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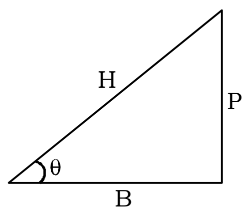
TRIGONOMETRY

TRIGONOMETRIC RATIOS & IDENTITIES

1. The meaning of Trigonometry

Tri	Gon	Metron
↓	↓	↓
3	sides	Measure

Hence, this particular branch in Mathematics was developed in ancient past to measure 3 sides, 3 angles and 6 elements of a triangle. In today's time–trigonometric functions are used in entirely different shapes. The 2 basic functions are sine and cosine of an angle in a right–angled triangle and there are 4 other derived functions.



sin θ	cos θ	tan θ	cot θ	sec θ	cosec θ
$\frac{P}{H}$	$\frac{B}{H}$	$\frac{P}{B}$	$\frac{B}{P}$	$\frac{H}{B}$	$\frac{H}{P}$

2. Basic Trigonometric Identities

(a) $\sin^2\theta + \cos^2\theta = 1 : -1 \le \sin\theta \le 1; -1 \le \cos\theta \le 1 \ \forall\ \theta \in R$

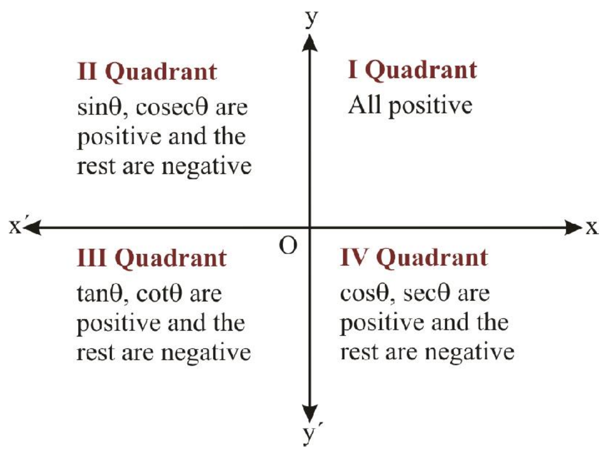
(b) $\sec^2\theta - \tan^2\theta = 1 : |\sec\theta| \ge 1 \ \forall\ \theta \in R$

(c) $\text{cosec}^2\theta - \cot^2\theta = 1 : |\text{cosec}\theta| \ge 1 \ \forall\ \theta \in R$

Trigonometric Ratios of Standard Angles

T–Ratio ↓	**Angle (θ)** 0°	**30°**	**45°**	**60°**	**90°**
sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞
cot	∞	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0
sec	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	∞
cosec	∞	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1

The sign of the trigonometric ratios in different quadrants are as under :



3. Trigonometric Ratios of Allied Angles

Using trigonometric ratio of allied angles, we could find the trigonometric ratios of angles of any magnitude.

sin (–θ) = – sin θ   cos (–θ) = cos θ

$\sin\left(\frac{\pi}{2}-\theta\right)=\cos\theta$ $\cos\left(\frac{\pi}{2}-\theta\right)=\sin\theta$

tan (–θ) = – tan θ   cot (–θ) = –cot θ

$\tan\left(\frac{\pi}{2}-\theta\right)=\cot\theta$ $\cot\left(\frac{\pi}{2}-\theta\right)=\tan\theta$

cosec (–θ) = – cosec θ   sec (–θ) = sec θ

$\sec\left(\frac{\pi}{2}-\theta\right)=\operatorname{cosec}\theta$ $\operatorname{cosec}\left(\frac{\pi}{2}-\theta\right)=\sec\theta$

$\sin\left(\frac{\pi}{2}+\theta\right)=\cos\theta$ $\cos\left(\frac{\pi}{2}+\theta\right)=-\sin\theta$

$\sin(\pi-\theta)=\sin\theta$ $\cos(\pi-\theta)=-\cos\theta$

$\tan\left(\frac{\pi}{2}+\theta\right)=-\cot\theta$ $\cot\left(\frac{\pi}{2}+\theta\right)=-\tan\theta$

$\tan(\pi-\theta)=-\tan\theta$ $\cot(\pi-\theta)=-\cot\theta$

$\sec\left(\frac{\pi}{2}+\theta\right)=-\operatorname{cosec}\theta$ $\operatorname{cosec}\left(\frac{\pi}{2}+\theta\right)=\sec\theta$

$\sec(\pi-\theta)=-\sec\theta$ $\operatorname{cosec}(\pi-\theta)=\operatorname{cosec}\theta$

$\sin(\pi+\theta)=-\sin\theta$ $\cos(\pi+\theta)=-\cos\theta$

$\sin\left(\frac{3\pi}{2}-\theta\right)=-\cos\theta$ $\cos\left(\frac{3\pi}{2}-\theta\right)=-\sin\theta$

$\tan(\pi+\theta)=\tan\theta$ $\cot(\pi+\theta)=\cot\theta$

$\tan\left(\frac{3\pi}{2}-\theta\right)=\cot\theta$ $\cot\left(\frac{3\pi}{2}-\theta\right)=\tan\theta$

$\sec(\pi+\theta)=-\sec\theta$ $\operatorname{cosec}(\pi+\theta)=-\operatorname{cosec}\theta$

$\sec\left(\frac{3\pi}{2}-\theta\right)=-\operatorname{cosec}\theta$ $\operatorname{cosec}\left(\frac{3\pi}{2}-\theta\right)=-\sec\theta$

$\sin\left(\frac{3\pi}{2}+\theta\right)=-\cos\theta$ $\cos\left(\frac{3\pi}{2}+\theta\right)=\sin\theta$

$\sin(2\pi-\theta)=-\sin\theta$ $\cos(2\pi-\theta)=\cos\theta$

$\tan\left(\frac{3\pi}{2}+\theta\right)=-\cot\theta$ $\cot\left(\frac{3\pi}{2}+\theta\right)=-\tan\theta$

$\tan(2\pi-\theta)=-\tan\theta$ $\cot(2\pi-\theta)=-\cot\theta$

$\sec\left(\frac{3\pi}{2}+\theta\right)=\operatorname{cosec}\theta$ $\operatorname{cosec}\left(\frac{3\pi}{2}+\theta\right)=-\sec\theta$

$\sec(2\pi-\theta)=\sec\theta$ $\operatorname{cosec}(2\pi-\theta)=-\operatorname{cosec}\theta$

$\sin(2\pi+\theta)=\sin\theta$ $\cos(2\pi+\theta)=\cos\theta$

$\tan(2\pi+\theta)=\tan\theta$ $\cot(2\pi+\theta)=\cot\theta$

$\sec(2\pi+\theta)=\sec\theta$ $\operatorname{cosec}(2\pi+\theta)=\operatorname{cosec}\theta$

4. Trigonometric Functions of Sum or Difference of Two Angles

(a) $\sin(A+B) = \sin A \cos B + \cos A \sin B$

(b) $\sin(A-B) = \sin A \cos B - \cos A \sin B$

(c) $\cos(A+B) = \cos A \cos B - \sin A \sin B$

(d) $\cos(A-B) = \cos A \cos B + \sin A \sin B$

(e) $\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$

(f) $\tan(A-B) = \dfrac{\tan A - \tan B}{1 + \tan A \tan B}$

(g) $\cot(A+B) = \dfrac{\cot A \cot B - 1}{\cot B + \cot A}$

(f) $\cot(A-B) = \dfrac{\cot A \cot B + 1}{\cot B - \cot A}$

(h) $\sin^2 A - \sin^2 B = \cos^2 B - \cos^2 A = \sin(A+B) \cdot \sin(A-B)$

(i) $\cos^2 A - \sin^2 B = \cos^2 B - \sin^2 A = \cos(A+B) \cdot \cos(A-B)$

(j) $\tan(A+B+C) = \dfrac{\tan A + \tan B + \tan C - \tan A \tan B \tan C}{1 - \tan A \tan B - \tan B \tan C - \tan C \tan A}$

5. Multiple Angles and Half Angles

(a) $\sin 2A = 2 \sin A \cos A$; $\sin\theta = 2 \sin\dfrac{\theta}{2} \cos\dfrac{\theta}{2}$

(b) $\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$;

$2\cos^2\dfrac{\theta}{2} = 1 + \cos\theta$, $2\sin^2\dfrac{\theta}{2} = 1 - \cos\theta$

(c) $\tan 2A = \dfrac{2\tan A}{1 - \tan^2 A}$; $\tan\theta = \dfrac{2\tan\frac{\theta}{2}}{1 - \tan^2\frac{\theta}{2}}$

(d) $\sin 2A = \dfrac{2\tan A}{1 - \tan^2 A}$; $\cos 2A = \dfrac{1 - \tan^2 A}{1 + \tan^2 A}$

(e) $\sin 3A = 3\sin A - 4\sin^3 A$

(f) $\cos 3A = 4\cos^3 A - 3\cos A$

(g) $\tan 3A = \dfrac{3\tan A - \tan^3 A}{1 - 3\tan^2 A}$

6. Transformation of Products into Sum or Difference of Sines & Cosines

(a) $2\sin A \cos B = \sin(A+B) + \sin(A-B)$

(b) $2\cos A \sin B = \sin(A+B) - \sin(A-B)$

(c) $2\cos A \cos B = \cos(A+B) + \cos(A-B)$

(d) $2\sin A \sin B = \cos(A-B) - \cos(A+B)$

7. Factorisation of the Sum or Difference of Two Sines or Cosines

(a) $\sin C + \sin D = 2\sin\frac{C+D}{2}\cos\frac{C-D}{2}$

(b) $\sin C - \sin D = 2\cos\frac{C+D}{2}\sin\frac{C-D}{2}$

(c) $\cos C + \cos D = 2\cos\frac{C+D}{2}\cos\frac{C-D}{2}$

(d) $\cos C - \cos D = -2\sin\frac{C+D}{2}\sin\frac{C-D}{2}$

8. Important Trigonometric Ratios

(a) $\sin n\pi = 0$; $\cos n\pi = (-1)^n$; $\tan n\pi = 0$ where $n \in Z$

(b) $\sin 15^\circ$ or $\sin\frac{\pi}{12} = \frac{\sqrt{3}-1}{2\sqrt{2}} = \cos 75^\circ$ or $\cos\frac{5\pi}{12}$;

$\cos 15^\circ$ or $\cos\frac{\pi}{12} = \frac{\sqrt{3}+1}{2\sqrt{2}} = \sin 75^\circ$ or $\sin\frac{5\pi}{12}$;

$\tan 15^\circ = \frac{\sqrt{3}-1}{\sqrt{3}+1} = 2-\sqrt{3} = \cot 75^\circ$;

$\tan 75^\circ = \frac{\sqrt{3}+1}{\sqrt{3}-1} = 2+\sqrt{3} = \cot 15^\circ$

(c) $\sin\frac{\pi}{10}$ or $\sin 18^\circ = \frac{\sqrt{5}-1}{4}$ &

$\cos 36^\circ$ or $\cos\frac{\pi}{5} = \frac{\sqrt{5}+1}{4}$

9. Conditional Identities

If $A + B + C = \pi$ then :

(i) $\sin 2A + \sin 2B + \sin 2C = 4\sin A\sin B\sin C$

(ii) $\sin A + \sin B + \sin C = 4\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}$

(iii) $\cos 2A + \cos 2B + \cos 2C = -1 - 4\cos A\cos B\cos C$

(iv) $\cos A + \cos B + \cos C = 1 + 4\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$

(v) $\tan A + \tan B + \tan C = \tan A\tan B\tan C$

(vi) $\tan\frac{A}{2}\tan\frac{B}{2} + \tan\frac{B}{2}\tan\frac{C}{2} + \tan\frac{C}{2}\tan\frac{A}{2} = 1$

(vii) $\cot\frac{A}{2} + \cot\frac{B}{2} + \cot\frac{C}{2} = \cot\frac{A}{2}.\cot\frac{B}{2}.\cot\frac{C}{2}$

(viii) $\cot A\cot B + \cot B\cot C + \cot C\cot A = 1$

10. Range of Trigonometric Expression

$E = a\sin\theta + b\cos\theta$

$E = \sqrt{a^2+b^2}\sin(\theta+\alpha), \left(\text{where } \tan\alpha = \frac{b}{a}\right)$

$E = \sqrt{a^2+b^2}\cos(\theta-\beta), \left(\text{where } \tan\beta = \frac{a}{b}\right)$

Hence for any real value of θ, $-\sqrt{a^2+b^2} \le E \le \sqrt{a^2+b^2}$

11. Sine and Cosine Series

(a) $\sin\alpha + \sin(\alpha+\beta) + \sin(\alpha+2\beta) + ... + \sin(\alpha + \overline{n-1}\beta)$

$$= \frac{\sin\frac{n\beta}{2}}{\sin\frac{\beta}{2}}\sin\left(\alpha+\frac{n-1}{2}\beta\right)$$

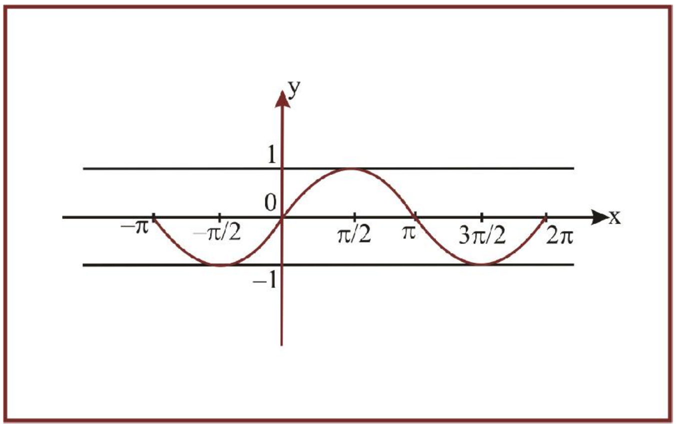
(b) $\cos\alpha + \cos(\alpha+\beta) + \cos(\alpha+2\beta) + ... + \cos(\alpha+\overline{n-1}\beta)$

$$= \frac{\sin\frac{n\beta}{2}}{\sin\frac{\beta}{2}}\cos\left(\alpha+\frac{n-1}{2}\beta\right)$$

12. Graphs of Trigonometric Functions

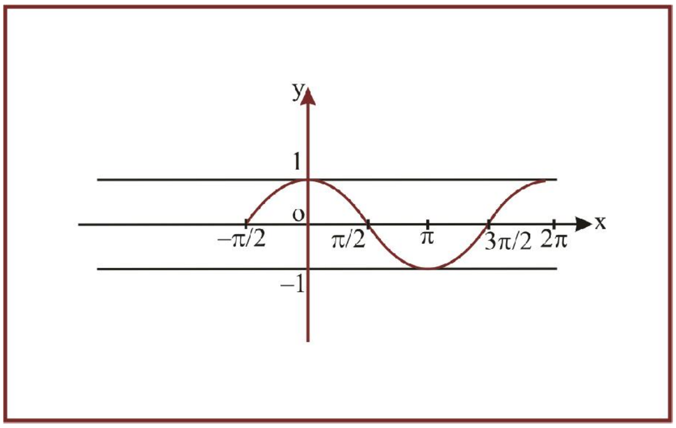
(a) $y = \sin x,$

$x \in R\ ;\ y \in [-1, 1]$



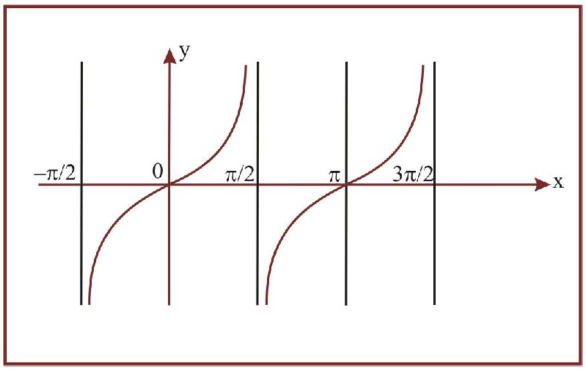
(b) $y = \cos x,$

$x \in R\ ;\ y \in [-1, 1]$



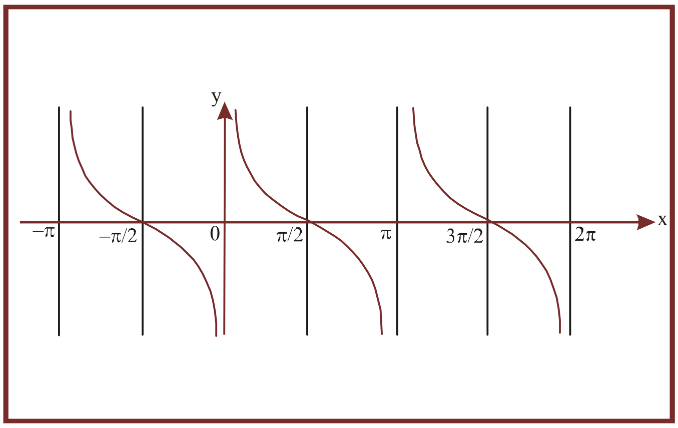
(c) $y = \tan x,$

$$x \in R - \left\{(2n+1)\frac{\pi}{2};\ n \in Z\right\};\ y \in R$$



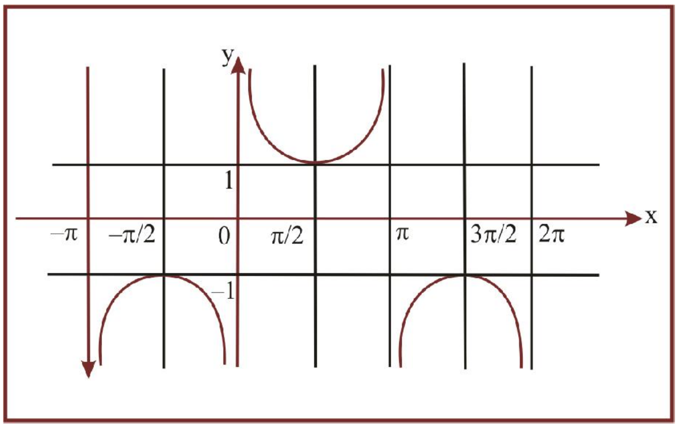
(d) $y = \cot x,$

$x \in R - \{n\pi;\ n \in z\};\ y \in R$



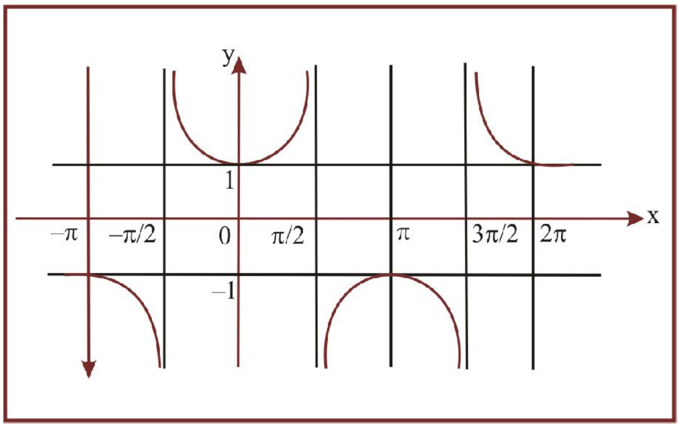
(e) $y = \text{cosec}\ x,$

$x \in R - \{n\pi;\ n \in Z\};\ \ y \in (-\infty, -1] \cup [1, \infty)$



(f) $y = \sec x,$

$$x \in R - \left\{(2n+1)\frac{\pi}{2};\ n \in Z\right\};\ y \in (-\infty, -1] \cup [1, \infty)$$



TRIGONOMETRIC EQUATIONS

13. Trigonometric Equations

The equations involving trigonometric functions of unknown angles are known as Trigonometric equations.

e.g., $\cos\theta = 0$, $\cos^2\theta - 4\cos\theta = 1$.

