 + x_2 + x_3)^n$	**(Q)** ${}^{n+2}C_3$
(C) $(1-x)^{-3}$ $(\lvert x \rvert < 1)$	**(R)** $\leq 2n+1$
(D) $(1+x+x^2)^n$	**(S)** ${}^{n+2}C_2$

The correct matching is

(a) A– Q; B–S; C–R; D–Q

(b) A –S; B–S; C–P; D–R

(c) A –Q; B –S; C–R; D–R

(d) A –Q; B –S; C–P; D–R

Text

45. Let n be a positive integer and

$(1+x+x^2)^n = a_0 + a_1 x + ... + a_{2n} x^{2n}$.

Show that $a_0^2 - a_1^2 + ... + a_{2n}^2 = a_n$.

46. Given $s_n = 1 + q + q^2 + ... + q^n$

$$S_n = 1 + \frac{q+1}{2} + \left(\frac{q+1}{2}\right)^2 + ... + \left(\frac{q+1}{2}\right)^n, q \neq 1 .$$

Prove that ${}^{n+1}C_1 + {}^{n+1}C_2 s_1 + {}^{n+1}C_3 s_2 + ... + {}^{n+1}C_{n+1} s_n = 2^n S_n$

47. Find the sum of the series :

$$\sum_{r=0}^{n} (-1)^r \; {}^nC_r \left[\frac{1}{2^r} + \frac{3^r}{2^{2r}} + \frac{7^r}{2^{3r}} + \frac{15^r}{2^{4r}} .... \text{upto m terms}\right]$$

48. Prove that $C_0 - 2^2 . C_1 + 3^2 . C_2 - ..... + (-1)^n (n+1)^2 . C_n = 0$, $n > 2$ where $C_r = {}^nC_r$.

49. Prove that : $({}^{2n}C_0)^2 - ({}^{2n}C_1)^2 + ({}^{2n}C_2)^2 + ... + ({}^{2n}C_{2n})^2 = (-1)^{n.2n} C_n$.

50. Prove that :

$$C_1^2 - 2.C_2^2 + 3.C_3^2 - ...... - 2n.C_{2n}^2 = (-1)^n n . {}^{2n}C_n$$

EXERCISE - 4 : PREVIOUS YEAR JEE ADVANCED QUESTIONS

Objective Questions I [Only one correct option]

1. For $2 \le r \le n$, $\binom{n}{r} + 2\binom{n}{r-1} + \binom{n}{r-2}$ is equal to : **(2000)**

(a) $\binom{n+1}{r-1}$ (b) $2\binom{n+1}{r-1}$

(c) $2\binom{n+2}{r}$ (d) $\binom{n+2}{r}$

2. In the binomial expansion of $(a-b)^n$, $n \ge 5$ the sum of the 5^{th} and 6^{th} terms is zero. Then a/b equals : **(2001)**

(a) $\frac{n-5}{6}$ (b) $\frac{n-4}{5}$

(c) $\frac{5}{n-4}$ (d) $\frac{6}{n-5}$

3. The sum $\sum_{i=0}^{m}\binom{10}{i}\binom{20}{m-i}$, where $\binom{p}{q} = 0$ if $p > q$, is maximum when m is : **(2002)**

(a) 5 (b) 10

(c) 15 (d) 20

4. Coefficient of t^{24} in $(1+t^2)^{12}(1+t^{12})(1+t^{24})$ is : **(2003)**

(a) ${}^{12}C_6 + 3$ (b) ${}^{12}C_6 + 1$

(c) ${}^{12}C_6$ (d) ${}^{12}C_6 + 2$

5. If ${}^{n-1}C_r = (k^2-3)\,{}^nC_{r+1}$, then k belong to : **(2004)**

(a) $(-\infty, -2]$ (b) $[2, \infty)$

(c) $\left[-\sqrt{3}, \sqrt{3}\right]$ (d) $(\sqrt{3}, 2]$

6. $\binom{30}{0}\binom{30}{10} - \binom{30}{1}\binom{30}{11} + ... + \binom{30}{20}\binom{30}{30}$ is equal to **(2005)**

