[SINGLE CORRECT CHOICE TYPE]

Q.1 to Q.10 has four choices (A), (B), (C), (D) out of which **ONLY ONE** is correct.

Q.1 If $\vec{V}_1 = \hat{i} + \hat{j} + \hat{k}$ and $\vec{V}_2 = a\hat{i} + b\hat{j} + c\hat{k}$ where a, b, c $\in \{-2, -1, 0, 1, 2\}$, then number of possible non-zero vectors $\vec{V}_2$ perpendicular to $\vec{V}_1$ is

(A) 10 (B) 13 (C) 15 (D) 18

Q.2 If a plane meets co-ordinate axes in A, B, C such that the centroid of the triangle is $(1, k, k^2)$ then the least distance of plane from origin, is

(A) $\left|\frac{3k^2}{\sqrt{1+k+k^2}}\right|$ (B) $\left|\frac{3k}{\sqrt{1+k+k^2}}\right|$ (C) $\left|\frac{3k^2}{\sqrt{1+k^2+k^4}}\right|$ (D) $\left|\frac{3k}{\sqrt{1+k^2+k^4}}\right|$

Q.3 Let $\hat{a}$ and $\hat{c}$ be unit vectors at an angle $\frac{\pi}{3}$ with each other. If $\left(\hat{a}\times(\hat{b}\times\hat{c})\right)\cdot(\hat{a}\times\hat{c}) = 5$, then the value of $[\hat{a}\ \hat{b}\ \hat{c}]$ is equal to

(A) 10 (B) –10 (C) 5 (D) –5

Q.4 If $\vec{a}$ and $\vec{b}$ are non-zero vectors such that $|\vec{a}+\vec{b}| = |\vec{a}-2\vec{b}|$, then

(A) $\vec{a}\cdot\vec{b} = 2|\vec{b}|^2$ (B) $\vec{a}\cdot\vec{b} = |\vec{b}|^2$

(C) least value of $\vec{a}\cdot\vec{b} + \frac{4}{|\vec{b}|+2}$ is $2\sqrt{2}$. (D) least value of $\vec{a}\cdot\vec{b} + \frac{4}{|\vec{b}|^2+2}$ is $2\sqrt{2}-1$.

Q.5 $P\left(1, 7, \sqrt{2}\right)$ be a point and line L is $2\sqrt{2}(x-1) = y-2,\ z=0$. If PQ is distance of plane $\sqrt{2}x + y - z = 1$ from P measured along a line inclined at an angle of $45°$ with L and is minimum then PQ is

(A) 2 (B) 3 (C) 4 (D) 5

Q.6 Let OABC be a regular tetrahedron of side length unity, where O is origin. If P is a point at a unit distance from origin such that OP is equally inclined to $\overrightarrow{OA}$, $\overrightarrow{OB}$ and $\overrightarrow{OC}$ at an angle α. Then $\cos^2\alpha$ equals

(A) 0 (B) $\frac{1}{3}$ (C) $\frac{2}{3}$ (D) $\frac{5}{6}$

Q.7 Volume of parallelopiped whose adjacent sides are given by vectors $\vec{\alpha}, \vec{\beta}$ and $\vec{\gamma}$, where $\vec{\alpha}, \vec{\beta}$ and $\vec{\gamma}$ are unit vectors which are bisecting the angle between three pair wise orthonormal vectors, is

(A) $\frac{1}{2\sqrt{2}}$ (B) $\sqrt{2}$ (C) $2\sqrt{2}$ (D) $\frac{1}{\sqrt{2}}$

Q.8 If $\vec{a} = \hat{i}+\hat{j}+\hat{k}$, $\vec{b} = 2\hat{i}-\hat{j}+\hat{k}$ and $\vec{c} = \hat{i}+x\hat{j}+y\hat{k}$ are linearly dependent and $|\vec{c}| = \sqrt{3}$, then (x, y) is (are)

(A) $\left(\frac{-7}{5}, \frac{-1}{5}\right)$ (B) (–2, 0) (C) $\left(\frac{1}{5}, \frac{-7}{5}\right)$ (D) (1, 1)

