3. MOTION IN A PLANE

1. INTRODUCTION

Motion in a plane is a two dimensional motion. The analysis of this type of motion becomes easy when we consider this motion as a combination of two straight line motions along two mutually perpendicular axes lying in the plane of motion. In Cartesian coordinate system the two mutually perpendicular axes are the x-axis and the y-axis respectively. The displacement, velocity and acceleration of the particle are resolved into components along the x and y axes and motion along each axis is studied independent of the other. The net displacement, velocity or acceleration is the vector sum of their respective components along the two axes. In this chapter we will discuss about the motion of a projectile, the motion of a body relative to another body, the motion of a body in a river, the motion of an airplane with respect to wind, and circular motion.

2. MOTION IN A PLANE

When a body moves in a straight line, we call it motion in a straight line or one dimension. For eg, a car going straight on a road. When you throw a ball towards your friend, the ball follows a non-linear path. This motion is termed as motion in two dimensions or motion in a plane.

The position of a particle that is free to move can be located by two coordinates in a plane. We choose the plane of motion as the X-Y plane. We choose a suitable instant as t = 0 and choose the origin at the place where the particle is situated at t = 0. Any two convenient mutually perpendicular directions in the X-Y plane are chosen as the X and Y-axes.

3. PROJECTILE MOTION

Projectile motion is a form of motion in which an object or particle (here called a projectile) is thrown in an oblique direction near the earth's surface, and it moves along a curved path under the action of a continuous motive force. The path observed during a projectile motion is called its trajectory. Projectile motion is possible only when there is one force applied at the beginning of the trajectory, after which there is no force in operation except a constant force.

3.1 Ground-To-Ground Projectile

In the Fig. 3.2 shown, let us consider the horizontal surface through the point O. Now, the point O here is called the point of projection, the angle θ is called the angle of projection and the distance OB is called the horizontal range or simply range. Further, the total time taken by the particle in describing the path OAB is called the time of flight.

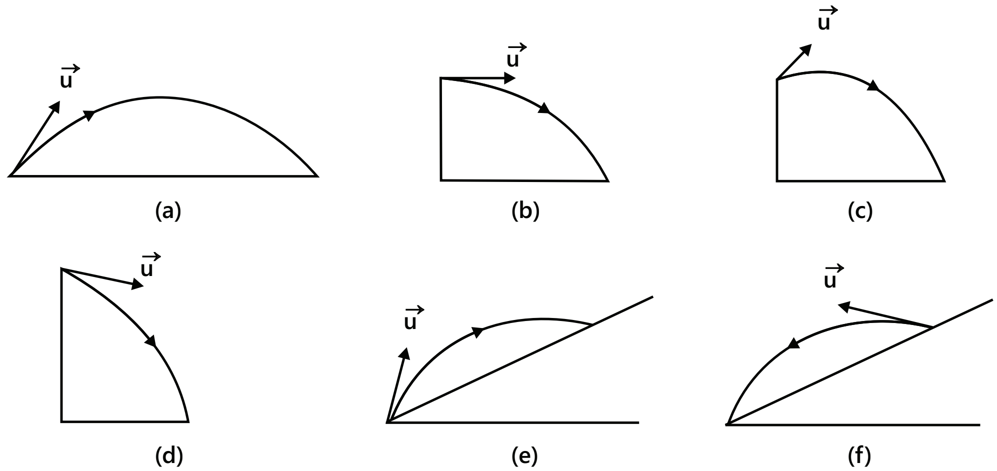

Figure 3.1

However, we can separately discuss the motion of the projectile for both the horizontal and vertical parts. In this regard, we begin by considering the origin as the point of projection.

Now, we have $u_x = u\cos\theta;\ a_x = 0;\qquad u_y = u\sin\theta;\ a_y = -g.$

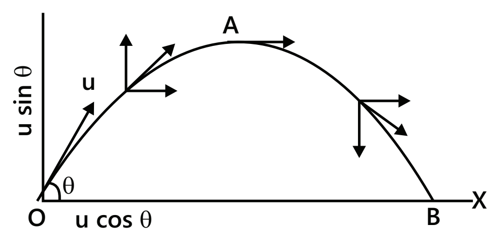

Figure 3.2

3.1.1 Horizontal Motion

As $\alpha_x = 0$, we have $\nu_x = u_x + \alpha_x t = u_x = u\cos\theta$ and $x = u_x t + \frac{1}{2}\alpha_x t^2 = u_x t = ut\cos\theta$

3.1.2 Vertical Motion

In the downward direction, we know that the acceleration of the particle is g. Thus, $\alpha_y = -g.$

Further, the y-component of the initial velocity is $u_{y.}$ Thus,

$\nu_y = u_y - gt$ and $y = u_y t - \frac{1}{2}gt^2;$ also we have, $\nu_y^2 = u_y^2 - 2gy.$

3.1.3 Time of Flight

Let us suppose that the particle is at B at time t. Therefore, the equation for horizontal motion gives OB = x = ut $\cos\theta$.

However, the y-coordinate at the point B is zero. Thus, from the equation of vertical motion,

$y = ut\sin\theta - \frac{1}{2}gt^2$ or, $0 = ut\sin\theta - \frac{1}{2}gt^2$ or, $t(u\sin\theta - \frac{1}{2}gt) = 0$

Thus, either t = 0 or, $t = \dfrac{2u\sin\theta}{g}$

Now, t = 0 exactly corresponds to the initial position O of the particle. Hence, the time at which it reaches B is thus,

$$T = \frac{2u\sin\theta}{g}$$

This equation helps us to exactly calculate the time of flight.

3.1.4 Range

Consider the distance OB covered by a particle, which is the horizontal range. It is the distance travelled by the particle in time $T = \frac{2u\sin\theta}{g}$

By the equation of horizontal motion, $x = (u\cos\theta) \times T$ or, $OB = \frac{2u^2 \sin\theta\cos\theta}{g} = \frac{u^2 \sin 2\theta}{g}$

3.1.5 Maximum Height

We have, $v_y = u_y - gt = u\sin\theta - gt$

However, at the maximum height, $0 = u\sin\theta - gt$ or, $t = \frac{u\sin\theta}{g}$

The actual maximum height is $H = u_y t - \frac{1}{2}gt^2 = (u\sin\theta)\left(\frac{u\sin\theta}{g}\right) - \frac{1}{2}g\left(\frac{u\sin\theta}{g}\right)^2$

$$= \frac{u^2\sin^2\theta}{g} - \frac{1}{2}\frac{u^2\sin^2\theta}{g} = \frac{u^2\sin^2\theta}{2g}$$

4. EQUATION OF TRAJECTORY OF A PROJECTILE

$x = (u\cos\alpha)t \qquad \therefore\ t = \frac{x}{u\cos\alpha}$

By substituting this value of t in $y = (u\sin\alpha)t - \frac{1}{2}gt^2$, we obtain

$$y = x\tan\alpha - \frac{gx^2}{2u^2\cos^2\alpha} = x\tan\alpha - \frac{gx^2}{2u^2}\sec^2\alpha = x\tan\alpha - \frac{gx^2}{2u^2}(1+\tan^2\alpha)$$

The above are the standard equations of trajectory of any projectile. Here, we should be aware of the fact that the equation is quadratic in x. This is why the path of a projectile is always a parabola. Further, the above equation can also be represented in terms of range (R) of the projectile as $y = x\left(1 - \frac{x}{R}\right)\tan\alpha$

Illustration 1: Assume that a ball is thrown from a field at a speed of 12.0 m/s and at an angle of 45° with the horizontal. At what distance will it hit the field again? Take g = 10.0 m/s^2. **(JEE MAIN)**

Sol: Use the formula for the range of a projectile.

The horizontal range $= \frac{u^2\sin 2\theta}{g} = \frac{(12\,m/s)^2 \times \sin(2\times 45^\circ)}{10\,m/s^2} = \frac{144\,m^2/s^2}{10.0\,m/s^2} = 14.4\,m$

Thus, the ball hits the field exactly at 14.4 m from the point of projection.

PLANCESS CONCEPTS

(i) Range is maximum where $2\alpha = 1$ or $\alpha = 45^\circ$ and this maximum range is:

$$R_{max} = \frac{u^2}{g} = 4H$$

(ii) For given value of u, range at α and range at ϕ are equal although times of flight and maximum heights may be different. Because

$$R_{90^o-\alpha} = \frac{u^2\sin 2(90^o-\alpha)}{g} = \frac{u^2\sin(180^o-2\alpha)}{g} = \frac{u^2\sin 2\alpha}{g} = R_\alpha$$

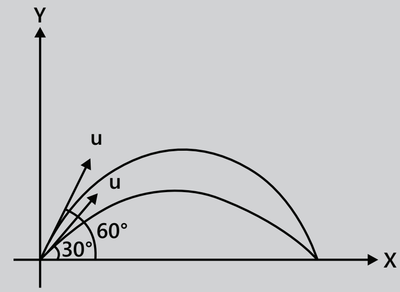

Figure 3.3

As we have seen in the above derivations that $a_x = 0$, i.e., motion of the projectile in the horizontal direction is uniform. Hence, horizontal component of velocity u $\cos\alpha$ does not change during its motion. Motion in the vertical direction is first retarded and then accelerated in opposite direction. As the equation of trajectory of projectile is of the form, $y = ax - bx^2$ (equation of parabola), therefore, the path followed by a projectile is a parabola.

B Rajiv Reddy (JEE 2012, AIR 11)

Illustration 2: Find the angle of a projectile for which both the horizontal range and maximum height are equal. **(JEE MAIN)**

Sol: Use the formula for the range and maximum height of a projectile.

Given, R = H

$$\therefore \quad \frac{u^2\sin 2\alpha}{g} = \frac{u^2\sin^2\alpha}{2g} \quad \text{or} \quad 2\sin\alpha\cos\alpha = \frac{\sin^2\alpha}{2} \quad \text{or} \quad \frac{\sin\alpha}{\cos\alpha} = 4 \text{ or } \tan\alpha = 4 \quad \alpha = \tan^{-1}(4)$$

Illustration 3: The given Fig. 3.4 shows a pirate ship 560 m from a fort defending a harbor entrance. A defense canon, located at sea level, fires balls at initial speed $v_o = 82 m/s$ **(JEE ADVANCED)**

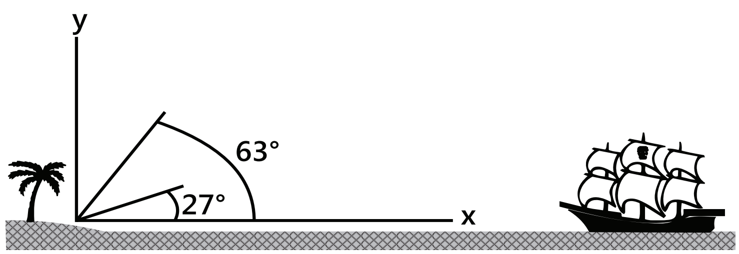

Figure 3.4

(a) At what angle θ_o from the horizontal must a ball be fired to hit the ship?

Sol: Use the formula for the range of a projectile to find the angle of projection.

We can relate the launch angle θ_o to the range R with Eq. $(R = (v_0^2/g)\sin 2\theta_o)$, which, after rearrangement, gives

$$\theta_o = \frac{1}{2}\sin^{-1}\frac{gR}{v_0^2} = \frac{1}{2}\sin^{-1}\frac{(9.8 m/s^2)(560 m)}{(82 m/s)^2} = \frac{1}{2}\sin^{-1}0.816$$

One solution of (54.7°) is worked out using a calculator; now, we subtract it from 180° to get the other solution (125.3°). This gives us $\theta_0 = 27^\circ$ and $\theta_0 = 63^\circ$.

Illustration 4: Suppose a batsman B hits a high-fly ball to the outfield, directly toward an outfielder F and with a launch speed of $v_\circ = 40\,m/s$ and a launch angle of $\theta_0 = 35^\circ$. During the flight, a line from the outfielder to the ball makes an angle ϕ with the ground. Based on the data provided, plot the elevation angle $\tan\theta = 2\cot\alpha$ versus time t, assuming that the outfielder is (a) already positioned to catch the ball, (b) is 6.0 m too close to the batsman and (c) is 6.0 m too far away. **(JEE ADVANCED)**

Sol: While trying to catch a ball which has gone to a great height you can imagine that the angle of line of sight increases as the ball moves. If we neglect air drag, then the ball is a projectile for which the vertical motion and the horizontal motion can be analyzed individually.

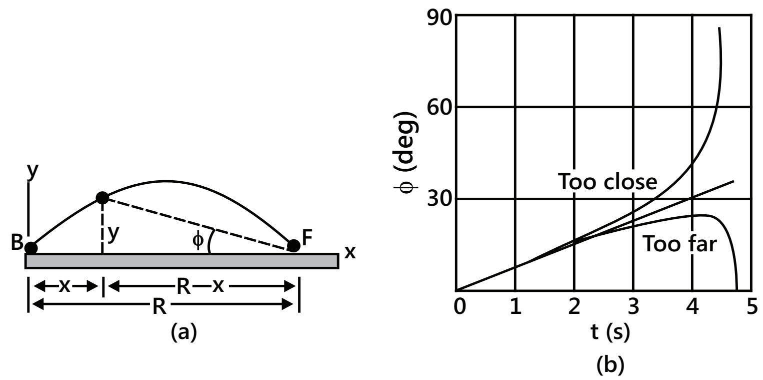

Figure 3.5

Assuming that the ball is caught at approximately the height it is hit, the horizontal distance traveled by the ball is the range R, given by Eq. $(R = (v_0^2/g)\sin 2\theta_\circ)$

The ball can be caught if the outfielder's distance from the batsman equals the range R of the ball. Using the above equation, we find the elevation angle ϕ for a ball that was hit toward an outfielder is (a) defined and (b) plotted versus time t.

$$R = \frac{v_0^2}{g}\sin 2\theta_\circ = \frac{(40\,m/s)^2}{9.8\,m/s^2}\sin(70^\circ) = 153.42\,m$$

Fig. 3.5 (a) above shows a snapshot of the ball in flight when the ball is at height y and horizontal distance x from the batsman (who is at the origin). The horizontal distance of the ball from the outfielder is R – x, and the elevation angle ϕ of the ball in the outfielder's view is given by $\tan\phi = y/(R-x)$.

Thus, using $v_0 = 40$ m/s and $\theta_0 = 35^\circ$, we have $\phi = \tan^{-1}\dfrac{(40\sin 35^\circ)t - 4.9t^2}{153.42 - (40\cos 35^\circ)t}$

By graphing this function versus t gives us the middle plot in b. We now see that the ball's angle in the outfielder's view increases at an almost steady rate throughout the flight.

If the outfielder is 6.0 m close to the batsman, then we replace the distance of 153.42 m in the given equation with 153.42 m – 6.0 m = 147.42 m. Further, regraphing the function gives the "Too close" plot as in Fig. 3.5 (b).

Now, we observe that the elevation angle of the ball rapidly increases toward the end of the flight as the ball soars over the outfielder's head. However, if the outfielder is 6.0 m too far away from the batsman then we replace the distance of 153.42 m in the equation with 159.42 m. The resulting plot is hence labeled "Too far" in the Fig. 3.5: The angle first increases and thereafter rapidly decreases.

Conclude: Thus, if a ball is hit directly toward an outfielder, then the player can tell from the change in the ball's elevation angle ϕ whether to stay put, run toward the batter, or back away from the batsman.

Illustration 5: Suppose that a projectile is fired horizontally with a velocity of 98 m/s from the top of a hill that is 490 m high. Find:

(a) The time taken by the projectile to reach the ground,

(b) The distance of the point where the particle hits the ground from the foot of the hill and

(c) The velocity with which the projectile hits the ground. (take $g = 9.8\,m/s^2$) **(JEE MAIN)**

Sol: Let x-axis be along the horizontal and the y-axis be along the vertical. The projectile will have uniform velocity along the positive x-axis and uniform acceleration along the negative y-axis.

In this problem, we cannot apply the formulae of R, H and T directly. Necessarily we have to follow the three steps discussed in the theory. Here, however, it will be more convenient to choose x and y directions as shown in the Fig. 3.6 provided.

Here, $u_x = 98\,m/s, a_x = 0, u_y = 0$ and $a_y = g$

(a) At A, $S_y = 490\,m$. Therefore, applying $s_y = u_y t + \frac{1}{2} a_y t^2$

$\therefore \quad 490 = 0 + \frac{1}{2}(9.8)t^2 \quad \therefore \quad t = 10s$

(b) $BA = s_x = u_x t + \frac{1}{2} a_x t^2$ or $BA = (98)(10) + 0$ or $BA = 980\,m$

(c) $\nu_x = u_x = 980 m/s; \quad v_y = u_y + a_y t = 0 + (9.8)(10) = 98 m/s$

$\therefore \quad \nu = \sqrt{\nu_x^2 + \nu_y^2} = \sqrt{(98)^2 + (98)^2} = 98\sqrt{2}\,m/s$

and $\tan\beta = \frac{v_y}{v_x} = \frac{98}{98} = 1 \quad \therefore \quad \beta = 45^\circ$

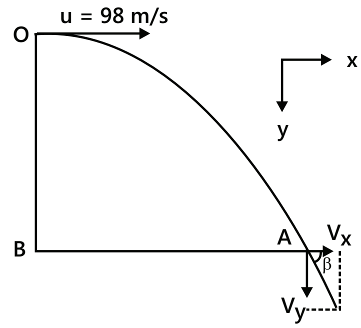

Figure 3.6

Thus, we show that the projectile hits the ground with a velocity $98\sqrt{2}\,m/s$ at an angle of $\beta = 45^\circ$ with horizontal as shown in the Fig. 3.6 provided.

5. PROJECTILE MOTION ON AN INCLINED PLANE

Projectile motion on an incline plane is one of the various types of projectile motion. However, the main distinguishing aspect is that points of projection and return are not on the same horizontal plane.

We know that there are two possibilities in this regard: (i) the point of return is at a higher level than the point of projection i.e., projectile is thrown up the incline and (ii) the point of return is at a lower level than the point of projection, i.e., the projectile is thrown down the inclined plane.

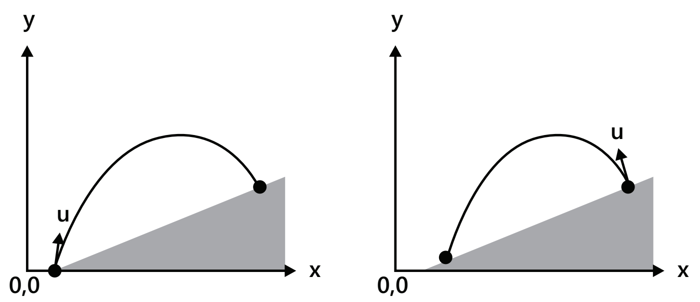

Figure 3.7

5.1 Analyzing Motion

We can make us of two different approaches of analyzing projectile motion on an inclined plane. The first approach preferably could be to continue analyzing motion in two mutually perpendicular horizontal and vertical directions. The second approach, therefore, could be to analyze motion by changing the reference orientation, i.e., we set up our coordinate system along the incline and a direction along the perpendicular to the incline.

Based on the analysis, alternatives are, therefore, distinguished on the basis of coordinate system that we choose to employ:

(a) Planar coordinates along the incline (x) and perpendicular to the incline (y)

(b) Planar coordinates in horizontal (x) and vertical (y) directions

However, we use the first approach for analyzing this kind of motion, i.e., coordinates along the incline (x) and perpendicular to the incline (y).

5.2 Projectile Motion Up an Inclined Plane

Based on the details provided in the Fig. 3.8, it is clear that the angle that the velocity of projection makes with the x-axis (i.e., incline) is " $\theta - a$ ".

Therefore, the components of initial velocity are

$u_x = u\cos(\theta-\alpha);\quad u_y = u\sin(\theta-\alpha)$

Hence, the components of acceleration are

$a_x = -g\sin\alpha;\quad a_y = -g\cos\alpha$

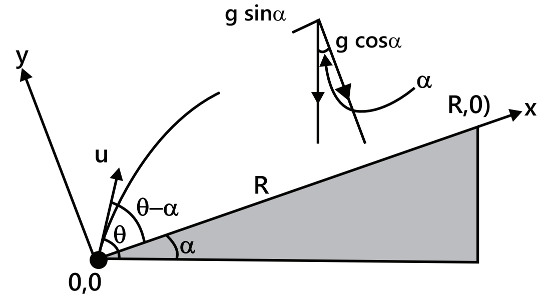

Figure 3.8

5.2.1 Time of Flight

The time of flight (T) is calculated by analyzing motion in y-direction (which is not vertical as in the normal case). However, the displacement in the y-direction after the projectile has returned to the incline is zero as in the normal case. Thus,

$$y = u_y T + \frac{1}{2}a_y T^2 = 0 \;\Rightarrow u\sin(\theta-\alpha)T + \frac{1}{2}(-g\cos\alpha)T^2 = 0 \;\Rightarrow T\left\{u\sin(\theta-\alpha)T + \frac{1}{2}(-g\cos\alpha)T\right\} = 0;\; T = 0$$

$$\Rightarrow T = \frac{2u\sin(\theta-\alpha)}{g\cos\alpha}$$

Here, the first value represents the initial time of projection. Hence, the second expression gives us the time of flight. However, we should note here that the expression of time of flight is as in a normal case albeit in a significant manner.

In the generic form, we can express the formula of the time of flight as: $T = \left|\frac{2u_y}{a_y}\right|$

5.2.2 Range of Flight

The first thing that we should note that we do not use the term "horizontal range" as the range on the inclined plane is no more horizontal. Rather, we simply refer the displacement along the x-axis as "range". Thus, we can find range of flight by considering motion in both "x" and "y" directions. We further note that we utilize the same approach even in the normal case. Now, let "R" be the range of the projectile motion.

Substituting the value of "T" as obtained earlier, we have $x = u_x T - \frac{1}{2}a_x T^2$

$$R = \frac{u\cos(\theta-\alpha)\times 2u\sin(\theta-\alpha)}{g\cos\alpha} - \frac{g\sin\alpha\times 4u^2\sin^2(\theta-\alpha)}{2g^2\cos^2\alpha}$$

$$\Rightarrow R = \frac{u^2}{g\cos^2\alpha}\left\{2\cos(\theta-\alpha)\sin(\theta-\alpha)\cos\alpha - \sin\alpha\times 2\sin^2(\theta-\alpha)\right\}$$

Using trigonometric relation, $2\sin^2(\theta-\alpha) = 1 - \cos2(\theta-\alpha)$,

$$\Rightarrow R = \frac{u^2}{g\cos^2\alpha}\left[\sin2(\theta-\alpha)\cos\alpha - \sin\alpha\{1-\cos2(\theta-\alpha)\}\right]$$

$$\Rightarrow R = \frac{u^2}{g\cos^2\alpha}\{\sin2(\theta-\alpha)\cos\alpha - \sin\alpha + \sin\alpha\cos2(\theta-\alpha)\}$$

Now, we use the trigonometric relation, $\sin(A+B) = \sin A\cos B + \cos A\sin B$

$$\Rightarrow R = \frac{u^2}{g\cos^2\alpha}\{\sin(2\theta-2\alpha+\alpha) - \sin\alpha\} \Rightarrow R = \frac{u^2}{g\cos^2\alpha}\{\sin(2\theta-\alpha) - \sin\alpha\}$$

This is the exact expression for the range of projectile on an inclined plane. We also note that this expression reduces to the one for the normal case, when $\alpha = 0 \quad \Rightarrow R = \frac{u^2 \sin 2\theta}{g}$

5.3 Projectile Motion down the Inclined Plane

The components of initial velocity: $u_x = u\cos(\theta+\alpha)$; $u_y = u\sin(\theta+\alpha)$

The components of acceleration: $a_x = g\sin\alpha$; $a_y = -g\cos\alpha$

Time of flight

The expression for the time of flight differs only with respect to angle of sine function in the numerator of the expression: $T = \frac{2u\sin(\theta+\alpha)}{g\cos\alpha}$

Range of flight

In the same way, the expression of range of flight differs only with respect to angle of sine function: $R = \frac{u^2}{g\cos^2\alpha}\{\sin(2\theta+\alpha)+\sin\alpha\}$

PLANCESS CONCEPTS

It is very handy to note that expressions have changed only with respect of the sign of "α" for the time of flight and the range. We only need to exchange "α" by"$-\alpha$ s".

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Illustration 6: Assume that a projectile is thrown from the base of an incline of angle 30° as shown in the Fig. 3.9 provided. It is thrown at an angle of 60° from the horizontal direction at a speed of 10 m/s. Calculate the total time of flight is (consider g = 10 m/s²). **(JEE MAIN)**

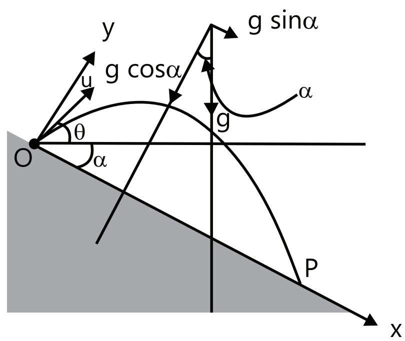


Figure 3.9

Sol: The x-axis has to be assumed along the inclined.

This problem can be handled with a reoriented coordinate system as shown in the Fig. 3.9 provided. Here, the angle of projection with respect to x-direction is ($\theta -$ a) and acceleration in y-direction is "g cosa". Now, the total time of flight for the projectile motion, when the point of projection and return are on the same level, is

$$\Rightarrow T = \frac{2u\sin(\theta - a)}{g\cos\alpha}$$

Now, $\theta = 60^\circ, a = 30^\circ$, u = 10m/s. Then, by substituting these values, we finally obtain

$$\Rightarrow T = \frac{2X10\sin(60^\circ - 30^\circ)}{g\cos 30^\circ} = \frac{20\sin 30^\circ}{10\cos 30^\circ} = \frac{2}{\sqrt{3}}$$

Illustration 7: Consider that two projectiles are thrown with the same speed from point "O" and "A" so that they hit the incline. If t_0 and t_A be the time of flight in two cases, then prove which option out of those given here is true. **(JEE MAIN)**

(A) $t_0 = t_A$ (B) t_0 & t_A (C) $t_0 > t_A$ (D) $t_0 = t_A = \frac{u\tan\theta}{g}$

Sol: The x-axis has to be assumed along the inclined. Use the formula for time of flight on an inclined. Let us first consider the projectile thrown from the point "O". Considering the angle the velocity vector makes with the horizontal, we represent the time of flight as:

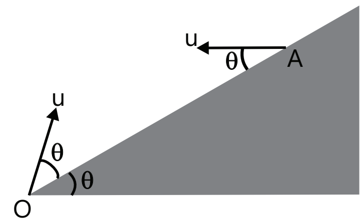


Figure 3.10

$$\Rightarrow t_0 = \frac{2u\sin(2\theta-\theta)}{g\cos\theta} \quad \Rightarrow t_0 = \frac{2u\tan\theta}{g}$$

Further, for the projectile thrown from the point "A", the angle with horizontal is zero. Hence, the time of flight is expressed as

$$\Rightarrow t_A = \frac{2u\sin(2X0-\theta)}{g\cos\theta} = \frac{2u\tan\theta}{g}$$

Thus, we observe that the times of flight in the two cases are equal.

$$\Rightarrow t_A = t_0$$

Hence, option (A) is correct.

Illustration 8: Two inclined planes of angles 30° and 60° are placed so that they touch each other at the base as shown in the Fig. 3.11 provided. Further, a projectile is projected at right angle at a speed of $10\sqrt{3}$ m/s from point "P" and hits the other incline at point "Q" normally. Then, the time of flight is: **(JEE ADVANCED)**

(A) 1 s (B) 2 s (C) 3 s (D) 4s

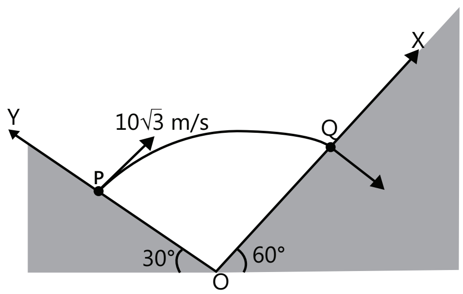


Figure 3.11

Sol: This problem is a specific case in which the inclined planes are right angles with respect to each other. Therefore, we actually take advantage of this fact in assigning our coordinates along the planes, say y-axis along first incline and x-axis against second incline.

Thus, in order to find the time of flight, we can further use the fact that projectile hits the other plane at right angle, i.e., parallel to the y-axis. This means that the component of velocity in x-direction, i.e., along the second incline is zero. This, in turn, suggests that we can analyze motion in x-direction to obtain the time of flight.

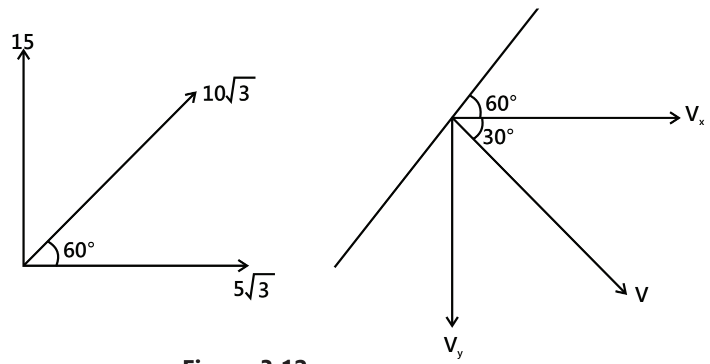


Figure 3.12

$V_x = 10\sqrt{3}\cos 60 = 5\sqrt{3}$ will remain constant

$\therefore$ Here, $V\cos 60 = 5\sqrt{3}$

$\therefore$ $V = \frac{5\sqrt{3}}{\sqrt{3}} X2 = 10 \quad \therefore V_y = -V\sin 30 = \frac{-V}{2} = -5$

$\therefore$ $V = u - gt \qquad \therefore -5 = 15 - 10t$

$\therefore$ $10t = 20 \qquad \therefore \boxed{t = 2s}$

PLANCESS CONCEPTS

Given are a simple set of guidelines in a very general way:

- Analyze motion independently along the selected coordinates for complicated problems. For simple cases, try remembering derived formula and use them directly to save time.
- Make note of information given in the question like angles, etc., which might render certain components of velocity zero in certain direction.
- If range of the projectile is given, we may try the trigonometric ratio of the incline itself to get the answer.
- If we use coordinate system along incline and in the direction perpendicular to it, then always remember that component of motion along both incline and in the direction perpendicular to it are accelerated motions. Ensure that we use appropriate components of acceleration in the equations of motion.
- The range is maximum for maximum value of "sin(2θ–α)". Thus, the range is maximum for the angle of projection as measured from horizontal direction, when

$$\sin(2\theta-\alpha)=1 \Rightarrow \sin(2\theta-\alpha)=\sin\pi/2$$

$$\Rightarrow\ 2\theta-\alpha=\pi/2\ \Rightarrow\ \theta=\pi/4+\alpha/2 \Rightarrow\ R_{max}=\frac{u^2}{g\cos^2\alpha}(1-\sin\alpha)$$

Anand K (JEE 2011, AIR 47)

6. RELATIVE MOTION

The measurements describing motion are generally subject to the state of motion of the frame of reference with respect to which measurements are taken about. Our day-to-day perception of motion is generally based on our earth's view—a view common to all bodies at rest with respect to earth. However, we come across cases when there is a subtle perceptible change in our view of earth. One such case is traveling in the city trains. We easily find that it takes lot longer to overtake another train on a parallel track. Also, we happen to see two people talking while driving separate cars in parallel lanes, as if they were stationary to each other! In terms of kinematics, as a matter of fact, they are actually stationary to each other even though each of them is in motion with respect to ground.

In this topic, we study motion from a perspective other than that of our earth. The only condition that we subject ourselves is that two references or two observers making the measurements of motion of an object, are moving at constant velocity.