A solution of a trigonometric equation is the value of the unknown angle that satisfies the equation.

e.g., $\sin\theta = \frac{1}{\sqrt{2}} \Rightarrow \theta = \frac{\pi}{4}$ or $\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{9\pi}{4}, \frac{11\pi}{4}, \ldots$

Thus, the trigonometric equation may have infinite number of solutions and can be classified as :

(i) Principal solution

(ii) General solution

14. General Solution

Since, trigonometric functions are periodic, a solution generalised by means of periodicity of the trigonometrical functions. The solution consisting of all possible solutions of a trigonometric equation is called its general solution.

14.1 Results

1. $\sin\theta = 0 \Leftrightarrow \theta = n\pi$
2. $\cos\theta = 0 \Leftrightarrow \theta\ (2n+1)\frac{\pi}{2}$
3. $\tan\theta = 0 \Leftrightarrow \theta = n\pi$
4. $\sin\theta = \sin\alpha \Leftrightarrow \theta = n\pi + (-1)^n\alpha$, where $\alpha \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
5. $\cos\theta = \cos\alpha \Leftrightarrow \theta = 2n\pi \pm \alpha$, where $\alpha \in [0, \pi]$.
6. $\tan\theta = \tan\alpha \Leftrightarrow \theta = n\pi + \alpha$, where $\alpha \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
7. $\sin^2\theta = \sin^2\alpha \Leftrightarrow \theta = n\pi \pm \alpha$.
8. $\cos^2\theta = \cos^2\alpha \Leftrightarrow \theta = n\pi \pm \alpha$.
9. $\tan^2\theta = \tan^2\alpha \Leftrightarrow \theta = n\pi \pm \alpha$.
10. $\sin\theta = 1 \Leftrightarrow \theta = (4n+1)\frac{\pi}{2}$.
11. $\cos\theta = 1 \Leftrightarrow \theta = 2n\pi$.
12. $\cos\theta = -1 \Leftrightarrow \theta = (2n+1)\pi$.
13. $\sin\theta = \sin\alpha$ and $\cos\theta = \cos\alpha \Leftrightarrow \theta = 2n\pi + \alpha$.

Note...

1. Every where in this chapter 'n' is taken as an integer, if not stated otherwise.
2. The general solution should be given unless the solution is required in a specified interval or range.
3. α is taken as the principal value of the angle. (i.e., Numerically least angle is called the principal value).

SOLVED EXAMPLES

Example – 1

Solve : If sec α and cosec α are the roots of $x^2 - px + q = 0$, then show $p^2 = q(q + 2)$.

Sol. Since, sec α and cosec α are roots of $x^2 - px + q = 0$

$\therefore$ sec α + cosec α = p and sec α. cosec α = q

$\therefore$ sin α + cos α = p sin α . cos α and $\sin\alpha . \cos\alpha = \frac{1}{q}$

$\therefore \quad \sin\alpha + \cos\alpha = \frac{p}{q}.$

Squaring both sides, we get

$$\sin^2\alpha + \cos^2\alpha + 2\sin\alpha.\cos\alpha = \frac{p^2}{q^2}$$

$$1 + 2\sin\alpha.\cos\alpha = \frac{p^2}{q^2}$$

or $\quad 1 + \frac{2}{q} = \frac{p^2}{q^2} \quad \Rightarrow \quad p^2 = q(q + 2).$

Example – 2

Solve : If α, β and γ are in A.P., show that

$$\cot\beta = \frac{\sin\alpha - \sin\gamma}{\cos\gamma - \cos\alpha}.$$

Sol. Since, α, β and γ are in A.P.

$2\beta = \alpha + \gamma$

$$\Rightarrow \cot\beta = \cot\left(\frac{\alpha+\gamma}{2}\right)$$

$$\Rightarrow \quad \cot\beta = \frac{\cos\left(\frac{\alpha+\gamma}{2}\right)}{\sin\left(\frac{\alpha+\gamma}{2}\right)}$$

Multiplying and dividing by $2\sin\left(\frac{\alpha-\gamma}{2}\right)$, we get

$$\cot\beta = \frac{2\cos\left(\frac{\alpha+\gamma}{2}\right).\sin\left(\frac{\alpha-\gamma}{2}\right)}{2\sin\left(\frac{\alpha+\gamma}{2}\right)\sin\left(\frac{\alpha-\gamma}{2}\right)} = \frac{\sin\alpha - \sin\gamma}{\cos\gamma - \cos\alpha}$$

$$\Rightarrow \quad \cot\beta = \frac{\sin\alpha - \sin\gamma}{\cos\gamma - \cos\alpha}.$$

Example – 3

Solve : Prove that

tan A + 2 tan 2A + 4 tan 4A + 8 cot 8A = cot A.

Sol. L.H.S. $= \tan A + 2\tan 2A + 4\tan 4A + 8\left(\frac{1-\tan^2 4A}{2\tan 4A}\right)$

$$= \tan A + 2\tan 2A + \left(\frac{4\tan^2 4A + 4 - 4\tan^2 4A}{\tan 4A}\right)$$

$$= \tan A + 2\tan 2A + 4\cot 4A$$

$$= \tan A + 2\tan 2A + 4\left(\frac{1-\tan^2 2A}{2\tan 2A}\right)$$

$$= \tan A + \left[\frac{2\tan^2 2A + 2 - 2\tan^2 2A}{\tan 2A}\right]$$

$$= \tan A + 2\cot 2A$$

$$= \tan A + 2\left(\frac{1-\tan^2 A}{2\tan A}\right) = \frac{\tan^2 A + 1 - \tan^2 A}{\tan A}$$

$$= \cot A = \text{R.H.S.}$$

Note: Students are adviced to learn above result as formulae.
i.e., tan A + 2 cot 2A = cot A

Example – 4

Solve : Evaluate :

cos 12° cos 24° cos 36°. cos 48°. cos 72°. cos 84°.

Sol. cos 12° cos 24° cos 36° cos 48°. cos 72°. cos 84°.

$\Rightarrow$ cos 12° cos 24° cos 48° . cos (180°–96°).

cos 36° . cos 72°

$\Rightarrow$ – (cos 12° . cos 24° . cos 48°. cos 96°) .

(cos 36° . cos 72°)

$$\Rightarrow -\frac{\sin(2^4.12^\circ)}{2^4.\sin(12^\circ)}\cdot\frac{\sin(2^2.36^\circ)}{2^2.\sin(36^\circ)}$$

using, $\cos A \cos 2A \ldots\ldots \cos 2^{n-1}A = \dfrac{\sin(2^n A)}{2^n \sin A}$

$$\Rightarrow -\frac{\sin(192^\circ)}{16.\sin(12^\circ)}\cdot\frac{\sin(144^\circ)}{4.\sin(36^\circ)}$$

$$\Rightarrow -\frac{\sin(180^\circ+12^\circ).\sin(180^\circ-36^\circ)}{64.\sin 12^\circ.\sin 36^\circ}$$

$$\Rightarrow \frac{\sin 12^\circ.\sin 36^\circ}{64\sin 12^\circ.\sin 36^\circ} \Rightarrow \frac{1}{64}.$$

Example – 5

Solve : Prove that :

tan A + tan (60° + A) – tan (60° – A) = 3 tan 3A

Sol. We have,

LHS = tan A + tan (60° + A) – tan (60° – A)

$$\Rightarrow \text{LHS} = \tan A + \frac{\sqrt{3}+\tan A}{1-\sqrt{3}\tan A} - \frac{\sqrt{3}-\tan A}{1+\sqrt{3}\tan A}$$

$$\Rightarrow \text{LHS} = \tan A + \frac{8\tan A}{1-3\tan^2 A}$$

$$\Rightarrow \text{LHS} = \frac{9\tan A - 3\tan^3 A}{1-3\tan^2 A}$$

$$\Rightarrow \text{LHS} = 3\left(\frac{3\tan A - \tan^3 A}{1-3\tan^2 A}\right) = 3\tan 3A = \text{RHS}$$

Example – 6

Solve : Prove that :

$$\cos^3 A + \cos^3(120^\circ + A) + \cos^3(240^\circ + A) = \frac{3}{4}\cos 3A$$

Sol. We know that

$$\cos 3A = 4\cos^3 A - 3\cos A \Rightarrow \cos^3 A = \frac{1}{4}(\cos 3A + 3\cos A)$$

$$\therefore \text{LHS} = \frac{1}{4}\{\cos 3A + 3\cos A\} + \frac{1}{4}\{\cos(360^\circ + 3A) +$$

$$3\cos(120^\circ + A)\} + \frac{1}{4}\{\cos(720 + 3A) + 3\cos(240^\circ + A)\}$$

$$\Rightarrow \text{LHS}\ \frac{1}{4}\{\cos 3A + 3\cos A\} + \frac{1}{4}$$

$$\{\cos 3A + 3\cos 120^\circ + A)\} + \frac{1}{4}\{\cos 3A + 3\cos(240^\circ + A)\}$$

$$\Rightarrow \text{LHS} = \frac{3}{4}\cos 3A + \frac{3}{4}\{\cos A + \cos(120^\circ + A) +$$

$$\cos(240^\circ + A)\}$$

$$\Rightarrow \text{LHS} = \frac{3}{4}\cos 3A + \frac{3}{4}\{\cos A + 2\cos(180^\circ - A)\cos 60^\circ\}$$

$$\Rightarrow \text{LHS} = \frac{3}{4}\cos 3A + \frac{3}{4}\left\{\cos A - 2\cos A \times \frac{1}{2}\right\} = \frac{3}{4}\cos 3A = \text{RHS}$$

ALITER

We have,

cos A + cos (120° + A) + cos (240° + A)

= cos A + 2 cos (180° + A) cos 120°

$$\left[\because \cos C + \cos D = 2\cos\frac{C+D}{2}\cos\frac{C-D}{2}\right]$$

= cos A – cos A = 0

$\therefore$ $\cos^3 A + \cos^3(120^\circ + A) + \cos^3(240^\circ + A)$

= 3 cos A cos (120° + A) cos (240° + A)

$[\because a + b + c = 0 \Rightarrow a^3 + b^3 + c^3 = 3abc]$

= 3 cos A cos (180° – 60° + A) cos (180° + 60° + A)

= 3 cos A cos {180° – (60° – A)} cos {180° + (60° + A)}

= 3 cos A cos (60° – A) cos (60° + A)

$$= 3 \times \frac{1}{4}\cos 3A = \frac{3}{4}\cos 3A$$

Example – 7

Solve : Prove that : $\sin 3A \sin^3 A + \cos 3A \cos^3 A = \cos^3 2A$

Sol. We have,

$$\cos^3 A = \frac{\cos 3A + 3\cos A}{4}$$

$\therefore$ LHS = $\sin 3A \sin^3 A + \cos 3A \cos^3 A$

$$\Rightarrow \text{LHS} = \sin 3A \left\{\frac{3\sin A - \sin 3A}{4}\right\} + \cos 3A \left\{\frac{\cos 3A + 3\cos A}{4}\right\}$$

$$\Rightarrow \text{LHS} = \frac{1}{4}\{3(\cos A \cos 3A + \sin A \sin 3A) + (\cos^2 3A - \sin^2 3A)\}$$

$$\Rightarrow \text{LHS} = \frac{1}{4}\{3\cos(3A - A) + \cos 2(3A)\}$$

$$\Rightarrow \text{LHS} = \frac{1}{4}\{3\cos 2A + \cos 3(2A)\}$$

$$\Rightarrow \text{LHS} = \frac{1}{4}\{3\cos 2A + (4\cos^3 2A - 3\cos 2A)\}$$

$$= \cos^3 2A = \text{RHS}$$

Example – 8

Solve : Prove that : $\tan 6° \tan 42° \tan 66° \tan 78° = 1$

Sol. We have,

$$\text{LHS} = \frac{\sin 6° \sin 42° \sin 66° \sin 78°}{\cos 6° \cos 42° \cos 66° \cos 78°}$$

$$\Rightarrow \text{LHS} = \frac{(2\sin 66° \sin 6°)(2\sin 78° \sin 42°)}{(2\cos 66° \cos 6°)(2\cos 78° \cos 42°)}$$

$$\Rightarrow \text{LHS} = \frac{(\cos 60° - \cos 72°)(\cos 36° - \cos 120°)}{(\cos 60° + \cos 72°)(\cos 36° + \cos 120°)}$$

$$\Rightarrow \text{LHS} = \frac{(\cos 60° - \sin 18°)(\cos 36° + \sin 30°)}{(\cos 60° + \sin 18°)(\cos 36° - \sin 30°)}$$

$$\Rightarrow \text{LHS} = \frac{\left(\frac{1}{2} - \frac{\sqrt{5}-1}{4}\right)\left(\frac{\sqrt{5}+1}{4} + \frac{1}{2}\right)}{\left(\frac{1}{2} + \frac{\sqrt{5}-1}{4}\right)\left(\frac{\sqrt{5}-1}{4} - \frac{1}{2}\right)}$$

$$= \frac{(3-\sqrt{5})(3+\sqrt{5})}{(\sqrt{5}+1)(\sqrt{5}-1)} = \frac{9-5}{5-1} = 1 = \text{RHS}$$

Example – 9

Solve : Prove that : $4\sin 27° = \sqrt{(5+\sqrt{5})} - \sqrt{(3-\sqrt{5})}$

Sol. We have,

$$16\sin^2 27° = 8(1 - \cos 54°)$$

$$\Rightarrow 16\sin^2 27° = 8(1 - \sin 36°)$$

$$\Rightarrow 16\sin^2 27° = 8\left\{1 - \frac{\sqrt{10-2\sqrt{5}}}{4}\right\}$$

$$\Rightarrow 16\sin^2 27° = 2\left\{4 - \sqrt{10-2\sqrt{5}}\right\}$$

$$\Rightarrow 16\sin^2 27° = 8 - 2\sqrt{10-2\sqrt{5}}$$

$$\Rightarrow 16\sin^2 27° = (5+\sqrt{5}) + (3-\sqrt{5}) - 2\sqrt{(5+\sqrt{5})(3-\sqrt{5})}$$

$$\Rightarrow 16\sin^2 27° = \left\{\sqrt{5+\sqrt{5}}\right\}^2 + \left\{\sqrt{3-\sqrt{5}}\right\}^2 - 2\sqrt{(5+\sqrt{5})(3-\sqrt{5})}$$

$$\Rightarrow 16\sin^2 27° = \left\{\sqrt{5+\sqrt{5}} - \sqrt{3-\sqrt{5}}\right\}^2$$

$$\sin 27° = \sqrt{5+\sqrt{5}} - \sqrt{3-\sqrt{5}}$$

Example – 10

Solve : Prove that $\frac{\tan 3x}{\tan x}$ never lies between $\frac{1}{3}$ and 3.

Sol. Let $y = \frac{\tan 3x}{\tan x}$. Then,

$$y = \frac{3\tan x - \tan^3 x}{\tan x(1 - 3\tan^2 x)}$$

$\Rightarrow$ $y = \frac{\tan^2 x}{1 - 3\tan^2 x}$

$\Rightarrow$ $(3y - 1)\tan^2 x = y - 3$

$\Rightarrow$ $\tan^2 x = \frac{y - 3}{3y - 1}$

Now,

$\tan^2 x \geq 0$ for all x

$\therefore$ $\frac{y-3}{3y-1} \geq 0$

$\Rightarrow$ $\frac{(y-3)(3y-1)}{(3y-1)^2} \geq 0$

$[\because (3y-1)^2 \geq 0]$

$\Rightarrow$ $(y - 3)(3y - 1) \geq 0$

$\Rightarrow$ $y \leq \frac{1}{3}$ or, $y \geq 3$

$\Rightarrow$ y does not lie between 1/3 and 3.

Example – 11

Solve : If $A + B + C = \pi$, then prove the following

(i) $\sin 2A + \sin 2B + \sin 2C = 4 \sin A \cdot \sin B \cdot \sin C$

(ii) $\sin^2 A + \sin^2 B + \sin^2 C = 2 + 2 \cos A \cdot \cos B \cdot \cos C$

(iii) $\cos^2\left(\frac{A}{2}\right) + \cos^2\left(\frac{B}{2}\right) + \cos^2\left(\frac{C}{2}\right)$

$= 2 + 2\sin\left(\frac{A}{2}\right)\sin\left(\frac{B}{2}\right)\sin\left(\frac{C}{2}\right)$

(iv) $\cos\left(\frac{A}{2}\right) + \cos\left(\frac{B}{2}\right) + \cos\left(\frac{C}{2}\right)$

$= 4\cos\left(\frac{\pi - A}{4}\right)\cos\left(\frac{\pi - B}{4}\right)\cos\left(\frac{\pi - C}{4}\right)$

(v) $\frac{\cos A - \cos B + \cos C + 1}{\cos A + \cos B + \cos C - 1} = \cot\left(\frac{A}{2}\right)\cos\left(\frac{C}{2}\right)$

Sol. L.H.S.

$= \sin 2A + \sin 2B + \sin 2C$

$= 2\sin\left(\frac{2A + 2B}{2}\right)\cos\left(\frac{2A - 2B}{2}\right) + 2 \sin C \cos C$

$= 2 \sin (A + B) \cdot \cos (A - B) + 2 \sin C [-\cos (A + B)]$

$= 2 \sin C \cdot \cos (A - B) - 2 \sin C \cdot \cos (A + B)$

$= 2 \sin C [\cos (A - B) - \cos (A + B)]$

$= 2 \sin C \times 2 \sin A \sin B$

$= 4 \sin A \sin B \sin C.$

= R.H.S.

(ii) L.H.S.