Q.9 If $\vec{\alpha}$ and $\vec{\beta}$ are any two vectors of magnitudes 2 and 3 respectively such that $2|\vec{\alpha}\times\vec{\beta}| + 3|\vec{\alpha}\cdot\vec{\beta}| = \gamma$, then the maximum value of γ, is

(A) $\sqrt{13}$ (B) $2\sqrt{13}$ (C) $6\sqrt{13}$ (D) $10\sqrt{13}$

Q.10 If $\vec{p}$, $\vec{q}$, $\vec{r}$ are unit vectors such that $\vec{p}\cdot\vec{q} = 0 = \vec{p}\cdot\vec{r}$ and the angle between $\vec{q}$ and $\vec{r}$ is $\frac{\pi}{3}$, then the value of $|\vec{p}\times\vec{q} - \vec{p}\times\vec{r}|$ is equal to

(A) 0 (B) $\frac{1}{2}$ (C) 1 (D) 2

[PARAGRAPH TYPE]

Q.11 to Q.13 has four choices (A), (B), (C), (D) out of which **ONLY ONE** is correct.

Paragraph for question nos. 11 to 13

Let $\vec{r}_1 = a^2\hat{i} - 2b\hat{j} - 6\hat{k}$ and $\vec{r}_2 = 2a\hat{i} - a\hat{j} - a^2\hat{k}$ be non-zero vectors and $\vec{r}_3 = \hat{i} - 2\hat{j} + a\hat{k}$ be a position vector of a point where $a, b \in R$. P_1 and P_2 are two planes for the largest integral value of b, which containing vector $\vec{r}_1$ and $\vec{r}_2$ is the normal vector of both the planes and passing through point $\vec{r}_3$.

Q.11 If θ is the acute angle between the planes then $\cos\theta$ is

(A) 0 (B) $\frac{7}{3\sqrt{6}}$ (C) $\frac{7}{18}$ (D) 1

Q.12 If d_1 and d_2 are perpendicular distances of the planes from the origin then $(d_1 + d_2)$ equals

(A) 0 (B) $\frac{5}{\sqrt{6}}$ (C) $\frac{7}{6\sqrt{6}}$ (D) $\frac{3}{\sqrt{6}}$

Q.13 Equation of the plane containing the line $P_1 = 0 = P_2$ and passing through the origin is

(A) $2x - y + 2z = 0$ (B) $2x + y - z = 0$ (C) $2x + y - 2z = 0$ (D) $x + y - 2z = 0$

[REASONING TYPE]

Q.14 to Q.16 has four choices (A), (B), (C), (D) out of which **ONLY ONE** is correct.

Q.14 Consider a plane $P : x + 2y - 3z + 4 = 0$ and three lines

$$L_1 : \frac{x-1}{1} = \frac{y-3}{2} = \frac{2-z}{3},\ L_2 : \frac{2x-1}{2} = \frac{y}{2} = \frac{z-1}{-3} \text{ and } L_3 : \frac{x-2}{2} = \frac{2y-10}{8} = \frac{z-5}{-6}$$

Statement-1 : Lines L_1, L_2 and L_3 lie in the same plane.

Statement-2 : Lines L_1, L_2 and L_3 are perpendicular to the plane P.

(A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.

(B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1.

(C) Statement-1 is true, statement-2 is false.

(D) Statement-1 is false, statement-2 is true.

Q.15 **Statement-1 :** Let $A(\vec{a})$, $B(\vec{b})$ and $C(\vec{c})$ be three points such that $\vec{a} = 2\hat{i} + \hat{j} + \hat{k}$, $\vec{b} = 3\hat{i} - \hat{j} + 3\hat{k}$ and $\vec{c} = -\hat{i} + 7\hat{j} - 5\hat{k}$, then OABC is a tetrahedron.

Statement-2 : Let $A(\vec{a})$, $B(\vec{b})$ and $C(\vec{c})$ be three points such that $\vec{a}$, $\vec{b}$ and $\vec{c}$ are non-coplanar then OABC is a tetrahedron, where O is the origin.

(A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.
(B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1.
(C) Statement-1 is true, statement-2 is false.
(D) Statement-1 is false, statement-2 is true.

