ALLEN CAREER INSTITUTE
KOTA (RAJASTHAN)
path to success

DISTANCE LEARNING PROGRAMME

(Academic Session : 2019 - 2020)

LEADER TEST SERIES / JOINT PACKAGE COURSE

TARGET : JEE (Main) 2020

Test Type : Unit Test TEST # 01 Test Pattern : JEE (Main)

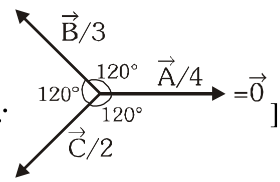
TEST DATE : 07 - 07 - 2019

ANSWER KEY

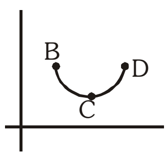
Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Ans.	2	3	2	3	2	2	2	4	3	2	3	2	4	1	1	2	4	3	2	3
Que.	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
Ans.	1	4	1	2	1	2	3	3	3	1	2	1	2	1	3	3	2	2	1	4
Que.	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
Ans.	3	4	1	3	4	1	2	2	4	3	3	3	3	1	3	2	2	3	2	1
Que.	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
Ans.	2	2	1	2	3	3	2	1	3	1	3	1	2	2	3	4	1	3	4	3
Que.	81	82	83	84	85	86	87	88	89	90										
Ans.	2	1	3	4	2	1	4	3	2	3										

HINT – SHEET

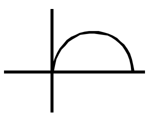
1. $\frac{3\vec{A}}{4}+\frac{\vec{B}}{3}+\frac{\vec{C}}{2}=\frac{\vec{A}}{2}+\left(\frac{\vec{A}}{4}+\frac{\vec{B}}{3}+\frac{\vec{C}}{2}\right)=\frac{\vec{A}}{2}+\vec{0}=\frac{\vec{A}}{2}$

[$\because$  $=\vec{0}$]

4. Continously increasing slope.



5. y = 2x – 4x^2
y = –4x^2 + 2x
y = x(–4x + 2)
y = 0 at x = 0
x = 1/2
also –4x^2 represent downward parabola

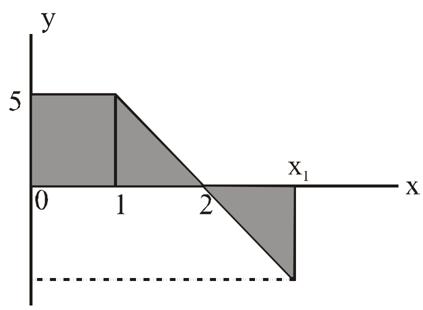


6. $\vec{F}_1=4\hat{j}$; $\vec{F}_2=3\hat{i}$

$\vec{F}_1+\vec{F}_2=3\hat{i}+4\hat{j}$

$\vec{F}_1+\vec{F}_2+\vec{F}$ should be along North–East direction

7.



$\int_0^{x_1} y\,dx$ = shaded area

$(5\times1) + \left(\frac{1}{2}\times1\times5\right) - A_3 = 5$

$A_3 = \frac{1}{2}\times1\times5$

Now at $x = x_1,\ y = y_1$

then $\frac{y_1}{x_1-2} = \frac{5}{1}$

$\Rightarrow \quad y_1 = 5(x_1 - 2)$

so $\quad A_3 = \frac{1}{2}\times(x_1-2)\times5(x_1-2) = \frac{1}{2}\times1\times5$

$\Rightarrow \quad x_1 = 3$

8. $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} \Rightarrow \frac{dR_{eq}}{R_{eq}^2} = \frac{dR_1}{R_1^2} + \frac{dR_2}{R_2^2}$

Given $dR_1/R_1 = 1/100$, $dR_2/R_2 = 2/100$

$$\frac{dR_{eq}}{R_{eq}} = R_{eq}\left\{\frac{dR_1}{R_1}\cdot\frac{1}{R_1} + \frac{dR_2}{R_2}\cdot\frac{1}{R_2}\right\}$$

% error $= 2\left\{\frac{1}{3} + \frac{2}{6}\right\} = 4/3\,\%$

9. $\rho = \frac{m}{v} = \frac{4.237g}{2.5cm^3} = 1.6948$

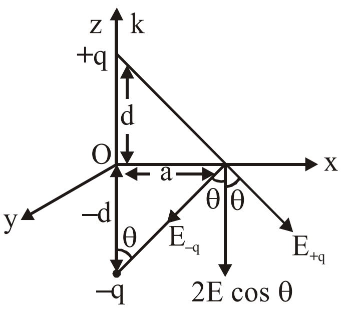
rounding of the number = 1.7

10. Charge on outer surface of shell is

$Q_0 + Q_0 - 2Q_0 = 0$

So potential at surface $= \frac{K3Q_0}{2R}$

11.