$= \sin^2 A + \sin^2 B + \sin^2 C$

$= \sin^2 A + \frac{1 - \cos 2B}{2} + \frac{1 - \cos 2C}{2}$

$= \frac{1}{2}[2 + 2\sin^2 A - (\cos 2B + \cos 2C)]$

$= \frac{1}{2}\ [2 + 2\sin^2 A - 2\cos\left(\frac{2B+2C}{2}\right)$

$\cos\left(\frac{2B-2C}{2}\right)]$

$= 1 + \sin^2 A - \cos(B + C) . \cos(B - C)$

$= 2 - \cos^2 A + \cos A . \cos(B - C)$

$= 2 + \cos A\ [-\cos A + \cos(B - C)]$

$= 2 + \cos A\ [\cos(B + C) + \cos(B - C)]$

$= 2 + \cos A \times 2\cos B . \cos C$

$= 2 + 2\cos A . \cos B . \cos C$

(iii) L.H.S.

$$= \cos^2\left(\frac{A}{2}\right) + \cos^2\left(\frac{B}{2}\right) + \cos^2\left(\frac{C}{2}\right)$$

$$= \cos^2\left(\frac{A}{2}\right) + \frac{1+\cos B}{2} + \frac{1+\cos C}{2}$$

$$= 1 + \cos^2\left(\frac{A}{2}\right) + \frac{1}{2}(\cos B + \cos C)$$

$$= 1 + \cos^2\left(\frac{A}{2}\right) + \frac{1}{2} 2\cos\left(\frac{B+C}{2}\right).\cos\left(\frac{B-C}{2}\right)$$

$$= 1 + 1 - \sin^2\left(\frac{A}{2}\right) + \sin\left(\frac{A}{2}\right).\cos\left(\frac{B-C}{2}\right)$$

$$= 2 - \sin\left(\frac{A}{2}\right)\left[\sin\left(\frac{A}{2}\right) - \cos\left(\frac{B-C}{2}\right)\right]$$

$$2 - \sin\left(\frac{A}{2}\right)\left[\cos\left(\frac{B+C}{2}\right) - \cos\left(\frac{B-C}{2}\right)\right]$$

$$= 2 - \sin\left(\frac{A}{2}\right) \times 2\sin\left(\frac{B}{2}\right).\sin\left(-\frac{C}{2}\right)$$

$$= 2 + 2\sin\left(\frac{A}{2}\right).\sin\left(\frac{B}{2}\right).\sin\left(\frac{C}{2}\right)$$

= R.H.S.

(iv) L.H.S.

$$= \cos\left(\frac{A}{2}\right) + \cos\left(\frac{B}{2}\right) + \cos\left(\frac{C}{2}\right)$$

$$= \sin\left(\frac{\pi - A}{2}\right) + 2\cos\left(\frac{\frac{B}{2}+\frac{C}{2}}{2}\right)\cos\left(\frac{\frac{B}{2}-\frac{C}{2}}{2}\right)$$

$$= 2\sin\left(\frac{\pi-A}{4}\right)\cos\left(\frac{\pi-A}{4}\right) + 2\cos\left(\frac{B+C}{4}\right)\cos\left(\frac{B-C}{4}\right)$$

$$= 2\sin\left(\frac{\pi-A}{4}\right)\cos\left(\frac{\pi-A}{4}\right) + 2\cos\left(\frac{\pi-A}{4}\right)\cos\left(\frac{B-C}{4}\right)$$

$$= 2\cos\left(\frac{\pi-A}{4}\right)\left[\sin\left(\frac{\pi-A}{4}\right) + \cos\left(\frac{B-C}{4}\right)\right]$$

$$= 2\cos\left(\frac{\pi-A}{4}\right)\left[\sin\left(\frac{B+C}{4}\right) + \cos\left(\frac{B-C}{4}\right)\right]$$

$$= 2\cos\left(\frac{\pi-A}{4}\right)\left[\cos\left[\left(\frac{\pi-B}{4}\right)+\left(\frac{\pi-C}{4}\right)\right] + \cos\left[\left(\frac{\pi-C}{4}\right)-\left(\frac{\pi-B}{4}\right)\right]\right]$$

$$= 2\cos\left(\frac{\pi-A}{4}\right).2\cos\left(\frac{\pi-B}{4}\right).\cos\left(\frac{\pi-C}{4}\right)$$

$$= 4\cos\left(\frac{\pi-A}{4}\right).\cos\left(\frac{\pi-B}{4}\right).\cos\left(\frac{\pi-C}{4}\right)$$

= R.H.S.

(v) L.H.S.

$$= \frac{\cos A - \cos B + \cos C + 1}{\cos A + \cos B + \cos C - 1}$$

$$= \frac{(\cos A + \cos C) + (1 - \cos B)}{(\cos A + \cos C) - (1 - \cos B)}$$

$$= \frac{2\cos\left(\frac{A+C}{2}\right)\cos\left(\frac{A-C}{2}\right) + 2\sin^2\left(\frac{B}{2}\right)}{2\cos\left(\frac{A+C}{2}\right)\cos\left(\frac{A-C}{2}\right) - 2\sin^2\left(\frac{B}{2}\right)}$$

$$= \frac{2\cos\left(\frac{A+C}{2}\right).\cos\left(\frac{A-C}{2}\right) + 2\cos^2\left(\frac{A+C}{2}\right)}{2\cos\left(\frac{A+C}{2}\right).\cos\left(\frac{A-C}{2}\right) - 2\cos^2\left(\frac{A+C}{2}\right)}$$

$$= \frac{2\cos\left(\frac{A+C}{2}\right)\left[\cos\left(\frac{A-C}{2}\right) + \cos\left(\frac{A+C}{2}\right)\right]}{2\cos\left(\frac{A+C}{2}\right)\left[\cos\left(\frac{A-C}{2}\right) - \cos\left(\frac{A+C}{2}\right)\right]}$$

$$= \frac{\cos\left(\frac{A-C}{2}\right) + \cos\left(\frac{A+C}{2}\right)}{\cos\left(\frac{A-C}{2}\right) - \cos\left(\frac{A+C}{2}\right)}$$

$$= \frac{2\cos\left(\frac{A}{2}\right).\cos\left(\frac{C}{2}\right)}{2\sin\left(\frac{A}{2}\right).\sin\left(\frac{C}{2}\right)}$$

$$= \cot\left(\frac{A}{2}\right).\cot\left(\frac{C}{2}\right)$$

= R.H.S.

Solution of Equations by Factoring

Example – 12

Solve : $2 \cos x \cos 2x = \cos x$.

Sol. The given equation is equivalent to the equation $\cos x\,(2 \cos 2x - 1) = 0$.

This equation is equivalent to the collection of equations.

$$\left[\begin{array}{l}\cos x = 0,\\ \cos 2x = \frac{1}{2},\end{array}\right. \Rightarrow \left[\begin{array}{ll}x = \frac{\pi}{2} + \pi n, & n \in Z,\\ 2x = \pm\frac{\pi}{3} + 2\pi k,\ \text{i.e. } x = \pm\frac{\pi}{6} + \pi k, & k \in Z.\end{array}\right.$$

Answer : $\frac{\pi}{2} + \pi n,\ \pm\frac{\pi}{6} + \pi k \quad (n, k \in Z)$

Solution of Equations Reducible to Quadratic Equations

Example – 13

Solve : $3 \cos^2 x - 10 \cos x + 3 = 0$.

Sol. Assume $\cos x = y$. The given equation assumes the form $3y^2 - 10y + 3 = 0$.

Solving it, we find that $y_1 = \frac{1}{3}$, $y_2 = 3$.

The value $y_2 = 3$ does not satisfy the condition since $|\cos x| \le 1$.

Consequently, $\cos x = \frac{1}{3}$, $x = \pm \cos^{-1} \frac{1}{3} + 2\pi n,\ n \in Z$

Answer : $\pm \cos^{-1}\left(\frac{1}{3}\right) + 2\pi n \ (n \in Z)$.

Solution of Homogeneous Equations and Equations Reducible to them

Equations of the form

$a_0 \sin^n x + a_1 \sin^{n-1} x \cos x + a_2 \sin^{n-2} x \cos^2 x + ... + a_{n-1} \sin x \cos^{n-1} x + a_n \cos^n x = 0,$

where $a_0, a_1, ..., a_n$ are real numbers, are said to be homogeneous with respect to sin x and cos x.

Example – 14

Solve : $6 \sin^2 x - \sin x \cos x - \cos^2 x = 3.$

Sol. $6 \sin^2 x - \sin x \cos x - \cos^2 x - 3 (\sin^2 x + \cos^2 x) = 0.$

Removing the brackets and collecting like terms, we get

$$3 \sin^2 x - \sin x \cos x - 4 \cos^2 x = 0.$$

Since the values $x = \frac{\pi}{2} + \pi n$ are not roots of the equation

and $\cos x \neq 0$,

we divide both sides of the equation by $\cos^2 x$

$3 \tan^2 x - \tan x - 4 = 0,$

whence $\tan x = -1, x = -\frac{\pi}{4} + \pi n, \quad n \in Z$

and $\tan x = \frac{4}{3}, x = \tan^{-1} \frac{4}{3} + \pi k, \quad k \in Z$

Answer : $-\frac{\pi}{4} + \pi n, \quad \tan^{-1} \frac{4}{3} + \pi k \quad (n, \quad k \in Z)$

Solving Equations by Introducing an Auxiliary Argument

Example – 15

Solve : $\frac{\sqrt{3}}{2} \cos x + \frac{1}{2} \sin x = 1$

Sol. $\cos \frac{\pi}{6} \cos x + \sin \frac{\pi}{6} \sin x = 1, \cos \left(x - \frac{\pi}{6}\right) = 1,$

$x - \frac{\pi}{6} = 2 \pi n \ (n \in Z), \quad x = \frac{\pi}{6} + 2\pi n \quad (n \in Z).$

Answer : $\frac{\pi}{6} + 2 \pi n \quad (n \in Z).$

Solving Equation by Transforming a Sum of Trigonometric Functions into a Product

Example – 16

Solve : $\cos 3x + \sin 2x - \sin 4x = 0$

Sol. $\cos 3x + (\sin 2x - \sin 4x) = 0$

Transforming the expression in brackets by formula

$$\sin \alpha - \sin \beta = 2 \sin \frac{\alpha - \beta}{2} \cos \frac{\alpha + \beta}{2}$$

we obtain

$$\cos 3x + (-2 \sin x \cos 3x) = 0,$$
$$\cos 3x (1 - 2 \sin x) = 0.$$

The last equation is equivalent to the collection of equation

$$\cos 3x = 0, \sin x = \frac{1}{2};$$

consequently, $x = \frac{\pi}{6} + \frac{\pi}{3} n, \ x = (-1)^k \frac{\pi}{6} + \pi k \ (n, k \in Z)$

The set of solution $x = (-1)^k \frac{\pi}{6} + \pi k \quad (k \in Z)$ belongs entirely to the set of solution $x = \frac{\pi}{6} + \frac{\pi n}{3} \quad (n \in Z).$

Therefore, this set alone remains as a set of solutions.

Answer : $\frac{\pi}{6} + \frac{\pi}{3} n \quad (n \in Z).$

Solving Equations by Transforming a Product of Trigonometric Functions into a sum

Example – 17

Solve : sin 5 x cos 3x = sin 6 x cos 2x.

Sol. We apply formula $\sin\alpha\cos\beta = \frac{1}{2}(\sin(\alpha-\beta)+\sin(\alpha+\beta))$ to both sides of the equation :

$$\frac{1}{2}(\sin 8x + \sin 2x) = \frac{1}{2}(\sin 8x + \sin 4x),$$

$$\sin 2x - \sin 4x = 0$$

Using formula $\sin\alpha - \sin\beta = 2\sin\frac{\alpha-\beta}{2}\cos\frac{\alpha+\beta}{2}$,

we obtain $-2\sin x\cos 3x = 0$.

$$\Rightarrow \left[\begin{array}{l}\sin x = 0,\\ \cos 3x = 0,\end{array}\right. \Rightarrow \left[\begin{array}{ll}x = \pi n, & n \in Z,\\ 3x = \frac{\pi}{2}+\pi k,\ x = \frac{\pi}{6}+\frac{\pi}{3}k, & k \in Z.\end{array}\right.$$

Answer : $\frac{\pi}{6}+\frac{\pi}{3}k$ (n, k ∈ Z).

Solving Equ. with the Use of $\cos^2\alpha = \sin^2\alpha$ = for Lowering a Degree

Example – 18

Solve : $\sin^2 x + \sin^2 2x = 1$

Sol. $\frac{1-\cos 2x}{2}+\frac{1-\cos 4x}{2}=1 \Rightarrow \cos 2x + \cos 4x = 0 \Rightarrow 2\cos 3x\cos x = 0$.

The last equation is equivalent to the collection of two equations.

(a) $\cos 3x = 0,\ 3x = \frac{\pi}{2}+\pi n,\ x = \frac{\pi}{6}+\frac{\pi}{3}n,\quad n \in Z$

(b) $\cos x = 0,\ x = \frac{\pi}{2}+\pi k,\ k \in Z$

The set of solutions of equation (b) is a subset of the set of solutions of (a) and, therefore, in the answer we write only roots equation (a).

Answer : $\frac{\pi}{6}+\frac{\pi n}{3}$ (n ∈ Z).

Solving Equation with the Use of $1 + \cos 2\alpha = 2\cos^2\alpha$, $1 - \cos 2\alpha = 2\sin^2\alpha$

Example – 19

Solve : $\cos x - 2\sin^2\frac{x}{2} = 0$.

Sol. $\cos x - (1-\cos x) = 0 \Rightarrow 2\cos x - 1 = 0$

$$\Rightarrow \cos x = \frac{1}{2} \Rightarrow x = \pm\frac{\pi}{3}+2\pi n\ (n\in Z)$$

Answer : $\pm\frac{\pi}{3}+2\pi n$ (n ∈ Z).

Solving Equations with the Use of Formulas for Double & Triple Arguments

Example – 20

Solve : $2\sin\frac{x}{2}\cos^2 x - 2\sin\frac{x}{2}\sin^2 x = \cos^2 x - \sin^2 x$.

Sol. On the left-hand side of the equation we put the factor $2\sin\frac{x}{2}$ before the parentheses :

$$2\sin\frac{x}{2}(\cos^2 x - \sin^2 x) = \cos^2 x - \sin^2 x.$$

Replacing the expression $\cos^2 x - \sin^2 x$ by $\cos 2x$ according to formula (2), we get

$$2\sin\frac{x}{2}\cos 2x = \cos 2x,$$

or $$2\sin\frac{x}{2}\cos 2x - \cos 2x = 0$$

$$\Rightarrow \cos 2x\left(2\sin\frac{x}{2}-1\right) = 0$$

$$\Rightarrow \left[\begin{array}{l}\cos 2x = 0,\\ \sin\frac{x}{2} = \frac{1}{2},\end{array}\right. \Rightarrow \left[\begin{array}{ll}x = \frac{\pi}{4}+\frac{\pi}{2}n, & n \in Z,\\ x = (-1)^k\frac{\pi}{3}+2\pi k, & k \in Z.\end{array}\right.$$

Answer : $\frac{\pi}{4}+\frac{\pi}{2}n$, $(-1)^k\frac{\pi}{3}+2\pi k$ (n, k ∈ Z).

Solving Equations by a Change of Variable

(a) Equations of the form P (sin x ± cos x, sin x cos x) = 0, where P (y, z) is a polynomial, can be solved by the change.

$\cos x \pm \sin x = t \Rightarrow 1 \pm 2 \sin x \cos x = t^2$.

Let us consider an example.

Example – 21

Solve : sin x + cos x = 1 + sin x cos x.

Sol. We introduce the designation sin x + cos x = t.
Then $(\sin x + \cos x)^2 = t^2$, $1 + 2 \sin x \cos x = t^2$,

$$\sin x \cos x = \frac{t^2-1}{2}.$$

In the new designations the initial equation looks like

$$t = 1 + \frac{t^2-1}{2} \text{ or } t^2 - 2t + 1 = 0, (t-1)^2 = 0, t = 1,$$

i.e.,

$$\sin x + \cos x = 1, \sqrt{2}\left(\frac{1}{\sqrt{2}}\sin x + \frac{1}{\sqrt{2}}\cos x\right) = 1,$$

$$\cos\frac{\pi}{4}\cos x + \sin\frac{\pi}{4}\sin x = \frac{1}{\sqrt{2}},$$

$$\cos\left(x - \frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$$

$$x - \frac{\pi}{4} = \pm\frac{\pi}{4} + 2\pi n, \ n \in Z,$$

$$x = \frac{\pi}{4} \pm \frac{\pi}{4} + 2\pi n, \ n \in Z.$$

Answer : $\frac{\pi}{2} + 2\pi n, 2\pi n \ (n \in Z)$.

(b) Equations of the form a sin x + b cos x + d = 0, where a, b, and d are real numbers, and a, b ≠ 0, can be solved by the change.

$$\cos x = \frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}, \ \sin x = \frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}},$$

$$x \neq \pi + 2\pi n \quad (n \in Z)$$

Example – 22

Solve : 3 cos x + 4 sin x = 5.

Sol. $$3\frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}} + 4\frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}} = 5,$$

$$3 - 3\tan^2\frac{x}{2} + 8\tan\frac{x}{2} = 5 + 5\tan^2\frac{x}{2},$$

$$4\tan^2\frac{x}{2} - 4\tan\frac{x}{2} + 1 = 0, \left(2\tan\frac{x}{2} - 1\right)^2 = 0$$

$$\tan\frac{x}{2} = \frac{1}{2}, x = 2\tan^{-1}\frac{1}{2} + 2\pi n, n \in Z$$

Answer : $2\tan^{-1}\frac{1}{2} + 2\pi n, (n \in Z)$.

(c) Many equations can be solved by introducing a new variable.

Solution of Trigonometric Eqn. of the Form

$$f(x) = \sqrt{\varphi(x)}$$

Example – 23

Solve : $\sqrt{1-\cos x} = \sin x, x \in [\pi, 3\pi]$

Sol. $$\begin{cases} 1 - \cos \geq 0, \\ \sin x \geq 0. \end{cases}$$

Under the condition that both sides of the equation are nonnegative, we square them:

$1 - \cos x = \sin^2 x$, $\quad 1 - \cos x = 1 - \cos^2 x$,

$\cos^2 x - \cos x = 0$, $\quad \cos x (\cos x - 1) = 0$.

(1) $\cos x = 0, x = \frac{\pi}{2} + \pi n, n \in Z$,

(2) $\cos x = 1, x = 2\pi k, k \in Z$. But since $\sin x \geq 0$ and $x \in [\pi, 3\pi]$, we leave $x = 2\pi, \frac{5\pi}{2}$.

Answer : $2\pi, \frac{5\pi}{2}$.