Q.16 **Statement-1:** If $\vec{b} \neq \vec{0}$, then $(\vec{r} - \vec{a}) \times \vec{b} = \vec{0}$ represents a straight line passing through the point whose position vector is $\vec{a}$ and the line is perpendicular to the plane $\vec{r} \cdot \vec{b} = \vec{a} \cdot \vec{b}$.

Statement-2: If $\vec{b}$ is a unit vector and $\vec{a} \cdot \vec{b} \neq 1$, then $\vec{r} \cdot \vec{b} = \vec{a} \cdot \vec{b}$ represents a plane passing through the point whose position vector is $\vec{b}$.

(A) Statement-1 is true, Statement-2 is true, Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is true, Statement-2 is true, Statement-2 is not a correct explanation for Statement-1.
(C) Statement-1 is true, Statement-2 is false.
(D) Statement-1 is false, Statement-2 is true.

[MATRIX TYPE]

Q.17 has **four** statements (A, B, C, D) given in **Column-I** and **five** statements (P, Q, R, S, T) given in **Column-II**. Any given statement in **Column-I** can have correct matching with one or more statement(s) given in **Column-II**.

Q.17 Let P be a variable point in 3-D space and A(5, 0, –3), B(0, –5, 4) and C(0, 0, 0) be three points on the plane $3x + 4y + 5z = 0$. If the volume of the tetrahedron P-ABC is equal to 25 cubic units, then locus of P is $ax + by + 5z \pm k = 0$.

	Column-I		**Column-II**
(A)	The value of k is	(P)	$3\sqrt{2}$
(B)	If A, B, C, D in order is a parallelogram, then area of the parallelogram is	(Q)	$15\sqrt{2}$
(C)	If G is the centroid of the ΔABC, then least value of PG is	(R)	$25\sqrt{2}$
(D)	If H is the orthocentre of the ΔABC, then least value of PH is	(S)	30
		(T)	34

[INTEGER TYPE]

Q.18 & Q.19 are "Integer Type" questions. (The answer to each of the questions **are upto 4 digits**)

Q.18 Let equation of plane be $x + 2y + z - 3 = 0$. An insect starts flying from point P(1, 3, 2) in straight line. It touches the plane at point R(a, b, c) and then goes to point Q(3, 5, 2) in straight line. If distance travelled PR + QR is minimum then find the value of $(a + b + c)$.

Q.19 Let $\vec{a}$ and $\vec{b}$ are two non-zero, non-collinear vectors $(|\vec{a}| = 1)$ such that vectors $3(\vec{a} \times \vec{b})$ and $2(\vec{b} - (\vec{a} \cdot \vec{b})\vec{a})$ represents two sides of a triangle. If area of the triangle is $\frac{3}{4}(|\vec{b}|^4 + 4)$, then find the value of $[|\vec{b}|]$.

[**Note :** [k] denotes greatest integer less than or equal to k.]

ANSWER KEY

Q.1	D	Q.2	C	Q.3	B	Q.4	D	Q.5	B
Q.6	C	Q.7	D	Q.8	D	Q.9	C	Q.10	C
Q.11	B	Q.12	D	Q.13	A	Q.14	D	Q.15	D
Q.16	C	Q.17	(A) S; (B) R; (C) P ; (D) P			Q.18	[2]	Q.19	[1]

[HINTS & SOLUTIONS]

Q.1 18

Sol. $\vec{V}_1 \cdot \vec{V}_2 = 0 \Rightarrow a + b + c = 0$

$\frac{a}{-2}$	$\frac{b}{1}$	$\frac{c}{1}$	$\Rightarrow$	$\frac{3!}{2!} = 3$
-2	0	2	$\Rightarrow$	$3! = 6$
-1	0	1	$\Rightarrow$	$3! = 6$
-1	-1	2	$\Rightarrow$	$\frac{3!}{2!} = 3$

Total number of vectors = 18

Q.2 $\left|\frac{3k^2}{\sqrt{1+k^2+k^4}}\right|$

Sol. Equation of plane is given by $\frac{x}{3} + \frac{y}{3k} + \frac{z}{3k^2} = 1$ or $k^2x + ky + z = 3k^2$.