We now consider two moving observers, "A" and "B":

The relative velocity of A with respect of B (written as $\vec{v}_{AB}$) is $\vec{v}_{AB}=\vec{v}_A-\vec{v}_B$

Similarly, the relative acceleration of A with respect to B is $\vec{a}_{AB}=\vec{a}_A-\vec{a}_B$

Illustration 9: Assume that two cars, standing apart, start moving toward each other at speeds of 1 m/s and 2 m/s along a straight road. What could be the speed with which they approach each other?

(JEE MAIN)

Sol: Let us consider that "A" denotes earth, "B" denotes the first car and "C" denotes the second car. Therefore, the equation of relative velocity for this case is: $v_{BA}=1\,m/s$ and $v_{CA}=-2\,m/s$.

$$v_{CA}=v_{BA}+v_{CB}\ \Rightarrow -2=1+v_{CB}\ \Rightarrow v_{CB}=-2-1=-3\,m/s$$

This implies that the car "C" is approaching "B" at a speed of –3 m/s along the straight road. Further, it also means that the car "B" is approaching

Relative velocity

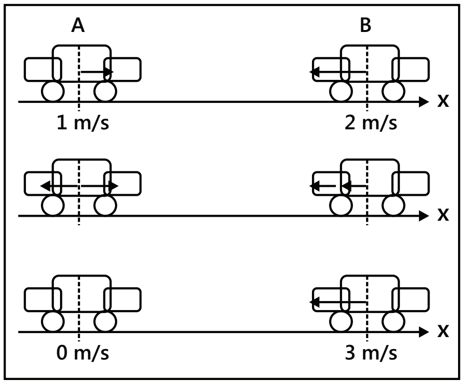


Figure 3.13

"C" at a speed of 3 m/s along the straight road. We, therefore, say that the two cars approach each other at a relative speed of 3 m/s.

To evaluate relative velocity, we proceed as follows:

- Apply velocity of the reference object (say object "A") to other object(s) and hence render the reference object at rest.
- The resultant velocity of the other object ("B") is therefore equal to relative velocity of "B" with respect to "A".

PLANCESS CONCEPTS

- The foremost thing in solving problems of relative motion is about visualizing measurement. If we say a body "A" has relative velocity "v" with respect to another moving body "B", then we simply mean that we are making measurement from the moving frame (reference) of "B".
- It is helpful in solving problem to make reference object stationary by applying negative of its velocity to both objects. The resultant velocity of the moving object is equal to the relative velocity of the moving object with respect to reference object. If we interpret relative velocity in this manner, it gives easy visualization as we are accustomed to observing motion from stationary state.

Nitin Chandrol (JEE 2012, AIR 134)

Illustration 10: Assume that a boy is riding a cycle at a speed of 5√3 m/s toward east along a straight line. It starts raining at a speed of 15 m/s in the vertical direction. What is the direction of rainfall as observed by the boy?

(JEE MAIN)

Sol: Let us denote earth, boy and rain with symbols A, B and C, respectively. The question here provides the velocity of B and C with respect to A (earth).

$\nu_{BA} = 5\sqrt{3}$ m/s; $\nu_{CA} = 15$ m/s

Now, we need to determine the direction of rain (C) with respect to boy (B), i.e., ν_{CB}. $\nu_{CA} = \nu_{BA} + \nu_{CB} \Rightarrow \nu_{CB} = \nu_{CA} - \nu_{BA}$

Thus, we now draw the vector diagram to evaluate the terms on the right side of the equation. Therefore, here, we need to evaluate "$\nu_{CA} - \nu_{BA}$", which is equivalent to "$\nu_{CA} + (-\nu_{BA})$". We now apply parallelogram theorem to obtain vector sum as represented in the Fig. 3.14 provided.

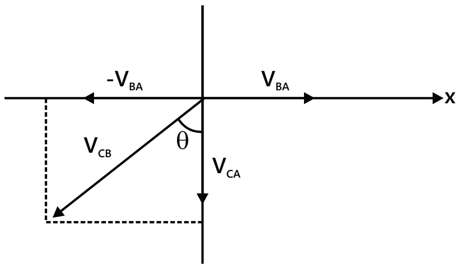


Figure 3.14

For the boy (B), the rain appears to fall, making an angle "θ" with the vertical (–y direction).

$$\Rightarrow \tan\theta = \frac{\nu_{BA}}{\nu_{CA}} = \frac{5\sqrt{3}}{15} = \frac{1}{\sqrt{3}} = \tan 30^\circ \Rightarrow \theta = 30^\circ$$

Illustration 11: Consider that a person is driving a car toward east at a speed of 80 km/hr. A train appears to move toward north with a velocity of 80√3 km/hr to this person. Find the speed of the train as measured with respect to earth. **(JEE ADVANCED)**

Sol: The velocity of the train with respect to earth is the vector sum of its velocity with respect to car and the velocity of car with respect to earth.

Let us first denote the car and train as "A" and "B," respectively. Here, we are provided with the speed of car ("A") with respect to earth, i.e., "ν_A" and speed of train ("B") with respect to "A," i.e., ν_{BA}. $\nu_A = 80\text{km/hr}$; $\nu_{BA} = 80\sqrt{3}\text{km/hr}$

Now, we are required to find the speed of train ("B") with respect to earth, i.e., ν_B. From the equation of relative motion, we have

$$\nu_{BA} = \nu_B - \nu_A \Rightarrow \nu_B = \nu_{BA} + \nu_A$$

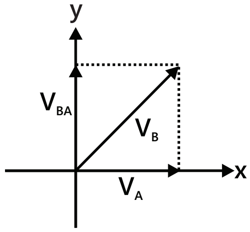


Figure 3.15

To evaluate the right-hand side of the equation, we draw vectors " ν_{BA} " and " ν_A " and use parallelogram law to find the actual speed of the train.

$$\Rightarrow \nu_B\sqrt{\left\{\left(\nu_{BA}\right)^2+\left(\nu_A\right)^2\right\}}=\sqrt{\left\{\left(80\sqrt{3}\right)^2+80^2\right\}}=160\,km/hr$$

6.1 Motion of Boat in a Stream

In this section, we consider a general situation of sailing of a boat in a moving stream of water. However, in order to keep our context simplified, we consider that the stream is unidirectional in x-direction and the width of stream, "d", is constant. Let the velocities of boat (A) and stream (B) be" ν_A " and " ν_B ," respectively with respect to ground. The velocity of boat (A) with respect to stream (B), therefore, is

$$\nu_{AB}=\nu_A-\nu_B \quad \Rightarrow \nu_A=\nu_{AB}+\nu_B$$

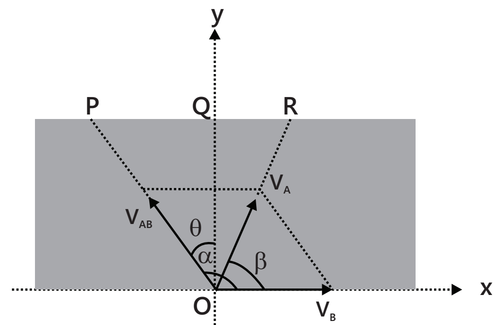


Figure 3.16

We represent these velocities in the Fig. 3.16 provided. It is clear from the Fig. 3.16 provided that boat sails in the direction, making an angle " θ " with y-direction, but reaches destination in different direction. The boat obviously is carried along the stream in x-direction. This displacement in x-direction (x = QR) from the directly opposite position to actual position on the other side of the stream is called the drift of the boat.

6.1.1 Resultant Velocity

We can calculate the magnitude of resultant velocity using the parallelogram theorem,

$$\nu_A=\sqrt{\left(\nu_{AB}^2+\nu_B^2+2\nu_{AB}\nu_B\cos\alpha\right)}$$

where " α " is the angle between ν_B and ν_{AB} vectors. The angle " β " formed by the resultant velocity with x-direction is given as: $\tan\beta=\dfrac{\nu_{AB}\sin\alpha}{\nu_B+\nu_{AB}\cos\alpha}$

6.1.2 Time to Cross the Stream

The boat covers a final distance equal to the width of stream "d" in the time "t" in y-direction. Now, by applying the concept of independence of motions in perpendicular directions, we can say that boat covers a final distance "OQ = d" with a speed equal to the component of resultant velocity in y-direction.

Now, the resultant velocity is composed of (i) velocity of boat with respect to stream and (ii) velocity of stream. Here, we observe that velocity of stream is perpendicular to y-direction. Therefore, it does not have any component in y-direction. We, therefore, conclude that the component of the resultant velocity is equal to the component of the velocity of boat with respect to stream in y-direction. Note that the two equal components shown in the Fig. 3.17 provided are geometrically equal as they are altitudes of same parallelogram. Hence, $\nu_{Ay}=\nu_{ABy}=\nu_{AB}\cos\theta$

where " θ " is the angle that relative velocity of boat w.r.t stream makes with the vertical. $t=\dfrac{d}{\nu_{Ay}}=\dfrac{d}{\nu_{AB}\cos\theta}$

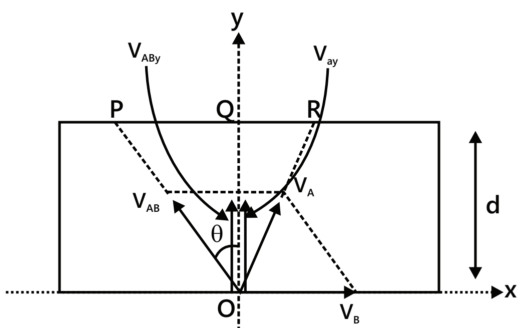


Figure 3.17

Thus, we can use either of these two expressions to calculate time to cross the river, depending on the inputs available.

6.1.3 Drift of the Boat

We now know that the displacement of the boat in x-direction is independent of motion in the perpendicular direction. Hence, displacement in x-direction is achieved with the component of resultant velocity in x-direction,

$$x = \left(\nu_{Ax}\right)t = \left(\nu_B - \nu_{ABx}\right)t = \left(\nu_B - \nu_{AB}\sin\theta\right)t$$

Then, substituting for time "t", we have: $x = \left(\nu_B - \nu_{AB}\sin\theta\right)\frac{d}{\nu_{AB}\cos\theta}$

6.1.4 Shortest Interval to Cross the Stream

The time taken by the boat to cross the river is given by: $t = \frac{d}{\nu_{Ay}} = \frac{d}{\nu_{AB}\cos\theta}$

Clearly, the time taken is minimum for the greatest value of denominator. The denominator is maximum for $\theta = 0°$, for this value, $t_{min} = \frac{d}{\nu_{AB}}$

This means that the boat needs to sail in the direction perpendicular to the stream to reach the opposite side in minimum time. The drift of the boat for this condition is: $x = \frac{\nu_B d}{\nu_{AB}}$

PLANCESS CONCEPTS

We have discussed motion with specific reference to boat in a water stream. However, the consideration is general and is applicable to the motion of a body in a medium. For example, the discussion and analysis can be extended to the motion of an aircraft, whose velocity is modified by the motion of the wind.

GV Abhinav (JEE 2012, AIR 329)

Illustration 12: An aircraft flies with velocity of 200 ($\sqrt{2}$)km/hr and the wind is blowing from the south. If the relative velocity of the aircraft with respect to wind is 1000 km/hr, then find the direction in which the aircraft should fly such that it reaches a destination in the north-east direction. **(JEE MAIN)**

Sol: The vector sum of the velocity of the airplane with respect to the wind and the velocity of the wind with respect to ground is equal to velocity of the aircraft with respect to ground. This net velocity should be in north-east direction.

We show the velocities pertaining to this problem in the Fig. 3.18 provided. In the Fig. 3.18 provided, OP denotes the velocity of the aircraft in still air or equivalently it represents the relative velocity of the aircraft with respect to air in motion; PQ denotes the velocity of the wind and OQ denotes the resultant velocity of the aircraft. However, it is clear that the aircraft should fly in the direction OP so that it is ultimately led to follow the north-east direction.

We should understand here that one of the velocities is the resultant velocity of the remaining two velocities. Therefore, it follows that the three velocity vectors are represented by the sides of a closed triangle.

We can now demonstrate the direction of OP, if we can find the angle" θ". The easiest way to determine the angle between vectors composing a triangle is to apply the sine law,

$$\frac{OP}{\sin 45^\circ} = \frac{PQ}{\sin\theta}$$

Therefore, by substituting these values, we obtain

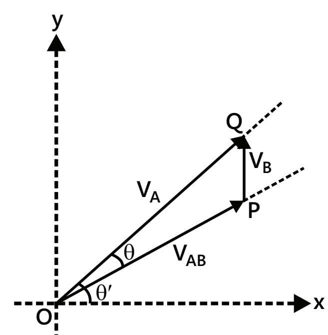


Figure 3.18

$\sin\theta = \frac{PQ\sin 45^\circ}{OP} = \frac{200\sqrt{2}}{1000x\sqrt{2}} = \frac{1}{5} = 0.2$

$\theta = \sin^{-1}(0.2)$

Hence, based on the above analysis, the aircraft should steer in the direction, making an angle with east as given by: $\theta' = 45^\circ - \sin^{-1}(0.2)$

Illustration 13: Assume that a boat, capable of sailing at 2 m/s, moves upstream in a river. The water in the stream flows at 1 m/s. A person walks from the front to the rear end of the boat at a speed of 1 m/s along the liner direction. What is the speed of the person (m/s) with respect to the ground? **(JEE MAIN)**

Sol: First find the velocity of boat with respect to ground. The velocity of man with respect to boat is added to the velocity of boat with respect to ground to get the velocity of man with respect to ground.

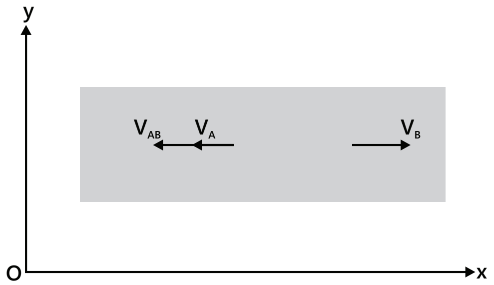


Figure 3.19

Let us assume that the direction of stream be in x-direction and the direction across stream be in y-direction. We further denote boat with "A", stream with "B", and the person with "C". We can now solve this problem in two parts. In the first part, we find out the velocity of boat (A) with respect to ground and then we calculate the velocity of the person (C) with respect to ground.

Here,

velocity of boat (A) with respect to stream (B): ν_{BA} = –2 m/s

Velocity of the stream (A) with respect to ground: ν_B = 1 m/s

Velocity of the person (C) with respect to boat (A): ν_{CA} = 1 m/s

Velocity of the person (C) with respect to ground: $\nu_C = ?$

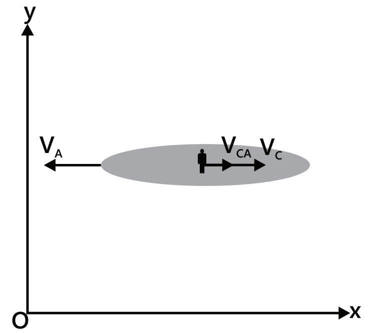


Figure 3.20

The velocity of boat with respect to ground is equal to the resultant velocity of the boat as given by: $\nu_A = \nu_{BA} + \nu_B \Rightarrow \nu_A = -2 + 1 = -1 m/s$

For the motion of person and boat, the velocity of the person with respect to ground is equal to the resultant velocity of (i) velocity of the person (C) with respect to boat (A) and (ii) velocity of the boat (A) with respect to ground. Hence, $\nu_C = \nu_{CA} + \nu_A \Rightarrow \nu_C = 1 + (-1) = 0$.

7. CIRCULAR MOTION

Circular motion is a movement of an object/particle along the circumference of a circle or motion along a circular path. However, it can be uniform or non uniform.

Familiar examples of circular motion include an artificial satellite orbiting the earth at constant height, a stone which is tied to a rope and is being swung in circles and a car turning through a curve in a race track.

Angular displacement of a body is the angle in radians (degrees, revolutions) through which a point or line has been turned in a specified sense about a specified axis. Angular displacement is denoted by θ.

The **angular velocity** is defined as the rate of change of angular displacement. The SI unit of angular velocity is radians per second. Angular velocity is usually represented by the symbol omega (ω). $\omega = \frac{d\theta}{dt}, \omega = \frac{\nu}{r}$ where v is linear velocity.

Angular acceleration is the rate of change of angular velocity. In SI units, it is measured in radians per second squared (rad/s^2), and is usually denoted by the Greek letter alpha (α).

$\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$, or $\alpha = \frac{a_T}{r}$

7.1 Uniform Circular Motion

Uniform Circular Motion, involves continuous change in the direction of velocity without any change in its magnitude (v). A change in the direction of velocity is a change in velocity (v). This implies that UCM is associated with acceleration and hence force. Thus, UCM signifies "presence" of force.

In other words, UCM requires a force, which is always perpendicular to the direction of velocity. Since the direction of velocity is continuously changing, the direction of force, being perpendicular to velocity, should also change continuously.

The direction of velocity along the circular trajectory is always tangential in nature. The perpendicular direction to the circular trajectory is, therefore, known as the radial direction. It implies that force (and hence acceleration) in uniform circular motion is radial. For this reason, acceleration in UCM is recognized to require center, i.e., centripetal (seeking center).

Irrespective of whether circular motion is uniform (constant speed) or non-uniform (varying speed), the circular motion inherently associates a radial acceleration to ensure that the direction of motion is continuously changed—at all instants. We learn about the magnitude of radial acceleration soon, but let us be emphatic to differentiate radial acceleration (accounting change in direction that arises from radial force) with tangential acceleration (accounting change in the speed that arises from tangential force).

The coordinates of the particle is given by the x- and y-coordinate pair as: $x = r\cos\theta;\ y = r\sin\theta$

The angle" θ " is measured anti-clockwise from the x-axis.

The position vector of the position of the particle, r, is represented in terms of unit vectors as:

$$r = x\hat{i} + y\hat{j} \Rightarrow r = r\cos\theta\hat{i} + r\sin\theta\hat{j} \Rightarrow r = r(\cos\theta\hat{i} + \sin\theta\hat{j})$$

The magnitude of velocity of the particle (v) is constant by the definition of UCM. In component form, however, the velocity (refer to the Fig. 3.21) is:

$$v = v_x\hat{i} + v_y\hat{j};\ v_x = -v\sin\theta;\ v_y = v\cos\theta$$

$$\sin\theta = \frac{y}{r};\ \cos\theta = \frac{x}{r};\ v = -\frac{vy}{r}\hat{i} + \frac{vx}{r}\hat{j}$$

Acceleration: Knowing that speed, "v" and radius of circle, "r" are constants, we easily differentiate the expression of velocity with respect to time to obtain expression for centripetal acceleration as:

$$a = -\frac{v}{r}\left(\frac{dy}{dt}\hat{i} - \frac{dx}{dt}\hat{j}\right) \Rightarrow a = -\frac{v}{r}\left(v_y\hat{i} - v_x\hat{j}\right)$$

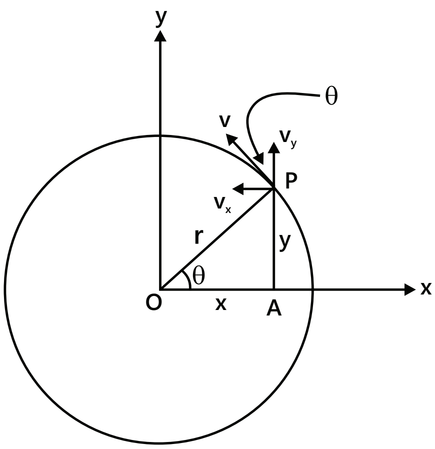


Figure 3.21

Substituting the value of component velocities in terms of angle, we obtain

$$\Rightarrow a = -\frac{v}{r}\left(v\cos\theta\hat{i} - v\sin\theta\hat{j}\right) = a_x\hat{i} + a_y\hat{j} \qquad \text{where } a_x = -\frac{v^2}{r}\cos\theta;\quad a_y = -\frac{v^2}{r}\sin\theta$$

It is evident from the equation of acceleration that it varies as the angle with horizontal, " θ " change. Therefore, the magnitude of acceleration is

$$a = |a| = \sqrt{\left(a_x^{\,2} + a_y^{\,2}\right)} \Rightarrow a = |a| = \frac{v}{r}\sqrt{\left\{v^2\left(\cos^2\theta + \cos^2\theta\right)\right\}} \Rightarrow a = \frac{v^2}{r}$$

Illustration 14: Assume that a cyclist negotiates the curvature of 20 m at a speed of 20 m/s. What is the magnitude of his acceleration? **(JEE MAIN)**

Sol: The speed of the cyclist moving along circular path is constant. So its acceleration is centripetal.

Let the speed of the cyclist be constant. Then, the acceleration of the cyclist is the centripetal acceleration that is required to move the cyclist along a circular path, i.e., the acceleration resulting from the change in the direction of motion along the circular path.

Hence, v = 20 m/s and r = 20 m $\Rightarrow a = \frac{v^2}{r} = \frac{20^2}{20} = 20\,m/s^2$

7.2 Non-Uniform Circular Motion

We are aware of the fact that the speed of a particle under circular motion is not constant.

A change in speed means that unequal length of arc (s) is covered in equal time intervals. It further means that the change in the velocity (v) of the particle is not limited to change in direction as in the case of UCM.

Radial or centripetal acceleration. Change in direction is due to radial acceleration (centripetal acceleration), which is given by $a_R = \frac{v^2}{r}$.

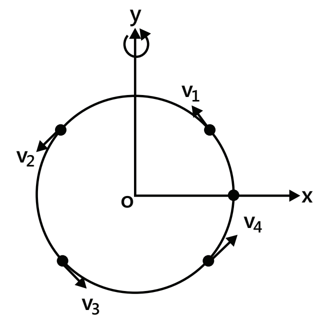


Figure 3.22

Tangential acceleration: The non-uniform circular motion basically involves a change in speed. This change is accounted by the tangential acceleration, which results due to a tangential force and which acts along the direction of velocity. $a_T = \frac{dv}{dt}$

7.3 Relation between Angular and Linear Acceleration

The relationship between angular and linear acceleration is shown hereunder.

$$a_T = \frac{dv}{dt} = \frac{d^2s}{dt^2} = \frac{d^2}{dt^2}(r\theta) = r\frac{d^2\theta}{dt^2} = r\alpha$$

Illustration 15: A particle, starting from the position (5 m, 0 m), is moving along a circular path about the origin in x–y plane. The angular position of the particle is a function of time as given here, $\theta = t^2 + 0.2t + 1$. Find (i) tangential acceleration **(JEE MAIN)**

Sol: Differentiate the expression for angular position with respect to time to get angular velocity. Tangential acceleration is the product of angular acceleration and the radius.

From the data on initial position of the particle, it is clear that the radius of the circle is 5 m.

(i) For determining tangential acceleration, we need to have expression of linear speed in time.

$$v = \omega r = (2t + 0.2) \times 5 = 10t + 1$$

We obtain tangential acceleration by differentiating the above function: $a_T = \frac{dv}{dt} = 10 m/s^2$

Illustration 16: At a particular instant, a particle is moving at a speed of 10 m/s on a circular path of radius 100 m. Its speed is increasing at the rate of 1 m/s^2. What is the acceleration of the particle? **(JEE MAIN)**

Sol: The acceleration of the particle is the vector sum of the centripetal acceleration and the tangential acceleration. The tangential acceleration is equal to the rate of change of speed.

The acceleration of a particle is the vector sum of mutually perpendicular radial and tangential accelerations. The magnitude of tangential acceleration given here is 1 m/s^2. Now, the radial acceleration at the particular instant is:

$$a_R = \frac{v^2}{r} = \frac{10^2}{100} = 1 m/s^2$$

Hence, the magnitude of the acceleration of the particle is: $a = |a| = \sqrt{(a_T{}^2 + a_R{}^2)} = \sqrt{1^2 + 1^2} m/s^2 = \sqrt{2} m/s^2$

Illustration 17: Which of the following expressions represent the magnitude of centripetal acceleration?:

(A) $\left|\frac{d^2r}{dt^2}\right|$ (B) $\left|\frac{dv}{dt}\right|$ (C) $r\frac{d\theta}{dt}$ (D) None of these **(JEE MAIN)**

Sol: The magnitude of centripetal acceleration depends on the square of the magnitude of velocity.

The expression $\left|\frac{dv}{dt}\right|$ represents the magnitude of tangential acceleration. The differential $\frac{d\theta}{dt}$ represents the magnitude of angular velocity. The expression $r\frac{d\theta}{dt}$ represents the magnitude of tangential velocity and the expression $\frac{d^2r}{dt^2}$ is second-order differentiation of position vector (r). This is the actual expression of acceleration of a particle under motion. Hence, the expression $\left|\frac{d^2r}{dt^2}\right|$ represents the magnitude of total or resultant acceleration.

Hence, option (d) alone is correct.

Illustration 18: A particle is executing circular motion. But the magnitude of velocity of the particle changes from zero to (0.3i + 0.4j) m/s in a period of 1 second. The magnitude of average tangential acceleration is:

(A) 0.1 m/s^2 (B) 0.2 m/s^2 (C) 0.3 m/s^2 (D) 0.5 m/s^2 **(JEE MAIN)**

Sol: Tangential acceleration is equal to the rate of change of speed. Average tangential acceleration is change in speed divided by total time.

The magnitude of average tangential acceleration is the ratio of change in speed and time as given by: $a_T = \frac{\Delta v}{\Delta t}$

Now, $\Delta v = \sqrt{\left(0.3^2 + 0.4^2\right)} = \sqrt{0.25} = 0.5\,m/s$; $a_T = 0.5\,m/s^2$

Hence, option (d) alone is correct.

PLANCESS CONCEPTS

Radial acceleration contributes in changing the direction of velocity of an object, but it does not affect the magnitude of velocity. However, tangential acceleration affects the speed of the object in motion.

Vaibhav Krishan (JEE 2009, AIR 22)

FORMULAE SHEET

(a) Projectile Motion

Time of flight: $T = \frac{2u\sin\theta}{g}$

Horizontal range: $R = \frac{u^2\sin 2\theta}{g}$

Maximum height: $H = \frac{u^2\sin^2\theta}{2g}$

Trajectory equation (equation of path):

$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta} = x\tan\theta\left(1 - \frac{x}{R}\right)$$

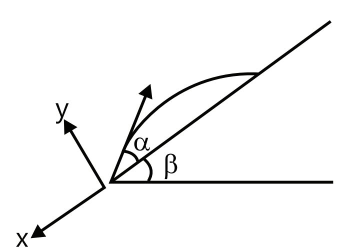


Figure 3.23

Projection on an inclined plane

(b) Relative Motion

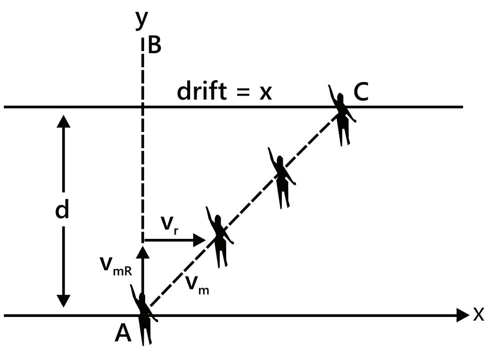


Figure 3.24

v_{AB} (velocity of A with respect to B) $= v_A - v_B$

a_{AB} (acceleration of A with respect to B) $= a_A - a_B$

Relative motion along straight line $= x_{BA} = x_B - x_A$

(c) Crossing River: A boat or man in a river always moves in the direction of resultant velocity of velocity of boat (or man) and velocity of the river flow.

(d) Shortest Time: Velocity along the river, $V_X = V_R$

Velocity perpendicular to the river, $V_f = V_{mR}$

The net speed is given by $V_m = \sqrt{V_{mR}^2 + V_R^2}$

(e) Shortest Path: Velocity along the river, $V_x = 0$

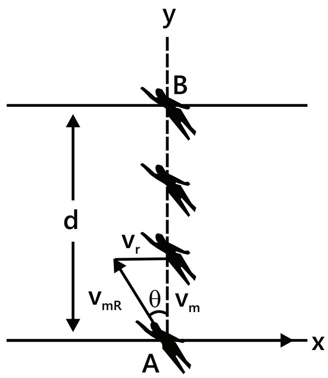


Figure 3.25

and velocity perpendicular to river $V_y = \sqrt{V_{mR}^2 - V_R^2}$

The net speed is given by $V_m = \sqrt{V_{mR}^2 - V_R^2}$

at an angle of $90°$ with the river direction.

velocity V_y is used only to cross the river, therefore time to cross the river,

$t = \frac{d}{v_y} = \frac{d}{\sqrt{v_{mR}^2 - v_R^2}}$ and velocity v_x is zero, therefore, in this case the drift should be zero.

$v_R = v_{mR}\sin\theta = 0$ or $v_R = v_{mR}\sin\theta$ or $\theta = \sin^{-1}\frac{v_R}{v_{mR}}$

(f) Rain Problems: $v_{Rm} = \vec{v}_R - \vec{v}_m$ or $v_{Rm} = \sqrt{v_R^2 + v_m^2}$

(g) Circular Motion

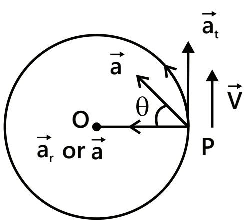


Figure 3.26

i. Average angular velocity $\omega_{av} = \frac{\theta_2 - \theta_1}{t_2 - t_1} = \frac{\Delta\theta}{\Delta t}$

ii. Instantaneous angular velocity $\omega = \frac{d\theta}{dt}$

iii. Average angular acceleration $\alpha_{av} = \frac{\omega_2 - \omega_1}{t_2 - t_1} = \frac{\Delta\omega}{\Delta t}$

iv. Instantaneous angular acceleration $\alpha = \frac{d\omega}{dt} = \omega\frac{d\omega}{d\theta}$

v. Relation between speed and angular velocity $v = r\omega$ and $v = \omega\ r$

vi. Tangential acceleration (rate of change of speed) $a_t = \frac{dV}{dt}$

vii. Radial or normal or centripetal acceleration $a_r = \frac{V^2}{r} = \omega^2 r$

viii. Total acceleration $\vec{a} = \vec{a}_t + \vec{a}_r$, $a = \left(a_t^2 + a_r^2\right)^{1/2}$

ix. Angular acceleration $\alpha = \frac{d\omega}{dt}$ (non-uniform circular motion)

x. Radius of curvature $R = \frac{v^2}{a_\perp} = \frac{mv^2}{F_\perp}$

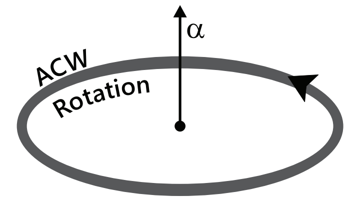


Figure 3.27

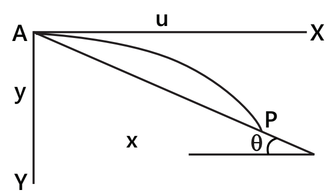
Solved Examples

JEE Main/Boards

Example 1: A particle is projected horizontally with a speed u from the top of a plane inclined at an angle θ with the horizontal. How far from the point of projection will the particle strike the plane?

Sol: Take the x-axis parallel to the horizontal. Take the y-axis along the vertical. Along x-axis velocity is uniform. Along y-axis initial velocity is zero and acceleration is uniform.

Take, X–Y axes as shown in Figure. Suppose that the particle strikes the plane at point P with coordinates (x and y). Consider the motion between A and P.



Motion in x-direction: initial velocity = u

Acceleration = 0; X = ut ... (i)

Motion in y-direction: initial velocity = 0

Acceleration = g; $y = \frac{1}{2}gt^2$ ... (ii)

Eliminating t from (i) and (ii)

$y = \frac{1}{2}g\frac{x^2}{u^2}$ Also, $y = x\tan\theta$.