Solving Equations with the Use of the Boundedness of the Functions sin x & cos x

Example – 24

Solve : $\left(\cos\frac{x}{4}-2\sin x\right)\sin x+\left(1+\sin\frac{x}{4}-2\cos x\right)\times\cos x=0.$

Sol. $\cos\frac{x}{4}\sin x-2\sin^2 x+\cos x+\sin\frac{x}{4}\cos x-2\cos^2 x=0.$

$\sin\left(x+\frac{x}{4}\right)+\cos x-2(\sin^2 x+\cos^2 x)=0, \sin\frac{5x}{4}+\cos x=2.$

Since the functions $\sin\frac{5x}{4}$ and cos x have the greatest value equal to 1, their sum is equal to 2 if $\sin\frac{5x}{4}=1$ and cos x = 1 simultaneously, i.e.

$$\Rightarrow\begin{cases}\sin\frac{5x}{4}=1,\\ \cos x=1,\end{cases}\quad\Rightarrow\begin{cases}\frac{5x}{4}=\frac{\pi}{2}+2\pi n,\\ x=2\pi k\ \ (n,\ k\in Z);\end{cases}$$

$$2\pi k=\frac{2\pi}{5}+\frac{8\pi}{5}n,\quad k=\frac{1+4n}{5}$$

Since $k\in Z$, it follows that $n = 1 + 5m$ $(m\in Z)$, and then $x = 2\pi + 8\pi m,\ m\in Z$

Answer : $2\pi + 8\pi m,\ m\in Z$

Trigonometric Systems

Example – 25

Solve : $\begin{cases}\sin x\ \cos y=\frac{1}{4},\\ 3\tan x=\tan y.\end{cases}$

Sol. We transform the second equation & get $3\sin x\cos y-\sin y\cos x=0$.
Substituting now the value of the product, sin x cos y from the first equation into the equation obtained, we get a system.

$$\begin{cases}\cos x\ \sin y=\frac{3}{4}\\ \sin x\cos y=\frac{1}{4}\end{cases}\qquad\text{.......(1)}$$

Adding together the equations of system (1) and then subtracting the first equation from the second, we get a system which is equivalent to system :

$$\begin{cases}\sin(x+y)=1,\\ \sin(x-y)=\frac{1}{2},\end{cases}\qquad\text{.......(2)}$$

whence we have

$$\begin{cases}x+y=\frac{\pi}{2}+2\pi k,\\ x-y=-\frac{\pi}{6}+2\pi l\end{cases}\qquad\text{.......(3)}$$

and

$$\begin{cases}x+y=\frac{\pi}{2}+2\pi k,\\ x-y=-\frac{5\pi}{6}+2\pi l\end{cases}\qquad\text{.......(4)}$$

From system (3) we find

$$x=\frac{\pi}{6}+\pi(k+l),\qquad y=\frac{\pi}{3}+\pi(k-l).$$

From system (4) we find

$$x=-\frac{\pi}{6}+\pi(k+l),\qquad y=\frac{2\pi}{3}+\pi(k-l).$$

EXERCISE - 1 : BASIC OBJECTIVE QUESTIONS

Basic System of Measurement

1. $10^\circ\, 40'\, 30''$ can be express in radians as follows

(a) $\frac{527\pi}{7200}$ (b) $\frac{427\pi}{7200}$

(c) $\frac{\pi}{7200}$ (d) $\frac{\pi}{427}$

2. The degree and radian measure of the angle between the hour-hand and the minute-hand of a clock at twenty minutes past seven is

(a) $\frac{5\pi}{4}$ (b) $\frac{5\pi}{8}$

(c) $\frac{5\pi}{7}$ (d) $\frac{5\pi^c}{9}$

3. If the perimeter of a sector of a circle, of area $25\,\pi$ sq. cms. is 20 cms then area of a sector is

(a) 20 sq. cms (b) 24 sq. cms

(c) 50 sq. cms (d) 25 sq. cms

4. Number of sides of regular polygon of interior angle $\frac{3\pi}{4}$ is

(a) 10 (b) 5

(c) 8 (d) 9

5. The sum of two angles is 5π and their difference is 60°. Then the angles are

(a) 480°, 420° (b) 470°, 450°

(c) 520°, 580° (d) 360°, 120°

Trigonometric Ratios

6. $\cos 24^\circ + \cos 5^\circ + \cos 175^\circ + \cos 204^\circ + \cos 300^\circ =$

(a) 1/2 (b) –1/2

(c) $\frac{\sqrt{3}}{2}$ (d) 1

7. If $\frac{2\sin\alpha}{1+\sin\alpha+\cos\alpha} = \lambda$ then $\frac{1+\sin\alpha-\cos\alpha}{1+\sin\alpha}$ is equal to

(a) $\frac{1}{\lambda}$ (b) λ

(c) $1-\lambda$ (d) $1+\lambda$

8. tan x is defined for all x in

(a) R (b) $R - \{n\pi : n \in I\}$

(c) $R - \{(2n+1)\frac{\pi}{2} : n \in I\}$ (d) none of these

9. cot x is defined for all x in

(a) R (b) $R - \{n\pi : n \in I\}$

(c) $R - \{(2n+1)\frac{\pi}{2} : n \in I\}$ (d) none of these

10. Which of the following is not correct ?

(a) $\sin\theta = -\frac{1}{5}$ (b) $\cos\theta = 1$

(c) $\sec\theta = \frac{1}{2}$ (d) $\tan\theta = 20$

11. If sin θ and cos θ are the roots of the equation $ax^2 - bx + c = 0$, then a, b and c satisfy the relation :

(a) $a^2 + b^2 + 2ac = 0$ (b) $a^2 - b^2 - 2ac = 0$

(c) $a^2 + c^2 - 2ab = 0$ (d) $a^2 - b^2 + 2ac = 0$

12. If $x \in R$ and $x \neq 0$, then which of the following is not possible ?

(a) $2\sin\theta = x + \frac{1}{x}$ (b) $2\cos\theta = x + \frac{1}{x}$

(c) $2\sin\theta = x - \frac{1}{x}$ (d) $\sin\theta = x + \frac{1}{x}$

13. In a trangle ABC, if cotA cotB cotC > 0, then the triangle is

(a) acute angled (b) right angled

(c) obtuse angled (d) does not exist

14. Which of the following is correct –

(a) $\sin 1^\circ > \sin 1$ (b) $\sin 1^\circ < \sin 1$

(c) $\sin 1^\circ = \sin 1$ (d) $\sin 1^\circ = \frac{\pi}{180}\sin 1$

15. If sin (x –y) = cos (x + y) = 1/2, then the values of x and y lying between 0º and 180º are given by

(a) $x = 45^\circ, y = 15^\circ$ (b) $x = 45^\circ, y = 135^\circ$

(c) $x = 165^\circ, y = 15^\circ$ (d) none of these

16. If 5 sin θ = 3, then $\frac{\sec\theta + \tan\theta}{\sec\theta - \tan\theta}$ is equal to

(a) $\frac{1}{4}$ (b) 4

(c) 2 (d) none of these

17. A value of θ satisfying $\cos\theta + \sqrt{3}\sin\theta = 2$ is

(a) $\frac{5\pi}{3}$ (b) $\frac{4\pi}{3}$

(c) $\frac{2\pi}{3}$ (d) $\frac{\pi}{3}$

18. Which of the following is correct ?

(a) cos 1 > cos 2 (b) cos 1 < cos 2

(c) cos 1 = cos 2 (d) none of these

19. If $\sqrt{\frac{1-\sin A}{1+\sin A}} + \frac{\sin A}{\cos A} = \frac{1}{\cos A}$, for all permissible values of A, then A may belongs to

(a) First Quadrant (b) Second Quadrant

(c) Third Quadrant (d) Fourth Quadrant

20. If 3 sin θ – 5 cos θ = a, then 5 sin θ + 3 cos θ is equal to

(a) 1 – a (b) $\sqrt{1-a^2}$

(c) $\sqrt{34-a^2}$ (d) $\sqrt{34-a^2}$ or $-\sqrt{34-a^2}$

21. The value of $\tan\frac{\pi}{8}\tan\frac{3\pi}{8}$ is

(a) 0 (b) 1

(c) $\frac{1}{2}$ (d) none of these

22. If tan θ = – 4/3, then sinθ is

(a) $\frac{-4}{5}$ but not $\frac{4}{5}$ (b) $\frac{-4}{5}$ or $\frac{4}{5}$

(c) $\frac{4}{5}$ but not $\frac{-4}{5}$ (d) none of these

23. The value of cos 1º cos 2º cos 3º .... cos 179º is

(a) $1/\sqrt{2}$ (b) 0

(c) 1 (d) None of these

24. If $\sin x + \sin^2 x = 1$, then the value of $\cos^{12}x + 3\cos^{10}x + 3\cos^8 x + \cos^6 x - 1$ is equal to

(a) 0 (b) 1

(c) –1 (d) 2

Trigonometric Identities

25. The value of $\frac{1-\tan^2 15^\circ}{1+\tan^2 15^\circ}$ is

(a) $\frac{\sqrt{3}}{2}$ (b) 1

(c) $\frac{1}{2}$ (d) $\sqrt{3}$

26. If $\sec A = \frac{17}{8}$ and $\text{cosec } B = \frac{5}{4}$; if A, B lies in first quadrant then sec (A + B) can have the value equal to

(a) $\frac{85}{36}$ (b) $-\frac{85}{36}$

(c) $-\frac{85}{84}$ (d) $\frac{85}{84}$

27. The two legs of a right triangle are $\sin\theta + \sin\left(\frac{3\pi}{2}-\theta\right)$ and $\cos\theta - \cos\left(\frac{3\pi}{2}-\theta\right)$. The length of its hypotenuse is

(a) 1 (b) 2

(c) $\sqrt{2}$ (d) some function of θ

28. Which of the following when simplified reduces to unity ?

(a) $\frac{1-2\sin^2\alpha}{2\cot\left(\frac{\pi}{4}+\alpha\right)\cos^2\left(\frac{\pi}{4}-\alpha\right)}$

(b) $\frac{\sin(\pi-\alpha)}{\sin\alpha - \cos\alpha\tan\frac{\alpha}{2}} + \cos(\pi-\alpha)$

(c) $\frac{1}{4\sin^2\alpha\cos^2\alpha} - \frac{(1-\tan^2\alpha)^2}{4\tan^2\alpha}$

(d) $\frac{1+\sin 2\alpha}{(\sin\alpha+\cos\alpha)^2}$

29. If $f(x) = 3\left[\sin^4\left(\frac{3\pi}{2}-x\right)+\sin^4(3\pi+x)\right]-2\left[\sin^6\left(\frac{\pi}{2}+x\right)+\sin^6(5\pi-x)\right]$ then, for all permissible values of x, f (x) is

(a) – 1

(b) 0

(c) 1

(d) not a constant function

30. The sines of two angles of a triangle are equal to $\frac{5}{13}$ & $\frac{99}{101}$. The cosine of the third angle can be :

(Assume that sum of all angles in a triangle are supplementary)

(a) $\frac{245}{1313}$ (b) $\frac{255}{1313}$

(c) $\frac{735}{1313}$ (d) $\frac{765}{1313}$

31. If tanx. tany = a and x + y = π/6, then tanx and tany satisfy the equation

(a) $x^2-\sqrt{3}(1-a)x+a=0$

(b) $\sqrt{3}x^2-(1-a)x+a\sqrt{3}=0$

(c) $x^2+\sqrt{3}(1+a)x-a=0$

(d) $\sqrt{3}x^2+(1+a)x-a\sqrt{3}=0$

32. If $\alpha+\beta=\frac{\pi}{2}$ and $\beta+\gamma=\alpha$, then tan α equals

(a) $\tan\beta + 2\tan\gamma$ (b) $2\tan\beta+\tan\gamma$

(c) $\tan\beta+\tan\gamma$ (d) none of these

33. $\tan 5x \tan 3x \tan 2x = ...$

(a) $\tan 5x - \tan 3x - \tan 2x$

(b) $\dfrac{\sin 5x - \sin 3x - \sin 2x}{\cos 5x - \cos 3x - \cos 2x}$

(c) 0

(d) None of these

34. If $\tan\alpha = \dfrac{m}{m+1}$, $\tan\beta = \dfrac{1}{2m+1}$, then $\alpha + \beta$ is equal to

(a) $\pi/4$ (b) $\pi/3$

(c) $\tan^{-1}\dfrac{m}{2m+1}$ (d) $\tan^{-1}\dfrac{m+1}{2m+1}$

35. If $\pi < 2\theta < \dfrac{3\pi}{2}$, then $\sqrt{2+\sqrt{2+2\cos 4\theta}}$ is equal to

(a) $-2\cos\theta$ (b) $-2\sin\theta$

(c) $2\cos\theta$ (d) $2\sin\theta$

36. If $A - B = \dfrac{\pi}{4}$, then $(1 + \tan A)(1 - \tan B) =$

(a) 1 (b) 2

(c) –1 (d) –2

37. If $\tan\dfrac{\alpha}{2}$ and $\tan\dfrac{\beta}{2}$ are the roots of the equation $8x^2 - 26x + 15 = 0$ then $\cos(\alpha + \beta)$ is equal to

(a) $-\dfrac{627}{725}$ (b) $\dfrac{627}{725}$

(c) –1 (d) none of these

38. If $\sin\alpha + \sin\beta = a$ and $\cos\alpha - \cos\beta = b$ then $\tan\dfrac{\alpha-\beta}{2}$ is equal to

(a) $-\dfrac{a}{b}$ (b) $-\dfrac{b}{a}$

(c) $\sqrt{a^2+b^2}$ (d) none of these

39. If $0 \le \beta \le \alpha \le \dfrac{\pi}{4}$, $\cos(\alpha+\beta) = \dfrac{3}{5}$ and $\cos(\alpha-\beta) = \dfrac{4}{5}$ then $\sin 2\alpha$ is equal to

(a) 1 (b) 0

(c) 2 (d) none of these

40. For all real values of θ, $\cot\theta - 2\cot 2\theta$ is equal to

(a) $\tan 2\theta$ (b) $\tan\theta$

(c) $-\cot 3\theta$ (d) none of these

41. If $\cos 20° - \sin 20° = p$ then $\cos 40°$ is equal to

(a) $-p\sqrt{2-p^2}$ (b) $p\sqrt{2-p^2}$

(c) $p+\sqrt{2-p^2}$ (d) none of these

42. If $\cos 2x + 2\cos x = 1$ then $\sin^2 x(2-\cos^2 x)$ is equal to

(a) 1 (b) –1

(c) $-\sqrt{5}$ (d) $\sqrt{5}$

43. If $A + C = B$, then $\tan A \tan B \tan C$ is

(a) $\tan A \tan B + \tan C$

(b) $\tan B - \tan C - \tan A$

(c) $\tan A + \tan C - \tan B$

(d) $-(\tan A \tan B + \tan C)$

44. If $x + \dfrac{1}{x} = 2\cos\theta$, then $x^3 + \dfrac{1}{x^3} =$

(a) $\cos 3\theta$ (b) $2\cos 3\theta$

(c) $\dfrac{1}{2}\cos 3\theta$ (d) $\dfrac{1}{3}\cos 3\theta$

45. If $\tan\alpha$, $\tan\beta$ are the roots of the equation $x^2 + px + q = 0$ $(p \ne 0)$, then

(a) $\sin(\alpha+\beta) = -p$ (b) $\tan(\alpha+\beta) = p/(q-1)$

(c) $\cos(\alpha+\beta) = 1 - q$ (d) none of these

46. The value of

$$\sin\frac{\pi}{14}.\sin\frac{3\pi}{14}.\sin\frac{5\pi}{14}.\sin\frac{7\pi}{14}.\sin\frac{9\pi}{14}.\sin\frac{11\pi}{14}.\sin\frac{13\pi}{14}$$

is equal to

(a) 1 (b) $\frac{1}{16}$

(c) $\frac{1}{64}$ (d) none of these

47. The numerical value of $\sin\frac{\pi}{18}.\sin\frac{5\pi}{18}.\sin\frac{7\pi}{18}$ is equal to

(a) 1 (b) $\frac{1}{8}$

(c) $\frac{1}{4}$ (d) $\frac{1}{2}$

48. The value of

cos 12° . cos 24° . cos 36° . cos 48° . cos 72° . cos 84° is

(a) $\frac{1}{64}$ (b) $\frac{1}{32}$

(c) $\frac{1}{16}$ (d) $\frac{1}{128}$

49. The value of sin 78° – sin 66° – sin 42° + sin 6° is

(a) $\frac{1}{2}$ (b) $-\frac{1}{2}$

(c) –1 (d) none of these

50. Let $0 \le \theta \le \frac{\pi}{2}$ and $x = X\cos\theta + Y\sin\theta$, $y = X\sin\theta - Y\cos\theta$ such that $x^2 + 4xy + y^2 = aX^2 + bY^2$, where a, b are constants. Then

(a) a = –1, b = 3 (b) $\theta = \frac{\pi}{4}$

(c) a = 3, b = –1 (d) $\theta = \frac{\pi}{3}$

Transformation of Product to Sum

51. sin x + sin 2x + sin 3x = 0 if

(a) sin x = 1/2 (b) sin 2x = 0

(c) sin 3x = $\sqrt{3}/2$ (d) cos x = – 1/2

52. If $(2^n + 1)\theta = \pi$, then $2^n \cos\theta \cos 2\theta \cos 2^2\theta \ldots \cos 2^{n-1}\theta =$

(a) –1 (b) 1

(c) 1/2 (d) none of these

53. If n = 1, 2, 3, ..., then $\cos\alpha \cos 2\alpha \cos 4\alpha \ldots \cos 2^{n-1}\alpha$ is equal to

(a) $\frac{\sin 2n\alpha}{2n\sin\alpha}$ (b) $\frac{\sin 2^n\alpha}{2^n \sin 2^{n-1}\alpha}$

(c) $\frac{\sin 4^{n-1}\alpha}{4^{n-1}\sin\alpha}$ (d) $\frac{\sin 2^n\alpha}{2^n\sin\alpha}$

54. tan 3 A – tan 2A – tan A is equal to

(a) – tan 3 A tan 2 A tan A

(b) tan A tan 2 A tan 3 A

(c) tan A tan 2A – tan 2 A tan 3 A – tan 3 A tan A

(d) none of these

55. $\cos\frac{\pi}{8}\cos\frac{3\pi}{8}\cos\frac{5\pi}{8}\cos\frac{7\pi}{8}$ is equal to

