Chapter 2 Nature of Proposition

Lg ic studies the preservatin f truth and prpo itin s are the b arers f truth and falsity.

Identify the following arguments.

EXAMPLE 1	*EXAMPLE 2*
All men are mortal.	All actors are handsome.
All artists are men.	Prasad is an actor.
Therefore, all artists are mortal.	Therefore, ----------

We have seen earlier that one of the functions of logic is to study arguments. However, to study the arguments, it is essential to understand the statements that constitute an argument.

We begin by examining propositions, the building blocks of every argument. An argument consists of premises and conclusion. These **premises** and **conclusion** are in the form of **propositions or statements**. Hence, a proposition is a basic unit of logic.

Find the premises and the conclusion from the following :

EXAMPLE 1

All monuments are beautiful.

Taj Mahal is a monument.

Therefore, Taj Mahal is beautiful.

EXAMPLE 2

All Mangoes are fruits

All fruits grows on trees.

Therefore, All mangoes grow on trees.

2.1 PROPOSITION (STATEMENT) AND SENTENCE

Definition of proposition –

A proposition is defined as a sentence, which is either true or false.

Activity : 1

Make a list of true or false propositions.

From the definition of proposition we can conclude that **all propositions are sentences but all sentences are not propositions**. Only those sentences which are either true or false will be propositions. Hence, the class of proposition is narrow, whereas the class of sentences is wider. This leads to a question that, which sentence can be true or false? To answer this question we shall have to consider various kinds of sentences.

Activity : 2

Make a list of sentences you know and state its kind.

Kinds of Sentences :

(1) **Interrogative Sentence :** This kind of sentence contains a question.

Example : What is your name?

(2) Exclamatory Sentence : It is a kind of sentence which expresses some kind of feelings.

Example : Oh! God

(3) Imperative Sentence : This sentence expresses a command or an order.

Example : Get Out.

(4) Optative Sentence : This sentence expresses a wish, desires, urges.

Example : May God bless us all.

(5) Assertive Sentence : It is a sentence which asserts something about an individual. This sentences can make positive or negative assertion. (It refers to identifiable particular individual possessing definite, particular property.)

The word "Individual" stands not only for persons but for anything like city, country, animal, or anything to which attributes can be significantly predicated and the property/ attribute may be an adjuctive, noun, or even a verb.

Example : Sanika visits her grandmother during the holidays. (Positive assertion)

Example : The tiger is not a domestic animal. (Negative assertion)

These kind of sentences can be either true or false. Hence, they are considered as statements or propositions in logic. They are also called as declarative sentences. They are informative sentences because they provide us with information. So declarative sentences can make logical propositions. So, we can conclude that all sentences are not propositions. Only those sentences which can be either true or false can be propositions.

Sometimes declarative sentence may be in the form of a question or an exclamation e g. (1) Do you feel you can fool your friend.

(2) Thief ..... Thief

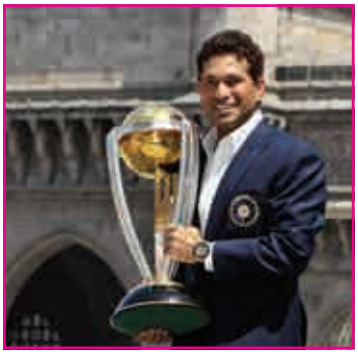
Grammatically given examples are Inerrogative and exclamatory sentences respectively but logically they are propositions .

Activity : 3

Make a list of Assertive / Declarative / Informative sentences.

PICTURE: 1.

PICTURE: 2



Activity : 4

Observe and describe these pictures and make a list of assertive propositions.

(positive assertion and negative assertion)

A proposition is expressed in the form of a sentence. But it is not the same as sentence. The same proposition may be expressed by different sentences.

Example : (1) This is a fish (English)

(2) Das ist ein fisch (German)

(3) यह मछली है। (Hindi)

(4) हा मासा आहे. (Marathi)

(5) kore wa sakana desu. (Japanese)

Here a sentence in English, Marathi, Hindi, German, Japanese may differ as sentence but they express the same proposition.

Anything that is known through sense organs has physical existence. A proposition refers to the meaning or content expressed in the form of a sentence. Therefore, it does not have a physical existence. It is expressed through the medium of a sentence.

On the other hand a sentence has a physical existence. A sentence when spoken, is in the form of sound waves. When written, it is a sign or a symbol on a surface. e.g. In five different sentences given above. The meaning expressed in these sentences is the proposition which does not have a physical existence because one cannot see it, touch it but one can understand it if and only if the language in which it is expressed is known.