Thus, $\frac{gx^2}{2u^2} = x\tan\theta$ giving x = 0, or, $\frac{2u^2\tan\theta}{g}$

Clearly the point P corresponds to $x = \frac{2u^2\tan\theta}{g}$

Then, $y = x\tan\theta = \frac{2u^2\tan^2\theta}{g}$

The distance $AP = l = \sqrt{x^2 + y^2}$

$$= \frac{2u^2}{g}\tan\theta\sqrt{1+\tan^2\theta} = \frac{2u^2}{g}\tan\theta\sec\theta$$

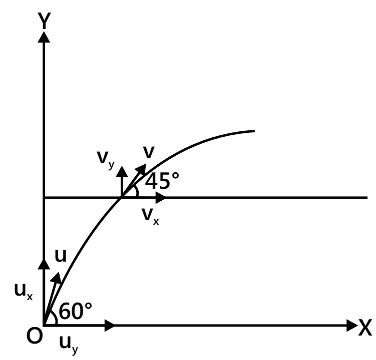
Examples 2: A projectile is projected at an angle 60° from the horizontal with a speed of $\left(\sqrt{3}+1\right)$ m/s. The time (in seconds) after which the inclination of the projectile with horizontal becomes 45° is:

Sol: Take the x-axis along the horizontal. Take the y-axis vertically upwards. Along x-axis velocity is uniform. Along y-axis initial velocity is positive and acceleration is uniform and negative.

Let "u" and "v" be the speed at the two specified angles. The initial components of velocities in horizontal and vertical directions are:

$u_x = u\cos 60^\circ$

$u_y = u\sin 60^\circ$



Similarly, the components of velocities, when the projectile makes an angle 45 with horizontal and vertical directions are:

$$v_x = v\cos 45^\circ;\ v_y = v\sin 45^\circ$$

But we know that horizontal component of velocity remains unaltered during motion. Hence,

$$v_x = u_x \Rightarrow v\cos 45^\circ = u\cos 60^\circ \Rightarrow v = \frac{u\cos 60^\circ}{\cos 45^\circ}$$

Here, we know initial and final velocities in vertical direction. We can apply v = u + at in vertical direction to know the time as required:

$$v\sin 45^\circ = u + at = u\sin 60^\circ - gt$$

$$\Rightarrow v\cos 45^\circ = u\cos 60^\circ \Rightarrow t = \frac{u\cos 60^\circ - v\sin 45^\circ}{g}$$

Substituting value of "v" in the above equation, we have:

$$\Rightarrow t = \frac{u\sin 60^\circ - u\left(\frac{\cos 60^\circ}{\cos 45^\circ}\right) X \sin 45^\circ}{g}$$

$$\Rightarrow t = \frac{u}{g}\left(\sin 60^\circ - \cos 60^\circ\right) \Rightarrow t = \frac{\left(\sqrt{3}+1\right)}{10}\left\{\frac{\left(\sqrt{3}-1\right)}{2}\right\}$$

$$\Rightarrow t = \frac{2}{20} = 0.1\text{s}$$

Example 3: A projectile is at an angle "θ" from the horizontal at the speed "u". If an acceleration of "g/2" is applied to the projectile due to wind in horizontal direction, then find the new time of flight, maximum height and horizontal range.

Sol: Take the x-axis along the horizontal. Take the y-axis vertically upwards. Along x-axis initial velocity is positive and acceleration is uniform and positive. Along y-axis initial velocity is positive and acceleration is uniform and negative.

The acceleration due to wind affects only the motion in horizontal direction. It would, therefore, not affect attributes like time of flight or maximum height that results exclusively from the consideration of motion in vertical direction. The generic expressions of time of flight, maximum height and horizontal range of flight with acceleration are given as under:

$$T = \frac{2u_y}{g};\ H = \frac{u_y^2}{2g} = \frac{gT^2}{4};\ R = \frac{u_x u_y}{g}$$

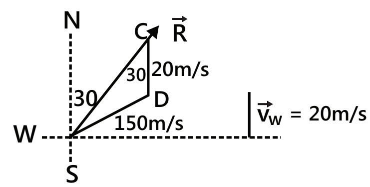
The expressions above revalidate the assumption made in the beginning. We can see that it is only the horizontal range that depends on the component of motion in horizontal direction. Now, considering accelerated motion in horizontal direction, we have:

$$x = R' = u_x T + \frac{1}{2}a_x T^2 \Rightarrow R' = u_x T + \frac{1}{2}\left(\frac{g}{2}\right)T^2;\ R' = R + H$$

Example 4: An airplane has to go from a point A to another point B, 500 km away due 30° east of north. A wind is blowing due north at a speed of 20 m/s. The air speed of the plane is 150 m/s. (i) Find the direction in which the pilot should head the plane to reach the point B. (ii) Find the time taken by the plane to go from A to B.

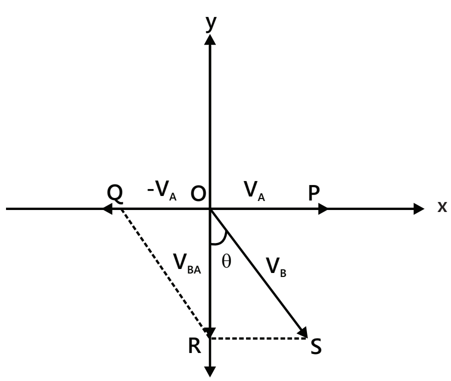
Sol: The vector sum of the velocity of the airplane with respect to the wind and the velocity of the wind with respect to ground is equal to velocity of the aircraft with respect to ground. This net velocity should be in the direction A to B.

In the resultant direction R, the plane reaches the point B.



Velocity of wind $\vec{V}_w$ =20 m/s

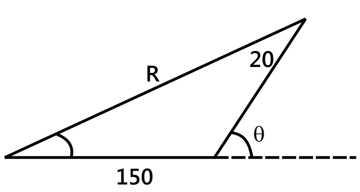
Velocity of aero plane $\vec{V}_a$ =150 m/s



In ΔACD, according to the sine formula

$$\therefore \frac{20}{\sin A} = \frac{150}{\sin 30^\circ} \Rightarrow \sin A = \frac{20}{150}\sin 30^\circ$$

$$= \frac{20}{150} \times \frac{1}{2} = \frac{1}{5} \Rightarrow A = \sin^{-1}(1/15)$$



(i) The direction is $\sin^{-1}(1/15)$ east of the line AB.

(ii) $\sin^{-1}(1/15) = 3^\circ 48' \Rightarrow 30^\circ + 3^\circ 48' = 33^\circ 48'$

$$R = \sqrt{150^2 + 20^2 + 2(150)20\cos 33^\circ 48'} = 167 m/s$$

$$\text{Time} = \frac{s}{v} = \frac{500000}{167} = 2994\,\text{sec} = 49 = 50\,\text{min}$$

Example 5: Rain drops appear to fall in vertical direction to a person, who is walking at a velocity 4 m/s in a given direction. When the person doubles his velocity in the same direction, the rain drops appear to come at an angle of 45° from the vertical. Find the speed of the rain drop.

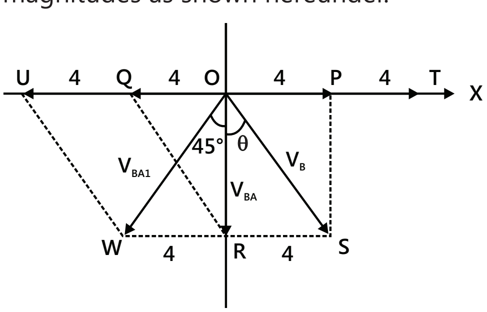
Sol: Velocity of rain with respect to the person is equal to the vector sum of the velocity of rain with respect to ground and the negative of the velocity of person with respect to ground. The direction of velocity of rain with respect to person is known in each case. Assume a direction of velocity of rain with respect to ground and draw the vector diagrams for velocity of rain with respect to person for both the cases.

This is a slightly tricky question. Readers may like to visualize the problem and solve on their own before going through the solution given here.

Let us consider the situation under two cases. Here, only the directions of relative velocities in two conditions are given. The Figure on the left represents initial situation. Here, the vector OP represents velocity of the person (V_A); OR represents relative velocity of rain drop with respect to person (V_A); OS represents velocity of rain drop.

The given figure represents situation when person starts moving with double velocity. Here, the vector OT represents velocity of the person (V_{A1}); OW represents relative velocity of rain drop with respect to person (V_{BA1}). We should note that velocity of rain (V_B) drop remains the same and as such, it is represented by OS represents as before.

According to the question, we are required to know the speed of the rain drop. It means that we need to know the angle "θ" and the side OS, which is the magnitude of velocity of rain drop. It is intuitive from the situation that it would help if we consider the vector diagram and carry out geometric analysis to find these quantities. For this, we substitute the vector notations with known magnitudes as shown hereunder.



We note here that WR = UQ = 4 m/s

Clearly, triangles ORS and ORW are congruent as two sides and one enclosed angle are equal.

WR = RS = 4 m/s; OR = OR; $\angle ORW = \angle ORS = 90°$

Hence, $\angle WOR = \angle SOR = 45°$

In triangle ORS, $\sin 45^\circ = \frac{RS}{OS} \Rightarrow OS = \frac{RS}{\sin 45^\circ} = 4\sqrt{2}\,m/s$

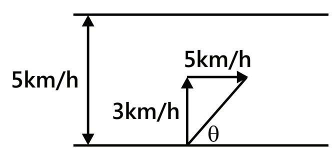
Example 6: A swimmer wishes to cross a 500 m wide river flowing at 5 km/h. His speed with respect to water is 3 km/h. (i) If he heads in a direction making an angle θ with the flow, find the time he takes to cross the river. (ii) Find the shortest possible time to cross the river.

Sol: Time taken to cross the river will depend on the component of velocity of swimmer which is perpendicular to the river flow. For shortest time this component should be maximum, i.e. $\theta = 90^o$.

(i) The vertical component 3 $\sin\theta$ takes him to the opposite side.

Distance = 0.5 km, velocity = 3 $\sin\theta$ km/h

$$\text{Time} = \frac{\text{Distance}}{\text{Velocity}} = \frac{0.5\text{km}}{3\sin\theta\,\text{km/h}} = \frac{10}{\sin\theta}\ \text{min.}$$



(ii) Here vertical component of the velocity, i.e., 3 km/hr takes him to the opposite side ($\theta = 90^o$).

$$\text{Time} = \frac{\text{Distance}}{\text{Velocity}} = \frac{0.5}{3} = 0.16\,\text{hr}$$

$$\therefore 0.16\,\text{hr} = 60 \times 0.16 = 9.6 = 10\,\text{minute}$$

Example 7: Two tall buildings are 200 m apart. With what speed must a ball be thrown horizontally from a window of one building 2 km above the ground so that it will enter a window 40 m from the ground in the other?

Sol: The time taken by ball to fall from the height of 2000 m to the height of 40 m (with zero initial velocity in vertical direction) should be equal to the time taken by ball to cover a horizontal distance of 200 m with constant velocity in horizontal direction.

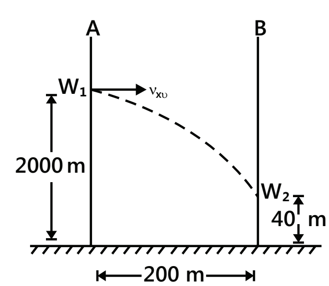


Figure shows the conditions of the problem. Here, A and B are the two tall buildings having windows W_1 and W_2, respectively. The window W_1 is 2 km (=2000 m) above the ground while window W_2 is 40 m above the ground. We want to throw the ball from window W_1 with such a horizontal speed (v_{x0}) so that it enters the window W_2. Note that the horizontal range of the ball is R = 200 m. Let t sec be the time taken by the ball to reach from window W_1 to window W_1. This time will depend upon the vertical motion (downward) alone.

For vertical motion:

h = 2000 – 40 = 1960 m; $g = 9.8\ ms^{-2}$: $v_{y0} = 0$

$$\therefore h = \frac{1}{2}gt^2 \text{ or } t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2\times 1960}{9.8}} = 20\text{ s}$$

$$\text{Now } R = v_{x0}t \therefore v_{x0} = \frac{R}{t} = \frac{200}{20} = 10\,ms^{-1}$$

Example 8: A particle moves in a circle of radius 20 cm. Its linear speed is given by v = 2t; t, is in second and v in metre/second. Find the radial and tangential acceleration at t = 3 s.

Sol: Radial acceleration depends on the square of instantaneous speed and the radius. Tangential acceleration is equal to the rate of change of instantaneous speed.

The linear speed at t = 3 s is V = 2 t = 6 m/s.

The radial acceleration at t = 3 s is

$$a_r = v^2/r = \frac{36 m^2/s^2}{0.20 m} = 180 m/s^2$$

The tangential acceleration is

$$a_t = \frac{dv}{dt} = \frac{d(2t)}{dt} = 2 m/s^2$$

Example 9: A boy wants to throw a letter wrapped over a stone to his friend across the street 40 m wide. The boy's window is 10 m below friend's window. How should he throw the ball?

Sol: We assume that the boy throws the ball such that the maximum height attained by the ball is H = 10 m. This implies that the range of the ball is R = 40×2 = 80 m. Thus from the formulae of H and R we can find the values of initial velocity and the angle of projection.

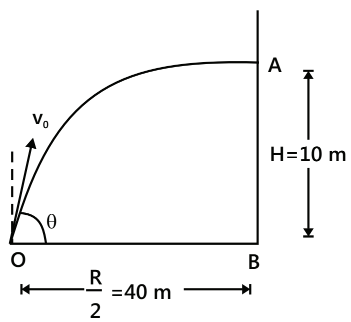


Figure shows the conditions of the problem. The boy's window is at O and friend's window is at A. Let the boy throw the stone with a velocity v_0 making an angle θ with the horizontal so as to enter the window at A. The stone will follow the parabolic path with A as the highest point on the trajectory of stone.

$\therefore \frac{R}{2} = 40$ or horizontal range, R = 2 × 40 = 80 m

Motion in a plane

$$R = \frac{V_0^2 \sin 2\theta}{g} = \frac{V_0^2\, 2\sin\theta\cos\theta}{g}$$

$$\text{and } H = \frac{V_0^2 \sin^2\theta}{2g} \therefore \frac{H}{R} = \frac{\tan\theta}{4} \text{ or}$$

$$\tan\theta = 4\times\frac{H}{R} = 4\times\frac{10}{80} = \frac{1}{2} \therefore \theta = 26.56°$$

Maximum height attained, H = 10 m

Now, the projection velocity v_0 can be found by substituting the value of θ in formula for H.

$$H = \frac{v_0^2 \sin^2\theta}{2g}$$

$$\therefore v_0^2 = \frac{2gH}{\sin^2\theta} = \frac{2\times 9.8\times 10}{\sin^2 26.56°} = 980 \text{ or}$$

$$v_0 = \sqrt{980} = 31.3\ ms^{-1}$$

Example 10: A body is projected with a velocity of $40\ ms^{-1}$. After 2 s, it crosses a vertical pole of height 20.4 m. Calculate the angle of projection and the horizontal range.

Sol: Use second equation of motion with constant acceleration in vertical direction.

Let θ be the angle of projection. For vertical motion,

$$h = (v_0 \sin\theta)t - \frac{1}{2}gt^2$$

$$\text{or } 20.4 = (40\sin\theta)\times 2 - \frac{1}{2}\times 9.8\times(2)^2$$

$$\text{or } 20.4 = 80\sin\theta - 19.6$$

$$\therefore \sin\theta = \frac{20.4+19.6}{80} = \frac{40}{80} = \frac{1}{2} \therefore \theta = 30°$$

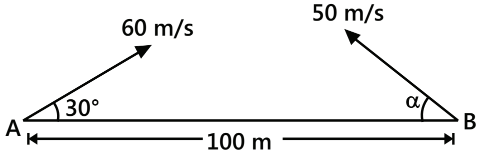
$$\text{Horizontal Range, } R = \frac{v_0^2 \sin 2\theta}{g}$$

$$= \frac{(40)^2 \times \sin 60°}{9.8}$$

$$= 141.1 \text{ m}$$

JEE Advanced/Boards

Example 1: A particle A is projected with an initial velocity of 60 m/s at an angle $30°$ to the horizontal. At the same time a second particle B is projected in opposite direction with initial speed of 50 m/s from a point at a distance of 100 m from A. If the particles collide in air, find (i) the angle of projection α of particle B, (ii) time when the collision takes place and (iii) the distance of P from A, where collision occurs. $\left(g = 10 m/s^2\right)$



Sol: This problem is best solved in the reference frame of one of the two particles, say particle B. The relative acceleration between the particles is zero. So in this reference frame, the particle A moves with uniform velocity.

(i) Taking x- and y-directions as shown in the figure.

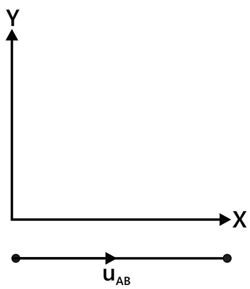
Here,

$\vec{a}_A = -g\hat{j};\ \vec{a}_g = g\hat{j}$

$u_{Ax} = 60\cos 30° = 30\sqrt{3} m/s$

$u_{Ay} = 60\sin 30° = 30 m/s$

$u_{Bx} = -50\cos\alpha;\ u_{By} = 50\sin\alpha$



and Relative acceleration between the two is zero as $\vec{a}_A = \vec{a}_B$. Hence, the relative motion between the two is uniform. It can be assumed that B is at rest and A is moving with $\vec{u}_{AB}$. Hence, the two particles will collide, if $\vec{u}_{AB}$ is along AB. This is possible only when $u_{Ay} = u_{By}$

i.e., component of relative velocity along the y-axis should be zero.

Or $30 = 50\sin\alpha\ \ \therefore \alpha = \sin^{-1}\left(3/5\right)$

(ii) Now,

$$\left|\vec{u}_{AB}\right| = u_{Ax} - u_{Bx} = \left(30\sqrt{3} + 50\cos\alpha\right) m/s$$

$$= \left(30\sqrt{3} + 50 \times \frac{4}{5}\right) m/s = \left(30\sqrt{3} + 40\right) m/s$$

Therefore, the time of collision is $t = \dfrac{AB}{\left|\vec{u}_{AB}\right|} = \dfrac{100}{30\sqrt{3} + 40}$

Or t = 1.09 s

(iii) Distance of point P from A where collision takes place is

$$s = \sqrt{\left(u_{Ax}t\right)^2 + \left(u_{Ay}t - \frac{1}{2}gt^2\right)}$$

$$= \sqrt{\left(30\sqrt{3} \times 1.09\right)^2 + \left(30 \times 1.09 - \frac{1}{2} \times 10 \times 1.09 \times 1.09\right)^2}$$

$s = 62.64 m$

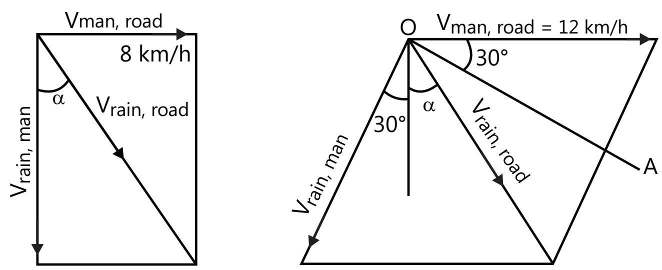
Example 2: A man running on a horizontal road at 8 km/h finds the rain falling vertically. He increases his speed to 12 km/h and finds that the drops make an angle $30°$ with the vertical. Find the speed and direction of the rain with respect to the road.

Sol: Velocity of rain with respect to the man is equal to the vector sum of the velocity of rain with respect to ground and the negative of the velocity of man with respect to ground. The direction of velocity of rain with respect to man is known in each case.

We have,

$$\vec{v}_{rain,road} = \vec{v}_{rain,man} + \vec{v}_{man,road} \qquad \text{... (i)}$$

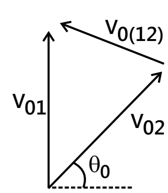
The two situations given in the problem may be represented by the following diagrams.



(a) (b)

$\vec{v}_{rain,road}$ is same in magnitude and direction in both the diagram. Taking horizontal components in E.q. (i) for the first diagram, $\vec{v}_{rain,road}\sin\alpha = 8 km/h$ ... (ii)

Now, consider given figure Draw a line $OA \perp U_{rain,man}$ as shown. Taking components in Eq. (i) along the line OA, we have



$$\vec{v}_{rain,road}\sin\left(30^o + \alpha\right) = 12\ km/h\cos 30^o \qquad \text{... (iii)}$$

From (ii) and (iii),

$$\frac{\sin(30°+\alpha)}{\sin\alpha}=\frac{12\times\sqrt{3}}{8\times 2}$$

or, $$\frac{\sin 30°\cos\alpha+\cos 30°\sin\alpha}{\sin\alpha}=\frac{3\sqrt{3}}{4}$$

or, $$\frac{1}{2}\cot\alpha+\frac{\sqrt{3}}{2}=\frac{3\sqrt{3}}{4}\quad \text{or,} \cot\alpha=\frac{\sqrt{3}}{2}$$

or, $$\alpha=\cot^{-1}\frac{\sqrt{3}}{2}$$

From(ii) $$v_{rain,road}=\frac{8km/h}{\sin\alpha}=4\sqrt{7}km/h$$

Example 3: Two bodies were thrown simultaneously from the same point; one, straight up, and the other, at an angle of $\theta=60°$ to the horizontal. The initial velocity of each body is equal to $v_0=25\ ms^{-1}$. Neglecting the air drag, find the distance between the bodies t = 1.70 s later.

Sol: The relative acceleration of the bodies is zero. The solution of this problem becomes interesting in the frame attached with one of the bodies.

Let the body thrown straight up be 1 and the other body be 2, then for the body 1 in the frame of 2 from the kinematical equation for constant acceleration (since both are moving under constant acceleration) is

$$r_{12}=r_{0(12)}+v_{0(12)}t+\frac{1}{2}w_{12}t^2$$

So, $r_{12}=v_{0(12)}t \qquad (\because w_{12}=0 \text{ and } r_{0(12)}=0)$

or, $|r_{12}|=|v_{o(12)}|t$ But, $|v_{01}|=|v_{02}|=v_0$

Therefore, from properties of triangle

$$|v_{0(12)}|=\sqrt{v_0^2+v_0^2-2v_0v_0\cos(\pi/2-\theta_0)}$$

Hence, the sought distance is

$$|r_{12}|=v_0t\sqrt{2(1-\sin\theta_0)}=22m$$

Example 4: The velocity of a projectile when it is at the greatest height is $\sqrt{2/5}$ times its velocity when it is at half of its greatest height. Determine its angle of projection.

Sol: The maximum height is known in terms of initial velocity and angle of projection. Horizontal component of velocity of projectile remains constant. Use the third equation of motion with uniform acceleration in vertical direction to find the vertical component of velocity at height equal to half of the maximum height.

Suppose the particle is projected with velocity u at an angle θ with the horizontal. Horizontal component of its velocity at all height will be $u\cos\theta$.

At the greatest height, the vertical component of velocity is zero, so the resultant velocity is

$$v_1=u\cos\theta$$

At half the greatest height during upward motion,

$$y=h/2, a_y=-g, u_y=u\sin\theta$$

Using $v_y^2-u_y^2=2a_yy$

we get, $$v_y^2-u^2\sin^2\theta=2(-g)\frac{h}{2}$$

or $$v_y^2=u^2\sin^2\theta-g\times\frac{u^2\sin^2\theta}{2g}=\frac{u^2\sin^2\theta}{2}$$

$$\left[\therefore h=\frac{u^2\sin^2\theta}{2g}\right]\quad \text{or } v_y=\frac{u\sin\theta}{\sqrt{2}}$$

Hence, the resultant velocity at half of the greatest height is

$$v=\sqrt{v_x^2+v_y^2}=\sqrt{u^2\cos^2\theta+\frac{u^2\sin^2\theta}{2}}$$

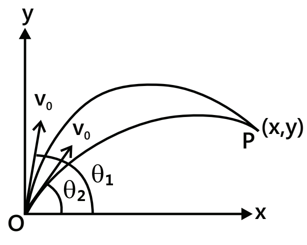
Given, $$\frac{v_1}{v_2}=\sqrt{\frac{2}{5}}$$

$$\therefore \frac{v_1^2}{v_2^2}=\frac{u^2\cos^2\theta}{u^2\cos^2\theta+\frac{u^2\sin^2\theta}{2}}=\frac{2}{5}\text{ or }\frac{1}{1+\frac{1}{2}\tan^2\theta}=\frac{2}{5}$$

$$2+\tan^2\theta=5, \text{or} \tan^2\theta=3;\quad \tan\theta=\sqrt{3};\quad \theta=60°$$

Example 5: A cannon fires successively two shells with velocity $v_0=250\ m/s$; the first at the angle $\theta_1=60°$ and the second at the angle $\theta_2=45°$ to the horizontal, the azimuth being the same. Neglecting the air drag, find the time interval between firings leading to the collision of the shells.

Sol: At the instant of collision, the horizontal and vertical distances covered by both the shells is will be equal respectively. Get two equations, one for horizontal distance and the other for vertical distance.



Let the shells collide at the point P(x, y). If the first shell takes t seconds to collide with second and Δt be the time interval between the firings, then

$$x=v_0\cos\theta_1 t=v_0\cos\theta_2(t-\Delta t) \qquad \text{... (i)}$$

and $$y=v_0\sin\theta_2(t-\Delta t)-\frac{1}{2}g(t-\Delta t)^2 \qquad \text{... (ii)}$$

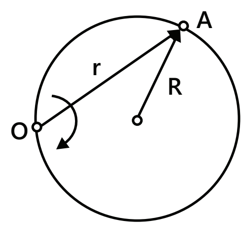
From Eq. (i) $$t=\frac{\Delta t\cos\theta_2}{\cos\theta_2-\cos\theta_1} \qquad \text{... (iii)}$$

From Eqs. (ii) and (iii)

$$\Delta t = \frac{2v_0 \sin(\theta_1 - \theta_2)}{g(\cos\theta_2 + \cos\theta_1)} \text{ as } \Delta t \neq 0$$

$= 11$ s (on substituting values)

Example 6: A particle A moves along a circle of radius R = 5 cm so that its radius vector r relative to the point O rotates with the constant angular velocity $\omega = 0.40 \text{ rad} s^{-1}$. Find the modulus of the velocity of the particle, and the modulus and direction of its acceleration.



Sol: Angular velocity about point O is given. We need to find the angular velocity about point C, ω_C. Once ω_C is known, velocity and acceleration can be found out from formulae of circular motion.

Angular velocity of point A, with respect to center C of the circle or turning rate of line CA taking the line OCX as reference line becomes $\omega_c = -\frac{d(2\theta)}{dt} = 2\left(\frac{-d\theta}{dt}\right) = 2\omega$

Because angular speed of line OA is

$\omega = -d\theta / dt$.

The turning rate of line CA is also the turning rate of velocity vector of point A, which is given by v_A / R.

Therefore, $v_A = \omega_C R = 2(\omega)R = 4$ cm / s (on substituting the values).

The acceleration of the particle will be centripetal as its speed is constant.

$$a = \frac{v^2}{R} = \frac{4^2}{5} \text{ cm/s}^2 = 3.2 \text{ cm/s}^2$$

Example 7: A point moves along a circle with a velocity v = kt, where k = 0.5 m/s^2. Find the total acceleration of the point at the moment when it has covered the n^{th} fraction of the circle after the beginning of motion, where n = $\frac{1}{10}$.

Sol: This is the case of circular motion with constant tangential acceleration. Use second equation of motion with constant acceleration and zero initial velocity to find the time required to cover 1/10 of the circle. Total acceleration is the vector sum of tangential acceleration and centripetal acceleration.

$$v = \frac{ds}{dt} = kt \quad \text{or} \int_0^s ds = k\int_0^t t\,dt \qquad \therefore \quad s = \frac{1}{2}kt^2$$

For completion of n^{th} fraction of the circle,

s = $(2\pi r)n$ or $t^2 = (4\pi nr)/k$ ... (i)

Tangential acceleration

$= \alpha_T = \frac{dv}{dt} = k$ ... (ii)

Normal acceleration, $\alpha_N = \frac{v^2}{r} = \frac{k^2t^2}{r}$ ... (iii)

or $\alpha_N = 4\pi nk$

$$\therefore \alpha = \sqrt{(\alpha_T^2 + \alpha_N^2)} = \left[k^2 + 16\pi^2n^2k^2\right]^{1/2}$$

$$= k\left[1 + 16\pi^2n^2\right]^{1/2} = 0.50\left[1 + 16 \times (3.14)^2 \times (0.10)^2\right]^{1/2}$$

$$= 0.8 \text{m}/\text{s}^2$$

Example 8: Two boats, A and B move away from a buoy anchored at the middle of a river along mutually perpendicular straight lines: the boat A along the river, and the boat B across the river. Having moved off an equal distance from the buoy the boats returned. Find the ratio of times of motion of boats $\frac{\tau_A}{\tau_B}$ if the velocity of each boat with respect to water is n = 1.2 times greater than the stream velocity.

Sol: The velocity of boat B will be vector sum of velocity of river flow and the velocity of B with respect to river. These three vectors form a right triangle. The velocity of boat B is the base, the velocity of river flow is the perpendicular and the velocity of B with respect to river is the hypotenuse.

Let l be the distance covered by the boat A along the river as well as by the boat B across the river. Let v_0 be the stream velocity and v' the velocity of each boat with respect to water. Therefore, the time taken by the boat A in its journey

$$t_A = \frac{l}{v' + v_0} + \frac{l}{v' - v_0} = \frac{2lv'}{v'^2 - v_0^2}$$

And for the boat B

$$t_B = \frac{l}{\sqrt{v'^2 - v_0^2}} + \frac{l}{\sqrt{v'^2 - v_0^2}} = \frac{2l}{\sqrt{v'^2 - v_0^2}}$$

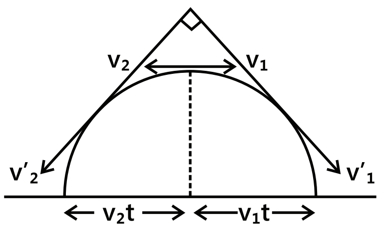
Hence, $\frac{t_A}{t_B} = \frac{v'}{\sqrt{v'^2 - v_0^2}} = \frac{\eta}{\sqrt{\eta^2 - 1}}$

$\left(\text{where } \eta = \frac{v'}{v_0}\right)$

On substitution, $\frac{t_A}{t_{B_1}} = 1.8$ (approx)

Example 9: Two particles move in a uniform gravitational field with an acceleration 'g'. At the initial moment the particles were located at one point and moved with velocities $v_1 = 3.0 \text{ ms}^{-1}$ and $v_2 = 4.0 \text{ ms}^{1}$ horizontally in opposite directions. Find the distance between the particles at the moment when their velocity vectors become mutually perpendicular.

Sol: The relative acceleration between the particles is zero. Initial relative distance is zero. So the final relative distance between them is equal to product of time and relative velocity. The time required can be found by using the equations of final velocities in Cartesian coordinates.



Let the velocities of the particles (say v'_1 and v'_2) become mutually perpendicular after time t. Then, their velocities become

$v'_1 = v_1\hat{i} + gt\hat{j}$; $v'_2 = -v_2\hat{i} + gt\hat{j}$

As $v'_1 \perp v'_2$, so, $v'_1 \cdot v'_2 = 0$

or $\left(v_1\hat{i} + gt\hat{j}\right)\cdot\left(v_2\hat{i} + gt\hat{j}\right) = 0$

or $-v_1v_2 + g^2t^2 = 0$

Hence, $t = \dfrac{\sqrt{v_1v_2}}{g}$

In the frame attached with 2 for the particle 1

$$r = r_0 + v_0t + \frac{1}{2}wt^2$$

Both the particles are initially at the same position and have same acceleration g, so

$r_0 = 0$, $w = 0$, and $v_0 = \left|v_1 - v_2\right|$.