(a) 1/2 (b) $\frac{1-\sqrt{2}}{2\sqrt{2}}$

(c) 1/8 (d) $\frac{1+\sqrt{2}}{2\sqrt{2}}$

Maximum and Minimum Values

56. Minimum value of $5\sin^2\theta + 4\cos^2\theta$ is

(a) 1 (b) 2

(c) 3 (d) 4

57. Maximum value of sin x + cos x is

(a) 1 (b) 2

(c) $\sqrt{2}$ (d) $\frac{1}{\sqrt{2}}$

58. Minimum value of $\sec^2\theta + \text{cosec}^2\theta$ is

(a) 2 (b) 4

(c) 1 (d) 3

59. Minimum value of $\sin^6\theta + \cos^6\theta$ is

(a) 0 (b) 1

(c) $\frac{1}{2}$ (d) $\frac{1}{4}$

60. If $A = 2\sin^2\theta - \cos 2\theta$, then A lies in the interval

(a) $[-1, 3]$ (b) $[1, 2]$

(c) $[-2, 4]$ (d) none of these

61. If $\sin\theta_1 + \sin\theta_2 + \sin\theta_3 = 3$, then $\cos\theta_1 + \cos\theta_2 + \cos\theta_3 =$

(a) 3 (b) 2

(c) 1 (d) 0

62. The maximum value of $12\sin\theta - 9\sin^2\theta$ is

(a) 3 (b) 4

(c) 5 (d) None of these

63. The number of solutions of $\cos\theta + \sqrt{3}\sin\theta = 5, 0 \le \theta \le 5\pi$, is

(a) 4 (b) 0

(c) 5 (d) None of these

Trigonometric Equations

64. A solution of the equation $\cos^2\theta + \sin\theta + 1 = 0$, lies in the interval

(a) $(-\pi/4, \pi/4)$ (b) $(\pi/4, 3\pi/4)$

(c) $(3\pi/4, 5\pi/4)$ (d) $(5\pi/4, 7\pi/4)$

65. If $4\sin^2\theta = 1$, then the values of θ are

(a) $2n\pi \pm \frac{\pi}{3}, n \in Z$ (b) $n\pi \pm \frac{\pi}{3}, n \in Z$

(c) $n\pi \pm \frac{\pi}{6}, n \in Z$ (d) $2n\pi \pm \frac{\pi}{6}, n \in Z$

66. The general solution of $\tan\left(\frac{\theta}{2}\right) = 0$ is

(a) $2n\pi; n \in I$ (b) $n\pi; n \in I$

(c) $(2n+1)\frac{\pi}{2}; n \in I$ (d) None of these

67. In $2\cos^2\theta + 3\sin\theta = 0$, then the general value of θ is–

(a) $n\pi + (-1)^n \frac{\pi}{6}; n \in I$ (b) $2n\pi \pm \frac{\pi}{6}; n \in I$

(c) $n\pi + (-1)^{n+1}\frac{\pi}{6}; n \in I$ (d) None of these

68. General solution of equation $\sqrt{3}\cos\theta + \sin\theta = \sqrt{2}$ is

(a) $n\pi \pm \frac{\pi}{4} + \frac{\pi}{6}; n \in I$

(b) $2n\pi \pm \frac{\pi}{4} + \frac{\pi}{6}; n \in I$

(c) $2n\pi \pm \frac{\pi}{4} - \frac{\pi}{6}; n \in I$

(d) None of these

69. The set of values of x for which $\frac{\tan 3x - \tan 2x}{1 + \tan 3x \tan 2x} = 1$ is

(a) ϕ

(b) $\{\pi/4\}$

(c) $\{n\pi + \pi/4 \mid n = 1, 2, 3.....\}$

(d) $\{2n\pi + \pi/4 \mid n = 1, 2, 3.....\}$

70. The number of solutions of the equation $\tan x + \sec x = 2\cos x$ lying in the interval $[0, 2\pi]$, is

(a) 0 (b) 1

(c) 2 (d) 3

EXERCISE - 2 : PREVIOUS YEAR JEE MAINS QUESTIONS

1. The number of solutions of tan x + sec x = 2 cos x in $[0, 2\pi)$ is **(2002)**

(a) 2 (b) 3
(c) 0 (d) 1

2. In a triangle ABC, $2ca \sin\left(\frac{A-B+C}{2}\right)$ is equal to **(2002)**

(a) $a^2+b^2-c^2$ (b) $c^2+a^2-b^2$
(c) $b^2-c^2-a^2$ (d) $c^2-a^2-b^2$

3. $\sin^2\theta = \frac{4xy}{(x+y)^2}$ is true, if and only if **(2002)**

(a) $x-y\neq 0$ (b) $x=-y$
(c) $x+y\neq 0$ (d) $x\neq 0, y\neq 0$

4. The value of $\frac{1-\tan^2 15°}{1+\tan^2 15°}$ is **(2002)**

(a) 1 (b) $\sqrt{3}$
(c) $\frac{\sqrt{3}}{2}$ (d) 2

5. If $\tan\theta = -\frac{4}{3}$, then $\sin\theta$ is **(2002)**

(a) $-\frac{4}{5}$ but not $\frac{4}{5}$ (b) $-\frac{4}{5}$ or $\frac{4}{5}$
(c) $\frac{4}{5}$ but not $-\frac{4}{5}$ (d) None of these

6. If $\sin(\alpha+\beta)=1$, $\sin(\alpha-\beta)=\frac{1}{2}$, then $\tan(\alpha+2\beta)\tan(2\alpha+\beta)$ is equal to
(2002)

(a) 1 (b) –1
(c) zero (d) None of these

7. If $y=\sin^2\theta+\operatorname{cosec}^2\theta$, $\theta\neq 0$, then **(2002)**

(a) $y=0$ (b) $y\leq 2$
(c) $y\geq -2$ (d) $y\geq 2$

8. In a triangle ABC, a = 4, b = 3, $\angle A = 60°$, then c is the root of the equation. **(2002)**

(a) $c^2-3c-7=0$ (b) $c^2+3c+7=0$
(c) $c^2-3c+7=0$ (d) $c^2+3c-7=0$

9. In a ΔABC, $\tan\frac{A}{2}=\frac{5}{6}$, $\tan\frac{C}{2}=\frac{2}{5}$, then **(2002)**

(a) a, c, b are in AP (b) a, b, c are in AP
(c) b, a c are in AP (d) a, b, c are in GP

10. The equation a sin x + b cos x = c where $|c|>\sqrt{a^2+b^2}$ has **(2002)**

(a) a unique solution
(b) infinite number of solutions
(c) no solution
(d) None of the above

11. If α is a root of $25\cos^2\theta+5\cos\theta-12=0$, $\frac{\pi}{2}<\alpha<\pi$, then $\sin 2\alpha$ is equal to **(2002)**

(a) $\frac{24}{25}$ (b) $-\frac{24}{25}$
(c) $\frac{13}{18}$ (d) $-\frac{13}{18}$

12. The sum of the radii of inscribed and circumscribed circles for an n sided regular polygon of side a, is **(2003)**

(a) $a\cot\left(\frac{\pi}{n}\right)$ (b) $\frac{a}{2}\cot\left(\frac{\pi}{2n}\right)$
(c) $a\cot\left(\frac{\pi}{2n}\right)$ (d) $\frac{a}{4}\cot\left(\frac{\pi}{2n}\right)$

13. If in a triangle ABC

$$a\cos^2\left(\frac{C}{2}\right)+c\cos^2\left(\frac{A}{2}\right)=\frac{3b}{2},$$

then the sides a, b and c **(2003)**

(a) are in AP (b) are in GP
(c) are in HP (d) satisfy a + b = c

14. In a triangle ABC, medians AD and BE are drawn. If AD = 4, $\angle DAB = \frac{\pi}{6}$ and $\angle ABE = \frac{\pi}{3}$, then the area of the ΔABC is **(2003)**

(a) $\frac{8}{3}$ sq unit (b) $\frac{16}{3}$ sq unit

(c) $\frac{32}{3\sqrt{3}}$ sq unit (d) $\frac{64}{3}$ sq unit

15. The upper $\left(\frac{3}{4}\right)$th portion of a vertical pole subtends an angle $\tan^{-1}\left(\frac{3}{5}\right)$ at a point in the horizontal plane through its foot and at a distance 40m from the foot. A possible height of the vertical pole is **(2003)**

(a) 20 m (b) 40 m

(c) 60 m (d) 80 m

16. Let α, β be such that $\pi < \alpha - \beta < 3\pi$. If $\sin\alpha + \sin\beta = -\frac{21}{65}$ and $\cos\alpha + \cos\beta = -\frac{27}{65}$, then the value of $\cos\frac{\alpha-\beta}{2}$ is **(2004)**

(a) $\frac{6}{65}$ (b) $\frac{3}{\sqrt{130}}$

(c) $-\frac{3}{\sqrt{130}}$ (d) $\frac{-6}{65}$

17. The sides of triangle are sin α, cos α and $\sqrt{1+\sin\alpha\cos\alpha}$ for some $0 < \alpha < \frac{\pi}{2}$. Then the greatest angle of the triangle is **(2004)**

(a) 120° (b) 90°

(c) 60° (d) 150°

18. A person standing on the bank of river observes that the angle of elevation of the top of a tree on the opposite bank of the river is 60° and when he retires 40 meters away from the tree the angle of elevation becomes 30°. The breadth of the river is **(2004)**

(a) 40 m (b) 30 m

(c) 20 m (d) 60 m

19. In a triangle PQR, if $\angle R = \frac{\pi}{2}$. If $\tan\left(\frac{P}{2}\right)$ and $\tan\left(\frac{Q}{2}\right)$ are the roots of $ax^2 + bx + c = 0, a \neq 0$ then **(2005)**

(a) b = a + c (b) b = c

(c) c = a + b (d) a = b + c

20. The number of value of x in the interval [0, 3π] satisfying the equation $2\sin^2 x + 5\sin x - 3 = 0$ is **(2006)**

(a) 4 (b) 6

(c) 1 (d) 2

21. A triangular park is enclosed on two sides by a fence and on the third side by a straight river bank. The two sides having fence are of same length x. The maximum area enclosed by the park is **(2006)**

(a) $\sqrt{\frac{x^3}{8}}$ (b) $\frac{1}{2}x^2$

(c) πx^2 (d) $\frac{3}{2}x^2$

22. If $0 < x < \pi$ and $\cos x + \sin x = \frac{1}{2}$, then tan x is **(2006)**

(a) $\frac{(1-\sqrt{7})}{4}$ (b) $\frac{(4-\sqrt{7})}{3}$

(c) $-\frac{(4+\sqrt{7})}{3}$ (d) $\frac{(1+\sqrt{7})}{4}$

23. A tower stands at the centre of a circular park. A and B are two points on the boundary of the park such that AB (=a) subtends an angle of 60° at the foot of the tower and the angle of elevation of the top of the tower from A or B is 30°. The height of the tower is **(2007)**

(a) $\frac{2a}{\sqrt{3}}$ (b) $2a\sqrt{3}$

(c) $\frac{a}{\sqrt{3}}$ (d) $\sqrt{3}$

24. AB is a vertical pole with B at the ground level and A at the top. A man finds that the angle of elevation of the point A from a certain point C on the ground is 60°. He moves away from the pole along the line BC to a point D such that CD = 7 m. From D the angle of elevation of the point A is 45°. Then the height of the pole is **(2008)**

(a) $\frac{7\sqrt{3}}{2}\left(\frac{1}{\sqrt{3}+1}\right)$m (b) $\frac{7\sqrt{3}}{2}\left(\frac{1}{\sqrt{3}-1}\right)$m

(c) $\frac{7\sqrt{3}}{2}(\sqrt{3}+1)$ m (d) $\frac{7\sqrt{3}}{2}(\sqrt{3}-1)$ m

25. Let A and B denote the statements

A : $\cos\alpha + \cos\beta + \cos\gamma = 0$

B : $\sin\alpha + \sin\beta + \sin\gamma = 0$

If $\cos(\beta-\gamma) + \cos(\gamma-\alpha) + \cos(\alpha-\beta) = -\frac{3}{2}$, then **(2009)**

(a) A is true and B is false (b) A is false and B is true

(c) both A and B are true (d) both A and B are false

26. Let $\cos(\alpha+\beta) = \frac{4}{5}$ and let $\sin(\alpha-\beta) = \frac{5}{13}$, where $0 \le \alpha, \beta \le \frac{\pi}{4}$. Then $\tan 2\alpha$ is equal to **(2010)**

(a) $\frac{25}{16}$ (b) $\frac{56}{33}$

(c) $\frac{19}{12}$ (d) $\frac{20}{7}$

27. For a regular polygon, let r and R be the radii of the inscribed and the circumscribed circles. A false statement among the following is **(2010)**

(a) there is a regular polygon with $\frac{r}{R} = \frac{1}{2}$

(b) there is a regular polygon with $\frac{r}{R} = \frac{1}{\sqrt{2}}$

(c) there is a regular polygon with $\frac{r}{R} = \frac{2}{3}$

(d) there is a regular polygon with $\frac{r}{R} = \frac{\sqrt{3}}{2}$

28. If $A = \sin^2 x + \cos^4 x$, then for all real x **(2011)**

(a) $\frac{13}{16} \le A \le 1$ (b) $1 \le A \le 2$

(c) $\frac{3}{4} \le A \le \frac{13}{16}$ (d) $\frac{3}{4} \le A \le 1$

29. The possible values of $\theta \in (0, \pi)$ such that $\sin(\theta) + \sin(4\theta) + \sin(7\theta) = 0$ are **(2011)**

(a) $\frac{2\pi}{9}, \frac{\pi}{4}, \frac{4\pi}{9}, \frac{\pi}{2}, \frac{3\pi}{4}, \frac{8\pi}{9}$ (b) $\frac{\pi}{4}, \frac{5\pi}{12}, \frac{\pi}{2}, \frac{2\pi}{3}, \frac{3\pi}{4}, \frac{8\pi}{9}$

(c) $\frac{2\pi}{9}, \frac{\pi}{4}, \frac{\pi}{2}, \frac{2\pi}{3}, \frac{3\pi}{4}, \frac{35\pi}{36}$ (d) $\frac{2\pi}{9}, \frac{\pi}{4}, \frac{\pi}{2}, \frac{2\pi}{3}, \frac{3\pi}{4}, \frac{8\pi}{9}$

30. In a Δ PQR, if 3 sin P + 4 cos Q = 6 and 4 sin Q + 3 cos P = 1, then the angle R is equal to **(2012)**

(a) $\frac{5\pi}{6}$ (b) $\frac{\pi}{6}$

(c) $\frac{\pi}{4}$ (d) $\frac{3\pi}{4}$

31. ABCD is a trapezium such that AB and CD are parallel and BC ⊥ CD. If ∠ADB = θ, BC = p and CD = q, then AB is equal to **(2013)**

(a) $\frac{(p^2+q^2)\sin\theta}{p\cos\theta + q\sin\theta}$ (b) $\frac{p^2+q^2\cos\theta}{p\cos\theta+q\sin\theta}$

(c) $\frac{p^2+q^2}{p^2\cos\theta+q^2\sin\theta}$ (d) $\frac{(p^2+q^2)\sin\theta}{(p\cos\theta+q\sin\theta)^2}$

32. The expression $\frac{\tan A}{1-\cot A} + \frac{\cot A}{1-\tan A}$ can be written as **(2013)**

(a) sin A cos A + 1 (b) sec A cosec A + 1

(c) tan A + cot A (d) sec A + cosec A

33. Let $f_k(x) = \frac{1}{k}(\sin^k x + \cos^k x)$ where $x \in R$ and $k \ge 1$. Then $f_4(x) - f_6(x)$ equals : **(2014)**

(a) $\frac{1}{12}$ (b) $\frac{1}{6}$

(c) $\frac{1}{3}$ (d) $\frac{1}{4}$

34. A bird is sitting on the top of a vertical pole 20 m high and its elevation from a point O on the ground is 45°. It flies off horizontally straight away from the point O. After one second, the elevation of the bird from O is reduced to 30°. Then the speed (in m/s) of the bird is : **(2014)**

(a) $20(\sqrt{3}-1)$ (b) $40(\sqrt{2}-1)$

(c) $40(\sqrt{3}-\sqrt{2})$ (d) $20\sqrt{2}$

35. The number of values of α in $[0, 2\pi]$ for which $2\sin^3\alpha - 7\sin^2\alpha + 7\sin\alpha = 2$, is: **(2014/Online Set–1)**

(a) 6 (b) 4

(c) 3 (d) 1

36. If $2\cos\theta + \sin\theta = 1\left(\theta \neq \frac{\pi}{2}\right)$ then $7\cos\theta + \sin\theta$ is equal to **(2014/Online Set–2)**

(a) $\frac{1}{2}$ (b) 2

(c) $\frac{11}{2}$ (d) $\frac{46}{5}$

37. If the angles of elevation of the top of a tower from three collinear points A, B and C on a line leading to the foot of the tower, are 30^0, 45^0 and 60^0 respectively, then the ratio, AB : BC, is: **(2015)**

(a) $1:\sqrt{3}$ (b) 2 : 3

(c) $\sqrt{3}:1$ (d) $\sqrt{3}:\sqrt{2}$

38. In a $\Delta ABC, \frac{a}{b} = 2+\sqrt{3}$ and $\angle C = 60°$. Then the ordered pair ($\angle A$, $\angle B$) is equal to : **(2015/Online Set–1)**

(a) (45°, 75°) (b) (75°, 45°)

(c) (105°, 15°) (d) (15°, 105°)

39. If $\cos\alpha + \cos\beta = \frac{3}{2}$ and $\sin\alpha + \sin\beta = \frac{1}{2}$ and θ is the arithmetic mean of α and β, then $\sin 2\theta + \cos 2\theta$ is equal to: **(2015/Online Set–2)**

(a) $\frac{3}{5}$ (b) $\frac{7}{5}$

(c) $\frac{4}{5}$ (d) $\frac{8}{5}$

40. If $0 \le x < 2\pi$, then the number of real values of x, which satisfy the equation $\cos x + \cos 2x + \cos 3x + \cos 4x = 0$, is : **(2016)**

(a) 5 (b) 7

(c) 9 (d) 3

41. The number of $x \in [0, 2\pi]$ for which $\left|\sqrt{2\sin^4 x + 18\cos^2 x} - \sqrt{2\cos^4 x + 18\sin^2 x}\right| = 1$ is : **(2016/Online Set–1)**

(a) 2 (b) 4

(c) 6 (d) 8

42. If A > 0, B > 0 and $A + B = \frac{\pi}{6}$, then the minimum value of tanA + tanB is : **(2016/Online Set–2)**

(a) $\sqrt{3}-\sqrt{2}$ (b) $2-\sqrt{3}$

(c) $4-2\sqrt{3}$ (d) $\frac{2}{\sqrt{3}}$

43. If $5(\tan^2 x - \cos^2 x) = 2\cos 2x + 9$, then the value of cos 4x is **(2017)**

(a) $-\frac{3}{5}$ (b) $\frac{1}{3}$

(c) $\frac{2}{9}$ (d) $-\frac{7}{9}$

44. Let a vertical tower AB have its end A on the level ground. Let C be the mid-point of AB and P be a point on the ground such that AP = 2AB. If $\angle BPC = \beta$, then $\tan\beta$ is equal to: **(2017)**

(a) $\frac{6}{7}$ (b) $\frac{1}{4}$

(c) $\frac{2}{9}$ (d) $\frac{4}{9}$

45. If sum of all the solutions of the equation $8\cos x.\left(\cos\left(\frac{\pi}{6}+x\right).\cos\left(\frac{\pi}{6}-x\right)-\frac{1}{2}\right)=1$ in $[0, \pi]$ is $k\pi$, then k is equal to : **(2018)**

(a) $\frac{20}{9}$ (b) $\frac{2}{3}$

(c) $\frac{13}{9}$ (d) $\frac{8}{9}$

46. If tanA and tanB are the roots of the quadratic equation, $3x^2 - 10x - 25 = 0$, then the value of $3\sin^2(A+B) - 10\sin(A+B).\cos(A+B) - 25\cos^2(A+B)$ is : **(2018/Online Set–1)**

(a) -10 (b) 10

(c) -25 (d) 25

47. An aeroplane flying at a constant speed, parallel to the horizontal ground, $\sqrt{3}$ km above it, is observed at an elevation of 60^0 from a point on the ground. If, after five seconds, its elevation from the same point, is 30^0, then the speed (in km/hr) of the aeroplane, is :

(2018/Online Set–1)

(a) 1500 (b) 1440

(c) 750 (d) 720

48. The number of solutions of sin 3x = cos 2x, in the interval $\left(\frac{\pi}{2}, \pi\right)$ is : **(2018/Online Set–2)**

(a) 1 (b) 2

(c) 3 (d) 4

49. A tower T_1 of height 60m is located exactly opposite to a tower T_2 of height 80m on a straight road. From the top of T_1, if the angle of depression of the foot of T_2 is twice the angle of elevation of the top of T_2, then the width (in m) of the road between the feet of the towers T_1 and T_2 is :

(2018/Online Set–2)

(a) $10\sqrt{2}$ (b) $10\sqrt{3}$

(c) $20\sqrt{3}$ (d) $20\sqrt{2}$

50. If an angle A of a ΔABC satisfies

5 cos A + 3 = 0, then the roots of the quadratic equation, $9x^2 + 27x + 20 = 0$ are : **(2018/Online Set–3)**

(a) sec A, cot A (b) sin A, sec A

(c) sec A, tan A (d) tan A, cos A

51. A man on the top of a vertical tower observes a car moving at a uniform speed towards the tower on a horizontal road. If it takes 18 minute for the angle of depression of the car to change from 30^0 to 45^0; then after this, the time taken (in minute) by the car to reach the foot of the tower, is :

(2018/Online Set–3)

(a) $9\left(1+\sqrt{3}\right)$ (b) $18\left(1+\sqrt{3}\right)$

(c) $18\left(\sqrt{3}-1\right)$ (d) $\frac{9}{2}\left(\sqrt{3}-1\right)$

EXERCISE - 3 : ADVANCED OBJECTIVE QUESTIONS

1. If $\cos A = \frac{3}{4}$ then the value of $\sin\frac{A}{2}\sin\frac{5A}{2}$ is

(a) 1/32 (b) 11/8
(c) 11/32 (d) 11/16

2. $\sin\theta(\sin\theta + \sin 3\theta)$ is

(a) ≥ 0 for all θ (b) ≥ 0 only when $\theta \geq 0$
(c) ≤ 0 for all θ (d) ≤ 0 only when $\theta \leq 0$

3. Given $A = \sin^2\theta + \cos^4\theta$, then for all real θ,

(a) $1 \leq A \leq 2$ (b) $\frac{3}{4} \leq A \leq 1$
(c) $\frac{13}{16} \leq A \leq 1$ (d) $\frac{3}{4} \leq A \leq \frac{13}{16}$.

