Sequence & Series

1. **Sequence :** A sequence is a function whose domain is the set N of natural numbers.
   A sequence whose range is a subset of R is called a real sequence.
2. **Series**
   By adding or subtracting the terms of a sequence, we get an expression which is called a series.
3. **An arithmetic progression (A.P.) :** a, a + d, a + 2d,....... a + (n – 1) d is an A.P.
   Let a be the first term and d be the common difference of an A.P., then n^{th} term = $t_n = a + (n-1)d$

   The sum of first n terms of are A.P.

   $$S_n = \frac{n}{2}\left[2a + (n-1)d\right] = \frac{n}{2}\left[a + \ell\right]$$

   r^{th} term of an A.P. when sum of first r terms is given is $t_r = S_r - S_{r-1}$.

   Properties of A.P.

   (i) If a, b, c are in A.P ⇒ 2b = a + c & if a, b, c, d are in A.P. ⇒ a + d = b + c.

   (ii) Three numbers in A.P. can be taken as a – d, a, a + d; four numbers in A.P. can be taken as a –
   3d, a – d, a + d, a + 3d; five numbers in A.P. are a – 2d, a – d, a, a + d, a + 2d & six terms in A.P.
   are a – 5d, a – 3d, a – d, a + d, a + 3d, a + 5d etc.

   (iii) sum of the terms of an A.P. equidistant from the beginning & end = sum of first & last term.
4. **Arithmetic Mean (Mean or Average) (A.M.):**
   If three terms are in A.P. then the middle term is called the A.M. between the other two, so if a, b, c
   are in A.P., b is A.M. of a & c.

   n – Arithmetic Means Between Two Numbers:

   If a, b are any two given numbers & a, $A_1, A_2, \ldots, A_n$, b are in A.P. then $A_1, A_2, \ldots A_n$ are the n A.M.'s between a & b.

   $$A_1 = a + \frac{b-a}{n+1}, A_2 = a + \frac{2(b-a)}{n+1}, \ldots\ldots, A_n = a + \frac{n(b-a)}{n+1}$$

   $\sum_{r=1}^{n} A_r = nA$ where A is the single A.M. between a & b.
5. **Geometric Progression:** a, ar, ar^2, ar^3, ar^4,...... is a G.P. with a as the first term & r as common ratio.

   (i) n^{th} term = $a\, r^{n-1}$ (ii) Sum of the first n terms i.e.

   $$S_n = \begin{cases} \dfrac{a(r^n - 1)}{r-1} & , \quad r \neq 1 \\ na & , \quad r = 1 \end{cases}$$

(iii) Sum of an infinite G.P. when $|r|<1$ is given by $S_\infty = \frac{a}{1-r}\left(|r|<1\right)$.

6. **Properties of G.P.**

(i) If a, b, c are in G.P. $\Rightarrow$ b^2 = ac, if $a_1, a_2, a_3, a_4, ...a_{n-1}, a_n$ are in G.P, then $a_1 a_n = a_1 a_{n-1} = a_3 a_{n-2} .....$

(ii) Any three consecutive terms of a G.P can be taken as $\frac{a}{r}, a, ar$.

(iii) Any four consecutive terms of a G.P. can be taken as $\frac{a}{r^3}, \frac{a}{r}, ar, ar^3$

(iv) If each term of a G.P. be multiplied or divided or raised to power by the some non-zero quantity, the resulting sequence is also a G.P..

(v) If $a_1, a_2, a_3, ...$ and $b_1, b_2, b_3, ...$ are two G.P's with common ratio r_1 and r_2 respectively then the
sequence $a_1b_1, a_2b_2, a_3b_3, ...$ is also a G.P. with common ratio r_1, r_2.

(vi) If $a_1, a_2, a_3, ...$ is a G.P. of positive term, then $\log a_1, \log a_2, \log a_3, ...$ are in A.P. and conversely.