Thus, the sought distance is

$\left|r\right| = \left|v_0\right|t = \left(v_1 + v_2\right)t$ (using the value of t)

$= \dfrac{v_1 + v_2}{g}\sqrt{v_1v_2}$

= 2.5 m, on substituting the values of v_1, v_2 and g.

Example 10: A boy whirls a stone in a horizontal circle of radius 1.5 m and at height 2.0 m above level ground. The string breaks, and the stone flies off horizontally and strikes the ground after travelling a horizontal distance of 10 m. What is the magnitude of the centripetal acceleration of the stone while in circular motion?

Sol: The time of fall of the stone depends on the height of the stone and can be found using the second equation of motion with constant acceleration and zero initial velocity. The horizontal component of stone's velocity remains constant is equal to the horizontal distance covered by the stone divided by the time of fall. The centripetal acceleration is equal to the square of the horizontal velocity divided by the radius of the horizontal circle.

$$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2\times 2}{9.8}} = 0.64\,s;\quad v = \frac{10}{t} = 15.63\,m/s$$

$$a = \frac{v^2}{R} = 163\,m/s^2$$

JEE Main/Boards

Exercise 1

Projectile Motion

Q.1 What do you understand by motion in two dimensions? When an object is moving with uniform velocity in two dimensions, explain displacement, velocity and find the equations of motion of the object.

Q.2 Find the relation for (i) velocity and time (ii) displacement and time, when an object is moving with uniform acceleration in two dimensions.

Q.3 What is a projectile? Give its examples. Show that the path of projectile is a parabolic path when projected horizontally from a certain height.

Q.4 Show that there are two angles of projection for which the horizontal range is the same.

Q.5 Find (i) time of flight, (ii) Max. height and (iii) horizontal range of projectile projected with speed

v_{AB} (velocity of A with respect to B) $= v_A - v_B$

a_{AB} (acceleration of A with respect to B) $= a_A - a_B$

making an angle θ with the horizontal direction from ground.

Q.6 Find the magnitude and direction of the velocity of an object at any instant during the oblique projection of a projectile.

Q.7 Find (i) the path of projectile, (ii) time of flight, (iii) horizontal range and (iv) maximum height, when a projectile is projected with velocity υ making an angle θ with the vertical direction.

Q.8 What is centripetal acceleration? Find its magnitude and direction in case of a UCM of an object.

Q.9 A stone is dropped from the window of a bus moving at 60 kmh^{-1}. If the window is 196 cm. high, find the distance along the track which the stone moves before striking the ground.

Q.10 A hiker stands on the edge of a cliff 490 m above the ground and throws a stone horizontally with an initial speed of 15 ms^{-1}. Neglecting air resistance, find the time taken by the stone to reach the ground, and the speed with which it hits the ground. Take g = 9.8 m/s^2.

Q.11 A ball is thrown horizontally from the top of a tower with a speed of $50\,ms^{-1}$. Find the velocity and position at the end of 3 second g = 9.8 ms^{-2}.

Q.12 A body is projected downward at an angle of 30° to the horizontal with a velocity of 9.8 m/s from the top of a tower 29.4 m high. How long will it take before striking the ground?

Q.13 Prove that a gun will shoot three times as high when its angle of elevation is $60°$ as when it is $30°$, but cover the same horizontal range.

Q.14 Prove that the maximum horizontal range is 4 times the maximum height attained by a projectile which is fired along the required oblique direction.

Q.15 Two particles are projected from the ground simultaneously with speeds of 30 m/s and 20 m/s at angles $60°$ and $30°$ with the horizontal on the same direction. Find maximum distance between them on ground where they strike. g = 10 m/s^2.

Q.16 A projectile has the same range when the maximum height attained by it is either H_1 or H_2. Find the relation between R, H_1 and H_2.

Q.17 A projectile is given an initial velocity of $\left(\hat{i}+2\hat{j}\right)$, where $\hat{i}$ is a unit vector along horizontal and $\hat{j}$ is unit vector vertically upward. Find the Cartesian equation of its path. (g = 10 m/s^2)

Q.18 Find the maximum horizontal range of a cricket ball projected with a velocity of 80 m/s. If the ball is to have a range of 100 $\sqrt{3}$ m, find the least angle of projection and the least time taken.

Q.19 A bullet fired from a rifle attains a maximum height of 5m and crosses a range of 200 m. Find the angle of projection.

Q.20 A target is fixed on the top of a pole 13 m high. A person standing at a distance 50 m from the pole is capable of projecting a stone with a velocity 10 $\sqrt{g}$ m/s. If he wants to strike the target in shortest possible time, at what angle should he project the stone.

Q.21 A particle is projected with a velocity u so that its horizontal range is twice the greatest height attained. Find the horizontal range of it.

Q.22 A drunkard walking in a narrow lane takes 5 steps forward and 3 steps backward, and so on. Each step is 1 m long and required 1s. Determine how long the drunkard takes to fall in a pit 13 m away from the start.

Q.23 A jet airplane travelling at the speed of 500km h^{-1} ejects its projects of combustion at the speed of $1500km\,h^{-1}$ relative to the jet plane. What is the speed of the later with respect to observer on the ground.

Q.24 A car moving along a straight highway with speed of 126 $km\,h^{-1}$ is brought to a stop within a distance of 200 m. What is the retardation of the car (assumed uniform) and how long does it take for the car to stop?

Q.25 Two trains A and B of length 400 m each are moving on two parallel tracks with a uniform speed of $72km\,h^{-1}$ in the same direction with A head of B. The driver of B decides to overtake A and accelerate by $1\,ms^{-2}$. If after 50 s, the guard of B just brushes past the driver of A, what was the original distance between them?

Q.26 Two towns A and B are connected by a regular bus service with a bus leaving in either direction every T min. A man cycling with a speed of 20km/h in the direction of A and B notices that a bus goes past him every 18 min in the direction of his motion, and every 6 min in the opposite direction. What is the period T of the bus service and with what speed (assumed constant) do the buses ply on the road?

Q.27 A ball is dropped from a height of 90 m on a floor. At each collision with the floor, the ball loses one tenth of its speed. Plot the speed-time graph of its motion between t=0 to 12s. ($g=10\,ms^{-2}$)

Q.28 A man walks on a straight road from his home to a market 2.5 km away with a speed of 5 km/h. Finding the market closed, he instantly turns and walks back with a speed of 7.5 km/h. What is the (a) magnitude of average velocity and (b) average speed of the man, over the interval of time (i) 0 to 30 min. (ii) 0 to 50 min (iii) 0 to 40 min?

Q.29 A dive bomber, diving at an angle of 53° with the vertical, released a bomb at an altitude of 2400ft. the bomb hits the ground 5.0 s after being released. (i) What is the speed of the bomber? (ii) How far did the bomb travel horizontally during its flight? (iii) What were the horizontal and vertical components of its velocity just before striking the ground?

Circular Motion

Q.30 Calculate the angular velocity of the minute's hand of a clock.

Q.31 What is the angular velocity in radian per second of a fly wheel making 300 r.p.m.?

Q.32 The wheel of an automobile is rotating with 4 rotations per second. Find its angular velocity. If the radius of the fly wheel is 50cm, find the linear velocity of a point on its circumference.

Q.33 The angular velocity of a particle moving in a circle of radius 50 cm is increased in 5 minutes from 100 revolutions per minute. Find (a) angular acceleration (b) linear acceleration.

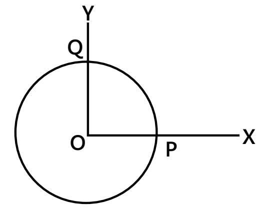
Q.34 A body is moving in a circle of radius 100 cm with a time period of 2 second. Find the acceleration.

Q.35 An insect trapped in a circular groove of radius 12cm moves along the grove steadily and completes 7 revolutions in 100s. (i) What is the angular speed, and the linear speed of the motion? (ii) Is the acceleration vector a constant vector? What is its magnitude?

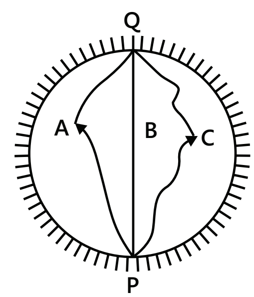
Q.36 Calculate the centripetal acceleration of a point on the equator of earth due to the rotation of earth due to the rotation of earth about its own axis. Radius of earth=6400 km.

Q.37 A cyclist is riding with a speed of $27\,kmh^{-1}$. As he approaches a circular turn on the road of radius 80.0m, he applies brakes and reduces his speed at a constant rate of 0.5 ms^{-1} per second. Find the magnitude of the net acceleration of the cyclist.

Q.38 A particle moves in a circle of radius 4.0 cm clockwise at constant speed of $2\ cm\,s^{-1}$. If $\hat{x}$ and $\hat{y}$ are unit acceleration vectors along X-axis and Y-axis respectively, find the acceleration of the particle at the instant half way between PQ figure.



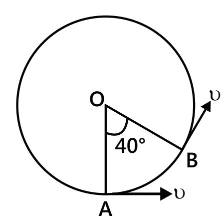
Q.39 Three girls skating on a circular ice ground of radius 200m start from a point P on the edge of the ground and reach a point Q diametrically opposite to P following different paths as shown in figure. What is the magnitude of the displacement vector for each? For which girl is this equal to the actual length of path skated?



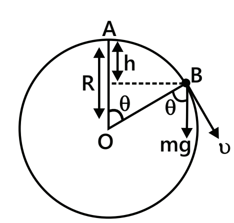
Q.40 A cyclist starts from the centre O of a circular park of radius 1m reaches the edge P of the park, then cycles along the circumference, and returns to the centre along QO. If the round trip takes 10 minutes, what is the (i) net displacement (ii) average velocity and (iii) average speed of the cyclist?

Q.41 A cyclist is riding with a speed of $36\ km\,h^{-1}$. As he approaches a circular turn on the road of radius 140m, he applies break and reduces his speed at the constant rate of $1\,m\,s^{-2}$. What is the magnitude and direction of the net acceleration of the cyclist on the circular turn?

Q.42 A particle is moving in a circle of radius r centres at O with constant speed υ. What is the change in velocity in moving from A to B? In the figure. Given $\angle AOB = 40°$.



Q.43 A particle originally at rest at the highest point of a smooth vertical circle of radius R, is slightly displaced. Find the vertical distance below the highest point where the particle will leave the circle.



Exercise 2

Projectile Motion

Single Correct Choice Type

Q.1 A particle is projected with a certain velocity at an angle θ above the horizontal from the foot of a given plane inclined at an angle of $45°$ to the horizontal. If the particle strikes the plane normally, then equals

(A) $\tan^{-1}(1/3)$ (B) $\tan^{-1}(1/2)$

(C) $\tan^{-1}(1/\sqrt{2})$ (D) $\tan^{-1}3$

Q.2. Two projectiles A and B are thrown with the same such that A makes angle θ with the horizontal and B makes angle θ with the vertical, then

(A) Both must have same time of fight

(B) Both must achieve same maximum height

(C) A must have more horizontal range than B

(D) Both may have same time of flight

Q.3 A projectile is fired with a speed u at an angle θ with the horizontal. Its speed when its direction of motion makes an angle $'\alpha'$ with the horizontal is

(A) $u\sec\theta\cos\alpha$ (B) $u\sec\theta\sin\alpha$

(C) $u\cos\theta\sec\alpha$ (D) $u\sin\theta\sec\alpha$

Q.4 A ball is projectile from top of tower with a velocity of 5 m/s at an angle of 53° to horizontal. Its speed when it is at a height of 0.45m from the point of projection is:

(A) 2 m/s (B) 3 m/s

(C) 4 m/s (D) Data insufficient

Q.5 particle is dropped from the height of 20m from horizontal ground. A constant force acts on the particle in horizontal direction due to which horizontal acceleration of the particle becomes 6 ms^{-2}. Find the horizontal displacement of the particle till it reaches ground.

(A) 6m (B) 10 m (C) 12 m (D) 24 m

Q.6 Find time of flight of projectile thrown horizontally with speed $10\,ms^{-1}$ from a long inclined plane which makes an angle of $\theta = 45°$ from horizontal.

(A) $\sqrt{2}$ sec (B) $2\sqrt{2}$ sec

(C) 2 sec (D) None of these

Q.7 A projectile is fired with a velocity at right angles to the slope which is inclined at an angle θ with the horizontal. The expression for the range R along the incline is

(A) $\frac{2v^2}{g}\sec\theta$ (B) $\frac{2v^2}{g}\tan\theta$

(C) $\frac{2v^2}{g}\tan\theta\sec\theta$ (D) $\frac{v^2}{g}\tan^2\theta$

Q.8 A hunter tries to hunt a monkey with a small, very poisonous arrow, blown from a pipe with initial speed v_0. The monkey is hanging on a branch of a tree at height H above the ground. The hunter is at a distance L from the bottom of the tree. The monkey sees the arrow leaving the blow pipe and immediately lose the grip on the tree, falling freely down with zero initial velocity. The minimum initial speed v_0 of the arrow for hunter to succeed while monkey is in air

(A) $\sqrt{\frac{g(H^2+L^2)}{2H}}$ (B) $\sqrt{\frac{gH^2}{H^2+L^2}}$

(C) $\sqrt{\frac{g(H^2+L^2)}{H}}$ (D) $\sqrt{\frac{2gH^2}{H^2+L^2}}$

Q.9 A swimmer swims in still water at a speed=5 km/hr. He enters a 200 m wide river, having river flow speed=4 km/hr at point A and proceeds to swim at an angle of $127°$ with the river flow direction. Another point B is located directly across A on the other side. The swimmer lands on the other bank at a point C, from which he walks the distance CB with a speed=3 km/hr. The total time in which he reaches from A to B is

(A) 5 minutes (B) 4 minutes

(C) 3 minutes (D) None

Q.10 A boat having a speed of 5 km/hr. in still water, crossed a river of width 1 km along the shortest possible path in 15 minutes. The speed of the river in Km/hr.

(A) 1 (B) 3 (C) 4 (D) $\sqrt{41}$

Q.11 A motor boat is to reach at a point $30°$ upstream (w.r.t. normal) on other side of a river flowing with velocity 5m/s. Velocity of motorboat w.r.t. water is $5\sqrt{3}$ m/s. The driver should steer the boat at an angle

(A) $120°$ w.r.t. stream direction

(B) $30°$ w.r.t. normal to the bank

(C) $30°$ w.r.t. the line of destination from starting point.

(D) None of these

Q.12 A flag is mounted on a car moving due north with velocity of 20 km/hr. Strong winds are blowing due East with velocity of 20km/hr. The flag will point in direction

(A) East (B) North-East

(C) South-East (D) South-West

Q.13 Three ships A, B & C are in motion. The motion of A as seen by B is with speed v towards north-east. The motion of B as seen by C is with speed v towards the north-west. Then as seen by A, C will be moving towards

(A) north (B) south (C) east (D) west

Q.14 Wind is blowing in the north direction at speed of 2 m/s which causes the rain to fall at some angle with the vertical. With what velocity should a cyclist drive so that the rain appears vertical to him?

(A) 2m/s south (B) 2m/s north

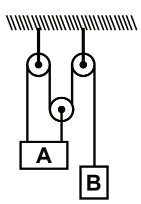
(C) 4m/s west (D) 4 m/s south

Q.15 When the driver of a car A sees a car B moving towards his car and at a distance 30m, takes a left turn of $30°$. At the same instant the driver of the car B takes a turn to his right at an angle $60°$. The two cars collides after two seconds, then the velocity (in m/s) of the car A and B respectively will be : [assume both cars to be moving along same line with constant speed]

(A) 7.5, 7.5 $\sqrt{3}$ (B) 7.5, 7.5

(C) 7.5 $\sqrt{3}$, 7.5 (D) None

Q.16 At a given instant, A is moving with velocity of 5m/s upwards. What is velocity of B at that time



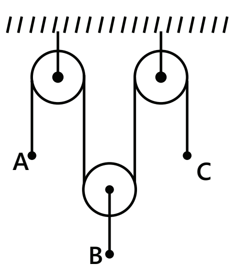
(A) 15 m/s↓

(B) 15 m/s↑

(C) 5 m/s ↓

(D) 5 m/s ↑

Q.17 The pulleys in the diagram are all smooth and light. The acceleration of A us a upwards and the acceleration of A is a upward and the acceleration of C is f downwards. The acceleration is



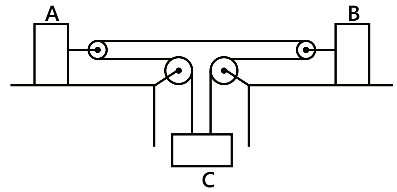
(A) $1/2(f-a)$ up

(B) $1/2(a+f)$ down

(C) $1/2(a+f)$ up

(D) $1/2(a-f)$ up

Q.18 If acceleration of A is 2 m/s² to left and acceleration of B is 1 m/s² to left, then acceleration C is



(A) 1 m/s² upwards (B) 1 m/s² downwards

(C) 2 m/s² downwards (D) 2m/s² upward

Circular Motion

Single Correct Choice Type

Q.19 Two racing cars of masses m_1 and m_2 are moving in circles of radii r_1 and r_2 respectively, their speeds are such that they each make a complete circle in the same time t. The ratio of the angular speed of the first to the second car is:

(A) $m_1 : m_2$ (B) $r_1 : r_2$ (C) 1:1 (D) $m_1r_1 : m_2r_2$

Q.20 A particle moves in a circle of radius 25 cm at two revolution per sec. The acceleration of the particle in m/s^2 is:

(A) π^2 (B) $8\pi^2$ (C) $4\pi^2$ (D) $2\pi^2$

Q.21 Two particle and Q are located at distance r_P and r_Q respectively from the centre of a rotating disc such that $r_P > r_Q$:

(A) Both P and Q have the same acceleration

(B) Both P and Q do not have any acceleration

(C) P has greater acceleration than Q

(D) Q has greater acceleration than P

Q.22 When particle moves in a circle with a uniform speed:

(A) Its velocity and acceleration are both constant

(B) Its velocity is constant but the acceleration changes

(C) Its acceleration is constant but the velocity changes

(D) Its velocity and acceleration both change

Q.23 If a particle moves in a circle described equal angles in equal times, its velocity vector:

(A) Remains constant

(B) Changes in magnitude

(C) Changes in direction

(D) Changes both in magnitude and direction

Q.24 If the equation for the displacement of a particle moving on a circular path is given by $(\theta) = 2t^3 + 0.5$, where θ is in radians and t in seconds, then the angular velocity of the particle after 2 sec from its start is:

(A) 8 rad/sec (B) 12 rad/sec

(C) 24 rad/sec (D) 36 rad/sec

Q.25 The second's hand of a watch has length 6cm. Speed of end point and magnitude of difference of velocities at two perpendicular ll be:

(A) 6.28 & 0 mm/s (B) 8.88 & 4.44 mm/s

(C) 8.88 & 6.28 m/s (D) 6.28 & 8.88 mm/s

Q.26 A fan is making 600 revolutions per minute. If after some time it makes 1200 revolutions per minute, then increases in its angular velocity is:

(A) 10π rad/sec (B) 20π rad/sec

(C) 40π rad/sec (D) 60π rad/sec

Q.27 A wheel completes 2000 revolutions to cover the 9.5 km. distance, then the diameter of the wheel is:

(A) 1.5 m (B) 1.5 cm (C) 7.5 cm (D) 7.5 m

Q.28 A body moves with constant angular velocity on a circle. Magnitude of angular acceleration is:

(A) $r\omega^2$ (B) Constant

(C) Zero (D) None of the above

Q.29 For a particle in a uniformly accelerated (speed increasing uniformly) circular motion:

(A) Velocity is radial and acceleration is transverse only.

(B) Velocity is transverse and acceleration is radial only

(C) Velocity is radial and acceleration has both radial and transverse components

(D) Velocity is transverse and acceleration has both radial and transverse components

Q.30 A particle moves in a circular orbit under the force proportional to the distance 'r'. The speed of the particle is:

(A) Proportional of r^2 (B) Independent of r

(C) Proportional to r (D) Proportional to 1/r

Previous Years' Questions

Q.1 A river is flowing from west to east at a speed of 5 m/min. A man on the south bank of the river, capable of swimming at 10m/min in still water, wants to swim across the river in the shortest time. He should swim in a direction ***(1983)***

Q.2 A boat which has a speed of 5 km/h in still water crosses a river of width 1 km along the shortest possible path in 15 mi. The velocity of the river water in km/h is ***(1988)***

Assertion Reasoning Type

(A) If statement-I is true, statement-II is true: statement-II is the correct explanation for statement-I

(B) if statement-I is true, Statement-II is true: statement-II is not a correct explanation of statement-I

(C) If statement-I is true: statement-II is false

(D) If Statement-I is false: statement-II is true

Q.3 Statement-I: For an observer looking out through the window of a fast moving train, the nearby objects appear to move in the opposite direction to the train, while the distant objects appear to be stationary.

Statement-II: If the observer and the object are moving at velocities $\vec{v}_1$ and $\vec{v}_2$ respectively with reference to a laboratory frame, the velocity of the object with respect to the observer is $\vec{v}_2 - \vec{v}_1$ ***(2008)***

Q.4 For a particle in uniform circular motion the acceleration $\vec{a}$ at a point P (R, θ) on the circle of radius R is (here θ is measures from the x-axis) ***(2010)***

(A) $-\frac{v^2}{R}\cos\theta\hat{i} + \frac{v^2}{R}\sin\theta\hat{j}$ (B) $-\frac{v^2}{R}\sin\theta\hat{i} + \frac{v^2}{R}\sin\theta\hat{j}$

(C) $-\frac{v^2}{R}\cos\theta\hat{i} - \frac{v^2}{R}\sin\theta\hat{j}$ (D) $\frac{v^2}{R}\hat{i} + \frac{v^2}{R}\hat{j}$

Q.5 A boy can throw a stone up to a maximum height of 10 m. The maximum horizontal distance that the boy can throw the same stone up to will be ***(2012)***

(A) $20\sqrt{2}$ m (B) 10 m (C) $10\sqrt{2}$ m (D) 20 m

Q.6 Two cars of masses m_1 and m_2 are moving in circles of radii r_1 and r_2, respectively. Their speeds are such that they make complete circles in the same time t. The ratio of their centripetal acceleration is ***(2012)***

(A) $m_1r_1 : m_2r_2$ (B) $m_1 : m_2$ (C) $r_1 : r_2$ (D) 1 : 1

Q.7 A projectile is given an initial velocity of $(\hat{i} + 2\hat{j})$ m/s, where $\hat{i}$ is along the ground and $\hat{j}$ is along the vertical. If g = 10 m/s^2, the equation of its trajectory is: ***(2013)***

(A) $y = 2x - 5x^2$ (B) $4y = 2x - 5x^2$

(C) $4y = 2x - 25x^2$ (D) $y = x - 5x^2$

JEE Advanced/Boards

Exercise 1

Projectile Motion

Q.1 A particle moves in the plane XY with constant acceleration a directed along the negative y axis. The equation of motion of the particle has the form $y = \alpha x - \beta x^2$, where α and β are positive constants. Find the velocity of the particle at the origin of coordinates.

Q.2 Two seconds after projection, a projectile is moving at $30°$ above the horizontal, after one more second it is moving horizontally. Find the magnitude and direction of its initial velocity. (g=10 m/s^2)

Q.3 A ball is projected from O with an initial velocity 700 cm/sec in a direction $37°$ above the horizontal. A ball B, 500 cm away from O on the line of the initial velocity of A, is released from rest at the instant A is projected. Find

(i) The height through which B falls, before it is hit by A.

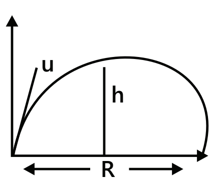
(ii) The direction of the velocity A at the time of impact (Given g=10 $m\,s^{-2}$, sin $37°$ =0.6)

Q.4 On a frictionless horizontal surface, assumed to be the x-y plane, a small trolley A is moving along a straight line parallel to the y-axis with a constant velocity of $\left(\sqrt{3}-1\right)$ m/s. At a particular instant, when the line OA makes an angle $45°$ with the x-axis, a ball is thrown along the surface from the origin O. Its velocity makes an angle ϕ with the x-axis and it hits the trolley.

(i) The motion of the ball is observed from the frame of the trolley. Calculate the angle θ made by the velocity vector of the ball with the x-axis in this frame.

(ii) Find the speed of the ball with respect to the surface if $\phi = 4\theta/3$.

Q.5 If R us the horizontal range and h, the greatest height of a projectile, find the initial speed. $\left[g = 10 m/s^2\right]$



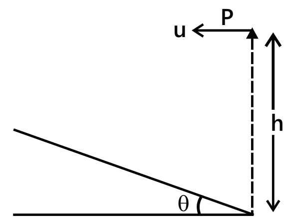
Q.6 A stone is thrown horizontally from a tower. In 0.5 second after the stone began to move, the numerical value of its velocity was 1.5 times its initial velocity. Find the initial velocity of stone.

Q.7 A shell is fired from a point O at an angle of $60°$ with a speed of 40 m/s & it strikes a horizontal plane through O. at a point A. The gun is fired a second time with the same angle of elevation but a different speed v. If it hits the target which starts to rise $9\sqrt{3}$ m/s at the same instant as the shell is fired, find v. (Take g=10 m/s^2)

Q.8 A cricket ball thrown from a height of 1.8 m at an angle of $30°$ with the horizontal at a speed of 18 m/s is caught by another field's man at a height of 0.6 m from the ground. How far were the two men apart?

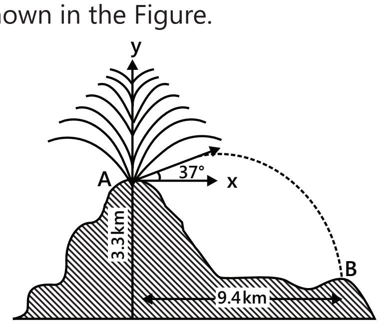
Q.9 A batsman hits the ball at a height 4.0 ft. from the ground at projection angle of $45°$ and the horizontal range is 350 ft. Ball falls on left boundary line, where a 24 ft height fence is situated at a distance of 320 ft. Will the ball clear the fence?

Q.10 (i) A particle is projected with a velocity of 29.4 m/s at an angle of $60°$ to the horizontal. Find the range on a plane inclined at $30°$ to the horizontal when projected from a point of the plane up the plane.



(ii) determine the velocity with which a stone must be projected horizontally from a point P, so that it may hit the inclined plane perpendicularly. The inclination of the plane with the horizontal is θ and P is h metre above the foot of the incline as shown in the Figure.

Q.11 During the volcanic eruption chunks of solid rock are blasted out of the volcano.



(i) At what initial speed would a volcanic object have to be ejected at $37°$ to the horizontal from the vent A in order to fall at B as shown in Figure.

(ii) What is the time of flight. (g=9.8 m/s^2)

Q.12 A projectile is projected with an initial velocity of $\left(6\hat{i}+8\hat{j}\right)ms^{-1}$, $\hat{i}$ = unit vector in horizontal direction and $\hat{j}$ = unit vector in vertical upward direction then calculate its horizontal range, maximum height and time of flight.

Q.13 An aeroplane is flying at a height of 1960 metre in a horizontal direction with a velocity of 100 m/s, when it is vertically above an object M on the ground it drops a bomb. If the bomb reaches the ground at the point N, then calculate the time taken by the bomb to reach the ground and also find the distance MN.

Q.14 A projectile is projected from the base of a hill whose slope is that of right circular cone, whose axis is vertical. The projectile grazes the vertex and strikes the hill again at a point on the base. If θ be the semi-vertical angle of the cone, h its height u the initial velocity of projection and α the angle of projection, show that

(i) $\tan\theta = 2\cot\alpha$ (ii) $u^2 = \frac{gh(4+\tan^2\theta)}{2}$

Q.15 A person is standing on a truck moving with a constant velocity of 14.7 m/s on a horizontal road. The man throws a ball in such a way that it returns to the truck after the truck has moved 58.8 m. Find the speed and the angle of projection (i) as seen from the truck, (ii) as seen from the road.

Q.16 Two bodies are thrown simultaneously from the same point. One thrown straight up and the other at an angle α with the horizontal. Both the bodies have equal velocity of v_0 Neglecting air drag, find the separation of the particle at time t.

Q.17 Two particles move in a uniform gravitational field with an acceleration g. At the initial moment the particles were located at one point and moved with velocities 3 m/s and 4 m/s horizontally in opposite directions. Find the distance between the particles at the moment when their velocity vectors become mutually perpendicular.

Q.18 A particle is projected from O at an elevation α and after t second it has an elevation β as seen from the point of projection. Prove that its initial velocity is $\frac{gt\cos\beta}{\sin(\alpha-\beta)}$.

Q.19 The velocity of a particle when it is at its greatest height is $\sqrt{\frac{2}{5}}$ of its velocity when it is at half its greatest height. Find the angle of projection of the particle.

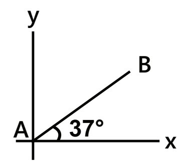
Q.20 A man crosses a river in a boat. If he crosses the river in minimum time he takes 10 minutes with a drift 120 m. If he crosses the river taking shortest path, he takes 12.5 minutes. Assuming $v_{b/r} > v_r$ find

(i) Width of the river

(ii) Velocity of the boat with respect to water,

(iii) Speed of the current.

Q.21 A butterfly is flying with velocity $10\hat{i}+12\hat{j}$ m/s and wind is blowing along x axis with velocity u. If butterfly starts motion from A and after some time reaches point B, find the value of u.



Q.22 Rain is falling vertically with a speed of 20 m/s^{-1} relative to air. A person is running in the rain with a velocity of 5 m/s^{-1} and a wind is also blowing with a speed of 15 m/s^{-1} (both towards east). Find the angle with the vertical at which the person should hold his umbrella so that he may not get drenched.

Circular Motion

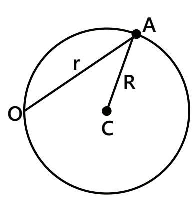
Q.23 A bullet is moving horizontally with certain velocity. It pierces two paper discs rotating co-axially with angular speed ω separated by a distance ℓ. If the hole made by the bullet on 2nd disc is shifted through an angle θ with respect to that in the first disc, find the velocity of the bullet, (change of velocity in the bullet is neglected)

Q.24 Position vector of a particle performing circular motion is given by $\vec{r} = 3\hat{i}+4\hat{j}$ m, velocity vector is $\vec{v} = -4\hat{i}+3\hat{j}$ m/s. If acceleration is $\vec{a} = -7\hat{i}-\hat{j}\,m/s^2$ find the radial and tangential components of acceleration.

Q.25 An astronaut is rotating in a rotor having vertical axis and radius 4m. If he can withstand upto acceleration of 10g. Then what is the maximum number of permissible revolution per second? $\left(g = m/s^2\right)$.

Q.26 A racing-car of 1000 kg moves round a banked track at a constant speed of 108 $km\,h^{-1}$. Assuming the total reaction at the wheels is normal to the track and the horizontal radius of the track is 90 m, calculate the angle of inclination of the track to the horizontal and the reaction at the wheels.

Q.27 A particle A moves along a circle of radius R=50 cm so that its radius vector r relative to the point O (see figure) rotates with the constant angular velocity $\omega = 0.40$ rad/sec. Find the modulus of the velocity of the particle and modulus and direction of its total acceleration.



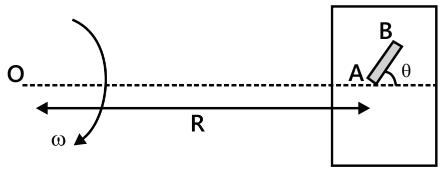
Q.28 A wet open umbrella is held upright and is rotated about the handle at uniform rate of 21 revolutions is 44s. If the rim of the umbrella circle is 1 meter in diameter and the height of the rim above the floor is 1.5m, find where the drops of water spun off the rim and hit the floor.