4. If $\cos\alpha = \frac{2\cos\beta - 1}{2 - \cos\beta}$, $(0 < \alpha < \pi, 0 < \beta < \pi)$, then $\tan\frac{\alpha}{2}\cot\frac{\beta}{2}$ is equal to

(a) 1 (b) $\sqrt{2}$
(c) $\sqrt{3}$ (d) none of these

5. If $\sin(\alpha+\beta) = 1$, $\sin(\alpha-\beta) = 1/2$, where $\alpha, \beta \in [0, \pi/2)$, then $\tan(\alpha + 2\beta)\tan(2\alpha + \beta)$ is equal to

(a) 1 (b) 0
(c) –1 (d) none of these

6. Which of the following statements are possible with a, b, m and n being non-zero real numbers :

(a) $4\sin^2\theta = 5$
(b) $(a^2 + b^2)\cos\theta = 2ab$
(c) $(m^2 + n^2)\operatorname{cosec}\theta = m^2 - n^2$
(d) none of these

7. $\cos^2 A(3 - 4\cos^2 A)^2 + \sin^2 A(3 - 4\sin^2 A)^2 =$

(a) cos 4A (b) sin 4A
(c) 1 (d) None of these

8. $2\sin^2\beta + 4\cos(\alpha+\beta)\sin\alpha\sin\beta + \cos 2(\alpha+\beta) =$

(a) $\sin 2\alpha$ (b) $\cos 2\beta$
(c) $\cos 2\alpha$ (d) $\sin 2\beta$

9. If $\sin\alpha$, $\sin\beta$ and $\cos\alpha$ are in G.P. then roots of the equation $x^2 + 2x\cot\beta + 1 = 0$ are always

(a) equal (b) real
(c) imaginary (d) greater than 1

10. If in a triangle ABC, $\sin^2 A + \sin^2 B + \sin^2 C = 2$, then the triangle is always

(a) isosceles triangle (b) right angled
(c) acute angled (d) obtuse angled

11. If $\sin\theta = 3\sin(\theta + 2\alpha)$, then the value of $\tan(\theta+\alpha) + 2\tan\alpha$ is

(a) 3 (b) 2
(c) 1 (d) 0.

12. The least value of $\cos^2\theta - (6\sin\theta \cdot \cos\theta) + 3\sin^2\theta + 2$ is

(a) $4 + \sqrt{10}$ (b) $4 - \sqrt{10}$
(c) 0 (d) 4

13. If $\sin\alpha + \sin\beta = a$ and $\cos\alpha - \cos\beta = b$, then $\tan\left(\frac{\alpha-\beta}{2}\right)$ is equal to

(a) $-\frac{a}{b}$ (b) $-\frac{b}{a}$
(c) $\sqrt{a^2 + b^2}$ (d) None of these

14. $\frac{1}{\sin 3\alpha}\left[\sin^3\alpha+\sin^3\left(\frac{2\pi}{3}+\alpha\right)+\sin^3\left(\frac{4\pi}{3}+\alpha\right)\right]$ is equal to

(a) $\frac{4}{3}$ (b) $\frac{3}{4}$

(c) $-\frac{3}{4}$ (d) none of these

15. The number of distinct solution of

$\sin 5\theta.\cos 3\theta=\sin 9\theta.\cos 7\theta$ in $\left[0,\frac{\pi}{2}\right]$ is

(a) 4 (b) 5

(c) 8 (d) 9

16. The value of θ satisfying

$3\cos^2\theta-2\sqrt{3}\sin\theta\cos\theta-3\sin^2\theta=0$ are

(a) $n\pi-\frac{2\pi}{3},\ n\pi+\frac{\pi}{6}$ (b) $n\pi-\frac{\pi}{3},\ n\pi+\frac{\pi}{6}$

(c) $2n\pi-\frac{\pi}{3},\ n\pi$ (d) $2n\pi+\frac{\pi}{3},\ n\pi$

17. If the equation $\tan\theta+\tan 2\theta+\tan\theta\tan 2\theta=1$, then θ is equal to

(a) $\frac{n\pi}{3}-\frac{\pi}{6}$ (b) $\frac{n\pi}{3}+\frac{\pi}{12}$

(c) $\frac{n\pi}{3}+\frac{\pi}{2}$ (d) None of these

18. The equation $k\cos x-3\sin x=k+1$ is solvable only if k belongs to the interval

(a) $[k,+\infty]$ (b) $[-4,4]$

(c) $(-\infty,4]$ (d) None of these

19. If $2\sin x+1\geq 0$ and $x\in[0,2\pi]$, then the solution set for x is

(a) $\left[0,\frac{7\pi}{6}\right]$ (b) $\left[0,\frac{7\pi}{6}\right]\cup\left[\frac{11\pi}{6},2\pi\right]$

(c) $\left[\frac{11\pi}{6},2\pi\right]$ (d) None of these

20. If the equation $\sin\theta=-1/2$ and $\tan\theta=\frac{1}{\sqrt{3}}$, then most common general values of θ is

(a) $2n\pi\pm\frac{7\pi}{6}$ (b) $2n\pi-\frac{7\pi}{6}$

(c) $2n\pi+\frac{7\pi}{6}$ (d) None of these

21. If $r>0$, $-\pi\leq\theta\leq\pi$ and r, θ satisfy $r\sin\theta=3$ and $r=4(1+\sin\theta)$, then the number of possible solutions of the pair (r, θ) is

(a) 2 (b) 4

(c) 0 (d) inifinite

22. If $x\in\left[-\frac{5\pi}{2},\frac{5\pi}{2}\right]$, the greatest positive solution of $1+\sin^4 x=\cos^2 3x$ is

(a) π (b) 2π

(c) 5π/2 (d) none of these

23. The number of real solutions of

$\sin e^x.\cos e^x=2^{x-2}+2^{-x-2}$ is

(a) zero (b) one

(c) two (d) infinite

24. All solutions of the equation, $2\sin\theta+\tan\theta=0$ are obtained by taking all integral values of m and n in :

(a) $2n\pi+\frac{2\pi}{3}$ (b) $n\pi\ \&\ 2m\pi\pm\frac{2\pi}{3}$

(c) $n\pi\ \&\ m\pi\pm\frac{\pi}{3}$ (d) $n\pi\ \&\ 2m\pi\pm\frac{\pi}{3}$

25. If $x\in\left[0,\frac{\pi}{2}\right]$, the number of solutions of the equation, $\sin 7x+\sin 4x+\sin x=0$ is :

(a) 3 (b) 5

(c) 6 (d) None

26. The number of solution in $[0, \pi/2]$ of the equation $\cos 3x \tan 5x = \sin 7x$ is

(a) 5 (b) 7

(c) 6 (d) none of these

27. $\sin x - \cos^2 x - 1$ assumes the least value for the set of values of x given by :

(a) $x = n\pi + (-1)^{n+1} (\pi/6)$

(b) $x = n\pi + (-1)^n (\pi/6)$

(c) $x = n\pi + (-1)^n (\pi/3)$

(d) $x = n\pi - (-1)^n (\pi/6)$ where $n \in Z$

28. If $2\cos^2(\pi + x) + 3\sin(\pi + x)$ vanishes then values of x lying in the interval from 0 to 2π are

(a) $x = \pi/6$ or $5\pi/6$ (b) $x = \pi/3$ or $5\pi/3$

(c) $x = \pi/4$ or $5\pi/4$ (d) $x = \pi/2$ or $5\pi/2$

29. $4\sin^4 x + \cos^4 x = 1$ if

(a) $x = n\pi$ (b) $n\pi \pm \sin^{-1}\sqrt{\frac{2}{5}}$

(c) $x = n\pi/2$ (d) none

30. The general solution of, $\sin x + \sin 5x = \sin 2x + \sin 4x$ is :

(a) $2n\pi$; $n \in 1$ (b) $n\pi$; $n \in 1$

(c) $n\pi/3$; $n \in 1$ (d) none

31. General solution of the equation, $\cot 3\theta - \cot\theta = 0$ is

(a) $\theta = (2n-1)\frac{\pi}{2}$ (b) $\theta = (2n-1)\frac{\pi}{4}$

(c) $\theta = (2n-1)\frac{\pi}{3}$ (d) none

32. $\frac{\sin 3\theta}{2\cos 2\theta + 1} = \frac{1}{2}$ if

(a) $\theta = n\pi + \frac{\pi}{6}$ (b) $\theta = 2n\pi - \frac{\pi}{6}$

(c) $\theta = n\pi + (-1)^n \frac{\pi}{6}$ (d) $\theta = n\pi - \frac{\pi}{6}$

33. $\cos 15x = \sin 5x$ if

(a) $x = -\frac{\pi}{20} + \frac{\pi}{5}n$ (b) $x = \frac{\pi}{40} + \frac{\pi}{10}n$

(c) $x = \frac{3\pi}{20} + \frac{\pi}{5}n$ (d) $x = -\frac{3\pi}{40} + \frac{\pi}{10}n$

34. $\sin^2 x + 2\sin x\cos x - 3\cos^2 x = 0$ if

(a) $\tan x = 3$ (b) $\tan x = -1$

(c) $x = n\pi + \pi/4$ (d) $x = n\pi + \tan^{-1}(-3)$

35. $5\sin^2 x + \sqrt{3}\sin x\cos x + 6\cos^2 x = 5$ if

(a) $\tan x = -1/\sqrt{3}$ (b) $\sin x = 0$

(c) $x = n\pi + \pi/2$ (d) $x = n\pi + \pi/6$

36. $\sin x + \sin 2x + \sin 3x = \cos x + \cos 2x + \cos 3x$ if

(a) $\cos x = -1/2$ (b) $\sin 2x = \cos 2x$

(c) $x = n\pi/2 + \pi/8$ (d) $x = 2n\pi \pm 2\pi/3$ $(n \in I)$

37. If $\sin x + 7\cos x = 5$ then $\cos(x - \phi) = 1/\sqrt{2}$ if

(a) $\phi = \cos^{-1} 7/\sqrt{50}$ (b) $\phi = \sin^{-1} 1/\sqrt{50}$

(c) $\phi = \cos^{-1} 1/7$ (d) $\phi = \sin^{-1} 5/7$

38. If $6\cos 2\theta + 2\cos^2\theta/2 + 2\sin^2\theta = 0$, $-\pi < \theta < \pi$, then $\theta =$

(a) $\pi/3$ (b) $\pi/3, \cos^{-1}(3/5)$

(c) $\cos^{-1}(3/5)$ (d) $\pi/3, \pi - \cos^{-1}(3/5)$

39. If $\sin^4 x + \cos^4 y + 2 = 4\sin x\cos y$, and $0 \le x, y \le \pi/2$ then $\sin x + \cos y$ is equal to :

(a) -2 (b) 0

(c) 2 (d) none of these

40. If $\sin\theta + 7\cos\theta = 5$, then $\tan(\theta/2)$ is a root of the equation

(a) $x^2 - 6x + 1 = 0$ (b) $6x^2 - x - 1 = 0$

(c) $6x^2 + x + 1 = 0$ (d) $x^2 - x + 6 = 0$

41. The value of $\cos y\cos(\pi/2 - x) - \cos(\pi/2 - y)\cos x + \sin y\cos(\pi/2 - x) + \cos x\sin(\pi/2 - y)$ is zero if

(a) $x = 0$ (b) $y = 0$

(c) $x = y + \pi/4$ (d) $y = x - 3\pi/4$

42. $\tan(p\pi/4) = \cot(q\pi/4)$ if

(a) $p + q = 0$

(b) $p + q = 2n + 1$

(c) $p + q = 2n$

(d) $p + q = 2(2n + 1)$ where n is any integer

43. If $\cos x - \sin x \geq 1$ and $0 \leq x \leq 2\pi$ then the solution set for x is

(a) $\left[0, \frac{\pi}{4}\right] \cup \left[\frac{7\pi}{4}, 2\pi\right]$ (b) $\left[\frac{3\pi}{2}, \frac{7\pi}{4}\right] \cup \{0\}$

(c) $\left[\frac{3\pi}{2}, 2\pi\right] \cup \{0\}$ (d) none of these

44. If $|\tan x| \leq 1$ and $x \in [-\pi, \pi]$ then the solution set for x is

(a) $\left[-\pi, -\frac{3\pi}{4}\right] \cup \left[-\frac{\pi}{4}, \frac{\pi}{4}\right] \cup \left[\frac{3\pi}{4}, \pi\right]$

(b) $\left[-\frac{\pi}{4}, \frac{\pi}{4}\right] \cup \left[\frac{3\pi}{4}, \pi\right]$

(c) $\left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$

(d) none of these

45. If $4\sin^2 x - 8\sin x + 3 \leq 0$, $0 \leq x \leq 2\pi$, then the solution set for x is

(a) $\left[0, \frac{\pi}{6}\right]$ (b) $\left[0, \frac{5\pi}{6}\right]$

(c) $\left[\frac{5\pi}{6}, 2\pi\right]$ (d) $\left[\frac{\pi}{6}, \frac{5\pi}{6}\right]$

46. $\cos 4x \cos 8x - \cos 5x \cos 9x = 0$ if

(a) $\cos 12x = \cos 14x$ (b) $\sin 13x = 0$

(c) $\sin x = 0$ (d) $\cos x = 0$

47. $\frac{\sin^3\theta - \cos^3\theta}{\sin\theta - \cos\theta} - \frac{\cos\theta}{\sqrt{1+\cot^2\theta}} - 2\tan\theta\cot\theta = -1$ if:

(a) $\theta \in \left(0, \frac{\pi}{2}\right)$ (b) $\theta \in \left(\frac{\pi}{2}, \pi\right)$

(c) $\theta \in \left(\pi, \frac{3\pi}{2}\right)$ (d) $\theta \in \left(\frac{3\pi}{2}, 2\pi\right)$

48. The least positive values of x satisfy the equation $8^{1+|\cos x|+\cos^2 x+|\cos^3 x|+......\infty} = 4^3$ will be (where $|\cos x| < 1$)

(a) $\frac{\pi}{3}$ (b) $\frac{2\pi}{3}$

(c) $\frac{\pi}{4}$ (d) None of these

49. If m and n (> m) are positive integers, the number of solutions of the equation $n|\sin x| = m|\cos x|$ in $[0, 2\pi]$ is

(a) m (b) n

(c) mn (d) none of these

50. The number of values of θ in the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ satisfying the equation $\left(\sqrt{3}\right)^{\sec^2\theta} = \tan^4\theta + 2\tan^2\theta$ is

(a) 1 (b) 2

(c) 3 (d) none of these

51. If $k_1 = \tan 27\theta - \tan\theta$ and

$k_2 = \frac{\sin\theta}{\cos 3\theta} + \frac{\sin 3\theta}{\cos 9\theta} + \frac{\sin 9\theta}{\cos 27\theta}$, then

(a) $k_1 = 2k_2$ (b) $k_1 = k_2 + 4$

(c) $k_1 = k_2$ (d) none of these

52. If $a = \sin\frac{\pi}{18}\sin\frac{5\pi}{18}\sin\frac{7\pi}{18}$, and x is the solution of the equation $y = 2[x] + 2$ and $y = 3[x - 2]$, where [x] denotes the integral part of x, then a is equal to

(a) [x] (b) $\frac{1}{[x]}$

(c) 2[x] (d) $[x]^2$

53. If sin x + cos y = a and cos x + sin y = b, then $\tan\frac{x-y}{2}$ is equal to

(a) a + b (b) a – b

(c) $\frac{a+b}{a-b}$ (d) $\frac{a-b}{a+b}$

54. If cot (θ – α), 3 cot θ, cot (θ + α) are in A.P., and θ is not an integral multiple of $\frac{\pi}{2}$, then sin θ cosec α is equal to :

(a) $\pm\sqrt{2}$ (b) $\pm\sqrt{\frac{3}{2}}$

(c) $\pm\sqrt{\frac{2}{3}}$ (d) none of these

55. Which of following functions have the maximum value unity?

(a) $\sin^2 x - \cos^2 x$

(b) $\sqrt{\frac{6}{5}}\left(\frac{1}{\sqrt{2}}\sin x + \frac{1}{\sqrt{3}}\cos x\right)$

