CURRENT ELECTRICITY

QUICK REVISION (Important Concepts & Formulas)

Electric current
The current is defined as the rate of flow of charges across any cross sectional area of a conductor. If a net charge *q* passes through any cross section of a conductor in time *t*, then the current is given by

$I = \frac{q}{t}$, where q is in coulomb and t is in second.

The S.I unit of current is called **ampere** (A) (coulomb/second).

If the rate of flow of charge is not uniform, the current varies with time and the instantaneous value of current I is given by $I = \frac{dq}{dt}$

Current is a scalar quantity. The direction of conventional current is taken as the direction of flow of positive charges or opposite to the direction of flow of electrons.

Electromotive force The emf (e) of the source is defined as the work done per unit charge in taking a positive charge through the seat of the emf from the low potential end to the high potential end. Thus, $e = \frac{W}{Q}$.

When no current flows, the emf of the source is exactly equal to the potential difference between its ends. The unit of emf is the same as that of potential, i.e. volt.

The average flow of electrons in the conductor not connected to battery is zero *i.e*. the number of free electrons crossing any section of the conductor from left to right is equal to the number of electrons crossing the section from right to left. Thus no current flows through the conductor until it is connected to the battery.

Drift velocity of free electrons in a metallic conductor In the absence of an electric field, the free electrons in a metal move randomly in all directions and therefore their average velocity is zero. When an electric field is applied, they are accelerated opposite to the direction of the field and therefore they have a net drift in that direction. However, due to frequent collisions with the atoms, their average velocity is very small. This average velocity with which the electrons move in a conductor under a potential difference is called the **drift velocity**.

If E is the applied field, e is the charge of an electron, m is the mass of an electron and t is the time interval between successive collisions (relaxation time), then the acceleration of the electron is $a = \frac{eE}{m}$.

Since the average velocity just after a collision is zero and just before the next collision, it is $a\tau$, the drift velocity must be

$$v_d = \frac{eE}{m}\tau$$

If I is the current through the conductor and n is the number of free electrons per unit volume, then it can be shown that $I = nAev_d$.

The **mobility** μ of a charge carrier is defined as the drift velocity per unit electric field.

$$\mu = \frac{v_d}{E}$$

Current density (*J*)

(i) $J = \frac{I}{area} = nev_d$

(ii) S.I. unit of J = $\mathrm{Am^{-2}}$.

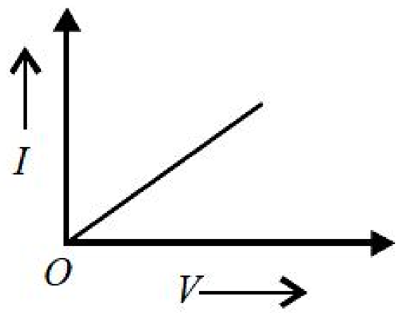
(iii) Current density is a vector quantity. Its direction is that of the flow of positive charge at the given point inside the conductor.

(iv) Dimensions of current density = $[M^0L^{-2}T^0A^1]$.

Current carriers: The current is carried by electrons in conductors, ions in electrolytes and electrons and holes in semiconductors.

Ohm's law Physical conditions (such as temperature) remaining unchanged, the current flowing through a conductor is proportional to the potential difference across its ends.
i.e. $V \propto I$ or $V = RI$.
The constant of proportionality R is called the resistance of the conductor. This law holds for metallic conductors.

According to Ohm's law, the graph between V and I is a straight line. Ohm's law is not valid for semiconductors, electrolytes and electronic devices etc. These are called non-ohmic or non-linear conductors.



The **resistance** of a conductor is a measure of the opposition offered by the conductor to the flow of current. This opposition is due to frequent collision of the electrons with the atoms of the conductor.

The resistance of a conductor is directly proportional to its length l and inversely proportional to the area of cross section A

$$R = \rho \frac{l}{A}$$

where the constant r depends on the nature of the material. It is called the **resistivity** (or specific resistance) of the material. The S.I. unit of resistance is ohm (W) and resistivity is ohm-metre (Ωm).

The resistivity of material of a conductor is given by $\rho = \frac{m}{ne^2\tau}$

where n is number of free electrons per unit volume and t is the relaxation time of the free electron. Its value depends on the nature of the material of the conductor and its temperature.
The inverse of resistance is called **conductance** G

$$G = \frac{1}{R}$$

Its S.I. unit Ω^{-1} is called **siemens** (S).

The inverse of resistivity is called **conductivity**, σ.

$$\sigma = \frac{1}{\rho} = \frac{ne^2\tau}{m}$$

Its S.I. unit is $\Omega^{-1}m^{-1}$ or Sm^{-1} or mho.m^{-1}.

The value of specific resistance or resistivity is low for metals, more for semiconductors and still greater for alloys like nichrome and manganin.

Magnetic field applied to metals increases the resistivity/specific resistance of material.

The exceptions are ferromagnetic materials like iron, cobalt and nickel wherein the resistivity decreases when magnetic field is applied.

Variation of resistivity of metals with temperature : If ρ_0 and ρ_t are the values of resistivities at 0°C and t°C respectively then over a temperature range that is not too large, we have approximately,

$$\rho_t = \rho_0(1+\alpha t)$$

where α is called the **temperature coefficient of resistivity** of the material.

Temperature coefficient of resistance (a)

(i) If R_2 and R_1 represent resistances at temperatures t_2°C and t_1°C, $\alpha = \dfrac{R_2 - R_1}{R_1(t_2 - t_1)}$

(ii) For metals, α is positive.
(iii) For semiconductors, α is negative.
(iv) For insulators, α is negative.
(v) For some alloys like nichrome, manganin and constantant, α is very low.

Non-Ohmic resistances

(i) Ohm's law does not hold good for some substances and conductors. These are called non-ohmic resistances.
(ii) Some examples are vacuum tubes (diode, triode), semiconductor diode, liquid electrolyte, transistor.

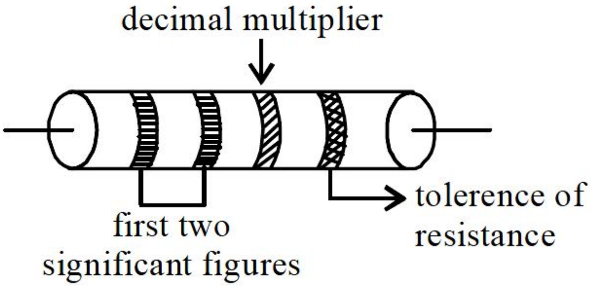
Superconductors A number of materials have the property that below a certain critical temperature, which is very close to absolute zero (0.1 K to 20 K), their resistivity suddenly drops to zero. Such a material is called a superconductor. A current once established in a superconductor continues for a long time without any driving field.
(i) Super conductors are those materials which offer almost zero resistance to the flow of current through them.
(ii) Some known examples are mercury at 4.2 K, lead at 7.25 K and niobium at 9.2 K.
(iii) Specific resistance of super conductor $\approx$ zero.
(iv) Specific conductance of super conductor $\approx$ infinite.
(v) Temperature coefficient of resistance a $\approx$ zero.

Applications of superconductors

(i) Superconductors form the basis of energy saving power systems, namely the superconducting generators, which are smaller in size and weight, in comparison with conventional generators.
(ii) Superconducting magnets have been used to levitate trains above its rails. They can be driven at high speed with minimal expenditure of energy.
(iii) Superconducting magnetic propulsion systems may be used to launch satellites into orbits directly from the earth without the use of rockets.

(iv) High efficiency ore–separating machines may be built using superconducting magnets which can be used to separate tumor cells from healthy cells by high gradient magnetic separation method.
(v) Since the current in a superconducting wire can flow without any change in magnitude, it can be used for transmission lines.
(vi) Superconductors can be used as memory or storage elements in computers.

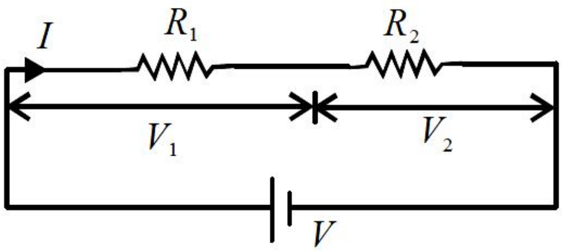
The value of resistances used in electric and electronic circuit vary over a very wide range. Such high resistances used are usually carbon resistances and the values of such resistance are marked on them according to a **colour code**.



Colour	tolerence
gold	5%
silver	10%
no colour	20%

Colour	Figure
black	0
brown	1
red	2
orange	3
yellow	4
green	5
blue	6
violet	7
grey	8
white	9

For resistances in series, $R_{eq} = R_1 + R_2$



In general, for n resistors in series, $R_{eq} = \sum_{i=1}^{n} R_i$

If $R_1 = R_2 = \ldots R_n = R$, then $R_{eq} = nR$.

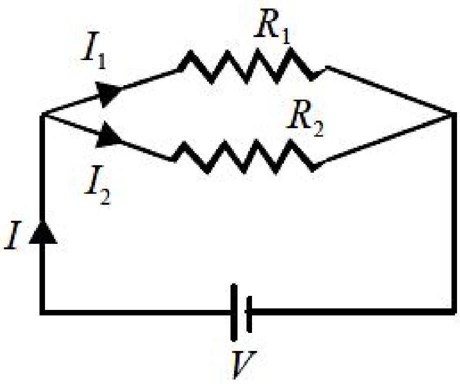
In case of resistances in series, $\frac{V_1}{V_1} = \frac{R_1}{R_2}$

$V_1 = \frac{VR_1}{R_1 + R_2}$; $V_2 = \frac{VR_2}{R_1 + R_2}$ as the current is the same.

In series combination of resistors,
(i) current through all the resistors is same
(ii) potential difference across a resistor is proportional to its resistance.
(iii) The equivalent resistance is greater than the greatest of the resistances connected in series.

For resistances in parallel,

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} \quad or \quad R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$



In general, for n resistors in parallel

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots\ldots + \frac{1}{R_n}$$

If $R_1 = R_2 = ... = R_n = R$, then $R_{eq} = \frac{R}{n}$.