7. **Geometric Means (Mean Proportional) (G.M.):**

If a, b, c > 0 are in G.P., b is the G.M. between a & c, then b^2 = ac

n-Geometric Means Between positive number a, b: If a, b are two given numbers & a, $G_1, G_2, ..., G_n$, b are in G.P.. Then $G_1, G_2, G_3, ...G_n$ are n G.M.s between a & b.
$G_1 = a(b/a)^{1/n+1}$, $G_2 = a(b/a)^{2/n+1}$,......, $G_n - a(b/a)^{n/n+1}$

8. **Harmonic Mean (H.M.):**

If a, b, c are in H.P., b is the H.M. between a & c, then b = $\frac{2ac}{a+c}$

H.M. H of $a_1, a_2, ...a_n$ is given by $\frac{1}{H} = \frac{1}{n}\left[\frac{1}{a_1} + \frac{1}{a_2} + ... + \frac{1}{a_n}\right]$

9. **Relation between means :**

G^2 = AH, A.M. $\geq$ G.M. $\geq$ H.M. and A.M. = G.M. = H.M. if $a_1 = a_2 = a_3 = ... = a_n$

10. **Arithmetico-Geometric Series:** a, (a + d) r, (a + 2d) r^2,..... , [a + (n – 1)d] r^{n-1} are in A.G.P.

$S_n = \frac{a}{1-r} + \frac{dr\left(1-r^{n-1}\right)}{\left(1-r\right)^2} - \frac{\left[a+\left(n-1\right)d\right]r^n}{1-r}, r \neq 1$. **Sum To Infinity :** If $|r|<1$, then

$$S_\infty = \frac{a}{1-r} + \frac{dr}{\left(1-r\right)^2}$$

Important Results

(i) $\sum_{r=1}^{n}\left(a_r \pm b_r\right) = \sum_{r=1}^{n} a_r \pm \sum_{r=1}^{n} b_r$, (ii) $\sum_{r=1}^{n} k\, a_r = k\sum_{r=1}^{n} a_r$ (iii) $\sum_{r=1}^{n} k = nk$; where k is a constant.

(iv) $\sum_{r=1}^{n} r = 1+2+3+....+n = \frac{n(n+1)}{2}$ (v)

$$\sum_{r=1}^{n} r^2 = 1^2+2^2+3^2+....+n^2 = \frac{n(n+1)(2n+1)}{6}$$

(vi) $\sum_{r=1}^{n} r^3 = 1^3+2^3+3^3+....+n^3 = \frac{n^2(n+1)^2}{4}$

(vii) $2\sum_{i<j=1}^{n} a_i a_j = (a_1+a_2+...+a_n)^2 - (a_1^{\,2}+a_2^{\,2}+....+a_n^{\,2})$

Binomial Theorem

1.Statement of Binomial theorem : If a, b ∈ R and n ∈ N, then

$$(a+b)^n = {}^nC_0\, a^n b^0 + {}^nC_1\, a^{n-1}b^1 + {}^nC_2\, a^{n-2}b^2 + ... + {}^nC_r\, a^{n-r}b^r + ... + ... + {}^nC_n a^0 b^n = \sum_{r=0}^{n} {}^nC_r a^{n-r} b^r$$

$$(1+x)^n = {}^nC_0 + {}^nC_1 x + {}^nC_2 x^2 + ... + {}^nC_r x^r + ... + {}^nC_n x^n$$

2.Properties of Binomial Theorem :

(i) The number of terms in the expansion is n + 1.

(ii) The sum of the indices of a and b in each term is n.

(iii) The binomial coefficients $({}^nC_0, {}^nC_1 ..... {}^nC_n)$ of the terms equidistant from the beginning and the end are equal, i.e. ${}^nC_0 = {}^nC_n, {}^nC_1 = {}^nC_{n-1}$ etc. $\{\because {}^nC_{n-1} = {}^nC_{n-r}\}$

(iv) **General term :** $T_{r+1} = {}^nC_r a^{n-r} b^r$

(v) **Middle term (s) :**

(a) If n is even, there is only one middle term, which is $\left(\frac{n+2}{2}\right)$th term.

(b) If n is odd, there are two middle terms, which are $\left(\frac{n+1}{2}\right)$th and $\left(\frac{n+1}{2}+1\right)$th terms.

(vi) **Numerically greatest term in the expansion of (x + y)^n, n ∈ N**

Let $r = \left[\frac{(n+1)|y|}{|x|+|y|}\right]$. If $\frac{(n+1)|y|}{|x|+|y|}$ is not an integer, then t_{r+1} is numerically greatest.

If $\frac{(n+1)|y|}{|x|+|y|}$ is an integer, then $t_r = t_{r+1}$ and both are numerically greatest.