(c) $\cos^6 + \sin^6 x$

(d) $\cos^2 x + \sin^4 x$

56. If sin θ + sin ϕ = a and cos θ + cos ϕ = b, then

(a) $\cos\left(\frac{\theta-\phi}{2}\right) = \pm\frac{1}{2}\sqrt{(a^2+b^2)}$

(b) $\cos\left(\frac{\theta-\phi}{2}\right) = \pm\frac{1}{2}\sqrt{(a^2-b^2)}$

(c) $\tan\left(\frac{\theta-\phi}{2}\right) = \pm\sqrt{\left(\frac{4-a^2-b^2}{a^2+b^2}\right)}$

(d) $\cos(\theta-\phi) = \frac{a^2+b^2-2}{2}$

57. If in Δ ABC if A > B, tan A + tan B + tan C = 6 and tan A tan B = 2, then $\sin^2 A : \sin^2 B : \sin^2 C$ is

(a) 8 : 9 : 5 (b) 8 : 5 : 9

(c) 5 : 9 : 8 (d) 5 : 8 : 5

58. If $\left(\frac{\sin\theta}{\sin\phi}\right)^2 = \frac{\tan\theta}{\tan\phi} = 3$, then

(a) $\tan\phi = 1/\sqrt{3}$ (b) $\tan\phi = -1/\sqrt{3}$

(c) $\tan\theta = \sqrt{3}$ (d) $\tan\theta = -\sqrt{3}$

59. Let $f_n(\theta) = \tan\frac{\theta}{2}$ (1 + sec θ) (1 + sec 2θ) (1 + sec 4θ)............ $(1 + \sec 2^n\theta)$, then

(a) $f_2\left(\frac{\pi}{16}\right) = 1$ (b) $f_3\left(\frac{\pi}{32}\right) = 1$

(c) $f_4\left(\frac{\pi}{64}\right) = 1$ (d) $f_5\left(\frac{\pi}{128}\right) = 1$

60. In a triangle ABC

(a) $\sin A \sin B \sin C \le \frac{3\sqrt{3}}{8}$

(b) $\sin^2 A + \sin^2 B + \sin^2 C \le \frac{9}{4}$

(c) sin A sin B sin C is always positive

(d) $\sin^2 A + \sin^2 B = 1 + \cos C$

61. The value(s) of the expression $\frac{\sin^3 x}{1+\cos x} + \frac{\cos^3 x}{1-\sin x}$ are

(a) $\sqrt{2}\cos\left(\frac{\pi}{4} - x\right)$ (b) $\sqrt{2}\cos\left(\frac{\pi}{4} + x\right)$

(c) $\sqrt{2}\sin\left(\frac{\pi}{4} - x\right)$ (d) $\sqrt{2}\sin\left(\frac{\pi}{4} + x\right)$

62. The equation sin 4x = a tan x, a > 0

(a) has no solution other than nπ if a > 4

(b) has solutions which are not of the type nπ if 0 < a < 4

(c) has solutions if a = 2009

(d) has no solutions if a = 2008

63. The equation $|\cot x| = \cot x + \frac{1}{\sin x}, (n \in Z)$

(a) has a general solution $\frac{2\pi}{3}(3n+1)$

(b) has a general solution $\frac{2\pi}{3}(3n-1)$

(c) is not defined if x = nπ

(d) cannot have a solution if cot x is positive

Assertion Reason

(A) If **ASSERTION** is true, **REASON** is true, **REASON** is a correct explanation for **ASSERTION**.

(B) If **ASSERTION** is true, **REASON** is true, **REASON** is not a correct explanation for **ASSERTION**.

(C) If **ASSERTION** is true, **REASON** is false

(D) If **ASSERTION** is false, **REASON** is true

(E) If both **ASSERTION** and **REASON** are false.

64. **Assertion :** If $2\sin\frac{\theta}{2} = \sqrt{1+\sin\theta} + \sqrt{1-\sin\theta}$, then $\frac{\theta}{2}$ lies between $2n\pi + \frac{\pi}{4}$ and $2n\pi + \frac{3\pi}{4}$

Reason : If $\frac{\theta}{2}$ runs from $\frac{\pi}{4}$ to $\frac{3\pi}{4}$, then $\sin\frac{\theta}{2} > 0$

(a) A (b) B (c) C

(d) D (e) E

65. **Assertion :** The numbers sin 18° and – sin 54° are the roots of same quadratic equation with integer co-efficients.

Reason : If x = 18°, then 5x = 90°, if y = – 54°, then 5y = –270°

(a) A (b) B (c) C

(d) D (e) E

66. **Assertion :** The function $f(x) = \min\{\sin x, \cos x\}$ takes the value $\frac{4}{5}$ twice when x varies from $\frac{20\pi}{3}$ to $\frac{43\pi}{6}$.

Reason : The periods of sin x and cos x are equal to 2π.

(a) A (b) B (c) C

(d) D (e) E

67. **Assertion :** The inequation $\tan x < \sqrt{3}$ is equivalent to $\cot x > \frac{1}{\sqrt{3}}$.

Reason : If a < b then $\frac{1}{a} > \frac{1}{b}$.

(a) A (b) B (c) C

(d) D (e) E

68. **Assertion :** The equation cos x + cos 2009 x = – 2 has infinite solution.

Reason : 2009 is an odd integer.

(a) A (b) B (c) C

(d) D (e) E

Using the following passage, solve Q.69 to Q.71

Passage –1

Given $\cos 2^m\theta \cos 2^{m+1}\theta \ldots\ldots\ldots\ldots \cos 2^n\theta$

$= \frac{\sin 2^{n+1}\theta}{2^{n-m+1}\sin 2^m\theta}$, where $2^m\theta \neq k\pi$, n, m, k ∈ I

Solve the following :

69. $\sin\frac{9\pi}{14} . \sin\frac{11\pi}{14} \sin\frac{13\pi}{14} =$

(a) $\frac{1}{64}$ (b) $-\frac{1}{64}$

(c) $\frac{1}{8}$ (d) $-\frac{1}{8}$

70. $\cos 2^3\frac{\pi}{10} \cos 2^4\frac{\pi}{10} \cos 2^5\frac{\pi}{10} \ldots\ldots\ldots \cos 2^{10}\frac{\pi}{10} =$

(a) $\frac{1}{128}$ (b) $\frac{1}{256}$

(c) $\frac{1}{512}\sin\frac{\pi}{10}$ (d) $\frac{\sqrt{5}-1}{512}\sin\frac{3\pi}{10}$

71. $\cos\frac{\pi}{11}\cos\frac{2\pi}{11}\cos\frac{3\pi}{11}\ ....\ \cos\frac{11\pi}{11}=$

(a) $-\frac{1}{32}$ (b) $\frac{1}{512}$

(c) $\frac{1}{1024}$ (d) $-\frac{1}{2048}$

Using the following passage, solve Q.72 to Q.74

Passage-2

If $P_n = \sin^n\theta + \cos^n\theta$ where $n \in W$ (whole number) and $\theta \in R$ (real number)

72. If $P_1 = m$, then the value of $4(1 - P_6)$ is

(a) $3(m-1)^2$ (b) $3(m^2-1)^2$

(c) $3(m+1)^2$ (d) $3(m^2+1)^2$

73. The value of $2P_6 - 3P_4 + 10$ is

(a) 0 (b) 6

(c) 9 (d) 15

74. If $P_{n-2} - P_n = \sin^2\theta\cos^2\theta\, P_\lambda$, then the value of λ is

(a) n – 1 (b) n – 2

(c) n – 3 (d) n – 4

Match the column

75.

	Column - I		Column - II
(A)	$\cos^2 52\frac{1°}{2} - \sin^2 22\frac{1°}{2}$	(P)	1
(B)	$\cos^2\frac{3\pi}{5} + \cos^2\frac{4\pi}{5}$	(Q)	$\frac{3-\sqrt{3}}{4\sqrt{2}}$
(C)	$\sin 24° + \cos 6°$	(R)	$\frac{3}{4}$
(D)	$\sin^2 50 + \cos^2 130°$	(S)	$\frac{\sqrt{15}+\sqrt{3}}{4}$

76. Match the following for the trigonometric equation

$\left|x+\frac{1}{4}\right| - \left|x-\frac{1}{4}\right| = \cos\pi x$, (n is an integer) :

	Column - I		Column - II
(A)	Over $\left(-\infty, -\frac{1}{4}\right)$	(P)	$\left\{\frac{1}{3}\right\} \cup \left\{2n \pm \frac{1}{3}, n > 0\right\}$
(B)	Over $\left[-\frac{1}{4}, \frac{1}{4}\right]$	(Q)	$\left\{-\frac{2}{3}\right\} \cup \left\{2n \pm \frac{2}{3}, n < 0\right\}$
(C)	Over $\left(\frac{1}{4}, \infty\right)$	(R)	No solution

Subjective

77. A solution of the equation $\sin 5x + \sin x + 2\sin^2 x = 1$ lying in the interval $\left(0, \frac{1}{5}\right)$ is $\frac{\pi}{\lambda}$ then λ must be

78. The smallest positive angle (in degrees) satisfying $\tan(x + 100°) = \tan(x + 50°)\tan x \tan(x - 50°)$ must be

79. If k is a positive integer, such that

(i) $\cos^2 x \sin x > -\frac{7}{k}$, for all x

(ii) $\cos^2 x \sin x < -\frac{7}{k+1}$ for some x, then k must be equal to

80. The value of $(1 - \cot 23°)(1 - \cot 22°)$ must be equal to

EXERCISE - 4 : PREVIOUS YEAR JEE ADVANCED QUESTIONS

Single Answer Question

1. If $\tan\theta = -\frac{4}{3}$, then $\sin\theta$ is **(1978)**

(a) $-\frac{4}{5}$ but not $\frac{4}{5}$ (b) $-\frac{4}{5}$ or $\frac{4}{5}$

(c) $\frac{4}{5}$ but not $-\frac{4}{5}$ (d) None of these

2. If $\alpha+\beta+\gamma=2\pi$, then **(1979)**

(a) $\tan\frac{\alpha}{2}+\tan\frac{\beta}{2}+\tan\frac{\gamma}{2}=\tan\frac{\alpha}{2}\tan\frac{\beta}{2}\tan\frac{\gamma}{2}$

(b) $\tan\frac{\alpha}{2}\tan\frac{\beta}{2}+\tan\frac{\beta}{2}\tan\frac{\gamma}{2}+\tan\frac{\gamma}{2}\tan\frac{\alpha}{2}=1$

(c) $\tan\frac{\alpha}{2}+\tan\frac{\beta}{2}+\tan\frac{\gamma}{2}=-\tan\frac{\alpha}{2}\tan\frac{\beta}{2}\tan\frac{\gamma}{2}$

(d) None of the above

3. Given $A=\sin^2\theta+\cos^4\theta$, then for all real values of θ : **(1980)**

(a) $1\le A\le 2$ (b) $\frac{3}{4}\le A\le 1$

(c) $\frac{13}{16}\le A\le 1$ (d) $\frac{3}{4}\le A\le\frac{13}{16}$

4. The equation $2\cos^2\left(\frac{1}{2}x\right)\sin^2 x = x^2+x^{-2},\ x\le\frac{\pi}{9}$ has : **(1980)**

(a) no real solution

(b) one real solution

(c) more than one real solution

(d) none of these

5. The general solution of the trigonometric equation $\sin x+\cos x=1$ is given by : **(1981)**

(a) $x=2n\pi;\ n=0,\pm1,\pm2.......$

(b) $x=2n\pi+\pi/2;\ n=0,\pm1,\pm2.......$

(c) $x=n\pi+(-1)^n\frac{\pi}{4}-\frac{\pi}{4};\ n=0,\pm1,\pm2.....$

(d) none of these

6. $\left(1+\cos\frac{\pi}{8}\right)\left(1+\cos\frac{3\pi}{8}\right)\left(1+\cos\frac{5\pi}{8}\right)\left(1+\cos\frac{7\pi}{8}\right)$

is equal to **(1984)**

(a) $\frac{1}{2}$ (b) $\cos\frac{\pi}{8}$

(c) $\frac{1}{8}$ (d) $\frac{1+\sqrt{2}}{2\sqrt{2}}$

7. The expression $3\left[\sin^4\left(\frac{3\pi}{2}-\alpha\right)+\sin^4(3\pi+\alpha)\right]$

$-2\left[\sin^6\left(\frac{\pi}{2}+\alpha\right)+\sin^6(5\pi-\alpha)\right]$ is equal to **(1986)**

(a) 0 (b) 1

(c) 3 (d) $\sin 4\alpha+\cos 6\alpha$

8. The number of all possible triplets (a_1, a_2, a_3) such that $a_1+a_2\cos(2x)+a_3\sin^2(x)=0$ for all x is : **(1987)**

(a) zero (b) one

(c) three (d) inifinite

9. The smallest positive root of the equation, $\tan x - x = 0$ lies in : **(1987)**

(a) $\left(0,\frac{\pi}{2}\right)$ (b) $\left(\frac{\pi}{2},\pi\right)$

(c) $\left(\pi,\frac{3\pi}{2}\right)$ (d) $\left(\frac{3\pi}{2},2\pi\right)$

10. The value of the expression $\sqrt{3}$ cosec 20° – sec 20° is equal to : **(1988)**

(a) 2 (b) 2 sin 20°/ sin 40°

(c) 4 (d) 4 sin 20°/ sin 40°

11. The general solution of sin x –3 sin 2x + sin 3x = cos x –3 cos 2x + cos 3x is : **(1989)**

(a) $n\pi + \frac{\pi}{8}$ (b) $\frac{n\pi}{2} + \frac{\pi}{8}$

(c) $(-1)^n \frac{n\pi}{2} + \frac{\pi}{8}$ (d) $2n\pi + \cos^{-1}\frac{3}{2}$

12. The equation (cos p –1) x^2 + (cos p) x + sin p = 0 in the variable x, has real roots. Then p can take any value in the interval : **(1990)**

(a) (0, 2π) (b) (–π, 0)

(c) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ (d) (0, π)

13. In a triangle ABC, angle A is greater than angle B. If the measures of angles A and B satisfy the equation 3 sin x –4 $\sin^3$ x –k = 0, 0 < k < 1, then the measure of angle C is : **(1990)**

(a) π/3 (b) π/2

(c) 2π/3 (d) 5π/6

14. The number of solutions of the equation $\sin(e)^x = 5^x + 5^{-x}$ is : **(1991)**

(a) 0 (b) 1

(c) 2 (d) infinitely many

15. Number of solution of the equation tan x + sec x = 2 cos x lying in the interval [0, 2π] is: **(1993)**

(a) 0 (b) 1

(c) 2 (d) 3

16. The general value of θ satisfying the equation 2 $\sin^2$ θ – 3 sin θ –2 = 0, is : **(1995)**

(a) $n\pi + (-1)^n \pi/6$ (b) $n\pi + (-1)^n \pi/2$

(c) $n\pi + (-1)^2 5\pi/6$ (d) $n\pi + (-1)^n 7\pi/6$

17. $3(\sin x - \cos x)^4 + 6(\sin x + \cos x)^2 + 4(\sin^6 x + \cos^6 x)$ equal to : **(1995)**

(a) 11 (b) 12

(c) 13 (d) 14

18. The number of values of x in the interval [0, 5π] satisfying the equation 3 $\sin^2$ x –7 sin x +2 = 0 is : **(1998)**

(a) 0 (b) 5

(c) 6 (d) 10

19. Which of the following numbers is rational ? **(1998)**

(a) sin 15° (b) cos 15°

(c) sin 15° cos 15° (d) sin 15° cos 75°

20. If $\alpha + \beta = \frac{\pi}{2}$ and $\beta + \gamma = \alpha$, then tan α equals : **(2001)**

(a) 2 (tan β + tan γ) (b) tan β + tan γ

(c) tan β + 2 tan γ (d) 2 tan β + tan γ

21. The number of integral values of k for which the equation 7 cos x + 5 sin x = 2k + 1 has a solution, is : **(2002)**

(a) 4 (b) 8

(c) 10 (d) 12

22. Given both θ and ϕ are acute angles $\sin\theta = \frac{1}{2}$, $\cos\phi = \frac{1}{3}$, then the value of θ + ϕ belongs to **(2004)**

(a) $\left(\frac{\pi}{3}, \frac{\pi}{6}\right]$ (b) $\left(\frac{\pi}{2}, \frac{2\pi}{3}\right)$

(c) $\left(\frac{2\pi}{3}, \frac{5\pi}{6}\right]$ (d) $\left(\frac{5\pi}{6}, \pi\right]$

23. The number of ordered pairs (α, β), where α, β ∈ (–π, π) satisfying cos (α – β) =1 and $\cos(\alpha+\beta) = \frac{1}{e}$ is : **(2005)**

(a) 0 (b) 1

(c) 2 (d) 4

24. The set of value of θ satisfying the inequation $2\sin^2\theta - 5\sin\theta + 2 > 0$, where $0 < \theta < 2\pi$, is: **(2006)**

(a) $\left(0, \frac{\pi}{6}\right) \cup \left(\frac{5\pi}{6}, 2\pi\right)$ (b) $\left[0, \frac{\pi}{6}\right] \cup \left[\frac{5\pi}{6}, 2\pi\right]$

(c) $\left[0, \frac{\pi}{3}\right] \cup \left[\frac{2\pi}{3}, 2\pi\right]$ (d) none of these

25. Let $\theta \in \left(0, \frac{\pi}{4}\right)$ and $t_1 = (\tan\theta)^{\cot\theta}$, $t_2 = (\tan\theta)^{\cot\theta}$, $t_3 = (\cot\theta)^{\tan\theta}$ and $t_4 = (\cot\theta)^{\cot\theta}$, then **(2006)**

(a) $t_1 > t_2 > t_3 > t_4$ (b) $t_4 > t_3 > t_1 > t_2$
(c) $t_3 > t_1 > t_2 > t_4$ (d) $t_2 > t_3 > t_1 > t_4$

26. The number of solutions of the pair of equations $2\sin^2\theta - \cos 2\theta = 0$ & $2\cos^2\theta - 3\sin\theta = 0$ in the interval $[0, 2\pi]$ is **(2007)**

(a) zero (b) one
(c) two (d) four

27. Let $P = \{\theta : \sin\theta - \cos\theta = \sqrt{2}\cos\theta\}$ and $Q = \{\theta : \sin\theta + \cos\theta = \sqrt{2}\sin\theta\}$ be two sets. Then **(2011)**

(a) $P \subset Q$ and $Q - P \neq \phi$ (b) $Q \not\subset P$
(c) $P \not\subset Q$ (d) $P = Q$

28. In a triangle the sum of two sides is x and the product of the same two sides is y. If $x^2 - c^2 = y$, where c is the third side of the triangle, then the ratio of the in-radius to the circum-radius of the triangle is **(2014)**

