

2. TOPIC – ARITHMETIC PROGRESSION

(2 MARKS QUESTIONS)

- Q1 The fee charged from a student every month by a school for the whole session, when the monthly fee is Rs 400, Is in the given situation do the list of numbers involved form an AP? If yes then find total fee for the year.
- Q 2. Given $a=5$, $d=3$, $a_n=50$, find S_n
- Q 3 Find 21st term of an AP whose first two terms are -3 and 4 .
- Q 4 Which terms of an AP $21, 42, 63 \dots$ is 210 , Solve.
- Q 5 Find 4th term from the end of the AP: $-11, -8, -5, \dots 49$.
- Q 6 Verify whether the given series $2, 2^2, 2^3, 2^4$ form an AP. If yes Find common difference.
- Q 7 If the first term of an AP is -5 and $d=2$, then find the sum of first six term.
- Q 8 Find $a_{30} - a_{20}$ in the given series $-3, -7, -11, \dots$
- Q 9 Is the given series $\sqrt{3}, \sqrt{12}, \sqrt{27}, \sqrt{48}, \dots$ form an AP. If yes find common difference.
- Q 10 In an AP $a=3.5$, $d=0$, $n=101$ find a_n .
- Q 11 Find the total number of terms the series $7 + 10\frac{1}{2} + 14 + \dots + 84$.
- Q 12 Verify that $a + b, \{(a + 1) + b\}, \{(a + 1) + (b + 1)\}, \dots$ is an AP.
Find Common difference in case of AP.
- Q 13 If $18, a, b, -3$ are in AP, then find $a + b$
- Q 14 Write down the first four terms of an AP when $a=-5$, $d=-3$
- Q 15 Is 0 a term of the AP: $31, 28, 25, \dots$? Justify your answer.

(3 MARKS QUESTIONS)

- Q 16 How many two-digits numbers are divisible by 3 .
- Q 17 Given $a_n=28$, $S_n=144$ and there are total 9 terms. Find a
- Q 18 **Two APs have the same common difference. The difference between their 100th term is 100, Find the difference between their 1000th terms.**
- Q 19 Find the 31st term of an A.P. whose 11th term is 38 and 16th term is 73 .
- Q 20 How many terms of the AP $9, 17, 25 \dots$ must be taken to give a sum of 636 ?
- Q 21 Find the sum of the first 15 multiples of 8 .
- Q 22 Find the sum of the first 40 positive integers divisible by 6 .
- Q 23 Find the sum of the odd numbers between 0 and 50 .
- Q 24 If the 9th term of an AP is zero, prove that its 29th term is twice its 19th term.

- Q25 Determine the AP whose 5th term is 19 and the difference of the 8th term from the 13th term is 20.

(4 MARKS QUESTIONS)

- Q 26 In a potato race a bucket is placed at the starting point, which is 5m from the first potato and the other potatoes are placed 3m apart in a straight line. There are ten potatoes in the line. A competitor starts from the bucket, picks up the nearest potato, runs back with it, drops it in the bucket, runs back to pick up the next potato, runs to the bucket to drop it in and she continues in the same way until all the potatoes are in the bucket, what is the total distance the competitor has to run?
- Q 27 Ramkali saves Rs 5 in the first week, of a year and increased her weekly savings by Rs 1.75. If in the n^{th} week her weekly savings became Rs 20.75, find n .
- Q 28 In an AP, if $S_5 + S_7 = 167$ and $S_{10} = 235$, then find the AP, where s , denotes the sum of its first n terms
- Q.29. The digits of a positive number of three digits are in A.P. and their sum is 15. The number obtained by reversing the digits is 594 less than the original number. Find the number.
- Q 30 Find the sum of all multiples of 7 lying between 500 and 900.
- Q 31 A thief runs with a uniform speed of 100 m/minute. After one minute a policeman runs after the thief to catch him. He goes with a speed of 100 m/minute in the first minute and increases his speed by 10 m/minute every succeeding minute. After how many minutes the policeman will catch the thief.
- Q 32 Divide 56 in four parts in A.P. such that the ratio of the product of their extremes (1st and 4th) to the product of means (2nd and 3rd) is 5: 6.
- Q 33 In a school, students decided to plant trees in and around the school to reduce air pollution. It was decided that the number of trees, that each section of each class will plant, will be double of the class in which they are studying. If there are 1 to 12 classes in the school and each class has two sections, find how many trees were planted by the students.
- Q 34 A sum of Rs 1600 is to be used to give ten cash prizes to students of a school for their overall academic performance. If each prize is Rs 20 less than its preceding prize, find the value of each of the prizes.
- Q 35 If the sum of the first n terms of an AP is $4n - n^2$, what is the first term (that is S_1)? What is the sum of first two terms? What is the second term? Similarly find the 3rd, the 10th and the n th terms.