3.Multinomial Theorem : $\left(x_1+x_2+x_3+.....x_k\right)^n=\sum_{r_1+r_2+...+r_k=n}\frac{n!}{r_1!r_2!...r_k!}x_1^{r_1}.x_2^{r_2}..x_k^{r_k}$

Here total number of terms in the expansion = ${}^{n+k-1}C_{k-1}$

4.Application of Binomial Theorem :

If $\left(\sqrt{A}+B\right)^n=I+f$ **where I and n are positive integers, n being odd and 0 < f < 1 then (I + f)**

f = k^n where A – B^2 = k > 0 and $\sqrt{A}-B$ **< 1.**

If n is an even integer, then $\left(I+f\right)\left(1-f\right)=k^n$

5.Properties of Binomial Coefficients :

(i) ${}^nC_0+{}^nC_1+{}^nC_2+......+{}^nC_n=2^n$

(ii) ${}^nC_0-{}^nC_1+{}^nC_2-{}^nC_3+......+\left(-1\right)^n\,{}^nC_n=0$

(iii) ${}^nC_0+{}^nC_2+{}^nC_4+....={}^nC_1+{}^nC_3+{}^nC_5+....=2^{n-1}$

(iv) ${}^nC_r+{}^nC_{r-1}={}^{n+1}C_r$ (v) $\frac{{}^nC_r}{{}^nC_{r-1}}=\frac{n-r+1}{r}$

6.Binomial Theorem For Negative Integer Or Fractional Indices

$$\left(1+x\right)^n=1+nx+\frac{n\left(n-1\right)}{2!}x^2+\frac{n\left(n-1\right)\left(n-2\right)}{3!}x^3+....+\frac{n\left(n-1\right)\left(n-2\right).....\left(n-r+1\right)}{r!}x^r+.....,\left|x\right|<1$$

.

$$T_{r+1}=\frac{n\left(n-1\right)\left(n-2\right).......\left(n-r+1\right)}{r!}x^r$$

Expansions to be remembered $\left(\left|x\right|<1\right)$

(a) $\left(1+x\right)^{-1}=1-x+x^2-x^3+....+\left(-1\right)^r x^r+....$

(b) $\left(1-x\right)^{-1}=1+x+x^2+x^5+....+x^r+....$

(c) $\left(1+x\right)^{-2}=1-2x+3x^2-4x^3+....+\left(-1\right)^r\left(r+1\right)x^r+....$

(d) $\left(1-x\right)^{-2}=1+2x+3x^2+4x^3+....+\left(r+1\right)x^r+....$

Practice Questions

1. Let A be the sum of the first 20 terms and B be the sum of the first 40 terms of the series $1^2 + 2.2^2 + 3^2 + 2.4^2 + 5^2 + 2.6^2 + \ldots$ If $B - 2A = 100\lambda$, then λ is equal to: (2018)

(a) 496
(b) 232
(c) 248
(d) 464

2. Let $a_1, a_2.a_3, \ldots, a_{49}$ be in A.P. such that $\sum_{k=0}^{12} a_{4k+1} = 416$ and $a_9, a_{43} = 66$. If $a_1^2 + a_2^2 + \ldots + a_{17}^2 = 140m$ then m is equal to: (2018)

(a) 33
(b) 66
(c) 68
(d) 34

3. If $x_1, x_2, x_3, \ldots . x_n$ and $\frac{1}{h_1}, \frac{1}{h_2}, \ldots ., \frac{1}{h_n}$ are two A.P.s such that $x_3 = h_2 = 8$ and $x_8 = h_7 = 20$, then $x_5 . h_{10}$ equals: (2018)

(a) 2650
(b) 2560
(c) 3200
(d) 1600

4. If b is the first term of an infinite G.P. whose sum is five, then b lies in the interval: (2018)

(a) (0, 10)
(b) [10, ∞]
(c) (-10, 0)
(d) (-∞, -10]

5. If the sum of first $2n$ terms of the AP series 2, 5, 8, …, is equal to the sum of the first n terms of the AP series 57, 59, 61, …, then n equals (2001)

(a) 10
(b) 12
(c) 11
(d) 13

6.If the 2^{nd}, 5^{th} and 9^{th} terms of a non-constant AP are in GP, then the common ratio of this GP is (2016)

(a) $\frac{8}{5}$

(b) $\frac{4}{3}$

(c) 1

(d) $\frac{7}{4}$

7.Let $f(x) = ax^2 + bx + c$, $a \neq 0$ and $\Delta = b^2 - 4ac$. If $\alpha + \beta$, $\alpha^2 + \beta^2$ and $\alpha^3 + \beta^3$ are in GP, then (2005)

