

# 2

## Fourier Series



### Multiple Choice Questions

**Q.1** Consider the following statements related to Fourier series of a periodic waveform:

1. It expresses the given periodic waveform as a combination of D.C. component, sine and cosine waveforms of different harmonic frequencies.
2. The amplitude spectrum is discrete.
3. The evaluation of Fourier coefficients gets simplified if waveform symmetries are used.
4. The amplitude spectrum is continuous.

Which of the above statements are correct?

- (a) 1, 2 and 4      (b) 2, 3 and 4  
(c) 1, 3 and 4      (d) 1, 2 and 3

[ESE-2002]

**Q.2** The Fourier series representation of an impulse train

denoted by  $s(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_0)$  is given by

- (a)  $\frac{1}{T_0} \sum_{n=-\infty}^{\infty} \exp\left(-\frac{j2\pi nt}{T_0}\right)$   
(b)  $\frac{1}{T_0} \sum_{n=-\infty}^{\infty} \exp\left(-\frac{j\pi nt}{T_0}\right)$   
(c)  $\frac{1}{T_0} \sum_{n=-\infty}^{\infty} \exp\left(\frac{j\pi nt}{T_0}\right)$   
(d)  $\frac{1}{T_0} \sum_{n=-\infty}^{\infty} \exp\left(\frac{j2\pi nt}{T_0}\right)$

[GATE-1999]

**Q.3** Which of the following cannot be the Fourier series expansion of a periodic signal?

- (a)  $x(t) = 2 \cos t + 3 \cos 3t$   
(b)  $x(t) = 2 \cos \pi t + 7 \cos t$   
(c)  $x(t) = \cos t + 0.5$   
(d)  $x(t) = 2 \cos 1.5 \pi t + \sin 3.5 \pi t$

[GATE-2002]

**Q.4** The Fourier series expansion of a real periodic signal with fundamental frequency  $f_0$  is given

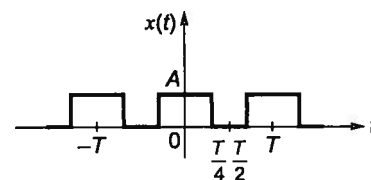
$$g_p(t) = \sum_{n=-\infty}^{\infty} C_n e^{j2\pi n f_0 t}$$

it is given that  $c_3 = 3 + j5$ . Then  $c_{-3}$  is

- (a)  $5 + j3$       (b)  $-3 - j5$   
(c)  $-5 + j3$       (d)  $3 - j5$

[GATE-2003]

**Q.5** Determine the Fourier series coefficient for given periodic signal  $x(t)$  is

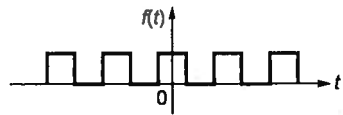


- (a)  $\frac{A}{j\pi k} \sin\left(\frac{\pi}{2}k\right)$       (b)  $\frac{A}{\pi jk} \cos\left(\frac{\pi}{2}k\right)$   
(c)  $\frac{A}{\pi k} \sin\left(\frac{\pi}{2}k\right)$       (d)  $\frac{A}{\pi k} \cos\left(\frac{\pi}{2}k\right)$

**Q.6** The Fourier expansion

$$f(t) = a_0 + \sum_{n=1}^{\infty} (a_n \cos n\omega t + b_n \sin n\omega t)$$

of the periodic signal shown below will contain the following nonzero terms



- (a)  $a_0$  and  $b_n$ ,  $n = 1, 3, 5, \dots \infty$   
 (b)  $a_0$  and  $a_n$ ,  $n = 1, 2, 3, \dots \infty$   
 (c)  $a_0$ ,  $a_n$  and  $b_n$ ,  $n = 1, 2, 3, \dots \infty$   
 (d)  $a_0$  and  $a_n$ ,  $n = 1, 3, 5, \dots \infty$

[ESE-2011]

Q.7 The signal  $x(t)$  has period 1 and the following Fourier co-efficients

$$C_k = \begin{cases} -\frac{1}{3} & k \geq 0 \\ 0 & k < 0 \end{cases}$$

What is  $x(t)$

- (a)  $x(t) = \frac{1}{1 - \frac{1}{3}e^{j2\pi t}}$  (b)  $x(t) = \frac{1}{1 + \frac{1}{3}e^{j2\pi t}}$   
 (c)  $x(t) = \frac{1}{1 + \frac{1}{3}e^{-j2\pi t}}$  (d)  $x(t) = \frac{1}{1 - \frac{1}{3}e^{-j2\pi t}}$

