CONTINUITY

SELECT THE CORRECT ALTERNATIVE (ONLY ONE CORRECT ANSWER)

1. If $f(x)=\begin{cases} x+2 & , \text{ when } x<1 \\ 4x-1 & , \text{ when } 1\le x\le 3 \\ x^2+5 & , \text{ when } x>3 \end{cases}$, then correct statement is -

(A) $\lim\limits_{x\to1} f(x)=\lim\limits_{x\to3} f(x)$
(B) f(x) is continuous at x = 3
(C) f(x) is continuous at x = 1
(D) f(x) is continuous at x = 1 and 3

2. If $f(x)=\begin{cases} \dfrac{1}{e^{1/x}+1} & , x\ne 0 \\ 0 & , x=0 \end{cases}$, then -

(A) $\lim\limits_{x\to0+} f(x)=1$
(B) $\lim\limits_{x\to0-} f(x)=0$
(C) f(x) is discontinuous at x = 0
(D) f(x) is continuous

3. If function f(x) = $\dfrac{\sqrt{1+x}-\sqrt[3]{1+x}}{x}$, is continuous function, then f(0) is equal to -

(A) 2 (B) 1/4 (C) 1/6 (D) 1/3

4. If $f(x)=\begin{cases} \dfrac{x^2-(a+2)x+2a}{x-2} & , x\ne 2 \\ 2 & , x=2 \end{cases}$ is continuous at x = 2, then a is equal to -

(A) 0 (B) 1 (C) –1 (D) 2

5. If f(x) = $\begin{cases} \dfrac{\log(1+2ax)-\log(1-bx)}{x} & , x\ne 0 \\ k & , x=0 \end{cases}$, is continuous at x = 0 , then k is equal to -

(A) 2a + b (B) 2a – b (C) b – 2a (D) a + b

6. If $f(x)=\begin{cases} [x]+[-x], & x\ne 2 \\ \lambda, & x=2 \end{cases}$, f is continuous at x = 2 then λ is (where [.] denotes greatest integer) -

(A) –1 (B) 0 (C) 1 (D) 2

7. If f(x) = $\begin{cases} \dfrac{1-\cos 4x}{x^2} & , x<0 \\ a & , x=0 \\ \dfrac{\sqrt{x}}{\sqrt{16+\sqrt{x}}-4} & , x>0 \end{cases}$, then correct statement is -

(A) f(x) is discontinuous at $x=0$ for any value of a
(B) f(x) is continuous at x = 0 when a = 8
(C) f(x) is continuous at x = 0 when a = 0
(D) none of these

8. Function $f(x) = \dfrac{1}{\log|x|}$ is discontinuous at -

(A) one point (B) two points (C) three points (D) infinite number of points

9. Which of the following functions has finite number of points of discontinuity in R (where [.] denotes greatest integer)

(A) $\tan x$ (B) $|x| / x$ (C) $x + [x]$ (D) $\sin[\pi x]$

10. If $f(x) = \dfrac{1-\tan x}{4x-\pi}, x \neq \dfrac{\pi}{4}, x \in \left[0, \dfrac{\pi}{2}\right)$ is a continuous functions, then $f(\pi/4)$ is equal to -

(A) $-1/2$ (B) $1/2$ (C) 1 (D) -1

11. The value of f(0), so that function, $f(x) = \dfrac{\sqrt{a^2-ax+x^2}-\sqrt{a^2+ax+x^2}}{\sqrt{a+x}-\sqrt{a-x}}$ becomes continuous for all x, is given by -

(A) $a\sqrt{a}$ (B) $-\sqrt{a}$ (C) $\sqrt{a}$ (D) $-a\sqrt{a}$

12. If $f(x) = \dfrac{x-e^x+\cos 2x}{x^2}$, $x \neq 0$ is continuous at $x = 0$, then -

(A) $f(0) = \dfrac{5}{2}$ (B) $[f(0)] = -2$ (C) $\{f(0)\} = -0.5$ (D) $[f(0)].\{f(0)\} = -1.5$

where [x] and {x} denotes greatest integer and fractional part function.