Resultant electric field = 2E cos θ

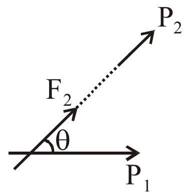
$$= \frac{1}{4\pi\varepsilon_0}\frac{2q}{(a^2+d^2)}\frac{d}{(a^2+d^2)^{1/2}}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{2qd}{(a^2+d^2)^{3/2}} \text{ in –ve Z-direction}$$

$\therefore \quad \vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{2qd}{(a^2+d^2)^{3/2}}(-\hat{k})$

12. $U = -P_2E_2\cos\theta$

$= -P_1\frac{2KP_2}{r^3}\cos\theta$



13. When electric field is switched ON, mg = qE

$m \rightarrow$ mass of oil drop $= \frac{4}{3}\pi r^3 \times \rho$, where r is radius of drop and ρ is density of oil.

When electric field is switched OFF mg = 6 πηrv where v is terminal velocity of oil drop.

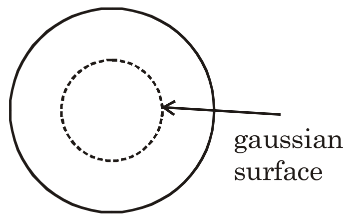
Solving above equation, we get $q = 8 \times 10^{-19}$ C

14. $E_{Arc} = \frac{2k\lambda_2}{R}\sin\frac{\theta}{2}$ (θ = 180°)

$E_{\text{Infinite wire}} = \frac{2k\lambda_1}{d}$ (d = R)

$\lambda_1 = \lambda_2$

15.



For spherical charge distribution we can apply Gauss theorem

$\int E\cdot ds = \frac{q_{in}}{\epsilon_0}$

$E4\pi r^2 = \frac{q_{in}}{\epsilon_0}$

$q_{in} = \int_0^{R/2}\rho dv$

$= \int_0^{R/2}\frac{Ar}{R}4\pi r^2 dr$

$q_{in} = \frac{A4\pi}{R}\left[\frac{r^4}{4}\right]_0^{R/2} = \frac{A\pi R^3}{16}$

$\frac{E4\pi R^2}{4} = \frac{A\pi R^3}{16\epsilon_0}$

$\frac{AR}{16\epsilon_0} = \frac{2R}{\epsilon_0} \Rightarrow A = 2$

16. $m = 10^{-3}$ kg

600V 0V

v_1 $v_2 = 20$ cm/s

$= 0.2$ m/s

$\Delta K = -q(\Delta V)$

$$\frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 = -q(V_B - V_A)$$

$$\frac{1}{2} \times 10^{-3}[(0.2)^2 - v_1^2] = -10^{-8}(0 - 600)$$

$(0.2)^2 - v_1^2 = 12 \times 10^{-3}$

$4 \times 10^{-2} - v_1^2 = 1.2 \times 10^{-2}$

$v_1^2 = 2.8 \times 10^{-2}$

$v_1 = 1.67 \times 10^{-1}$ m/s

$v_1 = 1.67 \times 10^{-1} \times 10^2$

$= 16.7$ cm/s

17. The electric potential at a point,

$V = -x^2y - xz^3 + 4$

The field $\vec{E} = -\left(\frac{\partial V}{\partial x}\hat{i} + \frac{\partial V}{\partial y}\hat{j} + \frac{\partial V}{\partial z}\hat{k}\right)$

$\therefore\ \vec{E} = \hat{i}(2xy + z^3) + \hat{j}x^2 + \hat{k}(3xz^2)$

18. In induction process, Net charge remain unchanged.

19. At $\frac{R}{\sqrt{2}}$, E is max

$\therefore$ F and hence a latill be max.