CASE STUDY QUESTIONS

Q 36 A road roller (sometimes called a roller-compactor, or just roller) is a compactor-type engineering vehicle used to compact soil, gravel, concrete, or asphalt in the construction of roads and foundations. Similar rollers are used also at landfills or in agriculture. Road rollers are frequently referred to as steamrollers, regardless of their method of propulsion. RCB Machine Pvt Ltd started making road roller 10 year ago. Company increased its production uniformly by fixed number every year. The company produces 800 rollers in the 6th year and 1130 rollers in the 9th year.



On the basis of the above information, answer any four of the following questions:

- (i) Find the company's production in first year.
- (ii) In which year the company's production was 1350 rollers?
 - (a) 5th
 - (b) 6th
 - (c) 11th
 - (d) 9

Q 37 Aditya is celebrating his birthday. He invited his friends. He bought a packet of toffees/candies which contains 120 candies. He arranges the candies such that in the first row there are 3 candies, in second there are 5 candies, in third there are 7 candies and so on.



On the basis of the above information, answer any four of the following questions:

- (i) Find the total number of rows of candies.
- (ii) Find the difference in number of candies placed in 7th and 3rd rows.

- Q 38 In a potato race, a bucket is placed at the starting point, which is 5 m from the first potato, and the other potatoes are placed 3 m apart in a straight line. There are 12 potatoes in the line (see Fig.).