The following are the **main characteristics of proposition :**

(1) Every proposition has a truth value :

The truth or falsity of proposition are called truth values. The truth value of a true proposition is true and that of a false proposition is false.

Now the question arises, "what determines the truth value of a proposition?

The answer is "The Fact".

If a proposition represents a fact or facts, it is true. It means a proposition is true when the assertion in a proposition {that which is said in a proposition} agrees with the facts.

Example : Butter melts in heat.

If a proposition does not represents a fact and if the claim is not justified then, the proposition is false.

Example : Mumbai is capital of India. (truth value of this proposition is false)

(2) A proposition has only one truth value.

A proposition cannot be true and false together.

E. g. Chalk is white. (This proposition cannot be both true and false.)

(3) The truth value of a proposition is definite :

A proposition has unique truth value. If a proposition is true, it is always true. If it is false, it is always false. In other words truth value of a proposition does not change.

Example : The earth is a flat disc.

Though, the truth value of the above proposition appears to have changed but in reality it not so. This proposition was believed to be true due to ignorance (lack of scientific knowledge) but it is proved to be false today.

Thus, all propositions are sentences but all sentences are not propositions. Only those sentences which are be either true or false are propositions.

Activity : 5 Look at the pictures carefully and construct the propositions describing the pictures.



There are important differences between the proposition and sentence. Yet they are interconnected.

Proposition (Statement)	Sentence
(1) It is sentence which is either true or false.	(1) It is a meaningful group of words in a grammatical order.
(2) A proposition is conveyed through a sentence.	(2) A sentence is a vehical through which a statement is expressed.
(3) Only declarative sentences are proposition.	(3) The sentences which expresses feeling, wish etc are sentences only.
(4) Every proposition has a truth value i.e it is either true or false.	(4) Sentence does not have a truth value. It is neither true nor false.
(5) A proposition does not have physical existence.	(5) A sentence has a physical existence.
(6) ***Example :*** Taj Mahal is white.	(6) ***Example :*** How are you?

2.2. Classification of proposition :

Classification of proposition can be done on the basics of whether the statement contains another statement as it's component propositions. Some propositions do not contain another proposition as a component, while others do. The former are called simple proposition and the later are compound propositions.

Simple Proposition :

It is a basic unit in logic. **Simple proposition is defined as a proposition that does not contain any other proposition / propositions as it's component.**

Example :

(1) Delhi is the capital of India.

(2) Peacock generally live in jungle.

(3) Polygon has six sides.

(4) Turmeric reduces my arthritis pain.

(5) Anil is eligible to drive.

(6) Mumbai is the capital of England

Activity : 6

Make a list of simple propositions.

Compound proposition :

Example :

(1) (Delhi is the capital of India) **and** (it is a crowded city).

(2) (Peacock generally lives in jungle) **or** (bushes.)

(3) **If** (polygon has six sides) **then** (it is a hexagon.)

(4) **If** (turmeric reduces my arthritis pain) **then** (I will eat turmeric everyday).

(5) (Anil is eligible to drive) **if and only** if (he is eighteen years old).

(6) **It is false** that (Mumbai is the capital of England).

Proposition that comes as a part of a proposition is called as **a component proposition.** The propositions in a compound statements are called its components.

Activity : 7

Identify Component proposition from the above example.

Thus, **a compound proposition is defined as a proposition which contains another proposition / propositions as its component.**

Activity : 8

Construct compound propositions using statements constructed by you in **Activity 6**.

Kinds of simple proposition :

There are four kinds of simple proposition. These are :

(1) Subjectless proposition :

The simplest kind of proposition is the subjectless proposition.

Example :

(1) Bomb!

(2) Fire!

Subjectless propositions make an assertion. They give information. Therefore they are propositions. However the subject of the assertion is not clear. They are primitive propositions.

(2) Subject – Predicate proposition :

A subject – predicate proposition states that an individual possesses a quality or attribute. A subject predicate proposition is that which has a subject , a predicate and a verb. An individual is a singular term. Therefore, **the subject of this kind of proposition is a singular term**.

Example : Ashok is intelligent.

(3) Relational Proposition :

A relational proposition states a relation between two subjects. The subjects between which a relation is stated are called **terms of relation.**

Example : Ram is taller than Shyam.

The above proposition expresses a relation between two subjects namely Ram and Shyam.

(4) Class membership proposition :

A class membership proposition asserts that an individual is a member of a class. Thus, it shows that the subject term belongs to the class indicated by predicate. So, here **predicate term is general.**

Example :

(1) Rani Lakshmi bai was a great warrior.

(2) Bhagat Singh was a freedom fighter.