but direction of EF is same

from centre to ∞

$\therefore$ V is max at centre

20. $\tau = -PE\sin\theta$

$= -PE\theta$, for small θ

$$T = 2\pi\sqrt{\frac{I}{C}} = 2\pi\sqrt{\frac{I}{PE}}$$

$$P = qL,\ I = m\left(\frac{L}{2}\right)^2 + m\left(\frac{L}{2}\right)^2 = \frac{mL^2}{2}$$

Required time

$$t = \frac{T}{4}$$

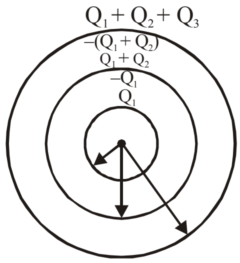
22. $E = \frac{kq}{r^2}$

where $q = \rho\frac{4}{3}\pi R^3$

23. $\phi = \vec{E}.\vec{S}$

$= (5\hat{i} + 4\hat{j} + 9\hat{k}).20\hat{i} = 100$ unit

24. $$\frac{Q_1}{4\pi R^2} = \frac{Q_1 + Q_2}{4\pi + (4R^2)}$$



$$= \frac{Q_1 + Q_2 + Q_3}{4\pi \times 9R^2}$$

$8Q_1 = Q_2 + Q_3$

$Q_3 = 5Q_1$

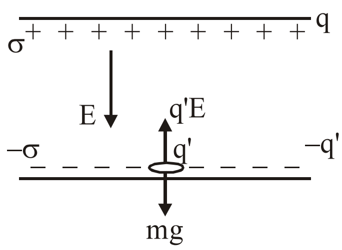
$3Q_1 = Q_2$

$Q_1 : Q_2 : Q_3 = Q_1 : 3Q_1 : 5Q_1 = 1 : 3 : 5.$

25. $$\frac{Kq^2}{r^2} = \frac{GM_eM_m}{r^2}$$

26. $$\sigma = \frac{q'}{\pi r^2}$$

$$E = \frac{V}{d} = \frac{\sigma}{\epsilon_0}$$



$$\frac{V}{d} = \frac{q'}{\epsilon_0\pi r^2}$$

$$q' = \epsilon_0\pi r^2\frac{V}{d}$$

$q'E = mg$

$$\left(\epsilon_0\pi r^2\frac{V}{d}\right)\frac{V}{d} = mg$$

$$V = \sqrt{\frac{mgd^2}{\epsilon_0\pi r^2}}$$

27. Due to external charge $\phi_{in} = \phi_{out}$

$\therefore$ No contribution in flux

28. $$\vec{E}_P = \left(\frac{\sigma}{2\epsilon_0} + \frac{2\sigma}{2\epsilon_0} + \frac{\sigma}{2\epsilon_0}\right)(-\hat{k})$$

29. $$V_C = \frac{3}{2}\frac{kq}{R}$$

$$V' = \frac{kq}{r} = \frac{V_C}{2} = \frac{3}{4}\frac{kq}{R}$$

$$r = \frac{4R}{3}$$

From surface $= \frac{4R}{3} - R = \frac{R}{3}$

30. $E_1 = \frac{kq}{r^2}$

$E_2 = \frac{2kq}{r^2} = 2E_1$

31. Moles of $Na_2SO_4.10H_2O \Rightarrow \frac{1.61}{322} = .005$ mole

weight of water = 10 × .005 × 18

= .9g

32. $2C_8H_{18} + 25O_2 \longrightarrow 16CO_2 + 18H_2O$

required oxygen $\Rightarrow \frac{25\times32}{2\times114}\times570 = 2kg$

35. $\frac{x\times18}{142 + x\times18}\times100 = 55.9$

x = 10

36. ∵ In 2 ml, no. of drops = 35

∴ Volume of 1 drop liquid = $\frac{2}{35}$

mass of 1 drop liquid = $\frac{2}{35}\times1.2 = \frac{2.4}{35}$ gm

moles of liquid in 1 drop = $\frac{2.4}{35\times70} = \frac{1.2}{35\times35} = \frac{1.2}{(35)^2}$

no. of molecules in 1 drop = $\frac{1.2\times N_A}{(35)^2}$

37. $4A + 2B + 3C \longrightarrow A_4B_2C_3$

1 mole 0.6 mole 0.72 mole

In the above reaction, reactant 'C' will be the limiting reactant and it will decide yield product.