(a) $\Delta \neq 0$

(b) $b\Delta = 0$

(c) $c\Delta = 0$

(d) $bc \neq 0$

8.Let a, b, c be in an AP and a^2, b^2, c^2 be in G.P. If $a < b < c$ and $a + b + c = \frac{3}{2}$, then the value of a is (2002)

(a) $\frac{1}{2\sqrt{2}}$

(b) $\frac{1}{2\sqrt{3}}$

(c) $\frac{1}{2} - \frac{1}{\sqrt{3}}$

(d) $\frac{1}{2} - \frac{1}{\sqrt{2}}$

9.Let α, β be the roots of $x^2 - x + p = 0$ and γ, δ be the roots of $x^2 - 4x + q = 0$. If $\alpha, \beta, \gamma, \delta$ are in GP, then the integer values of p and q respectively are (2001)

(a) -2, -32

(b) -2, 3

(c) -6, 3

(d) -6, -32

10. If a, b, c, d and p are distinct real numbers such that $(a^2 + b^2 + c^2) p^2 - 2 (ab + bc + cd) p + (b^2 + c^2 + d^2) \leq 0$, then a, b, c, d (1987)

(a) are in AP

(b) are in GP

(c) are in HP

(d) satisfy ab = cd

11. If a, b, c are in GP, then the equations $ax^2 + 2bx + c = 0$ and $dx^2 + 2ex + f = 0$ have a common root, if $\frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in (1985)

(a) AP

(b) GP

(c) HP

(d) None of these

12. The third term of a geometric progression is 4. The product of the first five terms is (1982)

(a) 4^3

(b) 4^5

(c) 4^4

(d) None of these

13. Three positive numbers form an increasing GP. If the middle term in this GP is doubled, then new numbers are in AP. Then, the common ratio of the GP is (2014)

(a) $\sqrt{2}+\sqrt{3}$

(b) $3+\sqrt{2}$

(c) $2-\sqrt{3}$

(d) $2+\sqrt{3}$

14. If $(10)^9 + 2(11)^1(10)^8 + 3(11)^2(10)^7 + \ldots + 10(11)^9 = k(10)^9$, then k is equal to (2014)

(a) $\frac{121}{10}$

(b) $\frac{441}{100}$

(c) 100

(d) 110

15. The sum of first 20 terms of the sequence 0.7, 0.77, 0.777, … , is (2013)

(a) $\frac{7}{81}\left(179-10^{-20}\right)$

(b) $\frac{7}{9}\left(99-10^{-20}\right)$

(c) $\frac{7}{81}\left(179+10^{-20}\right)$

(d) $\frac{7}{9}\left(99+10^{-20}\right)$

16. An infinite GP has first term x and sum 5, then x belongs to (2004)

(a) $x < -10$

(b) $-10 < x < 0$

(c) $0 < x < 10$

(d) $x > 10$

17. Consider an infinite geometric series with first term a and common ratio r. If its sum is 4 and the second term is ¾, then (2000)

(a) a = 4/7, r = 3/7

(b) a = 2, r = 3/8

(c) a = 3/2, r = 1/2

(d) a = 3, r = 1/4

18. Sum of the first n terms of the series $\frac{1}{2}+\frac{3}{4}+\frac{7}{8}+\frac{15}{16}+...$ is equal to (1988)

(a) $2^n - n - 1$

(b) $1 - 2^{-n}$

(c) $n + 2^{-n} - 1$

(d) $2^n + 1$

19. If $a_1, a_2, a_3, ...$ are in a harmonic progression with a_1 = 5 and a_{20} = 25. Then, the least positive integer n for which $a_n < 0$, is (2012)

(a) 22

(b) 23

(c) 24

(d) 25

20. If the positive numbers *a, b, c, d* are in AP. Then, *abc*, *abd*, *acd*, *bcd* are (2001)
(a) not in AP/GP/HP
(b) in AP
(c) in GP
(d) in HP

21. Let $a_1, a_2, ..., a_{10}$ be in AP and $h_1, h_2,$ equal to $..., h_{10}$ be in HP. If $a_1 = h_1 = 2$ and $a_{10} = h_{10} = 3$, then $a_4 h_7$ is (1999)
(a) 2
(b) 3
(c) 5
(d) 6