In parallel combination of resistors,

(i) potential difference across all of them is same

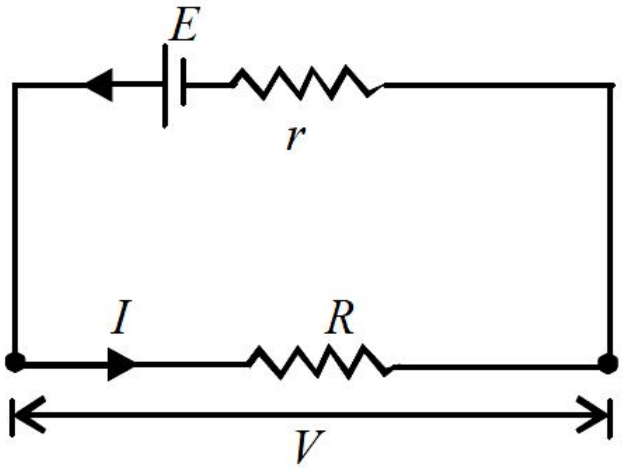
(ii) current through any resistance is inversely proportional to its resistance.

(iii) The equivalent resistance is less than the smallest of the resistances connected in parallel.

In case of resistances in parallel, $\frac{I_1}{I_2} = \frac{R_2}{R_1}$

- Acid and alkali accumulators or storage cells are the secondary cells which have a low internal resistance. Hence a large current can be drawn from such cells. These can be charged and used again and again.
- Acid accumulators have large e.m.f. but are delicate.
- Alkali accumulators have low e.m.f. but are robust.

Internal resistance of a cell and terminal voltage



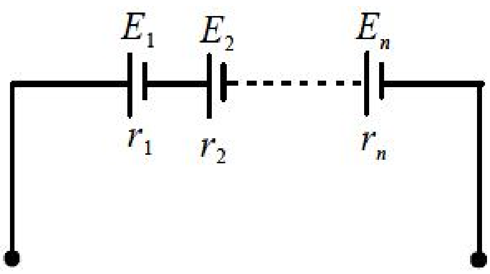
The resistance offered by the electrolyte to the flow of current through the cell is called the *internal resistance* of the cell.

$$V = E - Ir.$$

V is called the **terminal voltage**. If $I = 0$ then $V = E$.

When a cell is charged by an external source V, then $V = E + Ir$.

Cells in series :



If n cells having emfs E_1 , E_2 , ..., E_n and internal resistances r_1 , r_2 , ..., r_n are connected in series as shown, then

$E_{eq} = E_1 + E_2 + ... + E_n$.

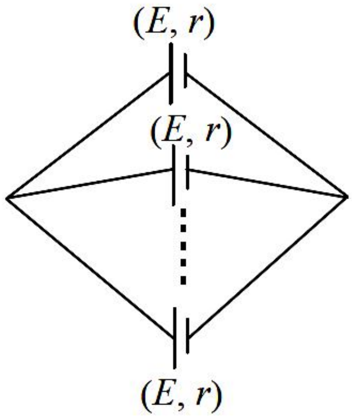
and $r_{eq} = r_1 + r_2 + ... + r_n$

In particular, if $E_1 = E_2 = ... = E_n = E$ and $r_1 = r_2 = ... = r_n = r$,

then $E_{eq} = nE$ and $r_{eq} = nr$.

Cells in parallel :

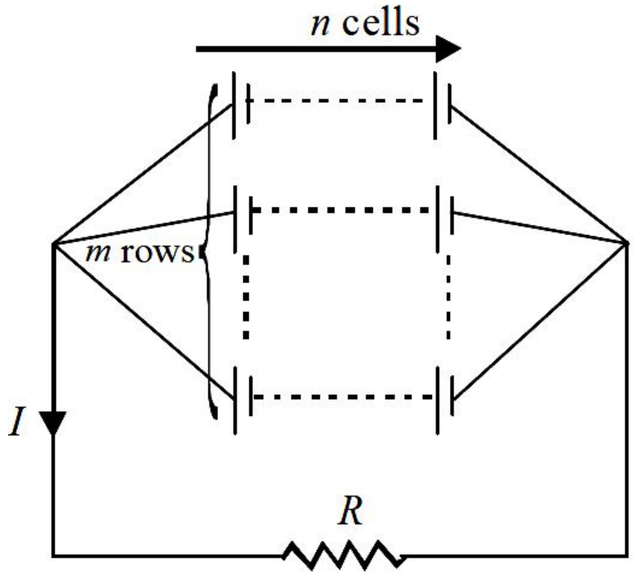
If n cells, each of emf E and internal resistance r are connected in parallel as shown, then



$$E_{eq} = E \quad and \quad r_{eq} = \frac{r}{n}$$

If the emfs of the cells are not all equal then we have to use Kirchhoff's rules.

Mixed combination of cells : Consider a combination of cells having m rows, each row
having n cells. Let the emf of each cell be E and its internal resistance be r.



We have $E_{eq} = nE$, and $r_{eq} = \frac{nr}{m}$, $I = \frac{nE}{R + \frac{nr}{m}} = \frac{mnE}{mR + nr}$

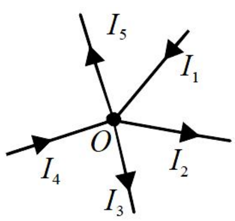
It can be shown that for I to be maximum, $mR = nr$.

Cells are grouped in series if $R >> r$.

Cells are grouped in parallel if $r >> R$.

In a mixed grouping of cell, maximum current is available if the total internal resistance of battery is equal to external resistance. $R = \frac{nr}{m}$ (m rows of n cells each)

Inside the cell, the current flows from negative plate to positive plate while outside the cell from positive plate to negative plate. But during charging of cell, this direction is reversed.



$\Sigma I = 0.$

When the cell is charged, then potential difference across the two plates (V) is greater than e.m.f. of cell. $V > E$

In open circuit when no current is drawn from a cell, $V = E$.

When current is drawn from a cell, $V < E$.

Kirchhoff's rules : All electrical networks cannot be reduced to simple series parallel combinations. Kirchhoff gave two simple and general rules which can be applied to find the currents flowing through or voltage drops across resistances in such networks.

First rule (junction rule) : The algebraic sum of the currents at a junction is zero.

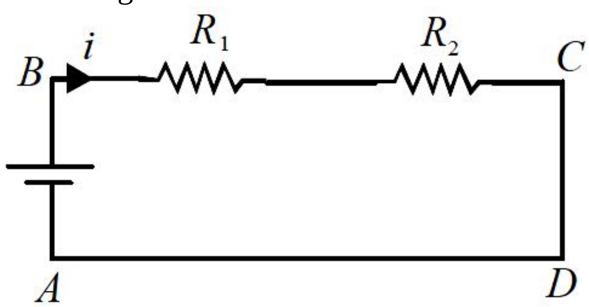
According to Kirchhoff's first law, $I_1 - I_2 - I_3 + I_4 - I_5 = 0$.

This rule follows the conservation of charge, since no charges can accumulate at a junction.
While applying this rule, we (arbitrarily) take the currents entering into a junction as positive and those leaving it as negative.

Second rule (loop rule) : According to this rule in any closed part of an electrical circuit, the algebraic sum of the emfs is equal to the algebraic sum of the products of the resistances and currents flowing through them.

$$\sum E = \sum IR$$

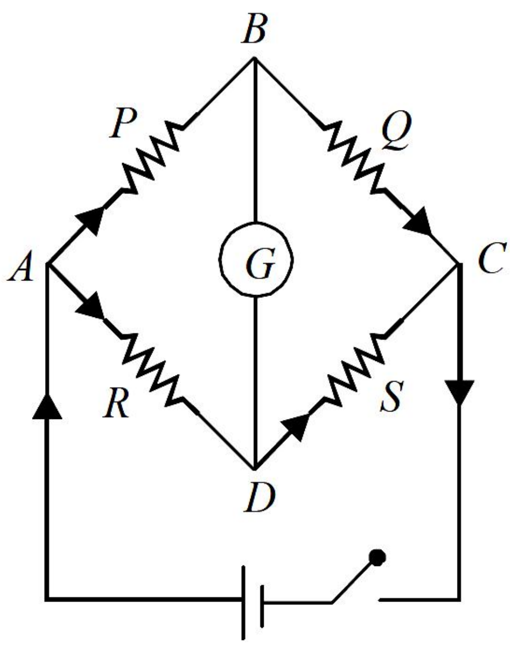
This rule follows from the law of conservation of energy.
The following procedure should be adopted while applying the rules to some network:
(i) In the resistors, $i \times R$ is +ve if it is against the current and –ve in the direction of the current.



(ii) Apply the junction rule for all junctions.
(iii) Choose any loop in the network and designate a direction, clockwise or anticlockwise, to traverse the loop.
(iv) In the cell, positive to negative terminals will be negative (down the potential) and positive to negative terminals will be positive if one takes from the negative terminal to the positive terminal (up the potential gradient).

(v) If necessary, choose another loop and repeat steps (iii) and (iv) until there are as many equations as unknowns.

The Wheatstone's bridge is an arrangement of four resistances *P*, *Q*, *R* and *S* connected as shown in the figure.



Their values are so adjusted that the galvanometer *G* shows no deflection. The bridge is then said to be balanced. When this happens, the points *B* and *D* are at the same potential and it can be shown that

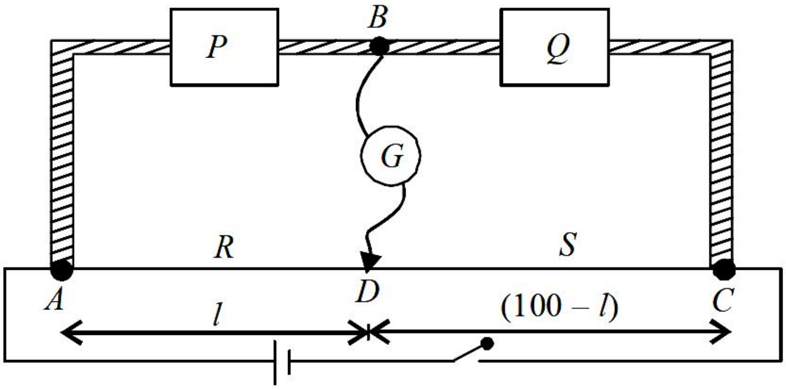
$$\frac{P}{Q} = \frac{R}{S}$$

This is called the **balancing condition**. If any three resistances are known, the fourth can be found.

Wheatstone's bridge is most sensitive when resistances in the four ratio arms are of the same order.