Q.8 Which of the following are Fourier co-efficient of an odd, real signal  $x(t)$

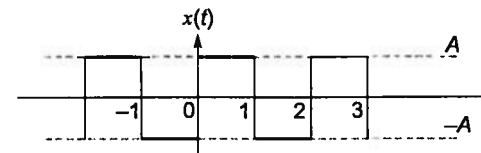
- (a)  $C_k = \frac{6j \cos \pi k}{\pi(9 - 4k^2)}$  (b)  $C_k = \frac{6jk \cos \pi k}{\pi(9 - 4k^2)}$   
 (c)  $C_k = \frac{6 \cos \pi k}{\pi(9 - 4k^2)}$  (d) None of the above

Q.9  $x(t) = C_n = \begin{cases} 2 & n = 0 \\ j\left(\frac{1}{2}\right)^{|n|} & \text{otherwise} \end{cases}$

Which of the following true?

- (a)  $x(t)$  is a real valued signal  
 (b)  $x(t)$  is an even signal  
 (c)  $\frac{dx(t)}{dt}$  is an even signal.  
 (d) both (b) and (c)

Q.10 The periodic signal  $x(t)$  is shown below, the exponential Fourier series coefficients ( $C_n$ ) of the signal is



- (a)  $\frac{A}{n\pi}[1 - (-1)^n]$  (b)  $\frac{A}{n\pi}[1 + (-1)^n]$   
 (c)  $\frac{A}{jn\pi}[1 - (-1)^n]$  (d)  $\frac{A}{jn\pi}[1 + (-1)^n]$

Q.11 Suppose we have given following information about a signal  $x(t)$ .

- (i)  $x(t)$  is real odd  
 (ii)  $x(t)$  is periodic with  $T = 2$   
 (iii) Fourier coefficients ( $x = 0, |x| > 1$ )

(iv)  $\frac{1}{2} \int_0^2 |x(t)|^2 dt = 1$

The signal that satisfy these conditions

- (a)  $\sqrt{2} \sin \pi t$  & unique  
 (b)  $\sqrt{2} \sin \pi t$  but not unique  
 (c)  $2 \sin \pi t$  & unique  
 (d)  $2 \sin \pi t$  but not unique

Q.12  $x(t)$  is a real valued function of a real variable with period  $T$ . Its trigonometric Fourier Series expansion contains no terms of frequency  $\omega = 2\pi(2k)/T$ ;  $k = 1, 2, \dots$ . Also, no sine terms are present. Then  $x(t)$  satisfies the equation

- (a)  $x(t) = -x(t - T)$   
 (b)  $x(t) = x(T - t) = -x(-t)$   
 (c)  $x(t) = x(T - t) = -x(t - T/2)$   
 (d)  $x(t) = x(t - T) = x(t - T/2)$  [GATE-2006]

Q.13 A signal  $x(t)$  is given by

$$x(t) = \begin{cases} 1, & -T/4 < t \leq 3T/4 \\ -1, & 3T/4 < t \leq 7T/4 \\ -x(t + T) \end{cases}$$

Which among the following gives the fundamental Fourier term of  $x(t)$ ?

- (a)  $\frac{4}{\pi} \cos\left(\frac{\pi t}{T} - \frac{\pi}{4}\right)$  (b)  $\frac{\pi}{4} \cos\left(\frac{\pi t}{2T} + \frac{\pi}{4}\right)$   
 (c)  $\frac{4}{\pi} \sin\left(\frac{\pi t}{T} - \frac{\pi}{4}\right)$  (d)  $\frac{\pi}{4} \sin\left(\frac{\pi t}{2T} + \frac{\pi}{4}\right)$

[GATE-2007]

Q.14 The Fourier Series coefficients, of a periodic signal  $x(t)$  expressed as

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{j2\pi kt/T}$$

are given by  $a_{-2} = 2 - j1$ ;  $a_{-1} = 0.5 + j0.2$ ;  $a_0 = j2$ ;  $a_1 = 0.5 - j0.2$ ;  $a_2 = 2 + j1$ ; and  $a_k = 0$ ; for  $|k| > 2$ . Which of the following is true?