22. If $x > 1$, $y > 1$, $z > 1$ are in GP, then $\frac{1}{1+\ln x}, \frac{1}{1+\ln y}, \frac{1}{1+\ln z}$ are in (1998)
(a) AP
(b) HP
(c) GP
(d) None of these

23. If the sum of the first ten terms of the series $\left(1\frac{3}{5}\right)^2 + \left(2\frac{2}{5}\right)^2 + \left(3\frac{1}{5}\right)^2 + 4^2 + \left(4\frac{4}{5}\right)^2 + ...,$ is $\frac{16}{5}m$, then m is equal to (2016)
(a) 102
(b) 101
(c) 100
(d) 99

24. If m is the AM of two distinct real numbers l and n (l, $n > 1$) and G_1, G_2 and G_3 are three geometric means between l and n, then $G_1^4 + 2G_2^4 + G_3^4$ equals (2015)
(a) $4l^2mn$
(b) $4lm^2n$
(c) lmn^2
(d) $l^2m^2n^2$

25. The sum of first 9 terms of the series $\frac{1^3}{1}+\frac{1^3+2^3}{1+3}+\frac{1^3+2^3+3^3}{1+3+5}+...$ is (2015)

(a) 71
(b) 96
(c) 142
(d) 192

26. If $\alpha \in \left(0, \frac{\pi}{2}\right)$, then $\sqrt{x^2+x}+\frac{\tan^2\alpha}{\sqrt{x^2+x}}$ is always greater than or equal to (2003)

(a) 2 tan α
(b) 1
(c) 2
(d) sec^2 α

27. If $a_1, a_2, ..., a_n$ are positive real numbers whose product is a fixed number *c*, then the minimum value of $a_1+a_2+...+a_{n-1}+2a_n$ is (2002)

(a) $n(2c)^{1/n}$
(b) $(n+1)c^{1/n}$
(c) $2nc^{1/n}$
(d) $(n+1)(2c)^{1/n}$

28. If *a*, *b*, *c* are positive real numbers such that $a + b + c + d = 2$, then M = (a + b) (c + d) satisfies the relation (2000)

(a) $0 < M \leq 1$
(b) $1 \leq M \leq 2$
(c) $2 \leq M \leq 3$
(d) $3 \leq M \leq 4$

29. The harmonic mean of the roots of the equation $\left(5+\sqrt{2}\right)x^2-\left(4+\sqrt{5}\right)x+8+2\sqrt{5}=0$ is (1999)

(a) 2
(b) 4
(c) 6
(d) 8

30. The product of n positive numbers is unity, then their sum is (1991)

(a) a positive integer

(b) divisible by n

(c) equal to $n+\frac{1}{n}$

(d) never less than n

31. If $\sum_{i=1}^{9}\left(x_i-5\right)=9$ and $\sum_{i=1}^{9}\left(x_i-5\right)^2=45$, then the standard deviation of the 9 items $x_1 x_2$, ….., x_9 is: (2018)

(a) 3

(b) 9

(c) 4

(d) 2

32. If n is the degree of the polynomial, $\left[\frac{2}{\sqrt{5x^3+1}+\sqrt{5x^3-1}}\right]^8+\left[\frac{2}{\sqrt{5x^3+1}+\sqrt{5x^3-1}}\right]$ and m is the coefficient of x^n in it, then the ordered pair (n, m) is equal to: (2018)

(a) $\left(24,(10)^8\right)$

(b) $\left(12,(20)^4\right)$

(c) $\left(8,5(10)^4\right)$

(d) $\left(12,8(10)^4\right)$

33. The coefficient of x^2 in the expansion of the product $\left(2-x^2\right).\left(\left(1+2x+3x^2\right)^6+\left(1-4x^2\right)^6\right)$ is: (2018)

(a) 107

(b) 106

(c) 108

(d) 155

34. If $X=\{4^n-3n-1:n\in N\}$ and $Y=\{9(n-1):n\in N\}$, where N is the set of natural numbers, then $X\cup Y$ is equal to (2014)

(a) N

(b) $Y-X$

(c) X

(d) Y

35. The term independent of x in expansion of $\left(\frac{x+1}{x^{2/3}-x^{1/3}+1}-\frac{x-1}{x-x^{1/2}}\right)^{10}$ is (2013)