(a) $\frac{3y}{2x(x+c)}$ (b) $\frac{3y}{2c(x+c)}$

(c) $\frac{3y}{4x(x+c)}$ (d) $\frac{3y}{4c(x+c)}$

29. For $x \in (0, \pi)$, then eqation $\sin x + 2\sin 2x - \sin 3x = 3$ has **(2014)**

(a) infinitely many solutions (b) three solutions
(c) one solution (d) no solution

30. Let $S = \left\{x \in (-\pi, \pi) : x \neq 0, \pm\frac{\pi}{2}\right\}$. The sum of all distinct solution of the equation $\sqrt{3}\sec x + \operatorname{cosec} x + 2(\tan x - \cot x) = 0$ in the set S is equal to **(2016)**

(a) $-\frac{7\pi}{9}$ (b) $-\frac{2\pi}{9}$

(c) 0 (d) $\frac{5\pi}{9}$

31. The value of $\sum_{k=1}^{13} \frac{1}{\sin\left(\frac{\pi}{4} + \frac{(k-1)\pi}{6}\right)\sin\left(\frac{\pi}{4} + \frac{k\pi}{6}\right)}$ is equal to **(2016)**

(a) $3 - \sqrt{3}$ (b) $2(3 - \sqrt{3})$

(c) $2(\sqrt{3} - 1)$ (d) $2(2 + \sqrt{3})$

Multiple Answer Question

32. The values of θ lying between θ = 0 and θ = π/2 and satisfying the equation

$$\begin{vmatrix} 1+\sin^2\theta & \cos^2\theta & 4\sin 4\theta \\ \sin^2\theta & 1+\cos^2\theta & 4\sin 4\theta \\ \sin^2\theta & \cos^2\theta & 1+4\sin 4\theta \end{vmatrix} = 0, \text{ is}$$ **(1988)**

(a) 7π/24 (b) 5π/24
(c) 11π/24 (d) π/24

33. For $0 < \phi < \pi/2$, if $x = \sum_{n=0}^{\infty} \cos^{2n}\phi$, $y = \sum_{n=0}^{\infty} \sin^{2n}\phi$, $z = \sum_{n=0}^{\infty} \cos^{2n}\phi \sin^{2n}\phi$, then **(1993)**

(a) $xyz = xz + y$ (b) $xyz = xy + z$
(c) $xyz = x + y + z$ (d) $xyz = yz + x$

34. $\sec^2\theta = \frac{4xy}{(x+y)^2}$ is true if and only if **(1996)**

(a) $x = y \neq 0$ (b) $x = y, x \neq 0$
(c) $x = y$ (d) $x \neq 0, y \neq 0$

35. For a positive integer n, let

$f_n(\theta) = \left(\tan\frac{\theta}{2}\right)(1+\sec\theta)(1+\sec 2\theta)$

$(1 + \sec 2^2\theta) \ldots (1 + \sec 2^n\theta)$, then **(1999)**

(a) $f_2\left(\frac{\pi}{16}\right) = 1$ (b) $f_3\left(\frac{\pi}{32}\right) = 1$

(c) $f_4\left(\frac{\pi}{64}\right) = 1$ (d) $f_5\left(\frac{\pi}{128}\right) = 1$

36. If $\frac{\sin^4 x}{2}+\frac{\cos^4 x}{3}=\frac{1}{5}$, then **(2009)**

(a) $\tan^2 x=\frac{2}{3}$ (b) $\frac{\sin^8 x}{8}+\frac{\cos^8 x}{27}=\frac{1}{125}$

(c) $\tan^2 x=\frac{1}{3}$ (d) $\frac{\sin^8 x}{8}+\frac{\cos^8 x}{27}=\frac{2}{125}$

37. For $0<\theta<\frac{\pi}{2}$, the solution(s) of $\sum_{m=1}^{6}\operatorname{cosec}\left(\theta+\frac{(m-1)\pi}{4}\right)\operatorname{cosec}\left(\theta+\frac{m\pi}{4}\right)=4\sqrt{2}$ is/are **(2009)**

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{6}$

(c) $\frac{\pi}{12}$ (d) $\frac{5\pi}{12}$

38. Let $\theta, \phi \in [0, 2\pi]$ be such that $2\cos\theta(1-\sin\phi)=\sin^2\theta\left(\tan\frac{\theta}{2}+\cot\frac{\theta}{2}\right)\cos\phi-1$, $\tan(2\pi-\theta)>0$ and $-1<\sin\theta<-\frac{\sqrt{3}}{2}$. Then, ϕ cannot satisfy **(2012)**

(a) $0<\phi<\frac{\pi}{2}$ (b) $\frac{\pi}{2}<\phi<\frac{4\pi}{3}$

(c) $\frac{4\pi}{3}<\phi<\frac{3\pi}{2}$ (d) $\frac{3\pi}{2}<\phi<2\pi$

39. In a ΔPQR, P is the largest angle and $\cos P=\frac{1}{3}$. Further in circle of the triangle touches the sides PQ, QR and RP at N, L and M respectively, such that the lengths of PN, QL and RM are consecutive even integers. Then, possible length(s) of the side(s) of the triangle is (are) **(2013)**

(a) 16 (b) 18

(c) 24 (d) 22

40. In a triangle XYZ, let x, y, z be the lengths of sides opposite to the angles X, Y, Z, respectively, and 2s = x + y + z. If $\frac{s-x}{4}=\frac{s-y}{3}=\frac{s-z}{2}$ and area of incircle of the triangle XYZ is $\frac{8\pi}{3}$, then **(2016)**

(a) area of the triangle XYZ is $6\sqrt{6}$

(b) the radius of circumcircle of the triangle XYZ is $\frac{35}{6}\sqrt{6}$

(c) $\sin\frac{X}{2}\sin\frac{Y}{2}\sin\frac{Z}{2}=\frac{4}{35}$

(d) $\sin^2\left(\frac{X+Y}{2}\right)=\frac{3}{5}$

41. Let α and β be non zero real numbers such that $2(\cos\beta-\cos\alpha)+\cos\alpha\cos\beta=1$. Then which of the following is/are true? **(2017)**

(a) $\tan\left(\frac{\alpha}{2}\right)+\sqrt{3}\tan\left(\frac{\beta}{2}\right)=0$

(b) $\sqrt{3}\tan\left(\frac{\alpha}{2}\right)-\tan\left(\frac{\beta}{2}\right)=0$

(c) $\tan\left(\frac{\alpha}{2}\right)-\sqrt{3}\tan\left(\frac{\beta}{2}\right)=0$

(d) $\sqrt{3}\tan\left(\frac{\alpha}{2}\right)+\tan\left(\frac{\beta}{2}\right)=0$

Match the Column

42. $(\sin 3\alpha)/(\cos 2\alpha)$ is **(1992)**

	Column I		Column II
(A)	positive	(p)	$(13\pi/48, 14\pi/48)$
(B)	negative	(q)	$(14\pi/48, 18\pi/48)$
		(r)	$(18\pi/48, 23\pi/48)$
		(s)	$(0, \pi/2)$

Integer Answer Type Question

43. The number of all possible values of θ, where $0 < \theta < \pi$, for which the system of equations

$(y+z)\cos 3\theta = (xyz)\sin 3\theta$

$$x\sin 3\theta = \frac{2\cos 3\theta}{y} + \frac{2\sin 3\theta}{z}$$

and $(xyz)\sin 3\theta = (y+2z)\cos 3\theta\ y\sin 3\theta$ have a solution (x_0, y_0, z_0) with $y_0 z_0 \neq 0$, is ..... **(2010)**

44. The number of distinct solution of the equation

$\frac{5}{4}\cos^2 2x + \cos^4 x + \sin^4 x + \cos^6 x + \sin^6 x = 2$ in the interval $[0, 2\pi]$ is **(2015)**

Analytical and Descriptive Questions

45. Prove that $\sin x \,.\, \sin y \,.\, \sin(x-y) + \sin y \,.\, \sin z \,.\, \sin(y-z)$ $+ \sin z \,.\, \sin x \,.\, \sin(z-x) + \sin(x-y) \,.\, \sin(y-z) \,.\, \sin(z-x) = 0$ **(1978)**

46. If $\cos(\alpha+\beta) = \frac{4}{5}$, $\sin(\alpha-\beta) = \frac{5}{13}$ and α, β lie between 0 and π/4, find tan 2α. **(1979)**

47. Suppose $\sin^3 x \sin 3x = \sum_{m=0}^{n} C_m \cos mx$ is an identity in x, where $C_0, C_1, ..., C_n$ are constants and $C_n \neq 0$. Then the value of n is ..... **(1981)**

Fill in the blanks

48. The larger of cos (log θ) and log (cos θ) if $e^{-\pi/2} < \theta < \frac{\pi}{2}$, is .... **(1983)**

49. The solution set of the system of equations $x + y = \frac{2\pi}{3}$, $\cos x + \cos y = \frac{3}{2}$, where x and y are real, is..... **(1986)**

50. The set of all x in the interval $[0, \pi]$ for which $2\sin^2 x - 3\sin x + 1 \geq 0$, is.... . **(1987)**

51. The value of $\sin\frac{\pi}{14}\sin\frac{3\pi}{14}\sin\frac{5\pi}{14}\sin\frac{7\pi}{14}\sin\frac{9\pi}{14}\sin\frac{11\pi}{14}\sin\frac{13\pi}{14}$ is equal to .... **(1991)**

52. If $k = \sin(\pi/18)\sin(5\pi/18)\sin(7\pi/18)$, then the numerical value of k is .... **(1993)**

53. General value of θ satisfying the equation $\tan^2\theta + \sec 2\theta = 1$ is.... . **(1996)**

True/False

54. If $\tan A = (1 - \cos B) / \sin B$, then $\tan 2A = \tan B$. **(1983)**

55. There exists a value of θ between 0 and 2π that satisfies the equation $\sin^4\theta - 2\sin^2\theta - 1 = 0$. **(1984)**

Subjective Question

56. Solve $2(\cos x + \cos 2x) + (1 + 2\cos x)\sin 2x = 2\sin x, -\pi \leq x \leq \pi$ **(1978)**

57. Prove that $5\cos\theta + 3\cos\left(\theta + \frac{\pi}{3}\right) + 3$ lies between –4 and 10. **(1979)**

58. Given $\alpha + \beta + \gamma = \pi$, prove that $\sin^2\alpha + \sin^2\beta - \sin^2\gamma = 2\sin\alpha\sin\beta\cos\gamma$. **(1980)**

59. For all θ in [0, π/2], show that $\cos(\sin\theta) \geq \sin(\cos\theta)$. **(1981)**

60. Find the coordinates of the points of intersection of the curves $y = \cos x$, $y = \sin 3x$, if $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$. **(1982)**

61. Show that the equation $e^{\sin x} - e^{-\sin x} - 4 = 0$ has no real solution. **(1982)**

62. Without using tables, prove that

$(\sin 12^\circ)(\sin 48^\circ)(\sin 54^\circ) = \frac{1}{8}$. **(1982)**

63. Show that $16\cos\left(\frac{2\pi}{15}\right)\cos\left(\frac{4\pi}{15}\right)\cos\left(\frac{8\pi}{15}\right)\cos\left(\frac{16\pi}{15}\right) = 1$. **(1983)**

64. Find all the solutions of $4\cos^2 x\sin x - 2\sin^2 x = 3\sin x$. **(1983)**

65. Find the values of x (–π, π) which satisfy the equation $2^{1+|\cos x|+|\cos^2 x|+....} = 4$ **(1984)**

66. Consider the system of linear equations in x, y, z

$(\sin 3\theta)x - y + z = 0$

$(\cos 2\theta)x + 4y + 3z = 0$

$2x + 7y + 7z = 0$

Find the values of θ for which this system has non-trivial solutions. **(1986)**

67. Prove that $\tan\alpha + 2\tan 2\alpha + 4\tan 4\alpha + 8\cot 8\alpha = \cot\alpha$. **(1988)**

68. ABC is a triangle such that

$$\sin(2A+B) = \sin(C-A) = -\sin(B+2C) = \frac{1}{2}.$$

If A, B and C are in Arithmetic Progression, determine the values of A, B and C. **(1990)**

69. If $\exp\{(\sin^2 x + \sin^4 x + \sin^6 x + ...\infty)\log_e 2\}$, satisfies the equation $x^2 - 9x + 8 = 0$, find the value of $\frac{\cos x}{\cos x + \sin x}, 0 < x < \frac{\pi}{2}$. **(1991)**

70. Determine the smallest positive value of x (in degrees) for which $\tan(x+100°) = \tan(x+50°)\tan(x)\tan(x-50°)$ **(1993)**

71. Find the smallest positive number p for which the equation $\cos(p\sin x) = \sin(p\cos x)$ has a solution $x \in [0, 2\pi]$. **(1995)**

72. Find number of values of θ in the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ satisfying the equation

$$(1-\tan\theta)(1+\tan\theta)\sec^2\theta + 2^{\tan^2\theta} = 0.$$

(1996)

73. Prove that the values of the function $\frac{\sin x\cos 3x}{\sin 3x\cos x}$ do not lie between $\frac{1}{3}$ and 3 for any real x. **(1997)**

74. In any triangle prove that

$$\cot\frac{A}{2} + \cot\frac{B}{2} + \cot\frac{C}{2} = \cot\frac{A}{2}\cot\frac{B}{2}\cot\frac{C}{2}$$ **(2000)**

ANSWER KEY

EXERCISE - 1 : BASIC OBJECTIVE QUESTIONS

1. (b)	2. (d)	3. (d)	4. (c)	5. (a)	6. (a)	7. (b)	8. (c)
9. (b)	10. (c)	11. (d)	12. (d)	13. (a)	14. (b)	15. (a)	16. (b)
17. (d)	18. (a)	19. (a,d)	20. (d)	21. (b)	22. (b)	23. (b)	24. (a)
25. (a)	26. (b)	27. (c)	28. (a,b,c,d)	29. (c)	30. (b,c)	31. (b)	32. (a)
33. (a)	34. (a)	35. (d)	36. (b)	37. (a)	38.(b)	39. (a)	40. (b)
41 (b)	42 (a)	43. (b)	44. (b)	45. (b)	46. (c)	47. (b)	48. (a)
49 (b)	50 (b,c)	51. (b,d)	52. (b)	53. (d)	54. (b)	55. (c)	56. (d)
57. (c)	58. (b)	59. (d)	60. (a)	61. (d)	62. (b)	63. (b)	64. (d)
65. (c)	66. (a)	67. (c)	68. (b)	69. (a)	70. (c)		

EXERCISE - 2 : PREVIOUS YEAR JEE MAINS QUESTIONS

1. (a)	2. (b)	3. (c)	4. (c)	5. (b)	6. (a)	7. (d)	8. (a)
9. (b)	10. (c)	11. (b)	12. (b)	13. (a)	14. (c)	15. (b)	16. (c)
17. (a)	18. (c)	19. (c)	20. (a)	21. (b)	22. (c)	23. (c)	24. (c)
25. (c)	26. (b)	27. (c)	28. (d)	29. (a)	30. (b)	31. (a)	32. (b)
33. (a)	34. (a)	35. (c)	36. (b)	37. (c)	38. (c)	39. (b)	40. (b)
41. (d)	42. (c)	43. (d)	44. (c)	45. (c)	46. (c)	47. (b)	48. (a)
49. (c)	50. (c)	51. (a)					

EXERCISE - 3 : ADVANCED OBJECTIVE QUESTIONS

1. (c)	2. (a)	3. (b)	4. (c)	5. (a)	6. (b)	7. (c)	8. (c)
9. (b)	10. (b)	11. (d)	12. (b)	13. (b)	14. (c)	15. (d)	16. (b)
17. (d)	18. (c)	19. (b)	20. (c)	21. (a)	22. (b)	23. (a)	24. (b)
25. (b)	26. (c)	27. (a,d)	28. (a)	29. (a,b)	30. (c)	31. (a)	32. (c)
33. (a,b,c,d)	34. (c,d)	35. (a,c)	36. (a,b,c,d)	37. (a,b)	38. (d)	39. (c)	40. (b)
41. (d)	42. (d)	43. (c)	44.(a)	45. (d)	46. (a,b,c)	47. (b)	48. (a,b)
49. (d)	50. (d)	51. (a)	52. (b)	53. (d)	54. (b)	55. (a,b,c,d)	56. (a,c,d)
57. (b)	58. (a,b,c,d)	59. (a,b,c,d)	60. (a,b,c)	61. (a,d)	62. (a,b,c)	63. (a,c,d)	64. (b)
65. (a)	66. (d)	67. (e)	68. (a)	69. (c)	70. (b)	71. (c)	72. (b)
73. (c)	74. (d)	75. $A \to Q; B \to R; C \to S; D \to P$			76. $A \to Q; B \to R; C \to P$		
77. 0018	78. 0030	79. 0018	80. 0002				

EXERCISE - 4 : PREVIOUS YEAR JEE ADVANCED QUESTIONS

1. (b)	2. (a)	3. (b)	4. (a)	5. (c)	6. (c)	7. (b)	8. (d)
9. (c)	10. (c)	11. (b)	12. (d)	13. (c)	14. (a)	15. (c)	16. (d)
17. (c)	18. (c)	19. (c)	20. (c)	21. (b)	22. (b)	23. (d)	24. (a)
25. (b)	26. (c)	27. (d)	28. (b)	29. (d)	30. (c)	31. (c)	32. (a,c)
33. (b,c)	34. (a,b)	35. (a,b,c,d)	36. (a,b)	37. (c,d)	38. (a,c,d)	39. (b,d)	40. (a,c,d)
41. (a,c)	42. A–r; B–p	43. (0003)	44. (0008)	46. $\frac{56}{33}$	47. 6	48. cos (log θ)	

49. no solution

50. $x \in \left[0, \frac{\pi}{6}\right] \cup \left\{\frac{\pi}{2}\right\} \cup \left[\frac{5\pi}{6}, \pi\right]$

51. $\frac{1}{64}$

52. $\frac{1}{8}$

53. $\theta = m\pi, n\pi \pm \frac{\pi}{3}$

54. True

55. False

56. $x = -\pi, -\frac{\pi}{2}, -\frac{\pi}{3}, \frac{\pi}{3}, \pi$

60. $\left(\frac{\pi}{8}, \cos\frac{\pi}{8}\right)\left(\frac{\pi}{4}, \cos\frac{\pi}{4}\right)\left(-\frac{3\pi}{8}, \cos\frac{3\pi}{8}\right)$

64. $\{x : x = n\pi\} \cup \left\{x : x = n\pi + (-1)^n \frac{\pi}{10}\right\} \cup \left\{x : x = n\pi + (-1)^n \left(\frac{-3\pi}{10}\right)\right\}$

65. $\left\{\pm\frac{\pi}{3}, \pm\frac{2\pi}{3}\right\}$

66. $\theta = n\pi \text{ or } n\pi + (-1)^n \left(\frac{\pi}{6}\right)$

68. $A = 45^\circ, B = 60^\circ, C = 75^\circ$

69. $\frac{\sqrt{3} - 1}{2}$

70. 30°

71. $\frac{\pi}{2\sqrt{2}}$

72. 4

Dream on !!