Kinds of compound proposition :

Compound proposition are further classified into two kinds –

(1) Truth -f unctional compound proposition

(2) Non Truth – functional compound proposition.

(1) Truth functional compound proposition:

In a compound proposition there are two or more component propositions that are connected by some expression like ‘and’, ‘or’ etc. These component propositions are either true or false. The component proposition as a whole also has some truth value.

Example : Sameer is intelligent and Sameer is smart.

In this proposition there are two propositions

(1) Sameer is intelligent.

(2) Sameer is smart.

Now when there are two component propositions, we get four possibilities as given below :

Sameer is intelligent	**And**	He is smart
TRUE	TRUE	TRUE
TRUE	FALSE	FALSE
FALSE	FALSE	TRUE
FALSE	FALSE	FALSE

The truth value of compound propositions (which is stated in the middle column) changes as per the truth value of its component proposition.

In the above example when both the components are true one can say that the compound proposition is true. Otherwise under other possibilities it is false.

Thus the **truth functional compound proposition is defined as a compound proposition whose truth value is determined by the truth value of its component proposition / propositions.**

(2) Non – truth functional compound proposition :

There are some compound propositions whose truth value is not determined by the truth value of its component proposition / propositions.

Such compound propositions are called **Non – truth functional compound proposition**

Example : I believe that Soul exist.

Here the **component proposition** "Soul exist" may be either true or false.

Whatever may be the truth value of the **component proposition**, the truth value of the compound proposition does not get affected.

If the proposition, 'I believe that Soul exit" is say true, then whether the component proposition "Soul exist" is true or false. The truth value of the compound proposition will remain true.

Hence, It is a Non Truth functional compound proposition.

Thus **Non – truth functional compound proposition is defined as a compound proposition whose truth value is not determined by the truth value of its component proposition / propositions.**

Types of truth functional compound proposition :

On the basis of the connectives which combine the components in truth functional propositions, we get five types of truth functional compound proposition.

(1) **Negative** proposition

Example : This book is **not** interesting.

(2) **Conjunctive** proposition

Example : This book is interesting **and** informative.

(3) **Disjunctive** proposition

Example : **Either** this book is interesting **or** informative.

(4) **Material Implicative** or conditional proposition –

Example : **if** this book is interesting **then** people will buy the book.

(5) **Material Equivalent** or Bi – conditional proposition –

Example : People will buy this book **if and only if** it is interesting.

2.3 Symbolization of proposition :

Need, uses and importance of symbolization.

Symbolization is necessary because arguments are expressed in language. The use of symbols is not misleading but it helps us to reason correctly.

There are certain defects of natural language as follows.

(1) use of ambiguous words and vague words.

(2) use of misleading idioms.

(3) confusing metaphorical style.

The symbolic language is free from the above mentioned defects.

Logic is concerned with arguments. Arguments contain propositions or statements as their premises and conclusion. Arguments may be valid or invalid. To determine the validity of the arguments we have to use certain logical procedures. These procedures can not be applied directly to the propositions with ordinary language. Logicians have developed specialized techniques to bring out the form of the proposition. It is done by symbolizing propositions.

Deductive Logic is concerned with the form of an argument and not with the content of argument. It is form of a proposition. This can be done by symbolization.

Use of symbols is convenient and advantageous, for better understanding of arguments and drawing of inference from it.

Significance of symbolization in Logic –

(1) It helps to focus on what is important in an argument and to ignore unnecessary details, thus helps to decide it's validity easily.

(2) It helps to understand the logical structure of propositions and arguments more clearly.

(3) It prevents confusion of vague and ambiguous words.

Symbols are kind of short – forms. In a natural language a proposition or an inference has a much longer expression. When we use symbols the expression becomes much more shorter.

For symbolizing of truth functional compound propositions. We need certain symbols. They are –

(1) Propositional Constant

(2) Propositional Variable

(3) Propositional Connective or Operator

(4) Brackets

(1) Propositional Constant :

Propositional constant is defined as a symbol, which stands for a specific (or particular) proposition as a whole. They are called constants because they have definite meaning. The capital letters from **A to Z** (English alphabet) are used as propositional constants. We are free to use any propositional constants for symbolizing of a proposition.

Example : Yogasanas act as bridges to unite the body with the mind.

The above proposition can be symbolized as **'A '** or by any other capital letter which will stand for the whole proposition.

When an argument contains more number of propositions as components we have to observe following **conditions or restrictions.**

(1) The same propositional constant is to be used for symbolizing a proposition if it occurs again in the same argument (or in the same compound propositions)

(2) The same propositional constant can not be used for different propositions in the same argument. (or in the same compound proposition)

Example : Santosh will take salad or sandwich.