Q.29 A spaceman in training is rotated in a seat at the end of horizontal rotating arm of length 5m. if he can withstand acceleration up to 9 g, what is the maximum number of revolutions per second permissible? Take $g = 10 m/s^2$

Q.30 An insect on the axle of a wheel observes the motion of a particle and 'find' it to take its place along the circumference of a circle of radius 'R' with a uniform angular speed ω. The axle is moving with a uniform speed 'v' relative to the ground. How will an observer on the ground describe the motion of the same point?

Q.31 A stone is thrown horizontally with a velocity 10 m/s. Find the radius of curvature of its trajectory in 3 second after the motion began. Disregard the resistance of air.

Q.32 A table with smooth horizontal surface is fixed in a cabin that rotates with a uniform angular velocity $\omega = 220 s^{-1}$ in a circular path of radius R=700m. A smooth groove AB of length L=7 m is made on the surface of the table. The groove makes an angle $\theta = 30°$ with the radius OA of the circle in which the cabin rotates. A small particle is kept at the point A in the groove and is released to move along AB. Find the time taken by the particle to reach the point B.



Q.33 A smooth sphere of radius R is made to translate in a straight line with a constant acceleration a. A particle kept on the top of the sphere is released from there at zero velocity with respect to the sphere. Find the speed of the particle with respect to the sphere as a function of the angle θ slides.

Q.34 If a particle is rotating in a circle of radius R with velocity at an instant v and the tangential acceleration is a. Find the net acceleration of the particle.

Exercise 2

Projectile Motion

Single Correct Choice Type

Q.1 A projectile of mass 1 kg is projected with a velocity of $\sqrt{20}$ m/s such that it strikes on the same level as the point of projection at a distance of $\sqrt{3}$ m. Which of the following options is incorrect?

(A) The maximum height reached by the projectile can be 0.25 m.

(B) The minimum velocity during its motion can be $\sqrt{15}$ m/s

(C) The time taken for the flight can be $\sqrt{\frac{3}{5}}$ sec.

(D) Minimum kinetic energy during its motion can be 6J.

Q.2 A particle is projected from the ground with velocity u at angle θ with horizontal. The horizontal range, maximum height and time of fight are R , H and T respectively. They are given by, $R = \frac{u^2 \sin 2\theta}{g}$, $H = \frac{u^2 \sin^2 \theta}{2g}$ and $T = \frac{2u\sin\theta}{g}$

Now keeping u as fixed, θ is varied from $30°$ to $60°$. Then,

(A) R will first increase then decrease, H will increase and T will decrease

(B) R will first increase then decrease while H and T both will increase

(C) R will first decrease while H and T will increase

(D) R will first increase while H and T will increase

Q.3 The trajectory of particle 1 with respect to particle 2 will be

(A) A parabola (B) A straight line

(C) A vertical straight line (D) A horizontal straight line

Q.4 If $v_1 \cos\theta_1 = v_2 \cos\theta_2$, then choose the incorrect statement.

(A) One particle will remain exactly below of above the other particle

(B) The trajectory of one with respect to other will be a vertical straight line

(C) Both will have the same range

(D) None of these

Q.5 If $v_1 \sin\theta_1 = v_2 \sin\theta_2$, then choose the incorrect statement.

(A) The time of flight of both the particle will be same

(B) The maximum height attained by the particles will be same

(C) The trajectory of one with respect to another will be a horizontal straight line

(D) None of these

Multiple Correct Choice Type

Q.6 Choose the correct alternative (s)

(A) If the greatest height to which a man can throw a stone is h, then the greatest horizontal distance upto which he can throw the stone is 2h.

(B) The angle of projection for a projectile motion whose range R is m times the maximum height is $\tan^{-1}$ (4/n)

(C) The time of flight T and the horizontal range R of a projectile are connected by the equation $gT^2 = 2R\tan\theta$ where θ is the angle of projection.

(D) A ball is thrown vertically up. Another ball is thrown at an angle θ with the vertical. Both of them remain in air for the same period of time. Then the ratio of heights attained by the two balls 1:1.

Q.7 If it is the total time of flight, h is the maximum height & R is the range for horizontal motion, the x & y co-ordinates of projectile motion and time t are related as:

(A) $y = 4th\left(\frac{t}{T}\right)\left(1-\frac{t}{T}\right)$ (B) $y = 4th\left(\frac{X}{R}\right)\left(1-\frac{X}{R}\right)$

(C) $y = 4th\left(\frac{T}{t}\right)\left(1-\frac{T}{t}\right)$ (D) $y = 4th\left(\frac{R}{X}\right)\left(1-\frac{R}{X}\right)$

Q.8 A particle moves in the xy plane with a constant acceleration 'g' in the negative y-direction. Its equation of motion is $y = ax - bx^2$, where a and b are constants. Which of the following is correct?

(A) The x-components of its velocity is constant.

(B) At the origin, the y-component of its velocity is a $\sqrt{\frac{g}{2b}}$.

(C) At the origin, its velocity makes an angle $\tan^{-1}$ (a) with the x-axis.

(D) The particle moves exactly like a projectile.

Q.9 A ball is rolled off along the edge of a horizontal table with velocity 4m/s. It hits the ground after time 0.4s. Which of the following are correct?

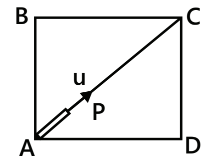
(A) The height of the table is 0.8 m.

(B) It hits the ground at an angle of 60° with the vertical

(C) It covers a horizontal distance 1.6 m from the table

(D) It hits the ground with vertical velocity 4m/s

Q.10 A large rectangular box moves vertically downward with an acceleration a. A toy gun fixed at A and aimed towards C fires a particle P.



(A) P will hit C if a=g

(B) P will hit the roof BC, if a>g

(C) P will hit the wall CD if a<g

(D) May be either (A), (B) or (C), depending on the speed of projection of P

Q.11 The vertical height of point P above the ground is twice that of point Q. A particle is projected downward with a speed of 5 m/s from P and at the same time another particle is projected upward with the same speed from Q. Both particles reach the ground simultaneously, if PQ is lie on same vertical line then

(A) PQ=30 m

(B) PQ=60 m

(C) Time of flight of the stones

(D) Time of flight of the stones=1/3s

Q.12 Two particles A & B projected along different directions from the same point P on the ground with the same speed of 70 m/s in the same vertical plane. They hit the ground at the same point Q such that PQ=480m. Then: (Use g=9.8 m/s^2, $\sin^{-1}0.96 = 74^\circ$, $\sin^{-1}0.6 = 37^\circ$)

(A) Ratio of their times of flights is 4:5

(B) Ratio of their maximum heights is 9:16

(C) Ratio of their minimum speeds during flight is 4:3

(D) The bisector of the angle between their directions of projection makes 45° with horizontal.

Comprehension Type

A projectile is thrown with a velocity of $50 m s^{-1}$ at an angle of 53° with the horizontal.

Q.13 Choose the incorrect statement

(A) It travels vertically with a velocity of $40 m s^{-1}$

(B) It travels horizontally with a velocity of $30 m s^{-1}$

(C) The minimum velocity of the projectile is $30\ m s^{-1}$

(D) None of these

Q.14 Determine the instants at which the projectile is at the same height.

(A) t=1s and t=7s (B) t=3s and t=5s

(C) t=2s and t=6s (D) all the above

Q.15 The equation of the trajectory is given by

(A) $180y = 240x - x^2$ (B) $180y = x^2 - 240x$

(C) $180y = 135x - x^2$ (D) $180y = x^2 - 135x$

Two projectile are thrown simultaneously in the same plane from the same point. If their velocities are v_1 and v_2 at angles θ_1 and θ_2 respectively from the horizontal, then answer the following questions

Match the Columns

Q.16 Match the quantities is column I with possible options from column II.

Particle's Motion	**Trajectory**
(A) Constant velocity	(p) straight line
(B) Constant speed	(q) Circular
(C) Variable acceleration	(r) Parabolic
(D) Constant acceleration	(s) Elliptical

Assertion Reasoning Type

(A) Statement-I is true, statement-II is true and statement-II is correct explanation for statement-I.

(B) Statement-I is true, statement-II is true and statement-II is NOT the correct explanation for statement-I

(C) Statement-I is true, statement-II is false.

(D) Statement-I is false, statement-II is true.

Q.17 Statement-I: The speed of a projectile is minimum at the highest point.

Statement-II: The acceleration of projectile is constant during the entire motion.

Q.18 Statement-I: Two stones are simultaneously projected from level ground from same point with same speeds but different angles with horizontal. Both stones move in same vertical plane. Then the two stones may collide in mid-air.

Statement-II: For two stones projected simultaneously from same point with same speed at different angles with horizontal, their trajectories may intersect at some point.

Q.19 Statement-I: If separation between two particles does not change then their relative velocity will be zero.

Statement-II: Relative velocity is the rate of change of position of one particle with respect to another.

Q.20 Statement-I: The magnitude of relative velocity of A with respect to B will be always less than V_A.

Statement-II: The relative velocity of A with respect to B is given by $V_{AB} = V_A - V_B$.

Q.21 Statement-I: Three projectiles are moving in different paths in the air. Vertical component of relative velocity between any of the pair does not change with time as long as they are in air. Neglect the effect of air friction.

Statement-II: Relative acceleration between any of the pair of projectile is zero.

Circular Motion

Q.22 An object follows a curved path. The following quantities may remain constant during the motion-

(A) Speed (B) Velocity

(C) Acceleration (D) Magnitude of acceleration

Q.23 The position vector of a particle in a circular motion about the origin sweeps out equal area in equal times-

(A) Velocity remains constant

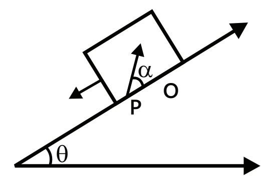
(B) Speed remains constant

(C) Acceleration remains constant

(D) Tangential acceleration remains constant

Previous Years' Questions

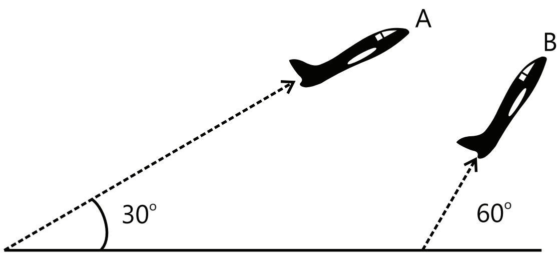
Q.1 A large heavy box is sliding without friction down a smooth plane of inclination θ. From a point P on the bottom of the box, a particle is projected inside the box. The initial speed of the particle with respect to the box is u and the direction makes an angle α with the bottom as shown in the Figure. ***(1998)***



(i) Find the distance along the bottom of the box between the point of projection P and the point Q where the particle lands (Assume that the particle does not hit any other surface of the box. Neglect air resistance.)

(ii) If the horizontal displacement of the particle as seen by an observer on the ground is zero, find the speed of the box with respect to the ground at the instant when the particle was projected.

Q.2 Airplanes A and B are flying with constant velocity in the same vertical plane at angles 30° and 60° with respect to the horizontal respectively as shown in the figure. The speed of A is $100\sqrt{3}$ ms^{-1}. At time t = 0 s, an observer in A finds B at a distance of 500 m. This observer sees B moving with a constant velocity perpendicular to the line of motion of A. If at $t = t_0$, A just escapes being hit by B, t_0 in seconds is ***(2014)***



Q.3 The distance r of the block at time t is ***(2016)***

(A) $\frac{R}{4}\left(e^{2\omega t}+e^{-2\omega t}\right)$ (B) $\frac{R}{2}\cos 2\omega t$

(C) $\frac{R}{2}\cos\omega t$ (D) $\frac{R}{4}\left(e^{\omega t}+e^{-\omega t}\right)$

PlancEssential Questions

JEE Main/Boards

Exercise 1

Q.15 Q.20 Q.22

Q.25 Q.26 Q.28

Q.29 Q.35 Q.40

Q.41

Exercise 2

Q.1 Q.5 Q.8

Q.9 Q.11 Q.15

Q.17 Q.27

JEE Advanced/Boards

Exercise 1

Q.3 Q.7 Q.10

Q.15 Q.17 Q.20

Q.22 Q.23 Q.32

Exercise 2

Q.9 Q.12 Q.15

Q.18 Q.20 Q.21

Answer Key

JEE Main/Boards

Exercise 1

Q.9 10.54 m

Q.10 10s; 99.1 $m\,s^{-1}$

Q.11 58 $m\,s^{-1}$; 30° 27′ with horizontal; 44.1 m below and 150 m horizontally away from the starting point

Q.12 2.0 s

Q.15 $25\sqrt{3}$ m.

Q.16 $R=4\sqrt{H_1H_2}$

Q.17 $y=2x-5x^2$

Q.18 653.06 m, 7° 42′ with horizontal, 2.19 s

Q.19 5° 43′

Q.20 30° 58′

Q.21 $4u^2/5g$

Q.22 37 seconds

Q.23 1000 km h^{-1}

Q.24 -3.06ms^{-2} ; 11.43 s

Q.25 450 m

Q.26 40 km h^{-1}; 9 min.

Q.28 (i) (a) 5km/h, (b) 5km/h; (ii) (a) 0, (b) 6 km/h; (iii) (a) 1.875 km/h, (b) 5.625 km/h

Q.29 (i) v_0=667 ft/s (ii) 2667 ft (iii) v_x=534 ft/s, v_y=560 ft/s

Circular Motion

Q.30 $\pi/1800$ rad. s^{-1}

Q.31 31.4 rad. s^{-1}

Q.32 8π rad.s^{-1} ; 1257.1 cm s^{-1}

Q.33 π /30 rad. s^{-2}, 5 π /3 cm s^{-2}

Q.34 987.7 cm s^{-2}

Q.35 (i) 0.44 rad/s; 5.3 cm s^{-1} (ii) Not constant, 2.3 cm s^{-2}

Q.36 0.03 m/s^2

Q.37 0.86 ms^{-2}

Q.38 $-(\hat{x}+\hat{y})/\sqrt{2}\text{cm}/s^2$

Q.39 400 m; girl B

Q.40 (i) Zero; (ii) 0; (iii) 6×10^{-3} m/s

Q.41 $\frac{5}{7}$ ms^{-2}; $\beta = \tan^{-1}\left(\frac{10}{7}\right)$

Q.42 $\Delta v = v(-0.24\hat{i} + 0.64\hat{j})$

Q.43 h=R/3

Exercise 2

Projectile Motion

Single Correct Choice Type

Q.1 D	**Q.2** D	**Q.3** C	**Q.4** C	**Q.5** C	**Q.6** C	**Q.7** C
Q.8 A	**Q.9** B	**Q.10** B	**Q.11** C	**Q.12** C	**Q.13** B	**Q.14** B
Q.15 C	**Q.16** A	**Q.17** A	**Q.18** A			

Circular Motion

Single Correct Choice Type

Q.19 C	**Q.20** C	**Q.21** C	**Q.22** D	**Q.23** C	**Q.24** C	**Q.25** D
Q.26 B	**Q.27** A	**Q.28** C	**Q.29** D	**Q.30** C		

Previous Years' Questions

Q.1 A	**Q.2** B	**Q.3** B	**Q.4** C	**Q.5** D	**Q.6** C	**Q.7** A

JEE Advanced/Boards

Exercise 1

Projectile Motion

Q.1 $v_0 = \sqrt{(1+\alpha^2)a/2\beta}$

Q.2 20 $\sqrt{3}$ m/s, $60°$

Q.3 2.55m, 27°43'

Q.4 (i) $45°$, (ii) 2m/sec

Q.6 4.4 m/s

Q.7 50 m/s

Q.8 30.55 m

Q.9 Yes

Q.10 (i) 58.8 m (ii) $\sqrt{\frac{2gh}{2+\cot^2\theta}}$

Q.11 (i) u=340m/s (ii) 46 s.

Q.12 9.8 m, 3.3 m, 1.6s.

Q.13 20 s, 2000 m

Q.15 (i) 19.6 m/s upward, (ii) 24.5 m/s at 53° with horizontal

Q.16 $v_0 t\sqrt{2(1-\sin\alpha)}$

Q.17 2.47 m

Q.19 60°

Q.20 200m, 20 m/min, 12 m/min

Q.21 6 m/s

Q.22 $\tan^{-1}(1/2)$

Circular Motion

Q.23 $v = \dfrac{\omega\ell}{\theta}$

Q.24 $\vec{a}_r = -3\hat{i} - 4\hat{j}\,m/s^2, \vec{a}_N = -4\hat{i} - 3\hat{j}\,m/s^2$

Q.25 $f_{max} = \dfrac{5}{2\pi}$ rev / sec

Q.26 $45^\circ, \sqrt{2}\times 10^4 N$

Q.27 $V = 40\times 10^{-2}\,m/s, a = 32\times 10^2\,m/s^2$

Q.28 0.83 metre on x-axis

Q.29 0.675 rev/s

Q.30 $x = R\cos\omega t + vt, y = R\sin\omega t$, cycloid

Q.31 334 m

Q.32 $\sqrt{\dfrac{2L}{\omega^2 R\cos\theta}}$

Q.33 $\left[2R(a\sin\theta + g - g\cos\theta)\right]^{1/2}$

Q.34 $\sqrt{a^2 + \left(\dfrac{v^2}{R}\right)^2}$

Exercise 2

Projectile Motion

Single Correct Choice Type

Q.1 D **Q.2** B **Q.3** B **Q.4** C **Q.5** D

Multiple Correct Choice Type

Q.6 A, B, C, D **Q.7** A, B **Q.8** A, B, C, D **Q.9** A, C, D **Q.10** A, B **Q.11** A, C **Q.12** B, C, D

Comprehension Type

Q.13 A **Q.14** D **Q.15** A

Match the Columns

Q.16 A $\rightarrow$ p; B $\rightarrow$ p, q, r, s; C $\rightarrow$ p, q, r, s; D $\rightarrow$ p, r

Assertion Reasoning Type

Q.17 B **Q.18** D **Q.19** D **Q.20** D **Q.21** A

Circular Motion

Multiple Correct Choice Type

Q.22 A, D **Q.23** B, D

Previous Years' Questions

Q.1 (i) $\dfrac{u^2\sin 2\alpha}{g\cos\theta}$ (ii) $\dfrac{u\cos(\alpha+\theta)}{\cos\theta}$ (down the plane) **Q.2** 5 **Q.3** D

Solutions

JEE Main/Boards

Exercise 1

Projectile Motion

Sol 1: Consider two perpendicular unit vectors i, j. At any instant, if we are able to express the position of an object w.r.t to initial position as a linear combination of i, j, then we say object is having two dimensional motion in the plane of $\hat{i}$, $\hat{j}$.

Displacement is the shortest distance between two points.

Velocity = $\dfrac{\text{Displacement}}{\text{time}}$

where 'time' is time taken to travel between the two points.

$\vec{v} = \vec{u} + \vec{a}.t$; $\vec{s} = \vec{u}.t + \frac{1}{2}\vec{a}t^2$

$v^2 - u^2 = 2\vec{a}\,\vec{s}$

Sol 2: We know that, $a = a_x\hat{i} + a_y\hat{j}$

$\vec{a} = \dfrac{d\vec{v}}{dt}$

Let $\vec{v} = v_x(t)\hat{i} + v_y(t)\hat{j}$

$\vec{a} = \dfrac{d\vec{v}}{dt} = \dfrac{dv_x(t)}{dt}\hat{i} + \dfrac{dv_y(t)}{dt}\hat{j}$

$\Rightarrow a_x = \dfrac{dv_x(t)}{dt}$ and $a_y = \dfrac{dv_y(t)}{dt}$

$\vec{x} = \int \vec{v}.dt + \vec{x}_0$

$\Rightarrow \vec{x} = \vec{x}_0 + \int v_x(t)\,dt\,\hat{i} + \int v_y(t)\,dt\,\hat{j}$

Sol 3: When a body is projected at an angle in a unit force field, then the body is called projectile

E.g.: Cannon ball shot from a cannon, bullet from a gun, droplets of water coming out from a piston, ball leaving a bat etc.

Let horizontal velocity = v

$x = vt \quad \Rightarrow t = \dfrac{x}{v}$ ... (i)

Let height of projection= h

Height $y = h - \frac{1}{2}gt^2$

$y = h - \frac{1}{2}g\dfrac{x^2}{v^2} \quad \therefore y = a - bx^2$

$a = h,\ b = \dfrac{g}{2v^2}$

$\Rightarrow$ Parabolic path

Sol 4: Range $r = \dfrac{v^2}{g}\sin 2\theta$

It has same value for θ, $90 - \theta$

Sol 5: Time of flight, $t = \dfrac{2v\sin\theta}{g}$

more height $h = \dfrac{v^2\sin^2\theta}{2g}$

range $= \dfrac{v^2}{g}\sin 2\theta$

Sol 6: $v_x = v_0\cos\theta$

$v_y = v_0\sin\theta - gt$ where v_0 is initial velocity

Angle of elevation = $\tan^{-1}\dfrac{v_0\sin\theta - gt}{v_0\cos\theta}$

Sol 7: $y = x\tan\theta - \dfrac{gx^2}{2v^2\cos^2\theta}$

Time of flight $t = \dfrac{2v\sin\theta}{g}$

Horizontal range $r = \dfrac{v^2}{g}\sin 2\theta$

More height $h = \dfrac{v^2\sin^2\theta}{2g}$

Sol 8: In case of uniform circular motion, the particles will have acceleration towards the centre only and is called centripetal acceleration.

$a_r = \dfrac{v^2}{R} = \omega^2 R$

Sol 9: Time of flight $t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2(1.96)}{9.8}} = 2\sqrt{0.1}$ s

Velocity of bus = 60 kmh^{-1} = $\frac{50}{3}$ ms^{-1}

Distance = velocity × time = $\frac{50}{3}(2\sqrt{0.1})$ = 10.54 m

∴ Distance travelled is 10.54 m

Sol 10: Time of flight = $\sqrt{\frac{2h}{g}} = \sqrt{\frac{2.490}{9.8}}$ = 10 s

Vertical velocity = gt = 10 × 9.8 = 98 ms^{-1}

Total velocity

$= \sqrt{(\text{horizontal velocity})^2 + (\text{vertical velocity})^2}$

$= \sqrt{(98)^2 + (15)^2}$ = 99.1 ms^{-1}

Sol 11: Vertical velocity v_y = gt = 9.8×3 = 29.4 ms^{-1}

Vertical displacement (y) = $\frac{1}{2}gt^2 = \frac{1}{2} \times 9.8 \times 3^2$

= 44.1 m

Horizontal displacements (x) = 50 × 3 = 150 m

Final velocity = $\sqrt{v_x^2 + v_y^2}$

$= \sqrt{(50)^2 + (29.4)^2}$ = 58 ms^{-1}

Total displacement = $\sqrt{x^2 + y^2} = \sqrt{(150)^2 + (44.1)^2}$

= 156.35 m

Angle of depression = $\tan^{-1}\frac{44.1}{150}$ = 16.38°

Its final velocity is 58 ms^{-1}

It is at a distance of 156.35 m at an angle of depression of 16.38°

Sol 12: Vertical velocity = v sinq = 9.8 (sin30°)= 4.9 ms^{-1}

$h = ut + \frac{1}{2}gt^2$

$\Rightarrow 29.4 = 4.9\,t + \frac{1}{2} \times 9.8t^2$; $\Rightarrow 29.4 = 4.9t + 4.9t^2$

$\Rightarrow 6 = t + t^2$; $\Rightarrow t^2 + t - 6 = 0$

$\Rightarrow (t - 2)(t + 3) = 0$

Since, t > 0

$\Rightarrow$ t = 2

It hits the ground in 2 seconds.

Sol 13: Let muzzle velocity be V

Velocity along vertical direction for 60°

$v_{1y} = v \sin 60° = \frac{\sqrt{3}}{2}V$

Velocity along vertical direction for 30°

$v_{2y} = v \sin 30° = \frac{V}{2}$

Max height = $\frac{v_y^2}{2g}$

Ratio of heights = $\frac{\frac{v_1^2}{2g}}{\frac{v_2^2}{2g}} = \left(\frac{v_1}{v_2}\right)^2 = \left(\frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}}\right)^2 = 3$

Hence if shoots thrice as high

Time of height $t = 2\left(\frac{v}{g}\right)$

$t_1 = 2\sqrt{\frac{3}{2}} \cdot \frac{v}{g} = \frac{\sqrt{3}v}{g}$

$t_2 = 2.\frac{1}{2}\frac{v}{g} = \frac{v}{g}$

Their horizontal velocity $v_x = v\cos\theta$

$v_{1x} = v\cos 60° = \frac{v}{2}$

$v_{2x} = v\cos 30° = \frac{\sqrt{3}}{2}v$

horizontal distance = $v_x t$

$d_1 = v_{1x} . t_1 = \frac{v}{2} . \frac{\sqrt{3}v}{g} = \frac{\sqrt{3}}{2}\frac{v^2}{g}$

$d_2 = v_{2x} . t_2 = \frac{\sqrt{3}}{2}v . \frac{v}{g} = \frac{\sqrt{3}}{2}\frac{v^2}{g}$

Hence their horizontal range is same.

Sol 14: Maximum horizontal range occurs when it is fixed at angle of 45°.

Let its initial velocity be v

Vertical velocity $v_y = v \sin 45° = \frac{v}{\sqrt{2}}$

Time of flight $t = 2\left(\frac{v_y}{g}\right) = \frac{2.v}{\sqrt{2}g} = \frac{\sqrt{2}v}{g}$

Maximum height (h) = $\frac{v_y^2}{2g} = \left(\frac{v}{\sqrt{2}}\right)^2 . \frac{1}{2g} = \frac{v^2}{4g}$

Horizontal velocity $v_x = v\cos 45° = \frac{v}{\sqrt{2}}$

Horizontal range $(r) = v_x.t = \frac{v}{\sqrt{2}}.\frac{\sqrt{2}v}{g} = \frac{v^2}{g}$

$$\frac{r}{h} = \frac{\frac{v^2}{g}}{\frac{v^2}{4g}} = 4$$

∴ Its maximum horizontal range is 4 times height.

Sol 15: Horizontal range is given by

$D = \frac{v^2 \sin 2\theta}{g}$ where θ is angle of projection

$D_1 = (30)^2.\frac{\sin 2(60^o)}{10} = 90 \sin 120^o = 45\sqrt{3}$ m

$D_2 = (20)^2.\frac{\sin 2(30)}{10} = 40 \sin 60° = 20\sqrt{3}$ m

Distance $= D_1 - D_2 = 45\sqrt{3} - 20\sqrt{3} = 25\sqrt{3}$ m

Sol 16: For a given angle of projection θ

Horizontal Range $r = \frac{v^2 \sin 2\theta}{g}$

Maximum Height $h = \frac{v^2 \sin^2\theta}{2g}$

given $r_1 = r_2$

$\Rightarrow \frac{v^2 \sin 2\theta_1}{g} = \frac{v^2 \sin 2\theta_2}{g}$

$\Rightarrow \sin 2\theta_1 = \sin 2\theta_2$

$\Rightarrow 2\theta_1 = 2\theta_2$ or $2\theta_1 = 180 - 2\theta_2$

$\because \theta_1 \neq \theta_2$

$\Rightarrow \theta_1 + \theta_2 = 90°$ ... (i)

$\Rightarrow h_1 = \frac{v^2 \sin^2\theta_1}{2g}$

$H_2 = \frac{v^2 \sin^2\theta_2}{2g} = \frac{v^2 \cos^2\theta_1}{2g}$ $(\theta_1 + \theta_2 = 90°)$

$H_1H_2 = \frac{v^2 \sin^2\theta_1}{2g}.\frac{v^2 \cos^2\theta_1}{2g}$

$= \frac{v^4}{4g^2}.\sin^2\theta_1 \cos^2\theta_1$

$= \frac{v^4}{16g^2}.\sin^2 2\theta_1$

$= \frac{1}{16}\left(\frac{v^2}{g}\sin 2\theta_1\right)^2 = \frac{1}{16}R^2$

$\therefore R^2 = 16H_1H_2 \Rightarrow R = 4\sqrt{H_1H_2}$

Sol 17: Given that, $v = \hat{i} + 2\hat{j}$

Horizontal displacement $x = v_x . t$; $x = 1\,t$

$x = t$ ... (i)

Vertical displacement $y = v_y t - \frac{1}{2}gt^2 = 2t - 5t^2$

$\Rightarrow y = 2x - 5x^2$

Sol 18: $r = \frac{v^2 \sin 2\theta}{g}$;

Max range $= \frac{v^2}{g} = \frac{80\times 80}{10} = 640$ m

$100\sqrt{3} = \frac{80\times 80}{10}.\sin 2\theta$

$\sin 2\theta = 0.27$

$\Rightarrow \theta = \frac{1}{2}\sin^{-1} 0.27 = 7.42°$

$t = \frac{2v\sin\theta}{g} = \frac{2\times 80}{10}\sin\theta = 16 \sin 7.42° = 2.19$ s

Sol 19: $h = \frac{v^2 \sin^2\theta}{2g}$

$\Rightarrow 5 = \frac{v^2 \sin^2\theta}{2g}$ ... (i)

$r = \frac{v^2 \sin 2\theta}{g} = \frac{2v^2 \sin\theta\cos\theta}{g}$

$\Rightarrow 200 = \frac{2v^2 \sin\theta\cos\theta}{g}$ ... (ii)

Divide (i) by (ii)

$$\frac{5}{200} = \frac{\frac{v^2\sin^2\theta}{2g}}{\frac{2v^2\sin\theta\cos\theta}{g}}$$

$\frac{1}{40} = \frac{1}{4}\tan\theta$; $\theta = \tan^{-1}\frac{1}{10}$

$\Rightarrow r = \frac{u^2 \sin 2\theta}{g} = \frac{2u^2 \sin\theta\cos\theta}{g}$ ... (iii)

$\Rightarrow H = \frac{u^2 \sin^2\theta}{2g}$

$\frac{r}{H} = \frac{\tan\theta}{4}$ Given $\frac{r}{H} = 2$ $\Rightarrow \tan\theta = 2$

$\Rightarrow \sin 2\theta = \frac{4}{3}$ $\Rightarrow r = \frac{4u^2}{5g}$

Sol 20: $y = x\tan\theta - \frac{gx^2}{2v^2\cos^2\theta}$

$$13 = 50\tan\theta - \frac{g(50)^2}{2\times(10\sqrt{g})^2\cos^2\theta}$$

$\Rightarrow 26 = 100\tan\theta - 25\sec^2\theta$

$\Rightarrow 25\tan^2\theta - 100\tan\theta + 51 = 0$ $(\sec^2\theta = 1 + \tan^2\theta)$

$$\Rightarrow \tan\theta = \frac{100 \pm \sqrt{(100)^2 - 4\times 25\times 51}}{2\times 25}$$

$$\tan\theta = \frac{100\pm 70}{50} = \frac{3}{5}, \frac{17}{5}$$

For t_{min}, $\tan\theta$ is minimum $\therefore \theta = \tan^{-1}\frac{3}{5} \simeq 30.1°$

Sol 21: Given, R=2H

$$\Rightarrow \frac{u^2\sin 2\theta}{g} = 2\frac{u^2\sin^2\theta}{2g}$$

$\Rightarrow 2\sin\theta\cos\theta = \sin\theta\sin\theta$

$\Rightarrow \tan\theta = 2$

$$\sin\theta = \frac{2}{\sqrt{3}}, \cos\theta = \frac{1}{\sqrt{3}}$$

$$\text{So, } R = \frac{u^2\sin 2\theta}{g} = \frac{u^2 2\sin\theta\cos\theta}{g} = \frac{u^2 2}{g}\times\frac{2}{3} = \frac{4u^2}{3g}$$

Sol 22: He moves 5 steps forward and 3 steps backward in 8 seconds.