The measurement of resistance by Wheatstone's bridge is not affected by the internal resistance of the cell.

The Metre Bridge : The metre bridge is the practical application of the Wheatstone network principle in which the ratio of two of the resistances, say R and S, is deduced from the ratio of their balancing lengths. AC is a 1 m long uniform wire. If $AD = l$ cm, then $DC = (100 - l)$ cm.



Clearly, $\frac{P}{Q} = \frac{l}{100 - l}$. If P is known then Q can be determined.

Applications of metre bridge:

(i) To measure unknown resistance, $X = \frac{R(100 - l)}{l}$.

Knowing the value of R and l, the value of X can be found.

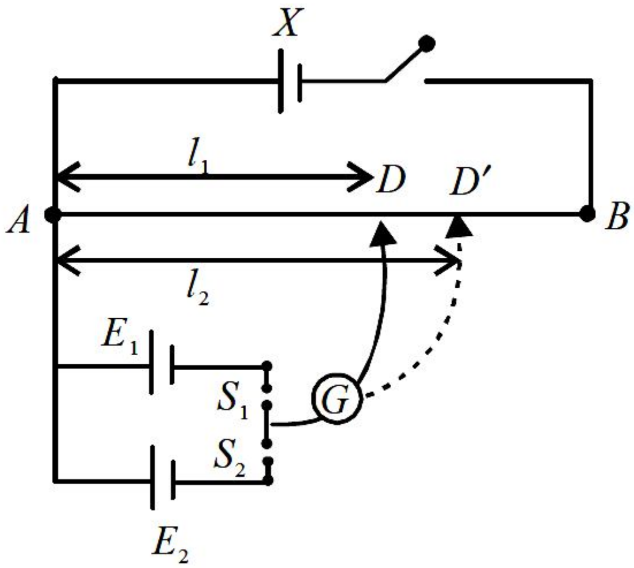
(ii) To compare two unknown resistances $\frac{R_1}{R_2} = \frac{l_2(100 - l_1)}{l_1(100 - l_2)}$

Knowing the value of l_1 and l_2 , the ratio $\frac{R_1}{R_2}$ can be found.

(iii) To measure the unknown temperature $\theta = \frac{R - R_0}{R_{100} - R_\theta} \times 100$

The unknown resistance X (in form of metallic wire) is immersed in ice and its resistance R_0 at 0°C is measured. The unknown resistance is then maintained at a temperature of 100°C (by placing the wire in steam) and its resistance R_{100} at 100°C is measured. At last resistance is immersed in hot bath at unknown temperature θ and experiment is repeated to measure its resistance R_θ at 0°C. After substituting the value of R_0 , R_{100} and R_θ in the equation, the unknown temperature of the hot bath can be found.

Potentiometer : It is a device commonly used for comparison of emfs of cells and for finding the internal resistance of a primary cell. A battery X is connected across a long uniform wire AB. The cells E_1 and E_2 whose emfs are to be compared are connected as shown along with a galvanometer G which detects the flow of current.



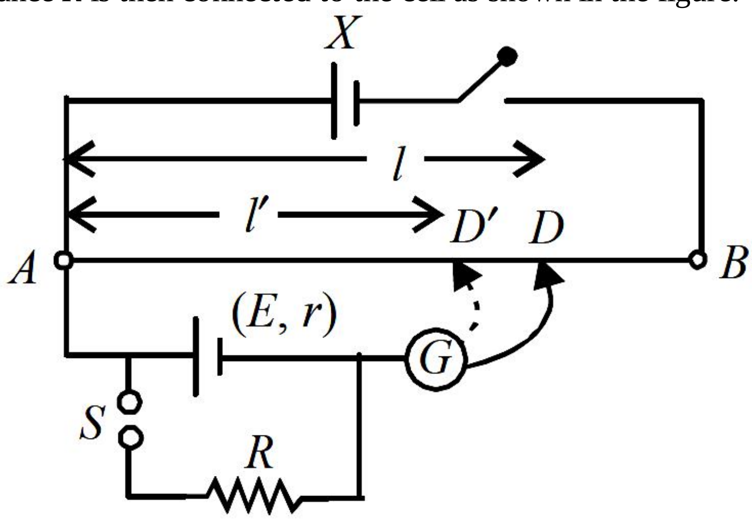
First the key S_1 is pressed which brings E_1 in the circuit. The sliding contact D is moved till the galvanometer shows no deflection. Let the length $AD = l_1$. Next S_1 is opened and S_2 is closed. This brings E_2 in the circuit. Let D' be the new null point and let $AD' = l_2$.

Then, clearly,

$$\frac{E_1}{E_2} = \frac{I_1}{I_2}$$

Potentiometer compares the true emfs of cells because no current flows through the cells at the balance points and therefore no errors are introduced due to internal resistances.

To find the internal resistance of a primary cell, first the emf E of the cell is balanced against a length $AD = l$. A known resistance R is then connected to the cell as shown in the figure.



The terminal voltage V is now balanced against a smaller length $AD' = l'$.

Then, $\frac{E}{V} = \frac{l}{l'}$

But we know that $\frac{E}{V} = \frac{R+r}{R}$

Therefore, $\frac{R+r}{R} = \frac{l}{l'} \Rightarrow r = \left(\frac{l}{r} - 1\right)R$

Potentiometer is an ideal voltmeter.

Sensitivity of potentiometer is increased by increasing length of potentiometer wire.

Resistance of wire

(i) If a resistance wire is stretched to a greater length, keeping volume (*V*) constant,

$$R = \rho \frac{l}{S} = \left(\frac{\rho}{V}\right).l^2$$

Suppose length is drawn to *n* times, *l'* = *nl* then

$$R' = \left(\frac{\rho}{V}\right).n^2 l^2 = n^2 R$$

(ii) If a resistance wire is drawn to *n* times its radius, *r'* = *nr*

$$R = \frac{\rho l}{S} = \frac{\rho l}{\pi r^2} = \frac{\rho}{\pi r^2}\left(\frac{Volume}{\pi r^2}\right) = \frac{\rho V}{\pi^2 r^4}$$

$$R' = \left(\frac{\rho V}{\pi^2}\right).\left(\frac{1}{nr}\right)^4 = \frac{\rho V}{\pi^2 r^4}.\frac{1}{n^4} = \frac{R}{n^4}$$

(iii) If a resistance wire is drawn to *n* times its area of cross section *S*, keeping volume *V* constant, then *S'* = *nS*.

$$R = \frac{\rho l}{S} = \frac{\rho}{S}\left(\frac{Volume}{S}\right) = \frac{\rho V}{S^2}$$

$$R' = \frac{\rho V}{(nS)^2} = \frac{\rho V}{S^2}.\frac{1}{n^2} = \frac{R}{n^2}$$

Electric Power

The rate at which work is done by the source of emf in maintaining the electric current in a circuit is called **electric power** of the circuit.

P = *VI*

where *V* is the potential difference across the conductor, *I* is current flowing through the conductor.

SI unit of electric power is watt.

1 W = 1 V × 1 A

Bigger unit of electric power

1 kilowatt (kW) = 10^3 W

1 megawatt (MW) = 10^6 W

Other expressions for power $P = I^2 R \Rightarrow P = \frac{V^2}{R}$

The total work done (or energy supplied) by the source of emf in maintaining the electric current in the circuit for a given time is called **electric energy** consumed in the circuit.

Electric energy = electric power × time

SI unit of electric energy is joule but another unit is **watt hour**.

The bigger unit of electric energy is kilowatt hour (kWh). It is known as **Board of Trade Unit** (BOT).

1 kilowatt hour = 1000 watt × 1 hour = (1000 J/s) × (3600 s) = 3.6 × 10^6 J

1 horse power = 746 watt.

The electric energy consumed in kWh is given by

$$\text{W (in kWh)} = \frac{\text{V (in volt)} \times \text{I(in ampere)} \times \text{t(in hour)}}{1000}$$

Important Questions & Answers

VERY SHORT ANSWER TYPE QUESTIONS (1 MARK)

1. **Two conducting wires *X* and *Y* of same diameter but different materials are joined in series across a battery. If the number density of electrons in *X* is twice that in *Y*, find the ratio of drift velocity of electrons in the two wires. [*AI 2010*]**
   Ans. Since the wires are connected in series, current *I* through both is same. Therefore ratio of drift velocities

   $$\frac{v_X}{v_Y} = \frac{\frac{I}{n_X e A_X}}{\frac{I}{n_Y e A_Y}} \Rightarrow \frac{v_X}{v_Y} = \frac{n_Y}{n_X} = \frac{1}{2} \quad (Given \;\; A_X = A_Y, n_X = 2n_Y)$$

   where, n_X , n_Y = respective electron densities, A_X , A_Y = cross sectional Areas
   $\therefore v_X : v_Y = 1:2$
2. **Define the term 'Mobility' of charge carries in a conductor. Write its SI unit.**
   Ans. Mobility is defined as the magnitude of the drift velocity per unit electric field.

   $$\mu = \frac{v_d}{E} = \frac{l\tau}{m}$$

   where τ is the average collision time for electrons.
   The SI unit of mobility is m^2/Vs or $m^2V^{-1}s^{-1}$.

3. **Define the term 'electrical conductivity' of a metallic wire. Write its S.I. unit.**
   Ans. The reciprocal of the resistivity of a material is called its conductivity and is denoted by σ.

   $$\text{Conductivity} = \frac{1}{\text{Resistivity}}$$

   The SI unit of conductivity is $ohm^{-1}m^{-1}$ or Sm^{-1}.

4. **Define the term 'drift velocity' of charge carriers in a conductor and write its relationship with the current flowing through it.**
   Ans. Drift velocity is defined as the average velocity acquired by the free electrons in a conductor under the influence of an electric field applied across the conductor. It is denoted by v_d.
   Current, $I = neA.\, v_d$

5. **Why are the connections between the resistors in a meter bridge made of thick copper strips?**
   Ans. A thick copper strip offers a negligible resistance, so does not alter the value of resistances used in the meter bridge.

6. **Why is it generally preferred to obtain the balance point in the middle of the meter bridge wire?**

Ans. If the balance point is taken in the middle, it is done to minimise the percentage error in calculating the value of unknown resistance.

7. **Which material is used for the meter bridge wire and why?**

Ans. Generally alloys magnin/constantan/nichrome are used in meter bridge, because these materials have low temperature coefficient of resistivity.