Santosh will not take salad.

Therefore, Santosh will take sandwich.

In the **above example** for the proposition "Santosh will take salad." we will choose the propositional constant "S" and for the proposition "Santosh will take sandwich" we cannot use the same propositional constant "S". (as per restriction no.2) so we will have to use different propositional constant, like "D"

Example :

The **first proposition (premise)** is

Santosh will take salad or sandwich

The **symbolization** of this proposition will be

S or D

The **second proposition (premise)** is

Santosh will not take salad.

The **symbolization** of this proposition will be

Not **S**

The **third proposition (conclusion)** is

Santosh will take sandwich

The **symbolization** of this proposition will be

Therefore **D**

Thus the argument may now will be symbolized thus:

S or D

Not S

Therefore D

(2) Propositional Variable :

Propositional variable is defined as a symbol which stand for any proposition whatsoever. Small latter p, q, r, s …….. (English alphabet) are used as propositional variable. Propositional variable does not stand for any specific proposition. It only marks or indicates the place of proposition.

Fo Example : The expression "if p then q" indicates that "p" stands for any proposition and "q" stands for any other proposition and these two different propositions are connected by the expression "if……then".

A propositional variable is a symbol used to substitute a proposition.

When an argument form contains more number of propositions as components we have to observe following **conditions or restrictions.**

(1) The same propositional variable is to be substituted by the same proposition if it occurs again in the same argument (or in the same compound proposition)

(2) The same propositional variable can not be substituted by different propositions in the same argument. (or in the same compound proposition)

In an argument of the following form, for instance by substituting any proposition for "p" and any other proposition for "q" we will get innumerable arguments.

Example : if p then q

Not q

Therefore Not p

Example No : 1

If a figure is a square then it has four sides.

The figure does not have four sides.

Therefore the figure is not a square.

Example No : 2

If you have a password then you can log on to the network.

You can not log on to the network.

Therefore, you do not have a password.

We can substitute any proposition for a propositional variable, it is therefore said to be a **place marker / place holder** or **dummy letter.**

Activity 9. Read the following arguments forms carefully and construct arguments form it.

(1) Either p or q	(2) If p then q	(3) If p then q
Not p	p	If q then r
Therefore q	Therefore q	Therefore If p then r

Propositional Connective	symbol	Name of the symbol
(1) Not	~	Tilde / Curl
(2) And	•	Dot
(3) Either ……. Or	∨	Wedge
(4) If …….. then	⊃	Horse - shoe
(5) If and only if then	≡	Tripple Bar

(3) Propositional connective – (Truth – Functional logical operator) –

Propositional connective is defined as an expression which operates on proposition or propositions or they connects two propositions in a truth functional compound proposition. There are five expressions which connect component or components in a truth functional compound proposition. The name of the symbols for five connectives, are given below. These symbols are also called **logical constants / operators.**

The propositional connective "not" operates on one proposition only.

Therefore it is known as **Monadic operator**.

On the other hand, the last four connectives or operators namely [and, either…….or, if….. then……, if and only if…… then…..] connects two propositions. Therefore, they are known as **Binary or Dyadic operators.**

Importance of Bracket (s) in symbolization –

In language punctuation is requires to make complicated statements clear.

Punctuation are a mark such as full stop, comma or question mark, exclamation mark, semicolon, inverted comma etc. which are used in writing to separate sentences and their elements and to clarify their meaning.

Example : (from English Language)

Why we need commas because
"I like cooked vegetables, fruits and dogs."
is not same as
"I like cooked vegetables fruits
and dogs."

In mathematics, to avoid ambiguity and to make meaning clear, punctuation marks appear in the form of brackets.

Example : $6 + 7 \times 8$

It could be $6 + (7 \times 8)$ or $(6 + 7) \times 8$

So, in Logic some punctuation marks are equally essential, to clear the complicated propositions. In symbolic Logic parentheses, brackets and braces are used as a punctuation marks.

(1) Parentheses : It is a symbol '( )' that is put around a word or a phrase or a sentence.

Example : $(p \bullet q) \supset r$

(2) Box Brackets : It is used to enclose words or figures. In logic it is used to group expressions that include parentheses.

Example : $[(p \bullet q) \vee (q \bullet p)] \equiv r$

(3) Braces : It is used to group expressions that include box brackets. Example: { }

Example : $\sim\{[(p \bullet q) \vee (q \bullet p)] \equiv p\}$

Truth functional compound propositions

On the basis of five propositional connectives, there are five types of Truth functional compound propositions. They are as follows –

(1) Negative proposition

(2) Conjunctive proposition

(3) Disjunctive proposition

(4) Material Implicative or Conditional proposition

(5) Material Equivalent or Biconditional proposition

(1) Negative proposition :

When any proposition is negated or denied we get negative proposition. Negation is commonly expressed in English language by the word **"Not"**. But a proposition can be negated with the help of words like **it is not the case that, it is not true that, it is false that, none, never.**

Example :

(1) Sadanand is **not** a mathematician.