13. Let $f(x) = \dfrac{x(1+a\cos x)-b\sin x}{x^3}$, $x \neq 0$ and $f(0) = 1$. The value of a and b so that f is a continuous function are -

(A) 5/2, 3/2 (B) 5/2, −3/2 (C) −5/2, −3/2 (D) none of these

14. 'f' is a continuous function on the real line. Given that $x^2+(f(x)-2)x-\sqrt{3}.f(x)+2\sqrt{3}-3=0$. Then the value of $f(\sqrt{3})$ is -

(A) $\dfrac{2(\sqrt{3}-2)}{\sqrt{3}}$ (B) $2(1-\sqrt{3})$ (C) zero (D) cannot be determined

SELECT THE CORRECT ALTERNATIVES (ONE OR MORE THAN ONE CORRECT ANSWERS)

15. The value(s) of x for which $f(x) = \dfrac{e^{\sin x}}{4-\sqrt{x^2-9}}$ is continuous, is (are) -

(A) 3 (B) −3 (C) 5 (D) all $x \in (-\infty, -3] \cup [3, \infty)$

16. Which of the following function(s) not defined at $x = 0$ has/have removable discontinuity at the origin ?

(A) $f(x) = \dfrac{1}{1+2^{\cot x}}$ (B) $f(x) = \cos\left(\dfrac{|\sin x|}{x}\right)$

(C) $f(x) = x\sin\dfrac{\pi}{x}$ (D) $f(x) = \dfrac{1}{\ell n|x|}$

17. Function whose jump (non-negative difference of LHL & RHL) of discontinuity is greater than or equal to one, is/are -

(A) $f(x) = \begin{cases} \dfrac{(e^{1/x}+1)}{(e^{1/x}-1)} & ; \quad x < 0 \\ \dfrac{(1-\cos x)}{x} & ; \quad x > 0 \end{cases}$

(B) $g(x) = \begin{cases} \dfrac{x^{1/3}-1}{x^{1/2}-1} & ; \quad x > 1 \\ \dfrac{\ell nx}{(x-1)} & ; \quad \dfrac{1}{2} < x < 1 \end{cases}$

(C) $u(x) = \begin{cases} \dfrac{\sin^{-1} 2x}{\tan^{-1} 3x} & ; \quad x \in \left(0, \dfrac{1}{2}\right] \\ \dfrac{|\sin x|}{x} & ; \quad x < 0 \end{cases}$

(D) $v(x) = \begin{cases} \log_3(x+2) & ; \quad x > 2 \\ \log_{1/2}(x^2+5) & ; \quad x < 2 \end{cases}$

18. If $f(x) = \dfrac{1}{x^2 - 17x + 66}$, then $f\left(\dfrac{2}{x-2}\right)$ is discontinuous at x =

(A) 2 (B) $\dfrac{7}{3}$ (C) $\dfrac{24}{11}$ (D) 6,11

19. Let f (x) = [x] & $g(x) = \begin{cases} 0; & x \in Z \\ x^2; & x \in R - Z \end{cases}$, then (where [.] denotes greatest integer function) -

(A) $\lim\limits_{x\to 1} g(x)$ exists, but g(x) is not continuous at x= 1.

(B) $\lim\limits_{x\to 1} f(x)$ does not exist and f (x) is not continuous at x=1.

(C) gof is continuous for all x.

(D) fog is continuous for all x.

ANSWER KEY

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	C	C	C	A	A	A	B	C	B	A
Que.	11	12	13	14	15	16	18	18	19	
Ans.	B	D	C	B	A,B	B,C,D	A,C,D	A,B,C	A,B,C	

EXTRA PRACTICE QUESTIONS ON CONTINUITY

SELECT THE CORRECT ALTERNATIVES (ONE OR MORE THAN ONE CORRECT ANSWERS)

1. Consider the piecewise defined function $f(x) = \begin{cases} \sqrt{-x} & \text{if } x < 0 \\ 0 & \text{if } 0 \le x \le 4 \\ x-4 & \text{if } x > 4 \end{cases}$ choose the answer which best describes the continuity of this function -

(A) the function is unbounded and therefore cannot be continuous

(B) the function is right continuous at x = 0

(C) the function has a removable discontinuity at 0 and 4, but is continuous on the rest of the real line

(D) the function is continuous on the entire real line

2. f(x) is continuous at x=0, then which of the following are always true ?

(A) $\lim\limits_{x\to 0} f(x) = 0$ (B) f(x) is non continuous at x=1

(C) $g(x) = x^2 f(x)$ is continuous at x = 0 (D) $\lim\limits_{x\to 0^+} (f(x) - f(0)) = 0$

3. Indicate all correct alternatives if, $f(x) = \frac{x}{2} - 1$, then on the interval $[0,\pi]$

(A) tan (f (x)) & $\frac{1}{f(x)}$ are both continuous (B) tan (f(x)) & $\frac{1}{f(x)}$ are both discontinuous

(C) tan (f (x))& $f^{-1}(x)$ are both continuous (D) tan (f(x)) is continuous but $\frac{1}{f(x)}$ is not

4. If f(x) = sgn(cos2x − 2 sinx + 3), where sgn ( ) is the signum function, then f(x) -

(A) is continuous over its domain (B) has a missing point discontinuity

(C) has isolated point discontinuity (D) has irremovable discontinuity.