(a) ${}^{30}C_{11}$ (b) ${}^{60}C_{10}$

(c) ${}^{30}C_{10}$ (d) ${}^{65}C_{55}$

7. For r = 0, 1, ..., 10, let A_r, B_r and C_r denote, respectively, the coefficient of x^r in the expansions of $(1+x)^{10}$, $(1+x)^{20}$ and $(1+x)^{30}$. Then $\sum_{r=1}^{10} A_r\,(B_{10}B_r - C_{10}A_r)$ is equal to **(2010)**

(a) $B_{10} - C_{10}$ (b) $A_{10}\left(B_{10}^2 - C_{10}A_{10}\right)$

(c) 0 (d) $C_{10} - B_{10}$

8. Coefficient of x^{11} in the expansion of $(1+x^2)^4(1+x^3)^7(1+x^4)^{12}$ is **(2014)**

(a) 1051 (b) 1106

(c) 1113 (d) 1120

Numerical Value Type Questions

9. The coefficient of three consecutive terms $(1+x)^{n+5}$ are in the ratio 5 : 10 : 14. Then, n is equal to **(2013)**

10. The coefficient of x^9 in the expansion of $(1+x)(1+x^2)(1+x^3)\ldots(1+x^{100})$ is **(2015)**

11. Let m be the smallest positive integer such that the coefficient of x^2 in the expansion of $(1+x)^2 + (1+x)^3 + \ldots .. + (1+x)^{49} + (1+mx)^{50}$ is $(3n+1)\,{}^{51}C_3$ for some positive integer n. Then the value of n is **(2016)**

12. Let $X = ({}^{10}C_1)^2 + 2({}^{10}C_2)^2 + 3({}^{10}C_3)^2 + \ldots .. + 10({}^{10}C_{10})^2$, where ${}^{10}C_r$, $r \in \{1, 2, ..., 10\}$ denote binomial coefficients. Then, the value of $\frac{1}{1430}X$ is ______ **(2018)**

13. Suppose $\det\begin{bmatrix} \sum_{k=0}^{n} k & \sum_{k=o}^{n} {}^nC_k k^2 \\ \sum_{k=0}^{n} {}^nC_k k & \sum_{k=0}^{n} {}^nC_k 3^k \end{bmatrix} = 0$ holds for some positive integer n. Then $\sum_{k=0}^{n} \frac{{}^nC_k}{k+1}$ equals. **(2019)**

Text

14. For any positive integers m, n (with $n \geq m$),

let $\binom{n}{m} = {}^{n}C_{m}$. Prove that

$$\binom{n}{m}+\binom{n-1}{m}+\binom{n-2}{m}+\ldots+\binom{m}{m}=\binom{n+1}{m+1}$$

Hence, or otherwise, prove that

$$\binom{n}{m}+2\binom{n-1}{m}+3\binom{n-2}{m}+\ldots.+(n-m+1)\binom{m}{m}=\binom{n+2}{m+2}$$

(2000)

15. Prove that

$$2^{k}\binom{n}{0}\binom{n}{k}-2^{k-1}\binom{n}{1}\binom{n-1}{k-1}+2^{k-2}\binom{n}{2}\binom{n-2}{k-2}-\ldots..$$

$$+(-1)^{k}\binom{n}{k}\binom{n-k}{0}=\binom{n}{k}$$

(2003)