(2) **It is false** that Ajit is taller than Rajesh.

(3) **It is not true** that Urmila is a magician.

(4) **It is not the case that** Ajay is a singer.

In Logic we use symbols for propositional connectives as well as propositions. For the connective "Negation" or the word "not" the symbol " ~ " is used.

This symbol is called as **"Tilde" or "Curl"**

Using the symbol "~" for negation and the propositional variable "p" for any proposition whatsoever, we get the form of negative propositions as follows :

$\sim p$

Symbolization :

Example : Sadanand is not a mathematician.

Step 1: The above example consists of **one proposition** and one propositional operator.

Underline the proposition and put a propositional operator in the **box**.

So we will get following expression:

Example :

Sadanand is **not** a mathematician.

$\sim S$

Thus the form of negative proposition is '~ p'. This is read as 'Not p'.

Always Remember :

~ Sign to be written before the letter or on the left hand side of the letter.

$\sim P$ ✓ $P \sim$ ×

Truth value for negation –

Negation is also known as contradictory function.

A negative statement is true when its component proposition is false and vice versa.

Basic truth table for negation :

~	P
F	T
T	F

(2) Conjunctive Proposition – (conjunction)

When two propositions are joined together by truth – functional connective **"and"** it is called a conjunctive proposition.

The components of conjunctive proposition are called as **Conjuncts**.

The word "and" is called dyadic connective or binary operator, as it connects two propositions.

Example : Be good and you will be happy.

The above example consists of two propositions –

(1) Be good

(2) You will be happy.

These are connected by the word "and".

Often we use word such as **but, though, although, while, yet, also, still, nevertheless, however, moreover, further, as well as, neither……. nor, in the conjunctive sense.**

Example :

(1) The lion is called king of the forest **and** it has a majestic appearance.

(2) I want to go to the party, **but** I am tired.

(3) Gauri is playing, **while** Varsha is studying.

(4) The couch was shouting, **yet** the players remained noisy.

(5) Hemangi kept working even **though** she was tired.

(6) It's a small house **still** it is spacious.

(7) Chocolates are **neither** nutritious **nor** good for teeth.

(8) Mr. Patil is a politician **and** Sai baba is a saint.

Symbolic form of conjunctive proposition will be as follows :

Example : Be good and you will be happy.

To symbolize a propositional operator "And" we can use symbol (•)

Symbolic from of conjunctive proposition is as follows:

'p • q'

Example : Sugandha is a mother and a grandmother.

Above proposition consists of two parts (components)

(1) Sugandha is a mother.

(2) Sugandha is a grandmother.

These two parts or components of a conjunctive proposition are called as **Conjuncts** in the language of Logic.

Sugandha is a mother | and | a grandmother.

(First conjunct) | | **(Second conjunct)**

Thus symbolization of above proposition is M • G

Thus the form of conjunctive proposition is 'p • q'. It is read as ' p and q'.

Truth Value :

A conjunctive proposition is a kind of truth functional compound proposition. Hence, the truth value of a conjunctive proposition depends on its components i.e. conjuncts.

A conjunctive proposition is true only when both the conjuncts are true otherwise it is false.

Basic truth table for conjunction :

p	**•**	**q**
T	**T**	**T**
T	F	F
F	F	T
F	F	F

(3) Disjunctive proposition – (Disjunction)

When two propositions are joined together by truth functional connective …. **'either ……. …… or'**, it is called a disjunctive proposition. The word "either …… Or" is called dyadic connective or binary operator, which connects two statements. The components of disjunctive proposition are called as **"Disjuncts"**.

Example :

(1) **Either** I will go to Prague **or** Vienna.

(2) **Either** she is weak **or** coward.

(3) The car is **either** blue **or** red.

Example :

Either he is rich or poor.

In the above example there are two propositions.

(1) he is rich

(2) he is poor

These two propositions are connected by truth – functional connective or logical connective" Either……..Or".

Form of the disjunctive proposition is "p ∨ q". This is read as "p or q".

Symbolization :

Either he is rich	or	he is poor.
Proposition	Logical connective or operator	Proposition
(First disjunct)		(Second disjunct)
R	∨	P

Therefore, symbolization of the above proposition will be:

R ∨ P

Form of disjunctive proposition is 'p ∨ q'. This is read as 'p or q'.