5. $f(x) = \frac{2\cos x - \sin 2x}{(\pi - 2x)^2}$; $g(x) = \frac{e^{-\cos x} - 1}{8x - 4\pi}$

h(x) = f(x) for $x<\pi/2$

= g(x) for $x>\pi/2$

then which of the followings does not holds ?

(A) h is continuous at $x = \pi/2$ (B) h has an irremovable discontinuity at $x=\pi/2$

(C) h has a removable discontinuity at $x = \pi/2$ (D) $f\left(\frac{\pi^+}{2}\right) = g\left(\frac{\pi^-}{2}\right)$

6. The number of points where f(x) = [sinx + cosx] (where [ ] denotes the greatest integer function), $x \in (0, 2\pi)$ is not continuous is -

(A) 3 (B) 4 (C) 5 (D) 6

7. On the interval I = [−2, 2], the function $f(x) = \begin{cases} (x+1)e^{-\left[\frac{1}{|x|}+\frac{1}{x}\right]} & (x \ne 0) \\ 0 & (x = 0) \end{cases}$

then which one of the following hold good ?

(A) is continuous for all values of $x \in I$ (B) is continuous for $x \in I - (0)$

(C) assumes all intermediate values from f(−2) & f(2) (D) has a maximum value equal to 3/e

8. If $f(x) = \cos\left[\frac{\pi}{x}\right] \cos\left(\frac{\pi}{2}(x-1)\right)$; where [x] is the greatest integer function of x, then f(x) is continuous at -

(A) x = 0 (B) x = 1 (C) x = 2 (D) none of these

9. Given $f(x) = \begin{bmatrix} 3 - \left[\cot^{-1}\left(\frac{2x^3-3}{x^2}\right)\right] & \text{for} \quad x > 0 \\ \left\{x^2\right\}\cos\left(e^{1/x}\right) & \text{for} \quad x < 0 \end{bmatrix}$ where { } & [ ] denotes the fractional part and the integral part functions respectively, then which of the following statement does not hold good -

(A) $f(0^-) = 0$ (B) $f(0^+)=3$

(C) $f(0)=0 \Rightarrow$ continuity of f at x = 0 (D) irremovable discontinuity of f at x = 0

10. Let 'f' be a continuous function on R. If $f(1/4^n) = (\sin e^n)e^{-n^2} + \frac{n^2}{n^2+1}$ then f(0) is -

(A) not unique (B) 1

(C) data sufficient to find f(0) (D) data insufficient to find f(0)

11. Given $f(x) = b([x]^2 + [x]) + 1$ for $x \geq -1$

$= \sin(\pi(x+a))$ for $x < -1$

where [x] denotes the integral part of x, then for what values of a, b the function is continuous at x = – 1 ?

(A) $a = 2n + (3/2); \; b \in R \; ; \; n \in I$ (B) $a = 4n + 2 \; ; \; b \in R \; ; \; n \in I$

(C) $a = 4n + (3/2) \; ; \; b \in R^+ \; ; \; n \in I$ (D) $a = 4n + 1 \; ; \; b \in R^+ \; ; \; n \in I$

12. Consider $f(x) = \begin{bmatrix} x[x]^2 \log_{(1+x)} 2 & \text{for} \quad -1 < x < 0 \\ \dfrac{\ln\left(e^{x^2} + 2\sqrt{\{x\}}\right)}{\tan\sqrt{x}} & \text{for} \quad 0 < x < 1 \end{bmatrix}$ where [*] & {*} are the greatest integer function & fractional part function respectively, then -

(A) $f(0) = \ln 2 \Rightarrow$ f is continuous at x = 0 (B) $f(0) = 2 \Rightarrow$ f is continuous at x = 0

(C) $f(0) = e^2 \Rightarrow$ f is continuous at x = 0 (D) f has an irremovable discontinuity at x = 0

13. Let $f(x) = \begin{bmatrix} a\sin^{2n} x & \text{for } x \geq 0 \text{ and } n \to \infty \\ b\cos^{2m} x - 1 & \text{for } x < 0 \text{ and } m \to \infty \end{bmatrix}$ then -

(A) $f(0^-) \neq f(0^+)$ (B) $f(0^+) \neq f(0)$ (C) $f(0^-) = f(0)$ (D) f is continuous at x = 0

14. Consider $f(x) = \lim_{n\to\infty} \frac{x^n - \sin x^n}{x^n + \sin x^n}$ for $x > 0, x \neq 1$ f(1)=0 then -

(A) f is continuous at x = 1

(B) f has a finite discontinuity at x = 1

(C) f has an infinite or oscillatory discontinuity at x = 1

(D) f has a removable type of discontinuity at x=1

ANSWER KEY

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	D	C,D	C,D	C	A,C,D	C	B,C,D	B,C	B,D	B,C
Que.	11	12	13	14						
Ans.	A,C	D	A	B						