8. A resistance *R* is connected across a cell of emf ε and internal resistance *r*. A potentiometer now measures the potential difference between the terminals of the cell as *V*. Write the expression for *r* in terms of ε, *V* and *R*. [*AI 2011*]

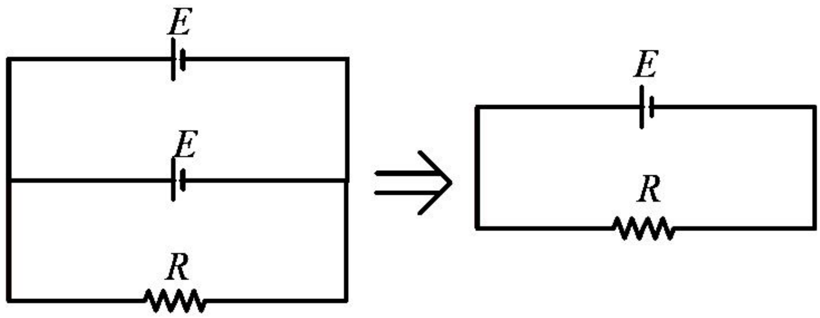
Ans. $\varepsilon = I(R+r)$ *and* $V = IR$

$$\therefore \frac{\varepsilon}{V} = \frac{R+r}{R}$$

We get, $r = \left(\frac{\varepsilon}{V} - 1\right)R$

9. Two identical cells, each of emf *E*, having negligible internal resistance, are connected in parallel with each other across an external resistance *R*. What is the current through this resistance?

Ans.



So, current $I = \frac{E}{R}$

10. Two wires of equal length, one of copper and the other of manganin have the same resistance. Which wire is thicker? [*AI 2012*]

Ans. $R_{Cu} = R_m$

$$\rho_{Cu}\frac{\rho_{Cu}}{A_{Cu}} = \rho_m\frac{\rho_m}{A_m}$$

Here, $\rho_{Cu} = \rho_m$ *as* $\rho_m > \rho_{Cu}$

$$\frac{\rho_{Cu}}{A_{Cu}} = \frac{\rho_m}{A_m}, \frac{\rho_m}{\rho_{Cu}} = \frac{A_m}{A_{Cu}} \quad as, \rho_m > \rho_{Cu}$$

So, $A_m > A_{Cu}$

Manganin wire is thicker than copper wire.

SHORT ANSWER TYPE QUESTIONS (2 MARKS/3 MARKS)

11. Two metallic wires of the same material have the same length but cross sectional area is in the ratio 1 : 2. They are connected (i) in series and (ii) in parallel. Compare the drift velocities of electrons in the two wires in both the cases (i) and (ii). [*AI 2008*]

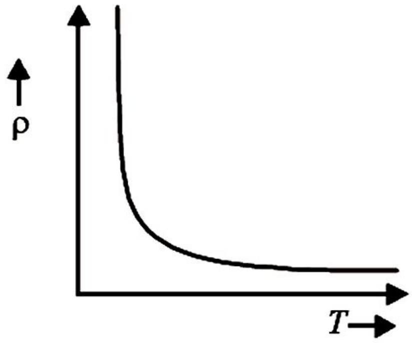
Ans. (i) In series, current *I* through both the metallic wires is same, so $\frac{v_1}{v_2} = \frac{\frac{I}{neA_1}}{\frac{I}{neA_2}} \Rightarrow \frac{v_1}{v_2} = \frac{A_2}{A_1} = \frac{2}{1}$

(ii) In Parallel, potential difference *V* applied across both of them is same, so $\frac{v_1}{v_2} = \frac{\frac{eV_\tau}{ml}}{\frac{eV_\tau}{ml}} = \frac{1}{1}$

12. Using the mathematical expression for the conductivity of a material, explain how it varies with temperature for (i) semiconductors, (ii) good conductors. [*AI 2008*]

Ans. (i) Variation of resistivity in semiconductors

Resistivity of semiconductors decrease rapidly with increase in temperature. This is because more and more electrons becomes free in semiconductors and insulators on heating, there by increasing number density *n* of free electrons. So, in insulators and semiconductors, it is not the relaxation time t but the number density '*n*' of free electrons that matters. An exponential relation exist between number of free electrons and temperature.



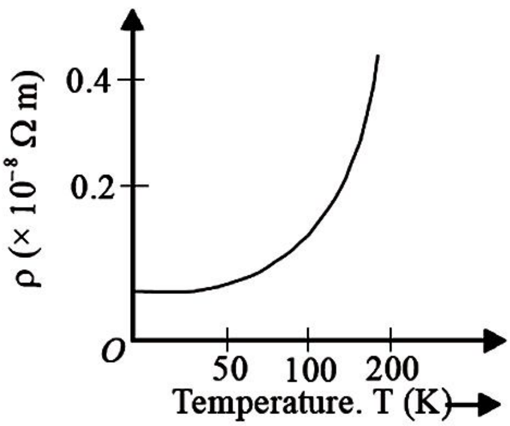
$$n(\tau) = n_0 e^{\frac{-E_g}{kT}}$$

Here E_g is the energy gap between valance and conduction band, n_0 is number of charge carriers at absolute zero temperature. So, the resistivity of insulators decreases exponentialy with increasing temperature.

$$\rho_\tau = \rho_0 e^{\frac{-E_g}{kT}}$$

Hence, temperature coefficient a of resistivity is negative in semiconductors and insulators.

(ii) Variation of resistivity in conductors

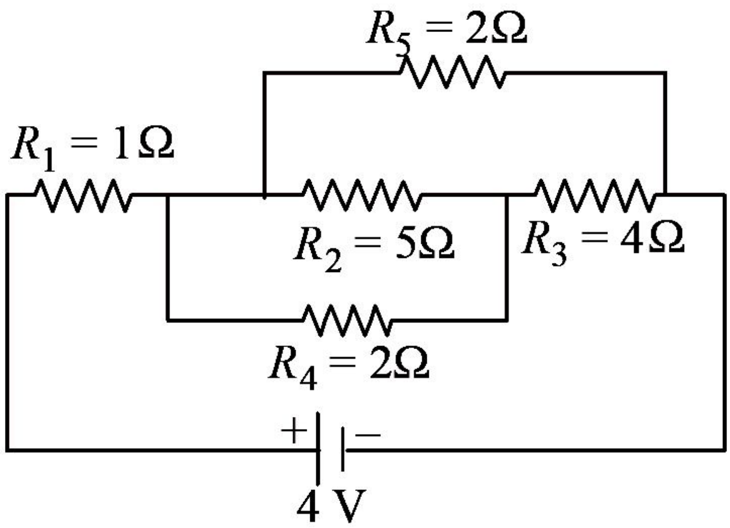


On increasing the temperature of a conductor its resistivity and resistance increases. In metals, the number of free electron is fixed. As the temperature is increased, the atom/ions vibrates with increasing amplitude also the kinetic energy of free electrons increases. Thus now the electrons collide more frequently with atoms and hence the relaxation time t decreases.

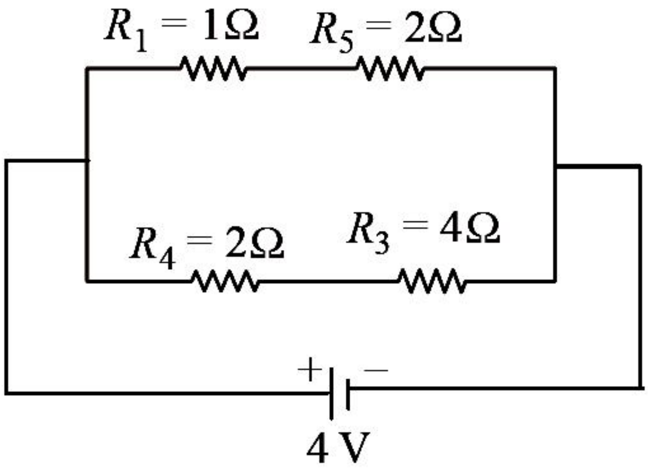
As resistivity of a conductor, $\rho = \frac{m}{ne^2\tau}$ *or* $\rho \propto \frac{1}{\tau}$

So the resistivity of a conductor increase, there by increasing the resistance of conductor. Resistivity increases non linearly in conductors. For example, variation of resistivity of copper is as shown in the graph.

13. Calculate the current drawn from the battery in the given network.



Ans. It is a balanced Wheatstone bridge, so it can be reduced to as shown below.



As R_1 and R_5 are in series, so their equivalent resistance is $R' = R_1 + R_5 = 1 + 2 = 3\,\Omega$
As R_4 and R_3 are in series, so their equivalent resistance is $R'' = R\,4 + R\,3 = 2 + 4 = 6\,\Omega$
So, net resistance of the network is

$$\frac{1}{R} = \frac{1}{R'} + \frac{1}{R''} = \frac{1}{3} + \frac{1}{6} = \frac{2+1}{6} = \frac{3}{6} = \frac{1}{2}$$

or $R = 2\,\Omega$
So, current drawn from the battery is

$I = \frac{V}{R}$ or $I = 2$ A.

14. Derive an expression for the current density of a conductor in terms of the drift speed of electrons. [*AI 2008*]

Ans. Current density $\vec{J}$ is the current flowing through a conductor per unit are of cross section, it is a vector quantity and has the direction same as current.