$\Rightarrow$ He moves 2 m in 8 second

Lets call it drunk movement (D.M)

i.e. in 1 D. M = 2 m in 8 seconds

$\therefore$ Distance travelled in n D.M = 2n m in 8n seconds.

Now 13m – 5m = 8 m

i.e. man will have complete D.M such that

$2n \geq 8$ for least possible n $\Rightarrow$ n = 4

$\Rightarrow$ he travelled 8 m in 32 seconds

he falls in pit in next 5 m, 5 seconds

$\therefore$ He falls in 32 + 5 = 37 seconds.

Sol 23: Velocity w.r.t ground

= Velocity w.r.t Jet + velocity of jet

= 1500 km h^{-1} – 500 km h^{-1}

= 1000 km h^{-1}

Sol 24: v = 126 km h^{-1} = 35 ms^{-1}

$$a = \frac{v^2}{2s} = \frac{35^2}{2\times 200} = 3.0625 \text{ ms}^{-1}$$

$$\Rightarrow t = \frac{v}{a} = \frac{v}{\frac{v^2}{2s}} = \frac{2s}{v} = \frac{2\times 200}{35} = 11.43 \text{ s}$$

Note: - here $t = \frac{2s}{v}$ can also be written as

$t = \frac{s}{\left(\frac{v}{2}\right)}$; $\frac{v}{2}$ is average velocity of motion.

So in a uniform accelerated motion,

$$t = \frac{\text{Distance}}{\text{Avg velocity}}$$

Try deriving the same assuming it has some final velocity.

Sol 25: Distance to be travelled by B relative to A

= 2(400) + x

x = initial separation (distance between them)

Relative velocity = $V_B - V_A$ = 72 – 72 = 0

Relative acceleration = 1 ms^{-2}

Time = 50

$\because \frac{1}{2}at^2 = S$

$\therefore 800 + x = \frac{1}{2} \times 1 \times (50)^2$

x = 450 m

Initial separation was 450 m.

Sol 26: Distance between two busses coming from same direction is VT; V is speed of bus.

Relative velocity of with buses coming from opposite direction = $V + V_C$; V_C is speed of man.

Relative velocity of man with buses coming from same direction as man = $V - V_C$

$$18 = \frac{VT}{V - V_C} \quad \Rightarrow \quad 1 - \frac{V_C}{V} = \frac{T}{18} \quad \text{... (i)}$$

$6 = \frac{Vt}{V+V_C} \Rightarrow 1 + \frac{V_C}{V} = \frac{T}{6}$ ... (ii)

equation (i) + equation (ii)

$\Rightarrow 2 = T\left(\frac{1}{6}+\frac{1}{18}\right)$

$T = \frac{2}{\frac{1}{6}+\frac{1}{18}} = 9$ min

$\Rightarrow V = 40$ km/h^{-1}

$V_C = 20$ km/h = velocity of cyclist

Sol 27: $t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2\times 90}{10}} = 3\sqrt{2}$ s

t = time of descent

$t_1 = \frac{2V_1}{g}$

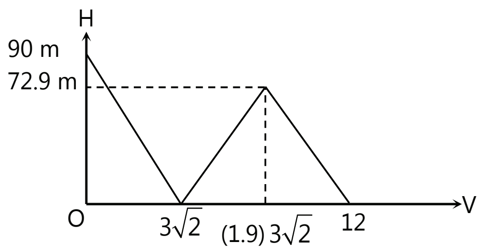
t_1 = time of flight between consecutive collisions with the floor

$V_1 = (0.9)V$ (i.e. velocity after collison)

$\Rightarrow t_1 = (1.8)\frac{V}{g}$; $t_1 = (1.8)\ t$

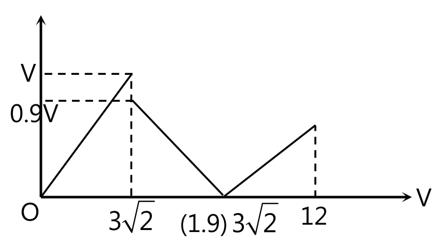
$t_1 + t_2 = t + 1.8t = 2.8t = (2.8)3\sqrt{2} \simeq 12$ s

$H_1 = \frac{V_1^2}{2g} = (0.9)^2.\ \frac{V^2}{20} = (0.81)90 = 72.9$ m



Note: - Here we have a sharp change at $3\sqrt{2}$ because ball got rebound from surface there.

There is a sharp change at $(1.9)\times 3\sqrt{2}$ because there is change in direction of motion and we took both sides positive (speed)



Sol 28: (i) 0 to 30

distance = $V \times t = 5 \times \frac{1}{2}$ (t = 30 min = $\frac{1}{2}$Hr) = 2.5 km

speed = $\frac{\text{distance}}{\text{time}} = \frac{2.5}{1/2} = 5$ km/h

velocity = $\frac{\text{displacement}}{\text{time}} = \frac{2.5}{1/2} = 5$ km/hr

(ii) 0 to 50

0 to 50 = 0 to 30 + 30 to 50

In 30 to 50

Distance = $V \times t = 7.5 \times \frac{2}{6} = 2.5$ km

(20 min = $\frac{2}{6}$ Hr)

Displacement (s) = $\sum Vt = 2.5 - 2.5 = 0$ km

Total distance (s) = $\sum |V| t = 2.5 + 2.5 = 5$ km

Velocity = $\frac{s}{t} = 0$

Speed = $\frac{D}{t} = \frac{5}{5/6} = 6$ km/hr (50 min = $\frac{5}{6}$ Hr)

(iii) 0 to 40

0 to 40 = 0 to 30 + 30 to 40

In 30 to 40, distance = $v \times t = 7.5 \times \frac{1}{6} = 1.25$ km

Displacement = 2.5 − 1.25 = 1.25 km

Total distance = 2.5 + 1.25 = 3.75 km

Velocity = $\frac{1.25}{4/6} = 1.875$ km/h

Speed = $\frac{3.75}{4/6} = 5.625$ km/h

Sol 29: $V_y = V\cos\theta$ (θ is angle with vertical)

$V_y = \frac{3}{5}V$; $S = V_y t + \frac{1}{2}gt^2$

$2400 = \frac{3}{5}V(5) + \frac{1}{2} \times 32.5 \times (5)^2$ (g= 32.5 ft. /m^2)

V = 664.6 ft. s^{-1}

$x = V_x.\ t = V$

$\sin\theta\ t = \frac{4}{5}.V.5 = 2658.4$ ft

$V_x = V\sin\theta = 534$ ft/s

$V_y = V\cos\theta + gt = 560$ ft/s

Circular Motion

Sol 30: Minutes hand of a clock completes one revolution in one hour i.e. 3600 second

So, $\omega = \frac{1}{3600}\frac{\text{Rev}}{\text{s}}$ and 1 revolution = 2π Rad

$\omega = \frac{2\pi}{3600}$ rad/s $\Rightarrow \omega = \frac{\pi}{18} \times 10^{-2}$ rad/s

Sol 31: A wheel making 300 rotation per minute

and one rotation = 2π rad.

1 minute = 60 sec

$\therefore \omega = \frac{300.2\pi}{60}$ rad/s; $\omega = 10\pi$ rad/s

Sol 32: 4 rotations per second

$\Rightarrow \omega = 4\frac{\text{rotations}}{\text{s}}$ and 1 rotation = 2π rad

$\Rightarrow \omega = 4.(2\pi)$ rad/s; $\omega = 8\pi$ rad/s

and the velocity of a point on its circumference

$v = R\omega$

$R = 50 \text{ cm} = \frac{1}{2}\text{m}.$

$v = \left(\frac{1}{2}\right)(8\pi)$ m/s

$v = 4\pi$ m/s

Sol 33: $\omega_{initial} = 100\frac{\text{revolutions}}{\text{minute}} = 100\frac{2\pi}{60}$ rad/s

$\omega_i = \frac{10\pi}{3}$ rad/s

$\omega_f = 400\frac{\text{revolutions}}{\text{minute}} = 400.\frac{2\pi}{60}$ rad/s

$\omega_f = \frac{40\pi}{3}$ rad/s

$\alpha = \frac{\Delta\omega}{\Delta t} = \frac{\omega_f - \omega_i}{t}$

$\alpha = \frac{\frac{40\pi}{3} - \frac{10\pi}{3}}{5\times 60}$ rad/s^2 = $\frac{30\pi}{3\times5\times60}$

$\alpha = \frac{\pi}{30}$ rad/s^2

and linear acceleration a = Ra

$R = 50 \text{ cm} = \frac{1}{2}\text{m}$

$\therefore a = \frac{1}{2}.\frac{\pi}{30}$ m/s^2 ; $a = \frac{\pi}{60}$ m/s^2

Sol 34: Time period $T = \frac{2\pi}{\omega}$

Given T = 2 s

$\therefore \omega = \frac{2\pi}{T} = \pi$ rad/s

and acceleration $a = R\omega^2$ m/s^2

R = 100 cm = 1 m

$\therefore a = \pi^2$ m/s^2

Sol 35: (i) Given that insect completes 7 revolutions in 100 seconds.

$\therefore \omega = 7 \text{ Rev/100s} = \frac{7.2\pi}{100}$ rad/s

$\omega = \frac{14\pi}{100}$ rad/s

$\omega = 0.44$ rad/s and $v = R\omega$

R = 12 cm = 0.12 m

v = (0.12) (0.44) m/s^2

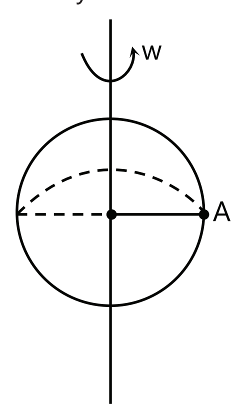
$v = 5.3 \times 10^{-2}$ m/s^2

(ii) Acceleration is not constant. Because the direction of the acceleration vector keeps on changing in direction. Hence acceleration vector in circular motion can never be a constant vector.

$|\vec{a}| = R\omega^2 = (0.12)(0.44)^2$ m/s^2

$|\vec{a}| = 2.3 \times 10^{-2}$ m/s^2

Sol 36: Earth completes 1 rotation in 1 day



i.e., $\omega = 1.\frac{\text{rotation}}{\text{day}}$

$\omega = 1.\frac{2\pi}{24\times60\times60}$ rad/s

$\omega = \frac{\pi}{432} \times 10^{-2}$ rad/s

and now acceleration at point A;

$a = r\omega^2$

$r = 6400 \text{ km} = 6400 \times 10^3$ m

$r = 64 \times 10^5$ m

$\therefore a = 64 \times 10^5 \times \frac{\pi^2}{(432)^2} \times 10^{-4}$ m/s^2

A = 0.03 m/s^2

Sol 37: $v = 27 \text{ km/h} = 27 \times \frac{5}{18}$ m/s

$v = \frac{15}{2}$ m/s

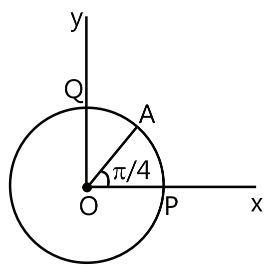
$\vec{a}_r = \frac{v^2}{R} = \frac{(15)^2}{4 \times 80} = 0.7$

$\vec{a}_t = 0.5 \text{ m/s}^2 = \frac{1}{2} \text{m/s}^2$

$\vec{a}_{net} = \vec{a}_r + \vec{a}_t = \sqrt{(0.7)^2 + (0.5)^2}$

$\vec{a}_{net} = 0.86 \text{ m/s}^2$

Sol 38: At point the acceleration will be centripetal acceleration which is radially directed towards point O. i.e.

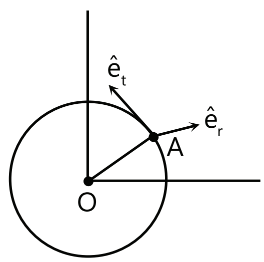


Physically: $\vec{a} = \frac{v^2}{r}(-\hat{e}_r)$

Remember $\hat{e}_r$ and $\hat{e}_t$ are the unit vectors along radial and tangential direction respectively.

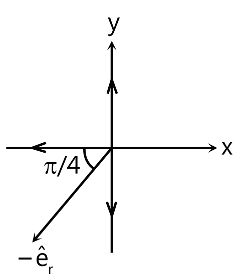
Refer to the figure.

So in this case also,



$\vec{a}_A = \frac{v^2}{r}(-\hat{e}_r)$

Now, since the point is in between the points P and Q,



Angle between $\overrightarrow{OA}$ and $\overrightarrow{OP}$ will be $\frac{\pi}{4}$

Now let us resolve $(-\hat{e}_r)$ into $\hat{i}$ and $\hat{j}$

$(-\hat{e}_r) = |-\hat{e}_r| \cos\frac{\pi}{4}(-\hat{i}) + |-\hat{e}_r| \sin\frac{\pi}{4}(-\hat{j})$

But since $\hat{e}_r$ and $\hat{e}_t$ are unit vectors;

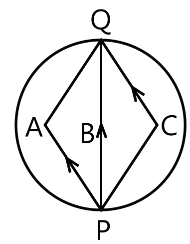
$|\hat{e}_r| = |\hat{e}_t| = 1$

$\because (-\hat{e}_r) = -\frac{1}{\sqrt{2}}\hat{i} - \frac{1}{\sqrt{2}}\hat{j} = -\frac{1}{\sqrt{2}}(\hat{i} + \hat{j})$

$\Rightarrow$ Now $\vec{a}_A = \frac{v^2}{r}\left(-\frac{1}{\sqrt{2}}(\hat{i} + \hat{j})\right)$ $\vec{a}_A = -\frac{v^2}{r\sqrt{2}}(\hat{i} + \hat{j})$

Put v = 2 cm/s and r = 4 cm to find $\vec{a}_A$

Sol 39: Now this tests your understanding of displacement vector



Displacement vector $\vec{r} = \vec{r}_f - \vec{r}_i$

where $\vec{r}_f$ is the co-ordinates of final position and $\vec{r}_i$ is the co-ordinates of initial position.

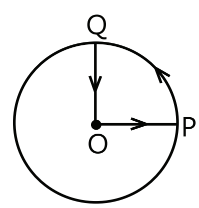
Now for all the three girls, final position is point Q and initial destination is point P. Hence displacement is same for all the three girls,

i.e. $2\vec{r} = 2(\vec{r}_Q - \vec{r}_P) = 2\overrightarrow{PQ} = 2 \times 200 = 400$ m

Distance is the total length of the path travelled.

Here for girl B; distance travelled is same as her displacement vector, since she travelled in the straight line connecting the points.

Sol 40: Now from the argument made above, displacement $\vec{r} = \vec{r}_f - \vec{r}_i$



Here the cyclist started from the point O and then finally reached the point O

Hence $\vec{r}_f = \vec{r}_i$

So $\vec{r}$ = zero

Hence net displacement is zero.

And average velocity = $\frac{\text{Total displacement}}{\text{Total time}} = \frac{0}{10} = 0$

and for average speed = $\frac{\text{total distance}}{\text{total time}}$

Total distance is |OP| + |PQ| + |QO|

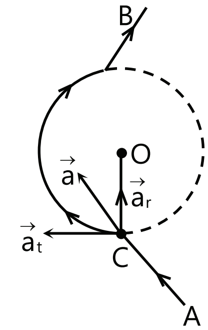
|OP| = |QO| = R = 1 m

And $|PQ| = \left(\frac{2\pi R}{4}\right) = \left(\frac{\pi}{2}\right)$ m

$\therefore$ Distance $= \left(2 + \frac{\pi}{2}\right)$ m.

Av. Speed $= \frac{\left(2 + \frac{\pi}{2}\right)}{10 \times 60}$ m/s $= 6 \times 10^{-3}$ m/s

Sol 41: Let us say the circular turn is of the shape AB.



Now at the starting point of the track

i.e. C; $\vec{a} = \vec{a}_r + \vec{a}_t$

$\vec{a}_r$ = centripetal acceleration

$= \frac{v^2}{R}(-\hat{e}_r)$

$v = 36 \text{ km/h} = 36 \times \frac{5}{18} \text{ m/s} = 10 \text{ m/s}$

R = 140 m

$\vec{a}_r = \frac{(10)^2}{140} = \frac{5}{7} \text{ m/s}^2 (-\hat{e}_r)$

and given that $\frac{dv}{dt} = 1$ m/s

$\therefore \vec{a}_t = \frac{dv}{dt}(\hat{e}_t)$; $\quad \vec{a}_t = 1 \text{ m/s}^2 (\hat{e}_t)$

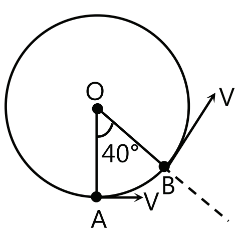
Now $\vec{a} = \vec{a}_r + \vec{a}_t$

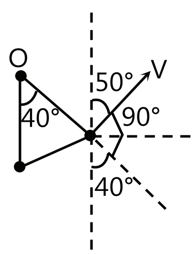
$\vec{a} = (0.7(-\hat{e}_r)) + 1\ \hat{e}_t)$ m/s²

$|a| = \sqrt{(0.7)^2+1} = \sqrt{0.49+1} = \sqrt{1.49}$ m/s² = 1.22 m/s²

and $\tan\beta = \left(\frac{1}{0.7}\right) \Rightarrow \beta = \tan^{-1}\left(\frac{10}{7}\right)$

Sol 42:





Velocity at point A, $\vec{V}_A = v\hat{i}$

Velocity at point B, $\vec{V}_B = v \sin 50\,\hat{i} + v \cos 50\,\hat{j}$

$\vec{V}_B = v(0.76\,\hat{i} + 0.64\,\hat{j})$

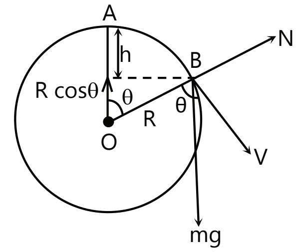
Now change in velocity $\Delta V = \vec{V}_B - \vec{V}_A$

$= v(0.76\,\hat{i} + 0.64\,\hat{j}) - v\hat{i}$

$\Delta V = v(-0.24\,\hat{i} + 0.64\,\hat{j})$

Sol 43: This is a very standard problem for a JEE aspirant.

Let us say at point B, the particle loses its contact. So let us write the equations of motions. At point B say the particle has velocity v.



$mg\cos\theta = N + \frac{mv^2}{R}$

$N = mg\cos\theta - \frac{mv^2}{R}$ ... (i)

Now when the particle is about to lose contact, the normal reaction between the particle and the surface becomes zero.

$\therefore N = 0$

$\Rightarrow mg\cos\theta = \frac{mv^2}{R}$ ... (ii)

Now energy at point A, taking O as reference

$E_A = 0 + mgR$ and $E_B = \frac{1}{2}mv^2 + mgR\cos\theta$

Using Energy conservation $E_A = E_B$

$\Rightarrow mgR = \frac{1}{2}mv^2 + mgR\cos\theta$

$\Rightarrow 2mg\ R\ (1-\cos\theta) = mv^2$

$2mg(1-\cos\theta) = \frac{mv^2}{R}$ ... (iii)

Putting this value of $\frac{mv^2}{R}$ in eqⁿ (ii)

$\Rightarrow mg\cos\theta = 2mg(1-\cos\theta) \Rightarrow 3\cos\theta = 2$

$\Rightarrow \cos\theta = \frac{2}{3} \Rightarrow \theta = \cos^{-1}\left(\frac{2}{3}\right)$

and now $h = R(1-\cos\theta) = R\left(1-\frac{2}{3}\right)$

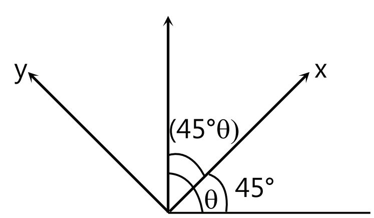
$\therefore h = \frac{R}{3}$.

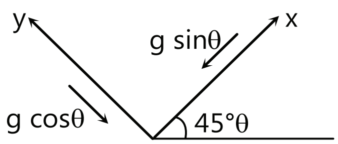
Exercise 2

Projectile Motion

Single Correct Choice Type

Sol 1: (D) Lets solve the problem taking along plane as x-axis and perpendicular to plane as y-axis





$V_x = V\cos(\theta - 45^o)$; $V_y = V\sin(\theta - 45^o)$

$g_y = g\cos\phi = \frac{g}{\sqrt{2}}$; $\phi = 45°$

$g_x = g\sin\phi = \frac{g}{\sqrt{2}}$

time of flight $= 2\frac{V_y}{g_y}$

$= \frac{2V\cos(\theta-45)}{\frac{g}{\sqrt{2}}} = \frac{2\sqrt{2}V\sin(45-\theta)}{g}$

It hits perpendicularly to plane

$\Rightarrow V_x = 0$

$\Rightarrow 0 = V\cos(\theta - 45) - g_x(t)$

$\Rightarrow V\cos(\theta-45) = \frac{g}{\sqrt{2}} \cdot \frac{2\sqrt{2}V\sin(\theta-45)}{g}$

$\Rightarrow \tan(\theta - 45) = \frac{1}{2} \Rightarrow \frac{\tan\theta - 1}{\tan\theta + 1} = \frac{1}{2}$

$\Rightarrow \tan\theta = 3$

$\Rightarrow \theta = \tan^{-1}3$

Sol 2: (D) Both the bodies have same horizontal range, since they form complementary angles with horizontal axis.

It $\theta = 45°$ they have same time of flight else differs.

Sol 3: (C) $u = u\cos\theta\,\hat{i} + u\sin\theta\,\hat{j}$

Let final velocity be v

$v = v\cos\alpha\,\hat{i} + v\sin\alpha\,\hat{j}$

$u\cos\theta = v\cos\alpha$

{∵ Horizontal components of velocity are same}

$\Rightarrow v = \frac{u\cos\theta}{\cos\alpha} = u\cos\theta\sec\alpha$

Sol 4: (C) $V = V\cos\theta\,\hat{i} + V\sin\theta\,\hat{j} = 5\left(\frac{3}{5}\right)\hat{i} + 5\left(\frac{4}{5}\right)\hat{j}$

$V = 3\hat{i} + 4\hat{j}$

$|V(t)| \geq V_x$

$\Rightarrow V \geq 3$

$V = V_x$ if $V_y(t) = 0$

$V_y(t) = 0 \Rightarrow H = \frac{V_y^2}{2g} = \frac{4^2}{2\times 10} = 0.8$ m

But given height is 0.45 m

$\therefore |V| > |V_x|$

The given data is sufficient to calculate $V_y(t)$

$0.45 = 4(t) - \frac{1}{2}.gt^2$

From this we can get t

And $V_y(t) = 4 - g(t)$

So data is sufficient.

Sol 5: (C) $t = \sqrt{\frac{2H}{g}}$

Horizontal displacement (x) $= \frac{1}{2}a_x t^2$

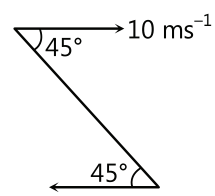
a_x = horizontal acceleration

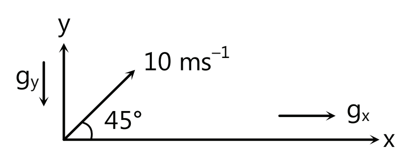
$x = \frac{1}{2}a_x\left(\sqrt{\frac{2H}{g}}\right)^2$

$x = H.\frac{a_x}{g} = 20 \times \frac{6}{10} = 12$ m

Sol 6: (C)

Let's solve the problem along the plane of inclined surface.





$g_x = g\sin45° = \frac{g}{\sqrt{2}}$

$g_y = g\cos45° = \frac{g}{\sqrt{2}}$

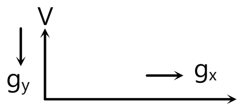
$V = V\cos\theta\,\hat{i} + V\sin\theta\,\hat{j}$

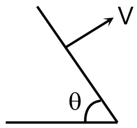
$V = \frac{10}{\sqrt{2}}\hat{i} + \frac{10}{\sqrt{2}}\hat{j}$

Time of flight $= \frac{2V_y}{g_y} = \frac{2.\frac{10}{\sqrt{2}}}{\frac{g}{\sqrt{2}}}$

Time of flight = 2 sec

Sol 7: (C) Let's solve along plane of incline.





Time of flight $t = \frac{2V}{g_y}$

Horizontal range along inclined $= \frac{1}{2} g_x t^2$

$= \frac{1}{2} g_x \cdot \frac{4.V^2}{g_y^2} = \frac{2V^2.g_x}{g_y^2}$

$g_x = g \sin\theta$; $g_y = g \cos\theta$

$r = \frac{2V^2.g\sin\theta}{g^2 \cos^2\theta}$; $\qquad r = \frac{2V^2 \tan\theta \sec\theta}{g}$

Sol 8: (A) Height of monkey above ground $= H - \frac{1}{2}gt^2$

Height of arrow above ground $= V_y t - \frac{1}{2}gt^2$

$\because$ At point of contact both are at same height $H - \frac{1}{2}$

$gt^2 = V_y t - \frac{1}{2}gt^2 \quad \Rightarrow H = V_y t$

$\Rightarrow t = \frac{H}{V_y}$

Now, here $L = V_x \cdot t$; $\Rightarrow L = V_x \cdot \frac{H}{V_y}$

$\Rightarrow V_x = V_y \left(\frac{L}{H}\right)$

$V = V_y \sqrt{1 + \frac{L^2}{H^2}}$ $(V = \sqrt{V_x^2 - V_y^2}) = \frac{V_y}{H}\sqrt{H^2 + L^2}$

Let vertical velocity at the time of impact be V_f

$V_f = V_y - gt$

Minimum value of $V_t = -V_y$

$\therefore -V_y = V_y - gt$

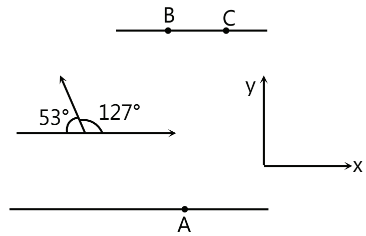
$2V_y = gt$; $\qquad t = \frac{H}{V_y}$

$\Rightarrow 2V_y = \frac{g.H}{V_y}$

$\Rightarrow V_y = \sqrt{\frac{gH}{2}}$

$\Rightarrow V = \frac{1}{H}\sqrt{\frac{gH}{2}} \cdot \sqrt{H^2 + L^2} = \sqrt{\frac{g(H^2 + L^2)}{2H}}$

Sol 9: (B)



$V_y = V\sin 53° = 5.\frac{4}{5} = 4$ km/hr

$V_x = V \cos 127° = -3$ km/hr

Resultant velocity along x-direction

$= V_R + V_x = 4 - 3 = 1$ km /hr

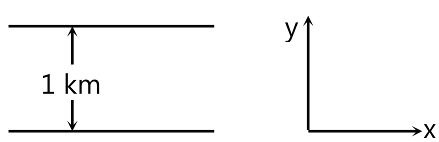
Time of swim $= \frac{\text{width}}{V_y} = \frac{200}{4.\frac{5}{18}} = 180$ seconds

Drift $= V_{Result} \times t = \left(\frac{1 \times 5}{18}\right) \times (180) = 50$ m

Time of walk $= \frac{\text{Drift}}{V} = \frac{50}{3 \times \frac{5}{18}} = 60$ second

Total time = 180 + 60 = 240 s = 4 minutes

Sol 10: (B)



Time = 15 min $= \frac{1}{4}$ hr

$V_y = \frac{\text{width}}{\text{time}} = \frac{1}{\frac{1}{4}} = 4$ km/hr

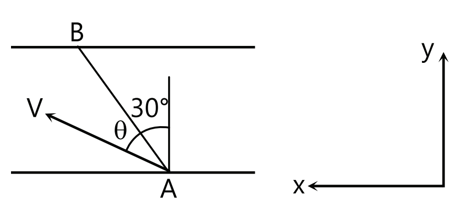
For shortest path, resultant speed along x-axis = 0

$\Rightarrow V_R - V_x = 0 \Rightarrow V_R = V_x$

$V = \sqrt{V_x^2 + V_y^2}$

$\Rightarrow V_x = 3$km/Hr $\Rightarrow V_y = 3$km/Hr

Sol 11: (C)



$V = V\sin\theta\,\hat{i} + V\cos\theta\,\hat{j}$

$V_{Result} = V - V_r = (V\sin\theta - V_r)\,\hat{i} + V\cos\theta\,\hat{j}$

$\tan 30^o = \dfrac{V\sin\theta - V_r}{V\cos\theta}$

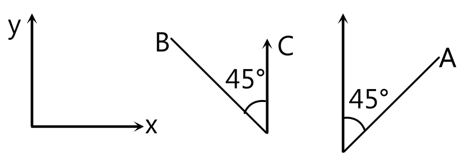
$\dfrac{1}{\sqrt{3}} = \dfrac{5\sqrt{3}\sin\theta - 5}{5\sqrt{3}\cos\theta}$

$\Rightarrow \theta = 60°$

$\Rightarrow$ He should steer at 30° w.r.t the line of destination from starting point

Sol 12: (C) Car moving north $\Rightarrow$ wind force acting south. Also normal winds are acting due east so flag will point south-east.

Sol 13: (C)



V_B w.r.t C $= -\dfrac{V}{\sqrt{2}}\hat{i} + \dfrac{V}{\sqrt{2}}\hat{j}$

V_A w.r.t B $= \dfrac{V}{\sqrt{2}}\hat{i} + \dfrac{V}{\sqrt{2}}\hat{j}$

V_A w.r.t C $= V_{AB} + V_{BC} = \dfrac{V}{\sqrt{2}}\hat{i} + \dfrac{V}{\sqrt{2}}\hat{j} - \dfrac{V}{\sqrt{2}}\hat{i} + \dfrac{V}{\sqrt{2}}\hat{j}$

$V_{AC} = \sqrt{2}V\hat{j}$

$V_{CA} = -V_{AC} = -\sqrt{2}V\hat{j}$

Sol 14: (B) $V_{rain} = V_x\hat{i} + V_y\hat{i}$

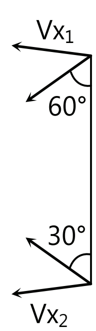
$V_{rain} = 2ms^{-1}$

$V_{rc} = V_{rain} - V_{cyclist}$

$V_{rc} = V_y\hat{j}$

$\Rightarrow V_{cyclist} = V_x\hat{i}$

Sol 15: (C)



$V_{x1} = V_{x2}$

$\Rightarrow V_1 \sin 60° = V_2 \sin 30°$

$V_2 = \sqrt{3}V_1$ ... (i)

Relative velocity linearly

$= V_1\cos 60 + V_2\cos 30$

$= \dfrac{V_1}{2} + \dfrac{\sqrt{3}}{2}V_2$

They collide in 2 second

$\Rightarrow \left(\dfrac{V_1}{2} + \dfrac{\sqrt{3}}{2}V_2\right)(2) = 30$

$V_1 + \sqrt{3}V_2 = 30$

$V_2 = \sqrt{3}V_1$

$\Rightarrow V_1 + \sqrt{3}(\sqrt{3}V_1) = 30$

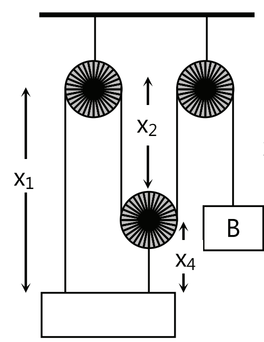
$\Rightarrow V_1 = 7.5\ ms^{-1}$

$V_2 = 7.5\ ms^{-1}$

V_1 is velocity of B

V_2 is velocity of A

Sol 16: (A)



Length of string 1: L_1

$= x_1 + 2x_2 + x_3$

Length of string 2: $L_2 = x_4$

$x_1 = x_2 + x_4$

Differentiate on both side

$dx_1 = dx_2 + dx_4$

$dx_4 = 0\ \therefore$ length of string is constant

$\Rightarrow dx_1 = dx_2$ ... (i)

$L_1 = x_1 + 2x_2 + x_3$

Differentiating $dL_1 = dx_1 + 2dx_2 + dx_3$

$dl_1 = 0$ as length of string is constant.