$\Rightarrow$ Distance of plane from origin equal to $\left|\frac{3k^2}{\sqrt{1+k^2+k^4}}\right|$

Q.3 -10

Sol. $\because \left(\hat{a} \times (\hat{b} \times \hat{c})\right) \cdot (\hat{a} \times \hat{c}) = 5$

$\Rightarrow \left((\hat{a} \cdot \hat{c})\hat{b} - (\hat{a} \cdot \hat{b})\hat{c}\right) \cdot (\hat{a} \times \hat{c}) = 5$

$\Rightarrow (\hat{a} \cdot \hat{c})\left(\hat{b} \cdot (\hat{a} \times \hat{c})\right) - 0 = 5 \Rightarrow \frac{1}{2}[\hat{b}\ \hat{a}\ \hat{c}] = 5$

$\Rightarrow [\hat{a}\ \hat{b}\ \hat{c}] = -10$

Q.4 least value of $\vec{a} \cdot \vec{b} + \frac{4}{|\vec{b}|^2 + 2}$ is $2\sqrt{2} - 1$.

Sol. $2\vec{a} \cdot \vec{b} = -4\vec{a} \cdot \vec{b} + 3\vec{b}^2$

$2\vec{a} \cdot \vec{b} = \vec{b}^2$

$\Rightarrow \vec{a} \cdot \vec{b} + \frac{4}{|\vec{b}|^2 + 2} = \left(1 + \vec{a} \cdot \vec{b}\right) + \frac{4}{2(\vec{a} \cdot \vec{b} + 1)} - 1 \geq 2\sqrt{2} - 1$

Q.5 3

Sol. Given line 'L' makes 45° with the given plane

Hence, $PQ = \frac{\left|\sqrt{2} + 7 - \sqrt{2} - 1\right|}{\sqrt{2+1+1}} = \frac{6}{2} = 3$

Q.6 $\frac{2}{3}$

Sol. Let $\overrightarrow{OA} = \hat{a}$, $\overrightarrow{OB} = \hat{b}$, $\overrightarrow{OC} = \hat{c}$ and $\overrightarrow{OP} = \hat{d}$

Here $\hat{a}\cdot\hat{b} = \hat{b}\cdot\hat{c} = \hat{c}\cdot\hat{a} = \frac{1}{2}$

and $\hat{d}\cdot\hat{a} = \hat{d}\cdot\hat{b} = \hat{d}\cdot\hat{c} = \cos\alpha$

Let $\hat{d} = x\hat{a} + y\hat{b} + z\hat{c}$ ......(i)

$$\hat{d}\cdot\hat{a} = x + \frac{y}{2} + \frac{z}{2} = \cos\alpha$$

$$\hat{d}\cdot\hat{b} = \frac{x}{2} + y + \frac{z}{2} = \cos\alpha$$

$$\hat{d}\cdot c = \frac{x}{2} + \frac{y}{2} + z = \cos\alpha$$

Adding we get $2(x + y + z) = 3\cos\alpha$

Taking dot product with $\hat{d}$ in (i) we get

$$1 = x\cos\alpha + y\cos\alpha + z\cos\alpha \Rightarrow x + y + z = \frac{1}{\cos\alpha}$$

$$\Rightarrow \frac{3\cos\alpha}{2} = \frac{1}{\cos\alpha} \Rightarrow \cos^2\alpha = \frac{2}{3}$$

Q.7 $\frac{1}{\sqrt{2}}$

Sol. Let three orthonormal vectors are $\hat{i}$, $\hat{j}$ and $\hat{k}$ then

$$\vec{\alpha} = \frac{\hat{i}+\hat{j}}{\sqrt{2}},\ \vec{\beta} = \frac{\hat{j}+\hat{k}}{\sqrt{2}},\ \vec{\gamma} = \frac{\hat{i}+\hat{k}}{\sqrt{2}}$$

$$\Rightarrow \text{volume} \left\| \left[\frac{\hat{i}+\hat{j}}{\sqrt{2}}, \frac{\hat{j}+\hat{k}}{\sqrt{2}}, \frac{\hat{i}+\hat{k}}{\sqrt{2}} \right] \right\| = \frac{1}{2\sqrt{2}} \times 2[\hat{i}\ \hat{j}\ \hat{k}] = \frac{1}{\sqrt{2}}$$