$I = \vec{J}.\vec{A}$

Magnitude of current density

$$J = \frac{I}{A} = nev_d$$

15. A wire of 15 Ω resistance is gradually stretched to double its original length. It is then cut into two equal parts. These parts are then connected in parallel across a 3.0 volt battery. Find the current drawn from the battery. [*AI 2009*]

Ans. When the wire of $15\,\Omega$ resistance is stretched to double its original length, then its resistance becomes

$R' = n^2 \times 15 = 2^2 \times 15 = 60\,\Omega$

When it cut into two equal parts, then resistance of each part becomes

$$R'' = \frac{R'}{2} = \frac{60}{2} = 30\Omega$$

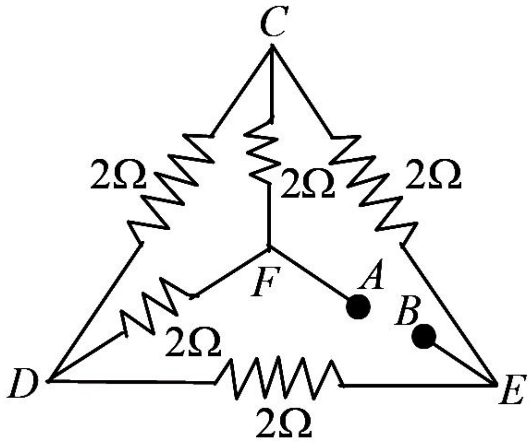
These parts are connected in parallel, then net resistance of their combination is

$$R = \frac{R''}{2} = \frac{30}{2} = 15\Omega$$

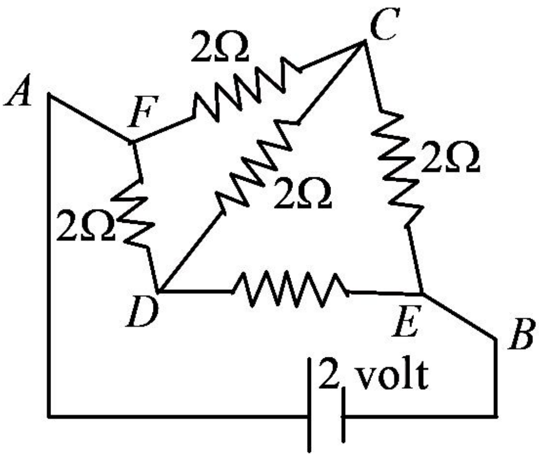
So, the current drawn from the battery

$I = \frac{V}{R} = \frac{3}{15} = \frac{1}{5}$ or $I = 0.2$ A.

16. A potential difference of 2 Volts is applied between the points *A* and *B* as shown in the network drawn in figure. Calculate (i) equivalent resistance of the network, across the point *A* and *B*, the (ii) the magnitudes of currents flowing in the arms *AFCEB* and *AFDEB*. (iii) current through *CD* and *ACB*, if a 10V d.c. source is connected between *A* and *B*. [*AI 2008*]



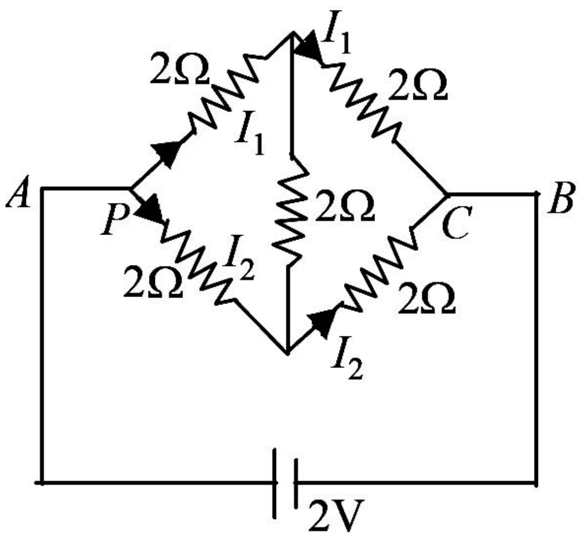
Ans. The given circuit, can also be opened up by stretching A and B as shown below.



Now it is a balanced wheatstone bridge. The potential at point C and D is the same. No current flows between C and D.

(i) The equivalent resistance is $R_{eq} = 2\,\Omega$

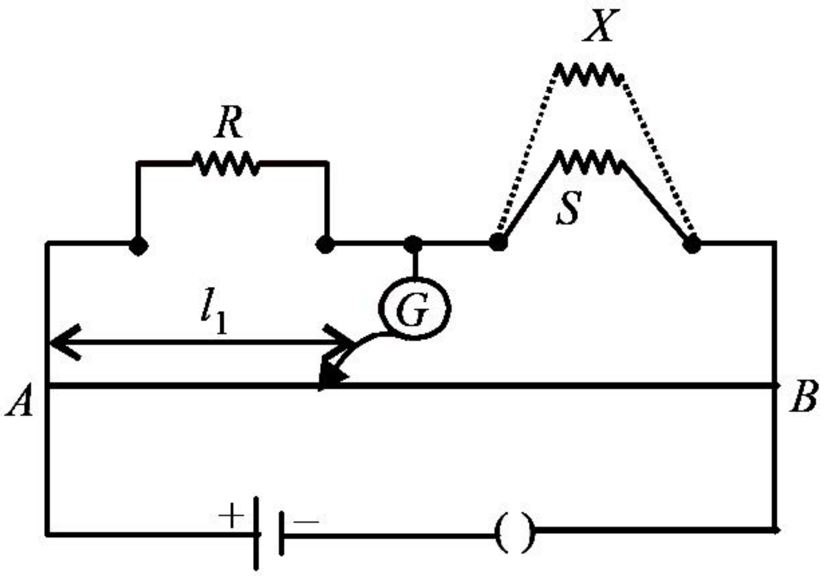
(ii) $2 = 4I_1 = 4I_2$



$I_1 = I_2 = 0.5$ A

Current through both the arms is 0.5 A.

17. (i)State the principle of working of a meter bridge. (ii) In a meter bridge balance point is found at a distance l_1 with resistances *R* and *S* as shown in the figure. When an unknown resistance *X* is connected in parallel with the resistance *S*, the balance point shifts to a distance l_2 . Find the expression for *X* in terms of l_1 , l_2 and *S*. [*AI 2009*]



Ans.
(i) Meter bridge works on the principle of Wheatstone bridge.

(ii) In first case, $\frac{R}{S} = \frac{l_1}{100 - l_1} \Rightarrow R = \frac{S.l_1}{100 - l_1}$ ----------------- (i)

In second case, $\frac{R}{\left(\frac{XS}{X+S}\right)} = \frac{l_2}{100 - l_s} \Rightarrow R = \frac{l_2 \times XS}{(100 - l_s)(X + S)}$ -------------- (ii)

By equations (i) and (ii), we get

$$\frac{S.l_1}{100 - l_1} = \frac{l_2 \times XS}{(100 - l_2)(X + S)}$$

$$\Rightarrow l_1(100 - l_2)(X + S) = Xl_2(100 - l_1)$$

$$\Rightarrow (100l_1 - l_1l_2)(X + S) = 100Xl_2 - Xl_1l_2$$

$$\Rightarrow 100Xl_1 - Xl_1l_2 + 100Sl_1 - Sl_1l_2 = 100Xl_2 - Xl_1l_2$$

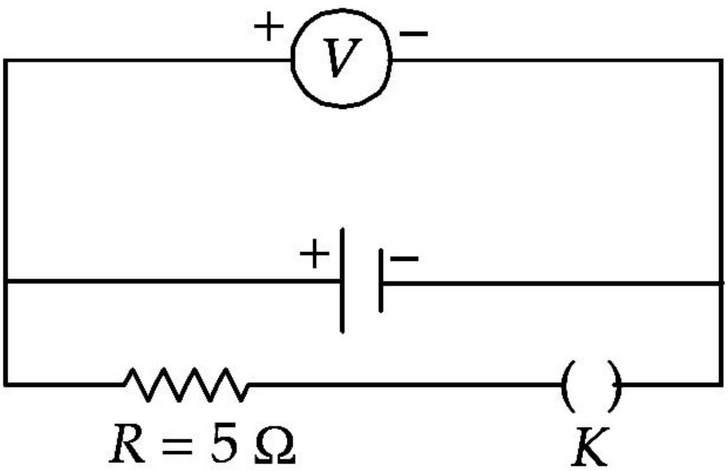
$$\Rightarrow 100Xl_1 - 100Xl_2 = Sl_1l_2 - 100Sl_1$$

$$\Rightarrow 100X(l_1 - l_2) = Sl_1(l_2 - 100)$$

$$\Rightarrow X = \frac{Sl_1(l_2 - 100)}{100(l_1 - l_2)}$$

18. Write any two factors on which internal resistance of a cell depends. The reading on a high resistance voltmeter, when a cell is connected across it, is 2.2 V. When the terminals of the cell are also connected to a resistance of $5\,\Omega$ as shown in the circuit, the voltmeter reading drops to 1.8 V. Find the internal resistance of the cell.

Ans. Internal resistance of a cell depends upon
(i) surface area of each electrode.
(ii) distance between the two electrodes.
(iii) nature, temperature and concentration of electrolyte.
Let internal resistance of cell be *r*.
Initially when *K* is open, voltmeter reads 2.2 V.
i.e. emf of the cell, ε = 2.2 V



Later when K is closed, voltmeter reads 1.8 V which is actually the terminal potential difference, V.
i.e. if I is the current flowing, then $\varepsilon = I(R + r)$

$\Rightarrow$ $2.2 = I(5 + r)$ ------------------- (i)

and $V = \varepsilon - Ir$

$1.8 = 2.2 - Ir$ ------------------- (ii)

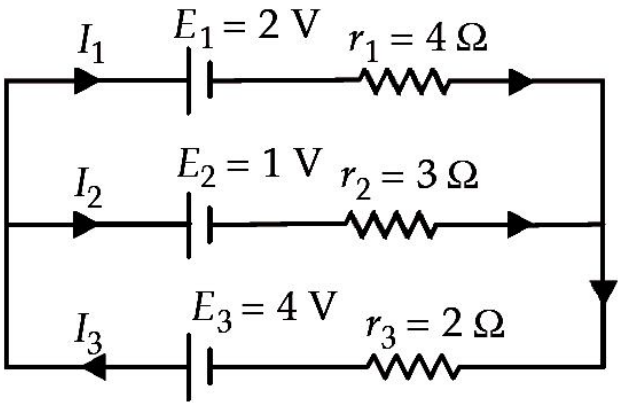
Solving (i) and (ii),

$I = 0.36$ A

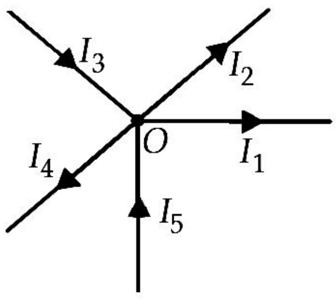
Substituting in (ii)

$$r = \frac{0.4}{0.36} = \frac{10}{9}\Omega$$

19. State Kirchhoff's rules. Use these rules to write the expressions for the currents I_1 , I_2 and I_3 in the circuit diagram shown.