Truth Value :

A disjunctive proposition is a kind of truth functional compound proposition. Hence, the truth value of a disjunctive proposition depends on its components i.e. disjuncts.

A disjunctive proposition is false, only when both the disjuncts are false otherwise it is true.

Basic truth table for disjunction :

p	∨	q
T	T	T
T	T	F
F	T	T
F	**F**	**F**

Disjunctive proposition may be used in the inclusive (weak) sense or exclusive (strong) sense.

(1) The Inclusive or weak sense of "OR" –

When both the disjuncts can be true, the word or is said to be used in inclusive sense.

Rajvi is either a mother or an actress.

In the above proposition there are two disjuncts.

(1) Rajvi is a mother.

(2) Rajvi is an actress.

Both these disjuncts can be true together because a person can be both a mother and an actress.

In other words the statement can be interpreted as "either p or q or both". Means "p" alone can be true, "q" alone can be true and both can be true together but cannot be false.

(2) Exclusive or strong sense of OR :

When both the disjuncts cannot be true together, the word "Or" is said to be used in exclusive sense.

Example : Either it is a sparrow or a crow.

In the above proposition there are two disjuncts.

(1) A bird is a sparrow.

(2) A bird is a crow.

Both these disjuncts cannot be true together. If one is true, other is necessarily (exclusively) false.

In other words this can be interpreted as, either "p" is true or "q" is true but both cannot be true together. i.e. if a bird is a sparrow then it cannot be a crow or vice versa.

In logic, disjunctive proposition is used in the inclusive sense only.

(4) Material Implicative or Conditional proposition –

When two propositions are joined together by truth functional connective **if …… then …….** it is called an implicative proposition.

Example :

(1) **If** you want a good pet **then** you should get a dog.

(2) **If** my car is out of fuel **then** it will not run.

(3) **If** a figure is a rhombus **then** it is not a rectangle.

4] **If** you do all the exercises in the book, you will get full marks in the exam.

5] **If** it is a molecule **then** it is made up of atoms.

(sometimes ' , ' (coma) is used instead of word "then")

Words indicating implicative proposition – The expression like "if ……then", **"in case", "unless" (if not)** indicate that the proposition is a conditional proposition.

If it rains then the trains will run late.

Proposition 1 Proposition 2

If …….. then ……. is logical operator.

The symbolic expression :

$$R \supset T$$

Thus, the form of the implicative proposition is " p ⊃ q. This is read as "if p then q" or "p implies q".

An **implicative proposition** is also called as **conditional proposition** because they state the condition and its consequences.

The proposition that **states the condition** is called as **antecedent** and the proposition that **states result** is called as **consequent**.

Example :

If she is tall then she can become a model.

Condition Result

antecedent consequent

Truth value :

An implicative proposition is false only when its antecedent is true and its consequent is false. Otherwise it is always true.

Basic truth table for material implication :

P	⊃	q
T	T	T
T	**F**	**F**
F	T	T
F	T	F

(5) Material Equivalent or Biconditional proposition. –

A biconditional proposition is a compound proposition in which two component propositions materially imply each other.

When two propositions are joined together by truth – functional connective **if and only if …. then…**, it is called as an material equivalent proposition.

Example :

(1) You can take a flight **if and only if** you buy a ticket.

(2) Two angles are congruent **if and only if** their measurements are equal.

(3) You can enter the theatre **if and only if** you have the entry pass.

(4) **If and only if** you study hard, you will pass.

Always remember

‘ ,’ (coma) is used to make the statement meaningful.

In a conditional statement antecedent implies consequent but consequent does not imply the antecedent. In biconditional statement, however, the first component implies the second and the second component also implies the first.

The expression “if and only if ” indicates that the statements is a biconditional statement.

Example :

Birds fly — if and only if — sky is clear.

Proposition 1 — logical connective — proposition 2

The symbolic expression :

B ≡ S

OR

S ≡ B

Thus, the form of the biconditional statement is **“ p ≡ q”**. This is read as “if and only if p then q “or” “p is materially equivalent to q”.

Truth value :

A biconditional proposition is true if and only if both the components have the same truth value. i.e. either both the components are true or both the components are false. Otherwise the statement is false.

Basic true table for material equivalance :

p	≡	q
T	**T**	**T**
T	F	F
F	F	T
F	**T**	**F**

Activity : 10

(1) I will go to a mall.

(2) I will go to a movie.

(3) I will go to gym

Use the above propositions and construct 5 types of truth functional propositions.

2.4 Symbolising compound proposition :

(1) Roses are red and Jasmines are white.