Q.8 (1, 1)

Sol. We must have $\begin{vmatrix} 1 & 1 & 1 \\ 2 & -1 & 1 \\ 1 & x & y \end{vmatrix} = 0$

$\Rightarrow$ $x - 3y + 2 = 0$ ......(1)

Also, $x^2 + y^2 + 1 = 3$

$\Rightarrow$ $x^2 + y^2 = 2$ ......(2)

$\therefore$ $(1) \cap (2)$

$\Rightarrow$ (1, 1)

Q.9 $6\sqrt{13}$ (D) $10\sqrt{13}$

Sol. $\gamma = 12 \sin\theta + 18\cos\theta$ where $\theta = \vec{\alpha} \wedge \vec{\beta}$

$\therefore\ \gamma \le \sqrt{(12)^2 + (18)^2} = \sqrt{468} = \sqrt{36\times 3} = 6\sqrt{13}$

Q.10 1

Sol. $|\vec{p}\times(\vec{q}-\vec{r})|^2 = |\vec{p}|^2|\vec{q}-\vec{r}|^2 - (\vec{p}\cdot(\vec{q}-\vec{r}))^2 = |\vec{p}|\ |\vec{q}-\vec{r}|^2 = (1)^2\left(1+1-2\cos\frac{\pi}{3}\right) = 1$

Q.11 $\frac{7}{3\sqrt{6}}$

Q.12 $\frac{3}{\sqrt{6}}$

Q.13 $2x - y + 2z = 0$

Sol. $\vec{r}_1 \cdot \vec{r}_2 = 0$

$2a^3 + 2ab + 6a^2 = 0$

$a^2 + b + 3a = 0$

$b = -a^2 - 3a = \frac{9}{4} - \left(a + \frac{3}{2}\right)^2 \Rightarrow b \le \frac{9}{4}$

$\therefore$ largest integral value of b is 2

Now, for $b = 2$, $a^2 + 3a + 2 = 0 \Rightarrow a = -1, -2$

For $a = -1, b = 2$

$\vec{r}_1 = \hat{i} - 4\hat{j} - 6\hat{k}$

$\vec{r}_2 = -2\hat{i} + \hat{j} - \hat{k}$

$\vec{r}_3 = -\hat{i} - 2\hat{j} - \hat{k}$

For $a = -2, b = 2$

$\vec{r}_1 = 4\hat{i} - 4\hat{j} - 6\hat{k}$

$\vec{r}_2 = -4\hat{i} + 2\hat{j} - 4\hat{k}$

$\vec{r}_3 = \hat{i} - 2\hat{j} - 2\hat{k}$

Plane P_1

$-2x + y - z + \lambda = 0$

$(1, -2, -1)$,

$-2 - 2 + 1 + \lambda = 0$

$\Rightarrow \lambda = 3$

$\therefore P_1 : -2x + y - z + 3 = 0$

Plane P_2

$-4x + 2y - 4z + \lambda' = 0$

$(1, -2, -2)$,

$-4 - 4 + 8 + \lambda' = 0$

$\Rightarrow \lambda' = 0$

$\therefore P_2 : 2x - y + 2z = 0$

(i) $\cos\theta = \left|\frac{(-2)(-4) + 1(2) + (-1)(-4)}{\sqrt{6}\ \sqrt{14+4+16}}\right| = \frac{14}{6\sqrt{6}} = \frac{7}{3\sqrt{6}}$

(ii) $d_1 = \frac{3}{\sqrt{6}}$, $d_2 = 0$

$\therefore\ d_1 + d_2 = \frac{3}{\sqrt{6}}$

(iii) $p_1 + \lambda p_2 = 0$

$-2x + y - 3 + 3 + \lambda(2x - y + 2z) = 0$

which passes through (0, 0, 0)

$3 + 0 = 0$ (absurd result)

It means p_2 is the required plane.