(a) 4

(b) 120

(c) 210

(d) 310

36. Coefficient of t^{24} in $(1+t^2)^{12}(1+t^{12})(1+t^{24})$ is (2003)

(a) ${}^{12}C_6+3$

(b) ${}^{12}C_6+1$

(c) ${}^{12}C_6$

(d) ${}^{12}C_6+2$

37. In the binomial expansion of $(a-b)^n, n\geq 5$ the sum of the 5th and 6th terms is zero. Then, $\frac{a}{b}$ equals (2001)

(a) $\frac{n-5}{6}$

(b) $\frac{n-4}{5}$

(c) $\frac{5}{n-4}$

(d) $\frac{6}{n-5}$

38. If in the expansion of $(1+x)^m (1-x)^n$, the coefficients of x and x^2 are 3 and –6 respectively, then m is equal to (1999)

(a) 6
(b) 9
(c) 12
(d) 24

39. The expansion $\left[x+\left(x^3-1\right)^{1/2}\right]^5+\left[x-\left(x^3-1\right)^{1/2}\right]^5$ is a polynomial of degree (1992)

(a) 5
(b) 6
(c) 7
(d) 8

40. The coefficient of x^4 in $\left(\frac{x}{2}-\frac{3}{x^2}\right)^{10}$ is (1983)

(a) $\frac{405}{256}$
(b) $\frac{504}{259}$
(c) $\frac{450}{263}$
(d) None of these

41. Given positive integers $r > 1$, $n > 2$ and the coefficient of $(3r)^{\text{th}}$ and $(r+2)^{\text{th}}$ terms in the binomial expansion of $(1+x)^{2n}$ are equal. Then, (1980)

(a) $n = 2r$
(b) $n = 2r + 1$
(c) $n = 3r$
(d) None of these

42. If the coefficients of x^3 and x^4 in the expansion of $(1 + ax + bx^2)(1 - 2x)^{18}$ in powers of x are both zero, then (a, b) is equal to

(a) $\left(16, \frac{251}{3}\right)$

(b) $\left(14, \frac{251}{3}\right)$

(c) $\left(14, \frac{272}{3}\right)$

(d) $\left(16, \frac{272}{3}\right)$

43. For $r = 0, 1, \dots, 10$, if A_r, B_r and C_r denote respectively the coefficient of x^r in the expansions of $(1 + x)^{10}$, $(1 + x)^{20}$ and $(1 + x)^{30}$. Then, $\sum_{r=1}^{10} A_r \left(B_{10}B_r - C_{10}A_r\right)$ is equal to (2010)

(a) $B_{10}C_{10}$

(b) $A_{10}\left(B_{10}^2 - C_{10}A_{10}\right)$

(c) 0

(d) $C_{10} - B_{10}$

44. If ${}^{n-1}C_r = \left(k^2 - 3\right){}^{n}C_{r+1}$, then k belongs to (2004)

(a) $(-\infty, -2]$

(b) $[2, \infty]$

(c) $\left[-\sqrt{3}, \sqrt{3}\right]$

(d) $\left(\sqrt{3}, 2\right]$

45. The sum $\sum_{i=0}^{m} \binom{10}{i}\binom{20}{m-i}$, where $\binom{p}{q} = 0$ if p > q, is maximum when m is equal to (2002)

(a) 5

(b) 10

(c) 15

(d) 20

46. For $2 \le r \le n, \binom{n}{r} + 2\binom{n}{r-1} + \binom{n}{r-2}$ is equal to (2000)

(a) $\binom{n+1}{r-1}$

(b) $2\binom{n+1}{r-1}$

(c) $2\binom{n+2}{r}$

(d) $\binom{n+2}{r}$

47. If $a_n = \sum_{r=0}^{n} \frac{1}{{}^nC_r}$, then $\sum_{r=0}^{n} \frac{r}{{}^nC_r}$ equals

(a) $(n-1)a_n$

(b) na_n

(c) $\frac{1}{2}na_n$

(d) none of these

48. If C_r stands for ${}^nC_r$, then the sum of the series

$\frac{2\left(\frac{n}{2}\right)!\left(\frac{n}{2}\right)!}{n!}\left[C_0^2 - 2C_1^2 + 3C_2^2 - ... + (-1)^n (n+1) C_n^2\right]$, where *n* is an even positive integer, is