Ans. Kirchhoff's first law of electrical network or junction rule states that at any junction of electrical network, sum of incoming currents is equal to the sum of outgoing currents *i.e.*,
$I_1 + I_2 + I_4 = I_3 + I_5$



Kirchhoff's second law of electrical network or loop rule states that in any closed loop, the algebraic sum of the applied emf's is equal to the algebraic sum of potential drops across the resistors of the loop *i.e.*, $\sum \varepsilon = \sum IR$

To find I_1 , I_2 , I_3 in the given diagram.
For loop *ABCFA*
$E_1 + I_1 r_1 - I_2 r_2 - E_2 = 0$
$\Rightarrow 2 + 4I_1 - 3I_2 - 1 = 0$
$\Rightarrow 4I_1 - 3I_2 + 1 = 0$ ---------------------- (i)
Using loop *FCDEF*
$E_2 + I_2 r_2 + I_3 r_3 - E_3 = 0$
$\Rightarrow 1 + 3I_2 + 2I_3 - 4 = 0$
$\Rightarrow 3I_2 + 2I_3 - 3 = 0$ ---------------------- (ii)
Also using junction rule $I_3 = I_1 + I_2$ ---------------------- (iii)
Using (ii) and (iii)
$3I_2 + 2I_1 + 2I_2 - 3 = 0$
$\Rightarrow 2I_1 + 5I_2 - 3 = 0$ ...(iv)
Solving (i) and (iv)

$$\begin{array}{r} 4I_1 - 3I_2 + 1 = 0 \\ -2\times(2I_1 + 5I_2 - 3) = 0I_1 \\ \hline 0 - 13I_2 + 7 = 0 \\ \hline \end{array}$$

$$\Rightarrow I_2 = \frac{7}{13}A$$

$$4I_1 - 3\times\frac{7}{13} + 1 = 0$$

$$\Rightarrow 4I_1 = \frac{8}{13} \Rightarrow I_1 = \frac{2}{13}A$$

$$\Rightarrow I_3 = I_1 + I_2 = \frac{2}{13} + \frac{7}{13} = \frac{9}{13}A$$

20. Define the terms (i) drift velocity, (ii) relaxation time. A conductor of length *L* is connected to a dc source of emf e. If this conductor is replaced by another conductor of same material and same area of cross section but of length 3*L*, how will the drift velocity change? [*AI 2011*]

Ans. (i) **Drift velocity :** It is defined as the average velocity of electrons with which they move along the length of the conductor when an electric field is applied across the conductor and is given by

$$v_d = \frac{e}{m}\left(\frac{V}{L}\right)\tau$$

where, V is potential difference across the conductor, L is length of conductor and τ is relaxation time m is the mass of the electron.

(ii) **Relaxation time :**

$$\text{Relaxation time} = \frac{\text{mean free path of electron}}{\text{drift speed of electron}}$$

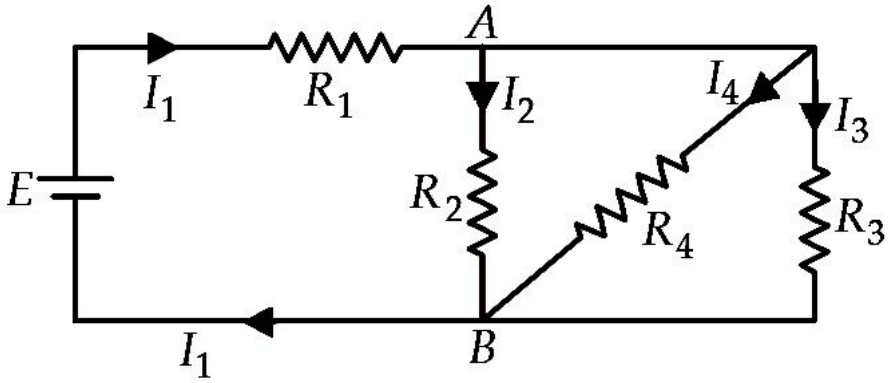
$$\text{Drift velocity, } v_d = \frac{I}{neA} = \frac{V}{neAR}\left(\because V = IR\right)$$

$$\Rightarrow v_d = \frac{V}{neA\left(\frac{\rho L}{A}\right)}\left(\because R = \frac{\rho L}{A}\right)$$

$$\Rightarrow v_d = \frac{V}{nAeL} \quad or \quad v_d \propto \frac{1}{L}$$

$$\therefore \frac{v'_d}{v_d} = \frac{L}{3L} \Rightarrow v'_d = \frac{v_d}{3}$$

21. In the circuit shown, R_1 = 4Ω, R_2 = R_3 = 15Ω, R_4 = 30Ω and E = 10 V. Calculate the equivalent resistance of the circuit and the current in each resistor.



Ans. From figure, R_2 , R_3 and R_4 are connected in parallel.

∴ Effective resistance R_p

$$\frac{1}{R_p}=\frac{1}{R_2}+\frac{1}{R_3}+\frac{1}{R_4}=\frac{1}{15}+\frac{1}{15}+\frac{1}{30}=\frac{5}{30}$$

$$\Rightarrow R_p = 6\Omega$$

Now, equivalent resistance of circuit,

$R = R_1 + R_p = 4 + 6 = 10\ \Omega$

Current, $I_1=\frac{10}{10}$ = 1 A

Potential drop across R_1 , = $I_1 R_1$ = 1 × 4 = 4 V

Potential drop across all other resistances = 10 – 4 = 6 V

Current through R_2 or R_3 ;

$$I_2=\frac{6}{15}A, I_3=\frac{6}{15}A$$

Current through R_4, $I_4=\frac{6}{30}A$

22. Define relaxation time of the free electrons drifting in a conductor. How is it related to the drift velocity of free electrons? Use this relation to deduce the expression for the electrical resistivity of the material. [*AI 2012*]

Ans. Relaxation time (τ): The average time interval between two successive collisions. For the free electrons drifting within a conductor (due to the action of the applied electric field), is called relaxation time.

$$v_d=\left(\frac{-eV\tau}{ml}\right)$$

Relation for drift velocity,

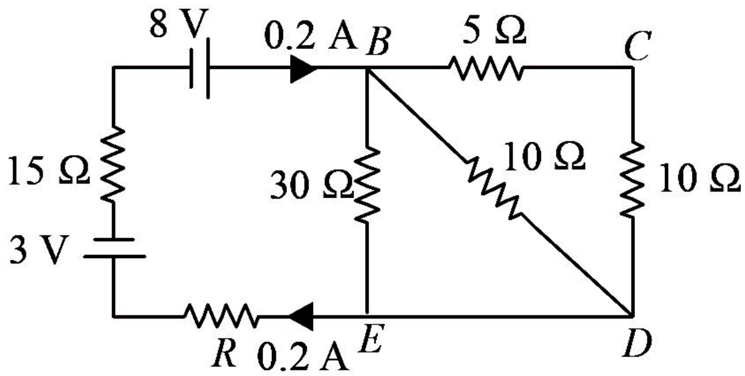
Since $I=-neAv_d$

$$\Rightarrow I=-neA\left(\frac{-eV\tau}{ml}\right)=\frac{ne^2A\tau V}{ml}$$

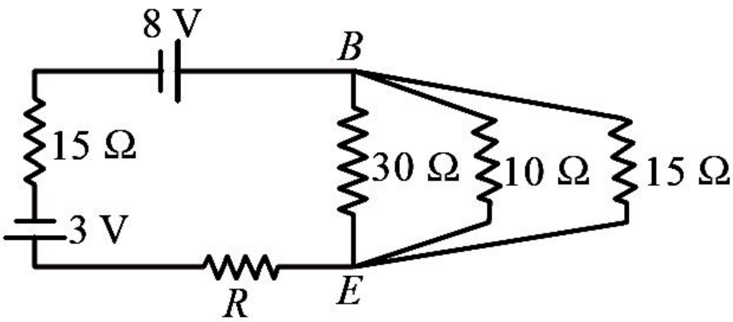
$$\therefore \frac{V}{I}=\frac{ml}{ne^2A\tau}=\frac{\rho l}{A}\left(\because \frac{V}{I}=R=\frac{\rho L}{A}\right)$$

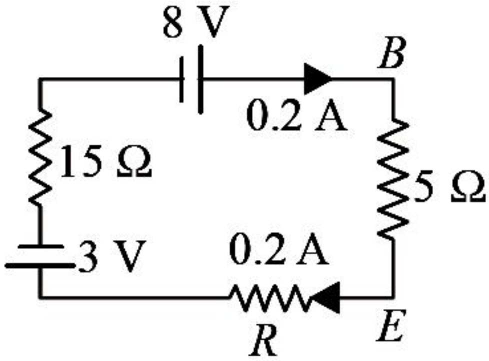
$$\therefore \rho=\frac{m}{ne^2\tau}$$

23. Calculate the value of the resistance *R* in the circuit shown in the figure so that the current in the circuit is 0.2 A. What would be the potential difference between points *B* and *E*? [*AI 2012*]



Ans. The given circuit can be simplified





In the given circuit

$$I = 0.2 = \frac{8-3}{5+15+R}$$

$$\Rightarrow 0.2 = \frac{5}{20+R} \Rightarrow 20+R = 25$$

$$\Rightarrow R = 5\Omega$$

$$V_{BE} = I(5) = 0.2 \times 5 = 1.0V$$

24. Explain the term 'drift velocity' of electrons in a conductor. Hence obtain the expression for the current through a conductor in terms of 'drift velocity'.

Ans. Drift velocity : It is the average velocity acquired by the free electrons superimposed over the random motion in the direction opposite to electric field and along the length of the metallic conductor.

Let n = number of free electrons per unit volume, v_d = Drift velocity of electrons

Total number of free electrons passing through a cross section in unit time

$$\frac{N}{t} = Anv_d$$

So, total charge passing through a cross section in unit time

i.e., current, $I = \frac{Q}{t} = \frac{Ne}{t} = Anev_d$

25. A potentiometer wire of length 1 m has a resistance of 10 Ω. It is connected to a 6 V battery in series with a resistance of 5 Ω. Determine the emf of the primary cell which gives a balance point at 40 cm.

