Quadratic Equations

Introduction

- In chapter2, we have studied about 'Quadratic Polynomials' of the form $ax^2 + bx + c, a \neq 0$ (i.e. whose highest degree is 2).
- When we equate this polynomial to zero, we get a 'Quadratic Equation'.

i.e $ax^2 + bx + c = 0$ gives a quadratic equation.

This is also called the Standard form of a quadratic equation.

Examples: (i) $7x^2 + 3x + 9 = 0$

(ii) $5x^2 + 3 = 0$

(iii) $2x^2 - 3x + 1 = 0$, etc....

Roots of a Quadratic Equation

It refers to those values of x that satisfies the equation.

Example: Consider an equation $x^2 - 3x - 10 = 0$

"5 is a root of the above equation"

because $(5)^2 - 3 \times 5 - 10$

$\Rightarrow$ $25 - 15 - 10 = 25 - 25 = 0 =$ R.H.S

Clearly, the equation gets satisfied.

How to identify Quadratic Equations? (1 ya 2 marks me pooch lete hai)

Example: Check whether the following are quadratic equations:

(i) $(x - 2)^2 + 1 = 2x - 3$.

(ii) $x(x + 1) + 8 = (x + 2)(x - 2)$

SOLUTION: (i) $(x^2 - 4x + 4) + 1 = 2x - 3$

$\Rightarrow x^2 - 4x + 5 = 2x - 3$

$\Rightarrow x^2 - 4x - 2x + 5 + 3 = 0$

$\Rightarrow x^2 - 6x + 8 = 0$

It is of the form $ax^2 + bx + c = 0$.

Therefore, the given equation is a quadratic equation.

SOLUTION: (ii) $x^2 + x + 8 = x^2 - 2x + 2x - 4$

$\Rightarrow x^2 + x + 8 = x^2 - 4$

$\Rightarrow x^2 - x^2 + x + 8 + 4 = 0$

$\Rightarrow x + 12 = 0$

It is not of the form $ax^2 + bx + c = 0$.

Therefore, the given equation is not a quadratic equation.

Solution of a Quadratic Equation

- Factorisation Method
- Completing the Square Method (not in syllabus)
- Quadratic Formula (Shri Dharacharya formula)

1. Factorisation Method :

(Middle term splitting)

Example: Find the roots of the following equations by factorisation:

(i) $2x^2 - 5x + 3 = 0$ (ii) $2x^2 + 7x - 9 = 0$ (iii) $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$

SOLUTION: (i) We need to split the middle term -5x into two numbers such that they give -5x on adding them and $6x^2$ on multiplying them(i.e $2x^2 \times 3 = 6x^2$).

$\Rightarrow \quad 2x^2 - 5x + 3 = 0$

$\Rightarrow \quad 2x^2 - 2x - 3x + 3 = 0$

$\Rightarrow \quad 2x(x - 1) - 3(x - 1) = 0$

$\Rightarrow \quad (x - 1)(2x - 3) = 0$

putting both brackets equal to 0

$x - 1 = 0 \quad ; \quad 2x - 3 = 0$

$x = 1 \quad ; \quad 2x = 3$

$x = 3/2$

Hence, the required roots of the given equation are x = 1 & x = 3/2

(ii) $2x^2 + 7x - 9 = 0$

$\Rightarrow \quad 2x + 9x - 2x - 9 = 0$

$\Rightarrow \quad x(2x + 9) - 1(2x + 9) = 0$

$\Rightarrow \quad (2x + 9)(x - 1) = 0$

putting both brackets equal to 0

$\Rightarrow \quad 2x + 9 = 0 \quad ; \quad x - 1 = 0$

$\Rightarrow \quad 2x = -9 \quad ; \quad x = 1$

$\Rightarrow \quad x = -9/2$

Hence, the required roots of the given equation are x = -9/2 & x = 1.