Proposition 1	logical connective	proposition 2
R	•	J

So, we will answer this with the help of two steps.

Symbolization : R • J

Kind of proposition : Conjunctive proposition.

(2) He is poor but not hardworking.

Proposition 1	logical connective	proposition 2
P	• ~	H

Symbolization : P • ~ H

Kind of proposition : Conjunctive proposition

(3) Mira is not both a good singer and a good actress.

Symbolization : ~ (S • A)

Kind of proposition : Negative proposition.

(4) **If** the road is wet, **then either** it has rained today **or** the fire truck spilled water on the road.

Symbolization : W ⊃ (R ∨ F)

Kind of proposition : Implicative or conditional proposition.

(5) He goes to play a match if and only if it does not rain.

Symbolization : ~ R ≡ M

Kinds of proposition : Equivqlent or Bi – conditional proposition.

(6) It is false that if and only if I will go to Australia, I will earn money.

Symbolization : ~ (A ≡ M)

Kind of proposition : Negative proposition.

(7) Either Sun is a star or not a star.

Symbolization : S ∨ ~ S

Kind of proposition : Disjunctive proposition

(8) Neither it is hot nor cold today.

Symbolization : ~ H • ~ C

Kind of proposition : Conjuctive proposition

(9) If fast food is not healthy then one must not eat it.

Symbolization : ~ H ⊃ ~ E

Kind of proposition : Implicative or Conditional proposition.

(10) A living being is either mortal or immortal.

Symbolization : M ∨ I

Kind of proposition : Disjunctive proposition.

Monadic operator	Dyadic operator
(1) It operates on one proposition.	(1) It operates on or connects two propositions.
(2) ~ is a monadic operator.	(2) •, ⊃, ∨, ≡ are dyadic or binary operator.

Always remember

All dyadic connectives are always placed in between the two component proposition.

p • q ✓	• p q ✗
p ∨ q ✓	∨ p q ✗
p ⊃ q ✓	⊃ p q ✗
p ≡ q ✓	≡ p q ✗

Summary

Proposition : A proposition is a sentences which is either true or false. Most logicians use the words, "proposition" and " statements" in the same sense. If a proposition represents facts, it is true, Otherwise, it is false.

Proposition and a sentence : A proposition is expressed in the form of a sentence. However, proposition differs from a sentence.

In /modern propositional logic, proposition are classified into –

(1) Simple Proposition :

(a) Subject less proposition

(b) Subject –P redicate Proposition

(c) Relational Proposition –

(d) Class – membership proposition

(2) Compound Proposition –

(a) Truth –f unctional compound proposition.

(b) Non –t ruth functional compound proposition.

Classification of Truth functional compound proposition –

(1) Negative proposition

(2) Conjunctive proposition

(3) Disjunctive proposition

(4) Material Implication or conditional proposition

(5) Material Equivalent or Biconditional proposition

Modern Logicians use constants, variables, logical operators and brackets for symbolizing propositions.

Exercises

Q. 1. Fill in the blanks with suitable words from those given in the brackets:

1. …… is a basic unit of Logic. *(Sentence / Proposition)*
2. A proposition is conveyed through a ……. *(Statement / Sentence)*
3. If a proposition represents a fact, it is said to be …… *(False / True)*
4. Only …… sentences are proposition. *(Declarative / Exclamatory)*
5. …… proposition does not contain any other proposition as its component. *(Simple / Compound)*
6. A, B, C, D are …….. *(Propositional Constant / Propositional Variable)*
7. ' • ' is a …… connective. *(Binary / Monadic)*
8. In logic disjunctive proposition is used in the …… sense only. *(Exclusive / Inclusive)*
9. An implicative proposition is false when its ….. is true and ….. is false. *(Consequent / Antecedent)*
10. The symbol used for a biconditional proposition is ….. . *(≡ / ∨)*

Q. 2. State whether the following statements are true or false.

1. The premise and conclusion are known as prpopsitions.
2. Every sentence asserts a proposition.
3. A proposition is false, if it stands for actual state of affairs.
4. When we negate simple proposition, we get a compound proposition.
5. A conjunctive proposition is false, when any one conjunct is false.
6. A variable is not proposition but is a "place holder" for any proposition.
7. The symbol ⊃ is a logical operator.
8. A proposition is neither true nor false.
9. In class – membership proposition, predicated is general.
10. The components of disjunctive proposition are called as disjuncts.