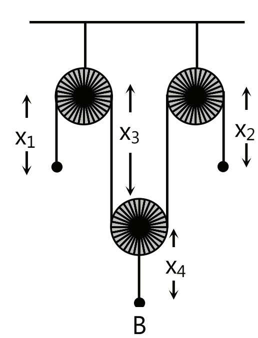
$dx_1 + 2dx_2 + dx_3 = 0$

$\Rightarrow 3dx_1 + dx_3 = 0 \quad (dx_1 = dx_2)$

$\Rightarrow dx_3 = -3dx_1 \Rightarrow \dfrac{dx_3}{dt} = -\dfrac{3dx_1}{dt}$

$V_B = -3V_A = -3(5) = -15\ ms^{-1}$

Sol 17: (A)



Length of string $L_1 = x_1 + 2x_3 + x_2$

$x_B = x_3 + x_4$

$dx_B = dx_3 + dx_4$

$dx_4 = 0$ (length of string constant)

$\Rightarrow dx_B = dx_3$

$x_1 = x_A, x_2 = x_C$

$L_1 = x_A + 2x_3 + x_C$

$dL_1 = dx_A + 2dx_3 + dx_C$

$dL_1 = 0$

$dx_A + 2dx_3 + dx_C = 0$

$dx_A + 2dx_B + dx_C = 0$

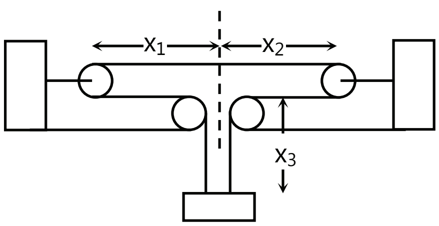
$dx_B = -\frac{1}{2}(dx_A + dx_C)$

$$\Rightarrow \frac{d^2x_B}{dt^2} = -\frac{1}{2}\left(\frac{d^2x_A}{dt^2} + \frac{d^2x_C}{dt^2}\right)$$

lets take upwards as positive

$\Rightarrow a_B = -\frac{1}{2}(a - f) \qquad \therefore \ a_B = \frac{1}{2}(f - a)$

Sol 18: (A) Length of string $L = 2x_1 + 2x_2 + 2x_3$



$$\frac{d^2L}{dt^2} = 2\left(\frac{d^2x_1}{dt^2} + \frac{2d^2x_2}{dt^2} + \frac{2d^2x_3}{dt^2}\right)$$

$0 = a_A + a_B + a_C$

$a_A = 2$ and $a_B = -1$

(B is moving away from central line)

$a_C = -(2 - 1) = -1$

$\therefore a_C = 1\ ms^{-2}$ upwards (A)

Note: - try understanding the sign convention used here. Positive was towards a reference point and negative was away.

Circular Motion

Single Correct Choice Type

Sol 19: (C) This is just a Kinematic problem. So nothing to do with the masses of the bodies.

And now given that both complete a circle in time 't'.

$\therefore$ Both of them have same time period.

$T_1 = T_2 = t$

and we know $T_1 = \frac{2\pi}{\omega_1}$ and $T_2 = \frac{2\pi}{\omega_2}$

$$\Rightarrow \boxed{\frac{\omega_1}{\omega_2} = 1}$$

Sol 20: (C) $r = 25$ cm $= \frac{1}{4}$ m.

And given $\omega = 2$ rev/s

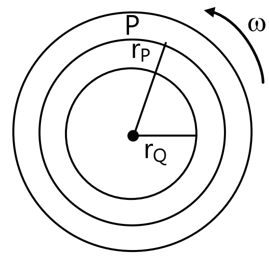
But 1 rev $= 2\pi$ rad

$\because \omega = 2(2\pi)$ rad/s $\qquad \therefore \ \omega = 4\pi$ rad/s

Now acceleration $= r\omega^2 = \frac{1}{4}(4\pi)^2\ m/s^2$

$a = 4p^2\ m/s^2$

Sol 21: (C)



Now acceleration of P is $r_P.\omega^2$ towards centre of disc and acceleration of Q is $r_Q.\omega^2$

Given $r_p > r_Q$

$\therefore \ a_P > a_Q$

Sol 22: (D) Velocity $V = \vec{r} \times \vec{\omega}$

Acceleration $= (\vec{r} \times \vec{\omega}) \times \vec{\omega}$

Now In uniform Circular motion, ω is constant and of course r is constant. Hence magnitude of both velocity and acceleration are constant. But the directions keep varying.

Hence both velocity and acceleration change.

Sol 23: (C) Equal angles in equal time implies ω is constant. Now follow the above argument

Sol 24: (C) $\theta = 2t^3 + 0.5$

$$\omega = \left.\frac{d\theta}{dt}\right|_{t_o} = \left.6t^2\right|_{t_o} = 6t_0^2$$

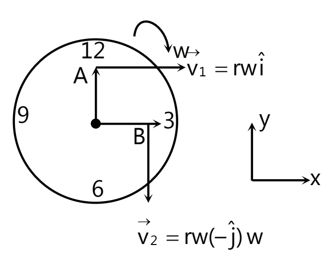
Now here $t_o = 2$ s

$\omega = 6(2)^2 = 24$ rad/s

Sol 25: (D)

A seconds hand completes one revolution in 60 seconds

i.e. 2π rad in 60 seconds



$\therefore \ \omega = \frac{2\pi}{60}$

$= \frac{\pi}{30}$ rad/sec

Speed of the end point = rω

$= 6.\ \frac{\pi}{30}$ cm/s = $\frac{\pi}{5}$ cm/s

=2π mm/s = 6.28 mm/s

Now consider the end point at point A; velocity of the end point would be

$\vec{v}_A = r\omega\hat{i}$ and now when the end point is at point B;

velocity of the end point is $\vec{v}_B = r\omega(-\hat{j})$

Now $\vec{v}_A - \vec{v}_B = r\omega\hat{i} - r\omega(-\hat{j})$

$\vec{v}_A - \vec{v}_B = r\omega(\hat{i} + \hat{j})$

$|\vec{v}_A - \vec{v}_B| = r\omega(\sqrt{2}) = \sqrt{2}\,rw$

$= \sqrt{2}\,(6.28)$ mm/s = 8.88 mm/s

Sol 26: (B) Initially the fan makes 600 revolutions per minute

$\therefore\ \omega = 600$ rev/min $= 600\left[\frac{2\pi}{60}\text{rad / sec}\right]$

$\therefore$ 1 rev = 2π rad

1 min = 60 sec

$\omega_i = 600\frac{2\pi}{60}$ rad/sec

$\omega_i = 20\pi$ rad/sec

and finally the fan makes 1200 revolutions per minute

$\therefore\ \omega_f = 1200\ \frac{2\pi}{60}$ rad/sec ; $\omega_f = 40\pi$ rad/sec

Increase in angular velocity = $\Delta\omega = \omega_f - \omega_i$

= (40π – 20π) rad/s = 20π rad/s

Sol 27: (A) Let 'R' be the radius of the wheel. In one revolution, the wheel completes a distance of 2πR.

And for 2000 revolutions, it is 2000× 2πR.

But given the distance is 9.5 km

$\because$ 2000 × 2πR = 9.5 × 10^3 m

$\therefore$ R = 0.75 m

Diameter d = 2R = 1.5 m

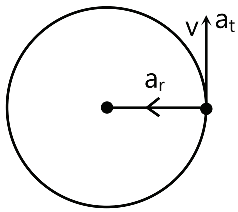
Sol 28: (C) Angular acceleration $\alpha = \frac{d\omega}{dt}$

Since ω is a constant

α = zero

Sol 29: (D)

For uniformly accelerated motion; velocity will be in tangential direction. And acceleration will have both the radial and tangential components.



$\vec{a}_r = \frac{v^2}{R}$ and $\vec{a}_t = \frac{d\vec{v}}{dt}$

Sol 30: (C) F ∝ r

⇒ F = kr (k is a constant)

But we also know that for a particle in circular orbit;

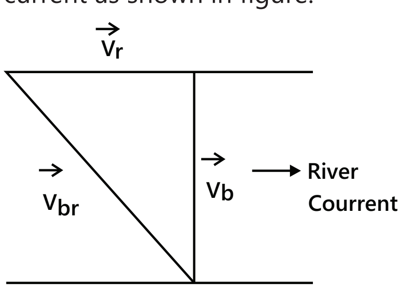
$F = \frac{mv^2}{r}$ $\qquad \therefore\ \frac{mv^2}{r} = kr$

$v = \sqrt{\frac{k}{m}}\,r$ $\qquad \Rightarrow v \propto r$

Previous Years' Questions

Sol 1: (A) To cross the river in shortest time one has to swim perpendicular to the river current.

Sol 2: (B) Shortest possible path comes when absolute velocity of boatman comes perpendicular to river current as shown in figure.



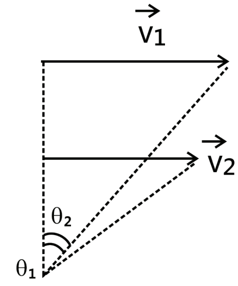
$t = \frac{\omega}{v_b} = \frac{\omega}{\sqrt{v_{br}^2 - v_r^2}}$; $\quad \frac{1}{4} = \frac{1}{\sqrt{25 - v_r^2}}$

Solving this equation we get v_r = 3 km/h

Sol 3: (B)

$\theta_2 > \theta_1 \qquad \therefore\ \omega_2 > \omega_1$

Statement-II, is formula of relative velocity. But it does not explain statement-I correctly. The correct explanation of statement-I is due to visual perception of motion. The object appears to be moving faster, when its angular velocity is greater w.r.t. observer.



Sol 5: (D) maximum vertical height = $\frac{u^2}{2g} = 10m$

Horizontal range of a projectile = $\frac{u^2 \sin 2\theta}{2g}$

Range is maximum when $\theta = 45^0$

Maximum horizontal range = $\frac{u^2}{g}$

Hence maximum horizontal distance = 20 m.

Sol 6: (C) $a \propto r$

Sol 7: (A) $x = t$; $y = 2t - 5t^2$

Equation of trajectory is $y = 2x - 5x^2$

JEE Advanced/Boards

Exercise 1

Projectile Motion

Sol 1: Let V_x V_y be velocities along x, y axis respectively.

$\frac{dV_y}{dt} = -a$ (given); $\qquad y = \alpha x - \beta x^2$

$V_y = \frac{dy}{dt} = \frac{d}{dt}(\alpha x - \beta x^2) = \alpha\frac{dx}{dt} - 2\beta x\frac{dx}{dt}$

$V_y = \alpha V_x - 2\beta x V_x$ ... (i)

at origin, (x, y) = (0, 0)

$\Rightarrow V_y(0, 0) = \alpha V_x - 2\beta(0)V_x$

$V_y = \alpha V_x$

$V^2 = V_x^2 + V_y^2 = V_x^2 + a^2 V_x^2$

$\Rightarrow V = \sqrt{1+\alpha^2}\, V_x$ ... (ii)

Coming back to equation (i)

$a_y = \frac{dV_y}{dt} = \frac{d}{dt}(\alpha V_x - 2\beta x V_x)$

$-a = \alpha\frac{d}{dt}(V_x) - 2\beta V_x\left(\frac{dx}{dt}\right) - 2\beta x\left(\frac{dV_x}{dt}\right)$

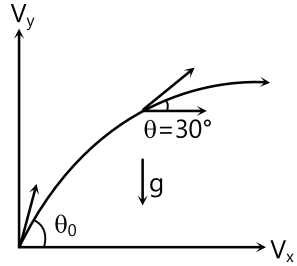
$\frac{d}{dt}V_x = 0$ $\because$ it is having acceleration only in x-direction

$\therefore -a = -2\beta V_x\left(\frac{dx}{dt}\right) \qquad \therefore -a = -2\beta(V_x)^2$

$\Rightarrow V_x = \sqrt{\frac{a}{2\beta}} \qquad \Rightarrow V = \sqrt{1+\alpha^2}\sqrt{\frac{a}{2\beta}}$

$V = \sqrt{(1+\alpha^2)\left(\frac{a}{2\beta}\right)}$

Sol 2: Let V_x be velocity along x-axis. Let $V_y(t)$ be velocity along y-axis at time t.



at t = 2, $\theta = 30°$

$\Rightarrow \frac{V_y(2)}{V_x} = \tan 30°$

$\Rightarrow V_y(2) = V_x\frac{1}{\sqrt{3}}$

At t = 3, $\theta = 0°$ (given moving horizontal)

$\Rightarrow V_y(3) = 0$

$V - u = at$

$\Rightarrow V_y(3) - V_y(2) = -g(3-2)$

$\Rightarrow 0 - \frac{V_x}{\sqrt{3}} = -g$; $\Rightarrow V_x = \sqrt{3}\,g$

Initial velocity of projectile

$V = \sqrt{V_x^2 + (V_y(0))^2}$

$V_y(2) - V_y(0) = -g(2-0)$

$V_y(0) = V_y(2) + 2g = \frac{V_x}{\sqrt{3}} + 2g = g + 2g$

$V_y(0) = 3g$

$\Rightarrow V = \sqrt{V_x^2 + (3g)^2} = \sqrt{(\sqrt{3}g)^2 + (3g)^2} = g\sqrt{12} = 10\sqrt{12}$

$V = 20\sqrt{3}\ ms^{-1}$

$\tan\theta_0 = \frac{V_y(0)}{V_x} = \frac{3g}{\sqrt{3}g} = \sqrt{3} \Rightarrow \theta = 60°$

Sol 3: $V = V\cos\theta\,\hat{i} + V\sin\theta\,\hat{j}$

$\theta = 37°$, $V = 700\ cms^{-1} = 7\ ms^{-1}$

$\therefore V = 7\cos 37°\,\hat{i} + 7\sin 37°\,\hat{j} = 7(0.8)\,\hat{i} + 7(0.6)\,\hat{j}$

$V = 5.6\,\hat{i} + 4.2\,\hat{j}$

Distance between the balls along the line of projection d = 500 cm

Distance between the balls along x-axis (d_x)

$= d\cos\theta = 500\cos 37° = 500(0.8) = 400\ cm = 4m$

When the two balls hit, their x-coordinates are same

$\Rightarrow t = \frac{d_x}{V_x} = \frac{4}{5.6}\ s$

Distance through which ball B falls is

$$= \frac{1}{2}gt^2 = \frac{1}{2} \times 10 \times \left(\frac{4}{5.6}\right)^2 = 2.55 \text{ m}$$

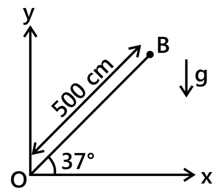
V_y of ball at O = 4.2 ms^{-1}

V_y at time of collision

V – u = at

$$V_y - 4.2 = -10\left(\frac{4}{5.6}\right)$$

$$V_y = -\frac{103}{35}$$



Angle of inclination = $\tan^{-1}\frac{V_y}{V_x} = \tan^{-1}\frac{-\frac{103}{35}}{5.6} = -27.72°$

Ball is directed at an angle 27.72° below x-axis.

Sol 4: (i) ∵ there is no friction and motion is taking plane in a horizontal plane,

Hence acceleration = 0 in all frames of reference (except some random accelerating frame of reference which we will not be using in this problem)

$V_{ball} = V\cos\phi\,\hat{i} + V\sin\phi\,\hat{j}$

$V_{ball-trolley}\,(V_{bT}) = V_{ball} - V_{trolley} = V\cos\phi\,\hat{i} + (V\sin\phi - V_{trolley})\,\hat{j}$

Hence motion of the ball is a straight line as observed by trolley.

In trolley's frame of reference, O moves downward let initial position of O be O_o. O_oA makes 45° with x-axis.

And the ball follows the path O_0A. Hence velocity vector of the ball makes 45° with the x-axis in this frame θ = 45°

(ii) $\phi = \frac{4\theta}{3} = \frac{4(45)}{3} = 60°$

$$\frac{V\sin\phi - V_{trolley}}{V\cos\phi} = \tan\theta = 1$$

$$\therefore \frac{V\sin 60 - (\sqrt{3}-1)}{V\cos 60} = 1$$

$$\frac{V}{2} = \frac{\sqrt{3}}{2}V - (\sqrt{3}-1) \qquad \Rightarrow V = 2\text{ ms}^{-1}$$

Sol 5: $R = \frac{V^2\sin 2\theta}{g} = \frac{2V^2}{g}\sin\theta\cos\theta$

$$H = \frac{V^2\sin^2\theta}{2g} \qquad \text{... (i)}$$

$$\frac{H}{R} = \frac{\tan\theta}{4} \Rightarrow \tan\theta = \frac{4H}{R}$$

$$\Rightarrow \sin\theta = \frac{4H}{\sqrt{(4H)^2 + R^2}}; \quad \sin^2\theta = \frac{16H^2}{16H^2 + R^2}$$

$$V^2 = \frac{2gH}{\sin^2\theta} \qquad \text{(from (i))}$$

$$= \frac{2\times 10\times H}{\frac{16H^2}{10H^2 + R^2}} = \frac{5(16H^2 + R^2)}{4H}$$

$$V = \sqrt{\frac{5(16H^2 + R^2)}{4H}}$$

Sol 6: $V_0 = V_x\hat{i}$, $V(t) = V_x\hat{i} + V_y\hat{j}$, $V_y = gt$

$$|V(t)| = \frac{3}{2}V_0 = \frac{3}{2}V_x$$

$$\sqrt{V_x^2 + V_y^2} = \frac{3}{2}V_x; \Rightarrow V_x^2 + (gt)^2 = \left(\frac{3}{2}V_x\right)^2$$

$$\Rightarrow (gt)^2 = \frac{5}{4}V_x^2; \Rightarrow V_x = \frac{2gt}{\sqrt{5}}$$

$$= \frac{2\times 10\times\frac{1}{2}}{\sqrt{5}} = 4.4 \text{ m/s}$$

Sol 7: $OA = \frac{V_2^2\sin 2\theta}{g}\,(V_2 = 40\text{ ms}^{-1})$

$$= \frac{40\times 40\sin 120}{10} = 80\sqrt{3}\text{ m}$$

$$V = V\cos\theta\,\hat{i} + V\sin\theta\,\hat{j} = \frac{V}{2}\,\hat{i} + \frac{\sqrt{3}}{2}V\,\hat{j}$$

$OA = V_x t$

$$\Rightarrow \frac{V}{2}.\,t = 80\sqrt{3}$$

$$Vt = 160\sqrt{3}$$

$$y = V_y t - \frac{1}{2}gt^2 = \frac{\sqrt{3}}{2}Vt - \frac{1}{2}gt^2$$

$$V = \frac{160\sqrt{3}}{t}$$

$y = a\sqrt{3}\,t$ (as they meet at same point)

$$a\sqrt{3}\,t = \frac{\sqrt{3}}{2}\,160\sqrt{3}\,t - \frac{1}{2}(10)t^2$$

$$\Rightarrow 5t^2 + a\sqrt{3}\,t - 240 = 0$$

$$t > 0 \Rightarrow t = \frac{16\sqrt{3}}{5}$$

$V = \frac{160\sqrt{3}}{t} = \frac{160\sqrt{3}}{\frac{10\sqrt{3}}{5}}$ $\therefore$ $V = 50ms^{-1}$

Sol 8: $D_y = 1.8 – 0.6 = 1.2$

$D_y = – ut + \frac{1}{2}gt^2$

$\Rightarrow 1.2 = – ut + \frac{1}{2}(10)t^2$

$u = V \sin30 = 18 \times \frac{1}{2} = 9 \text{ ms}^{-1}$

$\Rightarrow 1.2 = – 9t + 5 t^2$

Note: Try to understand the sign convention, here downward is taken positive,

Here $y = –ut + \frac{1}{2}gt^2)$

$5t^2 – 9t – 1.2 = 0$

$t \simeq 1.96$

$D = V_x t = 18 \cos 30t = 30.55$ m

Sol 9: $y = x \tan\theta - \frac{gx^2}{2(V\cos\theta)^2}$

$\theta = 45°$

Let 4 ft above the ground be taken as plane of referxe

$\Rightarrow y_1 = – 4$ ft

$x_1 = 350$ ft

$y_1 = x_1 \tan\theta - \frac{gx_1^2}{2V^2\cos^2\theta}$

$\Rightarrow \frac{gx_1^2}{2V^2\cos^2\theta} = x_1 \tan\theta – y_1$

$\Rightarrow \frac{g}{2V^2\cos^2\theta} = \frac{x_1\tan\theta - y_1}{x_1^2}$

We have $x_2 = 320$ ft

$\Rightarrow y_2 = x_2\tan\theta - \frac{gx_2^2}{2V^2\cos^2\theta}$

$= x_2 \tan\theta - x_2^2\left(\frac{x_1\tan\theta - y_1}{x_1^2}\right)$

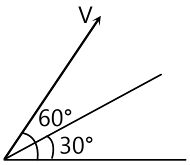
$= 320 \tan(45) - \frac{(320)^2}{(350)^2}(320\tan(45°)–(–4))$

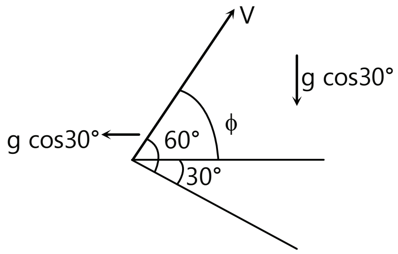
$\Rightarrow y_2 = 24.08$ ft

$y_2 > 24$ ft

$\therefore$ It will clear the fence

Sol 10: (i)





Let solve in the planes frame of reference

$\phi = 60 – 30 = 30°$

$V = V \cos30\,\hat{i} + V \sin30\,\hat{j}$

$V = \frac{\sqrt{3}V}{2}\hat{i} + \frac{V}{2}\hat{j}$

Time of flight $= \frac{2V_y}{g_y} = \frac{2.\frac{V}{2}}{g\cos30°} = \frac{V}{\frac{\sqrt{3}}{2}g}$

$t = \frac{2V}{\sqrt{3}g}$

$x = V_x t - \frac{1}{2}g_x t^2 = \frac{\sqrt{3}V}{2}\left(\frac{2}{\sqrt{3}}\frac{V}{g}\right) - \frac{1}{2}g\sin30\left(\frac{2V}{\sqrt{3}g}\right)^2$

$= \frac{V^2}{g} - \frac{V^2}{3g}$

$x = \frac{2V^2}{3g} = \frac{2}{3}.\frac{(29.4)^2}{9.8} = 58.8$ m

Sol 11: (i) $y = x\tan\theta - \frac{gx^2}{2V^2\cos^2\theta}$

Let's take A as origin

$y = x \tan(37) - \frac{gx^2}{2V^2\cos^2 37}$

$y = = – 3300$ m

$x = 9400$ m

$– 3300 = 94 \times 10^2 \times \frac{3}{4} - \frac{g\times(94)^2\times10^4}{2V^2.\frac{3^2}{5^2}}$

$\frac{10^4\times(94)^2 g}{V^2.\frac{18}{25}} = 10^2\left(94\times\frac{3}{4}+33\right) = 103.5$

$\Rightarrow V = \sqrt{\frac{(10)^2\times(94)^2\times9.8}{(103.5)}\times\frac{25}{18}} = 340.9 \text{ ms}^{-1}$

(ii) $t = \frac{9400}{V\cos37} = 46$ s

Sol 12: Angle of projection

$\theta = \tan^{-1}\frac{V_y}{V_x} = \tan^{-1}\frac{8}{6} = 53°$

$V = \sqrt{6^2+8^2} = 10\ ms^{-1}$

range $= \frac{V^2}{g}\sin 2\theta = \frac{2\times10\times10}{10} \times \frac{3}{5} \times \frac{4}{5} = 9.6$ m

max height $= \frac{V_y^2}{2g} = \frac{(8)^2}{2\times10} = 3.2$ m

Time of height $= \frac{2V_y}{g} = \frac{2\times8}{10} = 1.6$ s

Sol 13: $E = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2\times1960}{9.8}} = 20$ s

$MN = V_x \times t = 100 \times 20 = 2000$ m

Sol 14: (i) Let initial velocity be V

$H = \frac{V^2\sin^2\alpha}{2g}$

$t = \frac{V_y}{g} = \frac{V\sin\alpha}{g}$

Horizontal distance = H tanθ

Note: - body just gazes ⇒ $V_y = 0$ at the top.

$H\tan\theta = V_x.t$

$H\tan\theta = V\cos\alpha . \frac{V\sin\alpha}{g}$

$\frac{V^2\sin^2\alpha}{2g} . \tan\theta = \frac{V^2\sin\alpha\cos\alpha}{g}$

⇒ $\tan\theta = 2\cot\alpha$

(ii) $H = \frac{u^2\sin^2\alpha}{2g}$ (v = u)

$u^2 = \frac{2Hg}{\sin^2\alpha}$

$u^2 = 2Hg\ \text{cosec}^2\alpha = 2Hg(1+\cot^2\alpha)$

$= 2Hg\left(1+\left(\frac{\tan\theta}{2}\right)^2\right) = \frac{gh\,(4+\tan^2\theta)}{2}$

Sol 15: (i) The ball returns to him

⇒ there is no velocity in x-direction in the truck's frame of reference

⇒ Angle of projection = 90°

Time of flight $t = \frac{D}{V_{truck}} = \frac{58.8}{14.7} = 4$ s

$u = g\left(\frac{t}{2}\right) = 9.8 \times \frac{4}{2} = 19.6\ ms^{-1}$ vertically upwards

(ii) $V_{ball} = V_{bT} + V_{truck} = 19.6\hat{j} + 14.7\hat{i}$

$|V| = \sqrt{(19.6)^2+(14.7)^2} = 24.5\ ms^{-1}$

Angle of projection

$\theta = \tan^{-1}\frac{V_y}{V_x} = \tan^{-1}\frac{19.6}{14.7} = 53°$

Sol 16: $V_1 = V_0\hat{j}$

$V_2 = V_0\cos\alpha\,\hat{i} + V_0\sin\alpha\,\hat{j}$

$V_2 = V_0(\cos\alpha)\,\hat{i} + V_0(\sin\alpha - 1)\,\hat{j}$

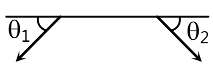
$a_1 = -g\hat{j};\ a_2 = -g\hat{j};\ a_2 = 0$

Separation $x(t) = V_{21}(t) - \frac{1}{2}a_{21}t^2$

$= V_0(\cos\alpha)t\,\hat{i} + V_0(\sin\alpha - 1)t\,\hat{j}$

$|x(t)| = t\,V_0\sqrt{(\cos\alpha)^2+(\sin\alpha-1)^2}$

$= V_0.t\sqrt{2-2\sin\alpha} = V_0.t + \sqrt{2(1-\sin\alpha)}$

Sol 17: 

Let the vertical components of their velocities be V_y. Let this angle of depression be θ_1, θ_2.

$\tan\theta_1 = \frac{V_y}{3}$

$\tan\theta_2 = \frac{V_y}{4}$

∵ They both are perpendicular, $\theta_1 + \theta_2 = 90°$

⇒ $\tan\theta_2 = \tan(90-\theta_1) = \cot\theta_1 = \frac{1}{\tan\theta_1}$

⇒ $\tan\theta_1 . \tan\theta_2 = 1$

⇒ $\frac{V_y^2}{12} = 1$

⇒ $V_y = \sqrt{12}\ ms^{-1}$

$V_y = gt;$ $t = \frac{V_y}{g}$

Separation = relative velocity × time = $(Vx_1 - Vx_2)$ t

$= [3-(-4)] \times \frac{V_y}{g} = \frac{7\times\sqrt{12}}{10} = 2.43$ m

Note: Here g is taken 10 ms^{-2}. You may take g = 9.8 ms^{-2} them separation = 2.47 m. The questions takes the value of g to intelligently manipulate the question.

Sol 18: Let initial velocity $V = V_x\hat{i} + V_y\hat{j}$

t = seconds $V(t) = V_x\hat{i} + (V_y - gt)\hat{j}$

$\tan\alpha = \dfrac{V_y}{V_x}$ $\quad \tan\beta = \dfrac{V_y - gt}{V_x}$

$V_y = V_x$ tana

$\tan\beta = \dfrac{V_x\tan\alpha - gt}{V_x}$

$V_x = \dfrac{gt}{\tan\alpha - \tan\beta}$

$V = \sqrt{V_x^2 + V_y^2} = \sqrt{1+\tan^2\alpha} \, . V_x$

$= \sqrt{1+\tan^2\alpha} \, . \dfrac{gt}{\tan\alpha - \tan\beta}$

$= \sec\alpha \, . \dfrac{gt}{\dfrac{\sin\alpha\cos\beta - \cos\alpha\sin\beta}{\cos\alpha\cos\beta}}$

$= \dfrac{1}{\cos\alpha} . \dfrac{gt}{\sin(\alpha-\beta)} . \cos\alpha$ cosb

$V = \dfrac{gt\cos\beta}{\sin(\alpha-\beta)}$

Sol 19: Let $V = V_x\hat{i} + V_y\hat{j}$

Velocity at maximum height $V_h = V_x$ $\quad (\because V_y = 0)$

Maximum height = $\dfrac{V_y^2}{2g}$

Velocity at half maximum height = V_{y_2}

$\dfrac{V_{y_2}^2}{2g} = \dfrac{1}{2}\dfrac{V_y^2}{29} \Rightarrow V_{y_2} = \dfrac{V_y}{\sqrt{2}}$

$V = \sqrt{V_x^2 + \left(\dfrac{V_y}{\sqrt{2}}\right)^2}$

Now as per given information,

$\sqrt{\dfrac{2}{5}}\sqrt{V_x^2 + \left(\dfrac{V_y}{\sqrt{2}}\right)^2} = V_x$

$\Rightarrow \dfrac{5}{2}V_x^2 = V_x^2 + \dfrac{V_y^2}{2}$

$\Rightarrow V_y = \sqrt{3}\,V_x \Rightarrow \sqrt{3} = \dfrac{V_y}{V_x}$

angle of projection = $\tan^{-1}\dfrac{V_y}{V_x} = \tan^{-1}\sqrt{3} = 60°$

Sol 20: If he takes minimum time $\Rightarrow$ he is always perpendicular w.r.t water

$\Rightarrow$ drift = velocity of water × time

$V_w = \dfrac{120}{10\times 60} = 0.2\ ms^{-1} = 12$ m/min

If he takes shortest path, his resultant velocity along the flow of river is 0 ms^{-1}

$\Rightarrow$ i.e. $V_w - V_x = 0$; $V_x = 0.2\ ms^{-1}$

Lets assume his velocity is V

V(10) = Vy(12.5)

$\Rightarrow V_y = \dfrac{4}{5}V$

$\Rightarrow V_x = \dfrac{3}{5}V \; (\sqrt{V_x^2 + V_y^2} = V)$

$\Rightarrow 0.2 = \dfrac{3}{5}V$

$\Rightarrow V = 0.33\ ms^{-1} = 20$ m/min

width = V × 10 = 200 m

Sol 21: Velocity of wind = $u\hat{i}$

$\Rightarrow$ V butterfly w.r.t earth = $V + V_{wind}$

$= (10 + u)\hat{i} + 12\hat{j}$

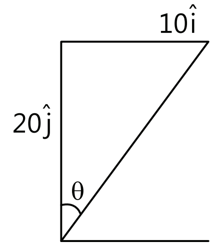
$\tan\theta = \dfrac{12}{10+u}$

$\dfrac{3}{4} = \dfrac{12}{10+u}$

$\Rightarrow u = 6ms^{-1}$

Note: - The resultant velocity is directed along AB.