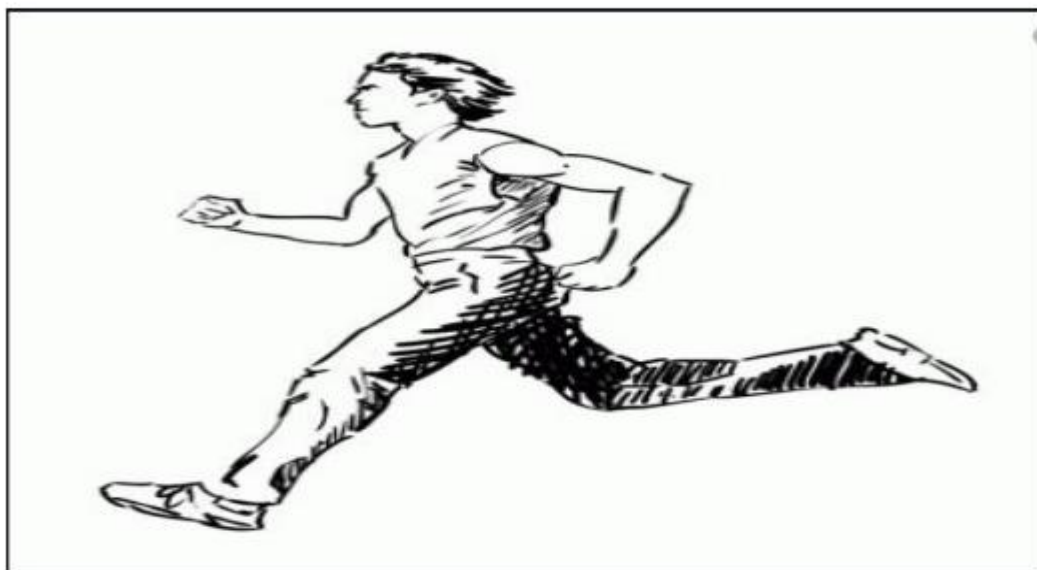


A competitor starts from the bucket, picks up the nearest potato, runs back with it, drops it in the bucket, runs back to pick up the next potato, runs to the bucket to drop it in, and she continues in the same way until all the potatoes are in the bucket.

- Find the total distance covered by the competitor after placed the second potato in the bucket?
- Calculate the total distance covered by the competitor?

Q 39.

Your friend Veer wants to participate in a 200m race. He can currently run that distance in 51 seconds and with each day of practice it takes him 2 seconds less. He wants to do in 31 seconds .



- Find the minimum number of days he needs to practice till his goal is achieved
- If n th term of an AP is given by $a_n = 2n + 3$ then find common difference of AP

Q 40

Your elder brother wants to buy a car and plans to take loan from a bank for his car. He repays his total loan of Rs 1,18,000 by paying every month starting with the first instalment of Rs 1000. If he increases the instalment by Rs 100 every month, answer the following:



- (i). Find the amount paid by him in the 30th installment.
- (ii). Find the total installments paid by him.

ANSWERS AND HINTS OF ARITHMETIC PROGRESSION

- (1) yes, Rs.4800
- (2) 440 [find $n=16$ by last term $= a+(n-1)d$, $50 = 5+(n-1)3$, then apply $S_n = \frac{n}{2}\{a + l\}$]
- (3) 137 [$a=-3$ and $d = 4-(-3) = 7$ then find 21st term $= -3+(21-1)7$]
- (4) 10 [$a=21$, $d=21$ then use formula of last term $210 = 21+(n-1)21$ & div.by 21]
- (5) 40 [rewrite AP in reverse order $a = 49$, $d = -11+8 = -3$
then 4th term from the end $= 49+(4-1)(-3) = 40$]
- (6) No because common difference is not same
- (7) 0 [$a=-5$, $d=2$ $n=6$ then use $s_6 = \frac{6}{2}\{2(-5) + (6-1)2\} = 3(0)$]
- (8) -40, [$a=-3$, $d=-4$
then $a_{30} - a_{20} = \{-3 + 29 \times (-4)\} - \{-3 + 19 \times (-4)\} = \{10\} \times (-4)$]
- (9) yes, [because common difference is same $= \sqrt{3} = 2\sqrt{3} - \sqrt{3} = 3\sqrt{3} - 2\sqrt{3} = \dots$]
- (10) 3.5 [$a=3.5$ and because common difference is zero so AP will not increase or decrease i.e. n th term $=0$]
- (11) 23 [$a=7$ $d=\frac{7}{2}$ use last term $l=a+(n-1)d$, $84=7+(n-1)\frac{7}{2}$]
- (12) Yes, [because common difference is same $=1$]
- (13) 15, [common difference $= a-18 = b-a = -3-b$ now take $a-18 = -3-b$ gives $a+b = 15$]
- (14) $-5, -8, -11, -14$ [use $a, a+d, a+2d, a+3d$ where $a=-5$ & $d=-3$]
- (15) No, [$\because a=31$, $d=-3$, Let n th term $=0$, then $31+(n-1)\times(-3)=0$
gives $n=34/3$ which is not a Positive Integer.]
- (16) 30, [because $a = 12$ $d = 3$ and $l = 99$ then use formula of last term $l = a + (n-1)d$]

$$(17) 4, [\text{ use } S_n = \frac{n}{2}\{a + (a + (n - 1)d)\}, 144 = \frac{9}{2}\{a + 28\}]$$