Q. 3. Match the columns :

	Group (A)		Group (B)
1.	Sentence	a.	v
2.	Dyadic Connective	b.	Negation
3.	Strong Disjunction	c.	Conjunctive proposition
4.	~ (p ∨ q)	d.	Either he is tall or short.
5.	And, yet, still etc.	e.	physical existence

Q. 4. Give logical terms for the following :

1. It is a meaningful group of words in a grammatical order.
2. A proposition asserts that an individual is a member of a class.
3. It is a symbol which stands for any proposition whatsoever.
4. The components of disjunctive proposition.
5. Truth or falsity of proposition.

Q. 5. Give reasons for the following :

1. '~' is called a monadic operator.
2. When we negate simple proposition we get compound proposition.
3. Equivalent proposition is also called as Bi-conditional proposition.
4. "Suresh is either a doctor or a teacher' is an inclusive sense of disjunction.
5. When we use symbols, the expression becomes much more shorter.

Q. 6. Explain the following :

1. Basic unit of logic
2. Conjunctive proposition
3. Logical operator.
4. Truth – functional compound proposition

Q. 7. Answer the following questions.

1. Explain the difference between proposition and sentence.
2. All propositions are sentences but all sentences are not proposition. Explain.
3. What are the restrictions on a propositional constants? Explain with example.
4. When a conjunctive proposition is true and false?
5. What is the difference between conditional proposition and Bi-conditional proposition.

Q. 8. Symbolize the following propositions using appropriate symbols given in the bracket and identify their kind:

1. He is creative and hardworking. (C, H)
2. If a student completes academic course then he will be graduated. (A, G)
3. It is false that parking is prohibited in this area. (P)
4. If and only if Viraj scores double century, we will win the match. (V, M)
5. This tour is not both safe and exciting. (S, E)
6. It is not the case that, the professor will take a leave if and only if the administration allow him. (P, A)
7. Pizza and Burger is a perfect combination (P, B)
8. She is neither well behaved nor humble. (W, H)
9. I will buy this dress if and only if it is not expensive. (D, E)
10. Puranpoli is delicious, but it is not good for diabetic patient. (P, D)
11. Either Danashree is a talented musician or he is not. (M)
12. If Ramesh was warm and caring person then I am an alien from outer space. (W, C, A)
13. B.E.S.T. is the heart of Mumbai city. (M)
14. If "Ted Talks' are informative and inspirational then people will follow it. (I, N, P)
15. She is simple yet presentable. (S, P)
16. If the road is wet then either it rains today or the water tanker spill water on the road. (R, T, W)
17. You are not allowed to take leave without permission. (L)
18. It is not the case that Bhalchandra is a superstar and not a superstar. (S)
19. Either cat fur or dog fur was found at the scene of the crime. (C, D)
20. Siddharth Mukherjee is a cancer physician and winner of the 2011 Pulitzer Prize. (P, W)
21. It is false that if Ranjit is a good singer then he will be a great musician. (G, M)
22. If company does not increase the salary of the workers then the union will go on strike. (S. U)
23. The Young inventor Richard Turere invented "lion lights" an elegant way to protect his family's cattle from lion attacks. (E)
24. Himalaya is snowy and majestic. (S. M)
25. If Mom's brinjal plants are ruined then an elephant was walking in her garden. (B, E)
26. Sujata will eat the fruit if and only if it is mango. (F, M)
27. If sharks are disturbed then they become aggressive. (D, A)
28. It is true that poverty is a worst enemy of man. (P)

29. Either no students are interested in giving feedback or it is not the case that administration requires the students' feedback. (F, A)

30. If hydrochloric acid (HCI) and sodium hydroxide (NaOH) are combined, then table salt (NaCI) will be produced. (H, S, T)

31. Success does not mean a lot of money or gaining lot of fame. (M,G)

32. If the pavement is **not** wet, then it did **not** rain. (W, R)

33. Cats are good pets and they are affectionate. (P, A)

34. Omkar ran fast but he missed the train. (F, T)

35. If Seema is in Denmark, then she is in Europe and If Seema is not in Europe, then she is not in Denmark. (S. E)

36. **Memory Banda** is Malawian children's rights activist who has drawn international attention for her work in opposition to child marriage.

37. If a triangle is equilateral then its angles all measure 60 and if all the angles of a triangle measure 60 then the triangle is equilateral. (T, A)

38. If I pass then I will have a party and if I fail then also I will have a party. (P, T, F)

39. Swayam talks is not just another talk series but its presentation make it a unique concept. (T, U)

40. Either Leena will learn music or dance. (M, D)

Activity 11 : Complete the following table

Sr. No.	Kinds of Proposition	Propositional connective	symbol
1.			' • '
2.	Conjunctive Proposition		
3.		Either ….. Or …..	
4.	Negative Proposition		
5.			≡

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