(iii) $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$

$\Rightarrow \quad \sqrt{2}x^2 + 2x + 5x + 5\sqrt{2} = 0$

$\Rightarrow \quad \sqrt{2}x^2 + \sqrt{2}.\sqrt{2}x + 5x + 5\sqrt{2} = 0$

$\Rightarrow \quad \sqrt{2}x(x + \sqrt{2}) + 5(x + \sqrt{2}) = 0$

$\Rightarrow \quad (x + \sqrt{2})(\sqrt{2}x + 5) = 0$

putting both brackets equal to 0

$\Rightarrow \quad x + \sqrt{2} = 0 \quad ; \quad \sqrt{2}x + 5 = 0$

$\Rightarrow \quad x = -\sqrt{2} \quad ; \quad \sqrt{2}x = -5$

$x = -5/\sqrt{2}$

Hence, the required roots of the given equation are $x = -\sqrt{2}$ & $x = -5/\sqrt{2}$.

2. Quadratic Formula:

Given a quadratic equation $ax^2 + bx + c = 0$, we can find the roots of this equation by using the formula given below:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

*Note that $b^2 - 4ac$ is called the discriminant of quadratic equation denoted by D.

Example: Find the roots of $2x^2 + 7x - 9 = 0$ by Quadratic Formula

SOLUTION: Comparing given equation $2x^2 + 7x - 9 = 0$ with $ax^2 + bx + c = 0$, we get

$$a = 2,\ b = 7\ \&\ c = -9$$

Now using the Quadratic Formula to find the roots, we have

$$\Rightarrow \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Rightarrow \quad x = \frac{-7 \pm \sqrt{(7)^2 - 4 \times 2 \times (-9)}}{2 \times 2}$$

$$\Rightarrow \quad x = \frac{-7 \pm \sqrt{49 + 72}}{4}$$

$$\Rightarrow \quad x = \frac{-7 \pm \sqrt{121}}{4}$$

$$\Rightarrow \quad x = \frac{-7 \pm 11}{4}$$

Now, $x = \frac{-7 + 11}{4}$ and $x = \frac{-7 - 11}{4}$

$x = 4/4$; $x = -18/4$

$x = 1$; $x = -9/2$

Nature of Roots

V.V.imp

Given a quadratic equation $ax^2 + bx + c = 0$, here $b^2 - 4ac$ is called the discriminant of this quadratic equation denoted by D. And a quadratic equation has

(i) two distinct real roots, if $b^2 - 4ac > 0$ (i.e D is positive)

(ii) two equal roots, if $b^2 - 4ac = 0$

(iii) no real roots, if $b^2 - 4ac < 0$ (i.e D is negative)

(*upar ki teeno conditions ko ratt Lena most imp. hai...Ok!

Kuch questions Mai tumhe karwa deta hun..par tumhe bhi baaki questions ki practice jarur karni hai apne end se theek hai..

Example: Find the discriminant of the quadratic equation $2x^2 - 4x + 3 = 0$, and hence find the nature of the roots

SOLUTION: We have given equation $2x^2 - 4x + 3 = 0$

here, $a = 2$, $b = -4$ and $c = 3$

We know Discriminant, $D = b^2 - 4ac$

$$= (-4)^2 - 4 \times 2 \times 3$$

$$= 16 - 24$$

$$= -8 < 0$$

Since $b^2 - 4ac < 0$, the given equation has no real roots.

Example: Find the values of K so that the following quadratic equations has two equal roots.