Q.14 Statement-1 is false, statement-2 is true.

Sol. Statement-1 : Lines L_1, L_2 and L_3 are parallel

Plane containing lines L_1 & L_2 is

$$\begin{vmatrix} x-1 & y-3 & z-2 \\ 1 & 2 & -3 \\ 1/2 & 3 & 1 \end{vmatrix} = 0$$

L_3 passing through the point (2, 5, 5)

$$\therefore \quad \begin{vmatrix} 1 & 2 & 3 \\ 1 & 2 & -3 \\ 1/2 & 3 & 1 \end{vmatrix} \neq 0$$

Hence, lines L_1, L_2 and L_3 are not coplanar.

Q.15 Statement-1 is false, statement-2 is true.

Sol. Since $\vec{a}\cdot(\vec{b}\times\vec{c}) = 0 = [\vec{a}\,\vec{b}\,\vec{c}]$

$\Rightarrow \overrightarrow{OA}, \overrightarrow{OB}$ and $\overrightarrow{OC}$ are coplanar.

So tetrahedron is not formed.

Q.16 Statement-1 is true, Statement-2 is false.

Sol. S-1 is true. $(\vec{r} - \vec{a}) \times \vec{b} = \vec{0}$ is the equation of the straight line passing through the point whose position vector is $\vec{a}$ and the vector $\vec{b}$ lies along the line. Again since vector $\vec{b}$ is perpendicular to the plane

$(\vec{r} - \vec{a}) \cdot \vec{b} = 0$

$\therefore$ the line is perpendicular to the given plane.

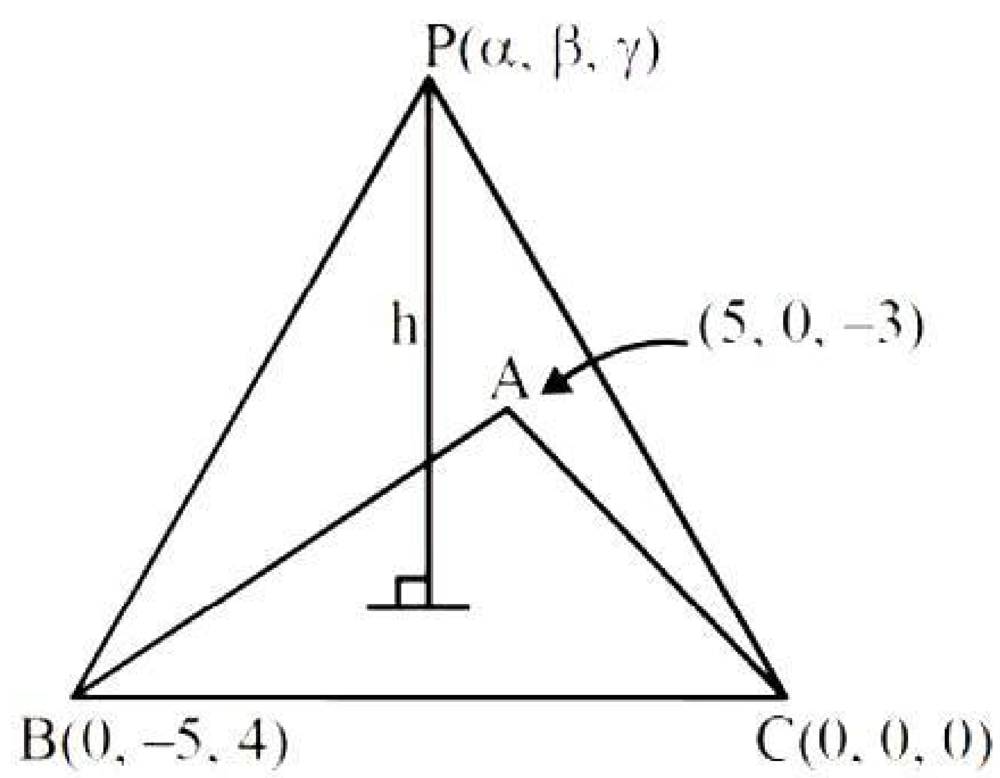
$\therefore$ Statement-1 is true.