- (a)  $x(t)$  has finite energy because only finitely many coefficients are non-zero  
 (b)  $x(t)$  has zero average value because it is periodic  
 (c) The imaginary part of  $x(t)$  is constant  
 (d) The real part of  $x(t)$  is even

[GATE-2009]

Q.15 The Fourier series of an odd periodic function, contains only

- (a) odd harmonics (b) even harmonics  
 (c) cosine terms (d) sine terms

[GATE-1994]

Q.16 The trigonometric Fourier series of an even function does not have the

- (a) DC term  
 (b) cosine terms  
 (c) sine terms  
 (d) odd harmonic terms

[GATE-2011]

Q.17 Consider two signals  $x_1(t) = e^{j20t}$  and  $x_2(t) = e^{(-2+j)t}$ .

Which one of the following statements is correct?

- (a) Both  $x_1(t)$  and  $x_2(t)$  are periodic  
 (b)  $x_1(t)$  is periodic but  $x_2(t)$  is not periodic  
 (c)  $x_2(t)$  is periodic but  $x_1(t)$  is not periodic  
 (d) Neither  $x_1(t)$  nor  $x_2(t)$  is periodic

[ESE-2007]



### Numerical Data Type Questions

Q.18 The signal  $x(t)$  has period 2 and the following Fourier co-efficients

$$C_k = \begin{cases} \left(\frac{1}{2}\right)^k & k \geq 0 \\ 0 & k < 0 \end{cases}$$

value of  $x(0)$  will be \_\_\_\_\_.

Q.19 Let  $x(t)$  be a periodic signal with time period  $T$ . Let  $y(t) = x(t - t_0) + x(t + t_0)$  for some  $t_0$ . The Fourier Series coefficients of  $y(t)$  are denoted by  $b_k$ . If  $b_k = 0$  for all odd  $k$ , then  $t_0$  can be equal to \_\_\_\_\_  $T$ .

[GATE-2008]

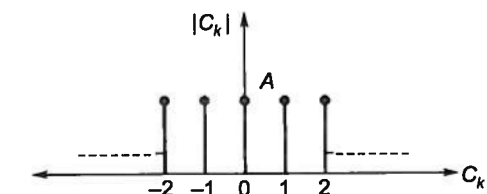
Q.20 A discrete time signal is given by

$$x[n] = \cos^2\left[\frac{\pi}{8}n\right]$$

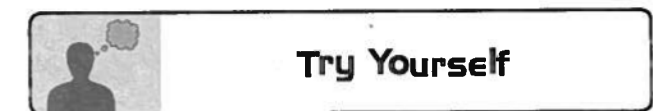
If the complex Fourier series coefficients of the signal are represented as  $C_k$ . Then the value of  $C_{15}$  is \_\_\_\_\_.

Q.21 Consider a sequence  $x[n] = \sum_{k=-\infty}^{\infty} \delta[n - 4k]$

then the complex exponential Fourier series coefficient  $C_k$  of  $x[n]$  can be represented as



then the value of  $A$  is \_\_\_\_\_?



### Try Yourself

T1. Let  $x(t)$  be a periodic signal with fundamental period  $T$  and Fourier series coefficients  $a_k$ . The Fourier series coefficient of  $\text{Re}\{x(t)\}$  (Where  $\text{Re}$  denotes the real part of the signal) is

- (a)  $\frac{a_k + a_{-k}^*}{2}$  (b)  $\frac{a_k - a_{-k}^*}{2}$   
 (c)  $\frac{a_k^* + a_{-k}}{2}$  (d)  $\frac{a_k^* - a_{-k}}{2}$

[Ans: (a)]

T2. Choose the function  $f(t)$ ,  $-\infty < t < \infty$ , for which a Fourier series cannot be defined.

- (a)  $3 \sin(25t)$   
 (b)  $4 \cos(20t + 3) + 2 \sin(710t)$   
 (c)  $\exp(-|t|) \sin(25t)$   
 (d) 1

[Ans: (c)]

T3. Which of the following statement is wrong about fourier series complex coefficient for a real time signal

- (a)  $C_n$  has even symmetric real part
- (b)  $C_n$  has odd symmetric imaginary part
- (c)  $C_n$  has even symmetric magnitude
- (d)  $C_n$  has even symmetric phase

[Ans: (c)]

T4. A signal is such that  $x(t) = -x(t + T_o/2)$ . It also given that it is even in nature. The fourier series expansion has

- (a) absent sine terms
- (b) absent cos terms
- (c) only odd harmonics
- (d) odd terms of cos as  $\sum a_n \cos n \omega$

[Ans: (d)]

T5. A signal has fouriers series coeffs

$$C_n \Rightarrow C_{-2} = 2 = C_2, C_{-1} = C_1 = 8, C_0 = 0.$$

Its power\_\_\_\_\_.

[Ans: (136)]