∵ from 3 moles of C; 1 mole product is formed

∴ from 0.72 moles of C = $\frac{1}{3}\times0.72$

= 0.24 moles of product

38. 1 mol $S_8 \longrightarrow$ 8 mol SO_2

= 8 × 80 = 640 gm

39. (I) 0.5 mole O_3 = 24 g CO_3;

(II) 0.5 g atom of oxygen = 8 g

(III) $\frac{3.011\times10^{23}}{6.022\times10^{23}}\times 32 = 16$ g O_2

(IV) $\frac{5.6}{22.4}\times44gCO_2 = 11\ gCO_2$

41. S.E. = 13.6 eV

$n_1 = 3 \qquad n_2 = \infty$

S.E. = $13.6 \times \frac{Z^2}{3^2}$

Z = 3, $\quad Li^{+2}$

43. $\frac{1}{\lambda} = R_H\left(\frac{1}{1^2} - \frac{1}{2^2}\right)$

$\lambda_{max} = \frac{4}{3}R_H$

$\frac{1}{\lambda}R_H\left(\frac{1}{1^2} - \frac{1}{\infty^2}\right)$

$\lambda_{min} = \frac{1}{R_H}$

$\frac{1}{R_H} \le \lambda \le \frac{4}{3}\frac{1}{R_H}$

46. $2\pi r = n\lambda$

for minimum n = 1

$2\pi r_{min} = \lambda$

51. H–C(=O)–NH–CH_2–CH_3 (1) main functional group is amide

52.
$\overset{4}{CH_3}-\overset{3}{C}(=O)-\overset{2}{CH_2}-\overset{1}{C}(=O)-O-C_2H_5$

AAE → Ethyl-3-oxo butanoate

53. $C - C\ \Delta^2\ C - C$

⇓

$C - C = C - C - C$

2-pentene

without bracket Δ represent double bond & no on it represent position.

54. Aceto nitrile → CH_3–C≡N but given CH_2 = CHCN so it is incorrectly matched.

57. $\overset{1}{CH_3}-\overset{2}{CH}(C_6H_5)-\overset{3}{CH_2}-\overset{4}{CH_3}$

58. Br–$\overset{2}{CH}$(Cl)–$\overset{1}{C}$(F)(F)–F

61. Given expression

$$= \frac{(1-\sin\alpha)-(1+\sin\alpha)}{\sqrt{1-\sin^2\alpha}}$$

$$= \frac{-2\sin\alpha}{|\cos\alpha|} = \frac{-2\sin\alpha}{-\cos\alpha}$$

$[\because \frac{\pi}{2} < \alpha < \pi \therefore \cos\alpha \text{ is } -\text{ve}] = 2\tan\alpha$

62. Since $0 \le \sin^{2n} x \le \sin^2 x$

$0 \le \cos^{2n} x \le \cos^2 x$

$[\because \sin^4 x = \sin^2 x \,.\, \sin^2 x \le \sin^2 x \,.\, 1$

$\therefore \sin^4 x \le \sin^2 x$ etc.]