Statement-2: $\vec{r}\cdot\vec{b} = \vec{a}\cdot\vec{b} \quad \Rightarrow \quad (\vec{r} - \vec{a})\cdot\vec{b} = 0$ represent a plane.

If point $\vec{b}$ lies on the plane then $\vec{b}\cdot\vec{b} = \vec{a}\cdot\vec{b} \quad \Rightarrow \quad \vec{a}\cdot\vec{b} = 1$ which is not rue.

$\therefore$ Statement-2 is false.

Q.1 (A) S; (B) R; (C) P ; (D) P



Sol. $$\overrightarrow{BC}\times\overrightarrow{BA} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 5 & -4 \\ 5 & 5 & -7 \end{vmatrix}$$

$$= \hat{i}(-15) - \hat{j}(20) + \hat{k}(-25)$$

$$= -15\hat{i} - 20\hat{j} - 25\hat{k}$$

Area $(\Delta ABC) = \dfrac{1}{2}\sqrt{(15)^2 + (20)^2 + (25)^2}$

$$= \frac{1}{2}\sqrt{225+400+625} = \frac{1}{2}\cdot 25\sqrt{2} = \frac{25}{\sqrt{2}}$$

Volume of tetrahedron $V = \frac{1}{3} \times \frac{25}{\sqrt{2}} \times h = 25 \Rightarrow h = 3\sqrt{2}$

Locus of P is the plane parallel to the given plane at a distance of $3\sqrt{2}$ units

$3x + 4y + 5z + d = 0$

$\therefore \quad \left|\frac{d}{\sqrt{9+16+25}}\right| = 3\sqrt{2} \Rightarrow \quad d = \pm 30$

$\therefore \quad 3x + 4y + 5z \pm 30 = 0$

(P) $k = 30$

(Q) Area of parallelogram ABCD $= \left|\overrightarrow{BC} \times \overrightarrow{BA}\right| = 25\sqrt{2}$

(R) Least value of $PG = h = 3\sqrt{2}$

(S) Least value of $PH = h = 3\sqrt{2}$

Q.18 [2]

Sol. Let M be foot of perpendicular from Q on plane : drs of QM $\Rightarrow$ 1, 2, 1

equation of QM is

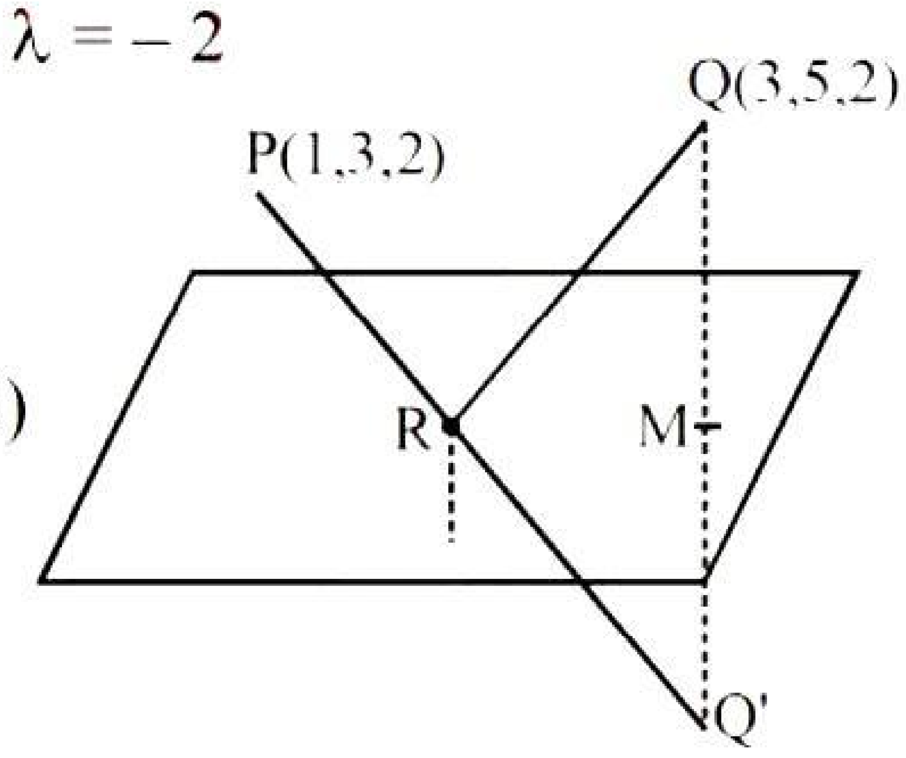
$$\frac{x-3}{1} = \frac{y-5}{2} = \frac{z-2}{1} = \lambda$$