Sol 22: $V_{rain/grd} = -20\hat{j}$



$V_m = 5\hat{i}$

$V_{air} = 15\hat{i}$

$V_{rain} - V_{ground} = -20\hat{j}$

$\Rightarrow V_{rain} = 15\hat{i} - 20\hat{j}$

$V_{man} = 5\hat{i}$

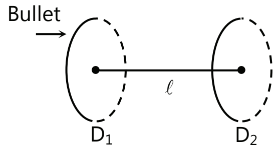
$\Rightarrow V_{rain/man} = 15\hat{i} - 20\hat{j} - 5z = 10\hat{i} - 20\hat{j}$

So $\tan\theta = \frac{10}{20} = \frac{1}{2}$

$\Rightarrow \theta = \tan^{-1}\left(\frac{1}{2}\right)$

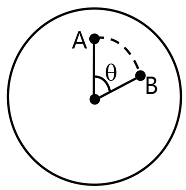
Circular Motion

Sol 23: Let the time = t_0 at the instant the bullet hits the first disc and makes a hole in it.



And time = t_1 when bullet makes hole in Disc-2. In this time interval $\Delta t = t_1 - t_0$,

An angular displacement of θ is made by point A w.r.t the point B.



At $t = t_1$

A is hole in Disc-I

B is hole in Disc-II

$\therefore \omega \Delta t = \theta \rightarrow$ (i)

And also in the same time interval Dt;

Bullet travelled a distance of 'ℓ'

$\therefore \ell = v\,\Delta t \rightarrow$ (ii)

Comparing eq^n $\rightarrow$ (i) and (ii); we get

$\frac{\omega\Delta t}{v\Delta t} = \frac{\theta}{\ell}$

$v = \frac{\ell w}{\theta}$

Sol 24: $\vec{r} = 3\hat{i} + 4\hat{j}$; $\vec{v} = -4\hat{i} + 3\hat{j}$

$|\vec{r}| = 5$m; $|\vec{v}| = 5$ m/s

We know that radial acceleration = $\frac{v^2}{r} = \frac{(5)^2}{5} = 5$ m/s^2

And this acceleration will be along the negative radial direction.

$\therefore \vec{r} = 3\hat{i} + 4\hat{j}$

Unit vector in the direction of $\vec{r}$

Is $\vec{r} = \frac{\vec{r}}{|r|} = \frac{1}{5}(3\hat{i} + 4\hat{j})$

$\therefore \hat{r} = \frac{3}{5}\hat{i} + \frac{4}{5}\hat{j}$ ... (i)

Now $\vec{a}_r = \frac{v^2}{5}(-\hat{r}) = 5\left(-\frac{3}{5}\hat{i} - \frac{4}{5}\hat{j}\right)$

$\vec{a}_r = -3\hat{i} - 4\hat{j}$ m/s^2

And also $\vec{a} = \vec{a}_r + \vec{a}_t$

Given $\vec{a} = -7\hat{i} - \hat{j}$

$\therefore \vec{a}_t = \vec{a} - \vec{a}_r = (-7\hat{i} - \hat{j}) - (-3\hat{i} - 4\hat{j})$

$\vec{a}_t = -4\hat{i} + 3\hat{j}$ m/s^2

Sol 25: Acceleration inside a rotor = $R\omega^2$

$\vec{a} = R\omega^2$

How for $\vec{a}_{max}$

$a_{max} = R\,\omega^2_{max}$

Given a_{max} = 10g = 100 m/s^2

$\omega_{max} = \sqrt{\frac{100}{4}} = \frac{10}{2}$ rad/s = 5 rad/s

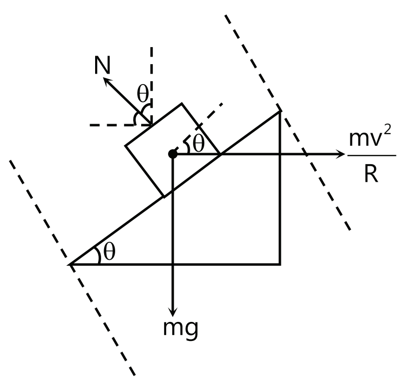
we know that 1 rad = $\frac{1}{2\pi}$ rev

$\therefore \omega_m = \frac{5}{2\pi}$ rev/s

Sol 26:

$N\sin\theta = \frac{mv^2}{R}$ ... (i)

$N\cos\theta = mg \rightarrow$ ... (ii)



Dividing (i) and (ii)

$\Rightarrow \tan\theta = \frac{v^2}{Rg}$

$\Rightarrow$ V = 108 Km/h = $108 \times \frac{5}{18}$ m/s

V = 30 m/s

R = 90 m

$\therefore \tan\theta = \frac{30.30}{90.10} = 1$

$\Rightarrow \theta = \frac{\pi}{4}$

Squaring (i) and (ii) and adding them;

$N^2 (\sin^2\theta + \cos^2\theta) = \left(\frac{mv^2}{R}\right)^2 + (mg)^2$

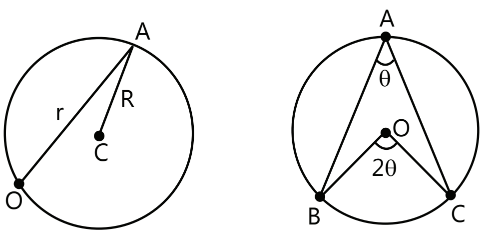
$N = \sqrt{(mg)^2 + \left(\frac{mv^2}{R}\right)^2}$

$N = m\sqrt{(10)^2 + (10)^2}$

$N = 10\ m\sqrt{2}$ Newton

$\Rightarrow N = 10^4.\sqrt{2}$ N.

Sol 27: In solving this question, we will use one of the most important theorems in circles



The figure explains us that for every θ traversed by $\overrightarrow{AB}$, $\overrightarrow{OB}$ traverses an angle of 2θ.

$\therefore \omega_{OB} = 2 \times \omega_{AB}$

Hence in this case ω w.r.t C is twice that of w.r.t point C.

$\therefore \omega = 2(0.4) = 0.8$ rad/sec.

$|\vec{v}| = R\omega$

$R = 50\text{ cm} = \frac{1}{2}\text{m}$

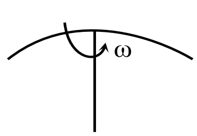
$|\vec{v}| = \frac{1}{2}(0.8)$ m/s

$|\vec{v}| = 0.4$ m/s and $\vec{a} = \vec{a}_r + \vec{a}_t$

But here $\vec{a}_t = 0$

$\therefore \vec{a} = \vec{a}_r = \left(\frac{v^2}{R}\right)(-\hat{e}_r)$

Sol 28: Angular velocity of the



umbrella $= \frac{21}{44}$ rev/s $= \frac{21}{44}.2\pi$ rad/s

$\omega = \frac{21\pi}{22}$ rad/s = 3 rad/s

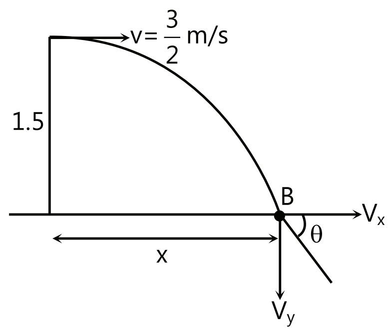
Now for a drop on the Rim; velocity

$|\vec{v}| = R\omega$

$|\vec{v}| = \left(\frac{1}{2}\right)(3)$ m/s

$|\vec{v}| = \frac{3}{2}$ m/s

Now this is fairly a kinematics problem;



$1.5 = 0.\ t - \frac{1}{2}gt^2$

$\Rightarrow t = \sqrt{0.3}$ and x = vt

$x = \frac{3}{2} \times \sqrt{0.3}$

x = 0.82 m

and $\tan\theta = \left(\frac{v_y}{v_x}\right)$

v_y at point B is $v_y = 0 - gt$

$v_y = -10\sqrt{0.3}$ m/s and $v_x = \frac{3}{2}$ m/s

$\therefore \tan\theta = \left(\frac{v_y}{v_x}\right) = \left(\frac{10\sqrt{0.3}}{3/2}\right)$

$\because \theta = 74.6°$

Sol 29: Acceleration inside a rotor = $R\omega^2$

$\vec{a} = R\omega^2$

Now for $\vec{a}_{max}$

$a_{max} = R\,\omega_{max}^2$

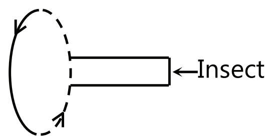
Given $a_{max} = 10g = 100\text{ m/s}^2$

$\omega_{max} = \sqrt{\frac{100}{4}} = \frac{10}{2}$ rad/s = 5 rad/s

We know that 1 rad = $\frac{1}{2\pi}$ rev

$\therefore\ \omega_m = \frac{5}{2\pi}$ Rev/s

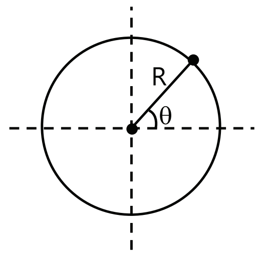
Sol 30: From the top view; The insect looks at the particle as



$\therefore$ x-co-ordinates of the Particle

= R cosθ But θ = ωt

$\therefore$ x = R cos ωt; y = R sin ωt



$\therefore\ \vec{r}_{particle,insect}$

$= R\cos\omega t\,\hat{i} + R\sin\omega t\,\hat{j}$

Now

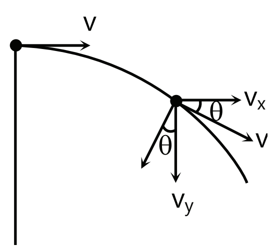
$\vec{r}_{particle,\,observer} = \vec{r}_{particle,insect} + \vec{r}_{insect,\,observer}$

$\vec{r}_{particle,\,observer} = vt\,\hat{i}$

$\therefore\ \vec{r}_{PO} = (R\cos\omega t + vt)\,\hat{i} + R\sin\omega t\,\hat{j}$

Hence the motion will be a cycloid.

Sol 31: Now we shall follow a standard procedure rather than a clumsy formula to find the radius of curvature.



Let us first find v_x and v_y at t = 3 s

$v_x = v_0 = 10$ m/s $\hat{i}$

$v_y = 0 - gt = -30$ m/s $\hat{j}$

$\tan\theta = \left|\left(\frac{v_y}{v_x}\right)\right|$ $\therefore$ $\tan\theta = 3$ ... (i)

Now we need to resolve the gravitational force normal to the curve at point P.

Hence it is equal to g cos θ and

$v = \sqrt{v_x^2 + v_y^2} = \sqrt{200 + 900} = 10\sqrt{10}$ m/s

Let 'R' be the radius of curvature,

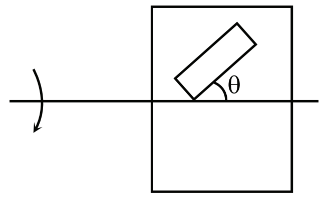
Then; $mg\cos\theta = \frac{mv^2}{R}$

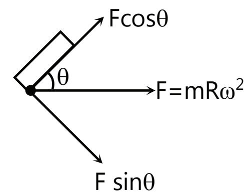
$R = \frac{v^2}{g\cos\theta}$; $\tan\theta = 3$

$\cos\theta = 0.3$

$R = \frac{1000}{(10)(0.3)} = 334$ m

Sol 32:





Now $mR\omega^2\cos\theta = ma$

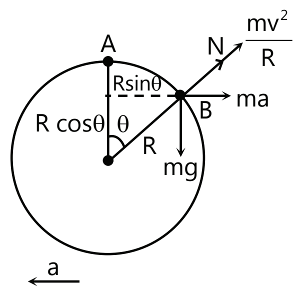
$\therefore\ a = R\omega^2\cos q$

Now $s = ut + \frac{1}{2}at^2$

$L = 0 + \frac{1}{2}R\omega^2\cos\theta\ t^2$

$t = \sqrt{\frac{2L}{R\omega^2\cos\theta}}$

Sol 33: In this case, there will be a pseudo force acting on the body. Now we use Work-Energy theorem, i.e. work done by all the forces is equal to change in kinetic energy. We know that, work done by normal force and centripetal force is zero.



Work done by pseudo force = ma.(R sin θ)

$W_{PF} = maR\sin\theta$

Work done by gravitational force = mg(R – R cos θ)

$W_{mg} = mgR(1 - \cos\theta)$

Net work done = maR sinθ + mgR (1–cos θ)

$\equiv \frac{1}{2}mv^2 = Rm(a\sin\theta + g(1 - \cos\theta))$

$v = \sqrt{2R(a\sin\theta + g(1-\cos\theta))}$

Sol 34: $\vec{a}_{net} = \vec{a}_{radial} + \vec{a}_{tangential}$

$\vec{a}_r = \frac{v^2}{R}.(-\hat{e}_r)$

$\vec{a}_t = a\,(\hat{e}_t)$

$$|\vec{a}_{net}| = \sqrt{a^2 + \left(\frac{v^2}{R}\right)^2} \text{ m/s}^2$$

Exercise 2

Projectile Motion

Single Correct Choice Type

Sol 1: (D) $V = \sqrt{20}\cos\theta\,\hat{i} + \sqrt{20}\sin\theta\,\hat{j}$

$$t = \frac{x}{V_x} = \frac{\sqrt{3}}{\sqrt{20}\cos\theta}$$

$$t = \text{time of flight} = \frac{2V_y}{g} = \frac{2\sqrt{20}\sin\theta}{g}$$

$$\Rightarrow \frac{\sqrt{3}}{\sqrt{20}\cos\theta} = \frac{2\sqrt{20}\sin\theta}{g}$$

$$\sin 2\theta = \frac{\sqrt{3}g}{20}$$

$$\sin 2\theta = \frac{\sqrt{3}}{2}$$

$2\theta = 120°, 60°$

$\Rightarrow\ \theta = 60°, 30°$

$$\Rightarrow \sin\theta = \left[\frac{\sqrt{3}}{2}, \frac{1}{2}\right]$$

$$t = \frac{2\sqrt{20}}{10}\left[\frac{\sqrt{3}}{2}, \frac{1}{2}\right]$$

$$t = \left[\sqrt{\frac{3}{5}}, \sqrt{\frac{1}{5}}\right]$$

$$\cos\theta = \left[\frac{1}{2}, \frac{\sqrt{3}}{2}\right]$$

$$V = \sqrt{20}\left[\frac{1}{2}\hat{i} + \frac{\sqrt{3}}{2}\hat{j}, \frac{\sqrt{3}}{2}\hat{i} + \frac{1}{2}\hat{j}\right]$$

$V_x = [\sqrt{5}, \sqrt{15}]\ \text{ms}^{-1}$

$$H = \frac{V_y^2}{2g} = \frac{1}{2g}\left[\left(\frac{\sqrt{3}}{2}\right)^2, \left(\frac{1}{2}\right)^2\right][\sqrt{20}]^2 = [0.75, 0.25]$$

$$P.E = mgH = m \times g \times \frac{V_y^2}{2g} = \frac{1}{2}mV_y^2$$

$$P.E. = \frac{1}{2}.1.\left[\left(\frac{\sqrt{3}}{2}\right)^2, \left(\frac{1}{2}\right)^2\right][\sqrt{20}]^2 = [7.5, 2.5]\text{ J}$$

$$K.E = \frac{1}{2}mV^2 - PE$$

$$= \frac{1}{2} \times 1 \times (\sqrt{20})^2 - [7.5, 2.5] = [2.5, 7.5]\text{ J}$$

Sol 2: (B) Its trivial.

Sol 3: (B) $V_1 = V_1\cos\theta_1\,\hat{i} + V_1\sin\theta_1\,\hat{j}$

$V_2 = V_2\cos\theta_2\,\hat{i} + V_2\sin\theta_2\,\hat{j}$

$V_1(t) = V_1 - gt\,\hat{j}$

$V_2(t) = V_2 - gt\,\hat{j}$

$V_{12} = V_1 - V_2$

V_{12} is independent of t.

i.e. $a_{12} = \frac{dV_{12}}{dt} = 0$

$\Rightarrow$ Trajectory of particle 1 w.r.t particle 2 is straight line along the direction of V_{12}.

Sol 4: (C) $V_{12} = V_1 - V_2$

$= (V_1\cos\theta_1 - V_2\cos\theta_2)\,\hat{i} + (V_1\sin\theta_1 - V_2\sin\theta_2)\,\hat{j}$

$\because\ V_1\cos\theta_1 = V_2\cos\theta_2$

$\Rightarrow V_{12} = (V_1\sin\theta_1 - V_2\sin\theta_2)\,\hat{j}$

This relative velocity is along $\hat{j}$.

$\Rightarrow$ Trajectory of 1 w.r.t 2 is a vertical straight line.

Note: They need not be one above the other because their initial x coordinates need not be same

$x = x_0 + V_x t$

$Vx_1 = Vx_2$

$\therefore V_1\cos\theta_1 = V_2\cos\theta_2$

but $x_{\theta_1} \neq x_{\theta_2}$ $\quad\Rightarrow x_1 \neq x_2$

Sol 5: (D) $V_y = V\sin\theta$

Time of flight $= \frac{2V_y}{t}$

Maximum height $= \frac{V_y^2}{2g}$

$\because V_1\sin\theta_1 = V_2\sin\theta_2$

$\Rightarrow Vy_1 = Vy_2$

$V_{12} = (V_1 \cos\theta_1 - V_2\cos\theta_2)\,\hat{i}$

$\Rightarrow$ Trajectory of one with respect to other is horizontal.

Multiple Correct Choice Type

Sol 6: (A, B, C, D) (a) $R_{max} = \dfrac{V^2}{g}$

$$h_{max} = \frac{V^2}{2g} = H$$

$\Rightarrow R_{max} = 2H$ (A)

(b) $R = \dfrac{V^2}{g}\sin 2\theta = \dfrac{2V^2}{g}\sin\theta\,\cos\theta$

$$H = \frac{V^2}{2g}\sin^2\theta$$

$$\frac{R}{h} = \frac{V^2}{2g}\sin^2\theta$$

$$\frac{2V^2\sin\theta\cos\theta}{gh} = \frac{V^2}{2g}\sin^2\theta$$

$$\tan\theta = \frac{4}{h}$$

$\theta = \tan^{-1}\left(\dfrac{4}{h}\right)$ (B)

(c) $T = \dfrac{2V\sin\theta}{g}$

$$gT^2 = \frac{4V^2\sin^2\theta}{g}$$

$$R = \frac{V^2}{g}\sin 2\theta$$

$$20 + m\theta = \frac{4V^2\sin^2\theta}{g}$$

$\therefore gT^2 = 2R + m\theta$ (C)

Sol 7: (A, B) $T = \dfrac{2V\sin\theta}{g}$

$$V_y = V\sin\theta = \frac{gT}{2}$$

$$y = V_y t - \frac{1}{2}gt^2 = \frac{gT}{2}t - \frac{1}{2}gt^2$$

$$= \frac{gT^2}{2}\left(\frac{t}{T}\right)\left(1 - \frac{t}{T}\right) \quad \text{... (i)}$$

$$h = \frac{V^2\sin^2\theta}{2g} = \frac{\left(\frac{gT}{2}\right)^2}{2g} = \frac{(gT)^2}{8g} = \frac{gT^2}{8}$$

$gT^2 = 8h$

substitute in (i)

$$y = 4h\left(\frac{t}{T}\right)\left(1 - \frac{t}{T}\right)$$

$x = V\cos\theta\ t$

$R = V\cos\theta\ T$

$$\frac{t}{T} = \frac{x}{R} \qquad \Rightarrow \qquad y = 4h\left(\frac{x}{R}\right)\left(1 - \frac{x}{R}\right)$$

Sol 8: (A, B, C, D) The equation is same as that of a projectile equation

$a_x = 0°$

$$y = ax - bx^2 \left(V_y = \frac{dy}{dx}\right)$$

$V_y = aV_x - 2b_xV_x$

$a_y = a(0) - 2b(V_x)^2 - 2bV_x(0)$

$$a_x = \frac{dV_x}{dx} = 0$$

$a_y = -2b\,V_x^2$

$$V_x = \sqrt{\frac{-a_y}{2b}} \qquad a_y = -g$$

$$\Rightarrow V_x = \sqrt{\frac{g}{2b}}$$

$V_y = aV_x - 2bxV_x$

$V_y(0, 0) = aV_x - 2b(0)V_x$

$V_y = aV_x$

$$V_y = a\sqrt{\frac{g}{2b}}$$

$$\frac{V_y}{V_x} = a$$

$\tan\theta = a$

$\therefore \theta = \tan^{-1}a$

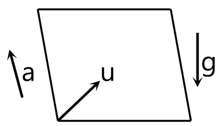
Sol 9: (A, C, D) Horizontal distance = $V_x \times t = 4 \times 0.4 = 1.6$ m

$V_y = gt = 10 \times 0.4 = 4\ \text{ms}^{-1}$

$\Rightarrow$ angle of impact $= \tan^{-1}\left(\dfrac{V_y}{V_x}\right) = \tan^{-1}1 = 45°$

$H = \frac{1}{2}gt^2 = \frac{1}{2} \times 10 \times (0.4)^2 = 0.8$ m

Sol 10: (A, B) In box frame of reference, a acts upwards



i.e. Resultant acceleration is (g – a) downwards . If g = a, resultant acceleration = 0, P will hit C.

For a > g it hits roof

for a < g , P will hit CD

Sol 11: (A, C) Q Goes up and reaches same point.

Time of flight till than $t = \frac{2V}{g} = \frac{2.(5)}{10} = 1$ s

Final velocity = – 5 + 10(1) = 5 ms^{-1} downwards

Distance to ground

$h = 5(t-1) + \frac{1}{2}g(t-1)^2 = 5(t-1) + 5(t-1)^2$

where t is time from starting (total time of flight)

Distance travelled by P = 2H (given)

$2H = 5t + \frac{1}{2}gt^2$

$\Rightarrow 2(5(t-1) + 5(t-1)^2) = 5t + 5t^2$

$10t - 10 + 10t^2 - 20t + 10 = 5t + 5t^2$

$5t^2 - 15t = 0$

$\Rightarrow t = 0, t = 3; t > 0 \Rightarrow t = 3$

$H = 5(3-1) + 5(3-1)^2 = 30$ m

Sol 12: (B, C, D) $R = \frac{V^2}{g}\sin 2\theta$

$480 = \frac{(70)^2}{10}\sin 2\theta \quad \Rightarrow \quad \sin 2\theta = \frac{4800}{4900} = 0.96$

$2\theta = \sin^{-1} 0.96 = 74°$

$\theta = 37°, \theta_2 = 90 - \theta = 53°$ (Complimentary angles)

θ , θ_2 are complimentary as they have same horizontal range.

Time of height $t = \frac{2V\sin\theta}{g}$

$\frac{t_1}{t_2} = \frac{\sin\theta_1}{\sin\theta_2} = \frac{\sin 37}{\sin 53} = \frac{3}{4}$

Max height $h = \frac{V^2\sin^2\theta}{2g}$

$\frac{h_1}{h_2} = \frac{\sin^2\theta_1}{\sin^2\theta_2} = \frac{(3/5)^2}{(4/5)^2} = 9 : 16$

$V_{min} = V_x = V\cos\theta$

$\frac{V_{min_1}}{V_{min_2}} = \frac{\cos\theta_1}{\cos\theta_2} = \frac{(4/5)}{(3/5)} = 4 : 3$

Angle bisector is 45° as θ_1, θ_2 are complementary angles.

Comprehension Type

Sol 13: (A) $V = V\cos\theta\,\hat{i} + V\sin\theta\,\hat{j}$

$\theta = 53°, V = 50\ ms^{-1}$

$\Rightarrow V = 50\cos(53)\,\hat{i} + V\sin(53)\,\hat{j} = 30\hat{i} + 40\hat{j}$

$\Rightarrow V_x = 30\ ms^{-1}$

$|v(t)| \geq |V_x|$

$\Rightarrow$ min velocity = 30 ms^{-1}

$V_y = 40\ ms^{-1}$ but V_y changes with time

$\therefore$ incorrect is (A).

Sol 14: (D) Time of flight $(T) = \frac{2V_y}{g} = \frac{2 \times 40}{10} = 8$ s

Now observe the vertical motion, the body ascends then it descends.

Now if we observe descent in reverse time, it looks like ascent,

Hence $t_{ascent} = t_{descent\ (reverse\ time)}$

$t_{ascent} = T - t_{descent} \Rightarrow t_{descent} = T - t_{ascent}$

T = 8

$\Rightarrow t_d = 8 - T_a$

$\Rightarrow t_a + t_d = 8 \quad (0 < (t_a, t_d) < 8)$

$\therefore$ All A, B, C satisfy

Sol 15: (A) trajectory equation

$y = x\tan\theta - \frac{gx^2}{2(V\cos\theta)^2}$

$\theta = 53°, V = 50\ ms^{-1}$

$y = x\tan 53° - \frac{10\,.\,x^2}{2(50\cos 53)^2}$

$y = x\left(\frac{4}{3}\right) - \frac{10x^2}{2\left(50 \times \frac{3}{5}\right)^2}$

$\Rightarrow 180y = 240x - x^2$

Since it is single correct you may as well solve by substituting any 3 points in motion. You may also eliminate option B, D as coefficient of x^2 should be negative, which is common knowledge to be known about trajectory equation.

Match the Columns

Sol 16: A → p; B → p, q, r, s; C → p, q, r, s; D → p, r

(A) Constant velocity ⇒ same direction ⇒ straight line

Answer is (A) (B)

(B) Constant speed ⇒ Constant magnitude of velocity

⇒ Variable direction of velocity ⇒ there is acceleration ⇒ It can follow any path.

B → p, q, r, s

(C) With variable acceleration, it can follow any path

C → p, q, r, s

(D) Consider a particle moving in circle with uniform velocity u.

Magnitude of acceleration = $\frac{mu^2}{r}$, directed toward canter.

This acceleration has constant magnitude, but variable direction.

Hence (q) false

Now circle is a special case of ellipse

⇒ (s) is also false

Straight line is a trivial example of constant acceleration. So p is true.

We know that trajectory of a projectile is parabola. Here acceleration is constant g towards ground.

Hence r is true.

D → p r.

Assertion Reasoning Type

Sol 17: (B) Speed of projectile is minimum because

$V_y = 0$

Sol 18: (D) $V_x = V\cos\theta$

∵ Angle of projections are different,

$V_{x_1} \neq V_{x_2}$ are $\theta_1 \neq \theta_2$

so they do not collide

Sol 19: (D) Consider two particles in circular motion

Sol 20: (D) $V_{AB} = V_A - V_B$

$V_{AB} > V_A$

$\Rightarrow V_B < 0$

Which is possible

Hence statement 1 false.

Sol 21: (A) Statement-II true

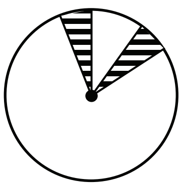
∵ Relative vertical acceleration is zero, relative vertical velocities don't charge.

Circular Motion

Sol 22: (A, D) In a curved path; the direction of velocity keeps on changing. So $\vec{v}$ cannot remain constant under any conditions. However $|\vec{v}|$ = Speed can remain constant.

And $\vec{a} = \frac{d\vec{v}}{dt}$; so it follows that acceleration also cannot remain constant. But still $|\vec{a}| = \left|\frac{d\vec{v}}{dt}\right|$ is possible

Sol 23: (B, D) For a circular motion



Sweeping equal area in equal time is only possible when ω is constant.

∴ Now $\vec{v} = \vec{r} \times \vec{\omega}$

So velocity is not constant

But speed = $|\vec{v}| = r\omega$ = constant

and $\vec{a} = \vec{a}_r + \vec{a}_t$

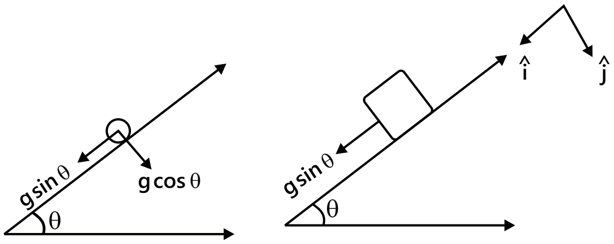
$\vec{a}_t = \frac{dv}{dt} = r.\frac{d\omega}{dt}$ = zero

and $\vec{a}_r = \vec{r} \times (\vec{r} \times \vec{\omega})$

∴ Acceleration is not constant.

Previous Years' Questions

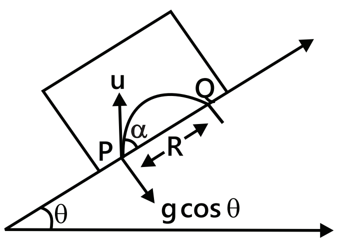
Sol 1: (i) Accelerations of particle and block are shown in figure.



Acceleration of particle with respect to block

= (Acceleration of particle) – (acceleration of block)

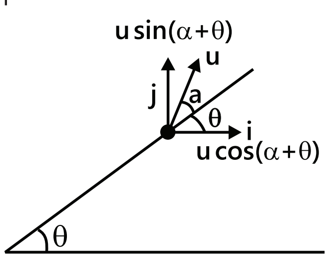
= $(g\sin\theta\ \hat{i} + g\cos\theta\ \hat{j}) - (g\sin\theta)\hat{i} = g\cos\theta\ \hat{j}$ Now motion of particle with respect to block will be a projectile as shown.

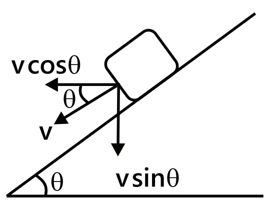


The only difference is, g will be replaced by gcosθ.

PQ = Range (R) = $\dfrac{u^2\sin 2\alpha}{g\cos\theta}$ PQ = $\dfrac{u^2\sin 2\alpha}{g\cos\theta}$

(ii) Horizontal displacement of particle with respect to ground is zero. This implies that initial velocity with respect to ground is only vertical, or there is no horizontal component of the absolute velocity of the particle.





Let v be the velocity of the block down the plane. Velocity of particle

= $u\cos(\alpha+\theta)\,\hat{i} + u\sin(\alpha+\theta)\,\hat{j}$

Velocity of block = $-v\cos\theta\,\hat{i} - v\sin\theta\,\hat{j}$

∴ Velocity of particle with respect to ground

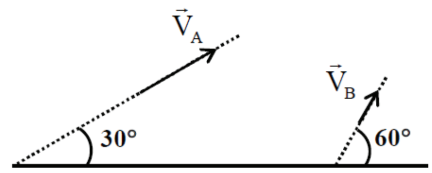
= $\{u\cos(\alpha+\theta) - v\cos\theta\}\,\hat{i} + \{u\sin(\alpha+\theta) - v\sin\theta\}\,\hat{j}$

Now, as we said earlier that horizontal component of absolute velocity should be zero.

Therefore, $u\cos(\alpha+\theta) - v\cos\theta = 0$

or $v = \dfrac{u\cos(\alpha+\theta)}{\cos\theta}$ (down the plane)

Sol 2:



The relative velocity of B with respect to A is perpendicular to line

of motion of A.

$\therefore\ A_B\ V\cos 30^\circ = V_A$

$\Rightarrow\ V_B = 200$ m/s

And time t_0 = (Relative distance) / (Relative velocity)

$= \dfrac{500}{V_B\sin 30^\circ} = 5\,\text{sec}$

Sol 3: (D)

$v\dfrac{dv}{dr} = \omega^2 r$, where v is the velocity of the block radially outward.

$$\int_0^v v\,dv = \omega^2\int_{R/2}^{r} r\,dr$$

$$\Rightarrow v = \omega\sqrt{r^2 - \frac{R^2}{4}}$$

$$\int_{R/2}^{r}\frac{dr}{\sqrt{r^2 - \frac{R^2}{4}}} = \omega\int_0^t dt$$

$$r = \frac{R}{4}\left(e^{\omega t} + e^{-\omega t}\right)$$