(i) $2x^2 + kx + 3 = 0$ (ii) $kx(x - 2) + 6 = 0$

SOLUTION: (i) On comparing with $ax^2 + bx + c = 0$, we get

$a = 2$, $b = k$ & $c = 3$

Dicriminant, $D = b^2 - 4ac$

$$= k^2 - 4 \times 2 \times 3$$

$$= k^2 - 24$$

For equal roots, $D = b^2 - 4ac = 0$

$$k^2 - 24 = 0$$

$$k^2 = 24$$

$$k = \pm\sqrt{24}$$

$$k = \pm 2\sqrt{6}$$

(ii) We have $kx(x - 2) + 6 = 0 \Rightarrow kx^2 - 2kx + 6 = 0$

here, $a = k$, $b = -2k$ & $c = 6$

Discriminant, $D = b^2 - 4ac$

$$= (-2k)^2 - 4 \times k \times 6$$

$$= 4k^2 - 24k$$

For equal roots, $D = b^2 - 4ac = 0$

$$4k^2 - 24k = 0$$

$$4k^2 = 24k$$

$$4k = 24$$

$$k = \frac{24}{6}$$

$$k = 4$$

*Tip: Practice some more questions of this topic from PYQ'S, reference books, etc because NCERT me kafi kaam questions diye hain.

SITUATIONAL or WORD PROBLEMS :

Example: The difference of squares of two numbers is 180. The square of the smaller number is 8 times the larger number. Find the two numbers.

SOLUTION: Let the larger number be x.

Then, as per given condition square of smaller no = 8 × larger no.

i.e. $(\text{smaller no})^2 = 8x$

or smaller no $= \sqrt{8x}$

Now, A.T.Q $(x)^2 - (\sqrt{8x})^2 = 180$

$\Rightarrow x^2 - 8x = 180$

$\Rightarrow x^2 - 8x - 180 = 0$

$\Rightarrow x^2 - 18x + 10x - 180 = 0$

$\Rightarrow x(x - 18) + 10(x - 18) = 0$

$\Rightarrow (x - 18)(x + 10) = 0$

$\Rightarrow x - 18 = 0 \quad ; \quad x + 10 = 0$

$\Rightarrow x = 18 \quad ; \quad x = -10$

Now, if $x = 18$, then smaller no $= \sqrt{8x} = \sqrt{8 \times 18} = \sqrt{144} = \pm 12$

or if $x = -10$, then smaller no $= \sqrt{8x} = \sqrt{8 \times -10} = \sqrt{-180}$ not possible

Now to find the numbers we have two cases :

CASE1 : When larger no(x) = 18 and smaller no = 12

CASE2 : When larger no(x) = 18 and smaller no = -12

Therefore the required numbers are 18, 12 or 18, -12.

Example: In a class test, the sum of Shefali's marks in Mathematics and English is 30. Had she got 2 marks more in Maths and 3 marks less in English, the product of their marks would have been 210. Find her marks in the two subjects

SOLUTION: Let her marks in Maths be x. Then, the marks in English will be 30-x.

A.T.Q $(x + 2)(30 - x - 3) = 210$

$\Rightarrow (x + 2)(27 - x) = 210$

$\Rightarrow 27x - x^2 + 54 - 2x - 210 = 0$

$\Rightarrow -x^2 + 25x - 156 = 0$

$\Rightarrow x^2 - 25x + 156 = 0$

$x^2 - 12x - 13x + 156 = 0$

$x(x - 12) - 13(x - 12) = 0$

$(x - 12)(x - 13) = 0$

$x - 12 = 0 \quad ; \quad x - 13 = 0$

$x = 12 \quad ; \quad x = 13$

If the marks in Maths are 12, then marks in English will be 30 - 12 = 18

If the marks in Maths are 13, then marks in English will be 30 - 13 = 17.

Example: A train travels 360 km at a uniform speed. If the speed had been 5 km/h more, it would have taken 1 hour less for the same journey. Find the speed of the train.

SOLUTION: Let the speed of the train be x km/hr.