A point M on line can be taken as $(\lambda + 3, 2\lambda + 5, \lambda + 2)$

$\because \quad$ M lies on the plane.

$\therefore \quad (\lambda + 3) + 2(2\lambda + 5) + (\lambda + 2) - 3 = 0 \Rightarrow 6\lambda = -12 \Rightarrow \lambda = -2$

$\therefore \quad M = (1, 1, 0)$



Let image of Q is plane be $Q'(x_1, y_1, z_1)$

$\therefore \quad \frac{x_1+3}{2} = 1, \frac{y_1+5}{2} = 1, \frac{z_1+2}{2} = 0. \quad \therefore Q' = (-1, -3, -2)$

$\because \quad PR + QR = PR + Q'R$

$PR + Q'R$ is min.

$\therefore \quad$ R must be point of intersection of line with the plane.

$\therefore \quad$ equation of $PQ' \Rightarrow \frac{x-1}{2} = \frac{y-3}{6} = \frac{z-2}{4} \Rightarrow \frac{x-1}{1} = \frac{y-3}{3} = \frac{z-2}{2} = \mu$

Any point R on this line can be taken as $(\mu + 1, 3\mu + 3, 2\mu + 2)$

$\because \quad$ Point R on the plane. $\therefore$ It will satisfy its equation.

$\therefore \quad (\mu + 1) + 2(3\mu + 3) + (2\mu + 2) - 3 = 0 \Rightarrow 9\mu = -6 \Rightarrow \mu = \frac{-2}{3}$

$\therefore \quad R\left(\frac{1}{3}, 1, \frac{2}{3}\right) = (a, b, c)$

$\therefore \quad a + b + c = 2.$

Q.19 [1]

Sol. $\vec{v}_1 = 3(\vec{a}\times\vec{b})$, $\vec{v}_2 = 2\left(\vec{b}-(\vec{a}\cdot\vec{b})\vec{a}\right)$

$\vec{v}_1\cdot\vec{v}_2 = 0 \Rightarrow \vec{v}_1 \perp \vec{v}_2$

$|\vec{v}_1| = 3|\vec{a}|\,|\vec{b}|\sin\theta = 3|\vec{b}|\sin\theta$

$|\vec{v}_2| = 2\sqrt{\left(\vec{b}-(\vec{a}\cdot\vec{b})\vec{a}\right)^2} = 2\sqrt{|\vec{b}|^2 - 2(\vec{a}\cdot\vec{b})^2 + (\vec{a}\cdot\vec{b})^2|\vec{a}|^2}$

$= 2\sqrt{\left|\vec{b}\right|^2 - (\vec{a}\cdot\vec{b})^2}$

$= 2\sqrt{\left|\vec{b}\right|^2 - a^2b^2\cos^2\theta} = 2\,|b|\sin\theta$

$$\text{Area} = \frac{1}{2}\cdot 3\left|\vec{b}\right|\sin\theta\cdot 2\left|\vec{b}\right|\sin\theta = \frac{3}{4}\left(\left|\vec{b}\right|^4 + 4\right)$$

$$|b|^2\sin^2\theta = \frac{1}{4}\left(|b|^4 + 4\right)$$

$$\underbrace{4\sin^2\theta}_{\le 4} = \underbrace{\left|\vec{b}\right|^2 + \frac{4}{\left|\vec{b}\right|^2}}_{\ge 4}$$

$\left|\vec{b}\right| = \sqrt{2}$

$\therefore \left[|\vec{b}|\right] = 1$