$\Rightarrow \quad 0 < \sin^{2n} x + \cos^{2n} x \le \sin^2 x + \cos^2 x = 1$

$\Rightarrow \quad 0 < \sin^{2n} x + \cos^{2n} x \le 1$

64. Let $f(\theta) = 12\sin\theta - 9\sin^2\theta$

$\therefore \quad f'(\theta) = 12\cos\theta - 18\sin\theta\cos\theta$

$= 6\cos\theta\,(2-3)\sin\theta)$

Now $f'(\theta) = 0$ gives $\cos\theta = 0$ or $\sin\theta = \frac{2}{3}$

$\Rightarrow \quad \sin\theta = 1$ or $\sin\theta = \frac{2}{3}$

$f''(\theta) = -12\sin\theta - 18[\cos^2\theta - \sin^2\theta]$

when $\sin\theta = 1$,

$f''(\theta) = -12 - 18[1-2] = +\text{ve}$

and when $\sin\theta = 2/3$

$f''(\theta) = -8 - 18\left[1-\frac{4}{9}\right] = -\text{ve}$

$\therefore \quad f(\theta)$ is Max. when $\sin\theta = 2/3$

$\therefore \quad$ Max. $f(\theta) = 8 - 4 = 4$

65. $K = (\sin^2 x + \cos^2 x)^3 - 3\sin^2 x\cos^2 x(\sin^2 x + \cos^2 x)$

$= (1)^3 - 3\sin^2 x\cos^2 x(1)$

$= 1 - \frac{3}{4}\sin^2 2x$

Now, $0 \le \sin^2 2x \le 1 \quad \Rightarrow \quad 0 \le \frac{3}{4}\sin^2 2x \le \frac{3}{4}$

$\Rightarrow \quad -\frac{3}{4} \le -\frac{3}{4}\sin^2 2x \le 0$

$\Rightarrow \quad 1-\frac{3}{4} \le 1-\frac{3}{4}\sin^2 2x \le 1$

$\Rightarrow \quad \frac{1}{4} \le 1-\frac{3}{4}\sin^2 2x \le 1$

$\Rightarrow \quad \frac{1}{4} \le K \le 1 \quad \Rightarrow \quad K \in \left[\frac{1}{4}, 1\right]$

66. $\sqrt{3} = \tan 60° = \tan(40° + 20°)$

$$= \frac{\tan 40° + \tan 20°}{1 - \tan 40°\tan 20°}$$

$\therefore \quad \sqrt{3} - \sqrt{3}\tan 40°\tan 20° = \tan 40° + \tan 20°$

Hence $\tan 40° + \tan 20° + \sqrt{3}\tan 40°\tan 20°$

$= \sqrt{3}$

67. $\sin\frac{7\pi}{8} = \sin\left(\pi - \frac{\pi}{8}\right) = \sin\frac{\pi}{8}$

$\sin\frac{5\pi}{8} = \sin\left(\pi - \frac{3\pi}{8}\right) = \sin\frac{3\pi}{8}$

$\therefore \quad$ The given value $= 2\left[\sin^2\frac{\pi}{8} + \sin^2\frac{3\pi}{8}\right]$

$= 2\left[\sin^2\frac{\pi}{8} + \cos^2\frac{\pi}{8}\right]$

$\left[\because \sin\frac{3\pi}{8} = \sin\left(\frac{\pi}{2} - \frac{\pi}{8}\right) = \cos\frac{\pi}{8}\right]$

$= 2(1) = 2$

68. $$\frac{3 + \cot 76°\cot 16°}{\cot 76° + \cot 16°} = \frac{3 + \frac{\cos 76°\cos 16°}{\sin 76°\sin 16°}}{\frac{\cos 76°}{\sin 76°} + \frac{\cos 16°}{\sin 16°}}$$

$$= \frac{3\sin 76°\sin 16° + \cos 76°\cos 16°}{\cos 76°\sin 16° + \sin 76°\cos 16°}$$

$$= \frac{2\sin 76°\sin 16° + \cos(76° - 16°)}{\sin(76° + 16°)}$$

$$= \frac{2\sin 76°\sin 16° + \frac{1}{2}}{\sin(92°)}$$

$$= \frac{\cos 60° - \cos 92° + \frac{1}{2}}{\sin(92°)} = \frac{1 - \cos 92°}{\sin(92°)}$$

$$= \frac{2\sin^2 46°}{2\sin 46°\cos 46°} = \tan(46°)$$

$= \cot(44°)$

69. $3\sin^2 x - 7\sin x + 2 = 0$

$\Rightarrow$ $(3\sin x - 1)(\sin x - 2) = 0$

$\Rightarrow$ $3\sin x = 1$ or $\sin x = 2$

$\Rightarrow$ $\sin x = \frac{1}{3}$ [$\because$ $\sin x = 2$ is not possible]

since $x \in [0, 5\pi]$

$\therefore$ 6 values of x will be possible.