$$(18) 100, [\text{ for first AP: } a_{100} = a_1 + 99d \text{ \& } a_{1000} = a_1 + 999d \text{ and for Second AP: } a_{100} = a_2 + 99d \text{ \& } a_{1000} = a_2 + 999d \text{ then } (a_1 + 999d) - (a_2 + 999d) = a_1 - a_2 \text{ \& then } (a_1 + 99d) - (a_2 + 99d) = 100 \text{ hence } a_1 - a_2 = 100}]$$

$$(19) 178, [a + 10d = 38 \text{ \& } a + 15d = 73, \text{ then } a = -32 \text{ \& } d = 7 \text{ so } 31^{\text{st}} \text{ term} = -32 + 30(7)]$$

$$(20) 12, [S_n = \frac{n}{2}\{2a + (n - 1)d\}, 636 = \frac{n}{2}\{2(9) + (n - 1)(8)\},$$

$$636 = n\{5 + 4n\} \text{ gives } n = 12]$$

$$(21) 960, [8 \times \{\text{sum of 15 natural Nos.}\}, 8 \times \left\{\frac{n(n+1)}{2}\right\}, 8 \times \left\{\frac{15(15+1)}{2}\right\}, 8\{120\}]$$

$$(22) 4920, [\text{Sum of first 40 positive integers which are divisible by 6} = 6, 12, 18, 24, \dots, 40$$

$$\text{terms} = 6 \times \{\text{sum of first 40 natural Nos.}\} = 6 \times \left\{\frac{40(40+1)}{2}\right\} = 4920]$$

$$(23) 625, [\text{use Sum of odd numbers} = n^2],$$

$$(24) \text{ do yourself}$$

$$(25) 3, 7, 11, 15 [a + 4d = 19 \text{ \& } \{(a + 12d) - (a + 7d)\} = 20, d = 4 \text{ \& } a = 3 \text{ then use } a, a + d, a + 2d, a + 3d]$$

$$(26) 370 \text{ m } [s = 2\{5 + 8 + 11 + \dots \text{ up to 10 terms}\} = 2 \times \frac{10}{2}\{2 \times 5 + (10 - 1) \times 3\} = 370 \text{ m}]$$

$$(27) 10 [\text{nth term} = a + (n - 1)d, 20.75 = 5 + (n - 1) \times 1.75, n = 10]$$

$$(28) \text{ A.P. is } 1, 6, 11 \dots [s_5 + s_7 = 167 \text{ means } 12a + 31d = 167$$

$$\text{ \& } s_{10} = 235 \text{ means } 2a + 9d = 47 \text{ then } a = 1 \text{ \& } d = 5]$$

$$(29) 852,$$

Soln: Let hundred's place digit = (a - d), ten's place digit = a, and unit's place digit = a + d

$$\text{ATQ, } a - d + a + a + d = 15 \Rightarrow 3a = 15 \Rightarrow a = 5$$

$$\text{Original number} = 100(a - d) + 10(a) + 1(a + d) = 100a - 100d + 10a + a + d = 111a - 99d$$

$$\text{Reversed number} = 1(a - d) + 10a + 100(a + d) = a - d + 10a + 100a + 100d = 111a + 99d$$

$$\text{Now, Original no.} - \text{Reversed no.} = 594$$

$$111a - 99d - (111a + 99d) = 594; -198d = 594 \Rightarrow d = -3$$

$$\therefore \text{The Original no.} = 111a - 99d = 111(5) - 99(-3) = 555 + 297 = 852$$

$$(30) 39900 [\text{AP: } 504, 511, 518, \dots, 896, \text{ apply last term} = a + (n - 1)d, 896 = 504 + (n - 1) \times 7$$

$$\text{ we get } n = 57 \text{ then sum of 57 terms} = \frac{57}{2}\{504 + 896\} = 39900]$$

$$(31) 5 \text{ minutes, Let the police catch the thief in } n \text{ min}$$

As the thief ran 1 min before the police

Time taken by the thief before being caught = (n + 1) min

Distance travelled by the thief in (n + 1) min = 100(n + 1) m

Speed of police in 1st min = 100m/min

Speed of police in 2nd min = 110m/min Speed of police in 3rd min = 120m/min. and so on