Ans. Here, l = 1 m, R_1 = 10Ω, V = 6 V, R_2 = 5 Ω

Current flowing in potentiometer wire,

$$I = \frac{V}{R_1+R_2} = \frac{6}{10+5} = \frac{6}{15} = 0.4A$$

Potential drop across the potentiometer wire

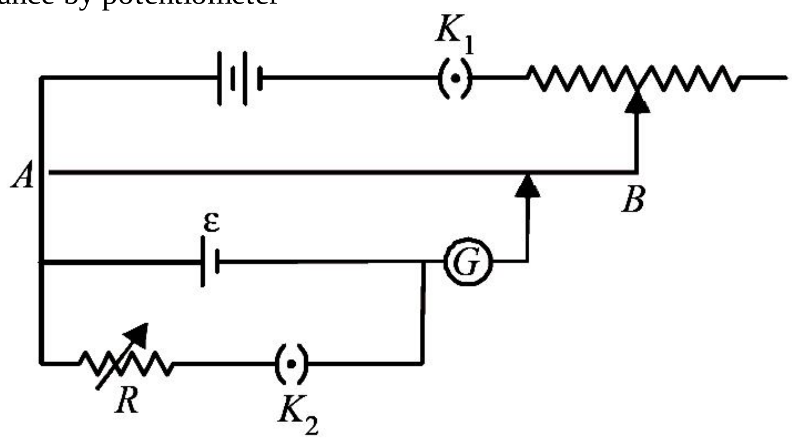
$V' = IR$ = 0.4 x 10 = 4V

Potential gradient, K = $\frac{V'}{} = \frac{4}{}$ = 4 V/m

Emf of the primary cell = $KI = 4 \times 0.4 = 1.6$ V

26. Describe briefly, with the help of a circuit diagram, how a potentiometer is used to determine the internal resistance of a cell. [*AI 2013*]

Ans. Internal resistance by potentiometer



Initially key K_2 is off

Then at balancing length l_1

$e = Kl_1$ ------------------ (i)

Now key K_2 is made on

At balancing length l_2

$V = Kl_2$ --------------- (ii)

So, $\frac{\varepsilon}{V} = \frac{l_1}{l_2}$ --------------- (iii)

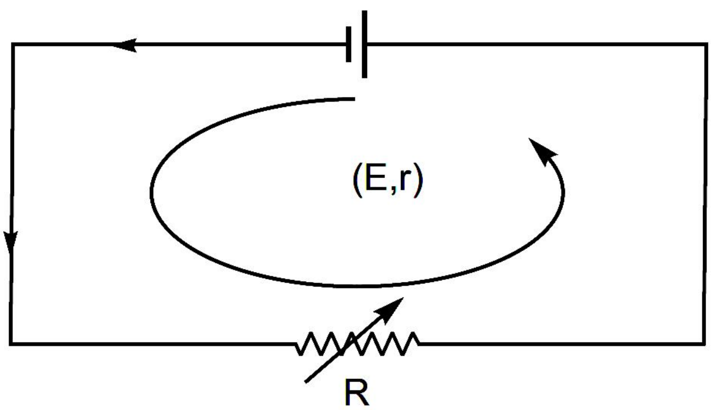
where internal resistance r is

$$r = \left(\frac{\varepsilon}{V} - 1\right)R$$

$$\Rightarrow r = \left(\frac{l_1}{l_2} - 1\right)R$$

27. A cell of emf '*E*' and internal resistance '*r*' is connected across a variable resistor '*R*'. Plot a graph showing variation of terminal voltage '*V*' of the cell versus the current '*I*'. Using the plot, show how the emf of the cell and its internal resistance can be determined.

Ans.



Suppose a current I flows through the circuit and using loop rule

$E - IR - Ir = 0$

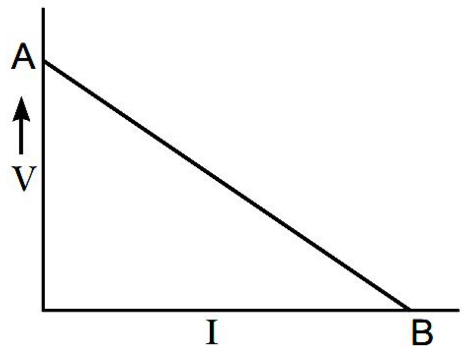
$\Rightarrow E - Ir = V\ [V = IR]$

$\Rightarrow V = E - Ir$ ……… (i)

If terminal voltage V is the function of current I, Reason – Equation of straight line, $y = -mx + c = c - mx$

Then,

Using the graph



For point A, $I = 0$ and on using equation (i)

$V = E - 0 \times r = E$

Hence voltage intercept (intercept on the vertical axis) measures emf of the cell.

For point B, $V = 0$, from equation (i)

$O = E - Ir$

$$\Rightarrow r = \frac{E}{I}$$

i.e., negative of the slope if V - I graph measures the internal resistance r.

28. Estimate the average drift speed of conduction electrons in a copper wire of cross-sectional area 1.0 × 10^{-7} m^2 carrying a current of 1.5 A. Assume the density of conduction electrons to be 9 × 10^{28} m^{-3}.

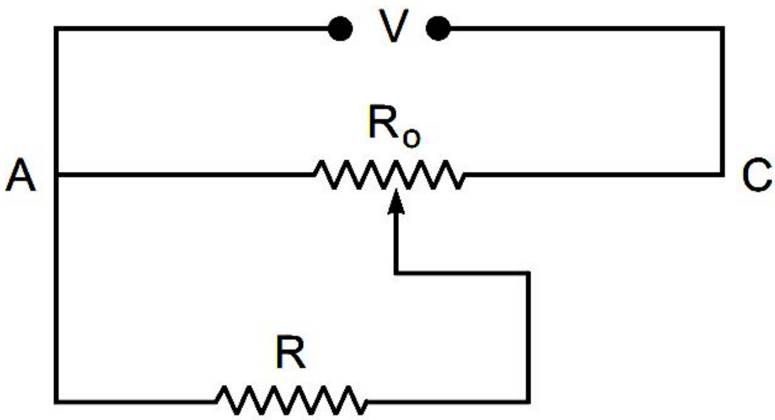
Ans. Flow of current in the conductor due to drift velocity of the free electrons is given by

$I = neAv_d$

$$v_d = \frac{I}{neA} = \frac{15}{9\times10^{28}\times1.6\times10^{-19}\times1.0\times10^{-7}}$$

$$\Rightarrow v_d = 1.048\times10^{-3} m/s \approx 1 mm/s$$

29. A resistance of RΩ draws current from a potentiometer as shown in the figure. The potentiometer has a total resistance *R*o Ω. A voltage V is supplied to the potentiometer. Derive an expression for the voltage across R when the sliding contact is in the middle of the potentiometer.

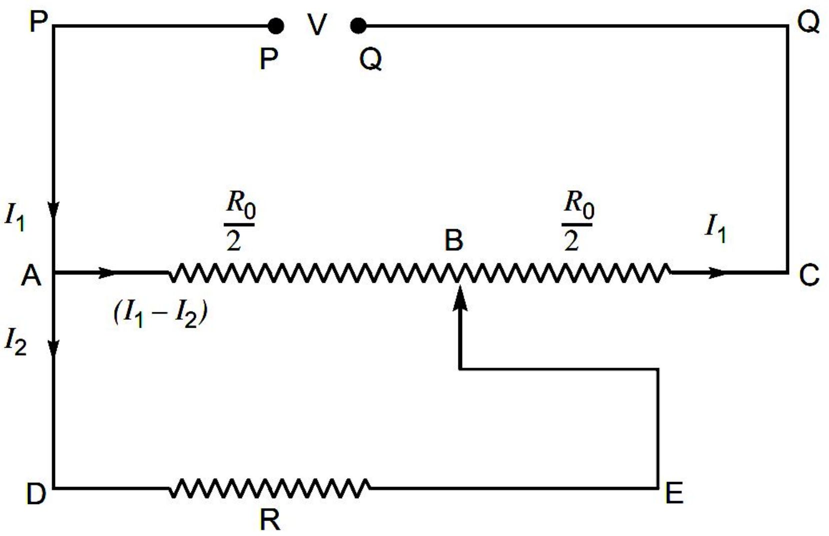


Ans. In loop *ABEDA*

$$(I_1 - I_2)\frac{R_0}{2} - I_2 R = 0$$

$$\Rightarrow I_1\frac{R_0}{2} = I_2\left(R + \frac{R_0}{2}\right) = \frac{I_2}{2}(R_0 + 2R)$$

$$\Rightarrow I_1 R_0 = I_2(R_0 + 2R) \qquad -------(i)$$



In Loop *PABCQP*

$$V = (I_1 - I_2) \times \frac{R_0}{2} + I_1 \frac{R_0}{2} = I_1 \frac{R_0}{2} - I_2 \frac{R_0}{2} + I_1 \frac{R_0}{2}$$

$$\Rightarrow V = I_1 R_0 - I_2 \frac{R_0}{2} \qquad ----------(ii)$$

From equation (*i*) and (*ii*)

$$V = R_0 \times \frac{I_2(R_0 + 2R)}{R} - I_2 \frac{R_0}{2}$$

$$\Rightarrow V = I_2 \left(\frac{R_0(R_0 + 2R)}{R} - \frac{R_0}{2} \right)$$

$$\Rightarrow V = \frac{I_2 R_0}{2R}(2(R_0 + 2R) - R) = I_2 \times \frac{R_0}{2R}(R_0 + 2R)$$

$$\Rightarrow I_2 = \frac{2VR}{R_0(R_0 + 2R)}$$

30. State the principle of a potentiometer. Define potential gradient. Obtain an expression for potential gradient in terms of resistivity of the potentiometer wire.

Ans. It is based on the fact that the fall of potential across any segment of the wire is directly proportional to the length of the segment of the wire, provided wire is of uniform area of cross-section and a constant current is flowing through it.

$$V \propto l$$

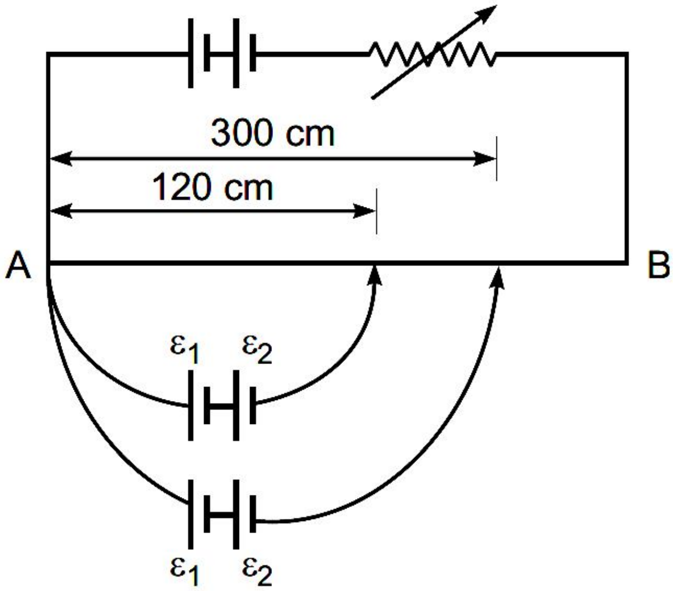
Potential gradient is the fall of potential per unit length of potentiometer wire.