[$\because$ x will lie in Ist or IInd quadrant]

71. $(I + A)^3 - 7A = I^3 + 3I^2A + 3IA^2 + A^3 - 7A$

$= I + 3A + 3A + A - 7A$

$= I + 0 = I$

72. Given, $|A| = 2^4$

$\Rightarrow |\,\text{adj}\,(\text{adj}\,A)| = (2^4)^9 = 2^{36}$

$$\Rightarrow \left\{\frac{\det(\text{adj}(\text{adj}A))}{7}\right\} = \left\{\frac{2^{36}}{7}\right\} = \left\{\frac{(7+1)^{12}}{7}\right\}$$

$$= \frac{1}{7}$$

73. $(-A)^{-1} = \frac{\text{adj}(-A)}{|-A|} = \frac{(-1)^{n-1}\text{adj}(A)}{(-1)^n|A|}$

$$= \frac{\text{adj}(A)}{-|A|} = -A^{-1}$$

74. $B = A_1 + 3A_3^3 + \ldots\ldots.. + (2n-1)(A_{2n-1})^{2n-1}$

$B^T = -[A_1 + 3A_3^3 + \ldots\ldots + (2n-1)(A_{2n-1})^{2n-1}]$

$= -B$

$\therefore$ B is skew - symmetric

75. $|A|\,|\text{adj}\,A| = |A\,\text{adj}\,A| = ||A|I|$

$$= \begin{vmatrix} |A| & 0 & 0 \\ 0 & |A| & 0 \\ 0 & 0 & |A| \end{vmatrix} = |A|^3 = (a^3)^3$$

$= a^9$

76. $A^2B = A(AB) = AA = A^2$,

$B^2A = B(BA) = BB = B^2$

$ABA = A(BA) = AB = A$

$\therefore$ All are correct

77. $M = \begin{bmatrix} 0 & -1 & 2 \\ 2 & -2 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 0 \\ 1 & 1 \end{bmatrix}$

$M = \begin{bmatrix} 1 & 2 \\ -2 & 2 \end{bmatrix}$

$\text{adj}\,M = \begin{bmatrix} 2 & -2 \\ +2 & 1 \end{bmatrix}$

$|M| = 6 \therefore M^{-1} = \begin{bmatrix} 1/3 & -1/3 \\ +1/3 & 1/6 \end{bmatrix}$

78. $\det(kA) = k^n\det A \neq k\det A$

79. $\text{tr}(A) = \sum_{i=j} a_{ij}$

$= a_{11} + a_{22} + a_{33} + \ldots\ldots\ldots + a_{1010}$

$= w^2 + w^4 + w^6 + \ldots\ldots + w^{20}$

$= w^2(1 + w^2 + w^4 + \ldots\ldots. + w^{18})$

$= w^2[(1 + w + w^2) + \ldots\ldots.. + (1 + w + w^2) + 1]$

$= w^2 \times 1 = w^2$

80. $\text{adj}(Q^{-1}BP^{-1}) = \text{adj}(P^{-1})\,\text{adj}(B)\,\text{adj}(Q^{-1})$

$$= \frac{P}{|P|}A\frac{Q}{|Q|} = PAQ$$

81. $\because 0 \le [x] < 2 \Rightarrow [x] = 0, 1$

$-1 \le [y] < 1 \Rightarrow [y] = -1, 0$

$1 \le [z] < 3 \Rightarrow [z] = 1, 2$

Now, $R_2 \to R_2 - R_1$; $R_3 \to R_3 - R_1$; then

$$\begin{vmatrix} [x]+1 & [y] & [z] \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{vmatrix}$$

$([x] + 1)(1 - 0) - [y](-1 - 0) + [z](0 + 1)$

$= [x] + [y] + [z] + 1$

$= 1 + 0 + 2 + 1 = 4$

($\because$ for max. value, $[x] = 1$, $[y] = 0$, $[z] = 2$)

82. Applying $C_1 \to C_1 + C_2 + C_3$

$$(a+b+c-x)\begin{vmatrix} 1 & c & b \\ 1 & b-x & a \\ 1 & a & c-x \end{vmatrix} = 0$$

$$\Rightarrow x\begin{vmatrix} 1 & c & b \\ 1 & b-x & a \\ 1 & a & c-x \end{vmatrix} = 0 \quad (\because a+b+c=0)$$

i.e., x = 0 is one root

83. $\Delta = (2\cos x + \sin x)$

$$\begin{vmatrix} 1 & \cos x & \cos x \\ 1 & \sin x & \cos x \\ 1 & \cos x & \sin x \end{vmatrix} (C_1 \to C_1 + C_2 + C_3)$$

$$= (2\cos x + \sin x)\begin{vmatrix} 0 & \cos x - \sin x & 0 \\ 1 & \sin x & \cos x \\ 1 & \cos x & \sin x \end{vmatrix} (R_1 \to R_1 - R_2)$$