$\therefore 100, 110, 120, \dots$ this forms an AP

Total distance travelled by the police in n min $= \frac{n}{2}(2 \times 100 + (n-1)10)$

On catching the thief by police, distance travelled by thief = distance travelled by the police

$$\Rightarrow 100(n+1) = \frac{n}{2}(2 \times 100 + (n-1)10)$$

$$\Rightarrow 100n + 100 = 100n + n(n-1)5 \Rightarrow 100 = n(n-1)5 \Rightarrow n^2 - n - 20 = 0 \Rightarrow (n-5)(n+4) = 0$$

$$\Rightarrow n-5=0, n+4=0 \Rightarrow n=5 \text{ OR } n=-4 (\text{but this is not possible}) \text{ so, } n=5$$

Time taken by the policeman to catch the thief = 5 min

(32) 8, 12, 16, 20

Hint: Take four parts of an AP as $a-3d, a-d, a+d, a+3d$ and their sum is 56

Then find $a = 14$ and $d = \pm 2$

(33) 312, $[2 \times \{2 \times (1 + 2 + 3 + \dots + 12)\}]$

(34) 250, 230, 210, 190, 170, 150, 130, 110, 90, 70

(35) **The second term is 1**, The 3rd, 10th, and n th terms are $-1, -15$ and $(5 - 2n)$ respectively

(36) (i) Production in 6th year = 800 9th year = 1130, means $a_6 = 800$ & $a_9 = 1130$

Means $a+5d=800$ & $a+8d=1130$ we get $a=250$ and $d=110$ First year production = 250

(ii) 11th year, [apply n th term = $a+(n-1)d$, $1350=250+(n-1)110$ we get $n=11$]

(37) (i) There is an AP: 3, 5, 7, ... $a=3$ & $d=2$ so apply let there are n rows

$$\text{sum of } n \text{ terms} = \frac{n}{2}\{2a + (n-1)d\}, 120 = \frac{n}{2}\{2 \times 3 + (n-1)2\}$$

we get $n^2 + 2n - 120 = 0$ then $n = 10$ & -12 So there are 10 rows

(ii) 7th row = $3+(7-1) \times 2 = 3+12 = 15$ and 3rd row = $3+(3-1) \times 2 = 7$ their diff. = 8

(38) (i) 26 m $[s = 2\{5+8+11+\dots \text{up to } 2 \text{ terms}\}] = 2\{\frac{2}{2}\{2 \times 5 + (2-1) \times 3\}\} = 26 \text{ m}]$

(ii) 516 m $[s = 2\{5+8+11+\dots \text{up to } 12 \text{ terms}\}] = 2 \times \frac{12}{2}\{2 \times 5 + (12-1) \times 3\} = 516 \text{ m}]$

(39) (i) $a=51$ & $d=(-2)$ so n th term = $a+(n-1)d$, since goal is 31 minutes

so $31=51+(n-1) \times (-2)$ hence $n=11$ th day

(ii) common difference = $a_n - a_{n-1} = (2n+3) - \{2(n-1)+3\} = 3-1=2$

(40) (i) AP: 1000, 1100, 1200, ... so $a=1000, d=100$ so 30th instalment = $1000+29 \times 100=3900$

(ii) Total Amount paid = 118000

$$S_n = \frac{n}{2}\{2a + (n-1)d\}, 118000 = \frac{n}{2}\{2 \times 1000 + (n-1) \times 100\} = \frac{n}{2}\{1900 + 100n\}$$

We get $100n^2 + 1900n - 236000 = 0$ means $n^2 + 19n - 2360 = 0$

$n^2 + 59n - 40n - 2360 = 0$ means $(n+59)(n-40) = 0$ so $n = 40$