Potential gradient $K = \frac{V}{l} = \frac{IR}{l} \qquad (\because V = IR)$

$$\Rightarrow K = \frac{I \frac{\rho l}{A}}{l} \qquad \left(\because R = \frac{\rho l}{A} \right)$$

$$\Rightarrow K = \frac{I\rho}{A}$$

31. Figure shows a long potentiometer wire *AB* having a constant potential gradient. The null points for the two primary cells of emfs ε_1 and ε_2 connected in the manner shown are obtained at a distance of l_1 = 120 cm and l_2 = 300 cm from the end *A*. Determine (*i*) $\varepsilon_1/\varepsilon_2$ and (*ii*) position of null point for the cell ε_1 only.



Ans. Let k = potential gradient in V/cm

$\varepsilon_1 + \varepsilon_2 = 300k$ ...(i)

$\varepsilon_1 - \varepsilon_2 = 120k$ ...(ii)

Adding (i) and (ii), we get $2\,\varepsilon_1 = 420k$

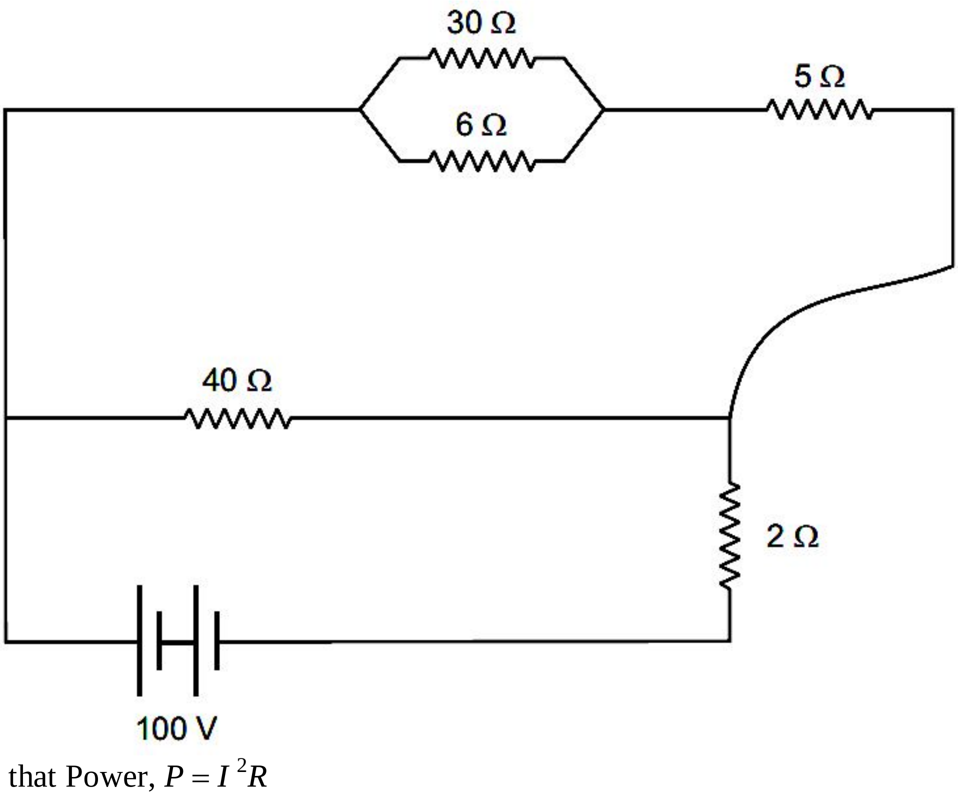
$\Rightarrow \varepsilon_1 = 210k$

Substituting the value of ε_1 in equation (i), we get $\varepsilon_2 = 90k$

Therefore, $\frac{\varepsilon_1}{\varepsilon_2} = \frac{210}{90} = \frac{7}{3}$

Balancing length for cell ε_1 is 210 cm.

32. A 100 V battery is connected to the electric network as shown. If the power consumed in the 2Ω resistor is 200 W, determine the power dissipated in the 5Ω resistor.



Ans. We know that Power, $P = I^2R$

$\Rightarrow 200 = I^2 \times 2$

$I^2 = \frac{200}{2} = 100$

$\Rightarrow I = \sqrt{100} = 10A$

$\therefore$ Current flowing through 2Ω resistor = 10 A

Potential drop across 2Ω resistor, $V = IR$

= 10 × 2 = 20 V

Equivalent resistance of 30Ω and 6Ω

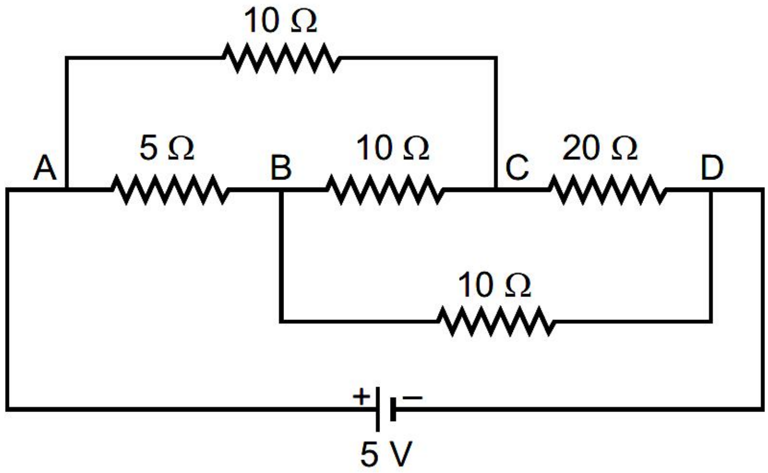
$\frac{30 \times 6}{30 + 6} = \frac{180}{36} = 5\Omega$

$\therefore$ Therefore, potential across parallel combination of 40Ω and 10Ω = 10 × 8 = 80 V

$\therefore$ Current through 5Ω resistor, $I = \frac{80}{10} = 8A$

$\therefore$ Power dissipated in 5Ω resistor = I^2R = 82 × 5 = 320 W

33. Calculate the value of the current drawn from a 5 V battery in the circuit as shown.



Ans.

In case of balanced Wheatstone bridge, no current flows through the resistor 10Ω between points B and C.

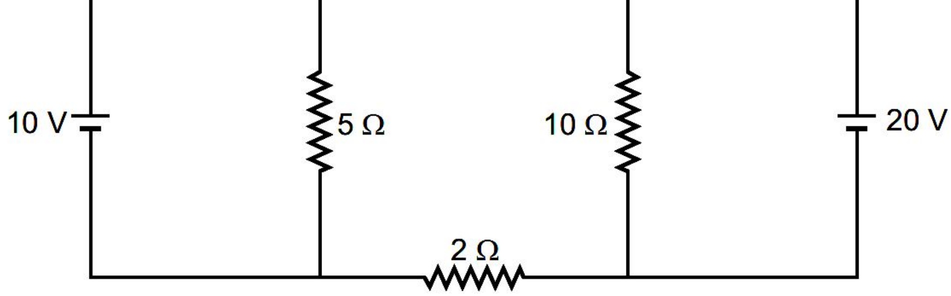
The resistance of arm ACD, R_1 =10 + 20 =30Ω

The resistance of arm ABD, R_2 = 5 +10 =15Ω

Equivalent resistance $R_{eq} = \frac{R_1 \times R_2}{R_1 + R_2} = \frac{30 \times 15}{30 + 15} = \frac{450}{45} = 10\Omega$

Current drawn from the source, $I = \frac{V}{R_{eq}} = \frac{5}{10} = 0.5A$

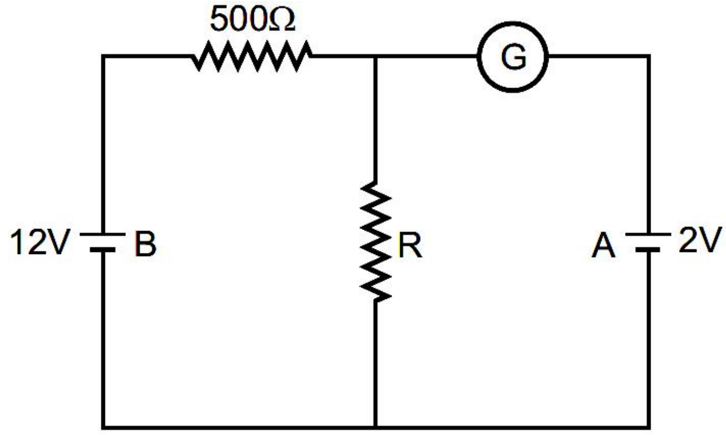
34. What will be the value of current through the 2Ω resistance for the circuit shown in the figure? Give reason to support your answer.



Ans. No current will flow through 2Ω resistor, because in a closed loop, total p.d. must be zero. So

$10\text{ V} - 5I_1 = 0 \ldots (1)$
$20\text{ V} - 10I_2 = 0 \ldots (2)$
Equation (1) and (2) have no solutions and resistor 2Ω is not part of any loop ABCD and EFGH.

35. In the circuit shown in the figure, the galvanometer 'G' gives zero deflection. If the batteries A and B have negligible internal resistance, find the value of the resistor R.



Ans. If galvanometer G gives zero deflection, than current of source of 12V flows through R, and voltage across R becomes 2V.

Current in the circuit, $I = \dfrac{\varepsilon}{R_1 + R_2} = \dfrac{12.0V}{500 + R}$

and V = IR = 2.0V

$\left(\dfrac{12.0V}{500 + R}\right) R = 2.0$

$\Rightarrow$ 12R = 1000 + 2R
$\Rightarrow$ 10R = 1000
$\Rightarrow$ R = 100 Ω