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(a) $(-1)^{n/2}(n+2)$

(b) $(-1)^n (n+1)$

(c) $(-1)^{n/2}(n+1)$

(d) none of these

49. If n ∈ N, then $11^{n+2} + 12^{2n+1}$ is divisible by

(a) 113

(b) 123

(c) 133

(d) none of these

50. The term independent of x in the expansion of $\left(x+\frac{1}{x}+2\right)^{21}$ is

(a) T_{22}
(b) T_{21}
(c) T_{23}
(d) T_{20}

51. If a, b, c represents the sides and A, B, C represent the angels of a Δ ABC, then the value of the expression $\sum_{r=0}^{n} {}^{n}C_{r} a^{r}.b^{n-r}.\cos\left(rB-(n-r)A\right)$ is equal to:

(a) b^2
(b) c^n
(c) a^n
(d) none of these

52. In the expansion of $\left(2^x+\frac{1}{4^x}\right)^n, n \in N$ if sum of the coefficients of 2nd and 3rd term is 36, then which of these are correct?

(a) n = 8
(b) n = 9
(c) $\frac{T_3}{T_2}=\frac{7}{4}\ when\ x=-\frac{1}{3}$
(d) $\frac{T_3}{T_2}=7\ when\ x=\frac{1}{3}$

53. The coefficient of x^{160} in the expansion of $\left(x^8+1\right)^{60}\left(x^{12}+3x^4+\frac{3}{x^4}+\frac{1}{x^{12}}\right)^{-10}$ is

(a) ${}^{30}C_5$
(b) ${}^{30}C_6$
(c) ${}^{30}C_{24}$
(d) ${}^{30}C_{26}$

54. The number of dissimilar terms in the expansion of
$\left[{}^{4}C_0\, a^4b^2+{}^{4}C_1\, a^3b^{3/2}+{}^{4}C_2\, a^2b+{}^{4}C_3\, ab^{1/2}+{}^{4}C_4\right]^{20}$
(a) 61
(b) 81
(c) 41
(d) none of these

55. The number of rational terms in the expansion of $\left(\sqrt{2}+3^{1/3}\right)^{100}$ is

(a) 34
(b) 51
(c) 17
(d) 16

56. The greatest coefficient of x in the expansion of $(3 + 2x)^{50}$ is

(a) $^{50}C_{25}.3^{25}.2^{25}$
(b) $^{50}C_{25}$
(c) $^{50}C_{20}\times 3^{30}\times 2^{20}$
(d) none of these

57. The numerically greatest term in the expansion of $\left(3-4x\right)^{17}$, when $x=\frac{3}{2}$, is

(a) T_{12}
(b) T_{13}
(c) both T_{12} and T_{13}
(d) T_9

58. The remainder when 3^{91} divided by 80 is

(a) 3
(b) 1
(c) 80
(d) 27

59. The remainder when $x=5^{5^{5^5}}$ is divided by 24 is

(a) 1
(b) 3
(c) 5
(d) 23

60. If $\left(1+x\right)^n = C_0 + C_1 x + C_2 x^2 + \ldots + C_n x^n$, then $2C_0 + 2^2.\frac{C_1}{2} + 2^3.\frac{C_2}{3} + \ldots + 2^{n+1}\frac{C_n}{n+1} =$

(a) $\frac{3^{n+1}-1}{n+1}$

(b) $\frac{3^n-1}{n}$

(c) $\frac{3^{n+1}}{n+1}$

(d) $\frac{3^{n+2}-1}{n+2}$

Answer key

61. (c) 62. (d) 63. (b) 64. (a) 65. (c) 66. (b) 67. (c) 68. (d) 69. (a) 70. (b) 71. (a) 72. (b)
73. (d) 74. (c) 75. (c) 76. (c) 77. (d) 78. (c) 79. (d) 80. (d) 81. (d) 82. (b) 83. (b) 84. (b)
85. (b) 86. (a) 87. (a) 88. (a) 89. (b) 90. (d) 91. (d) 92. (b) 93. (b) 94. (d) 95. (c) 96. (d)
97. (b) 98. (c) 99. (c) (a) 101. (a) 102. (d) 103. (d) 104. (d) 105. (c) 106. (d) 107. (c)
108. (a) 109. (c) 110. (a) 111. (b) 112. (b) 113. (a) 114. (b) 115. (c) 116. (c) 117. (c) 118. (d)
119. (c) 120. (a)

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