We know that Time = $\frac{\text{Distance}}{\text{Speed}}$

A.T.Q CASE1 : $t_1 = \frac{360}{x}$ | CASE2 : $t_2 = \frac{360}{x+5}$

It is given that train would have taken 1 hour less if speed was x+5 km/h.

$$t_1 - t_2 = 1$$

$$\Rightarrow \frac{360}{x} - \frac{360}{x+5} = 1$$

$$\Rightarrow \frac{360(x+5) - 360x}{x(x+5)} = 1$$

$$\Rightarrow \cancel{360}x + 1800 - \cancel{360}x = x(x+5)$$

$$1800 = x^2 + 5x$$

$$x^2 + 5x - 1800 = 0$$

$$x^2 + 45x - 40x - 1800 = 0$$

$$x(x+45) - 40(x+45) = 0$$

$$(x-40)(x+45) = 0$$

$$\Rightarrow x = 40, -45 \text{ (not possible)}$$

Hence, the speed of train is 40 km/h.

Example: A motor boat whose speed is 18 km/h in still water takes 1 hour more to go 24 km upstream than to return downstream to the same spot. Find the speed of the stream.

*Upstream matlab paani ki stream ke against jaana.
Downstream matlab paani ki stream ke saath jaana.

SOLUTION: Let the speed of the stream be x km/hr.

Now, the speed of the boat upstream = (18 - x) km/h and
the speed of the boat downstream = (18 + x) km/h

We know that Time = $\frac{\text{Distance}}{\text{Speed}}$

CASE1 : UPSTREAM

$t_1 = \frac{24}{18 - x}$

CASE2 : DOWNSTREAM

$t_2 = \frac{24}{18 + x}$

It is given that the boat takes 1 hour more to go 24 km upstream than to return downstream

$$t_1 = t_2 + 1$$

or $$t_1 - t_2 = 1$$

$$\Rightarrow \frac{24}{18-x} - \frac{24}{18+x} = 1$$

$$\Rightarrow \quad \frac{24(18 + x) - 24(18 - x)}{(18 - x)(18 + x)} = 1$$

$\Rightarrow \quad \cancel{432} + 24x - \cancel{432} + 24x = (18 - x)(18 + x)$

$\Rightarrow \quad 48x = 18^2 - x^2 \quad [(a - b)(a + b) = a^2 - b^2]$

$\Rightarrow \quad 48x = 324 - x^2$

$\Rightarrow \quad x^2 + 48x - 324 = 0$

$x^2 + 54x - 6x - 324 = 0$

$x(x + 54) - 6(x + 54) = 0$

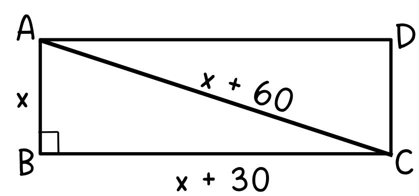
$(x + 54)(x - 6) = 0$

$\Rightarrow x = -54$ or $x = 6$

(Np)

Since x is the speed of stream, it can't be negative. Therefore, the speed of the stream is 6 km/hr.

Example: The diagonal of a rectangular field is 60 metres more than the shorter side. If the longer side is 30 metres more than the shorter side, find the sides of the field.



SOLUTION: Let the shorter side of rectangle be x m.

Then, larger side of rectangle = (x + 30)m

and diagonal of the rectangle = (x + 60)m.

Now, In ΔABC, by Pythagoras theorem, we have

$$(AC)^2 = (AB)^2 + (BC)^2$$

$\Rightarrow \quad (x + 60)^2 = x^2 + (x + 30)^2$

$\Rightarrow \quad x^2 + 3600 + 120x = x^2 + x^2 + 900 + 60x$

$\Rightarrow \quad 2x^2 + 900 + 60x - x^2 - 3600 - 120x = 0$

$\Rightarrow \quad x^2 - 60x - 2700 = 0$

$\Rightarrow \quad x^2 - 90x + 30x - 2700 = 0$

$\Rightarrow \quad x(x - 90) + 30(x - 90) = 0 \quad \Rightarrow \quad (x - 90)(x + 30) = 0$

$\Rightarrow \quad x = 90$ or $x = -30$

Since x is side of rectangle, it can't be negative. Therefore, the length of the shorter side will be 90m. And, length of larger side = 90+30 = 120m.