Answer Key

CHAPTER -12 | BINOMIAL THEOREM

EXERCISE - 1 : BASIC OBJECTIVE QUESTIONS

1. (a)	**2.** (b)	**3.** (c)	**4.** (a)	**5.** (c)
6. (b)	**7.** (d)	**8.** (b)	**9.** (c)	**10.** (c)
11. (d)	**12.** (c)	**13.** (c)	**14.** (d)	**15.** (a)
16. (c)	**17.** (b)	**18.** (c)	**19.** (a)	**20.** (d)
21. (c)	**22.** (c)	**23.** (b)	**24.** (b)	**25.** (d)
26. (c)	**27.** (d)	**28.** (d)	**29.** (b)	**30.** (c)
31. (d)	**32.** (a)	**33.** (c)	**34.** (b)	**35.** (b)
36. (d)	**37.** (d)	**38.** (b)	**39.** (d)	**40.** (b)
41. (c)	**42.** (b)	**43.** (5)	**44.** (51)	**45.** (17)
46. (12)	**47.** (540)	**48.** (210)	**49.** (9)	**50.** (252)
51. (5)	**52.** (5)	**53.** (1120)	**54.** (60)	**55.** (41)

EXERCISE - 2 : PREVIOUS YEAR JEE MAIN QUESTIONS

1. (c)	**2.** (a)	**3.** (a)	**4.** (d)	**5.** (a)
6. (a)	**7.** (c)	**8.** (a)	**9.** (b)	**10.** (b)
11. (24)	**12.** (b)	**13.** (d)	**14.** (b)	**15.** (c)
16. (a)	**17.** (84)	**18.** (b)	**19.** (c)	**20.** (8)
21. (b)	**22.** (a)	**23.** (100)	**24.** (a)	**25.** (d)
26. (0)	**27.** (c)	**28.** (d)	**29.** (c)	**30.** (d)
31. (b)	**32.** (b)	**33.** (118)	**34.** (c)	**35.** (c)
36. (8)	**37.** (c)	**38.** (13)	**39.** (120)	**40.** (b)
41. (d)	**42.** (30)	**43.** (b)	**44.** (b)	**45.** (615)
46. (a)	**47.** (51)	**48.** (c)	**49.** (21)	**50.** (d)
51. (a)	**52.** (210)	**53.** (1)	**54.** (a)	**55.** (d)
56. (b)	**57.** (96)	**58.** (8)	**59.** (d)	**60.** (a)
61. (a)	**62.** (55)	**63.** (98)	**64.** (924)	**65.** (49)
66. (b)	**67.** (b)	**68.** (15)	**69.** (55)	**70.** (315)
71. (6)	**72.** (b)	**73.** (d)	**74.** (4)	**75.** (4)
76. (210)	**77.** (19)	**78.** (c)	**79.** (b)	**80.** (12)
81. (d)	**82.** (1)	**83.** (b)	**84.** (45)	**85.** (b)

ANSWER KEY

CHAPTER -12 | BINOMIAL THEOREM

EXERCISE - 3 :
ADVANCED OBJECTIVE QUESTIONS

1. (b)	**2.** (c)	**3.** (a)	**4.** (a)	**5.** (b)
6. (c)	**7.** (c)	**8.** (b)	**9.** (c)	**10.** (a)
11. (d)	**12.** (c)	**13.** (a)	**14.** (c)	**15.** (b)
16. (c)	**17.** (a)	**18.** (b)	**19.** (b)	**20.** (b)
21. (b)	**22.** (a)	**23.** (c)	**24.** (a)	**25.** (b)
26. (c)	**27.** (a)	**28.** (c)	**29.** (a)	**30.** (a)
31. (c)	**32.** (c)	**33.** (c)	**34.** (b)	**35.** (a,b)
36. (a,c,d)	**37.** (a,c)	**38.** (3)	**39.** (5.06)	**40.** (34)
41. (5.50)	**42.** (-5050)		**43.** (a)	**44.** (d)

47. $\frac{2^{mn}-1}{2^{mn}(2^n-1)}$

EXERCISE - 4 :
PREVIOUS YEAR JEE ADVANCED QUESTIONS

1. (d)	**2.** (b)	**3.** (c)	**4.** (d)	**5.** (d)
6. (c)	**7.** (d)	**8.** (c)	**9.** (6)	**10.** (8)
11. (5)	**12.** (646)	**13.** (6.20)		