$= (2\cos x + \sin x)(\cos x - \sin x)^2 = 0$

$\therefore$ $\tan x = -2$ or 1; Hence one solution

84. $\because f(-x) = \begin{vmatrix} -x & \cos x & e^{x^2} \\ -\sin x & x^2 & \sec x \\ -\tan x & 1 & 2 \end{vmatrix} = -f(x)$

$\therefore \int_{-\pi/2}^{\pi/2} f(x)dx = 0$ [$\because$ f(x) is an odd function]

85. Applying $C_1 \to aC_1$ & then $C_1 \to C_1 + b_2 + cC_3$ & taking $(a^2 + b^2 + c^2)$ common from C_1, we get

$$\Delta = \left(\frac{a^2 + b^2 + c^2}{a}\right)\begin{vmatrix} 1 & b - c & c + b \\ 1 & b & c - a \\ 1 & b + a & c \end{vmatrix}$$

$$= \left(\frac{a^2 + b^2 + c^2}{a}\right)\begin{vmatrix} 1 & b - c & c + b \\ 0 & c & -a - b \\ 0 & a + c & -b \end{vmatrix}$$

$R_2 \to R_2 - R_1$; $R_3 \to R_3 - R_1$

$$\left(\frac{a^2 + b^2 + c^2}{a}\right)(-bc + a^2 + ab + ac + bc)$$

$$= (a^2 + b^2 + c^2)(a + b + c)$$

Hence, $\Delta = 0 \Rightarrow a + b + c = 0$

$\therefore$ Line $ax + by + c = 0$ Passes through

fixed pt. (1, 1)

86. Determinant formed by the cofactors of

$$\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} \text{ is } \begin{vmatrix} bc - a^2 & ac - b^2 & ab - c^2 \\ ac - b^2 & ab - c^2 & bc - a^2 \\ ab - c^2 & bc - a^2 & ac - b^2 \end{vmatrix}$$

$$\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}^2$$

$\therefore$ Always +ve

87. $$\begin{vmatrix} 1+1+1 & \alpha+\beta+\gamma & \alpha^2+\beta^2+\gamma^2 \\ \alpha+\beta+\gamma & \alpha^2+\beta^2+\gamma^2 & \alpha^3+\beta^3+\gamma^3 \\ \alpha^2+\beta^2+\gamma^2 & \alpha^3+\beta^3+\gamma^3 & \alpha^4+\beta^4+\gamma^4 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 1 & 1 \\ \alpha & \beta & \gamma \\ \alpha^2 & \beta^2 & \gamma^2 \end{vmatrix}\begin{vmatrix} 1 & \alpha & \alpha^2 \\ 1 & \beta & \beta^2 \\ 1 & \gamma & \gamma^2 \end{vmatrix}$$

$= (\alpha - \beta)^2 (\beta - \gamma)^2 (\gamma - \alpha)^2$

88. $\Delta = 0 \Rightarrow bc + ab = 2ac$

$\therefore \frac{1}{a} + \frac{1}{c} = \frac{2}{b} \Rightarrow a, b, c$ are in H.P

89. $d_1 = d_2 = d_3 = 0 \Rightarrow D_1 = D_2 = D_3 = 0$

$$D = \begin{vmatrix} \lambda & 0 & 1 \\ 2 & \lambda & 1 \\ 0 & -1 & 1 \end{vmatrix} = 0$$

$\Rightarrow \lambda(\lambda + 1) + (-2) = 0$

$\Rightarrow \lambda^2 + \lambda - 2 = 0$

$\Rightarrow \lambda = -2, 1$

So, quadratic equation is $x^2 + x - 2 = 0$

90. Taking x^5 common from R_3, then

$$x^5\begin{vmatrix} x^n & x^{n+2} & x^{2n} \\ 1 & x^a & a \\ x^n & x^{a+1} & x^{2n} \end{vmatrix} = 0 \ \forall x \in R$$

$\Rightarrow a + 1 = n + 2 \Rightarrow a = n + 1$